CENG 424
Fall 2024
Homework 1

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Q1

a)

A	B	¬ B	A ∧¬ B	A → B	¬ (A ∧¬ B)
0	0	1	0	1	1
0	1	0	0	1	1
1	0	1	1	0	0
1	1	0	0	1	1

A → B and ¬ (A ∧¬ B) are logically equivalent.

b)

A	B	¬ A	¬ B	¬ A ∨ B	¬ B ∨ A	A ↔ B	(¬ A ∨ B) ∧ (¬ B ∨ A)
0	0	1	1	1	1	1	1
0	1	1	0	1	0	0	0
1	0	0	1	0	1	0	0
1	1	0	0	1	1	1	1

A ↔ B and (¬ A ∨ B) ∧ (¬ B ∨ A) are logically equivalent.

c)

A	B	¬ A	¬ A → B	A → (¬ A → B)	1
0	0	1	0	1	1
0	1	1	1	1	1
1	0	0	1	1	1
1	1	0	1	1	1

A → (¬ A → B) and 1 are logically equivalent.

d)

A	B	C	¬ A	¬ B	A ∨¬ B	¬ A ∧ B	(A ∨¬ B) → C	(¬ A ∧ B) ∨ C
0	0	0	1	1	1	0	0	0
0	0	1	1	1	1	0	1	1
0	1	0	1	0	0	1	1	1
0	1	1	1	0	0	1	1	1
1	0	0	0	1	1	0	0	0
1	0	1	0	1	1	0	1	1
1	1	0	0	0	1	0	0	0
1	1	1	0	0	1	0	1	1

(A ∨¬ B) → C and (¬ A ∧ B) ∨ C are logically equivalent.

Q2

a)

$$
\begin{aligned}
&A \wedge (\neg A \rightarrow A) \\
&A \wedge (\neg\neg A \vee A) \\
&A \wedge (A \vee A) \\
&A \wedge A \\
&A
\end{aligned}
$$

b)

$$
\begin{aligned}
&(A \to B) \to ((A \to \neg B) \to \neg A)\\
&\neg\,(A \to B) \vee ((A \to \neg B) \to \neg A)\\
&\neg(A \to B) \vee ((A \to \neg B) \to \neg A)\\
&\neg(\neg A \vee B) \vee ((A \to \neg B) \to \neg A)\\
&(A \wedge \neg B) \vee ((A \to \neg B) \to \neg A)\\
&(A \wedge \neg B) \vee (\neg(A \to \neg B) \vee \neg A)\\
&(A \wedge \neg B) \vee (\neg(A \to \neg B) \vee \neg A)\\
&(A \wedge \neg B) \vee (\neg(\neg A \vee \neg B) \vee \neg A)\\
&(A \wedge \neg B) \vee ((A \wedge B) \vee \neg A)\\
&(A \wedge \neg B) \vee (A \wedge B) \vee \neg A\\
&\neg\,(A \wedge \neg B) \wedge \neg(A \wedge B) \wedge \neg\neg A\\
&(\neg A \vee \neg\neg B) \wedge (\neg A \vee \neg B) \wedge A\\
&(\neg A \vee B) \wedge (\neg A \vee \neg B) \wedge A
\end{aligned}
$$

c)

$$
\begin{aligned}
&(A \to (B \vee \neg C)) \wedge \neg A \wedge B\\
&(\neg A \vee (B \vee \neg C)) \wedge \neg A \wedge B\\
&(\neg A \vee B \vee \neg C) \wedge \neg A \wedge B
\end{aligned}
$$

Q3

1. $(\neg A \wedge B) \wedge \neg(B \wedge C) \wedge (C \vee D) \wedge \neg(\neg A \to D) \equiv 1$
2. $\quad\vert$
3. $\neg A \wedge B \equiv 1$ (1.)
4. $\neg(B \wedge C) \equiv 1$ (1.)
5. $C \vee D \equiv 1$ (1.)
6. $\neg(\neg A \to D) \equiv 1$ (1.)
7. $\quad\vert$
8. $\neg A \equiv 1$ (3.)
9. $B \equiv 1$ (3.)
10. $\neg B \equiv 1$ (4.) $\qquad$ $\neg C \equiv 1$ (4.)
11. $\square$
12. $C \equiv 1$ (5.) $\qquad$ $D \equiv 1$ (5.)
13. $\square$ $\qquad$ $\vert$
14. $\neg A \equiv 1$ (6.)
15. $\neg D \equiv 1$ (6.)
16. $\square$

The logical formulas $\neg A \wedge B$, $\neg(B \wedge C)$, $C \vee D$ and $\neg(\neg A \rightarrow D)$ are not mutually consistent.