PRACTICAL SHEET - 10

CONTINUOUS PROBABILITY DISTRIBUTION

1. The variable $X \sim N(45, 10)$. Find the probability that

(i) $P(X \geq 60)$

(ii) $P(40 < X < 56)$

(iii) $P(25 < X < 73)$

Solution:

(i) $P(X \geq 60) = P\left(\frac{X-45}{10} \geq \frac{60-45}{10}\right)$

$= P(Z \geq 1.5)$

$= 0.5 - P(0 < Z < 1.5)$

$= 0.0668$

(ii) $P(40 < X < 56) = P(-0.5 < Z < 1.1)$

$= P(0 < Z < 0.5) + P(0 < Z < 1.1)$

$= 0.5558$

(iii) $P(25 < X < 73) = P(-2 < Z < 2.8)$

$= P(0 < Z < 2) + P(0 < Z < 2.8)$

$= 0.9746$

Program

```
midx <- seq(from=17.05, to=86.35, by=7.7); midx
f<-c(2,10,16,34,43,49,29,13,6,5); f
N<-sum(f); N
mean <-sum(midx*f)/N; mean
variance <-sum(((midx-mean)^2)*f)/N; variance
sd <-sqrt(variance); sd
l <- c(13.2,20.9,28.6,36.3,44,51.7,59.4,67.1,74.8,82.5); l
cdf <-pnorm(l,mean,sd); cdf
cdf <- c(0,cdf,1); cdf
d <-diff(cdf); d
ef <-round(N*d); ef
tf <-c(3,8,19,33,44,43,31,17,7,3); tf
df <- data.frame(midx,f,tf); df
```

Output

midx	f	tf
17.05	2	3
24.75	10	8
32.45	16	18
40.15	34	33
47.85	43	44
55.55	49	43
63.25	29	31
70.95	13	17
78.65	6	7
86.35	5	3

2. The following table displays the frequency table of heights of trees in a locality. Fit a normal distribution for the data.

Class	13.2-20.9	20.9-28.6	28.6-36.3	36.3-44	44-51.7	51.7-59.4
f	2	10	16	34	43	49
	59.4-67.1	67.1-74.8	74.8-82.5	82.5-90.2		
	29	13	6	5		

Solution:

Class	Midvalue(x_i)	f_i	$x_i f_i$	x_i^2	$x_i^2 f_i$
13.2-20.9	17.05	2	34.1	290.7025	581.405
20.9-28.6	24.75	10	247.5	612.5625	6125.625
28.6-36.3	32.45	16	519.2	1053.0025	16848.04
36.3-44	40.15	34	1365.21	1612.0225	54808.765
44-51.7	47.85	43	2057.55	2289.6225	98453.7675
51.7-59.4	55.55	49	2721.95	[illegible]085.8025	151204.3225
59.4-67.1	63.25	29	1834.25	4000.5625	116016.3125
67.1-74.8	70.95	13	922.35	5033.9025	65440.7325
74.8-82.5	78.65	6	471.9	6185.8225	37114.935
82.5-90.2	86.35	5	431.75	7456.3225	37281.6125
		207	10605.65		583875.5175

$$\mu = \frac{\Sigma x_i f_i}{\Sigma f_i}$$

$$= 51.235$$

$$V(X), \sigma^2 = \frac{1}{N}\Sigma x_i^2 f_i - (\mu)^2$$

$$= 195.63$$

S.D, $\sigma = 13.986$

$$\therefore f(x) = \frac{1}{13.986\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-51.235}{13.986}\right)^2}, \; -\infty < x < \infty$$

True Class	Upper Class Boundary (x)	$z = (x-\mu)/\sigma$	$\Phi(z)$	$\Delta\Phi(z)$	Expected freq ($N \times \Delta\Phi(z)$)
13.2 - 20.9	$-\infty$	$-\infty$	0.5	0.015	3
20.9 - 28.6	20.9	-2.17	0.4850	0.0376	8
28.6 - 36.3	28.6	-1.62	0.4474	0.0897	18
36.3 - 44	36.3	-1.07	0.3577	0.1592	33
44 - 51.7	44	-0.52	0.1985	0.2105	44
51.7 - 59.4	51.7	0.03	0.0120	0.207	43
59.4 - 67.1	59.4	0.58	0.2190	0.1518	31
67.1 - 74.8	67.1	1.13	0.3708	0.0827	17
74.8 - 82.5	74.8	1.68	0.4535	0.034	7
82.5 - 90.2	82.5	2.24	0.4875	0.0125	3
	∞	∞	0.5		

Program

```
p1<-pnorm(35,30,4) -pnorm(30,30,4); p1   # P(30<X<35)
p2<- 1-pnorm(40,30,4); p2   # P(X>40)
```

Output

p1 = 0.3943502

p2 = 0.006209665

Program

```
p1 <- pnorm(2,0,1); p1    # P(X<2)
p2 <- pnorm(2.5,0,1) - pnorm(0.84,0,1); p2  # P(0.84<X<2.5)
p3 <- 1-p1; p3  # P(X>2)
```

Output

p1 = 0.9772499

p2 = 0.1942445

p3 = 0.02275013

4. Suppose X is a standard normal variate. Find

(i) $P(X \le 2)$

(ii) $P(0.84 \le X \le 2.5)$

(iii) $P(X \ge 2)$

Solution:

Given $X \sim N(0,1)$.

(i) $P(X \le 2) = 0.5 + P(0 < X < 2)$

$= 0.5 + 0.4772$

$= 0.9772$

(ii) $P(0.84 \le X \le 2.5) = P(-\infty < X \le 2.5) - P(-\infty < X \le 0.84)$

$= 0.5 + P(0 < X < 2.5) - (0.5 + P(0 < X < 0.84))$

$= 0.5 + 0.4938 - 0.5 - 0.29955$

$= 0.19425$

(iii) $P(X \ge 2) = 1 - P(X \le 2)$

$= 1 - 0.9772$

$= 0.0228$

Program

```
values <- rnorm(20,5,2); values
mean(values)
var(values) #variance
```

Output

mean(values) = 4.709833

var(values) = 5.314299