PRACTICAL SHEET - 9

DISCRETE PROBABILITY DISTRIBUTION

1. 8 unbiased coins are tossed simultaneously. Find the probability of getting

(i) Exactly 4 heads

(ii) No heads at all.

(iii) 6 or more heads

(iv) Atmost 2 heads

(v) No. of heads ranging from 3 to 5.

Solution:

Let X denote no. of heads in throw of 8 unbiased coins.

$$f(x) = \begin{cases} \binom{8}{x}\left(\frac{1}{2}\right)^8 & ; x = 0,1,2,\ldots,8 \\ 0 & ; \text{otherwise} \end{cases}$$

(i) $P(X=4) = \binom{8}{4}\left(\frac{1}{2}\right)^8$

$= \underline{\underline{0.273}}$

(ii) $P(X=0) = \binom{8}{0}\left(\frac{1}{2}\right)^8$

$\underline{\underline{0.0039}}$

(iii) 6 or more heads : $P(X \geq 6) = \left(\frac{1}{2}\right)^8 \sum_{x=6}^{8} \binom{8}{x}$

$= \underline{\underline{0.1445}}$

(iv) Atmost 2 heads : $P(X \leq 2) = \left(\frac{1}{2}\right)^8 \sum_{x=0}^{2} \binom{8}{x}$

$= \underline{\underline{0.1445}}$

(v) No. of heads ranging from 3 to 5, $P(3 \leq X \leq 5) = P(X=3,4,5)$

$= \left(\frac{1}{2}\right)^8 \sum_{x=3}^{5} \binom{8}{x}$

$= \underline{\underline{0.7109}}$

Program

```
p1 <- dbinom(2,3,1/3); p1
p2 <- dbinom(0,3,1/3); p2
p3 <- 1-p2; p3
p4 <- 1-dbinom(3,3,1/3); p4
```

Output

p1 = 0.2222222 ($P(X=2)$)

p2 = 0.2962963 ($P(X=0)$)

p3 = 0.7037037 ($P(X\geq 1)$)

p4 = 0.962963 ($P(X\leq 2)$)

2. Probability that a batsman score century in a cricket match is $\frac{1}{3}$. Find the probability that out of 3 matches he may score century in:

(i) exactly 2 matches

(ii) no matches

(iii) atleast one match

(iv) atmost two matches

Solution:

Let X denote no. of matches in which the batsman scored a century.

$$f(x) = \begin{cases} \binom{3}{x}\left(\frac{1}{3}\right)^x\left(\frac{2}{3}\right)^{3-x} & ; x = 0,1,2,3 \\ 0 & ; \text{otherwise} \end{cases}$$

(i) Probability that he scores a century in exactly 2 matches,

$$P(X=2) = \binom{3}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)$$

$$= 0.22$$

(ii) Probability that he scores a century in no matches,

$$P(X=0) = 0.296$$

(iii) Probability that he scores a century in atleast one match,

$$P(X\geq 1) = 1 - P(X<1)$$

$$= 1 - P(X=0)$$

$$= 1 - 0.296$$

$$= 0.7037$$

(iv) Probability that he scores a century in atmost 2 matches,

$$P(X\leq 2) = 1 - P(X>2)$$

$$= 1 - P(X=3)$$

$$= 1 - \binom{3}{3}\left(\frac{1}{3}\right)^3$$

$$= 0.962$$

Program

```
p1<- dpois(0,1.5);p1
p2<- 1-(p1 +dpois(1,1.5) + dpois(2,1.5));p2
```

Output

p1 = 0.22310302 (P(X=0))

P2 = 0.1911532 (P(X>2))

3. A car firm has 2 cars which it hires out day by day. The no. of demands for a car in each day is distributed as a Poisson variate with mean 1.5. Calculate proportion of day in which

(i) neither car is used

(ii) some demand is refused

Solution:

Let X denote no. of demands.

$$f(x) = \begin{cases} \frac{e^{-1.5}(1.5)^x}{x!} & ; x = 0,1,2,\dots \\ 0 & ; \text{otherwise} \end{cases}$$

(i) Probability that neither car is used,

$P(X=0) = e^{-1.5}$

$= \underline{\underline{0.223}}$

(ii) Probability that demands are refused,

$P(X>2) = 1 - P(X \le 2)$

$= 1 - e^{-1.5}(1 + 1.5 + 1.125)$

$= \underline{\underline{0.191}}$

Program

```
x<-0:6;x
f<-c(7,12,18,20,16,11,8);f
N<-sum(f);N
mean<-sum(x*f)/N;mean
n<-6;n
p<-mean/n;p
pvalues<-dbinom(x,n,p);pvalues
ef<-round(N*pvalues);ef   # expected frequencies
df<-data.frame(x,f,ef);df
```

