PRACTICAL SHEET - 4

MOMENTS, SKEWNESS, KURTOSIS

1. Find the four moments about 60.

x	45	50	55	60	65	70	75	80
f	3	5	8	7	9	7	4	7

Solution:

x_i	f_i	$(x_i-60)f_i$	$(x_i-60)^2 f_i$	$(x_i-60)^3 f_i$	$(x_i-60)^4 f_i$
45	3	-45	675	-10125	151875
50	5	-50	500	-5000	50000
55	8	-40	200	-1000	5000
60	7	0	0	0	0
65	9	45	225	1125	5625
70	7	70	700	[illegible]00	70000
75	4	60	900	13500	202500
80	7	140	2800	56000	1120000
	50	180	6000	61500	1605000

$\mu_1' = \frac{1}{N} \Sigma (x_i - 60) f_i$

$= 3.6$

$\mu_2' = \frac{1}{N} \Sigma (x_i - 60)^2 f_i$

$= 120$

$\mu_3' = \frac{1}{N} \Sigma (x_i - 60)^3 f_i$

$= 1230$

$\mu_4' = \frac{1}{N} \Sigma (x_i - 60)^4 f_i$

$= 32100$

Program

```
x <- c(72,74,40,60,82,115,41,61,65,83,53,110,46,84,50,67,78,79,56,65,68,69,
   104,78,59,81,66,49,77,90,84,76,42,64,64,70,72,50,79,52,103,96,51,86,
   78,94,80,79,79,82); x
summary(x)
mean <- 72.06 ; mean
median <- 73; median
n<-length(x);n
mu2 <- sum((x-mean)^2)/n; mu2
sd <- sqrt(mu2); sd
mu3 <- sum (((x-mean)^3))/n; mu3
mskew <- (mu3^2)/(mu2^3); mskew   #moment measure of skewness
pskew <- (3*(mean-median))/sd; pskew   # Pearson's measure of skewness
```

Output

mean = 72.06

median = 73

mu2 = 304.5364

mu3 = 1225.403

sd = 17.45097

mskew (β_1) = 0.05316679

pskew (Pearson's measure of skewness) = -0.1615956

2. Calculate the moment measure of skewness for the data. Also calculate Pearson's measure of skewness.

72, 74, 40, 60, 82, 115, 41, 61, 65, 83, 53, 110, 46, 84, 50, 67, 78, 79, 56, 65, 68, 69, 104, 78, 59, 81, 66, 49, 77, 90, 84, 76, 42, 64, 64, 70, 72, 50, 79, 52, 103, 96, 51, 86, 78, 94, 80, 79, 79, 82.

Solution:

$n = 50$

Mean, $\bar{x} = \dfrac{\Sigma x_i}{n}$

$= \underline{\underline{72.06}}$

After arranging in ascending order, Median = average of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ observations

$= \dfrac{72+74}{2}$

$= \underline{\underline{73}}$

x_i	$(x_i-\bar{x})^2$	$(x_i-\bar{x})^3$
72	0.0036	-0.0002
74	3.7636	7.3014
40	1027.8436	-32952.6658
60	145.4436	-1754.0498
82	98.8036	982.1078
115	1843.8436	79174.6442
41	964.7236	-29964.3150
61	122.3236	-1352.8990
65	49.8436	-351.8958
83	119.6836	1309.3386
53	363.2836	-6924.1854
110	1439.4436	54612.4902
46	679.1236	-17697.961

[illegible]	$(x_i-\bar{x})^2$	$(x_i-\bar{x})^3$
84	142.5636	334.2554
50	486.6436	-4142.2530
67	25.6036	-351.8958
78	35.2836	209.5846
79	48.1636	334.2554
56	257.9236	-4142.2530
65	49.8436	-351.8958
68	16.4836	-66.9234
69	9.3636	-28.6526
104	1020.1636	32584.0254
78	35.2836	209.5846
59	170.5636	-2227.5606
81	79.9236	714.5170

x_i	$(x_i-\bar{x})^2$	$(x_i-\bar{x})^3$
66	36.7236	-222.5450
49	531.7636	-12262.4686
77	24.4036	120.5538
90	321.8436	5773.8742
84	142.5636	1702.2094
76	15.5236	61.1630
42	903.6036	-27162.3242
64	64.9636	-523.6066
64	64.9636	-523.6066
70	4.2436	-8.7418
72	0.0036	-0.0002
50	486.6436	-10735.3578

x_i	$(x_i-\bar{x})^2$	$(x_i-\bar{x})^3$
79	48.1636	334.2554
52	402.4036	-8072.2162
103	957.2836	29618.3546
96	573.1236	13720.5790
51	443.5236	-9340.6070
86	194.3236	2708.8710
78	35.2836	209.5846
94	481.3636	10561.1174
80	63.0436	500.5062
79	48.1636	334.2554
79	48.1636	334.2554
82	98.8036	982.1078

