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A modified monkey algorithm for optimal sensor placement in structural health monitoring

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Abstract

Proper placement of sensors plays a key role in construction and implementation of an effective structural health monitoring (SHM) system. This paper outlines a novel methodology called the modified monkey algorithm (MA) for the optimum design of SHM system sensor arrays, which is very different from the conventional method and is simple to implement. The integer coding method instead of the binary coding method is proposed to code the solution. The Euclidean distance operator and the stochastic perturbation mechanism of the harmony search algorithm are employed to improve the local and global search capability. A computational case of a high-rise building has been implemented to demonstrate the effectiveness of the modified method. The obtained sensor placements using the modified MA are compared with those gained by the existing MA using the integer coding method and the famous forward sequential sensor placement algorithm. Results showed that the innovations in the MA proposed in this paper could improve the convergence of the algorithm and the method is effective in solving combinatorial optimization problems such as optimal sensor placement.

(Some figures may appear in colour only in the online journal)

1. Introduction

Large and complex civil infrastructures are especially susceptible to random vibrations, whether it is from high ground accelerations, strong wind forces, or abnormal loads such as explosions [1]. Thus, it is imperative that the robust and continuous health monitoring system needs to be designed and implemented so as to meet the demanding goals of increasing structural safety and reliability, while reducing its operating and maintenance costs. The efficiency of an structural health monitoring (SHM) system relies on the sensitivity of the acquired data to structural changes that may be obtained by an extended sensor network on the structures [2]. For this reason, the more locations sensors are placed at on a structure, the more detailed information on the stress, strain, deformation, acceleration etc of the structure can be obtained. On the other hand, advances in sensing technology have also enabled the use of large numbers of sensors for the SHM. However, taking the cost of sensors and their supporting instruments into account, it is not realistic and efficient to install sensors in every part of the system. In such cases, a practical and challenging question that naturally arises is how to judiciously select a set with a minimum number of sensor locations from all possibilities, such that the data collected may provide the greatest opportunities for achieving the best identification of structural behaviors.

Numerous techniques have been advanced for solving the optimal sensor placement (OSP) problem and are widely reported in the literature [3, 4]. The problem of OSP was perhaps first investigated by Shah and Udwadia [5]. They considered a linear relation between small perturbations in a finite dimensional representation of the structural parameters and a finite sample of observations of the system time response, and formulated the optimal sensor locations as an optimization problem to minimize the error in the parameter estimates. One of the most significant OSP approaches, called the effective independence (EfI) method, which tends to maximize the trace and determinant and minimize the condition number of the Fisher information matrix (FIM) corresponding to the target modal partitions, was developed by Kammer in 1991 [6]. Salama *et al* [7] proposed using modal kinetic energy (MKE) as a means of ranking the importance of candidate sensor locations. Li *et al* [8] studied the relation between the EfI method and MKE method. Carne and Dohmann [9] gave another famous method, called MAC, which used the minimization of the off-diagonal terms in the modal assurance criterion (MAC) matrix as a measure of the utility of a sensor configuration. Chang and Markmiller [10], as a measurement for quantifying the reliability of a sensor network, defined the probability of detection (POD). The optimal sensor network was introduced as the network sensor configuration that could achieve the target probability of detection. Work by Azarbayejani *et al* [11] showed the importance of addressing the issue of uncertainty in handling the optimal sensor configuration and presented a probabilistic approach. Starting from a general formulation of Bayes risk, Flynn and Todd [12] derived a global optimality criterion to quantify damage in the structure based on the change in modal information. Yi *et al* [13] presented a method based on the simplified finite element (FE) model which was able to avoid the problem of choosing the high order mode accurately based on the modal participation mass ratios. Work by Chow *et al* [14], and earlier work by Papadimitriou [15], introduced the information entropy norm as the measure that best corresponded to the objective of structural testing, which was to minimize the uncertainty in the model parameter estimates. Far too many sensor placement approaches exist to mention them all in this paper.

Therefore, the sensor placement for SHM must be run with care. However, for a structure that has simple geometry, or smaller number of degrees of freedom (DOF), traditional approaches may suffice to solve the problem. For a large-scale complicated structure, candidate sensor positions may be counted in the order of thousands to tens of thousands; the computational intelligence approach is thus needed to solve such a computationally demanding problem. Among the algorithms that have emerged, the genetic algorithm (GA) has been proved as an effective alternative to the previous heuristic algorithm [16]. However, the GA also has some faults that need to be improved. When the GA is used to solve the OSP, the general crossover and mutation operators may generate chromosomes which do not satisfy the constraints. One location should not have two or more sensors or sensor number is equal to a certain number [17]. Different genetic operators, such as the dual-structure coding method [18], have been used in order to overcome these faults. However, the processes of these particular genetic operators are complex and the computational efficiency is low. Another drawback of the GA is that it may spend much time on complex optimization problems as a result of the repeated evolution of the objective function and the population-based nature of the search. Some attempts have been made to overcome this fault. For example, Javadi *et al* [19] presented a hybrid intelligent GA which was based on a combination of a neural network and a GA. In order to improve the convergence speed and avoid premature convergence, virus evolutionary theory [20, 21] was introduced into the GA. The successful application of GAs in the sensor network design for the SHM system led to the development of several other intelligent approaches, such as particle swarm optimization (PSO) and ant colony optimization (ACO). However, in most of these approaches, either very limited network characteristics are considered, or several requirements of the application cases are not incorporated into the performance measure of the algorithm.

