An Iron Man Helmet For My Dog

Introduction
As a student in the diploma programme, stress is a huge factor of my life. Therefore, I have the necessity to find any means to deal with this factor. My family dog is one factor that lowers my stress and maintains my positivity both in and out of school. While everything may seem fine to me at home, I find that my dog seems to be afraid of slight sounds or movements of objects, in which I would like to find a solution that would prevent him from feeling fear by providing some sort of psychological protection. Fortunately, I think I have found a solution to that.

As a growing teenager, I also often found superhero movies an excellent form of entertainment to help cope with the stress that I deal with. It was most appealing to watch a futuristic, but possible concept of Iron Man in the cinema. At first, I thought that the actor playing Tony Stark was wearing a plastic suit and enhanced through cinematic editing. However, after I watching the "behind the scenes" of these movies, I found out that the iron suit of armor was actually generated through 3D graphics ("Iron Man 3" 00:00:11-00:00:21). This made me wonder whether it is possible to use this concept to provide an armor that would provide protection for my dog to prevent fear. Knowing this, would it be possible to replicate this 3D model on my dog to provide a sense of safety?

Aim
In this exploration, I will be creating a 3D model of an Iron Man helmet for my dog, calculating the volume of the helmet.

Methodology:
1. Formulate equations that outline my dog's head
2. Evaluate the volume of my dog's head
3. Formulate equations that outline the helmet
4. Evaluate the equations to calculate the total volume of the helmet

Formulate equations that outline my dog's head
In order to calculate the volume, the 3D shape of my dog's head must be formed using Cartesian coordinates—"coordinates" used to "describe the position of points" (Harcet et al. 738). This will allow for evaluating the volume through the use of coordinate points, approximation, and integration. I first took three different pictures in different perspectives in relation to the three planes—xy, yz, zx— on a three-dimensional plane. The metric measurements of each perspective was found using LoggerPro, helping in scaling each perspective appropriately. With the information gathered, coordinates were plotted on each perspective individually using Geogebra:

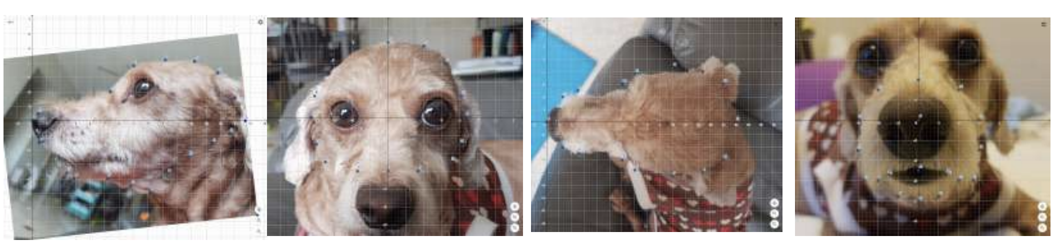

Figure 1: Different Perspectives of My Dog

These coordinates are then plotted connected together on a 3D graph to form a 3D model of my dog' head. Additional coordinates are plotted through inspection and approximation using the pictures shown in Figure 1 in order to help find the volume of my dog's head. Segments are drawn to form triangles between three nearest points connect the coordinates on the 3D model. Using this information, equations will be formulated so that the volume under each triangle on the 3D model can be evaluated. To do so, any three coordinate points that form a triangle are selected. Since the purpose of the triangles were to find the volume under them, it is appropriate to integrate a graph of an equation that passes through all three coordinate points of the triangle.

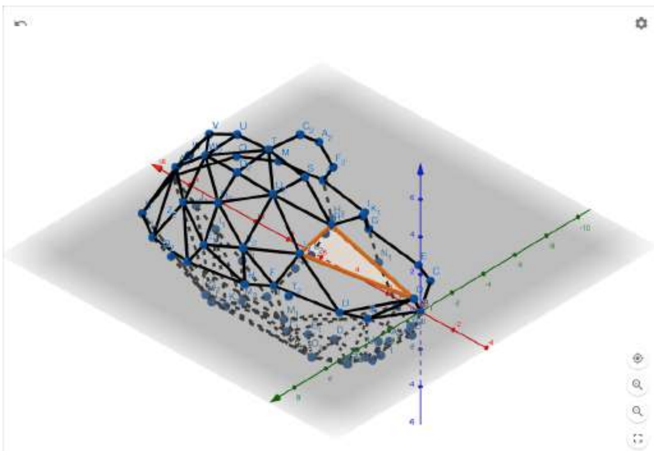

Figure 2: Orange region of the 3D Model

An example of how to find the equation for each triangle will be provided with explanation. Let's say that the volume under the orange triangle is to be found on Figure 2 on the left. The triangle has three coordinates: $(0.4, 0, 0.5), (5.4, 0, 2), (5.4, 2, 1.5)$

Since the 3D Cartesian coordinate system consists of three different axes, the equations will consist of three different variables:

$$ax + by + cz = 0$$

The coefficients—a, b, c—will allow the two-variable linear equations to intersect with all three coordinates, which will allow us to evaluate the volume under the triangle. First, pick any coordinate among the three provided. In this case, I will use the coordinate: $(0.4, 0, 0.5)$. Using this, translate equation so it intersects with the coordinate:

$$ax + by + cz = 0$$
$$a(x - 0.4) + by + c(z - 0.5) = 0$$

In order to allow the graph of the equation to also intersect the remaining to coordinates, the values of the variables in each coordinate will be substituted into the translated equation separately:

$$a(5.4 - 0.4) + b(0) + c(2 - 0.5) = 0$$
$$5a + 0b + 1.5c = 0$$
$$a(5.4 - 0.4) + b(2) + c(1.5 - 0.5) = 0$$
$$5a + 2b + c = 0$$

