MATHS

FOUNDATIONS OF MATHEMATICAL LITERACY

A COMPREHENSIVE GUIDE

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CL/OTHMFL3/11/90

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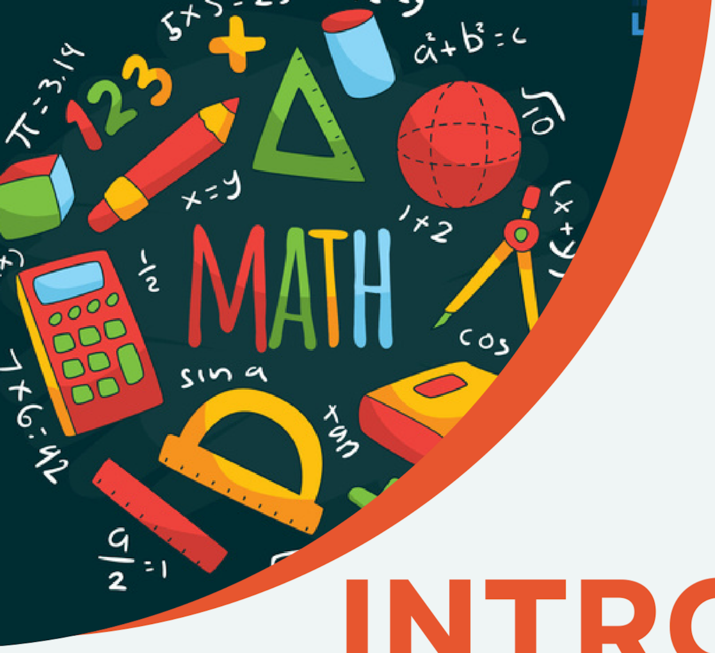


INTRODUCTION

Welcome to the comprehensive guidebook on mathematical literacy, designed to equip learners with essential skills and understanding in numeracy and data interpretation. This guide covers a wide array of topics, including the application of the four rules of numeracy, expressing numbers in standard form, and performing operations with negative numbers. Through practical examples and clear explanations, readers will gain proficiency in comparing numbers, interpreting approximate data, and executing simple transactions with confidence.

Furthermore, this guide delves into problem-solving methodologies, offering a diverse range of approaches to tackle mathematical challenges effectively. From basic algebraic manipulation to graphical representation of data through charts and diagrams, learners will learn how to analyze and interpret various types of data. Additionally, the guide explores the fundamentals of differentiation, integration, and probability theory, empowering individuals to calculate expected outcomes and understand the principles behind the normal distribution. Whether you're a student, educator, or professional seeking to enhance your mathematical skills, this guidebook serves as a valuable resource for mastering essential mathematical concepts and techniques.

ACKNOWLEDGEMENT

I would like to extend my heartfelt gratitude to Mr. Janaka Upendra, my esteemed lecturer, whose guidance and expertise have been invaluable in shaping this revision guide. Mr. Upendra's dedication to teaching and commitment to excellence have inspired me to strive for the highest standards in mathematics education. His unwavering support and encouragement have played a pivotal role in my professional development, and I am deeply grateful for his mentorship.

I am also thankful to the school administration for providing me with the opportunity to create this revision guide for our students undertaking external examinations. Their commitment to fostering academic excellence and supporting the educational endeavors of both students and staff is truly commendable. Additionally, I would like to express my appreciation to my colleagues for their collaboration and input throughout the process of developing this guide. Together, we aim to empower our students with the knowledge and skills they need to succeed in their examinations and beyond.

Lastly, I extend my sincere thanks to the students who will utilize this revision guide. Your dedication to learning and commitment to academic achievement are truly admirable. It is my hope that this guide will serve as a valuable resource in your preparation for the upcoming examinations, and I wish you all the best in your academic endeavors.

NUMERACY

BASIC RULES OF NUMERACY

(+) Addition
(-) Subtraction
(x) Multiplication
(÷) Division

Addition: Addition is combining two or more numbers to find their total sum.

- 5 + 3 = 8
- 10.25 + 4.75 = 15

Subtraction: Subtraction is taking one number away from another to find the difference.

- 10 - 7 = 3
- 15.5 - 8.25 = 7.25

Multiplication: Multiplication is repeated addition. It's the process of combining equal groups to find a total.

- 10 - 7 = 3
- 15.5 - 8.25 = 7.25

Division: Division is the process of splitting a number into equal parts or finding out how many times one number is contained within another.

- 12 ÷ 4 = 3
- 15 ÷ 5 = 3

ACTIVITY

ADDITION

$$\begin{array}{r} 95 \\ +\,13 \\ \hline \end{array} \quad \begin{array}{r} 28 \\ +\,99 \\ \hline \end{array} \quad \begin{array}{r} 11 \\ +\,59 \\ \hline \end{array} \quad \begin{array}{r} 45 \\ +\,32 \\ \hline \end{array} \quad \begin{array}{r} 25 \\ +\,30 \\ \hline \end{array}$$

SUBTRACTION

$$\begin{array}{r} 186 \\ -\,113 \\ \hline \end{array} \quad \begin{array}{r} 590 \\ -\,271 \\ \hline \end{array} \quad \begin{array}{r} 804 \\ -\,491 \\ \hline \end{array} \quad \begin{array}{r} 236 \\ -\,117 \\ \hline \end{array} \quad \begin{array}{r} 373 \\ -\,108 \\ \hline \end{array}$$

MULTIPLICATION

$$\begin{array}{r} 44 \\ \times\,24 \\ \hline \end{array} \quad \begin{array}{r} 17 \\ \times\,23 \\ \hline \end{array} \quad \begin{array}{r} 85 \\ \times\,6 \\ \hline \end{array} \quad \begin{array}{r} 96 \\ \times\,41 \\ \hline \end{array} \quad \begin{array}{r} 58 \\ \times\,20 \\ \hline \end{array} \quad \begin{array}{r} 45 \\ \times\,0 \\ \hline \end{array}$$

$$\begin{array}{r} 47 \\ \times\,27 \\ \hline \end{array} \quad \begin{array}{r} 38 \\ \times\,4 \\ \hline \end{array} \quad \begin{array}{r} 84 \\ \times\,14 \\ \hline \end{array} \quad \begin{array}{r} 77 \\ \times\,36 \\ \hline \end{array} \quad \begin{array}{r} 27 \\ \times\,10 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ \times\,56 \\ \hline \end{array}$$

DIVISION

$$4\overline{)28} \quad 8\overline{)40} \quad 4\overline{)8} \quad 1\overline{)3} \quad 1\overline{)1}$$

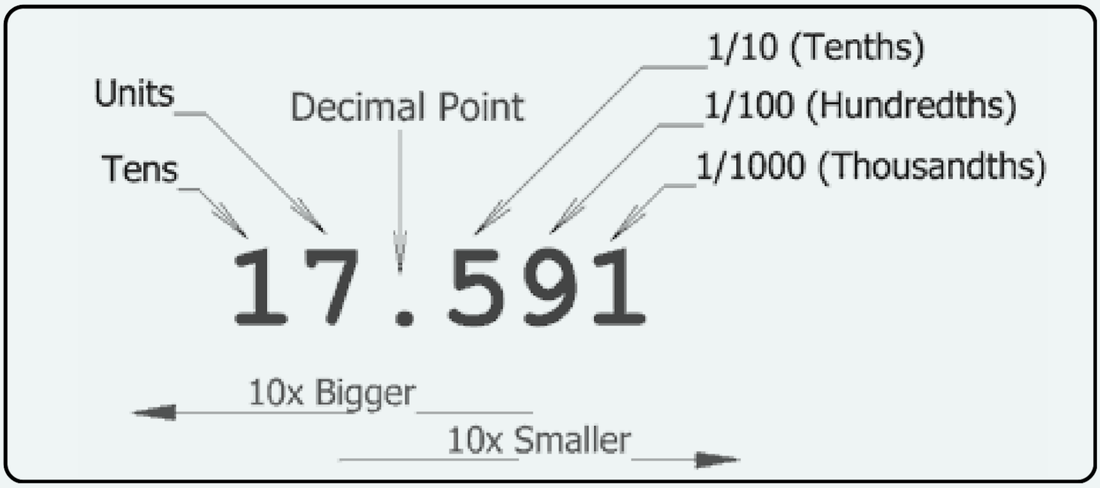
$$8\overline{)64} \quad 1\overline{)3} \quad 4\overline{)16} \quad 7\overline{)7} \quad 8\overline{)48}$$

NUMBERS IN STANDARD FORM

Standard form, also known as scientific notation, is a way of expressing very large or very small numbers in a concise format. In standard form, a number is expressed as the product of a coefficient and a power of 10. The coefficient is a number between 1 and 10, and the power of 10 represents how many places the decimal point must be moved to obtain the original number.

Here's how to express a number in standard form:

- **Identify the coefficient:** The coefficient is a number between 1 and 10.
- **Determine the exponent:** Count the number of places the decimal point must be moved to make the coefficient a number between 1 and 10.
- Write the coefficient multiplied by 10 raised to the exponent.



For example:

- 14,300,000 in the standard form is 1.43×107.
- 3000 in the standard form is 3×10^{3}.
- Some other examples are 1.98×10^{13} and 0.76×10^{13}.

ACTIVITY

Solve the following. Give your answers in standard form:

a. $(5 \times 10^3) + (2 \times 10^3)$

b. $(1.7 \times 10^5) + (3.1 \times 10^5)$

c. $(4.2 \times 10^2) + (2.5 \times 10^3)$

d. $(7.8 \times 10^5) + (1.4 \times 10^4)$

e. $(7.24 \times 10^6) + (3 \times 10^6)$

f. $(8.01 \times 10^8) + (4 \times 10^3)$

g. $(4.27 \times 10^5) + (9.3 \times 10^2)$

h. $(2.9 \times 10^{-5}) + (6 \times 10^{-4})$

i. $(2.01 \times 10^{-2}) + (9.3 \times 10^{-4})$

j. $(4.12 \times 10^{-7}) + (8.2 \times 10^{-5})$

a. $(8 \times 10^6) - (3 \times 10^6)$

b. $(4.81 \times 10^4) - (4.5 \times 10^4)$

c. $(7.51 \times 10^8) - (4.4 \times 10^7)$

d. $(8.32 \times 10^7) - (2.1 \times 10^6)$

e. $(2.7 \times 10^{12}) - (3 \times 10^{10})$

f. $(8.44 \times 10^{-8}) - (3.4 \times 10^{-9})$

g. $(1.6 \times 10^{21}) - (3.3 \times 10^{19})$

h. $(8.132 \times 10^2) - (9.9 \times 10^{-1})$

i. $(8.01 \times 10^{-4}) - (3.3 \times 10^{-3})$

j. $(5.8311 \times 10^8) - (5.42 \times 10^6)$

In a science lab there are three petri dishes. The first dish contains 2.75×10^8 bacteria, the second dish contains 6.12×10^5 bacteria and the third dish contains 8.345×10^6 bacteria.

In standard form, how much bacteria is there in the three petri dishes in total?

MULTIPLICATION AND DIVISION OF NEGATIVE NUMBERS

In order to multiply or divide negative numbers you must remember:

If the **signs are the same**, the answer is **positive**.
If the **signs are different**, the answer is **negative**.

When multiplying negative numbers:

+	x	+	=	+	Same signs, answer is positive
-	x	-	=	+	
+	x	-	=	-	Different signs, answer is negative
-	x	+	=	-	

The same rules apply for dividing negative numbers:

+	÷	+	=	+	Same signs, answer is positive
-	÷	-	=	+	
+	÷	-	=	-	Different signs, answer is negative
-	÷	+	=	-	

Multiplying two negative numbers

When you multiply two negative numbers together, the result is positive.
example: **(−2)×(−3)=6(−2)×(−3)=6**

Multiplying a negative and a positive number

When you multiply a negative number by a positive number, the result is negative.
example: **(−2)×3=−6(−2)×3=−6**

Dividing two negative numbers

When you divide a negative number by another negative number, the result is positive.
example: **(−12)÷(−4)=3**

Dividing a negative number by a positive number

When you divide a negative number by a positive number, the result is negative.
example: **(−12)÷4=−3**

ACTIVITY

1) (+3) x (+7) =	2) (+7) x (-2) =
3) (+6) x (-2) =	4) (+7) x (+6) =
5) (-8) x (+6) =	6) (+7) x (+9) =
7) (+5) x (0) =	8) (-4) x (0) =
9) (-6) x (0) =	10) (0) x (+7) =
11) (+5) x (+4) =	12) (+7) x (+5) =
13) (+4) x (+2) =	14) (0) x (-3) =
15) (-1) x (+5) =	16) (0) x (+3) =
17) (-7) x (+4) =	18) (+4) x (+8) =
19) (0) x (-2) =	20) (+3) x (-5) =

ALGEBRA EQUATIONS

COMPARISON OF NUMBERS AND APPROXIMATE DATA

COMPARISON OF NUMBERS

It is good to know if one number is the same as, smaller than, or bigger than another number:

(=) **Same**
(<) **Less**
(>) **More**

When two values are equal, we use the "equals" sign.
example: **2+2 = 4**

When one value is smaller than another, we can use a "less than" sign.
example: **3 < 5**

When one value is bigger than another, we can use a "greater than" sign
example: **9 > 6**

The "less than (<)" sign and "greater than (>)" sign look like a "V" on its side, don't they?

To remember which way around the "<" and ">" signs go, remember this:

- **BIG > small**
- **small < BIG**

ACTIVITY

> is greater than	< is less than	= is equal to
10 > 7	3 < 6	7 = 7

1) 15 __ 18

2) 20 __ 14

3) 12 __ 12

4) 17 __ 20

5) 24 __ 32

6) 27 __ 16

7) 19 __ 12

8) 11 __ 16

9) 18 __ 18

10) 24 __ 20

11) 31 __ 25

12) 46 __ 38

13) 37 __ 39

14) 41 __ 41

15) 39 __ 43

16) 50 __ 37

17) 26 __ 22

18) 50 __ 80

19) 24 __ 37

20) 70 __ 40

APPROXIMATE DATA

To compare numbers and approximate data, let's first understand what each represents. Numbers are exact values, while approximate data gives us an estimate or a close value. For example, if we have the number 10 and an approximate value of 9.5, we know that 10 is exact, but 9.5 is close to 10, but not exactly 10.

When comparing them, we need to consider the context and the level of precision required. Sometimes, exact numbers are necessary, especially in mathematical equations or when dealing with precise measurements like in physics or engineering. In other situations, approximate data is sufficient, such as when making estimations or predictions in statistics or economics.

Suppose we are comparing the population of two cities: City A and City B.

- City A has an exact population of **1,250,000** people.
- City B has an approximate population of **1.3 million** people.

In this scenario, City A provides an exact number, so we know precisely how many people live there. However, City B gives us **an approximate value, indicating that the population is around 1.3 million, but not an exact count.**

when comparing these populations, I would recognize that City A's population is known precisely, while City B's population is estimated. Depending on the context, both types of data could be useful. If we need to know the exact number of people, City A's population would be more reliable. However, if we're making general comparisons or predictions, City B's approximate population might be sufficient. We would also consider factors such as the margin of error in City B's estimate and the sources of data to make informed decisions.