In this paper, a new algorithm, called the modified monkey algorithm (MA), is proposed for design of a sensor system, which is a useful approach that specifically addresses structural health monitoring application goals: maximize performance (detection) while minimizing the cost of the sensor system. The remainder of the paper is organized as follows. Section 2 presents the main features of the algorithm under study and gives a detailed description of the modified optimization approach. In the following section, the effectiveness of the novel algorithm is demonstrated via a comprehensive analysis for a high-rise structure. Finally, a few concluding remarks are given.

2. Description of modified method

2.1. Introduction to monkey algorithm

The MA was first designed by Zhao and Tang [22] from the inspiration of mountain-climbing processes of monkeys. It assumes that there are many mountains in a given field (i.e. in the feasible space of the optimization problem); in order to find the highest mountaintop (i.e. find the maximal value of the objective function), monkeys need to climb up from their respective positions. The algorithm mainly consists of the climb process, watch–jump process, and somersault process, in which the climb process is employed to search the local optimal solution, the watch–jump process to look for other points whose objective values exceed those of the current solutions so as to accelerate the monkeys' search courses, and the somersault process to make the monkeys transfer to new search domains rapidly. After much repetitious iteration of the three processes, the monkeys could find the highest mountaintop (i.e. find the optimal value). Figure 1 displays a schematic drawing of the MA.

The MA is a kind of evolutionary algorithm, similar to other population-based algorithms, such as GA and PSO, which can be adopted in various fields needing optimization. As one of the novel evolutionary algorithms, the MA can solve a variety of difficult optimization problems featuring non-linearity, non-differentiability, and high dimensionality. The difference from the other algorithms is that the time

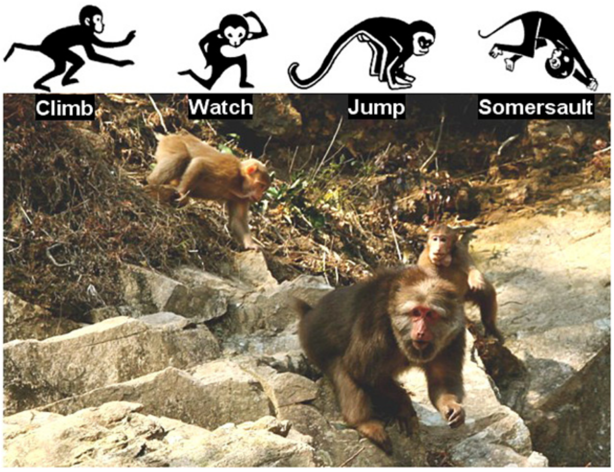

Figure 1. Schematic drawing of the MA.

consumed by the MA mainly lies in using the climb process to search local optimal solutions. The essential characteristic of this process is the calculation of the pseudo-gradient of the objective function, that only requires two measurements of the objective function, regardless of the dimension of the optimization problem. This feature allows a significant decrease in the cost of optimization and gives the MA a faster convergence rate, especially in optimization with a large dimension and large numbers of local optima. Another advantage of the MA is that it only has a few parameters that need to be adjusted, which makes it particularly easy to implement.

2.2. Objective function

It is known from the structural dynamic principle that the structural inherent modes should comprise a group of orthogonal vectors at the nodes. But in fact, it is impossible to guarantee the measured modal vector is orthogonal, because of the problems of the measured freedoms being less than those of structures and of measuring accuracy limitation. Further, it is even possible to lose many important modes owing to the space angles between vectors being too small. Larger space angles among the measured modal vectors should be guaranteed while choosing measuring points in order to keep the original properties of the structure if possible. Carne and Dohmann [9] thought that the MAC was an ideal scalar constant relating to the causal relationship between two modal vectors

$$\mathrm{MAC}_{ij} = \frac{(\Phi_i^{\mathrm{T}}\Phi_j)^2}{(\Phi_i^{\mathrm{T}}\Phi_i)(\Phi_j^{\mathrm{T}}\Phi_j)} \tag{1}$$

where Φ_i and Φ_j represent the ith and jth column vectors in matrix Φ, respectively, and the superscript T denotes the transpose of the vector.

In equation (1), the element values of the MAC matrix range between zero and one, where zero indicates that there is little or no correlation between the off-diagonal element MAC_{ij} $(i \neq j)$ (i.e. the modal vector is easily distinguishable) and one denotes that there is a high degree of similarity between the modal vectors (i.e. the modal vector is fairly indistinguishable). For an optimal (orthogonal) set the MAC matrix would be diagonal, thus the size of the off-diagonal elements could be an indication of an optimal result. In this paper, the MAC matrix is adopted to construct two objective functions.

