# CENG 424
# Fall 2024
# Homework 4

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## Q1)

### Definitions

- $h$ : the horse
- $A(x)$ : $x$ is an animal
- $P(x)$ : $x$ is a plant
- $G(x, y)$ : $x$ grooms $y$
- $S(x)$ : $x$ is a stableman

### Premises

- $(A(h) \rightarrow \exists y(S(y) \wedge G(y, h)))$
- $(P(h) \rightarrow \neg\exists y(S(y) \wedge G(y, h)))$

### Goal

- $A(h) \rightarrow \neg P(h)$

#### Negated Goal

- $\neg(A(h) \rightarrow \neg P(h))$

## Conversion to CNF

| | |
|---|---|
| $A(h) \rightarrow \exists y(S(y) \wedge G(y,h))$ | Premise #1 |
| $\neg A(h) \vee \exists y(S(y) \wedge G(y,h))$ | I |
| $\neg A(h) \vee \exists y(S(y) \wedge G(y,h))$ | N |
| $\neg A(h) \vee \exists y(S(y) \wedge G(y,h))$ | S |
| $\neg A(h) \vee (S(c) \wedge G(c,h))$ | E |
| $\neg A(h) \vee (S(c) \wedge G(c,h))$ | A |
| $(\neg A(h) \vee S(c)) \wedge (\neg A(h) \vee G(c,h))$ | D |
| $\{\neg A(h), S(c)\}$ | O |
| $\{\neg A(h), G(c,h)\}$ | O |

| | |
|---|---|
| $P(h) \rightarrow \neg\exists y(S(y) \wedge G(y,h))$ | Premise #2 |
| $\neg P(h) \vee \neg\exists y(S(y) \wedge G(y,h))$ | I |
| $\neg P(h) \vee \forall y\neg(S(y) \wedge G(y,h))$ | N |
| $\neg P(h) \vee \forall y(\neg S(y) \vee \neg G(y,h))$ | N |
| $\neg P(h) \vee \forall y(\neg S(y) \vee \neg G(y,h))$ | S |
| $\neg P(h) \vee \forall y(\neg S(y) \vee \neg G(y,h))$ | E |
| $\neg P(h) \vee (\neg S(y) \vee \neg G(y,h))$ | A |
| $\neg P(h) \vee \neg S(y) \vee \neg G(y,h)$ | D |
| $\{\neg P(h), \neg S(y), \neg G(y,h)\}$ | O |

| | |
|---|---|
| $\neg(A(h) \rightarrow \neg P(h))$ | Negated Goal |
| $\neg(\neg A(h) \vee \neg P(h))$ | I |
| $\neg\neg A(h) \wedge \neg\neg P(h)$ | N |
| $A(h) \wedge P(h)$ | N |
| $A(h) \wedge P(h)$ | S |
| $A(h) \wedge P(h)$ | E |
| $A(h) \wedge P(h)$ | A |
| $A(h) \wedge P(h)$ | D |
| $\{A(h)\}$ | O |
| $\{P(h)\}$ | O |

### Resolution

| | | |
|---|---|---|
| $\{\neg A(h), S(c)\}$ | Premise #1 | (1) |
| $\{\neg A(h), G(c,h)\}$ | Premise #1 | (2) |
| $\{\neg P(h), \neg S(y), \neg G(y,h)\}$ | Premise #2 | (3) |
| $\{A(h)\}$ | Negated Goal | (4) |
| $\{P(h)\}$ | Negated Goal | (5) |
| $\{S(c)\}$ | $1, 4$ | (6) |
| $\{G(c,h)\}$ | $2, 4$ | (7) |
| $\{\neg P(h), \neg S(c)\}$ | $3, 5\{y \leftarrow c\}$ | (8) |
| $\{\neg P(h)\}$ | $6, 8$ | (9) |
| $\{\}$ | $5, 9$ | (10) |
| | | (11) |

# Q2)

### Premises

- $T$
- $\neg S \vee \neg T \vee \neg R$
- $\neg T \vee R$
- $S \vee \neg R$

### Conversion to CNF

| | |
|---|---|
| $T$ | Premise #1 |
| $T$ | I |
| $T$ | N |
| $T$ | S |
| $T$ | E |
| $T$ | A |
| $T$ | D |
| $\{T\}$ | O |

| | |
|---|---|
| $\neg S \vee \neg T \vee \neg R$ | Premise #2 |
| $\neg S \vee \neg T \vee \neg R$ | I |
| $\neg S \vee \neg T \vee \neg R$ | N |
| $\neg S \vee \neg T \vee \neg R$ | S |
| $\neg S \vee \neg T \vee \neg R$ | E |
| $\neg S \vee \neg T \vee \neg R$ | A |
| $\neg S \vee \neg T \vee \neg R$ | D |
| $\{\neg S, \neg T, \neg R\}$ | O |

| | |
|---|---|
| $\neg T \vee R$ | Premise #3 |
| $\neg T \vee R$ | I |
| $\neg T \vee R$ | N |
| $\neg T \vee R$ | S |
| $\neg T \vee R$ | E |
| $\neg T \vee R$ | A |
| $\neg T \vee R$ | D |
| $\{\neg T, R\}$ | O |