This will create a simultaneous equation that can be solved in order to find the coefficients of the equations.

$$5a + 0b + 1.5c = 0$$
$$5a + 2b + c = 0$$
$$a = 0.3$$
$$b = -0.25$$

$$c = -1$$

Substitute the coefficients back into the translated equation:

$$f(x, y) = 0.3x - 0.25y + 0.38$$

Evaluate the volume of my dog's head

The volume of under a triangular region of the equation will be found using double integration:

$$Volume = \int_c^d \int_a^b f(x, y)\, dA$$

(Dawkins)

However, simply using rational numbers as upper and lower bounds to evaluate the double integral would lead to finding the volume under a rectangle instead of a triangle. Thus, functions will be used to set upper and lower bounds of the definite integral. Since the equation will be integrated in terms of x and y, the upper and lower bounds will be set relative to the xy plane.

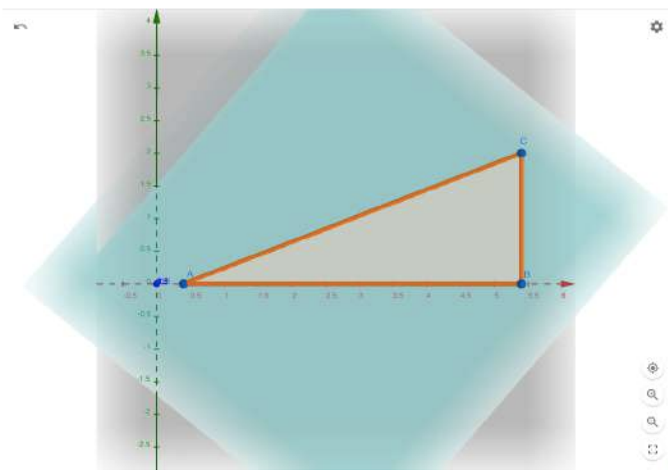
Figure 3: Orange region on the graph

Figure 3 on the left is the graph of the equation shown in relation to the xy plane:

From this, we can gather information on the upper and lower bounds of the definite integral:

$$0.4 \le x \le 5.4$$
$$0 \le y \le g(x)$$

Using the coordinates $(0.4, 0, 0.5)$ and $(5.4, 2, 1.5)$, the function $g(x)$ can be formulated:

$$g(x) = \frac{\Delta y}{\Delta x}(x - a)$$
$$g(x) = 0.4(x - 0.4)$$

Thus, the definite integral is evaluated:

$$Volume = \int_{0.4}^{5.4} \int_0^{0.4(x-0.4)} 0.3x - 0.25y + 3.8 \ dy\, dx$$

$$= \int_{0.4}^{5.4} [0.3xy - 0.125y^2 + 3.8y]_{y=0}^{y=0.4(x-0.4)}\, dx$$

$$= \int_{0.4}^{5.4} (0.3x)\,[0.4(x - 0.4)] - (0.125)[0.4(x - 0.4)]^2 + (3.8)[0.4(x - 0.4)]\, dx$$

To simplify the equation, the sum rule is applied:

$$\int [f(x) \pm g(x)]\, dx = \int f(x)\, dx \pm \int g(x)\, dx$$

(Harcet et al. 346)

Therefore:

$$= 0.12\int_{0.4}^{5.4} x^2 - 0.4x\,dx - 0.02\int_{0.4}^{5.4} (x-0.4)^2\,dx + 1.52\int_{0.4}^{5.4} x - 0.4\,dx$$
$$= (0.12)\left[\frac{x^3}{3} - 0.2x^2\right]_{x=0.4}^{x=5.4} - (0.02)\left[\frac{(x-0.4)^3}{3}\right]_{x=0.4}^{x=5.4} + (1.52)\left[\frac{x^2}{2} - 0.4x\right]_{x=0.4}^{x=5.4}$$
$$= (0.12)(46.656 - 0.011) - (0.02)(41.667) + (1.52)(12.42 + 0.08)$$
$$= 23.767cm^3$$

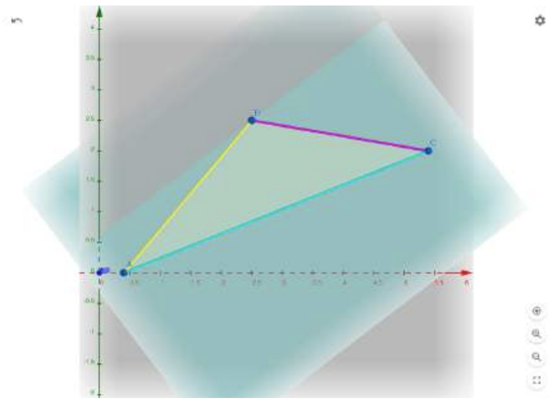

Figure 4: A region on the 3D model

The method above is appropriate for calculating the volume under most of the triangles on the 3D model, but may not work for all of them. Take Figure 4 an example:

Each side of the triangle, or three different functions has to be set as upper and lower bounds in order to find the volume under the triangle (each equation is highlighted in color in reference to Figure 4):

$$g_1(x) = 1.19(x-0.4)$$
$$g_2(x) = -0.172(x-2.5) + 2.5$$
$$g_3(x) = 0.4(x-0.4)$$

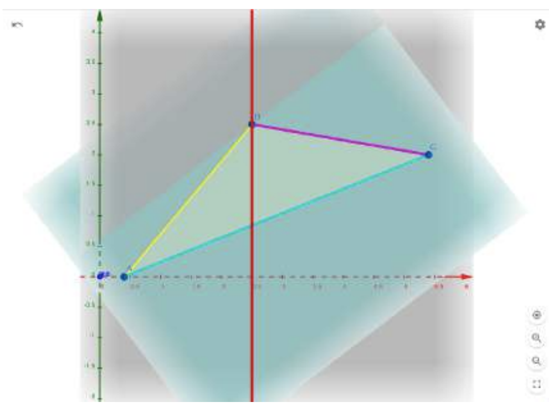

Figure 5: Region on 3D model divided

In order to do so, the triangle is split through the intermediate coordinate among the three coordinates, parallel to the y-axis, similar to what is shown on Figure 5 on the right. This will form to different integral equations that solve for each half of the volume under the triangle. Therefore, two different integral equations are formed and evaluated:

$$Volume = \int_{0.4}^{2.5}\int_{0.4(x-0.4)}^{1.19(x-0.4)} 0.422(x-0.4) - 0.554y + 0.5\,dy\,dx$$
$$+ \int_{2.5}^{5.4}\int_{0.4(x-0.4)}^{-0.172(x-2.5)+2.5} 0.422(x-0.4) - 0.554y + 0.5\,dy\,dx$$
$$= \int_{0.4}^{2.5} [-0.277y^2 + 0.422xy + 0.331y]_{y=0.4(x-0.4)}^{y=1.19(x-0.4)}\,dx$$
$$+ \int_{2.5}^{5.4} [-0.277y^2 + 0.422xy + 0.331y]_{y=0.4(x-0.4)}^{y=-0.172(x-2.5)+2.5}\,dx$$
$$= \int_{0.4}^{2.5} -0.0146x^2 + 0.407x + 0.160\,dx + \int_{2.5}^{5.4} -0.205x^2 + 1.358x - 1.348\,dx$$

$$= [-0.00487x^3 + 0.204x^2 + 0.160x]_{x=0.4}^{x=2.5} + [-0.0683x^3 + 0.679x^2 - 1.348x]_{x=2.5}^{x=5.4}$$
$$= 2.773cm^3$$

The two methods above are applied for all triangles on the 3D model appropriately. Only one half of the triangles were are integrated in order to save time and reduce redundancies. The sum of all volumes under the triangles were doubled to find the total volume of my dog's head:

$$Total\ Volume = 444.915cm^3$$

Formulate equations that outline the helmet

In the movies, Iron Man's helmet has a very smooth surface with minimal physical features in the face highlighted. For my dog, I will be using two-variable mathematical equations to approximate the shape of the helmet. As an example, two-variable equation that covers the shaded region shown in Figure 6 on the left will be found:

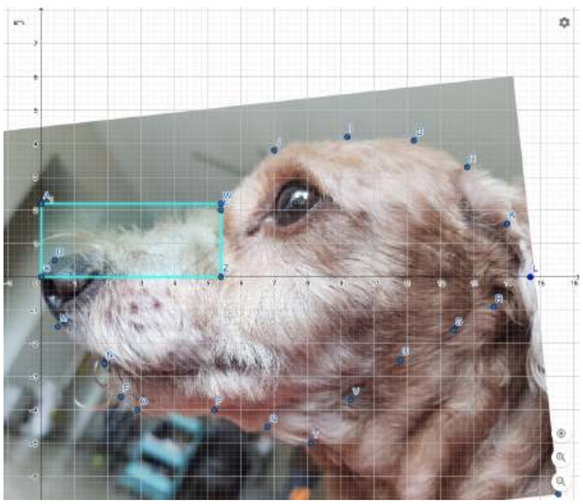

Figure 6: General region covering the front of the helmet

Since the goal is to formulate a two-variable equation that is ultimately relative to z, the two perspectives that reflect the yz and zx plane will be used. In this example, the perspective on the zx plane will be shown, since the method is the same for both perspectives. First, pick two coordinates that will cover the general region that will be covered by the helmet. This one step requires more thinking as a designer rather than a mathematician since it does not require mathematical knowledge, but rather a more artistic sense. For this case, the two coordinates will be used:

$$(0,0), (5.4, 2.2)$$

The next step is to approximate the exact region of the helmet using a single variable equation. Looking at the shape of my dog in the region and comparing it to Iron Man's helmet, it is appropriate to use a reciprocal equation of a certain power to cover the region of the helmet. When formulating inverse equations, we know that:

$$f(x) = a(x-b)^n + c$$

$$f^{-1}(x) = \sqrt[n]{\frac{1}{a}(x-c)} + b$$

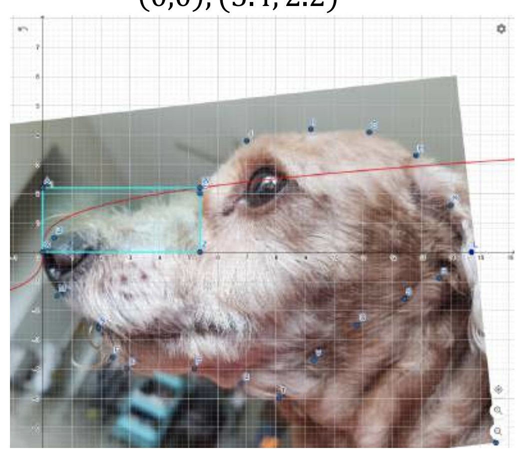

Figure 7: Region that covering the front of the helmet

The constant a is the gradient of the graph. Therefore, we can first say that the gradient of the graph is:

$$a = \frac{\Delta\, y}{\Delta\, x}$$

However, we know that we are not working with a linear function, but rather a function to the power of n. Therefore, the gradient of the function can be rewritten as:

$$a = \frac{\Delta\, y}{(\Delta\, x)^n}$$

The gradient a is written this way since the gradient decreases exponentially as the change in x increases in $f(x)$. Knowing this, we can now substitute a into the inverse equation $f^{-1}(x)$:

$$f^{-1}(x) = \sqrt[n]{\frac{(\Delta\, x)^n}{\Delta\, y}(x - c)} + b$$

Using this formula, values from the coordinates can be substituted into the formula:

$$y = \sqrt[n]{\frac{2.2^n}{5.4}x}$$

This next step is also less of math and more of approximating the shape of the graph, determining which value (n) forms the most appropriate shape for the helmet in the region. For this example, the equation that covered the shape of the helmet in the region is shown with a visual representation on Figure 7 on the right:

$$y = \sqrt[3]{\frac{2.2^3}{5.4}x} = 1.254\sqrt[3]{x}$$

The equations for other regions of the helmet were covered either by using the same method as above, or by using linear, polynomial, circle equations:

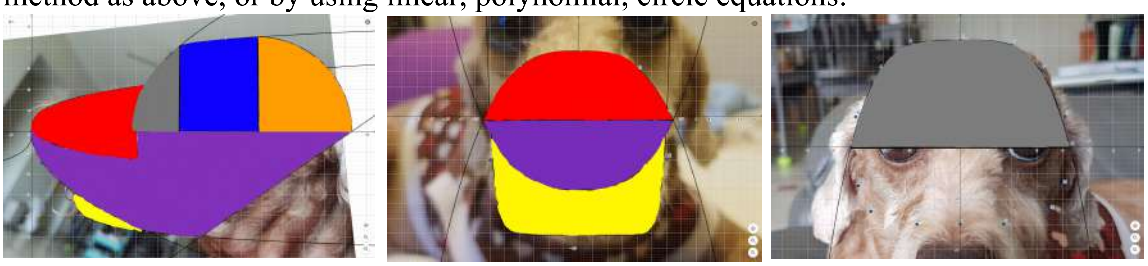

Figure 8: All regions of the helmet on each appropriate perspective

The final step is to formulate these equations into two-variable equations. To do so, simply add the equations from each plane that would cover the same region of the helmet. This would create a three dimensional model as such shown in Figure 9 on the right:

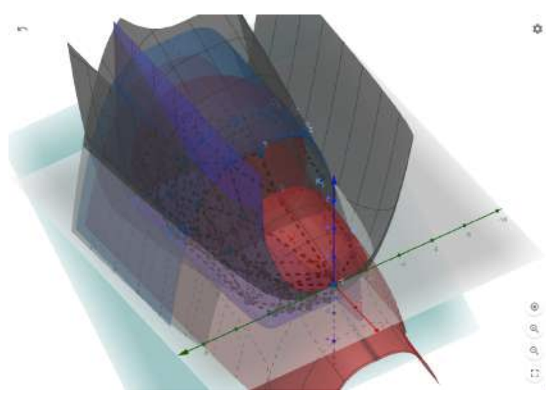

Figure 9: 3D model of helmet

All the two variable equations that form the three dimensional figure are displayed in the table below:

Table 1: Two-Variable Equations that Cover Each Region of the Helmet (see Figure 8)

Color of the Region	Equation of the Region
Red (Upper Mouth)	$f(x,y) = 1.254\sqrt[3]{x} - 0.07y^3$
Grey (Front Head)	$f(x,y) = 2.556\sqrt{(x-4.79)} - 0.0123y^4 + 0.2$
Blue (Middle Head)	$f(x,y) = 0.256\sqrt{(x-7)} - 0.0110y^4 + 4$
Orange (Back Head)	$f(x,y) = \sqrt{4.5^2 - (x-10.8)^2 - y^2}$
Purple (Lower Head)	$f(x,y) = -2.031\sqrt{x} - 0.07y^3$
Yellow (Chin)	$f(x,y) = -3.763\sqrt[10]{x-2} + (2.788\times10^{-7})y^{15} - 0.5$