The first is the biggest value in all the off-diagonal elements in the MAC matrix

$$f_1(x) = \max_{i \neq j} \left\{\mathrm{MAC}_{ij}\right\} \tag{2}$$

where x means the current position of the monkey (i.e. the scheme of the sensor placement), which is explained in section 2.3. Many researchers have adopted the largest value in all the off-diagonal elements in the MAC matrix as the criterion in the optimal sensor placement problem [22, 23].

The second is the average value of all the off-diagonal elements in the MAC matrix

$$f_2(x) = \operatorname*{mean}_{i \neq j} \left\{\mathrm{MAC}_{ij}\right\}. \tag{3}$$

2.3. Coding method

To implement the MA, it is necessary first to devise a general coding system for the representation of the design variables. In [24], the MA was originally designed to solve global numerical optimization problems with continuous variables. In [25], the author proposed a kind of discrete version of the MA for the first time and it has been successfully applied in solving the transmission network expansion planning problem. However, for the problem at hand, the optimization variables are the sensor's locations, which could be either the spatial coordinate or the number of nodes on the FEM mesh excluding the constrained nodes. Most commonly, the design variables in the OSP are coded by a simple one dimensional binary coding method that is very simple and intuitionistic. A key problem with binary coding is that it requires increased string length and computational time, especially in the large-scale structures where possible sensors are large. In order to overcome this difficulty, the integer coding method, which is also more efficient than the binary coding method because it does not need coding/decoding of real/binary variables, is adopted here.

First an integer M is defined as the population size of monkeys. Then, for monkey i, its position is denoted as a vector $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})$, and this position will be employed to express a string of the sensor location optimization problem, where $i = 1, 2, \ldots, M$, n is the number of sensors needed to be placed. If there are n sensors to place in the total m degrees of freedom (DOFs) (i.e. the number of candidate sensor positions), the coding length of the string is n. Every value of the string is the sorted DOF on which the sensor is located. For example, (3, 6, 12, 14, 22, 26, 31, 35, 38, 43) is a string, which denotes that the sensors are uniquely located on the third, sixth, 12th, 14th, etc ten DOFs.

2.4. Initial population

Having decided on a representation, the next step is to generate, at random, an initial population of possible solutions. The number of monkeys depends on several factors, including the size of each individual monkey, which itself depends on the size of the solution space. Like the GA, the MA also incorporates the stochastic operations during the optimization process, thus the quality of the stochastically generated initial population may drastically affect the final performance. However, the random initialization process may generate similar positions for different monkeys (i.e. similar scheme of sensor placement). In order to avoid this issue, the Euclidean distance, which can increase the diversity of the monkey's position, is adopted here.

The Euclidean distance of two monkeys' positions x_i and x_j can be defined as

$$|x_i - x_j| = \sqrt{\sum_{s=1}^{n}(x_{is} - x_{js})^2} \tag{4}$$

where x_{is} and x_{js} represent the sth component in the vectors x_i and x_j, respectively.

From equation (4), it can be seen that the smaller the Euclidean distance of the two monkeys' positions is, the bigger similarity the two monkeys' positions have, and the more redundant the information is. That is to say, if the available and useful information is much less in the iterative process due to the small Euclidean distance of the two monkeys' positions, the algorithm trends to the local optimum, and then the global search capability will be decreased. Therefore, a threshold *hm* is proposed for the Euclidean distance between the monkeys' positions in the initialization. Herein, the Euclidean distance between the monkeys' positions should be bigger than or equal to *hm*; otherwise, the re-initialization needs to be performed, by which the diversity of the monkeys can be enhanced and the global search capability will be improved. Thus, in the following exploration and then further application of the algorithm that are presented, several runs are needed so as to keep the ideal initial populations.

2.5. Climb process

The climb process is a step-by-step procedure to change the monkeys' positions from the initial positions to new ones that can make an improvement in the objective function. The original climb process in the MA is designed to use the idea of pseudo-gradient-based simultaneous perturbation stochastic approximation (SPSA), a kind of recursive optimization algorithm. Analysis has shown that general sensor selection problems addressing diagnosability, or observability, are *NP-complete* and are therefore computationally intractable [26]. These types of problem require fast approximate search solutions in order to generate acceptable results in reasonable time. In [25], two kinds of climb process with large and small steps are introduced to avoid the disordered search direction of primary MA in solving discrete optimization problems. A cooperation process is also introduced to enhance the monkeys' interaction. The large-step climb process makes the monkeys' positions greatly change before/after updating, which can expand the search extent of the potential solution. However, the disadvantages of this process are that the larger pace of the climbing may skip the global optimal solution; therefore, to a certain extent, to obtain the optimal solution a greater number of iterations is needed, and for some complex problems it may not be able to search for the optimal solution. Considered during the climb process to the mountaintop (optimal value), the diversity of the monkeys will gradually decrease, trending to a local optimal solution (i.e. the so called 'premature convergence' phenomenon). The stochastic perturbation mechanism of the harmony search algorithm (HAS) is employed after the large-step climb process to overcome the problem.