ACTIVITY

a. 760 + 290 b. 700 + 300 c. 760 + 300 d. 770 + 300

2. Circle the best approximation for 5 045 – 256.

a. 5 000 - 200 b. 5 050 - 260 c. 5 500 - 260 d. 6 000 - 300

3. Circle the best approximation for 38 x 94.

a. 40 x 90 b. 30 x 90 c. 40 x 100 d. 30 x 100

4. Circle the best approximation for 253 ÷37.

a. 250 ÷ 30 b. 260 ÷ 30 c. 250 ÷ 40 d. 260 ÷40

5. Work out approximate answers to the following sums by rounding to the nearest hundred:

a. 6 018 + 2 919 ....

b. 1 354 + 5 732 ....

c. 5 447 + 3 558 ....

d. 2 505 + 209 ....

e. 2 879 + 1 004 ....

6. What is 7 532 to the nearest whole ten? ....

7. What is 7 532 to the nearest whole one hundred? ....

DETERMINATION OF VALUES FOR SIMPLE TRANSACTIONS

Determining the values for simple transactions often involves solving algebraic equations.

Here's an example

Let's say you have \$20 and you want to buy some apples and oranges. Each apple costs \$2, and each orange costs \$1. You want to buy a total of 15 fruits.

\$2

\$1

Let's represent the number of apples you buy as '**X**' and the number of **oranges** as '**Y**'.

We can set up two equations based on the given conditions

- The total cost of the apples and oranges should be equal to \$20
  $2X+1Y=20$
- The total number of fruits should be 15
  $X+Y=15$

Now, you can solve this system of equations to find the values of 'X' and 'Y', which represent the number of apples and oranges you need to buy.

To find the values of 'X' and 'Y', let's solve the system of equations

- $2X+1Y=20$ -- equation 1
- $X+Y=15$ -- equation 2

We can solve equation 2 for 'Y'

- $Y=15-X$

Now, substitute this expression for 'Y' into equation 1

- $2X+1(15-X)=20$

Simplify

- **2X+15-X=20**

Combine like terms

- **X+15=20**

Subtract 15 from both sides

- **X+15-15=20-15**
- **X=5**

Now that we have found the value of '**X**', we can substitute it back into equation 3 to find 'o'

- **X+Y=15**
- **Y=15-X**
- **Y=15-(5)**
- **Y=10**

So, the solution is **X=5** and **Y=10**. This means you should **buy 5 apples and 10 oranges to spend $20 on 15 fruits**.

SOLVE MATHEMATICAL PROBLEMS USING A RANGE OF METHODS

Three distinct approaches for solving mathematical problems

1. **Indices**
2. **Logarithms**
3. **Fractions**

INDICES

Laws of indices provide us with rules for simplifying calculations or expressions involving powers of the same base. They are:

Product Rule: When multiplying powers with the same base, you add the exponents.

$$a^m \times a^n = a^{m+n}$$

Example

- 2^3×2^4=2^(3+4)=27=128

Quotient Rule: When multiplying powers with the same base, you add the exponents.

$$a^m \div a^n = a^{m-n}$$

Example

- 3^5×3^2=3^(5+2)=3^3=27

Power Rule: When raising a power to another power, you multiply the exponents.

$$(a^m)^n = a^{m \times n} = a^{mn}$$

Example

- (4^2)^3=4^2x3=4^6=4096

Negative Exponent Rule: A negative exponent means to take the reciprocal of the base raised to the positive exponent.

$$\boxed{a^{-m} = \frac{1}{a^m}}$$

Example
- 2^-3=1/(2^3)=1/8

Zero Exponent Rule: Any nonzero number raised to the power of zero equals 1.

$$\boxed{a^0 = 1}$$

Example
- 2^0=1

ACTIVITY

1. $7^5 \times 7^2$	**2.** $2^3 \times 2^4$	**3.** $8^5 \times 8^2$	**4.** $9^2 \times 9$
5. $12^4 \times 12^3$	**6.** $10 \times 10^2 \times 10^3$	**7.** $3^5 \times 3^{-2}$	**8.** $5^7 \times 5^{-3}$
9. $8^{-3} \times 8^9$	**10.** $6^{-2} \times 6^{-2}$	**11.** $2^3 \times 2^4 \times 2^{-5}$	**12.** $3^{-1} \times 3^4 \times 3^5$
13. $x^4 \times x^2$	**14.** $x^3 \times x^5$	**15.** $y^7 \times y^1$	**16.** $y^7 \times y$
17. $y^3 \times y^2 \times y$	**18.** $t \times t^2 \times t^3$	**19.** $a^4 \times a^2 \times a^3$	**20.** $b^4 \times b^4 \times b^4$

a) $2(x^3y)^2$ **b)** $(4a^2)^3$ **c)** $(8t^2u^9v^4)^0$

1. Simplify the following expressions <u>without</u> using a calculator. Leave your answers in index form:

a. $5^2 \times 5^4 =$	b. $3^5 \times 3^3 =$	c. $10^8 \times 10^7 =$
d. $2^{12} \times 2^3 =$	e. $a^6 \times a^{-3} =$	f. $f^{21} \times f^{13} =$
g. $6^9 \div 6^4 =$	h. $12^{15} \div 12^2 =$	i. $4^5 \div 4^3 =$
j. $20^8 \div 20^5 =$	k. $m^{15} \div m^8 =$	l. $n^4 \div n^2 =$
m. $(4^2)^5 =$	n. $(8^4)^3 =$	o. $(p^{12})^4 =$
p. $(30^5)^{10} =$	q. $(13^7)^{11} =$	r. $(t^9)^6 =$

2. Simplify the following expressions <u>without</u> using a calculator. Leave your answers in index form:

a. $9^2 \times 9^4 \times 9^3 =$	b. $3^5 \times 3^6 \div 3^2 =$	c. $(4^5)^3 \times 4^{10} =$
d. $(d^7 \times d^2)^6 =$	e. $(y^{12} \div y^4)^5 =$	f. $(k^8 \times k^{21} \div k^7)^2 =$
g. $r^{23} \div (r^3)^5 =$	h. $(s^0 \times s^4)^2 \div s^5 =$	i. $p^{18} \times (p^4 \div p^2)^3 =$
j. $\dfrac{w^9 \times w^2}{w^5 \times w^3} =$	k. $\dfrac{(g^5 \times g^2)^3}{g^4 \times g^{10}} =$	l. $\left[\dfrac{j^4 \times j^8}{j^9 \div j^2}\right]^3 =$

3. <u>Without</u> using a calculator, simplify these expressions to find the missing power:

a. $9^2 \times 3^6 = 3^{\square}$	b. $10\,000 \div 10^2 = 10^{\square}$	c. $25^2 \times 5^3 = 5^{\square}$
d. $64^2 \div 4^3 = 4^{\square}$	e. $144 \times 12^5 = 12^{\square}$	f. $(3^4)^2 \times 27 = 3^{\square}$
g. $4^5 \div 2 = 2^{\square}$	h. $11^5 \times 121^2 = 11^{\square}$	i. $(16^3 \times 2^4) \div 2^3 = 2^{\square}$

LOGARITHMS

Logarithms are mathematical functions that describe the relationship between exponential growth and decay. They're essentially the inverse operation of exponentiation.

For example, if **10^2 = 100** then **log10(100)= 2**.

There are certain rules based on which logarithmic operations can be performed. The names of these rules are:

Product Rule: In this rule, the multiplication of two logarithmic values is equal to the addition of their individual logarithms.

$$\boxed{\log_b (mn) = \log_b m + \log_b n}$$

Example

- log3 (2y) = log3 (2) + log3 (y)

Division Rule: The division of two logarithmic values is equal to the difference of each logarithm.

$$\boxed{\log_b (m/n) = \log_b m - \log_b n}$$

Example

- log3 (2/ y) = log3 (2) -log3 (y)

Exponential Rule: In the exponential rule, the logarithm of m with a rational exponent is equal to the exponent times its logarithm.

$$\boxed{\log_b (m^n) = n \log_b m}$$

Example

- logb(2^3) = 3 logb 2

Change of Base Rule

$$\log_b m = \log_a m / \log_a b$$

Example

- logb 2 = loga 2/loga b

Base Switch Rule

$$\log_b (a) = 1 / \log_a (b)$$

Example

- logb 8 = 1/log8 b

Logarithmic Formulas

$$\log_b(mn) = \log_b(m) + \log_b(n)$$

$$\log_b(m/n) = \log_b (m) - \log_b (n)$$

$$\log_b (xy) = y \log_b(x)$$

$$\log_b m\sqrt{n} = \log_b n/m$$

$$m \log_b(x) + n \log_b(y) = \log_b(x^m y^n)$$

$$\log_b(m+n) = \log_b m + \log_b(1+nm)$$

$$\log_b(m - n) = \log_b m + \log_b (1-n/m)$$

ACTIVITY

1) Evaluate $\log_{10} 6 + \log_{10} 45 - \log_{10} 27$

2) Simplify $\dfrac{\log\sqrt{8}}{\log 8}$

3) Simplify $\dfrac{\log 27^{\frac{1}{3}}}{\log 18}$

4) Evaluate $2\log_3 6 + \log_3 16$

5) Evaluate $\log_{10} 4 + \log_{10} 25$

Expand each logarithm.

$$\log_5\left(\frac{m}{s}\right)^4 =$$

$$\log_2\left(\frac{n^5}{w^4}\right) =$$

$$\log_8\left(a^3 b\right)^2 =$$

$$\log_4\left(\frac{p^3 m}{n^2}\right) =$$

$$\log_9\left(\frac{w^2 t^3}{r s^2}\right) =$$

ACTIVITY

1. Find the domain and range of the function. Use inequality notation.

$$f(x) = -3\log(x+2) - 4$$

2. Find the domain and range of the function. Use interval notation.

$$f(x) = -\log(x-6) - 1$$

3. Find the domain and range of the function. Use interval notation.

$$f(x) = -4\log(x-6) - 6$$

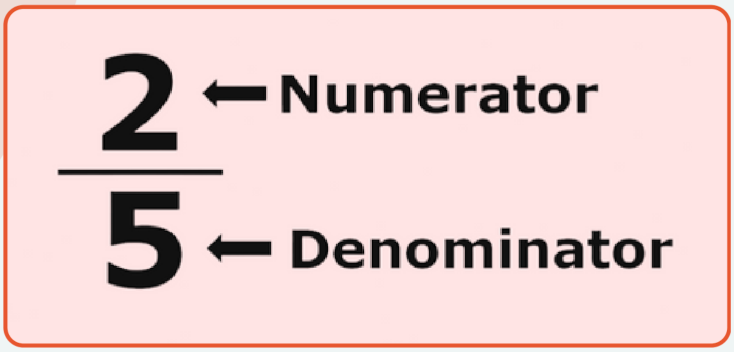
4. Find the domain and range of the function. Use inequality notation.

$$f(x) = -2\log(x-1) + 4$$

Use a calculator to approximate to the nearest hundredth.

1) $\ln 139$
2) $\log_5 44.3$
3) $\log_9 63$
4) $\log_4 23$
5) $\log_6 14.2$
6) $\log_2 12.9$
7) $\log_9 185$
8) $\log_{14} 17.2$
9) $\log_6 12.8$
10) $\log_{12} 108$
11) $\log 5$
12) $\log 4.4$
13) $\log_3 34.2$
14) $\log_8 37.2$
15) $\log_{16} 34.6$
16) $\log_{11} 161$

A fraction is a mathematical expression representing a part of a whole. It consists of **two numbers separated by a line or slash (/)**. The number above the line is called the **numerator**, and the number below the line is called the **denominator**. The numerator represents how many parts of the whole are being considered, while the denominator represents the total number of equal parts into which the whole is divided. Fractions can represent values that are not whole numbers, such as 1/2, 3/4, or 5/8. They are used in various mathematical operations, such as addition, subtraction, multiplication, and division.

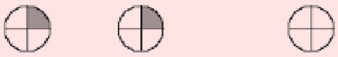


There are six types of fractions.

1. **Proper Fractions**: Fractions where the numerator is smaller than the denominator, such as 1/2 or 3/5.
2. **Improper Fractions**: Fractions where the numerator is equal to or greater than the denominator, such as 5/4 or 7/3.
3. **Mixed Fractions**: Also known as mixed numbers, these are combinations of whole numbers and proper fractions, such as 1 1/2 or 2 3/4.
4. **Equivalent Fractions**: Fractions that represent the same value but are written in different forms. For example, 1/2 and 2/4 are equivalent fractions because they both represent one-half.
5. **Like Fractions**: Fractions with the same denominators, such as 2/5 and 3/5.
6. **Unlike Fractions**: Fractions with different denominators, such as 1/3 and 2/5.

1) 

$\frac{2}{9} + \frac{2}{9} =$ ______

2) 

$\frac{1}{4} + \frac{1}{4} =$ ______

3) 

$\frac{3}{10} + \frac{4}{10} =$ ______

4) 

1 1

What is the Fraction of the Shaded Area ?

1)  ______

6)  ______

2)  ______

7)  ______

3)  ______

8)  ______

4)  ______

9)  ______

5)  ______

10)  ______

Simplify the fractions.

1. $\frac{6}{9} =$ ______
2. $\frac{5}{10} =$ ______
3. $\frac{9}{15} =$ ______
4. $\frac{26}{28} =$ ______
5. $\frac{4}{14} =$ ______

Write the Correct Comparison Symbol (>, < or =) in Each Box

1) $\frac{1}{3} \square \frac{7}{8}$

2) $\frac{1}{6} \square \frac{3}{4}$

3) $\frac{3}{8} \square \frac{2}{4}$

4) $\frac{4}{5} \square \frac{4}{8}$

5) $\frac{1}{4} \square \frac{1}{2}$

6) $\frac{1}{6} \square \frac{4}{5}$

7) $\frac{1}{2} \square \frac{2}{6}$

8) $\frac{3}{4} \square \frac{2}{6}$

9) $\frac{1}{3} \square \frac{1}{3}$

10) $\frac{2}{3} \square \frac{2}{4}$

11) $\frac{1}{8} \square \frac{3}{4}$

12) $\frac{2}{4} \square \frac{1}{3}$

13) $\frac{1}{6} \square \frac{3}{5}$

14) $\frac{5}{8} \square \frac{4}{8}$

15) $\frac{1}{3} \square \frac{2}{3}$

16) $\frac{3}{5} \square \frac{1}{2}$

17) $\frac{2}{5} \square \frac{2}{3}$

18) $\frac{1}{2} \square \frac{5}{6}$

19) $\frac{1}{6} \square \frac{2}{5}$

20) $\frac{1}{8} \square \frac{1}{2}$

BASIC ALGEBRAIC MANIPULATION

EXPLORING SIMPLE LINEAR EQUATIONS

Basic algebra involves using such equations to solve for unknown variables. This often involves manipulating equations using properties such as the distributive property, combining like terms, and isolating variables. Algebra allows us to solve problems involving unknown quantities and to represent relationships between quantities using symbols and equations.

A simple linear equation is an algebraic equation that represents a straight line when graphed on a coordinate plane. It has the general form:

- **ax+b=0**

Here, **a** and **b** are constants, and **x** is the variable. The variable **x** is raised to the power of 1, which means it is not squared or cubed, just raised to the first power.