Finding the Total Volume of the Helmet

Finding the volume of the helmet requires the use of double integration, requiring some upper and lower bounds to be functions. However, the upper and lower bounds in this step are not linear, but have similar steps in finding them to when finding the total volume of my dog's head. As an example, the equation of the red region will be evaluated:

$$f(x,y) = 1.254\sqrt[3]{x} - 0.07y^3$$

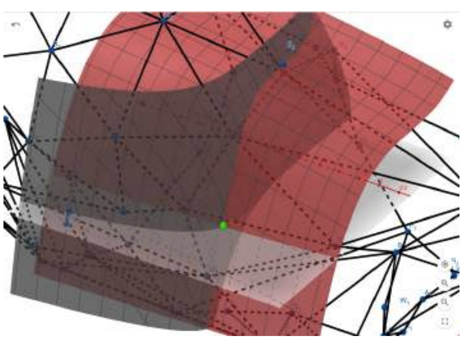

Figure 10: Red and grey region on 3D model

The upper bound and lower bounds of the integral equations will be found by also using the equation of the grey region (which is another region of the helmet that intersects with the red region):

$$f(x,y) = 2.556\sqrt{(x-4.79)} - 0.0123y^4 + 0.2$$

The upper and lower bounds will be found with the aid of the Figure 10 shown above. The equation will be divided to form two different integral equations to prevent having two sets of upper and lower bounds in a single integral equation. In order to know where to do so, the intersection between the two equations must be found:

$$1.254\sqrt[3]{x} - 0.07y^3 = 2.556\sqrt{(x-4.79)} - 0.0123y^4 + 0.2$$

Use Numeric Solver from GDC to solve for when $z = 0$:

$$x = 4.94$$
$$y = 3.1$$

This will find the coordinates of the intersection between the two regions, which will help in calculating the upper and lower bounds of the region. This coordinate is shown visually through the green dots in Figure 10. Using the information provided, we can gather set the upper and lower bounds of the integral equations. The functions in the

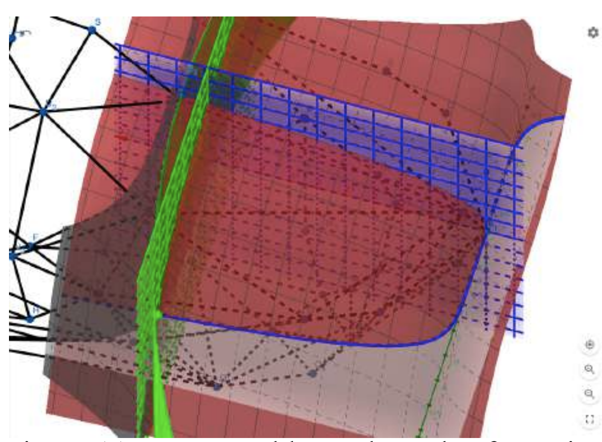

Figure 11: Upper and lower bounds of a region

upper and lower bounds are formulated using the same method as when finding the outline of the helmet. Thus, the upper and lower bounds of the equation is found.

1st upper and lower bounds:

$$0 \leq x \leq 4.94$$

$$0 \leq y \leq \sqrt[9]{\frac{3.1^9}{4.94}x} = 2.596x$$

2nd upper and lower bounds:

$$4.94 \leq x \leq -\frac{0.4}{3.13^2}y^2 + 5.4$$

$$0 \leq y \leq 3.13$$

A visual representation of these upper and lower bounds are shown in Figure 11 on the right. This method is applied for all other regions of the helmet. The next step is to finally integrate the two variable equations of each region. However, there are a few extra steps needed to calculate the volume under the regions. Take the integral equation of the blue region:

$$Volume = \int_0^{4.5} \int_{0.0514y^2+7}^{10.8} 0.256\sqrt{(x-7)} - 0.0110y^4 + 4\, dx\, dy$$

Before integrating, it is important to understand that the variable with functions set as their upper and lower bounds must be integrated first in order to end with a numeric value. Using the already shown methods of double integration, the volume of the region is found:

$$Volume = 55.27cm^3$$

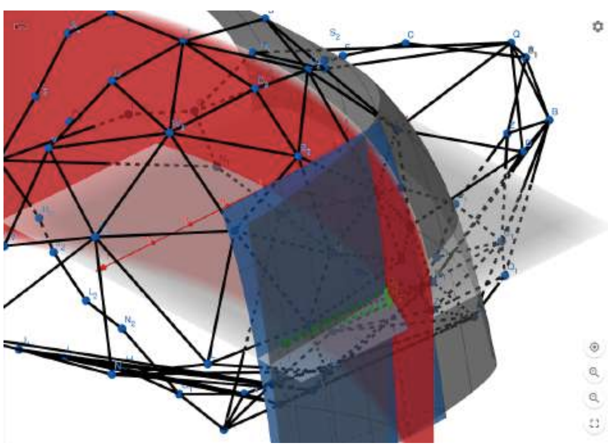

Figure 12: Additional volume found under the region

However, the volume integrated from the equation is slightly greater than the actual volume of the region due to the additional volume shown as a green triangle in Figure 12 on the left. This additional volume can be integrated and subtracted from the total volume by using the same method. However, it is very inaccurate to use functions for additional upper and lower bounds with the same integral equation. Therefore, the equation will first be expressed in terms of x and z. This will rotate graph so that the volume is calculated with greater accuracy. To do so, the inverse equation is found in terms of x and z:

$$f^{-1}(x,y) = 3.0896\sqrt[4]{0.256\sqrt{(x-7)} + 4 - y}$$

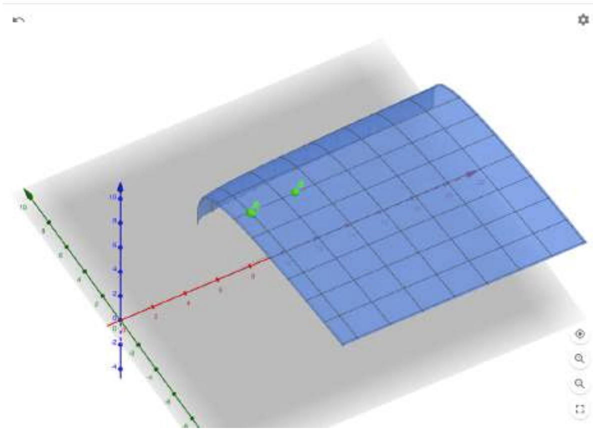

Figure 13: Inverse equation of the blue region

The graph of this equation is shown in Figure 13 on the left. This will allow the volume to be integrated using the same method used previously. With the coordinates:

$$(8, 0, 4.39), (10.8, 0, 4.5), (8, -0.24, 4.5)$$

The integral equation is formulated and evaluated:

$$Volume = \int_{8}^{10.8} \int_{0.143\sqrt{x-8}-0.24}^{0} 3.0896 \sqrt[4]{0.256\sqrt{x-7} + 4 - y}\, dy\, dx$$

Using u-substitution and the sum rule, the integral equation, in terms of x, can be split into two integrands:

$$= 3.0896 \left[(8\times10^{-4}) \int_{8}^{10.8} \left(0.256\sqrt{x-7} - 0.143\sqrt{x-8} + 4.24\right)^{\frac{5}{4}} dx \right.$$
$$\left. - (4.785\times10^{-4}) \int_{8}^{10.8} \left(\sqrt[4]{32\sqrt{x-7} + 500}\right) \left(128\sqrt{x-7} + 2000\right) dx \right]$$

The definite integral of the second integrand can be evaluated by continuing utilize the similar methods:

$$(4.785\times10^{-4}) \int_{8}^{10.8} \left(\sqrt[4]{32\sqrt{x-7} + 500}\right) \left(128\sqrt{x-7} + 2000\right) dx = 14.236cm^3$$

However, unlike the second integrand, the first integrand cannot be integrated through methods such as u-substitution. Instead, the definite integral of the integrand will be approximated using the trapezoid rule:

$$\int_{a}^{b} f(x)\, dx \approx \frac{\triangle x}{2} [(fx_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + \cdots + 2f(x_{n-1}) + 2f(x_n)]$$

$$Where\ \ \triangle x = \frac{b-a}{n}$$

(Simmons)

In this step, the volume will be divided into n pieces, integrating each piece separately by substituting the upper and lower bounds of the piece in the volume. For this calculation, the volume will be divided into 28 pieces, with each piece having the width of a $0.1cm$. This will reduce the amount of errors from the actual answer of this definite integral. Therefore:

$$\triangle x = \frac{10.8 - 8}{28} = 0.1$$

$$f(x_0) = \left(0.256\sqrt{(8-7)} - 0.143\sqrt{(8-8)} + 4.24\right)^{\frac{5}{4}} = 6.54672$$
$$f(x_1) = \left(0.256\sqrt{(8.1-7)} - 0.143\sqrt{(8.1-8)} + 4.24\right)^{\frac{5}{4}} = 6.48733$$
$$f(x_2) = \left(0.256\sqrt{(8.2-7)} - 0.143\sqrt{(8.2-8)} + 4.24\right)^{\frac{5}{4}} = 6.47499$$
$$f(x_3) = \left(0.256\sqrt{(8.3-7)} - 0.143\sqrt{(8.3-8)} + 4.24\right)^{\frac{5}{4}} = 6.46963$$
$$\vdots$$
$$f(x_{27}) = \left(0.256\sqrt{(10.7-7)} - 0.143\sqrt{(10.7-8)} + 4.24\right)^{\frac{5}{4}} = 6.54950$$
$$f(x_{28}) = \left(0.256\sqrt{(10.8-7)} - 0.143\sqrt{(10.8-8)} + 4.24\right)^{\frac{5}{4}} = 6.55360$$

Thus:

$$(8\times 10^{-4})\int_{8}^{10.8}\left(0.256\sqrt{x-7} - 0.143\sqrt{x-8} + 4.24\right)^{\frac{5}{4}} dx$$
$$\approx \frac{0.1}{2}[6.54672 + 2(6.48733) + 2(6.47499) + 2(6.46963) + \cdots + 2(6.54950) + 2(6.55360)]$$
$$\approx 14.57453 cm^3$$

Using the approximation, the additional volume below under the equation is evaluated:

$$\approx 3.0896(14.57453 - 14.236)$$
$$\approx 1.0459 cm^3$$

This is volume then subtracted from the total volume to find an accurate calculation of the volume under the equation. This value is doubled since the volume is half of volume needed to evaluate:

$$55.271 - 1.0459 \approx 54.225$$
$$54.225\times 2 \approx 108.450 cm^3$$

Apart from using the trapezoid rule, there is another two-variable equation that covers the region of the helmet that requires more complex steps when integrating. Take the integral equation of the grey region:

$$\int_{0}^{3.13}\int_{4.79}^{-0.0408y^2+5.4} 2.556\sqrt{x-4.79} - 0.0123y^4 + 0.2\, dx\, dy$$