For monkey i with position $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})$, $i = 1, 2, \ldots, M$, an outline of the improved climb process is given as follows:

Step (1). Randomly generate a integer vector $\Delta x_i = (\Delta x_{i1}, \Delta x_{i2}, \ldots, \Delta x_{in})^{\mathrm{T}}$ in the range $[-a, a]$, $j = 1, 2, \ldots, n$, respectively, where the parameter a ($a > 0$) is called the step length of the climb process.

The step length a plays a crucial role in the precision of the approximation of the local solution in the climb process. Usually, the smaller the parameter a is, the more precise the solutions are. Considering the characteristics of the OSP problem, a should be defined as 1, 2, or another positive integer.

Step (2). Calculate $f(x_i + \Delta x_i)$, update the monkeys' positions x_i with $x_i + \Delta x_i$ only if $f(x_i + \Delta x_i) < f(x_i)$, otherwise keep x_i unchanged; if the positions between any two monkeys are similar, go back to step (1);

Step (3). Adapt the pitch adjusting rate (par) in the HSA to the new components x_{ij} of the vector x_i after the large-step climb process according to the following equation:

$$x'_{ij} = x_{ij} + \mathrm{round}(2 * u * \mathrm{rand} - u) \tag{5}$$

where x_{ij} is the j component of the vector x_i, u denotes the arbitrary distance bandwidth, rand means a random number between 0 and 1, and round indicates the rounding operation.

Thus, the new position $x' = (x'_{i,1}, x'_{i,2}, \ldots x'_{i,n})^{\mathrm{T}}$ can be reached; if the new components of x' are different from each other, calculate $f(x'_i)$; update the monkeys' position $x_i = x'_i$ only if $f(x'_i) < f(x_i)$; otherwise, keep the monkeys' positions unchanged.

Step (4). Repeat steps (1)–(3) until there is little change on the values of objective function in the neighborhood iterations or until the maximum allowable number of iterations (called the climb number, denoted by N_c) has been reached.

The purpose of checking how the objective function changes during the iterations is to see whether convergence is being achieved. This could allow the climb process to stop immediately before the maximum number of iterations is reached. In addition, it has to be noted that the 'spillover' phenomenon may occur in step (2) and other steps sometimes (i.e. the new components in $x_i + \Delta x_i$ may exceed the range

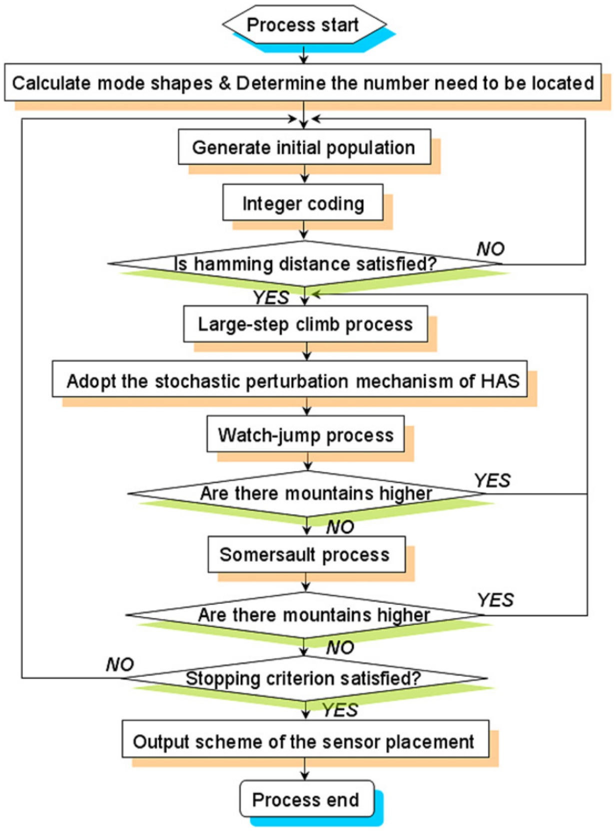

Figure 2. Flowchart of the modified MA for OSP.

of the sensor's candidate location $1 \sim m$). Thus here, if a new component exceeds the upper limit m, then take the component to m; if a new component is below the lower limit 1, then take the component to 1.

2.6. Watch–jump process

For each monkey, when it gets on the top of the mountain, it is natural to have a look and to find out whether there are other mountains around it higher than its present positions. If yes, it will jump to some place on the mountain watched by it from the current position (this action is called the 'watch–jump process') and then repeat the climb process until it reaches the top of the mountain.

For monkey i with position $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})$, $i = 1, 2, \ldots, M$, the outline of the proposed watch–jump process is as follows.

Step (1). Randomly generate integer numbers y_{ij} from $[x_{ij} - b, x_{ij} + b]$, $j = 1, 2, \ldots, n$, respectively, where the parameter b is a positive integer which represents the eyesight of the monkey (i.e. the maximal distance that the monkey can watch).

