

## Solution

### Approach 1: Backtracking

#### Intuition

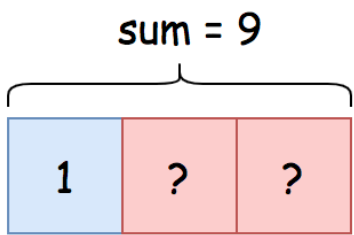
The problem asks us to come up with some fixed-length combinations that meet certain conditions.

To solve the problem, it would be beneficial to build a combination by hand.

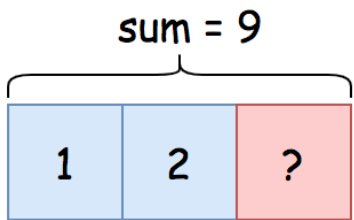
If we represent the combination as an array, we then could fill the array **one element at a time**.

For example, given the input  $k = 3$  and  $n = 9$ , *i.e.* the size of the combination is 3, and the sum of the digits in the combination should be 9. Here are a few steps that we could do:

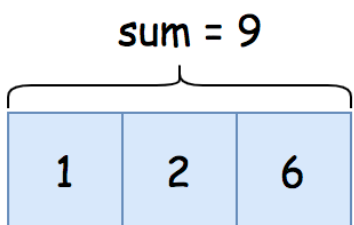
- 1). We could pick a digit for the **first** element in the combination. Initially, the list of candidates is `[1, 2, 3, 4, 5, 6, 7, 8, 9]` for any element in the combination, as stated in the problem. Let us pick `1` as the first element. The current combination is `[1]`.



- 2). Now that we picked the first element, we have two more elements to fill in the final combination. Before we proceed, let us review the conditions that we should fulfill for the next steps.
  - Since we've already picked the digit `1`, we should exclude the digit from the original candidate list for the remaining elements, in order to ensure that the combination does not contain any **duplicate** digits, as required in the problem.
  - In addition, the sum of the remaining two elements should be  $9 - 1 = 8$ .
- 3). Based on the above conditions, for the second element, we could have several choices. Let us pick the digit `2`, which is not a duplicate of the first element, plus it does not exceed the desired sum (*i.e.* 8) neither. The combination now becomes `[1, 2]`.

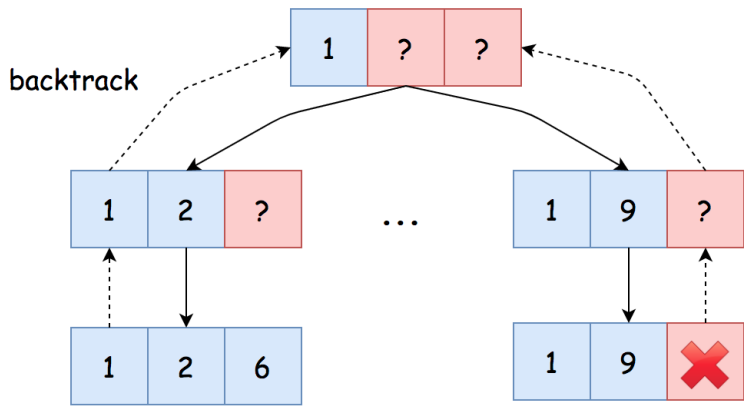


- 4). Now for the third element, with all the constraints, it leaves us no choice but to pick the digit `6` as the final element in the combination of `[1, 2, 6]`.



- 5). As we mentioned before, for the second element, we could have several choices. For instance, we could have picked the digit `3`, instead of the digit `2`. Eventually, it could *lead* us to another solution as `[1, 3, 5]`.
- 6). As one can see, for each element in the combination, we could **revisit** our choices, and **try out** other possibilities to see if it leads to a valid solution.

If you have followed the above steps, it should become *evident* that **backtracking** would be the technique that we could use to come up an algorithm for this problem.



Indeed, we could resort to **backtracking**, where we try to fill the combination **one element at a step**. Each choice we make at certain step might lead us to a final solution. If not, we simply revisit the choice and try out other choices, *i.e.* backtrack.

#### Algorithm

There are many ways to implement a backtracking algorithm. One could also refer to our Explore card (<https://leetcode.com/explore/learn/card/recursion-ii/472/backtracking/>) where we give some examples of backtracking algorithms.

To implement the algorithm, one could literally follow the steps in the Intuition section. However, we would like to highlight a key **trick** that we employed, in order to ensure the **non-redundancy** among the digits

within a single combination, as well as the ***non-redundancy*** among the combinations.