The solution to a simple linear equation is the value of **x** that makes the equation true. To find the solution, you usually isolate **x** on one side of the equation by performing operations to both sides that maintain equality. For example, you might add or subtract the same value from both sides, or multiply or divide both sides by the same value.

For instance, if you have the equation **3x+2=8**, you can isolate **x** by subtracting 2 from both sides to get **3x=6**, and then dividing both sides by 3 to find that **x=2**.

- **3x+2=8**
- **3x+2-2=8-2**
- **3x=6**
- **3x/3=6/3**
- **x=2**

ACTIVITY

Solve the following equations

1) $y+4=9$

2) $k+5=7$

3) $m-7=10$

4) $v-20=1$

5) $2x=6$

6) $3z=24$

7) $8g=64$

8) $\frac{h}{2}=7$

Solve the following equations:

$2m+1=\frac{4}{3}$

$\frac{2}{3}m+3=\frac{4}{3}$

$\frac{2}{3}m+3=\frac{4}{9}$

$\frac{2}{3}m+3=\frac{4}{7}$

$\frac{2v-1}{3}=-7$

$\frac{2v-1}{3}=-\frac{7}{4}$

$\frac{19}{r}=7$

$\frac{19}{2r-1}=7$

$\frac{5}{9}(t+2)=\frac{3}{2}$

CHARTS AND DIAGRAMS

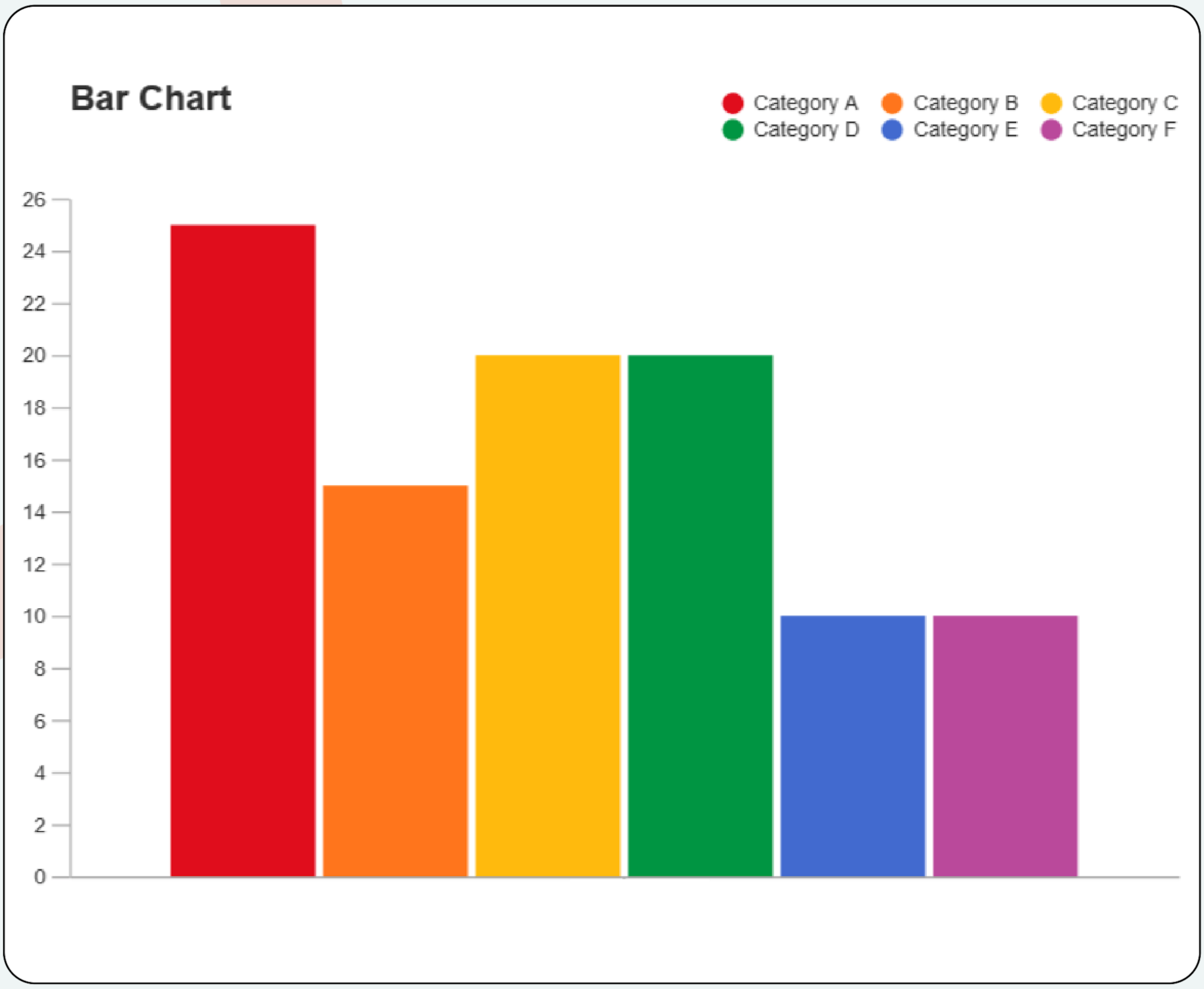
CREATING CHARTS FROM TABULAR DATA

Graphs and charts streamline the communication of complex information, making your meetings and presentations more effective and clear. Incorporating them enhances the impact of your message. Various formats exist for graphs and charts, serving as graphical representations of statistical data.

There are four primary types of charts.

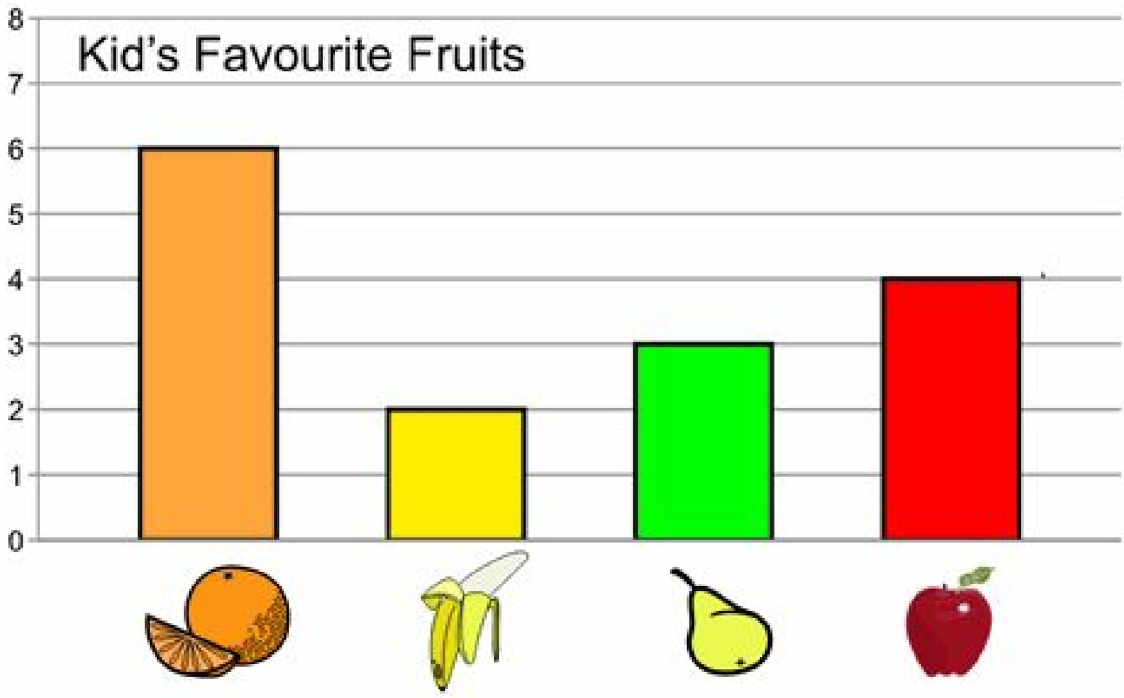
1. **Bar Charts**
2. **Line Charts**
3. **Histogram Charts**
4. **Pie Charts**

Displays data using rectangular bars with lengths proportional to the values they represent. It's effective for comparing values across categories.



ACTIVITY

Study the bar graph and answer the questions.



How many kids liked apples ? ____________

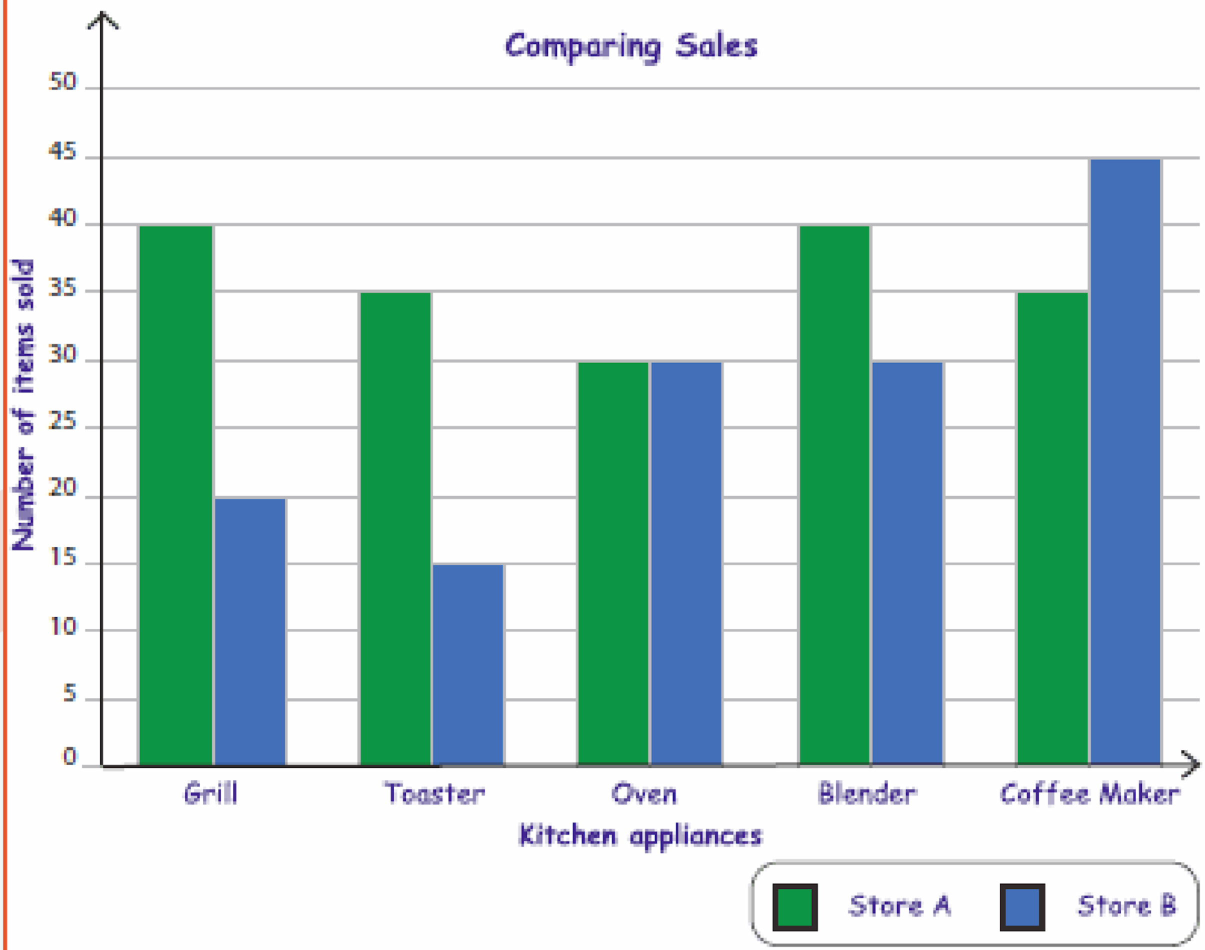
Which fruit did the most kids like ? ____________

Which fruit did the kids like the least ? ____________

How many kids liked bananas? ____________

ACTIVITY

Mr. Peter owns two kitchen appliance stores. He compares the sales of two stores and recorded the information in a bar graph. Use the bar graph to answer the questions.



1) Which item sold the most in Store B? __________

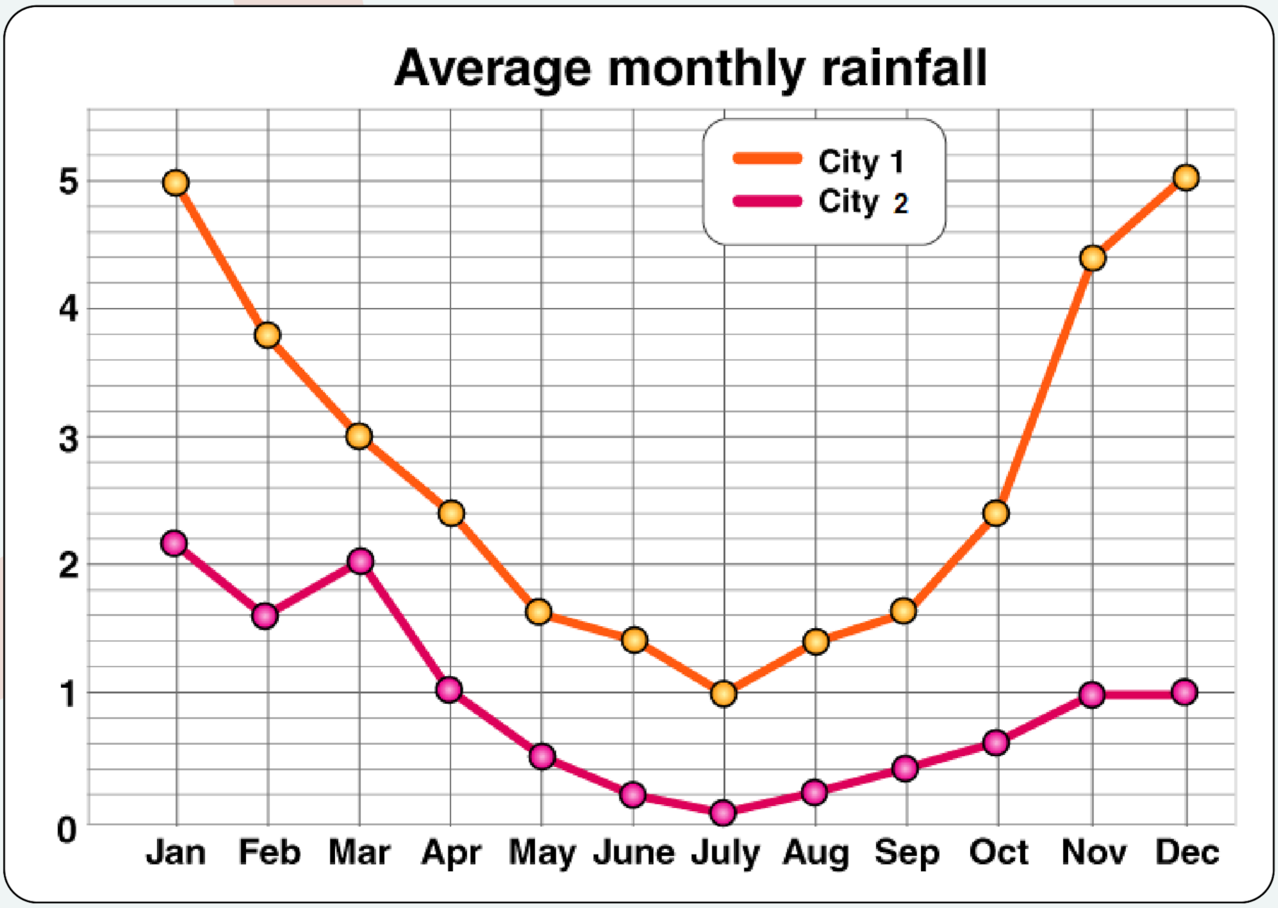
2) Which store sold the least number of toasters? __________

3) The number of grills sold by Store A is twice of Store B. Is it correct? __________

4) What is the difference on sales of blenders between Store A and Store B? __________

5) How many total appliances were sold by Store A? __________

Used to display data points over time. It's great for showing trends and patterns, especially when there are continuous data points.

