

Program

x ← choose (7, 1); x

y ← choose (12, 1); y

z ← choose (4, 1); z

t ← choose (23, 3); t

prob ← (x * y * z) / t; prob

Output

prob = 0.1897233

PRACTICAL SHEET - 06

PROBABILITY

"A bag contains 7 red, 12 white, 4 blue balls. What is the probability that 3 balls are drawn, one of each color?"

Solution:

Probability of drawing 3 balls, one of each colour

$$= \frac{7C_1 \times 12C_1 \times 4C_1}{23C_3}$$

$$= \underline{\underline{0.1897}}$$

Program

PA \leftarrow choose (10, 4) | choose (18, 4); PA

PB \leftarrow choose (8, 4) | choose (18, 4); PB

probrept \leftarrow PA * PB; probrept

PBA \leftarrow choose (8, 4) | choose (14, 4); PBA

probnorept \leftarrow PA * PBA; probnorept

Output

probrept (probability when coins are replaced) = 0.001569409

probnorept (probability when coins are not replaced) = 0.004299122

2. A bag contains 10 gold and 8 silver coins. 2 successive drawings from coins are made such that the coins are

(i) replaced before the second draw

(ii) not replaced

Find the probability that the first drawing will give 4 gold and second drawing will give 4 silver

Solution

let A \rightarrow drawing 4 gold, and B \rightarrow drawing 4 silver

(i) If coins are replaced before second draw, then A and B are mutually exclusive events.

$$\begin{aligned}\therefore P(A \cap B) &= P(A) \cdot P(B) \\ &= \frac{10C4}{18C4} \times \frac{8C4}{18C4} \\ &= 0.0016\end{aligned}$$

(ii) If coins are not replaced

$P(A \cap B) = P(A) \cdot P(B/A)$ (not mutually exclusive)

$$\begin{aligned}&= \frac{10C4}{18C4} \times \frac{8C4}{14C4} \\ &= 0.0048\end{aligned}$$

Program

$$P_{E1} \leftarrow 0.8 ; P_{E1}$$

$$P_{E2} \leftarrow 0.2 ; P_{E2}$$

$$P_{A|E1} \leftarrow 0.85 ; P_{A|E1}$$

$$P_{A|E2} \leftarrow 0.65 ; P_{A|E2}$$

$$P_{E1|A} \leftarrow (P_{A|E1} \cdot P_{E1}) / (P_{A|E1} \cdot P_{E1} + P_{A|E2} \cdot P_{E2}) ; P_{E1|A}$$

$$P_{E2|A} \leftarrow (P_{A|E2} \cdot P_{E2}) / (P_{A|E1} \cdot P_{E1} + P_{A|E2} \cdot P_{E2}) ; P_{E2|A}$$

Output

$$P_{E1|A} (P_{E1|A}) = 0.8395062$$

$$P_{E2|A} (P_{E2|A}) = 0.1604938$$

3. A company has 2 plants to manufacture a scooter. Plant I manufactures 80% of the scooter and plant II manufactures 20%. At plant I, 85% out of 100 are noted standard quality is better. At plant II, 65% out of 100 are noted a standard quality or better. What is the probability that

- (i) Scooter selected at random came from plant I if it is known that scooter is of standard quality?
- (ii) Scooter selected at random came from plant II, if it is known that scooter of standard quality?

Solution: \rightarrow Let $E_1 \rightarrow$ Scooter came from plant I
 $E_2 \rightarrow$ scooter came from plant II
 $A \rightarrow$ scooter is of standard quality

$$P(E_1) = 0.8, P(E_2) = 0.2, P(A|E_1) = 0.85, P(A|E_2) = 0.65$$

- (i) Probability that scooter came from plant I given it is of standard quality, $P(E_1|A)$, by Bayes theorem

$$P(E_1|A) = \frac{P(A|E_1) \cdot P(E_1)}{P(A|E_1) \cdot P(E_1) + P(A|E_2) \cdot P(E_2)} = 0.8395$$

- (ii) Probability that scooter came from plant II given it is of standard quality,

$$P(E_2|A) = \frac{P(A|E_2) \cdot P(E_2)}{P(A|E_1) \cdot P(E_1) + P(A|E_2) \cdot P(E_2)} = 0.16$$

Program

$$PE1 \leftarrow 0.2; PE1$$

$$PE2 \leftarrow 0.6; PE2$$

$$PE3 \leftarrow 0.2; PE3$$

$$PAE1 \leftarrow \text{choose}(7, 2) / \text{choose}(10, 2); PAE1$$

$$PAE2 \leftarrow \text{choose}(4, 2) / \text{choose}(10, 2); PAE2$$

$$PAE3 \leftarrow \text{choose}(2, 2) / \text{choose}(10, 2); PAE3$$

$$PE3A \leftarrow (PAE3 + PE3) / (PAE1 + PE1 + PAE2 + PE2 + PAE3 + PE3); PE3A$$

Output

$$PE3A(PE3A) = 0.025$$

1) The contents of 3 urns are as follows

urn 1: 7 white, 3 black balls

urn 2: 4 white, 6 black balls

urn 3: 2 white, 8 black balls

One of these urn is chosen at random with probabilities 0.2, 0.6, 0.2 from the chosen urn 2 balls are drawn at random without replacement. If both these balls are white, what is the probability that these are from urn 3?

solution >

$$E_1 \rightarrow \text{urn 1 are selected} \Rightarrow P(E_1) = 0.2$$

$$E_2 \rightarrow \text{urn 2 are selected} \Rightarrow P(E_2) = 0.6$$

$$E_3 \rightarrow \text{urn 3 are selected} \Rightarrow P(E_3) = 0.2$$

A \rightarrow both balls are selected white

$$P(A|E_1) = 7C_2/10C_2$$

$$= 21/45$$

$$P(A|E_2) = 4C_2/10C_2$$

$$= 6/45$$

$$P(A|E_3) = \frac{2C_2}{10C_2}$$

$$= 1/45$$

Probability that balls are drawn from urn 3 given both balls are white, $P(E_3|A)$, by Bayes' theorem

$$P(E_3|A) = \frac{P(A|E_3)P(E_3)}{P(A|E_1)P(E_1) + P(A|E_2)P(E_2) + P(A|E_3)P(E_3)}$$

$$= 0.02$$

5. A problem in statistics is given to students whose chance of solving the problem are $\frac{1}{2}$, $\frac{1}{2}$ and $\frac{1}{3}$. What is the chance that the problem will be solved?

solution >

A \rightarrow 1st student solves the problem

B \rightarrow 2nd student solves the problem

C \rightarrow 3rd student solves the problem

Probability that the problem is solved = $P(A \cup B \cup C)$

$$\begin{aligned} P(A \cup B \cup C) &= P(A) + P(B) + P(C) - P(AB) - P(BC) - P(AC) + P(ABC) \\ &= \frac{1}{2} + \frac{1}{2} + \frac{1}{3} - \frac{1}{4} - \frac{1}{6} - \frac{1}{6} + \frac{1}{2} \\ &= \underline{\underline{0.83}} \end{aligned}$$