$\mu_3 = \frac{1}{N}\Sigma(x_i-\bar{x})^3$

$= 1225.4032$

$\mu_2 = \frac{1}{N}\Sigma(x_i-\bar{x})^2$

$= 304.5364$

$S.D = \sqrt{\mu_2}$

$= 17.45$

Moment measure of skewness, $\beta_1 = \frac{\mu_3^2}{\mu_2^3}$

$= 0.0532$

Pearson's measure of skewness $= \frac{3(\text{mean}-\text{median})}{S.D}$

$= -0.1616$

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<u>Program</u>

```
x <- c(62,45,59,32,51,56,60,51,49,25,42,54,54,58,70,43,58,50,52,38,67,50,
    59,48,65,71,30,46,55,82,51,63,45,53,40,36,56,70,52,67,55,57,30,63,
    42,74,58,44,55) ; x
n <- length(x); n
summary(x)
mean <- sum(x)/n ; mean
mu2 <- sum((x-mean)^2)/n ; mu2
mu4 <- sum((x-mean)^4)/n ; mu4
beta2 <- mu4/(mu2^2); beta2
gamma2 <- beta2 - 3; gamma2
```

<u>Output</u>

mu2 = 140.4831

mu4 = 58997.56

beta2 (β_2) = 2.989411

gamma2 (γ_2) = -0.01058864

3. Calculate the kurtosis.

62, 45, 59, 32, 51, 56, 60, 51, 49, 25, 42, 54, 54, 58, 70, 43, 58, 50, 52, 38, 67, 50, 59, 48, 65, 71, 30, 46, 55, 82, 51, 63, 45, 53, 40, 36, 56, 70, 52, 67, 55, 57, 30, 63, 42, 74, 58, 44, 55.

Solution:

$n = 49$

Mean, $\bar{x} = \frac{\sum x_i}{n}$

$= 52.91$

x_i	$(x_i - 52.91)^2$	$(x_i - 52.91)^4$
62	82.6281	6827.4029
45	62.5681	3914.7671
59	37.0881	1375.5272
32	437.2281	191168.4114
51	3.6481	13.3086
56	9.5481	91.1662
60	50.2681	2526.8819
51	3.6481	13.3086
49	15.2881	233.7260
25	778.9681	606791.3008
42	119.0281	14167.6886
54	1.1881	1.4116
54	1.1881	1.4116
58	25.9081	671.2296
70	292.0681	85303.7750
43	98.2081	9644.8309
58	25.9081	671.2296
50	8.4681	71.7087
52	0.8281	0.6857
38	222.3081	49420.8913
67	198.5281	39413.4065
50	8.4681	71.7087
59	37.0881	1375.5272

x_i	$(x_i - 52.91)^2$	$(x_i - 52.91)^4$
48	24.1081	581.2005
65	146.1681	21365.1135
71	327.2481	107091.319
30	524.8681	275486.5224
46	47.7481	2279.8811
55	4.3681	19.0803
82	846.2281	716101.9972
51	3.6481	13.3086
63	101.8081	10364.8592
45	62.5681	3914.7671
53	0.0081	0.00007
40	166.6681	27778.2556
36	285.9481	81766.3159
56	9.5481	91.1662
70	292.0681	85303.7750
52	0.8281	0.6857
67	198.5281	39413.4065
55	4.3681	19.0803
57	16.7281	279.8293
30	524.8681	275486.5224
63	101.8081	10364.8892
42	119.0281	14167.6886

x_i	$(x_i - 52.91)^2$	$(x_i - 52.91)^4$
74	444.7881	197836.4539
58	25.9081	671.2296
44	79.3881	6302.4704
55	4.3681	19.0803

$$\mu_2 = \frac{1}{n}\Sigma(x_i - \bar{x})^2$$

$$= \frac{6883.6769}{49}$$

$$= \underline{\underline{140.483}}$$

$$\mu_4 = \frac{1}{n}\Sigma(x_i - \bar{x})^4$$

$$= \underline{\underline{58997.56}}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$= \underline{\underline{2.989}}$$