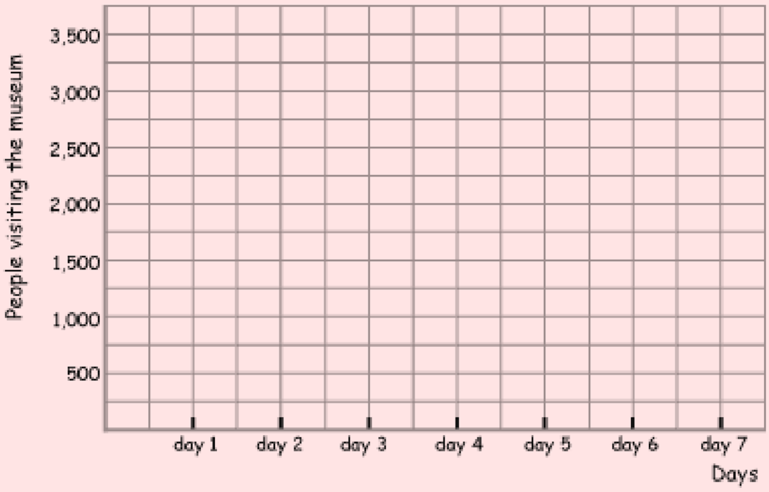


ACTIVITY

Use the data tables to complete the line graphs, including the labels and scales. Round the data in the tables off to the nearest hundred first.

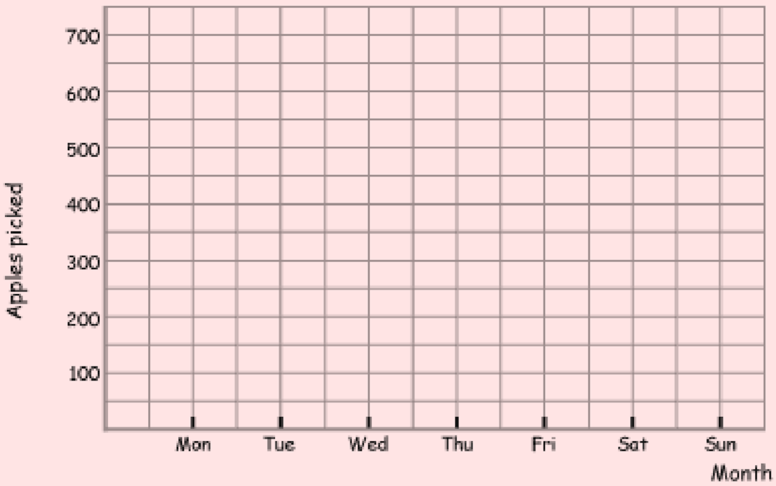
1) People visiting the museum per day

day 1	day 2	day 3	day 4	day 5	day 6	day 7
1,000	500	2,000	1,500	3,000	1,000	2,500



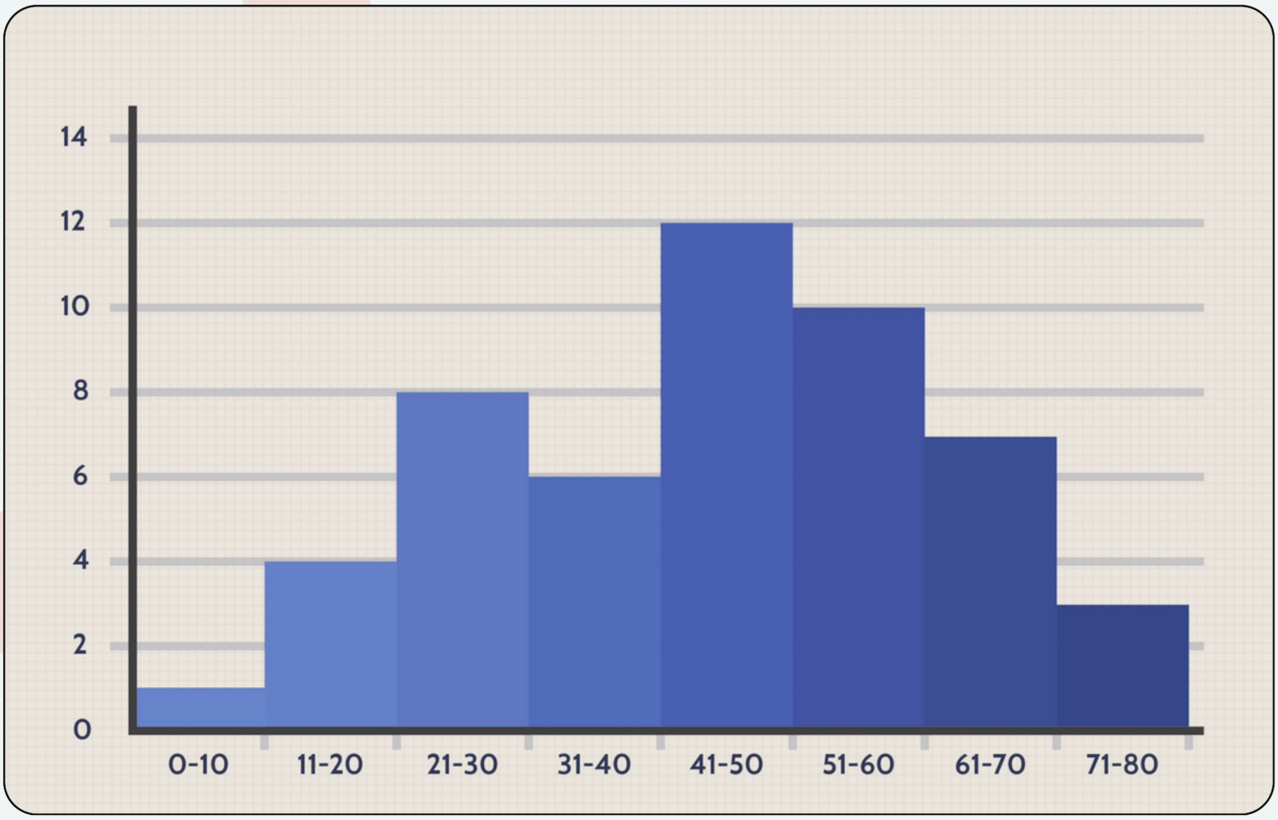
2) Apples picked per day

200	250	600	300	450	350	650
Mon	Tue	Wed	Thu	Fri	Sat	Sun



HISTOGRAM CHARTS

Similar to a bar chart but used for frequency distributions. It displays the distribution of numerical data by dividing it into bins and showing the number of observations in each bin.

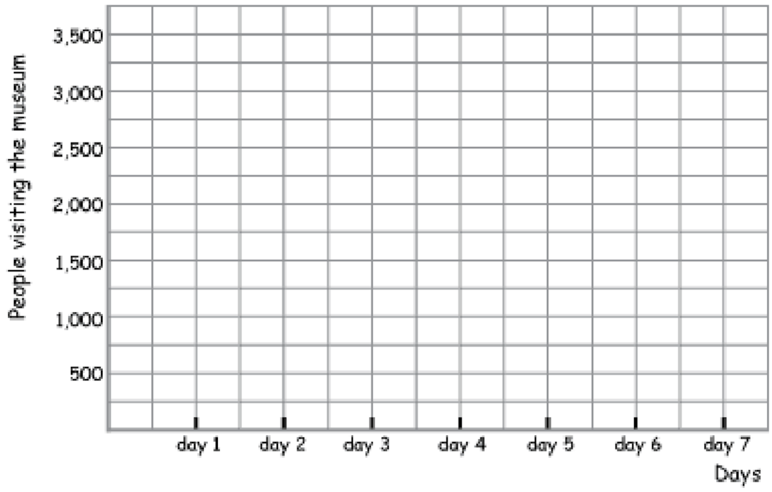


ACTIVITY

Use the data tables to complete the line graphs, including the labels and scales.
Round the data in the tables off to the nearest hundred first.

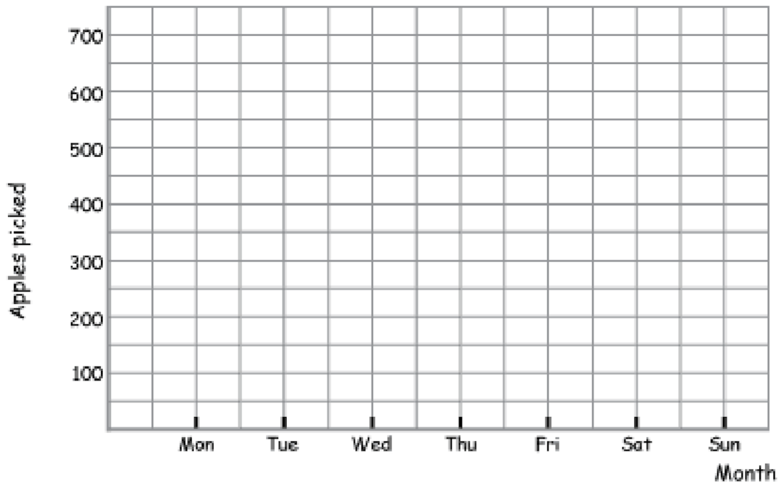
1) People visiting the museum per day

day 1	day 2	day 3	day 4	day 5	day 6	day 7
1,000	500	2,000	1,500	3,000	1,000	2,500

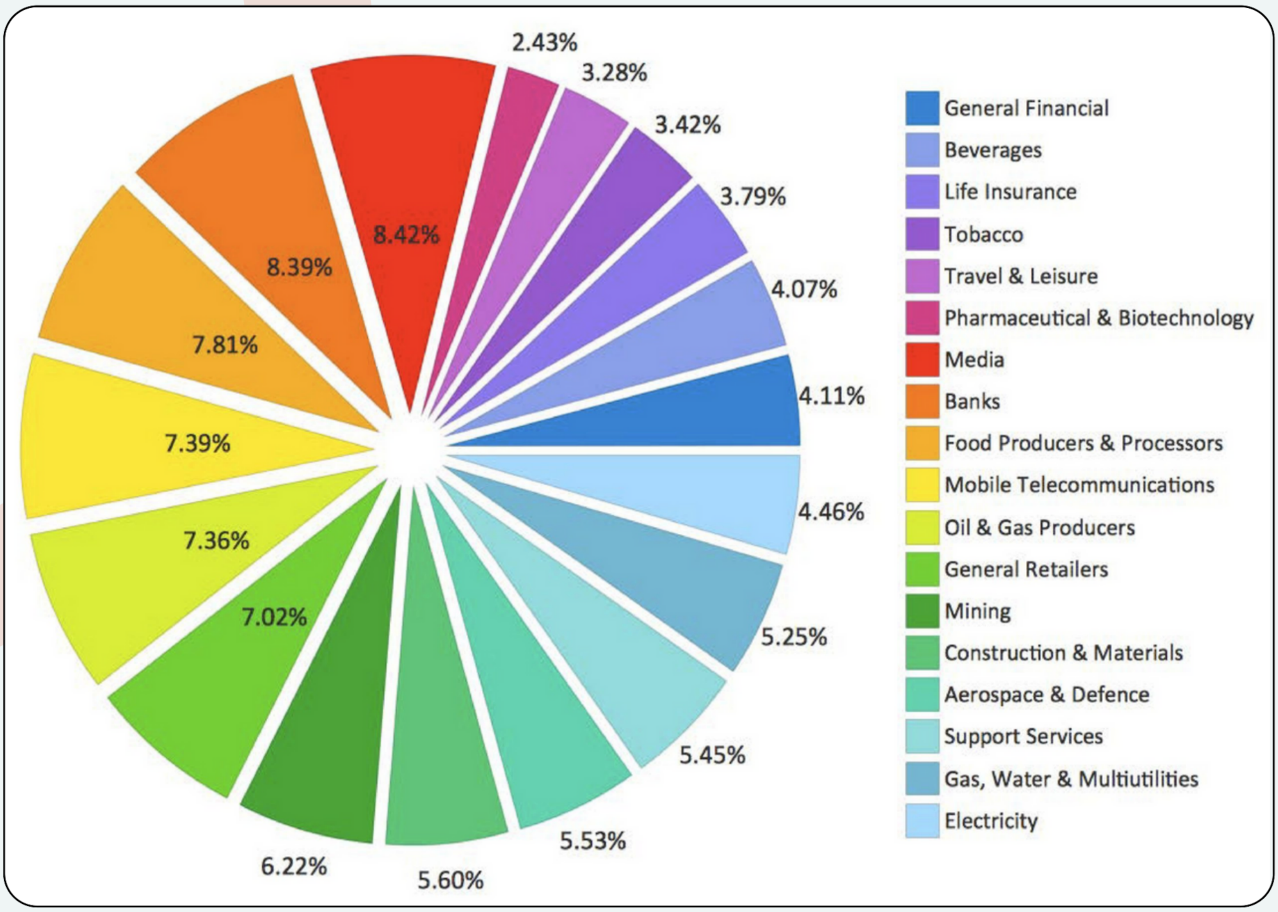


2) Apples picked per day

200	250	600	300	450	350	650
Mon	Tue	Wed	Thu	Fri	Sat	Sun

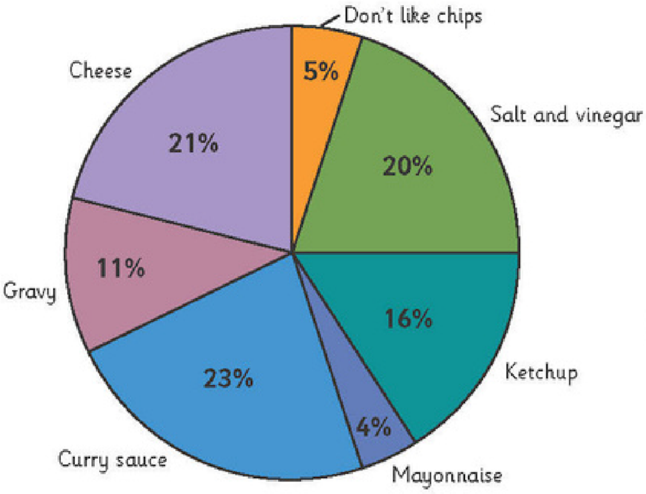


Represents data as slices of a circular pie. It's useful for showing proportions or percentages of a whole, but can be less effective when comparing many categories or when precise values are needed.



