Middle East Technical University | Department of Computer Engineering

CENG 424

Logic for Computer Science

Fall 2023 - Homework 3

Relational Logic & Resolution

Due date: 19 November 2023, Sunday, 23:59 (No Late Allowed!)

1 Specifications

1. Your work must be on PDF file preferably outputted by a LaTeX file.
2. Your work must be of your own. This is an individual homework, no collaboration is allowed.
3. Your work must obey, of course, **zero tolerance policy for cheating**.
4. Your work must be submitted before the deadline. There is **no late submission policy**.
5. Your work must be submitted as specified in the section 3, otherwise there is a penalty of 10 points.
6. You may ask your questions by sending an email to "adhd@ceng.metu.edu.tr".

2 Questions

1. Let D(x) be "x is a dog", C(x) be "x is a cat", and Friends(x, y) be "x and y are friends", where x and y represent animals.

   Translate the following into English statements.

   (a) $\forall x.(C(x) \Rightarrow \exists y.(D(y) \wedge Friends(x, y)))$

   (b) $\exists x.(C(x) \wedge \forall y.(D(y) \Rightarrow Friends(x, y)))$

2. Show whether the following relational logic sentences are valid, contingent, or unsatisfiable.

   (a) $\exists x.(\forall y.p(x, y) \Rightarrow p(z, z))) \Leftrightarrow (\exists x.p(x, x) \Rightarrow \exists y.p(y, y)$

   (b) $(\forall x.(p(x) \vee q(x)) \Rightarrow (\exists y.p(y) \Rightarrow (p(x) \Rightarrow \forall y.p(y)))$

   (c) $\exists y.(p(y) \Rightarrow \exists x.q(x, y)) \Rightarrow \neg\exists x.q(y, x)$

3. Prove the following sentence by modus ponens and the standard axiom schemata.

   $$\forall x.(p(x) \Rightarrow q(x)), \neg\exists z.r(z), \exists y.p(y) \vee r(a), \neg\exists z.r(z) \Rightarrow \forall z.(\neg p(z)) \vdash \exists z.q(z)$$

   hint: you can also use mendelson's corrolary of replacement.

4. Prove the validity of the following sentence using resolution.

   $$\{\forall y.A(a, y), \forall x.\forall y.(A(x, y) \Rightarrow A(B(x), B(y)))\} \vdash \exists z.(A(a, z) \wedge A(z, B(B(a)))$$

3 Submission

Please submit a PDF file named `hw2_e1234567.pdf` to gradescope.com, where 1234567 refers to your student identification number.