Program

```r
x <- c(4, 4.4, 3.9, 4, 4.2, 4.4, 5, 4.8, 4.6, 3.9); x
y <- c(5.3, 4.3, 4.1, 4.1, 4.4, 5.5, 4.2, 3.8, 5.4, 4.6); y
cor(x,y)
```

Output

r = -0.100242

PRACTICAL SHEET - 5

FITTING OF CURVE

1. Calculate the correlation coefficient.

Matrix A	4.0	4.4	3.9	4.0	4.2	4.4	5.0	4.8	4.6	3.9
Matrix B	5.3	4.3	4.1	4.1	4.4	5.5	4.2	3.8	5.4	4.6

Solution:

Correlation coefficient, $r = \frac{cov(X,Y)}{\sqrt{V(X)}\sqrt{V(Y)}}$

x_i	x_i^2	y_i	y_i^2	$x_i y_i$
4.0	16	5.3	28.09	21.2
4.4	19.36	4.3	18.49	18.92
3.9	15.21	4.1	16.81	15.99
4.0	16	4.1	16.81	16.4
4.2	17.64	4.4	19.36	18.48
4.4	19.36	5.5	30.25	24.2
5.0	25	4.2	17.64	21
4.8	23.04	3.8	14.44	18.24
4.6	21.16	5.4	29.16	24.84
3.9	15.21	4.6	21.16	17.94
43.2	187.98	45.7	212.21	197.21

$E(x) = \frac{\Sigma x_i}{n}$

$= 4.32$

$E(y) = \frac{\Sigma y_i}{n}$

$= 4.57$

$E(x^2) = \frac{\Sigma x_i^2}{n}$

$= 18.798$

$E(y^2) = \frac{\Sigma y_i^2}{n}$

$= 21.221$

$Var(x) = E(x^2) - [E(x)]^2$

$= 0.1356$

$Var(y) = E(y^2) - [E(y)]^2$

$= 0.3361$

$cov(x,y) = E(xy) - E(x).E(y)$

$= -0.0214$

correlation coefficient, $r = \dfrac{-0.0214}{\sqrt{0.1356 \times 0.3361}}$

$r = -0.1002$

Program

```r
x <- c(3,5,6,7,10,11); x
y <- c(8,12,11,14,16,17); y
lm(y~x)     # Y on X
lm(x~y)     # X on Y
```

Output

```
Call:
lm(formula = y ~ x)

Coefficients:
(Intercept)           x
     5.543        1.065

Call:
lm(formula = x ~ y)

Coefficients:
(Intercept)           y
    -4.375        0.875
```

2. Fit a regression line of Y on X and X on Y for the following:

length (x)	3	5	6	7	10	11
weight (y)	8	12	11	14	16	17

Solution:

x_i	y_i	x_i^2	y_i^2	$x_i y_i$
3	8	9	64	24
5	12	25	144	60
6	11	36	121	66
7	14	49	196	98
10	16	100	256	160
11	17	121	289	187
42	78	340	1070	595

$\bar{x} = E(X) = 7$

$\bar{y} = E(y) = 13$

$E(X^2) = \frac{\sum x_i^2}{n}$

$= 56.667$

$E(Y^2) = \sum y_i^2$

$= 178.333$

$E(XY) = \frac{\sum x_i y_i}{n}$

$= 99.167$

$cov(X,Y) = E(XY) - E(X)E(Y)$

$= 8.17$

$V(X) = E(X^2) - [E(X)]^2$

$= 7.667$

$V(Y) = E(Y^2) - [E(Y)]^2$

$= 9.333$

Regression line of Y on X $\Rightarrow (y-\bar{y}) = b_{yx}(x-\bar{x})$

$\Rightarrow (y-\bar{y}) = \frac{cov(X,Y)}{V(X)}(x-\bar{x})$

$\Rightarrow y = 1.065x + 5.543$

Regression line of X on Y $\Rightarrow (x-\bar{x}) = b_{xy}(y-\bar{y})$

$$\Rightarrow (x-\bar{x}) = \frac{cov(X,Y)}{V(Y)} (y-\bar{y})$$

$$\Rightarrow x = 0.875y - 4.376$$

Program

```
x<- c(1,6,5,8,3,2,4,7); x
y<- c(3,5,7,4,6,3,2,1); y
cor(x,y, method="spearman")
```