ACTIVITY

A takeaway shop surveyed **200** of its customers to ask what was their favourite topping to go with chips. The results are presented below in a **pie chart**.



1. Complete the table:

Topping	Don't like chips	Salt and vinegar	Ketchup	Mayonnaise	Curry sauce	Gravy	Cheese
Percentage	5%						
Number of Customers	10						

2. How many customers chose either **cheese** or **ketchup**?

....

3. How many more customers chose **curry sauce** than **salt and vinegar**?

....

4. Do more customers prefer **curry sauce** and **gravy**, or **salt and vinegar** and **ketchup**?

....

5. The manager decided that the customers who **don't like chips** should choose their favourite topping anyway. 4 people chose **cheese** and 6 chose **ketchup**. What percentage now prefer **ketchup**?

....

GRAPH PLOTTING AND CONSTRUCTION PRINCIPLES

LINEAR GRAPHS

Linear graphs represent relationships between two variables where one variable, typically denoted as **x**, changes at a constant rate, causing the other variable, typically denoted as **y**, to change in a proportional manner. The general equation for a linear relationship is:

$$y=mx+c$$

Where:

- ***y*** is the dependent variable (often called the "response" variable).
- ***x*** is the independent variable (often called the "predictor" variable).
- **m** is the slope of the line, representing the rate of change of **y** with respect to **x**.
- **c** is the y-intercept, which is the value of **y** when **x=0**

Linear graphs can take various forms depending on the values of **m** and **c**:

1. **Positive Slope**: If ***m*>0**, the line slopes upward from left to right, indicating that as ***x* increases**, ***y* increases**.
2. **Negative Slope**: If ***m*<0**, the line slopes downward from left to right, indicating that as ***x* increases**, ***y* decreases**.
3. **Zero Slope**: If ***m*=0**, the line is horizontal, indicating that ***y* remains constant regardless of changes in *x***.
4. **Undefined Slope**: If ***x*** is constant (vertical line), the slope is undefined because division by **zero is not defined**. This type of graph is not a function.

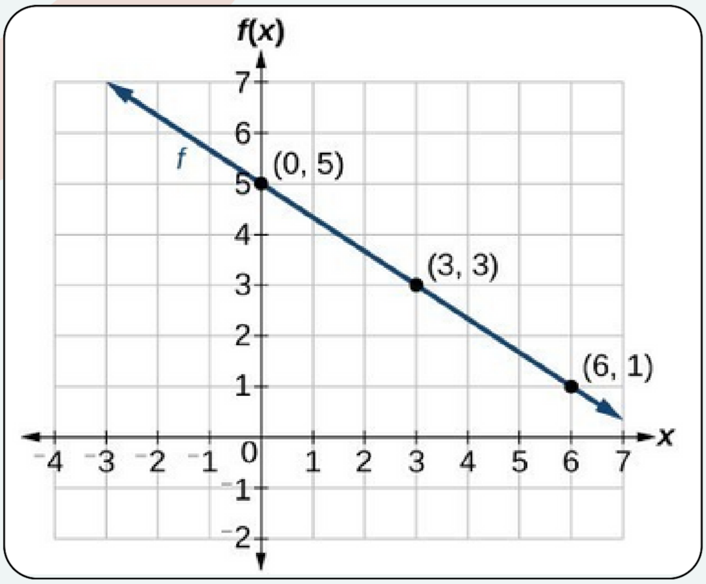
Example: Graphing by Plotting Points

Graph f(x)=−2/3x+5 by plotting points.

Begin by choosing input values. This function includes a fraction with a denominator of 3, so let’s choose multiples of 3 as input values. We will choose 0, 3, and 6. Evaluate the function at each input value, and use the output value to identify coordinate pairs.

x=0 f(0)=(−2/3)(0)+5 =5 ⇒(0,5)
x=3 f(3)=(−2/3)(3)+5 =3 ⇒(3,3)
x=6 f(6)=(−2/3)(6)+5 =1 ⇒(6,1)

Plot the coordinate pairs and draw a line through the points. The graph below is of the function

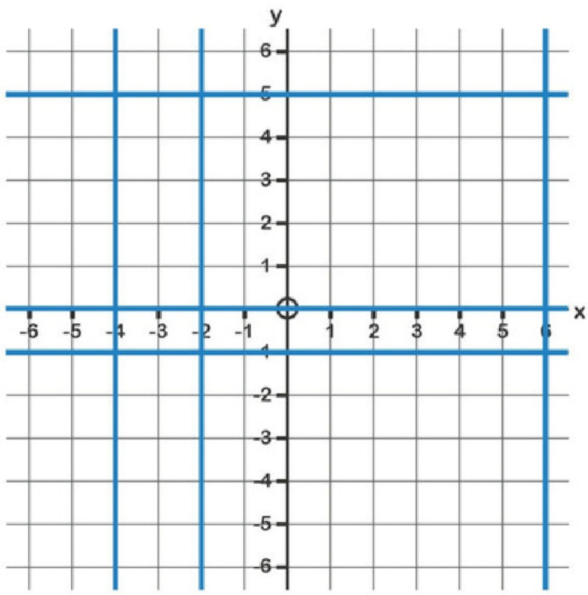


Analysis of the Solution

The graph of the function is a line as expected for a linear function. In addition, the graph has a downward slant, which indicates a negative slope. This is also expected from the negative constant rate of change in the equation for the function.

ACTIVITY

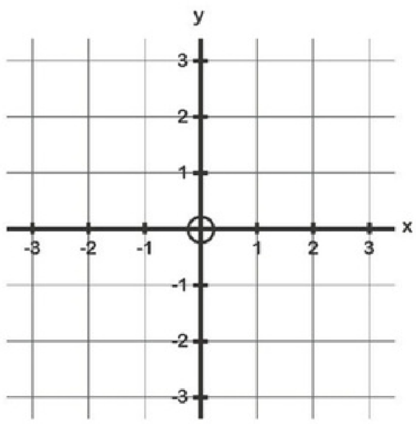
Section A *Label each straight line with its equation.*

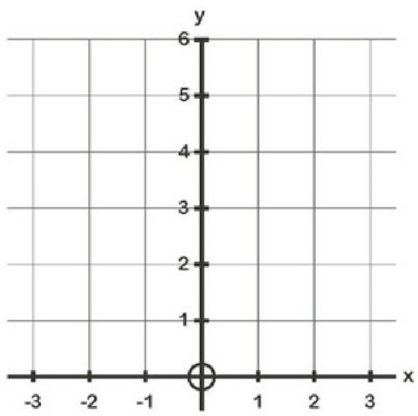


Section B *Plot the straight lines using the tables of values.*

x	-2	-1	0	1	2	3
y = x – 1						

x	-2	-1	0	1	2	3
y = x + 3						



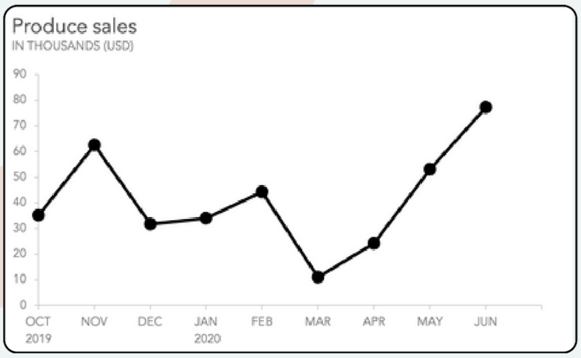


ANALYZE DATA VISUALS

Graphs, charts, and tables are powerful tools for visualizing data and understanding patterns. Here's a breakdown of each and how to interpret them:

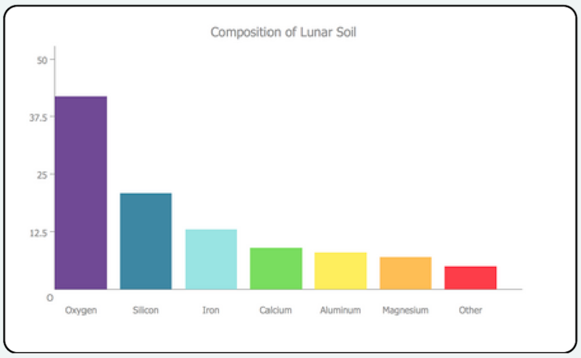
1. Line Graphs

- Plot data points on a Cartesian plane with a line connecting them.
- Typically used to show trends over time or continuous data.
- Interpretation: Look for trends, patterns, and changes over time. Rising lines indicate increasing values, while falling lines indicate decreasing values.



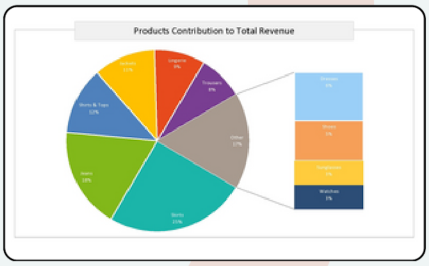
2. Bar Charts

- Represent data using rectangular bars where the length or height of the bar corresponds to the value being represented.
- Useful for comparing discrete categories.
- Interpretation: Compare the lengths or heights of bars to understand relative sizes or quantities. Longer bars represent larger values.



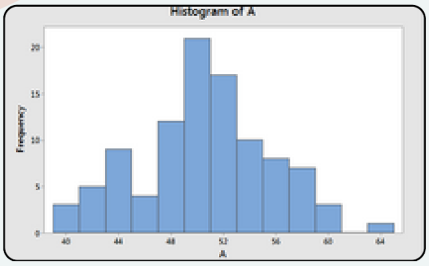
3. Pie Charts

- Represent data as slices of a circle, with each slice representing a proportion of the whole.
- Useful for showing parts of a whole.
- Interpretation: Look at the size of each slice relative to the whole pie to understand the proportion of each category.



4. Histograms

- Similar to bar charts, but used for representing the distribution of continuous data.
- Bars touch each other to indicate that the data is continuous.
- Interpretation: Examine the distribution of data across different intervals. Histograms show frequency or density of values within each interval.



5. Tables

- Organize data into rows and columns.
- Can display raw data or summary statistics.
- Interpretation: Look for specific values, trends, or patterns within the data. Compare values across rows or columns to draw conclusions.

STATISTICS

DIFFERENT TYPES OF DATA

Data can be broadly categorized into several types based on various characteristics such as format, structure, and intended use. Here are some common types of data

- **Nominal Data:** Nominal data is also a type of categorical data, but the categories have no natural order or ranking. They are purely labels used to classify data into mutually exclusive groups. Examples include colors, shapes, types of fruit, and customer IDs.

- **Ordinal Data:** Ordinal data is a type of categorical data where the categories have a natural order or ranking. The intervals between the categories may not be uniform or measurable. Examples include ratings (e.g., on a scale of 1 to 5), educational levels (e.g., elementary, middle, high school), and socioeconomic status (e.g., low, middle, high).

- **Categorical Data:** Categorical data represents categories or groups and can take on values that are distinct and separate. It is often represented by labels or names and is used to organize data into groups. Examples include gender, ethnicity, type of vehicle, and marital status.

- **Qualitative Data:** This type of data describes qualities or characteristics and is often non-numeric. It deals with descriptions, observations, and subjective attributes. Examples include colors, textures, tastes, and opinions.

- **Quantitative Data**: Quantitative data consists of numerical measurements or counts and is typically expressed in terms of numbers. It deals with quantities, amounts, or sizes and can be further categorized as discrete or continuous.
  - **Discrete Data**: Discrete data can only take specific values and usually represents counts or whole numbers. Examples include the number of students in a class or the number of cars in a parking lot.
  - **Continuous Data**: Continuous data can take any value within a range and can be measured to any level of precision. Examples include height, weight, temperature, and time.

- **Time-Series Data**: Time-series data consists of observations or measurements taken at multiple points in time and is typically arranged chronologically. It is often used to analyze trends and patterns over time, such as stock prices, weather data, and sales figures.

- **Spatial Data**: Spatial data refers to data that represents the physical location and characteristics of geographic features, such as maps, GPS coordinates, and satellite images. It is used in various fields like geography, urban planning, and environmental science.

UNDERSTANDING SUMMARY STATISTICS

Summary statistics are numerical values that summarize the main features of a dataset. The most common summary statistics include measures of central tendency (mean, median, mode) and measures of dispersion (range, variance, standard deviation).

UNGROUPED DATA

Let's say we have a dataset of exam scores:

“85,90,92,88,78,95,85,92,88,90"

Here are some summary statistics we can calculate:

- **Mean:** The mean is the average of all the values. It's calculated by summing up all the values and dividing by the total number of values

  Mean = (85+90+92+88+78+95+85+92+88+90)/10=88.3

  On average, the exam scores in the dataset are 88.3.

- **Median:** The median is the middle value when the data is arranged in ascending order. If there's an even number of observations, it's the average of the two middle values.

  To get median place in dataset

  Median = “(78,85,85,88,88,90,90,92,92,95"observations count +1)/2
  = (10 + 1) / 2
  = 11 /2
  = 5.5th place

  - The median will be the average of the 5th and 6th values.

  Median = (88+90)/2 = 178/2 = 89

  50% of the exam scores are below 89, and 50% are above 89.

- **Mode:** The mode is the value that appears most frequently in the dataset.

  Mode = 88, 90, and 92 (since each appears twice)

  The most common exam scores are 88, 90, and 92.

- **Range:** The range is the difference between the highest and lowest values in the dataset.

  Range = 95 - 78 = 17

  The spread of exam scores ranges from 78 to 95.

- **Interquartile Range(IQR):** we first need to find the first quartile (Q1) and the third quartile (Q3).

  Q1 is the median of the lower half of the data, and Q3 is the median of the upper half of the data.

  “78,85,85,88,88,90,90,92,92,95"

  Q1 = median of the **lower half** = **median of {78,85,85,88,88}**
  Q1 = **(85+88)/2=86.5**

  Q3 = median of the **upper half** = **median of {90,90,92,92,95}**
  Q3 = (90+92)/2=91

  Now, we can calculate the Interquartile Range (IQR) as the difference between **Q3** and **Q1**:

  IQR=Q3−Q1=91−86.5=4.5

  So, the Interquartile Range (IQR) for the given dataset is 4.5.

  The middle 50% of the exam scores fall within a range of 4.5 points.

ACTIVITY

1) 78 , 66 , 64 , 72 , 81 , 77 , 65 , 66

2) 73 , 56 , 63 , 55 , 57 , 46 , 63

3) 34 , 47 , 42 , 50 , 48 , 34 , 36 , 46

4) 85 , 81 , 78 , 78 , 78

5) 61 , 47 , 55 , 57 , 58 , 47 , 59 , 63

Answers

	Mean	Median	Mode
1			
2			
3			
4			
5			

ACTIVITY

Find the mean, median, mode and range in each of the sets of data.