Usually, the bigger the feasible space of the optimal problem is, the bigger the value of the parameter b should be taken. The eyesight b can be determined by specific situations; like the step length a, the eyesight b should also be defined as 1, 2, or another positive integer in the sensor location problem.

Step (2). If the components between two monkeys in vector y_i are different, the new position x_i'' could be obtained after the large-step climb process again; calculate $f(x_i'')$, update the monkeys' positions x_i with x_i'' provided that $f(x_i'') < f(x_i)$, otherwise keep x_i unchanged; if the components between any two monkeys in vector y_i are similar, go back to step (1).

Step (3). Repeat the climb process steps (1) and (2) until the maximum allowable number of iterations (called the watch–jump number, denoted by N_w) has been reached.

2.7. Somersault process

After repetitions of the climb process and the watch–jump process, each monkey will find a locally maximal mountaintop around its initial point. In order to find a much higher mountaintop, it is natural for each monkey to somersault to a new search domain (this action is called the 'somersault process'). In the MA, the monkeys will somersault along the direction pointing to the pivot which is equal to the barycenter of all monkeys' current positions.

For monkey i with position $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})$, $i = 1, 2, \ldots, M$, the outline of the proposed somersault process is as follows.

Step (1). Randomly generate real numbers θ from the interval $[c, d]$ (called the somersault interval, which governs the maximum distance that monkeys can somersault).

Step (2). Obtain the monkeys' pivot $p = (p_1, p_2, \ldots, p_n)$ by calculating all the monkeys' barycenter $p_j = \sum_i^M x_{ij}/M$, $j = 1, 2, \ldots, n$, respectively.

Step (3). Calculate $x_{ij}''' = x_{ij} + \mathrm{round}(\theta|p_j - x_{ij}|)$, update the monkeys' position x_i with x_i''' provided that the new components of x_i''' are different from each other, and then return to the large-step climb process; otherwise, go back to step (1).

2.8. Termination condition

Following the climb process, the watch–jump process, and the somersault process, all monkeys are ready for their next actions. The condition for terminating the modified MA iteration could either be when the iteration accuracy has been achieved, or when a relatively large number of iterations N has been reached. For the problem considered in this paper, the ending condition of the modified MA is chosen to be the latter one to avoid redundant iteration. It is known that the best position does not necessarily appear in the last iteration, so it should be kept from the beginning. If a monkey finds a better one in the new iteration, the old one will be replaced by it. This position will be reported as an optimal solution at the end of the iterations. In addition, the results obtained from an MA process with a limited number of iterations might be a suboptimal solution. To get a result with higher confidence, one has to run the modified MA process either several times, each with a randomly generated initial condition, or with sufficient number of iterations.

To sum up, the whole flowchart of the modified MA to find the optimal sensor locations presented herein is shown in figure 2, that can be fully implemented easily with the commercial software MATLAB (MathWorks, Natick, MA, USA) [27].

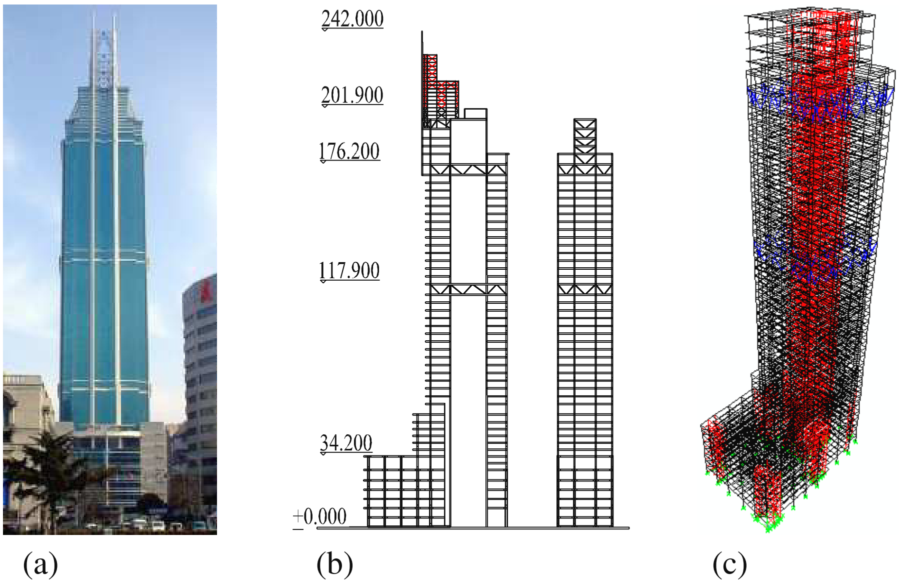

Figure 3. Dalian world trade building. (a) Bird's eye view; (b) sectional drawing; (c) FE model.

3. Demonstration case

To demonstrate the effectiveness of the modified method, a case study to determine the sensor configuration on a high-rise building is considered.