The trick is that we pick the candidates ***in order***. We treat the candidate digits as a list with order, *i.e.* [1, 2, 3, 4, 5, 6, 7, 8, 9]. At any given step, once we pick a digit, *e.g.* 6, we will not consider any digits before the chosen digit for the following steps, *e.g.* the candidates are reduced down to [7, 8, 9].

With the above strategy, we could ensure that a digit will never be picked twice for the same combination. Also, all the combinations that we come up with would be unique.

Here are some sample implementations based on the above ideas.

JavaPython3

Copy

```
1 class Solution {
2     protected void backtrack(int remain, int k,
3         LinkedList<Integer> comb, int next_start,
4         List<List<Integer>> results) {
5
6         if (remain == 0 && comb.size() == k) {
7             // Note: it's important to make a deep copy here,
8             // Otherwise the combination would be reverted in other branch of backtracking.
9             results.add(new ArrayList<Integer>(comb));
10            return;
11        } else if (remain < 0 || comb.size() == k) {
12            // Exceed the scope, no need to explore further.
13            return;
14        }
15
16        // Iterate through the reduced list of candidates.
17        for (int i = next_start; i < 9; ++i) {
18            comb.add(i + 1);
19            this.backtrack(remain - i - 1, k, comb, i + 1, results);
20            comb.removeLast();
21        }
22    }
23
24    public List<List<Integer>> combinationSum3(int k, int n) {
25        List<List<Integer>> results = new ArrayList<List<Integer>>();
26        LinkedList<Integer> comb = new LinkedList<Integer>();
27    }
```

Complexity Analysis

Let  $K$  be the number of digits in a combination.

- Time Complexity:  $\mathcal{O}\left(\frac{9! \cdot K}{(9-K)!}\right)$ 
  - In a worst scenario, we have to explore all potential combinations to the very end, *i.e.* the sum  $n$  is a large number ( $n > 9 * 9$ ). At the first step, we have 9 choices, while at the second step, we have 8 choices, so on and so forth.
  - The number of exploration we need to make in the worst case would be  $P(9, K) = \frac{9!}{(9-K)!}$ , assuming that  $K \leq 9$ . By the way,  $K$  cannot be greater than 9, otherwise we cannot have a combination whose digits are all unique.
  - Each exploration takes a constant time to process, except the last step where it takes  $\mathcal{O}(K)$  time to make a copy of combination.
  - To sum up, the overall time complexity of the algorithm would be  $\frac{9!}{(9-K)!} \cdot \mathcal{O}(K) = \mathcal{O}\left(\frac{9! \cdot K}{(9-K)!}\right)$ .
- Space Complexity:  $\mathcal{O}(K)$ 
  - During the backtracking, we used a list to keep the current combination, which holds up to  $K$  elements, *i.e.*  $\mathcal{O}(K)$ .
  - Since we employed recursion in the backtracking, we would need some additional space for the function call stack, which could pile up to  $K$  consecutive invocations, *i.e.*  $\mathcal{O}(K)$ .
  - Hence, to sum up, the overall space complexity would be  $\mathcal{O}(K)$ .
  - Note that**, we did not take into account the space for the final results in the space complexity.

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Post



(/cedrus)

cedrus (/cedrus) ★12 ⌚ September 12, 2020 4:21 AM

I think this problem involves combination. In terms of combination, the time complexity is  $O(C(9,k)) = O(9!/(k! * (9-k)!))$ . There is no duplicate in each combination, and the order of numbers doesn't matter. It can be faster since all numbers sum up to n, and the case numbers added up more than n can return earlier.

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(/clock330)

clock330 (/clock330) ★49 ⌚ September 6, 2020 10:12 AM

I am confused by the time complexity analysis:

At the first step, we have 9 choices, while at the second step, we have 8 choices, so on and so forth.

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(/magks)

magks (/magks) ★158 September 13, 2020 8:41 PM

In the python version, line 7 -- why is it necessary to `append(list(comb))` rather than just `append(comb)` ?

I tried it without wrapping comb and see that the result is appending an empty list to results but does anyone know whv this is? Both `type(comb)` and `type(list(comb))` return `<class 'list'>` and also

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1 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100 101 102 103 104 105 106 107 108 109 110 111 112 113 114 115 116 117 118 119 120 121 122 123 124 125 126 127 128 129 130 131 132 133 134 135 136 137 138 139 140 141 142 143 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183 184 185 186 187 188 189 190 191 192 193 194 195 196 197 198 199 200 201 202 203 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223 224 225 226 227 228 229 230 231 232 233 234 235 236 237 238 239 240 241 242 243 244 245 246 247 248 249 250 251 252 253 254 255 256 257 258 259 260 261 262 263 264 265 266 267 268 269 270 271 272 273 274 275 276 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