Output

x	f	ef
0	7	1
1	12	9
2	18	22
3	20	29
4	16	21
5	11	9
6	8	1

4. Fit a binomial distribution to the following data and calculate expected frequencies.

x	0	1	2	3	4	5	6
f	7	12	18	20	16	11	8

Solution:

x	f	xf
0	7	0
1	12	12
2	18	36
3	20	60
4	16	64
5	11	55
6	8	48

Mean, $\bar{x} = \frac{\Sigma xf}{\Sigma f}$

$= \frac{275}{92}$

$= 2.99$

$p = \frac{\bar{x}}{n}$ ($\because$ mean of Binomial distribution $= np$)

$= \frac{2.99}{6}$

$= 0.498$

$q = 1-p$

$= 0.502$

$$f(x) = \begin{cases} \binom{6}{x}(0.498)^x(0.502)^{6-x} & ; x = 0,1,2,\dots,6 \\ 0 & ; \text{otherwise} \end{cases}$$

x	Expected Frequencies $[N \times f(x)]$
0	1
1	9
2	22
3	29
4	21
5	9
6	1

Program

```
x <- 0:4 ; x
f <- c(123,59,14,3,1); f
N <- sum(f); N
mean <- sum(x*f)/N; mean
pvalues <- dpois(x,mean); pvalues
ef <- round (N*pvalues); ef
df <- data.frame(x,f,ef); df
```

Output

x	f	ef
0	123	121
1	59	61
2	14	15
3	3	3
4	1	0

5. Fit a Poisson distribution for the following data and calculate the expected frequencies.

x	0	1	2	3	4
f	123	59	14	3	1

Solution:

x	f	xf
0	123	0
1	59	59
2	14	28
3	3	9
4	1	4
	200	100

$$\lambda = \frac{\Sigma xf}{\Sigma f}$$

$$\lambda = 0.5$$

$$\therefore f(x) = \begin{cases} \frac{e^{-0.5}(0.5)^x}{x!} & ; x = 0,1,2,\ldots \\ 0 & , \text{otherwise} \end{cases}$$

x	Expected frequencies [N×f(x)]
0	121
1	61
2	15
3	3
4	0

Program

```
x <- 0:4 ; x
f <- c(123,59,14,3,1); f
N<- sum(f); N
mean <- sum(x*f)/N, mean
pvalues <- dpois(x, mean); pvalues
ef <- round (N*pvalues); ef
df <- data.frame(x,f,ef); df
```

Output

x	f	ef
0	123	121
1	59	61
2	14	15
3	3	3
4	1	0

5. Fit a Poisson distribution for the following data and calculate the expected frequencies.

x	0	1	2	3	4
f	123	59	14	3	1

Solution:

x	f	xf
0	123	0
1	59	59
2	14	28
3	3	9
4	1	4
	200	100

$$\lambda = \frac{\Sigma xf}{\Sigma f}$$

$$\lambda = 0.5$$

$$\therefore f(x) = \begin{cases} \dfrac{e^{-0.5}(0.5)^x}{x!} & ; x = 0,1,2,\ldots \\ 0 & , \text{otherwise} \end{cases}$$

x	Expected frequencies [N × f(x)]
0	121
1	61
2	15
3	3
4	0

Program

```
values <- rbinom(100, 10, 0.5); values
mean(values)
var(values)
```

Output

mean(values) = 4.81

var(values) = 2.276667

6. Draw a random sample of size 100 from binomial distribution with parameters p = 0.5 and n = 10. Also find its mean and variance.

Solution:

A random variable X following the binomial distribution with parameters n = 10 and p = 0.5, has the p.d.f:

$$f(x) = \begin{cases} \binom{10}{x}(0.5)^x(0.5)^{10-x} & ; x = 0, 1, 2, \ldots, 10 \\ 0 & ; \text{otherwise} \end{cases}$$

Program

```
data <- rpois(250,1.5); data
mean(data)
var(data)
```

Output

mean(data) = 1.388

var(data) = 1.186201

7. Draw a random sample of size 250 from Poisson distribution with mean=1.5; also find mean and variance of the samples drawn.

Solution:

A random variable X following Poisson distribution with parameter $\lambda=1.5$, has the p.d.f:

$$f(x) = \begin{cases} \dfrac{e^{-1.5}(1.5)^x}{x!} & ; x=0,1,2,\ldots \\ 0 & ; \text{otherwise} \end{cases}$$