$$\gamma_2 = \beta_2 - 3$$

$$= \underline{\underline{-0.011}}$$

$\therefore$ Platykurtic

Program

```
x <- c(45, 28, 13, 8, 55, 67, 76, 42, 50, 60, 60, 62, 68, 70, 42, 75, 75, 80, 72, 79, 85,
   81, 25, 26, 31, 32, 78, 45, 37, 31, 45, 42, 43, 55, 56); x
n <- length(x); n
rm1 <- sum(x)/n; rm1
rm2 <- sum(x^2)/n; rm2
rm3 <- sum(x^3)/n; rm3
rm4 <- sum(x^4)/n; rm4
mu2 <- rm2 - (rm1^2); mu2
mu3 <- rm3 - (3*rm2*rm1) + (2*(rm1^3)); mu3
mu4 <- rm4 - (4*rm3*rm1) + (6*rm2*(rm1^2)) - (3*(rm1^4)); mu4
```

Output

Raw moments:

rm1 = 52.54286

rm2 = 3180.371

rm3 = 209115.2

rm4 = 14503851

Central moments:

mu1 = 0

mu2 = 419.6196

mu3 = -2086.655

mu4 = 369862.6

4. Calculate the first four raw moments and hence deduce the central moments.

45, 28, 13, 8, 55, 67, 76, 42, 50, 60, 60, 62, 68, 70, 42, 75, 75, 80, 72, 79, 85, 81, 25, 26, 31, 32, 78, 45, 37, 31, 45, 42, 43, 55, 56,

Solution:

$n = 35$

Raw moments:

$\mu_1' = \frac{\Sigma x_i}{n}$

$= 52.54$

$\mu_2' = \frac{1}{n} \Sigma x_i^2$

$= 3180.37$

$\mu_3' = \frac{1}{n} \Sigma x_i^3$

$= 209115.17$

$\mu_4' = \frac{1}{n} \Sigma x_i^4$

$= 14503851.46$

Central moments:

$\mu_1 = 0$

$\mu_2 = \mu_2' - (\mu_1')^2$

$= 419.92$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$= -2086.655$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$

$= 369862.6$

Program

```
x <- c(7,14,21,28,35,42,49,56); x

f <- c(29,57,92,134,216,287,314,350); f

N <- sum(f); N

mean <- sum(x*f)/N; mean

variance <- sum(((x-mean)^2)*f)/N; variance

sd <- sqrt(variance); sd

newx <- rep(x,f); newx

median <- median(x); median

pskew <- (3*(mean-median))/sd; pskew

mu3 <- sum(((x-mean)^3)*f)/N; mu3

beta1 <- (mu3^2)/(variance^3); beta1
```

Output

mean = 41.43678

median = 42

sd = 12.769

pskew (Pearson's measure) = -0.132324

mu3 = -1563.63

beta1 (β_1) = 0.564021

5. Find Pearson's measure of skewness and moment measure of skewness for the following data.

x	7	14	21	28	35	42	49	56
f	29	57	92	134	216	287	314	350

Solution:

x_i	f_i	$x_i f_i$	$(x_i-\bar{x})^2 f_i$	$(x_i-\bar{x})^3 f_i$	cf
7	29	203	34391.302	-1184333.27	29
14	57	798	42908.973	-1177293.444	86
21	92	1932	38425.732	-785306.627	178
28	134	3752	24194.102	-325096.093	312
35	216	7560	8949.96	-57610.849	528
42	287	12054	90.979	51.216	815
49	314	15386	17960.486	135835.082	1129
56	350	19600	74228.35	1080987.303	1479
	1479	61285	241149.884	-2312766.682	

Mean, $\bar{x} = \frac{\sum x_i f_i}{N}$

$= \frac{61285}{1479}$

$= 41.437$

variance, $\mu_2 = \frac{1}{N} \Sigma (x_i-\bar{x})^2 f_i$

$= 163.049$

Standard deviation, S.D $= \sqrt{\text{variance}}$

$= 12.769$

$\mu_3 = \frac{1}{N} \Sigma (x_i-\bar{x})^3 f_i$

$= -1563.737$

Moment measure of skewness, $\beta_1 = \mu_3^2 / \mu_2^3$

$\beta_1 = 0.564$

Median = value at $\left(\frac{N+1}{2}\right)^{th}$ position

$= 42$

$\therefore$ Pearson's measure of skewness $= \frac{3(\text{Mean} - \text{Median})}{S.D}$

$= \frac{3(41.437 - 42)}{12.769}$

$= -0.13227$