Program

```
x<-choose(7,1); x
y<-choose(12,1); y
z <-choose(4,1); z
t<-choose(23,3); t
prob<- (x*y*z)/t ;prob
```

Output

prob = 0.1897233

PRACTICAL SHEET - 6

PROBABILITY

1. A bag contains 7 red, 12 white, 4 blue balls. What is the probability that 3 balls are drawn, one of each color?

Solution:

Probability of drawing 3 balls, one of each color $= \dfrac{{}^{7}C_1 \times {}^{12}C_1 \times {}^{4}C_1}{{}^{23}C_3}$

$= 0.1897$

Program

```
PA <- choose(10,4)/choose(18,4); PA
PB <- choose(8,4)/ choose(18,4); PB
probrepl <- PA*PB; probrepl
PBA <- choose (8,4)/choose(14,4); PBA
probnorepl <- PA*PBA; probnorepl
```

Output

probrepl (required probability when coins are replaced) = 0.001569909

probnorepl (required probability when coins are not replaced) = 0.004799122

2. A bag contains 10 gold and 8 silver coins. 2 successive drawings from coins are made such that the coins are:

(i) replaced before the second draw

(ii) not replaced.

Find the probability that the first drawing will give 4 gold and second drawing will give 4 silver?

Solution:

Let A → drawing 4 gold, and B → drawing 4 silver

(i) If coins are replaced before second draw, then A and B are mutually exclusive events.

$$\therefore P(A \cap B) = P(A) \cdot P(B)$$

$$= \frac{{}^{10}C_4}{{}^{18}C_4} \times \frac{{}^{8}C_4}{{}^{18}C_4}$$

$$= \underline{0.0016}$$

(ii) If coins are not replaced,

$$P(A \cap B) = P(A) \cdot P(B|A) \quad \text{(not mutually exclusive)}$$

$$= \frac{{}^{10}C_4}{{}^{18}C_4} \times \frac{{}^{8}C_4}{{}^{14}C_4}$$

$$= \underline{0.0048}$$

Program

```
PE1 <- 0.8 ; PE1

PE2 <- 0.2; PE2

PAE1 <- 0.85; PAE1

PAE2 <- 0.65; PAE2

PE1A <- (PAE1 * PE1)/(PAE1* PE1 + PAE2 * PE2) ; PE1A

PE2A <- (PAE2 * PE2)/(PAE1* PE1 + PAE2* PE2) ; PE2A
```

Output

PE1A ($P(E_1|A)$) = 0.8395062

PE2A ($P(E_2|A)$) = 0.1604938

3. A company has 2 plants to manufacture a scooter. Plant I manufactures 80% of the scooter and plant II manufactures 20%. At plant I, 85 out of 100 are noted standard quality or better. At plant II, 65 out of 100 are noted as standard quality or better. What is the probability that

(i) scooter selected at random came from plant I if it is known that scooter is of standard quality?

(ii) scooter selected at random came from plant II, if it is known that scooter is of standard quality?

Solution:

Let $E_1 \rightarrow$ Scooter came from plant I

$E_2 \rightarrow$ scooter came from plant II

$A \rightarrow$ scooter is of standard quality

$P(E_1) = 0.8$, $P(E_2) = 0.2$, $P(A|E_1) = 0.85$, $P(A|E_2) = 0.65$

(i) Probability that scooter came from plant I given it is of standard quality, $P(E_1|A)$, by Bayes theorem:

$$P(E_1|A) = \frac{P(A|E_1)\cdot P(E_1)}{P(A|E_1)\cdot P(E_1) + P(A|E_2)\cdot P(E_2)}$$

$$= \underline{0.8395}$$

(ii) Probability that scooter came from plant II given it is of standard quality, $P(E_2|A) = \frac{P(A|E_2)\cdot P(E_2)}{P(A|E_1)\cdot P(E_1) + P(A|E_2)\cdot P(E_2)}$

$$= \underline{0.16}$$

Program

```
PE1<- 0.2; PE1
PE2<- 0.6; PE2
PE3<- 0.2; PE3
PAE1 <- choose(7,2)/choose(10,2); PAE1
PAE2 <-choose(4,2)/choose(10,2); PAE2
PAE3 <- choose(2,2)/ choose(10,2); PAE3
PE3A <- (PAE3*PE3)/(PAE1*PE1 + PAE2*PE2 + PAE3*PE3); PE3A
```

Output

PE3A [$P(E_3|A)$] = 0.025

4. The contents of 3 urns are as follows:

Urn 1: 7 white, 3 black balls

Urn 2: 4 white, 6 black balls

Urn 3: 2 white, 8 black balls

One of these urns is chosen at random with probabilities 0.2, 0.6, 0.2. From the chosen urn 2 balls are drawn at random without replacement. If both these balls are white, what is the probability that these are from urn 3?

Solution:

Let $E_1 \rightarrow$ urn 1 is selected $\Rightarrow P(E_1) = 0.2$

$E_2 \rightarrow$ urn 2 is selected $\Rightarrow P(E_2) = 0.6$

$E_3 \rightarrow$ urn 3 is selected $\Rightarrow P(E_3) = 0.2$

$A \rightarrow$ both balls selected are white

$P(A|E_1) = \frac{^7C_2}{^{10}C_2} = \frac{21}{45}$

$P(A|E_2) = \frac{^4C_2}{^{10}C_2} = \frac{6}{45}$

$P(A|E_3) = \frac{^2C_2}{^{10}C_2} = \frac{1}{45}$

Probability that balls are drawn from urn 3 given both balls are white, $P(E_3|A)$, by Bayes' theorem:

$$P(E_3|A) = \frac{P(A|E_3) \cdot P(E_3)}{P(A|E_1) \cdot P(E_1) + P(A|E_2) \cdot P(E_2) + P(A|E_3) \cdot P(E_3)}$$

$$= 0.02$$

5. A problem in statistics is given to students whose chance of solving the problem are $\frac{1}{2}$, $\frac{1}{2}$ and $\frac{1}{3}$. What is the chance that the problem will be solved?

Solution:

Let $A \rightarrow 1^{st}$ student solves the problem $\Rightarrow P(A) = \frac{1}{2}$

$B \rightarrow 2^{nd}$ student solves the problem $\Rightarrow P(B) = \frac{1}{2}$

$C \rightarrow 3^{rd}$ student solves the problem $\Rightarrow P(C) = \frac{1}{3}$

Probability that the problem is solved $= P(A \cup B \cup C)$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(AB) - P(BC) - P(AC) + P(ABC)$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{1}{3} - \frac{1}{4} - \frac{1}{6} - \frac{1}{6} + \frac{1}{12}$$

$$= \underline{\underline{0.83}}$$