

Program

$x \leftarrow c(14.4, 13.9, 14.1, 14.4, 15.1, 14.1, 3.9); x$

$y \leftarrow c(5.3, 14.3, 14.1, 14.1, 15.5, 14.3, 8.5, 4.1, 4.6); y$

$\text{cor}(x, y)$

Output

$r = -0.100292$

PRACTICAL SHEET-05 (Fitting of curve)

1. Calculate the correlation coefficient

Matrix A	4.0	4.4	3.9	4.0	4.2	4.4	5.0	4.8	4.6	3.9
Matrix B	5.3	4.3	4.1	4.1	4.4	5.5	4.2	3.8	5.4	4.6

Solution >

$$\text{correlation coefficient, } r = \frac{\text{cov}(x, y)}{\sqrt{V(x)} \sqrt{V(y)}}$$

x_i	x_i^2	y_i	y_i^2	$x_i y_i$
4	16	5.3	28.09	21.2
4.4	19.36	4.3	18.49	18.92
3.9	15.21	4.1	16.81	15.99
4.0	16	4.1	16.81	16.4
4.2	17.64	4.4	19.36	18.48
4.4	19.36	5.5	30.25	24.2
5.0	25	4.2	17.64	21
4.8	23.04	3.8	14.44	18.24
4.6	21.16	5.4	29.16	24.84
3.9	15.21	4.6	21.16	17.94
43.2	187.98	45.7	208.81	197.21

$$E(x) = \frac{\sum x_i}{n} = 4.32$$

$$E(y) = \frac{\sum y_i}{n} = 4.57$$

$$E(x^2) = \frac{\sum x_i^2}{n} = 18.798$$

$$E(y^2) = \frac{\sum y_i^2}{n} = 21.221$$

$$\text{var}(x) = E(x^2) - [E(x)]^2$$

$$= 0.1356$$

$$\text{var}(y) = E(y^2) - [E(y)]^2$$

$$= 0.3361$$

$$\text{cov}(x, y) = E(xy) - E(x)E(y)$$

$$= -0.0214$$

$$\text{correlation coefficient, } r = \frac{-0.0214}{\sqrt{0.1356 \times 0.3361}}$$

$$= -0.1002$$

Program

$x \leftarrow c(3, 5, 6, 7, 10, 11); x$

$y \leftarrow c(8, 12, 11, 14, 16, 17); y$

$\text{lm}(y \sim x)$

$\text{lm}(x \sim y)$

output

Call:

$\text{lm}(\text{formula} = y \sim x)$

Coefficient:

(intercept)

x

5.543

1.067

Call:

$\text{lm}(\text{formula} = x \sim y)$

Coefficient:

(intercept)

y

-4.375

0.875

2. Fit a regression line of $\log x$ vs x on y for the following

length(x)	3	5	6	7	10	11
weight(y)	8	12	11	14	16	17

Solution >

x_i	y_i	x_i^2	y_i	$x_i y_i$
3	8	9	64	24
5	12	25	144	60
6	11	36	121	66
7	14	49	196	98
10	16	100	256	160
11	17	121	289	187
42	78	340	1620	595

$$\bar{x} = E(x) = 7$$

$$E(x^2) = 56.667$$

$$E(xy) = \sum xy / n$$

$$= 99.167$$

$$v(x) = E(x^2) - (E(x))^2$$

$$= 7.667$$

$$\bar{y} = E(y) = 13$$

$$E(y^2) = 56.667$$

$$\text{cov}(x, y) = E(xy) - E(x)E(y)$$

$$= 8.17$$

$$v(y) = E(y^2) - [E(y)]^2$$
$$= 1.333$$

Regression line of y on x

$$\Rightarrow (y - \bar{y}) = b_{yx} (x - \bar{x})$$

$$(y - \bar{y}) = \frac{\text{cov}(x, y)}{v(x)} (x - \bar{x})$$

$$y = 1.065x + 5.543$$

Regression line of x on y

$$\Rightarrow (x - \bar{x}) = b_{xy} (y - \bar{y})$$

$$x - \bar{x} = \frac{\text{cov}(x, y)}{v(y)} (y - \bar{y})$$

$$x = \underline{\underline{0.875y - 4.376}}$$

3. Find Spearman's rank correlation for the following

Rank given by 1st judge	1	6	5	8	3	2	4	7
Rank given by 2nd judge	3	5	7	4	6	3	2	1

x	y	Rank x	Rank y	d	d ²
1	3	3.5	3.5	-2.5	6.25
6	5	6	6	0	0
5	7	5	8	-3	9
8	4	8	5	3	9
3	6	3	7	-4	16
2	3	2	3.5	-1.5	2.25
4	2	4	2	2	4
7	1	7	1	6	36