## a) Unit Resolution

| | | |
|---|---|---|
| $\{T\}$ | Premise #1 | (1) |
| $\{\neg S, \neg T, \neg R\}$ | Premise #2 | (2) |
| $\{\neg T, R\}$ | Premise #3 | (3) |
| $\{S, \neg R\}$ | Premise #4 | (4) |
| $\{R\}$ | $1, 3$ | (5) |
| $\{\neg S, \neg T\}$ | $2, 5$ | (6) |
| $\{\neg S\}$ | $1, 6$ | (7) |
| $\{\neg R\}$ | $4, 7$ | (8) |
| $\{\neg T\}$ | $3, 8$ | (9) |
| $\{\}$ | $1, 9$ | (10) |

### b) Input Resolution

| | | |
|---|---|---|
| $\{T\}$ | Premise #1 | (1) |
| $\{\neg S, \neg T, \neg R\}$ | Premise #2 | (2) |
| $\{\neg T, R\}$ | Premise #3 | (3) |
| $\{S, \neg R\}$ | Premise #4 | (4) |
| $\{R\}$ | $1, 3$ | (5) |
| $\{\neg S, \neg T\}$ | $2, 5$ | (6) |
| $\{\neg S\}$ | $1, 6$ | (7) |
| $\{\neg R\}$ | $4, 7$ | (8) |
| $\{\neg T\}$ | $3, 8$ | (9) |
| $\{\}$ | $1, 9$ | (10) |

### c) Linear Resolution

| | | |
|---|---|---|
| $\{T\}$ | Premise #1 | (1) |
| $\{\neg S, \neg T, \neg R\}$ | Premise #2 | (2) |
| $\{\neg T, R\}$ | Premise #3 | (3) |
| $\{S, \neg R\}$ | Premise #4 | (4) |
| $\{R\}$ | $1, 3$ | (5) |
| $\{\neg S, \neg T\}$ | $2, 5$ | (6) |
| $\{\neg S\}$ | $1, 6$ | (7) |
| $\{\neg R\}$ | $4, 7$ | (8) |
| $\{\neg T\}$ | $3, 8$ | (9) |
| $\{\}$ | $1, 9$ | (10) |

## Q3)

### Premises

- $R \vee P \vee \neg Q$
- $\neg P \vee R$
- $\neg Q \vee \neg R$
- $Q$

## Conversion to CNF

| | |
|---|---|
| $R \vee P \vee \neg Q$ | Premise #1 |
| $R \vee P \vee \neg Q$ | I |
| $R \vee P \vee \neg Q$ | N |
| $R \vee P \vee \neg Q$ | S |
| $R \vee P \vee \neg Q$ | E |
| $R \vee P \vee \neg Q$ | A |
| $R \vee P \vee \neg Q$ | D |
| $\{R, P, \neg Q\}$ | O |

| | |
|---|---|
| $\neg P \vee R$ | Premise #2 |
| $\neg P \vee R$ | I |
| $\neg P \vee R$ | N |
| $\neg P \vee R$ | S |
| $\neg P \vee R$ | E |
| $\neg P \vee R$ | A |
| $\neg P \vee R$ | D |
| $\{\neg P, R\}$ | O |

| | |
|---|---|
| $\neg Q \vee \neg R$ | Premise #3 |
| $\neg Q \vee \neg R$ | I |
| $\neg Q \vee \neg R$ | N |
| $\neg Q \vee \neg R$ | S |
| $\neg Q \vee \neg R$ | E |
| $\neg Q \vee \neg R$ | A |
| $\neg Q \vee \neg R$ | D |
| $\{\neg Q, \neg R\}$ | O |

| | |
|---|---|
| $Q$ | Premise #4 |
| $Q$ | I |
| $Q$ | N |
| $Q$ | S |
| $Q$ | E |
| $Q$ | A |
| $Q$ | D |
| $\{Q\}$ | O |

## Ordered Resolution

$$\text{Order} = P > R > Q$$

| | | |
|---|---|---|
| $\{R, P, \neg Q\}$ | Premise #1 | (1) |
| $\{\neg P, R\}$ | Premise #2 | (2) |
| $\{\neg Q, \neg R\}$ | Premise #3 | (3) |
| $\{Q\}$ | Premise #4 | (4) |
| $\{R, \neg Q\}$ | $1, 2$ | (5) |
| $\{\neg Q\}$ | $3, 5$ | (6) |
| $\{\}$ | $4, 6$ | (7) |
| | | (8) |