Boundedness and Stability

第五次　**SDEM 5.4**

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全章结构

稳定性种类	节	定义	V 函数判别法	系数判别法	例子
p 阶矩渐近有界	5.2	1	2	3	4,5
p 阶矩指数稳定	5.3	7	8	10,12,16	25,26,27
p 阶矩渐近稳定	5.4	28	29,30,31		32,33
a.s. 指数稳定	5.3	7	9	10,12,13,14,16	
a.s. 渐近稳定	5.4	28	29		
依概率稳定	5.5	34	35		
依概率渐近稳定	5.5	34	36		38
依概率渐近大范围稳定	5.5	34	37		
依分布渐近稳定	5.6	40	43	44	45,46

Theorem 5.24（部分思路）

$$\mathbb{E}|x(t\wedge\tau_z)|^p = \mathbb{E}\left(\xi(t\wedge\tau_z)\prod_{k=0}^{n-1}\zeta_k\right) = \mathbb{E}\left(\left(\xi(t\wedge\tau_z)\prod_{k=0}^{n-2}\zeta_k\right)\zeta_{k-1}\right) \tag{1}$$

$$= \mathbb{E}\left(\left(\xi(t\wedge\tau_z)\prod_{k=0}^{n-2}\zeta_k\right)\mathbb{E}(\zeta_{k-1}|\mathscr{G})\right) \tag{2}$$

$$= \mathbb{E}\left(\left(\xi(t\wedge\tau_z)\prod_{k=0}^{n-2}\zeta_k\right)\sum_{i\in\mathbb{S}} I_{\{r(t\wedge\tau_z)=i\}}\mathbb{E}(\zeta_{k-1}(i)|\mathscr{G})\right) \tag{3}$$

$$= \mathbb{E}\left(\left(\xi(t\wedge\tau_z)\prod_{k=0}^{n-2}\zeta_k\right)\sum_{i\in\mathbb{S}} I_{\{r(t\wedge\tau_z)=i\}}\mathbb{E}\zeta_{k-1}(i)\right) \tag{4}$$

$$= \mathbb{E}\left(\left(\xi(t\wedge\tau_z)\prod_{k=0}^{n-2}\zeta_k\right)\sum_{i\in\mathbb{S}} I_{\{r(t\wedge\tau_z)=i\}}\right) \tag{5}$$

$$= \mathbb{E}\left(\xi(t\wedge\tau_z)\prod_{k=0}^{n-2}\zeta_k\right) \tag{6}$$

$$= \cdots = \mathbb{E}\xi(t\wedge\tau_z) \tag{7}$$

5.4 Asymptotic Stability

- Definition 5.28: p 阶矩/a.s. 渐近稳定性定义
- Theorem 5.29: p 阶矩/a.s. 渐近稳定性 V 函数判别法（Step 1：与 Theorem 5.9 类似的估计方法；Step 2：区间放缩，积分转化为级数；Step 3：类似控制收敛下的求和积分换序）
- Theorems 5.30,31: p 阶矩渐近稳定性 V 函数判别法的两个推广（反证法，L^p 估计，$\mathscr{K},\Psi$ 类函数性质，Jensen 不等式）
- Examples 5.32,33: 数值算例

Theorem 5.29

已知条件

$$\to \int_0^\infty \mathbb{E}|x(t)|^p \mathrm{d}t < \infty \tag{1}$$

$$\to \int_0^\infty \mathbb{E} \sup_{t \le s \le t+\delta} |x(s)|^p \mathrm{d}t < \infty \tag{2}$$

$$\to \sum_{k=1}^\infty \mathbb{E} \sup_{\frac{k\delta}{2} \le s \le \frac{(k+1)\delta}{2}+\delta} |x(s)|^p < \infty \tag{3}$$

$$\to \mathbb{E} \sum_{k=1}^\infty \sup_{\frac{k\delta}{2} \le s \le \frac{(k+1)\delta}{2}+\delta} |x(s)|^p < \infty \tag{4}$$

Theorem 5.30

已知条件

$$\to \mathbb{E}|x(t)|^p < K \tag{1}$$

$$\to |\mathbb{E}|x(t)|^p - \mathbb{E}|x(s)|^p| \leq C(t-s) \tag{2}$$

$$\to \lim_{t\to\infty} \mathbb{E}|x(t)|^p = 0 \tag{3}$$

上上上上次的 LV 计算 (SDEM p.159)

By (5.7) and (5.9) we compute the operator LV from $\mathbb{R}^n \times \mathbb{R}_+ \times \mathbb{S}$ to $\mathbb{R}$ as follows:

$$
\begin{aligned}
LV(x,t,i) &= pq_i|x|^{p-2}x^T f(x,t,i) + \tfrac{1}{2}pq_i|x|^{p-2}|g(x,t,i)|^2 \\
&\quad + \tfrac{1}{2}p(p-2)q_i|x|^{p-4}|x^T g(x,t,i)|^2 + \sum_{j=1}^{N}\gamma_{ij}q_j|x|^p \\
&\le pq_i|x|^{p-2}\left[x^T f(x,t,i) + \frac{p-1}{2}|g(x,t,i)|^2\right] + \sum_{j=1}^{N}\gamma_{ij}q_j|x|^p \\
&\le pq_i|x|^{p-2}(\beta_i|x|^2+\alpha) + \sum_{j=1}^{N}\gamma_{ij}q_j|x|^p \\
&\le \left(p\beta_i q_i + \sum_{i=1}^{N}\gamma_{ij}q_j\right)|x|^p + \alpha p q_i|x|^{p-2} \\
&= -\lambda_i|x|^p + \alpha p q_i|x|^{p-2}.
\end{aligned}
\tag{5.10}
$$

BDG 不等式 (SDE p.127)

Theorem 7.3 *Let* $g \in \mathcal{L}^2(R_+; R^{d\times m})$. *Define, for* $t \geq 0$,

$$x(t) = \int_0^t g(s)dB(s) \qquad \text{and} \qquad A(t) = \int_0^t |g(s)|^2 ds.$$

Then for every $p > 0$, *there exist universal positive constants* c_p, C_p *(depending only on* p*), such that*

$$c_p E|A(t)|^{\frac{p}{2}} \leq E\Big(\sup_{0\leq s\leq t} |x(s)|^p\Big) \leq C_p E|A(t)|^{\frac{p}{2}} \tag{7.5}$$

for all $t \geq 0$. *In particular, one may take*

$$\begin{array}{lll} c_p = (p/2)^p, & C_p = (32/p)^{p/2} & \text{if } 0 < p < 2; \\ c_p = 1, & C_p = 4 & \text{if } p = 2; \\ c_p = (2p)^{-p/2}, & C_p = \left[p^{p+1}/2(p-1)^{p-1}\right]^{p/2} & \text{if } p > 2. \end{array}$$