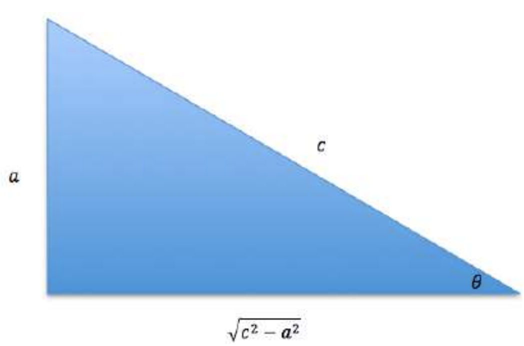

Figure 14: Visual representation of trigonometric substitution

After the integration the application of the sum rule, one of the integrands will lead to:

$$1.704\int_{0}^{3.13}\left(\sqrt{0.61 - 0.0408y^2}\right)^3 dy$$

The equation above cannot be evaluated through u-substitution due to the power of y in the integrand. Therefore, trigonometric substitution is used instead. Imagine a right-angle triangle (“Trig Substitutions”):

The lengths of the triangle can be calculated similarly using the figure on the left. Using trigonometry, can be calculated that:

$$\sin\theta = \frac{a}{c}$$

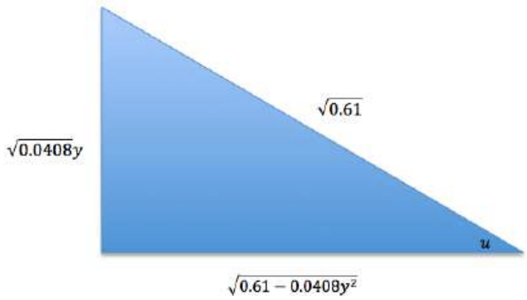

Figure 15: Trigonometric substitution for the integrand of grey region

The substitution for evaluating the integrand can be done in a similar way:

$$\sin u = \frac{\sqrt{0.0408}y}{\sqrt{0.61}} = 0.259y$$

$$u = \sin^{-1} 0.259y$$

Therefore:

$$y = \frac{\sin u}{0.259} = 3.865\sin u$$

$$dy = 3.865\cos u\, du$$

Thus:

$$1.704\int_0^{3.13} 3.865\cos u\,(0.61 - 0.61\sin^2 u)^{\frac{3}{2}}\, du$$

Using the understanding of the Pythagorean identities, Double angle identities and u-substitution, the volume of the region is calculated:

$$Volume = 1.857cm^3$$

The last method is finding the volume under the region of the sphere equation. Take the equation of the orange region:

$$f(x,y) = \sqrt{4.5^2 - (x-10.8)^2 - y^2}$$

Instead of integrating the equation as it is, it will be more efficient to use the formula used to evaluate the volume of a revolution.

$$Volume = \int_a^b \pi[f(x)]^2\, dx$$

Since the formula requires a single variable equation, the sphere equation will be used to find the circle equation in order to find the volume of the circle in a revolution, or otherwise the volume of the sphere:

$$f(x) = \sqrt{4.5^2 - (x-10.8)^2}$$

Therefore:

$$= \int_{10.8}^{15.3} \pi\left[\sqrt{4.5^2 - (x-10.8)^2}\right]^2 dx$$

$$= 190.852$$

Since the region is only half of the calculated value:

$$Volume = 190.852 \div 2 = 95.426cm^3$$

The remaining integrands of each region are evaluated and displayed in the table below: (reciprocated equations are **bolded** and the equations that are approximated using the trapezoid rule are highlighted):

Table 2: Integral Equations Used to Evaluate the Volume Under Each Region

Color of Regio	Integral Equation(s) for the Volume under the Region	Volume

n		
Red	$\int_0^5 \int_0^{2.7\sqrt[10]{x}} 1.254\sqrt[3]{x} - 0.07y^3 \, dy \, dx$ $\int_0^{3.13} \int_5^{-0.408y^2} 1.254\sqrt[3]{x} - 0.07y^3 \, dx \, dy$ $\boldsymbol{\int_0^5 \int_{-0.537x}^0 2.426\sqrt[3]{1.254\sqrt[3]{x} - y} \, dy \, dx}$	$53.620 cm^3$
Grey	$\int_5^8 \int_0^{3.794\sqrt[8]{x-4.79}} 2.556\sqrt{(x-4.79)} - 0.0123y^4 + 0.2 \, dy \, dx$ $\int_0^{3.13} \int_{4.79}^{-0.0408y^2+5.4} 2.556\sqrt{x-4.79} - 0.0123y^4 + 0.2 \, dx \, dy$	$86.703 cm^3$
Blue	$\int_0^{4.5} \int_{0.0514y^2+7}^{10.8} 0.256\sqrt{(x-7)} - 0.0110y^4 + 4 \, dx \, dy$ $\boldsymbol{\int_8^{10.8} \int_{0.143\sqrt{(x-8)}-0.24}^0 3.0896\sqrt[4]{0.256\sqrt{(x-7)} + 4 - y} \, dy \, dx}$	$108.450 cm^3$
Orange	$\int_{10.8}^{15.3} 20.25\pi - (x-10.8)^2\pi \, dx$	$95.426 cm^3$
Purple	$\boldsymbol{\int_0^{5.5} \int_{-2.031x}^{-0.537x} 2.426\sqrt[3]{2.031\sqrt{x} + y} \, dy \, dx}$ $\boldsymbol{\int_{5.5}^{8.1} \int_{-3.763\sqrt[10]{x-2}-0.5}^0 2.426\sqrt[3]{2.031\sqrt{x} + y} \, dy \, dx}$ $\boldsymbol{\int_{-5}^0 \int_{8.1}^{1.44y+15.3} 2.426\sqrt[3]{2.031\sqrt{x} + y} \, dx \, dy}$	$185.794 cm^3$
Yellow	$\boldsymbol{\int_2^{5.5} \int_{-3.763\sqrt[10]{x-2}-0.5}^{-2.031\sqrt{x}} 2.735\sqrt[15]{3.763\sqrt[10]{x-2} + y + 0.5} \, dy \, dx}$	$1.768 cm^3$
Total Volume under the Regions		$533.327 cm^3$

