Middle East Technical University   Department of Computer Engineering

CENG 424

Logic for Computer Science

Fall 2023 - Homework 1

First-Order Logic

Due date: 22 October 2023, Sunday, 23:59 (No Late Allowed!)

1 Specifications

1. Your work must be on PDF file preferably outputted by a LaTeX file.
2. The homework must be submitted before the deadline. There is no late submission policy.
3. This is an individual homework. All solutions must be your own, otherwise is considered as cheating.

2 Questions

1. Draw truth tables to show which of the following pairs of expressions are logical equivalences.
   (a) $\mathrm{A} \rightarrow \mathrm{B}$ and $\neg(\mathrm{A} \wedge \neg \mathrm{B})$
   (b) $\mathrm{A} \leftrightarrow \mathrm{B}$ and $(\neg \mathrm{A} \vee \mathrm{B}) \wedge (\neg \mathrm{B} \vee \mathrm{A})$
   (c) $\mathrm{A} \rightarrow (\neg \mathrm{A} \rightarrow \mathrm{B})$ and $\mathbf{1}$
   (d) $(\mathrm{A} \vee \neg \mathrm{B}) \rightarrow \mathrm{C}$ and $(\neg \mathrm{A} \wedge \mathrm{B}) \vee \mathrm{C}$
2. Convert each of the following logical forms to conjunctive normal form (CNF):
   (a) $\mathrm{A} \wedge (\neg \mathrm{A} \rightarrow \mathrm{A})$
   (b) $(\mathrm{A} \rightarrow \mathrm{B}) \rightarrow ((\mathrm{A} \rightarrow \neg \mathrm{B}) \rightarrow \neg \mathrm{A})$
   (c) $(\mathrm{A} \rightarrow (\mathrm{B} \vee \neg \mathrm{C})) \wedge \neg \mathrm{A} \wedge \mathrm{B}$
3. Construct a semantic tableaux (use only rules defined in section 4) to show that the following logical forms mutually consistent or not.

$$\neg \mathrm{A} \wedge \mathrm{B}, \quad \neg(\mathrm{B} \wedge \mathrm{C}), \quad \mathrm{C} \vee \mathrm{D}, \quad \neg(\neg \mathrm{A} \rightarrow \mathrm{D})$$

3 Submission

Please submit a PDF file named `hw1_e1234567.pdf` to gradescope.com, where 1234567 refers to your student identification number.

4 Semantic Tableaux

If a tableau contains;

1. $A \wedge B$: it can be extended to form a new tableau by adding both A and B below to the branch containing $A \wedge B$.
2. $A \vee B$: it can be extended to form a new tableau by adding two new branches, one containing A and the other containing B.
3. A $\rightarrow$ B: it can be extended to form a new tableau by adding two new branches, one containing $\neg$A and the other containing B.
4. A $\leftrightarrow$ B: it can be extended to form a new tableau by adding two new branches, one containing A $\wedge$ B and the other containing $\neg$A $\wedge$ $\neg$B.
5. $\neg\neg$A: it can be extended to form a new tableau by adding A below to the branch containing $\neg\neg$A.
6. $\neg$(A $\wedge$ B): it can be extended to form a new tableau by adding two new branches, one containing $\neg$A and the other containing $\neg$B.
7. $\neg$(A$\vee$B): it can be extended to form a new tableau by adding both $\neg$A and $\neg$B below to the branch containing $\neg$(A $\vee$ B).
8. $\neg$(A $\rightarrow$ B): it can be extended to form a new tableau by adding both A and $\neg$B below to the branch containing $A \wedge B$.
9. $\neg$(A $\leftrightarrow$ B): it can be extended to form a new tableau by adding two new branches, one containing A $\wedge$ $\neg$B and the other containing $\neg$A $\wedge$ B.
10. Finally, and most importantly, whenever a logical form A and its negation $\neg$A appear in a branch of a tableau, an inconsistency is indicated in that branch and it is said to be 'closed', i.e. it is not further extended. This is because A and $\neg$A cannot both be true at the same time.