1)	{31, 27, 19, 22, 21, 18, 19, 25, 29, 34, 30}		
order	{18, 19, 19, 21, 22, 25, 27, 29, 30, 31, 34}		
Mean ________	Median ________	Mode ________	Range ________
2)	{8, 14, 7, 15, 14, 11, 10, 9, 19, 11, 14}		
order			
Mean ________	Median ________	Mode ________	Range ________
3)	{106, 112, 98, 102, 112, 95, 106, 101, 98, 103, 117, 98}		
order			
Mean ________	Median ________	Mode ________	Range ________
4)	{142, 353, 271, 396, 217, 92, 198, 271, 313, 502, 424}		
order			
Mean ________	Median ________	Mode ________	Range ________
5)	{96, 103, 106, 98, 95, 97, 101, 105, 103, 98, 101, 95, 101, 117, 99}		
order			
Mean ________	Median ________	Mode ________	Range ________
6)	{12, 22, 8, 4, 11, 9, 15, 9, 11, 10, 8, 12, 11, 18, 8, 10, 12, 8}		
order			
Mean ________	Median ________	Mode ________	Range ________

- **Variance:** we need to find the mean of the dataset first, then compute the squared differences between each data point and the mean, and finally average those squared differences.

  We already calculated the mean to be 88.3. Now, let's compute the variance

  Variance = 〖Σf (m - X̄)〗 ^2/(n-1)

  = [(85−88.3)^2 + (90−88.3)^2+ (92−88.3)^2 + (88−88.3)^2 + (78−88.3)^2 + (95−88.3)^2 + (85−88.3)^2 + (92−88.3)^2 + (88−88.3)^2 + (90−88.3)^2] / 10
  = [(−3.3)^2+(1.7)^2+(3.7)^2+(−0.3)^2+(−10.3)^2+(6.7)^2+(−3.3)^2+(3.7)^2+(−0.3)^2+(1.7)^2] / 10
  = [10.89 + 2.89 + 13.69 + 0.09 + 106.09 + 44.89 + 10.89 + 13.69 + 0.09 + 2.89] / 10
  = 205.01 / 10
  = 20.501

  So, the variance for the given dataset is approximately 20.501

  The variance measures how much the exam scores deviate from their mean, with higher variance indicating greater variability around the mean. In this case, the variance is 20.501, suggesting a moderate level of variability in the exam scores.

- **Standard Deviation:** we take the square root of the variance. Given that we've already calculated the variance as approximately 20.501, we can now find the standard deviation.

  Standard Deviation = √(〖Σ (X -μ)〗 ^2/N)
  = √20.501
  = 4.53

  So, the standard deviation for the given dataset is approximately 4.53.

  The standard deviation measures the average deviation of data points from the mean. In this case, a standard deviation of approximately 4.53 indicates that, on average, exam scores deviate from the mean of 88.3 by approximately 4.53 points. This suggests the spread or variability in the exam scores.

ACTIVITY

An artefact is being manufactured in quantity on a machine tool. To check the length of the artefacts, 100 of them are measured and the following results were obtained:

Length (mm)	22.36	22.37	22.38	22.39	22.40	22.41	22.42	22.43
Frequency	1	4	9	24	30	26	5	1

Calculate for the artefact:

a) the mean

b) the variance

c) the standard deviation

Given the scores of the students: 7, 11, 8, 8, 19, 15, 7, 9, 9, 20, 17, 14.

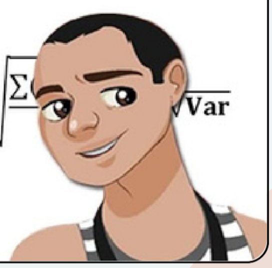
Find the Mean(μ), Population Variance(σ^2) and Standard Deviation(σ).

x	μ	x − μ	$(x-\mu)^2$
7			
11			
8			
8			
19			
15			
7			
9			
9			
20			
17			
14			

Mean =

Variance = $\frac{\Sigma(x-\mu)^2}{N}$

Standard Deviation = $\sqrt{\Sigma(}$ $\sqrt{Var}$



GROUPED DATA

The following data represents the survey regarding the heights (in cm) of 51 girls of Class x

Heigh(in cm)	Number of Girls
Less than 140 but higher than 135	4
Less than 145	11
Less than 150	29
Less than 155	40
Less than 160	46
Less than 165	51

first, we need to find the class intervals and their corresponding frequencies.

- 135 - 140 is the first Class Interval
- 4 girls in that class is the Frequency
- Midpoint is the middle value of each class interval.

class interval	Mid point	Frequency	
(x)	(m)	(f)	fm
135 - 140	137.5	4	550
140 - 145	142.5	7	997.5
145 - 150	147.5	18	2655
150 - 155	152.5	11	1677.5
155 - 160	157.5	6	945
160 - 165	162.5	5	812.5

- **Mean:** The mean (average) for group data can be calculated using the midpoint of each class interval and the frequency of each interval.

  The formula for calculating the mean for group data is:

  Mean = Σ(Midpoint×Frequency) / ΣFrequency

  Where
  - Midpoint is the middle value of each class interval.
  - Frequency is the number of observations in each class interval.

  For our example
  - Midpoint for each class interval
    - **Midpoint=Lower Limit+Upper Limit/2**
  - Frequency is given in the table.

  So, let's calculate the mean:

  Mean = (137.5 x 4 + 142.5 x 7 + 147.5 x 18 + 152.5 x 11 + 157.5 x 6 + 162.5 x 5) / 4 + 7 + 18 + 11 + 6 + 5
  = 7637.5 / 51
  = 149.75
  = 145 - 150 (mean class)

- **Median :** To find the median for grouped data, we first need to determine the median class interval, which is the class interval that contains the median value.

  We start by calculating the cumulative frequency. Then, we identify the class interval where the cumulative frequency exceeds half of the total frequency.

class interval	Frequency	cumulative frequency
(x)	(f)	
135 - 140	4	4
140 - 145	7	11
145 - 150	18	29
150 - 155	11	40
155 - 160	6	46
160 - 165	5	51

The median class interval is where the cumulative frequency first exceeds 51/2 = 26.5 , which is the third class interval (145 - 150).

Now, we can use the formula for the median for grouped data:

$$M = L_1 + \left(\frac{\left(\frac{n}{2}\right) - c}{f}\right) \times w$$

where

- ***L*** is the lower boundary of the median class interval (145 in our case).
- **n** is the total frequency (51 in our case).
- **c** is the cumulative frequency of the class before the median class interval (11 in our case)
- ***f*** is the frequency of the median class interval (18 in our case).
- **w** is the width of the median class interval (5 in our case).

So, the median of the given group data is approximately **149.02**

Median = L_1+(((n/2)-c)/f)×w
Median = 145+(((51/2)-11)/18)×5
Median = 149.027777778

So, the median of the given group data is approximately **149.02**

- **Mode:** To find the mode for grouped data, we simply identify the class interval with the highest frequency.

  The mode is the class interval "145 - 150" because it has the highest frequency of 18.
  So, the mode of the given group data is the class interval "145 - 150".

- **Range:** To find the range for the given group data, we simply subtract the lowest value from the highest value.

  For the class intervals "135 - 140" to "160 - 165", the lowest value is 135 and the highest value is 165.

  Range = Highest Value−Lowest Value
  Range = 165−135
  Range = 30

- **Interquartile Range (IQR):** To find the Interquartile Range (IQR) for the given group data, we first need to determine the quartiles.

  Identify the quartiles:
  - **Q1**: First quartile, which is the 25th percentile.
  - **Q2**: Second quartile, which is the 50th percentile (median).
  - **Q3:** Third quartile, which is the 75th percentile.

  For the 25th percentile (**Q1**), we find the class interval where the cumulative frequency just exceeds. **n/4 = 51/4 = 12.75**, which is the third class interval **(145 - 150)**.

  For the 50th percentile (**Q2**), we already calculated it earlier, which is **149.02**, which is the third class interval **(145 - 150)**.

  For the 75th percentile (**Q3**), we find the class interval where the cumulative frequency just exceeds. 3**n/4 = (3x51)/4 = 38.25**, which is the third class interval **(150 - 155)**.

Now, we can use interpolation to find the exact values of **Q1** and **Q3**.

$$Q_1 = L_1 + \frac{(\frac{n}{4} - c)}{f} \times w$$

$$Q_3 = L_1 + \frac{(\frac{3n}{4} - c)}{f} \times w$$

Where

- **L** = real lower limit of the quartile class
- **n** = total number of observations in the entire data set
- **c** = cumulative frequency in the class immediately before the quartile class
- **f** = frequency of the relevant quartile class
- **w** = the length of the real class interval of the relevant quartile class

For **Q1**

- **Q1 = L_1+ ((n/4 -c))/f ×w**
- **Q1 = 145 + ((51/4 -18))/18 ×5**
- **Q1 = 143.54**

For **Q3**

- **Q1 = L_3+ ((3n/4 -c))/f ×w**
- **Q1 = 150 + (((3*51)/4 -29))/11 ×5**
- **Q1 = 154.2**

IQR = Q3 - Q1
IQR = 154.2 - 143.54
IQR = 10.66

So, the interquartile range (IQR) for the given group data is approximately **10.66**.

ACTIVITY

Give answers to 2 decimal places where necessary.

Section A *Work out the modal class, median class and an estimate of the mean for the following data.*

Weight (kg)	Frequency
40 - 49	13
50 - 59	28
60 - 69	37
70 - 79	44
80 - 89	21

Estimate of the mean:
Modal class:
Median class:

Distance (d km)	Frequency
$0 \le d < 15$	2
$15 \le d < 30$	7
$30 \le d < 45$	19
$45 \le d < 60$	30
$60 \le d < 75$	1

Estimate of the mean:
Modal class:
Median class:

Section B

The daily rainfall, in millimetres, was recorded over a one month period and the results are shown in the frequency table.

Rainfall (mm)	Frequency
0 - 10	2
10 - 20	6
20 - 30	7
30 - 40	11
40 - 50	5

1) Estimate the amount of rainfall there was over the one month period.

2) As a percentage, how much of the month had more than 30 mm of rainfall?

3) Which is the modal class?

4) In which class is the median value?

5) Work out an estimate of the mean.

6) During the following month there were two days of heavy rainfall. Which average would you choose to represent the amount of rainfall in the following month?

ACTIVITY

1) The table gives the number of words in each sentence of a page of writing.

Number of words	Frequency	Mid point	
1 – 5	6		
6 – 10	5		
11 – 15	4		
16 – 20	2		
21 – 25	3		
Totals			

Estimate for the mean =

Modal group =

2) The results of 24 students in a test are given below.

Mark	Frequency	Mid point	
40 – 54	5		
55 – 69	8		
70 – 84	7		
85 – 99	4		
Totals			

Estimate for the mean =

Modal group =

3) A survey was made to see how quickly the AA attended calls which were not on a motorway. The following table summarises the results:

Time (mins)	Frequency	Mid point	
0 – 15	2		
16 – 30	23		
31 – 45	48		
46 – 60	31		
61 – 75	27		
76 – 90	18		
91 – 105	11		
Totals			

Estimate for the mean =

Modal group =

4) The number of letters delivered to 26 houses in a street was as follows:

Grouping Data - Worksheet

1. An airline company records the weights of the bags that passengers check in (in kg to 1 dp).

7.3	8.2	9.1	9.0	6.2	6.1
7.0	8.4	4.9	5.4	6.4	7.9
10.8	11.2	13.4	11.2	7.8	4.3
7.9	12.4	11.6	9.4	9.0	6.7
10.5	9.5	4.7	5.6	7.1	8.2

Weight (kg)	Tally	Frequency
$4 \le w < 6$		
$6 \le w < 8$		
$8 \le w < 10$		
$10 \le w < 12$		
$12 \le w < 14$		

A) Copy and complete the grouped frequency table.
B) What is the modal class?

2. Mr Jackson times the students in class 7C doing a Maths puzzle (in seconds to 1dp).

22.5	12.3	6.3	9.9	29.0	22.2	37.0	17.2	17.2	28.4
37.0	29.7	4.1	28.8	27.0	23.8	14.8	10.0	20.0	33.8
30.1	32.2	20.1	21.6	10.5	23.7	36.7	25.5	18.8	25.6

A) Copy and complete the grouped frequency table.
B) What is the modal class?
C) The cut off point for Bronze medal was 30 seconds. How many students got a medal?

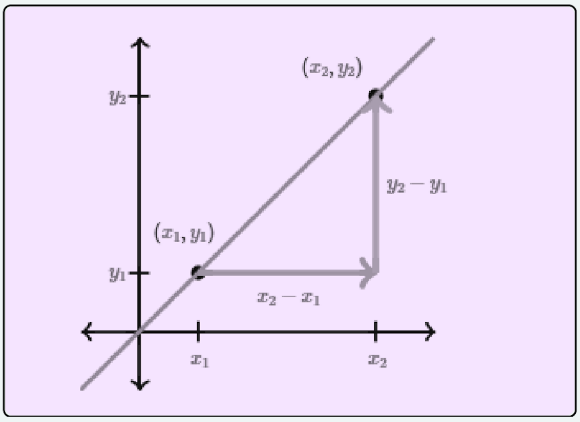
Time (s)	Tally	Frequency
$0 \le t < 10$		
$10 \le t < 20$		

ELEMENTARY FUNCTION DIFFERENTIATION AND INTEGRATION

Differentiation and integration are fundamental concepts in calculus, a branch of mathematics concerned with rates of change and accumulation.

DIFFERENTIATION

$$\text{Slope} = \frac{\text{Change in Y}}{\text{Change in X}} = \frac{y2 - y1}{x2 - x1}$$



- Differentiation is the process of finding the rate at which a quantity changes with respect to another quantity.
- In simpler terms, it's about finding the slope of a curve at a particular point. This slope represents the rate of change of the function at that point.
- The derivative of a function represents its rate of change. It tells us how the function behaves as its input (usually denoted as x) changes.
- For example, if you have a function that represents the position of an object over time, its derivative would give you the velocity of the object at any given time.

Rules:

Constant Rule	$\frac{d}{dx}[c] = 0$
Constant Multiple Rule	$\frac{d}{dx}[cf(x)] = cf'(x)$
Sum Rule	$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$
Difference Rule	$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$
Product Rule	$\frac{d}{dx}[f(x) \cdot g(x)] = f'(x)g(x) + f(x)g'(x)$
Quotient Rule	$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$
Chain Rule	$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$

Example Calculation:

Let's find the derivative of $\boldsymbol{f(x) = 3x^2 + 2x + 1}$

Using the power rule and sum rule:

$$f'(x) = 2 \cdot 3x^{2-1} + 1 \cdot 2x^{1-1} + 0.$$
$$f'(x) = 6x + 2.$$

ACTIVITY

Use differentiation rules to solve each problem.

1)

x	f(x)	f '(x)	g(x)	g'(x)
-8	6	-7	-8	-9
0	-2	-3	-8	1
-4	9	8	7	3

Given $h(x) = (f(x))^2$, find $h'(0)$.

2)

x	f(x)	f '(x)	g(x)	g'(x)
4	1	4	-1	-3
-2	5	-4	4	9
-4	-2	8	-6	-9

Given $h(x) = 4f(x) - 5g(x)$, find $h'(-2)$.

3)

x	f(x)	f '(x)	g(x)	g'(x)
-5	-5	5	-2	-1
-2	6	3	-7	4
-7	-4	9	-3	7

Given $h(x) = \frac{f(x)}{g(x)}$, find $h'(-5)$.