3.1. Description of the building

The Dalian World Trade Building, comprising office, commerce, finance and insurance parts, is a super-high-rise structure (figure 3(a)). It has four stories under the ground level and 50 stories above. The main structure is about 201.9 m high from ground level. With the top tower, the total height is about 242 m and there is also an eight-story commercial building (locally nine stories) 34.2 m in height around it (figure 3(b)). Up to now, the building has still been the tallest in the northeast of China. The structural system utilizes both steel and reinforced concrete, including core wall systems and perimeter steel frame coupled by outrigger trusses at two levels (the 30th and 45th floors) [13].

3.2. Calculation model

In order to provide input data for the modified OSP method, a three-dimensional FE model of the building is built using the ETABS software (CSI, Berkeley, CA, USA) [28], as shown in figure 3(c). All the beams and columns are simulated using 'Frame Elements' in the ETABS element library. The beam and column properties are input by defining the relevant cross-sectional shape from the pre-defined ETABS cross-section library. The slab and core wall are simulated using the 'Shell Element' having bending and membrane stiffness terms available in the ETABS library. The model is supported at the bottom using a 'Link Element'. The FE model is built considering the bending and shearing deformation of the beam and column, and also the axial deformation of the column. The rigid floor assumption is used. To the 30th and 45th strengthened stories, as the axial deformation of the column needs to be considered, the corresponding floors are computed as flexible floors. The model has 13 324 node elements, 90 062 frame elements and 22 967 shell elements, considering 31 section types and eight materials' properties.

3.3. Optimization process, results and discussion

Although the structure has a large number of DOFs, only translational DOFs are considered for possible sensor installation in this case study, as rotational DOFs are usually difficult to measure. Since the stiffnesses of the DWTB in two translational directions are different, the structural vibration monitoring is taken into account in the direction with weaker stiffness. Consequently, a total of 50 DOFs are available for sensor installation (i.e. $m = 50$). It is known that recovering a higher order of mode shape will lead to more accurate representation of the structure; yet this requests higher numbers of sensors. In view of this, the first 10 modes in the weak axis of the DWTB are selected for calculation. To start with, an initial monkeys' position is randomly generated to represent various possible sensor configurations. In this case, the monkeys' population size M is set to 5 first. With the increase of the number of sensors, the initialization process may generate similar positions for different monkeys. In order to avoid this problem, M is set to 3 when 40–50 sensors are added. Considering the characteristics of the OSP problem and the integer coding method, the step length a and the eyesight b are defined as 1 and 2, respectively, and the somersault interval is set to $[-20, 20]$. Using trial and error, $N_c = 500$ and $N_w = 20$ are found to be sufficient for achieving the best performance.

To show the performance improvement achieved by the modified method, three cases are investigated carried out and their performances are compared. What needs to be mentioned

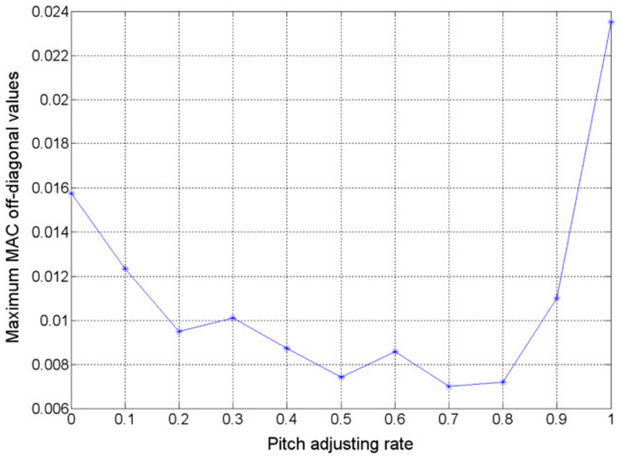

Figure 4. Variation curve of maximum MAC off-diagonal element with change of pitch adjusting rate.

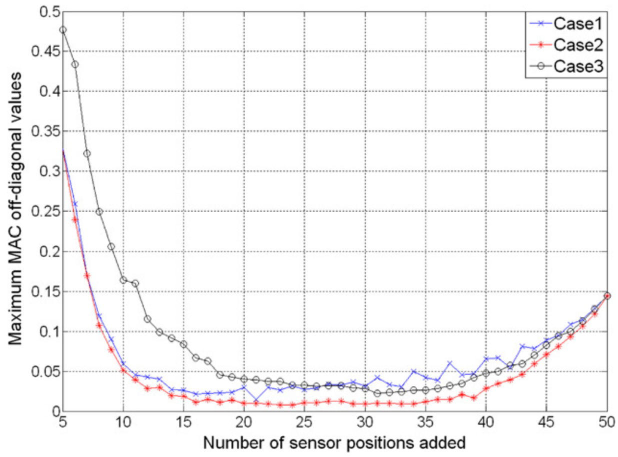

Figure 5. Variation curve of maximum MAC off-diagonal element with more sensors added.