Output

ρ = 0.01197626

3. Find Spearman's rank correlation for the following:

Rank given by 1st judge	1	6	5	8	3	2	4	7
Rank given by 2nd judge	3	5	7	4	6	3	2	1

Solution:

Spearman's rank correlation coefficient, $\rho = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$

Rank X	Rank Y	$d = x-y$	d^2
1	3	-2	4
6	5	1	1
5	7	-2	4
8	4	4	16
3	6	-3	9
2	3	-1	1
4	2	2	4
7	1	6	36

$n = 8$

$\therefore \rho = 0.0119$

4. Find the equation of the form $y = ab^x$ to the data.

x	2	3	4	5	6
y	144	172.8	207.4	248	298.6

Solution:

$y = ab^x$

Taking log on both sides:

$\log y = \log a + x \log b$

Let $Y = \log y$, $A = \log a$, $B = \log b$. Then we get : $Y = A + xB$

$n = 5$

x_i	y_i	$Y_i = \log y_i$	x_i^2	$x_i Y_i$
2	144	2.1584	4	4.3168
3	172.8	2.2375	9	6.7125
4	207.4	2.3168	16	9.2672
5	248	2.3945	25	11.9725
6	298.6	2.4751	36	14.8506
20	1070.8	11.5823	90	47.1196

$$\sum_{i=1}^{5} x_i Y_i = B \sum_{i=1}^{5} x_i^2 + A \sum_{i=1}^{5} x_i$$

$\Rightarrow 47.1196 = 90B + 20A$ —— (1)

$$\sum_{i=1}^{5} Y_i = B \sum_{i=1}^{5} x_i + 5A$$

$\Rightarrow 11.5823 = 20B + 5A$ —— (2)

Solving (1) and (2):

$A = 2$, $B = 0.079$

$\therefore Y = 2 + 0.079x$

Then $\log a = 2$

$a = 10^2 = 100$

$\log b = 0.079$

$b = 10^{0.079}$

$\underline{b = 1.2}$

Required equation is : $\underline{y = (100)(1.2)^x}$

Program

```
x <- c(5,10,15,20,25,30) ; x
y <- c(7,17,27,37,47,57); y
lm(y~x)  # Y on X
```

Output

```
Call:
lm(formula = y ~ x)

Coefficients:
(Intercept)          x
         -3          2
```

5. Fit a straight line of the form $y = ax + b$.

x	5	10	15	20	25	30
y	7	17	27	37	47	57

Solution:

Let $u = \frac{x-15}{5}$ and $v = \frac{y-27}{10}$

Then the equation becomes $v = a'u + b'$ — ⓐ

x_i	$u_i = (x_i - 15)/5$	u_i^2	y_i	$v_i = (y_i - 27)/10$	$u_i v_i$
5	-2	4	7	-2	4
10	-1	1	17	-1	1
15	0	0	27	0	0
20	1	1	37	1	1
25	2	4	47	2	4
30	3	9	57	[illegible]	9
	3	19		3	19

$$\sum_{i=1}^{6} u_i v_i = a' \sum_{i=1}^{6} u_i^2 + b' \sum_{i=1}^{6} u_i$$

$$\Rightarrow 19 = 19a' + 3b' \quad \text{—— (1)}$$

$$\sum_{i=1}^{6} v_i = a' \sum_{i=1}^{6} u_i + nb'$$

$$\Rightarrow 3 = 3a' + 6b' \quad \text{—— (2)}$$

Solving (1) and (2):

$a' = 1$, $b' = 0$

Substituting in ⓐ:

$v = u$

i.e., $\frac{y-27}{10_2} = \frac{x-15}{5}$

$\therefore$ $y = 2x - 3$ with $a = 2$ and $b = -3$ is the required equation.