4)

x	f(x)	f '(x)	g(x)	g'(x)
1	-6	7	9	-2
-2	1	6	1	3
-5	8	-7	-9	-4

Given $h(x) = \frac{f(x)}{g(x)}$, find $h'(-2)$.

5)

x	f(x)	f '(x)	g(x)	g'(x)
-5	1	8	-1	7
4	4	-9	-2	6
5	-4	-5	-5	-8

Given $h(x) = f(g(x))$, find $h'(5)$.

6)

x	f(x)	f '(x)	g(x)	g'(x)
4	8	5	6	-5
1	-8	-6	-1	7
3	-4	-7	1	4

Given $h(x) = f(x) \cdot g(x)$, find $h'(3)$.

Differentiate the following functions. Show all your steps. Answer must be simplified.

1. $f(x) = 3x^5 - 5x^3$

2. $f(x) = 2x^2(x^3+2)^4$

3. $f(x) = e^x \sin(2x)$

4. $f(x) = (4x^3 - 2x)^{-2}$

5. $f(x) = \cot^4 x - 4\cos x$

6. $f(x) = \dfrac{2x}{1+x^2}$

7. $f(x) = \dfrac{3-2x}{x^3}$

8. $f(x) = 5x^3\sqrt{x}$

12. $f(x) = \dfrac{2x^4 - 3x^2 - 1}{x^3}$

13. $f(x) = 5\sqrt{3x^2 - 6}$

14. $f(x) = \dfrac{10}{(2x^3 - 1)^2}$

15. $f(x) = (xe^x)^2$

16. $f(x) = \dfrac{\sin^{-1}(2x)}{3}$

17. $f(x) = (\tan^{-1}(2x))^3$

18. $f(x) = \sqrt[3]{x^2} + \dfrac{2}{\sqrt[3]{x}}$

22. $f(x) = \tan(\sin(2x))$

23. $f(x) = \tan(x)\sin(2x)$

24. $f(x) = \tan^{-1}(2^x)$

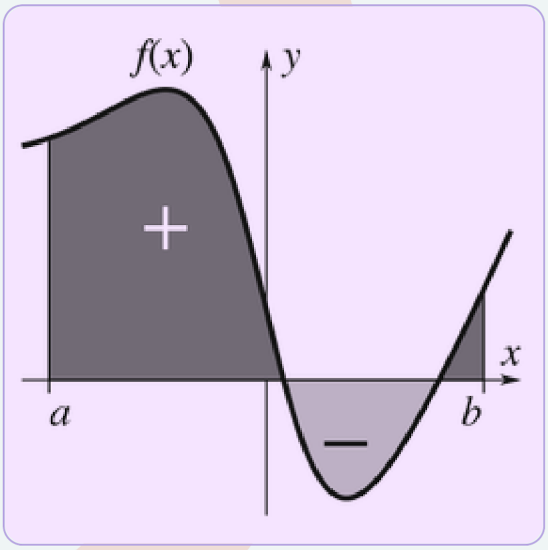
25. $f(x) = \left(6x - \sqrt{x^2+1}\right)^4$

26. $f(x) = \dfrac{\cos x}{\sin^2 x}$

27. $f(x) = \log_3(4x^2 + 5x^4)$

28. $f(x) = (2x+1)\sec^2 x$

INTEGRATION



- Integration is the reverse process of differentiation. It's about finding the accumulation of quantities.
- Given a function representing a rate of change, integrating it with respect to its variable yields the total change or accumulation of that quantity over a specified interval.
- In simpler terms, integration finds the area under a curve between two points on the graph of a function.
- The integral of a function represents the accumulated effect of the function over a given range or interval.
- For example, if you have a function representing the velocity of an object over time, integrating it would give you the total displacement or distance traveled by the object over a certain period.

Rules:

Power Rule

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

Constant Coefficent Rule

$$\int c\, f(x) dx = c \int f(x) dx$$

Sum and Difference Rule

$$\int [f(x) + g(x)] dx = \int f(x)\, dx + \int g(x) dx$$

$$\int [f(x) - g(x)] dx = \int f(x)\, dx - \int g(x) dx$$

Example Calculation:

∫(x – x2)dx

Using the Difference Rule of Integration

= ∫x dx – ∫x2 dx

= x2/2 – x3/3 + C

ACTIVITY

Find the indefinite integral of each function.

1) $\int (2x - 1)dx$

2) $\int (-4x^3 + 36x^2 - 72x)dx$

3) $\int (2x + 3)dx$

4) $\int (\frac{-1}{x^2})dx$

5) $\int (\frac{-57}{x^4})dx$

6) $\int (\frac{-28}{x^3})dx$

7) $\int (\frac{12}{169x^{\frac{9}{13}}})dx$

8) $\int (\frac{-3}{11x^{\frac{7}{11}}})dx$

9) $\int (2x + 5)dx$

10) $\int (\frac{-10}{133x^{\frac{9}{7}}})dx$

Find each indefinite integral.

1) $\int (9x^{-1})dx$

2) $\int (\frac{-7}{x})dx$

3) $\int (12e^x)dx$

4) $\int (4 \cdot 16^x)dx$

5) $\int (\frac{8}{x})dx$

6) $\int (6e^x)dx$

7) $\int (7 \cdot 3^x)dx$

8) $\int (5x^{-1})dx$

9) $\int (-8e^x)dx$

10) $\int (-9 \cdot 13^x)dx$

ACTIVITY

Find each indefinite integral using inverse trigonometric functions.

1) $\int \left(\frac{7}{16\sqrt{1-(\frac{7}{6}x)^2}}\right)dx$

2) $\int \left(\frac{9}{5x\sqrt{(\frac{3}{8}x)^2-1}}\right)dx$

3) $\int \left(\frac{35}{24(1+(\frac{7}{8}x)^2)}\right)dx$

4) $\int \left(\frac{1}{40(1+(\frac{1}{8}x)^2)}\right)dx$

5) $\int \left(\frac{-15}{28(1+(\frac{5}{8}x)^2)}\right)dx$

6) $\int \left(\frac{63}{2\sqrt{1-(\frac{7}{2}x)^2}}\right)dx$

7) $\int \left(\frac{-1}{18\sqrt{1-(\frac{1}{3}x)^2}}\right)dx$

8) $\int \left(\frac{-3}{x\sqrt{(\frac{3}{4}x)^2-1}}\right)dx$

9) $\int \left(\frac{-7}{x\sqrt{(\frac{8}{7}x)^2-1}}\right)dx$

10) $\int \left(\frac{-12}{35\sqrt{1-(\frac{3}{10}x)^2}}\right)dx$

Evaluate each sum.

1) $\sum_{w=1}^{n} w + w^2$

2) $\sum_{a=1}^{n} a + a^3$

3) $\sum_{r=1}^{n} r^2$

4) $\sum_{k=1}^{n} 6k^3$

5) $\sum_{c=1}^{n} c^3$

6) $\sum_{z=1}^{n} 11 - 7z^3$

7) $\sum_{u=1}^{n} 23u + 7u^2$

8) $\sum_{x=1}^{n} 14x$

9) $\sum_{m=1}^{n} 3m^2$

10) $\sum_{f=1}^{n} f^2 - f^3$

11) $\sum_{v=1}^{n} 7v + 5v^3$

12) $\sum_{i=1}^{n} 18 - 13i$

PROBABILITY

DETERMINING PROBABILITIES

WHAT IS PROBABILITIES

Determining probabilities involves assessing the likelihood of various outcomes occurring in a given situation. This process relies on gathering relevant information, understanding the context, and applying mathematical or statistical methods to quantify the chances of different events happening.

The probability of all the events in a sample space adds up to 1.

Probability of event to happen P(E) = Number of favourable outcomes/ Total Number of outcomes

Let's consider flipping a fair coin. When you flip a coin, there are two possible outcomes: it can land either heads (H) or tails (T). Since it's a fair coin, the probability of getting heads or tails is equal.

So, the probability of getting heads (H) is 1/2 or 0.5, and the probability of getting tails (T) is also 1/2 or 0.5.

In this case:

- Probability of getting heads (H) = 0.5
- Probability of getting tails (T) = 0.5

This means if you were to flip the coin many times, you would expect to get heads approximately 50% of the time and tails approximately 50% of the time.

INDEPENDENT EVENTS

Independent events are events where the occurrence of one event does not affect the occurrence of another event. In other words, the outcome of one event has no influence on the outcome of the other event.

Mathematically, two events A and B are considered independent if and only if:

$$P(A \cap B) = P(A) \times P(B)$$

Where:

- **$P(A \cap B)$** is the probability of both events A and B occurring.
- **$P(A)$** is the probability of event A occurring.
- **$P(B)$** is the probability of event B occurring.

In simpler terms, if events A and B are independent, then the probability of both events happening is simply the product of the probabilities of each individual event happening.

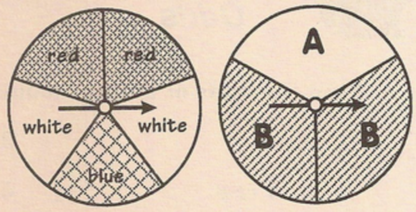
For example, if you flip a fair coin twice, the outcome of the first flip (e.g., getting heads) does not affect the outcome of the second flip. Therefore, the events "getting heads on the first flip" and "getting heads on the second flip" are independent events.

Similarly, if you roll a fair six-sided die and flip a fair coin, the outcome of rolling the die does not affect the outcome of flipping the coin. Thus, rolling the die and flipping the coin are independent events.

ACTIVITY

Find each answer in the set of answers under the exercise. Write the exercise letter in that box.

1 **Find each probability if you spin both spinners.**



T. P(blue, A)

A. P(not red, A)

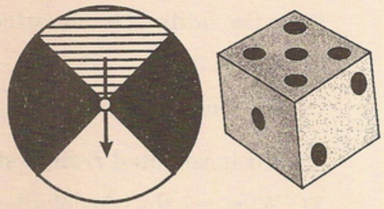
E. P(red, A)

E. P(not white, B)

O. P(white, B)

D. P(not blue, B)

2 **Find each probability if you spin the spinner and roll the die.**



A. P(white, 2)

T. P(striped, less than 5)

H. P(black, 6)

K. P(not striped, odd)

E. P(white, even)

W. P(green, odd)

3 **Solve.**

M. Suppose the probability that a new spark plug is defective is $\frac{1}{24}$. And suppose you buy two new spark plugs for a motorcycle. What is the probability that both of them are defective?

N. A test includes several multiple choice questions, each with five choices. Suppose you don't know the answers for three of these questions, so you guess. What is the probability of getting all three correct?

$\frac{1}{12}$	$\frac{2}{15}$	$\frac{1}{496}$	0	$\frac{1}{5}$	$\frac{1}{125}$	$\frac{1}{15}$	$\frac{1}{8}$	$\frac{8}{15}$	$\frac{1}{75}$	$\frac{1}{6}$	$\frac{4}{15}$	$\frac{5}{8}$	$\frac{1}{576}$	$\frac{1}{24}$	$\frac{3}{8}$	$\frac{2}{5}$

4 **"ARKANSAS": Find each probability if you pick a card, do NOT replace it, then pick a second card.**

O. P(N, then K)

B. P(S, then A)

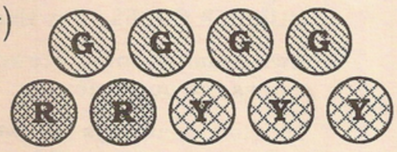
A. P(R, then S)

Y. P(S, then not S)

I. P(A, then N)

A. P(A, then not A)

5 **Find each probability if you pick two marbles without replacing the first (G = green; R = red; Y = yellow).**



O. P(red, then green)

N. P(yellow, then not yellow)

A. P(red, then yellow)

T. P(green, then not green)

W. P(green, then green)

D. P(not red, then not red)

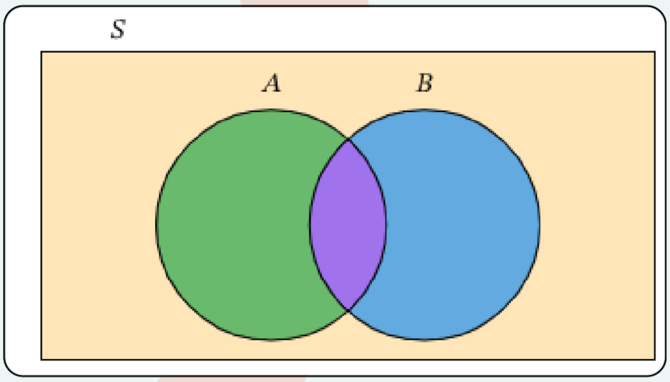
6 **Solve.**

H. Two students are chosen at random from a class of 30. What is the probability that both you and your best friend are chosen?

R. Two cards are drawn at random from a standard deck of 52 cards. What is the probability that both cards are aces?

$\frac{1}{12}$	$\frac{7}{18}$	$\frac{1}{435}$	$\frac{3}{56}$	$\frac{5}{18}$	$\frac{2}{869}$	$\frac{1}{56}$	$\frac{1}{4}$	$\frac{3}{220}$	$\frac{3}{28}$	$\frac{1}{221}$	$\frac{1}{9}$	$\frac{15}{56}$	$\frac{7}{12}$	$\frac{1}{6}$	$\frac{1}{28}$	$\frac{3}{14}$

In probability, a Venn diagram is a figure with one or more circles inside a rectangle that describes logical relations between events. The rectangle in a Venn diagram represents the sample space or the universal set, that is, the set of all possible outcomes. A circle inside the rectangle represents an event, that is, a subset of the sample space.



In the diagram above, we have two events A and B within the sample space (or universal set) S.Sometimes, the sample space is denoted by σ or ξ instead of S.Colored regions in this Venn diagram represent the following events:

Green and purple regions : $\mathbf{A}$

Blue and purple regions : $\mathbf{B}$

Purple region : $\mathbf{A \cap B}$

Green, purple, and blue regions : $\mathbf{A \cup B}$

Yellow region : $\overline{\mathbf{A \cup B}}$ or $\mathbf{(A \cup B)'}$

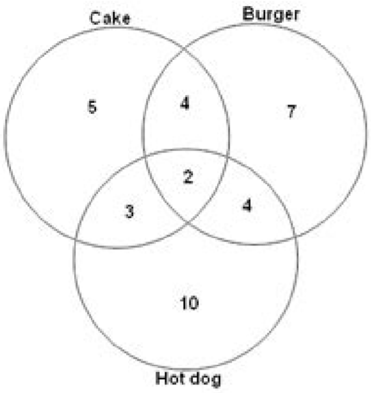
Two-Event Venn Diagrams

Let **A** and **B** be events described in a Venn diagram. Then,

- The circles do not overlap if **A** and **B** are mutually exclusive events, that is, $\mathbf{A \cap B = \varnothing}$;
- The circles overlap if $\mathbf{A \cap B \neq \varnothing}$,in which case the intersection $\mathbf{A \cap B}$ is represented by the overlapping region;
- The region outside both circles but within the rectangle represents the complement of the union of both events, that is, $\mathbf{A \cup B}$ or, alternatively, $\mathbf{(A \cup B)'}$.