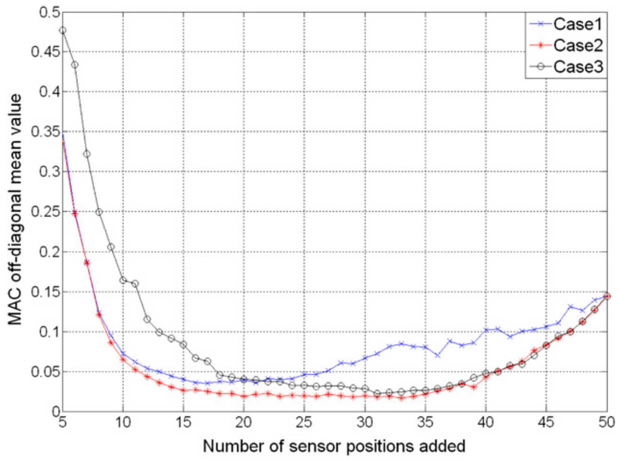

Figure 6. Variation curve of the mean value of MAC off-diagonal element with more sensors added.

is that each case is tested 10 times, and the best results are selected.

Case (1): the original MA, i.e. set par = 0 and $hm = 0$.

Case (2): the modified MA.

Case (3): the existing FSSP algorithm [13, 15].

What needs to be mentioned is that the stochastic perturbation mechanism of the HAS employed in the modified method is important to overcome the 'premature convergence' phenomenon. To find out the most appropriate pitch adjusting rate (par), many simulations with different pitch adjusting rates ranging from 0.0 to 1.0 were first made. According to the variation curve of the maximum MAC off-diagonal element with the change of pitch adjusting rate shown in figure 4, 0.8 is found to be the favorable value. Thus, the pitch adjusting rate is set to 0.8 and the threshold *hm* of Euclidean distance is correspondingly set to 20 here.

Figures 5 and 6 show the variation curves of the maximum and mean off-diagonal element for adding one more sensor to the initial placement using different methods, respectively. In order to obtain the two figures, full searches are performed each time with respect to all of the sensors. It is expected that the maximum MAC off-diagonal terms will decrease with more sensors included. This is in fact the case, more or less a decreasing trend as shown in the three curves of the three cases in figure 5. However, an interesting phenomenon is that the maximum MAC off-diagonal terms become unexpectedly larger even when further sensors are added. The same phenomenon was also observed in [29], where decreasing sensor number led to smaller MAC off-diagonal terms. The reason for such an increasing contradiction is that a newly included sensor may conflict with other previously selected sensors. Mathematically speaking, the row vector determined at this newly included sensor position has a strong linear relationship with the previous whole sensor set. Namely, this row vector is nearly a linear combination of other row vectors of the mode shape matrix specified by previous sensors.

At the same time, the optimal values are compared with each other although they have similar trends as mentioned above. From figure 5, it can be easily found that the maximum off-diagonal elements of the MAC matrix obtained by the original MA, the modified MA and the FSSP algorithm are 0.014 59, 0.0049 and 0.0222, respectively. This means that the modified MA can improve the performance by 66.42% and 77.93% compared with the original MA and the FSSP algorithm. Comparing the computational results of the two objective functions as shown in figures 5 and 6, it can be remarked that the results are almost identical and thus the analysis for figure 6 is omitted here.

Figure 7 shows the MAC values of all of the 50 DOFs. As expected, there are many higher off-diagonal elements, so the results are not so good. From figure 5, it can be obtained that the lowest point is the value for adding 20 DOFs (i.e. deploy 24 sensors). As shown from figure 8, after adding 20 measurement points, the off-diagonal elements of the MAC matrix obtained by the modified MA are dramatically reduced, that means the identified modal vector would be more distinguishable than the former one. Accordingly, the optimal value for the FSSP algorithm is obtained when selecting 31 DOFs. However, this is not economic due to high cost of the data acquisition systems (sensors and their supporting instruments). As depicted from figure 5, the

Table 1. Maximum off-diagonal element of MAC of each kind of sensor placement.

Scheme selection of the sensor placement	All of the 50 DOFs	31 DOFs obtained by the FSSP method	18 DOFs obtained by the FSSP method	24 DOFs obtained by the modified MA	18 DOFs obtained by the modified MA
Maximum MAC off-diagonal value	0.014 42	0.0222	0.0458	0.0049	0.0114

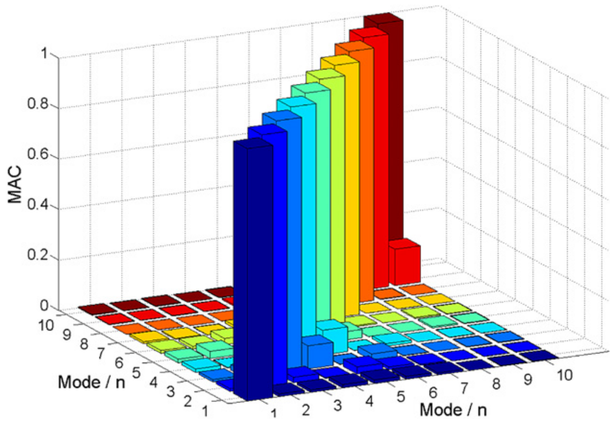

Figure 7. MAC values of all of 50 DOFs.