By subtracting the total volume of my dog's head, the total volume of the helmet is calculated:

$$533.327 - 443.350 \approx 89.977 cm^3$$

Conclusion

By the end of this exploration, I was able to successfully design the base of the Iron Man helmet for my dog and also calculate its volume mainly through the use of mathematics. By choosing to design an Iron Man helmet for my dog, I was able to learn more about the uses of mathematics and especially the extensive uses of integration. Whether it was the trapezoid rule or the method of trigonometric substitution, I was able to understand that there is much more to learn outside of the IB curriculum in order to fully understand the concept of integration. However through every process, there were several limitations in the methodology that affected my thought process and the exploration itself.

The most obvious limitation was the approximations made in the methodology. Apart from using the trapezoid rule, the model made to evaluate the volume of my dog's head was the largest approximation made in the exploration. The pictures taken from each perspective was not perfectly aligned with every other perspective, which caused a few errors when creating the 3D model of my dog's head. This would consequently affect the total volume of my dog's head. Also, the method used to create a 3D model of my dog's head also may have affected the calculated volume of my dog's head. Since using only several coordinate points formed the 3D model, the model and my dog's head is not identical.

The second major limitation was when forming the equations that cover the region of the helmet. No matter how applicable mathematics can be in many situations, using mathematics is not better than sketching when it comes to creating certain designs. When forming the equations, I was only able to utilize the single-variable equations from the perspectives that reflect the yz and xz plane, but not the xy plane. My aim in this exploration was to first create an Iron Man helmet appropriate for my dog, in which there were some problems in replicating the helmet due to the limitations of mathematics.

The limitations aside, I felt that my exploration was very satisfying to work on. However, I also felt that there would have been several extensions and additions to improve the outcome of my exploration. The most wanted improvement to this exploration was using double integration to find the surface area of the helmet. This will provide an extension to not only find the volume of the helmet, but also approximate the amount of paint needed to complete the Iron Man-look on the helmet. Using functions to set the upper and lower bounds, I could have also found the ratio between red and gold paint, possibly completing the exploration with a fully designed Iron Man helmet.

Applications

Precision and accuracy in any project on a massive scale plays a crucial role in determining its success. A single error in calculation or an issue in rounding can lead to a failure of a project such as from a decade old space mission all the way to as small as a an issue in manufacturing a product that can lead to problems in its ergonomics. By using the mathematics expressed in this exploration, these errors can be minimized drastically, which can increase the probability of succeeding in a certain project.

Though it was mentioned that precision and accuracy was considered a limitation for this exploration, the use of mathematical functions and double integration can not only decrease the error, but also increase the efficiency in providing a prototype or an initial plan for a project if a proficient program is used. Take a project as simple as designing a case for a smartphone. By using the mathematics shown in the exploration, an algorithm can be written for a program to automatically assign several different two-variable equations that cover the volume of the smartphone case, which can largely reduce the approximations that were made in this exploration. Then, the equations can be integrated appropriately to find a very accurate volume for the smartphone case, allowing an economic way to reduce costs for a business making these smartphone cases. Though this is a simple example, I believe that this method can be implemented similarly into other larger scale projects, using mathematical algorithms and equations to increase the accuracy of models.

Works Cited

Dawkins, Paul. "Double Integrals." *Paul's Online Math Notes*, edited by Paul Dawkins, tutorial.math.lamar.edu/Classes/CalcIII/DoubleIntegrals.aspx. Accessed 27 June 2018.

---. "Trig Substitutions." *Paul's Online Math Notes*, edited by Paul Dawkins, tutorial.math.lamar.edu/Classes/CalcII/TrigSubstitutions.aspx. Accessed 8 July 2018.

Harcet, Josip, et al. *IB Higher Level Mathematics Course Book: For the IB Diploma*. Oxford, Oxford UP, 2012.

"Iron Man 3 - VFX Breakdown by Trixter (2013)." YouTube, uploaded by FilmIsNow Movie Bloopers & Extras, 6 Feb. 2017, www.youtube.com/watch?v=NBr_vlOEe2w. Accessed 23 Oct. 2018.

Simmons, Bruce. "Trapezoid Rule." *Mathwords*, edited by Bruce Simmons, 19 July 2017, www.mathwords.com/t/trapezoid_rule.htm. Accessed 7 July 2018.