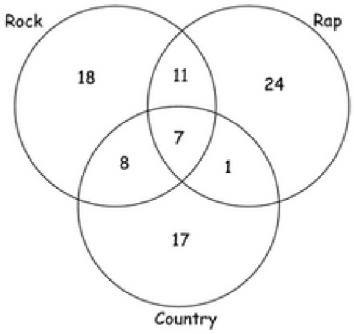
ACTIVITY

In a birthday party, cakes, burgers and hotdogs are served and it is represented by a Venn diagram below:



1. How many had hot dogs?
   Answer: _______________
2. How many had only cakes?
   Answer: _______________
3. How many had both cakes and burgers?
   Answer: _______________
4. How many had hot dog and cake but not burger?
   Answer: _______________
5. How many had at least two items?
   Answer: _______________

Use the Venn Diagram below to answer question #1 - 5.



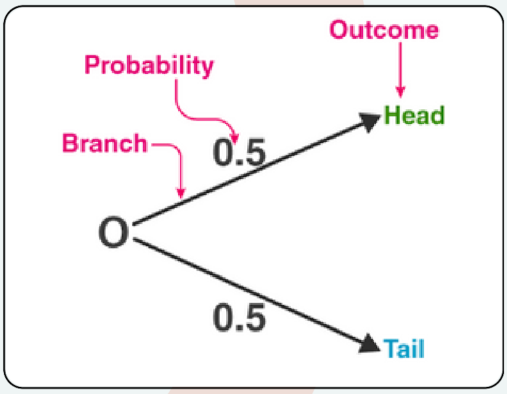
1. How many total people are represented in the diagram? _____
2. How many people like country? ______
3. If one person is chosen at random, what is the probability that that person will like rap music?

   P (rap) =
4. If one person is chosen at random, what are the odds for picking a person who likes country?

   Odds <u>for</u> country =
5. If one person is chosen at random, what are the odds against picking a person who likes all three types of music?

   Odds <u>against</u> all three =

Tree diagrams are a useful tool for visualizing and calculating probabilities, especially in situations involving multiple stages or outcomes. Let's go through an example to demonstrate how to use a tree diagram for probability calculation.

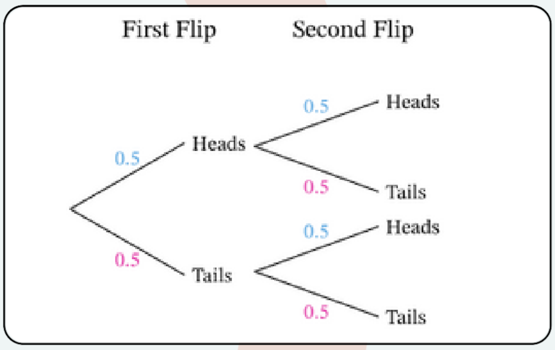


Branch: In a tree diagram, a branch signifies a possible outcome at a specific stage of a process. Each branch extends from a node and leads to another node or outcome, illustrating the different paths the process can take and the corresponding results at each stage.

Probability: Probability quantifies the likelihood of an event occurring, typically expressed as a number between 0 and 1. In the context of a tree diagram, probabilities assigned to branches represent the chances of specific outcomes happening, enabling the assessment of uncertainty and making informed decisions based on the likelihood of different scenarios.

Outcome: An outcome is a possible result or occurrence of an event. In a tree diagram, endpoints of branches represent outcomes, depicting the specific results of the process being analyzed. By considering all potential outcomes, individuals can evaluate the possible consequences of their actions and assess the overall risk associated with different choices.

let's calculate the probability of flipping a coin two times and getting exactly one head.



As before, since the coin is fair, the probability of getting Heads (H) or Tails (T) at each flip is 0.5.

Calculate the Probability of Each Outcome

- The probability of getting one head and one tail is: **(0.5×0.5)=0.25**

Find the Probability of the Desired Outcome

- In this case, we want exactly one head. There are two ways this can happen:
  - **HT**
  - **TH**
- so, add the probabilities of these two outcomes together:

P(Exactly 1 Head)=P(HT)+P(TH)

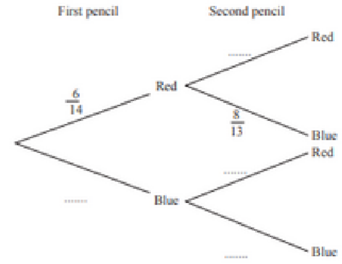
  - **=(0.5×0.5)+(0.5×0.5)**
  - **=2×(0.5×0.5)**
  - **=2×0.25**
  - **=0.5**

So, the probability of flipping a coin two times and getting exactly one head is **0.5**, or **50%**.

ACTIVITY

23 A box contains 6 red pencils and 8 blue pencils.
A pencil is chosen at random and not replaced.
A second pencil is then chosen at random.

(a) Complete the tree diagram.



[2]

(b) Calculate the probability that

(i) both pencils are red,

Answer(b)(i) .... [2]

(ii) at least one of the pencils is red.

Answer(b)(ii) .... [3]

8 **In all parts of this question give your answer as a fraction in its lowest terms.**

(a) **(i)** The probability that it will rain today is $\frac{1}{3}$.

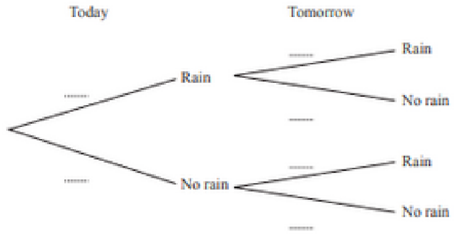
What is the probability that it will not rain today?

Answer(a)(i) .... [1]

(ii) If it rains today, the probability that it will rain tomorrow is $\frac{2}{5}$.

If it does not rain today, the probability that it will rain tomorrow is $\frac{1}{6}$.

Complete the tree diagram.



[2]

(b) Find the probability that it will rain on at least one of these two days.

Answer(b) .... [3]

(c) Find the probability that it will rain on only one of these two days.

Answer(c) .... [3]

EXPECTED VALUE CALCULATION

Determining probabilities involves assessing the likelihood of various outcomes occurring in a given situation. This process relies on gathering relevant information, understanding the context, and applying mathematical or statistical methods to quantify the chances of different events happening.

The formula for calculating the expected value (EV) of an outcome, denoted as E(X), is:

$$\mu = E(X) = \sum_{i=1}^{N} X_i P(X_i)$$

Where:

- **E(X)** is the expected value of the outcome.
- **x_i** represents each possible outcome.
- **P(X=x_i)** is the probability of each outcome.

Suppose you roll a fair six-sided die. The possible outcomes are the numbers 1 through 6, each with a probability of 1/6 To find the expected value of this scenario, you would use the formula:

E(X) = (1×1/6) + (2×1/6) + (3×1/6) + (4×1/6) + (5×1/6) + (6×1/6)

E(X) = 1/6+2/6+3/6+4/6+5/6+6/6

E(X) = 21/6

E(X) = 3.5

So, the expected value of rolling a fair six-sided die is 3.5. This means that if you were to roll the die many times and average the outcomes, you would expect to get a value close to 3.5.

1. You draw one card from a standard deck of playing cards. If you pick a heart, you will win \$10. If you pick a face card, which is not a heart, you win \$8. If you pick any other card, you lose \$6. Do you want to play? Explain.

2. The world famous gambler from Philadelphia, Señor Rick, proposes the following game of chance. You roll a fair die. If you roll a 1, then Señor Rick pays you \$25. If you roll a 2, Señor Rick pays you \$5. If you roll a 3, you win nothing. If you roll a 4 or a 5, you must pay Señor Rick \$10, and if you roll a 6, you must pay Señor Rick \$15. Is Señor Rick loco for proposing such a game? Explain.

3. You pay \$10 to play the following game of chance. There is a bag containing 12 balls, five are red, three are green and the rest are yellow. You are to draw one ball from the bag. You will win \$14 if you draw a red ball and you will win \$12 is you draw a yellow ball. How much do you expect to win or loss if you play this game 100 times?

4. A detective figures that he has a one in nine chance of recovering stolen property. His out-of-pockets expenses for the investigation are \$9,000. If he is paid his fee only if he recovers the stolen property, what should he charge clients in order to breakeven?

5 At Tucson Raceway Park, your horse, Soon-to-be-Glue, has a probability of 1/20 of coming in first place, a probability of 1/10 of coming in second place, and a probability of ¼ of coming in third place. First place pays \$4,500 to the winner, second place \$3,500 and third place \$1,500. Is it worthwhile to enter the race if it costs \$1,000?

6. Your company plans to invest in a particular project. There is a 35% chance that you will lose \$30,000, a 40% chance that you will break even, and a 25% chance that you will make \$55,000. Based solely on this information, what should you do?

7. A manufacturer is considering the manufacture of a new and better mousetrap. She estimates the probability that the new mousetrap is successful is $\frac{3}{4}$. If it is successful it would generate profits of \$120,000. The development costs for the mousetrap are \$98,000. Should the manufacturer proceed with plans for the new mousetrap? Why or why not?

8. A grab bag contains 12 packages worth 80 cents apiece, 15 packages worth 40 cents apiece and 25 packages worth 30 cents apiece. Is it worthwhile to pay 50 cents for the privilege of picking one of the packages at random?

PROBABILITIES AND NORMAL DISTRIBUTION

Determining probabilities involves assessing the likelihood of various outcomes occurring in a given situation. This process relies on gathering relevant information, understanding the context, and applying mathematical or statistical methods to quantify the chances of different events happening.

NORMAL DISTRIBUTION FORMULA

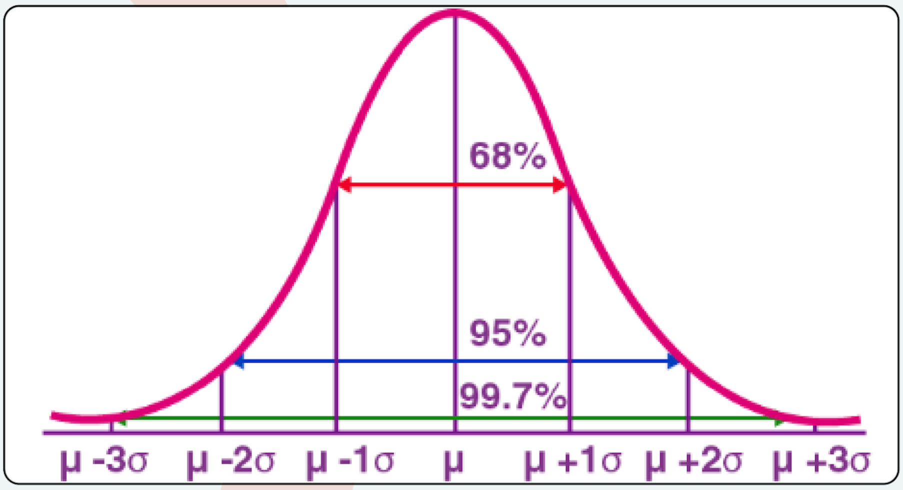
The probability density function of normal or gaussian distribution is given by

$$f(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

Where:

- **x** is the variable
- **μ** is the mean
- **σ** is the standard deviation

The normal distribution curve, often referred to as the bell curve because of its characteristic shape, is a probability distribution that is symmetric around its mean. It's one of the most important concepts in statistics and probability theory



- **Shape**: The normal distribution curve is symmetric, forming a bell-shaped curve when plotted. The highest point on the curve represents the mean, and the curve tapers off symmetrically on both sides.

- **Mean, Median, Mode**: In a normal distribution, the mean, median, and mode are all equal, and they are located at the center of the distribution.

- **Standard Deviation:** The spread of data around the mean is measured by the standard deviation. The standard deviation determines the width of the curve. A larger standard deviation results in a wider curve, indicating greater variability in the data.

- **68-95-99.7 Rule:** This rule, also known as the empirical rule, states that in a normal distribution:
    - Approximately **68%** of the data falls within one standard deviation of the mean.
    - Approximately **95%** of the data falls within two standard deviations of the mean.
    - Approximately **99.7%** of the data falls within three standard deviations of the mean.

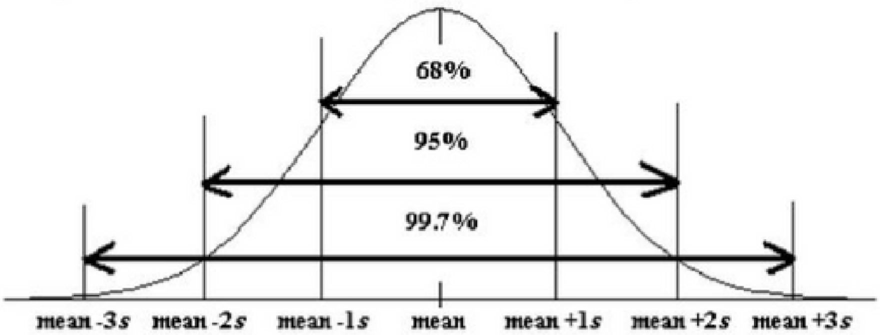
- **Probability Density Function:** The equation for the normal distribution is given by the probability density function (PDF), which is defined by two parameters: the mean (μ) and the standard deviation (σ). The equation for the standard normal distribution (where $\mu=0$ and $\sigma=1$) is often denoted as $\varphi(z)$, where z represents the standardized value.

- **Applications:** The normal distribution is widely used in various fields such as natural sciences, social sciences, engineering, finance, and more. It serves as a fundamental tool for modeling and analyzing data due to its ubiquity in real-world phenomena.

- **Central Limit Theorem:** One of the reasons why the normal distribution is so important is because of the Central Limit Theorem, which states that the distribution of the sum (or average) of a large number of independent, identically distributed random variables approaches a normal distribution, regardless of the original distribution of the variables.

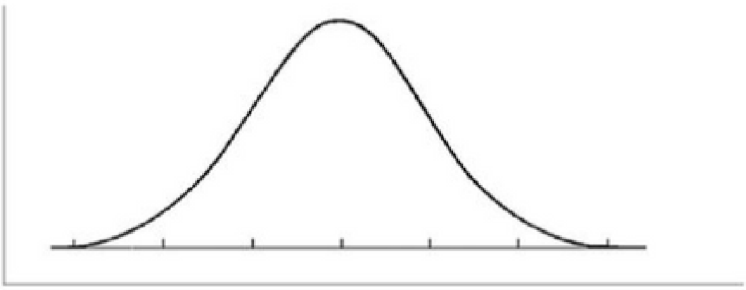
ACTIVITY

The Empirical Rule For Normal Distributions (a.k.a. the 68-95-99.7 Rule)



We use the Empirical Rule to analyze data when original values are unknown.

Example 1: Suppose the scores on a test are normally distributed, that the mean score is 80 and the standard deviation is 7. Draw a normal curve to represent this scenario.



a. What percent scored less than 87?

b. What percent scored less than 73?

c. What percent scored more than 94?

d. 2.5% scored less than what value?

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END !

Thank you for choosing this mathematics guidebook. Whether you're a student delving into the intricacies of calculus or an educator seeking comprehensive resources, this book aims to be your trusted companion. With clear explanations, illustrative examples, and practical exercises, it endeavors to demystify mathematical concepts and foster a deeper understanding of the subject. Here's to embarking on a journey of mathematical discovery together.

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