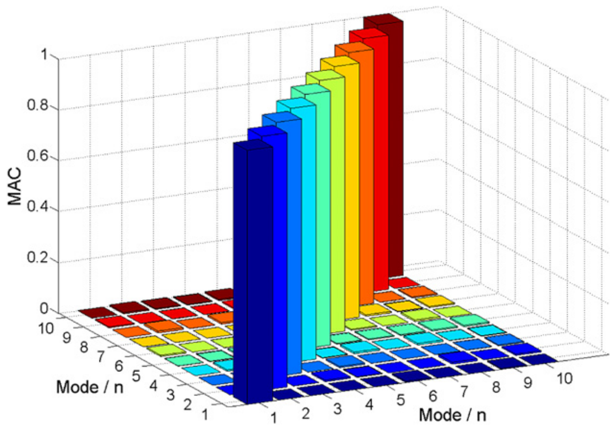

Figure 8. Optimal values of MAC obtained by modified MA.

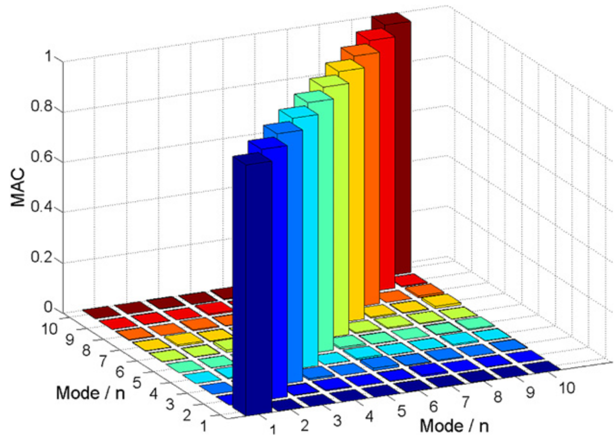

Figure 9. MAC values of 18 DOFs obtained by modified MA.

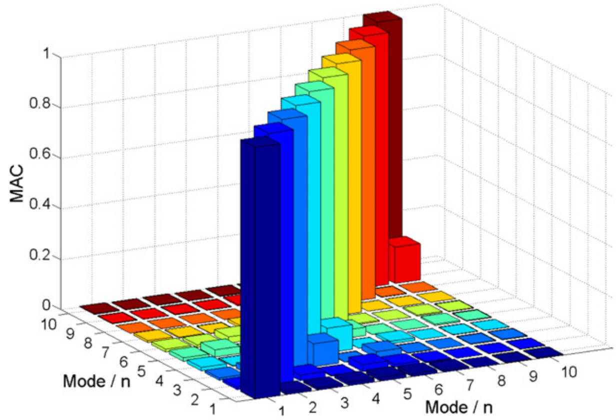

Figure 10. MAC values of 18 DOFs obtained by FSSP method.

maximum off-diagonal element varies gently with 18 sensors. Therefore, 18 DOFs are selected as the sensor quantity and the MAC values determined by the modified MA and the FSSP are shown in figures 9 and 10, respectively. Table 1 gives the maximum off-diagonal elements of the MAC matrix for various sensor locations, which indicates that the modified method is greatly effective. The final sensor placement result of the DWTB obtained is given in table 2.

Table 2. Sensor placements of the DWTB.

Sensor number	1	2	3	4	5	6	7	8	9
Story	35	29	6	17	18	5	23	13	45
Sensor number	10	11	12	13	14	15	16	17	18
Story	3	8	4	31	36	12	41	25	48

4. Conclusions

Considering the characteristics of the OSP techniques in a large-scale civil structure, this paper outlines an efficient methodology for the optimal design of SHM system sensor arrays, and some innovations in the MA such as the coding method, the Euclidean distance and the stochastic perturbation mechanism are proposed to enlarge the local and global search capability, as well as improving the convergence of the algorithm. With the case analysis, some conclusions and recommendations are summarized as follows.

(1) The OSP is a discrete combinatory integer problem, that means the original MA cannot be implemented

directly. In order to overcome this difficulty, the integer coding method is adopted in the modified MA and the corresponding algorithm process for discrete variables is also presented, which is proved more efficient than the binary coding method because it does not need coding/decoding of real/binary variables.

(2) Due to the random nature of the initial population used in the MA, the proposed Euclidean distance operator is effective to increase the diversity of the monkeys' positions so as to avoid similar schemes of sensor placement as well as improving the global search capability of the algorithm.

(3) In order to improve the local search capability of the MA, the stochastic perturbation mechanism of HAS is employed skillfully after the large-step climb process, which is certified effective to increase the diversity of the monkeys' positions.

(4) The modified MA is particularly effective in solving a combinatorial optimization problem such as the OSP problem when the performance tradeoffs are not unbearable and when the number of combinations is too large to preclude enumeration. The optimization problem in this paper is relatively small and could be solved by the conventional method too. However, even in this case the modified MA has outperformed the famous FSSP algorithm, as demonstrated in the numerical example.

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