Spearman's rank correlation, $\rho = 1 - \frac{6 \sum d^2}{n^2(n^2-1)}$

$$\begin{aligned}
 \rho &= 1 - \frac{6 \times 82.5}{8(64-1)} \\
 &= 1 - 495/504 \\
 &= \underline{\underline{0.01964}}
 \end{aligned}$$

4. Find the equation of the form $y = ab^x$ to the data

x	2	3	4	5	6
y	144	172.8	207.4	248	298.6

Solution $y = ab^x$

Taking log on both sides

$$\log y = \log a + x \log b$$

Let $x = \log y$, $A = \log a$, $B = \log b$.

$$Y = nA + Bx$$

x_i	y_i	$Y = \log y_i$	x_i^2	$x_i Y_i$
2	144	2.1584	4	4.3168
3	172.8	2.2375	9	6.7125
4	207.4	2.3168	16	9.2675
5	248	2.3945	25	11.9725
6	298.6	2.4751	36	14.8506
20	1070.8	11.5823	90	47.1196

$$\sum_{i=1}^5 x_i Y_i = B \sum_{i=1}^5 x_i^2 + A \sum_{i=1}^5 x_i$$

$$\Rightarrow 47.1196 = 90B + 20A \quad \text{--- (1)}$$

$$\sum_{i=1}^5 Y_i = 5A + B \sum_{i=1}^5 x_i$$

$$\Rightarrow 11.5823 = 20B + 5A \text{ --- (2)}$$

Solving (1) & (2)

$$A = 2, B = 0.079$$

$$\therefore Y = 2 + 0.079x$$

$$\text{Then } \log a = 2$$

$$a = \text{Antilog}(2) \\ = 100$$

$$\log b = 0.079$$

$$b = \text{Antilog}(0.079) \\ = 1.2$$

Required equation

$$y = (100)(1.2)^x$$

Program

$x \leftarrow c(5, 10, 15, 20, 25, 30); x$

$y \leftarrow c(7, 17, 27, 37, 47, 57); y$

$\text{lm}(y \sim x) \# Y \text{ on } X$

output

call:

$\text{lm}(\text{formula} = y \sim x)$

coefficients:

intercept x
-3 2

5. Fit a straight line of the form $y = ax + b$.

x	5	10	15	20	25	30
y	7	17	27	37	47	57

Solution >

x_i	$u_i = (x - 15)/5$	u_i^2	y_i	$v_i = (y_i - 27)/10$	$u_i v_i$
5	-2	4	7	-2	-4
10	-1	1	17	-1	-1
15	0	0	27	0	0
20	1	1	37	1	1
25	2	4	47	2	4
30	3	9	57	3	9
	3	19		3	19

let $u = \frac{x-15}{5}$ and $v = \frac{y-27}{10}$

Then, the equation becomes $v = a'u + b' \rightarrow (1)$

$$\sum_{i=1}^6 u_i v_i = a' \sum_{i=1}^6 u_i^2 + b' \sum_{i=1}^6 u_i$$

$$\Rightarrow 19 = 19a' + 3b' \rightarrow (1)$$

$$\sum_{i=1}^6 v_i = a' \sum_{i=1}^6 u_i + 6b'$$

$$\Rightarrow 3 = 3a' + 6b' \rightarrow (2)$$

Solving (1) & (2)

$$a' = 1, \quad b' = 0$$

Substituting in (a)

$$v = u$$

$$\text{i.e. } \frac{y-27}{10} = \frac{x-15}{8}$$

$y = 2x - 3$ with $a = 2$ and $b = -3$ is the required equation