The final answer is highlighted with color yellow answer
The work is presented with text color dark blue work

Question 1 :

A. Convert the following numbers to their decimal representation. Show your work.

1. 10011011_2 = **155_{10}**

   $10011011_2 = (2^0)\times 1 + (2^1)\times 1 + (2^3)\times 1 + (2^4)\times 1 + (2^7)\times 1 = 1 + 2 + 8 + 16 + 128 =$ **155_{10}**

2. 456_7 = **237_{10}**

   $456_7 = (7^0)\times 6 + (7^1)\times 5 + (7^2)\times 4 = 6 + 35 + 196 =$ **237_{10}**

3. $38A_{16}$ = **906_{10}**

   $38A_{16} = (16^0)\times 10 + (16^1)\times 8 + (16^2)\times 3 = 10 + 128 + 768 =$ **906_{10}**

4. 2214_5 = **309_{10}**

   $2214_5 = (5^0)\times 4 + (5^1)\times 1 + (5^2)\times 2 + (5^3)\times 2 = 4 + 5 + 50 + 250 =$ **309_{10}**

B. Convert the following numbers to their binary representation:

1. 69_{10} = **1000101_2**

   69 / 2 = 34 R **1** ;
   34 / 2 = 17 R **0**;
   17 / 2 = 8 R **1**;
   8 / 2 = 4 R **0**;
   4 / 2 = 2 R **0**;
   2 / 2 = 1 R **0**;
   1 / 2 = 0 R **1**;
   Then, we have **1000101_2**

2. 485_{10}= **111100101_2**
   485 / 2 = 242 R **1**;
   242 / 2 = 121 R **0**;
   121 / 2 60 R **1**;
   60 / 2 30 R **0**;
   30 / 2 = 15 R **0**;
   15 / 2 = 7 R **1**;
   7 / 2 = 3 R **1**;
   3 / 2 = 1 R **1**;

$1 / 2 = 0$ R **1**;
Then, we have **111100101_2**

3. $6D1A_{16}$ = **110110100011010_2**

6 D 1 A convert each bit to the binary representation separately
$6 / 2 = 3$ R **0**;
$3 / 2 = 1$ R **1**;
$1 / 2 = 0$ R **1**;
Therefore, 6_{16} = **110_2**

$13 / 2 = 6$ R **1;**
$6 / 2 = 3$ R **0**;
$3 / 2 = 1$ R **1**;
$1 / 2 = 0$ R **1**;
Therefore, D_{16} = **1101_2**

$1 / 2 = 0$ R **1;**
Therefore, 1_{16} = **0001_2**

$10 / 2 = 5$ R **0;**
$5 / 2 = 2$ R **1**;
$2 / 2 = 1$ R **0**;
$1 / 2 = 0$ R **1**;
Therefore, A_{16} = **1010_2**
Combine all of the binary representations above = **110 1101 0001 1010_2**

C. Convert the following numbers to their hexadecimal representation:

1. 1101011_2 = **$6B_{16}$**

110 1011 convert 4 bits to the hexadecimal representation separately
$110_2 = (2^0)\times 0 + (2^1)\times 1 + (2^2)\times 1 = 2 + 4 = 6 = 6_{16}$
$1011_2 = (2^0)\times 1 + (2^1)\times 1 + (2^2)\times 0 + (2^3)\times 1 = 1 + 2 + 8 = 11 = B_{16}$
Combine all of the hexadecimal representations above = **$6B_{16}$**

2. 895_{10} = **$37F_{16}$**

$895 / 16 = 55$ R **15**;
$55 / 16 = 3$ R **7**;
$3 / 16 = 0$ R **3;**
Then we have **$37F_{16}$**

Question 2 :

Solve the following, do all calculations in the given base. Show your work.

1. $7566_8 + 4515_8 =$ **14303_8**

$$\begin{array}{rccccc} & 1 & 1 & 1 & 1 & \\ & & 7 & 5 & 6 & 6 \\ + & & 4 & 5 & 1 & 5 \\ \hline & \mathbf{1} & \mathbf{4} & \mathbf{3} & \mathbf{0} & \mathbf{3}_8 \end{array}$$

2. $10110011_2 + 1101_2 =$ **11000000_2**

$$\begin{array}{rcccccccc} & & 1 & 1 & 1 & 1 & 1 & 1 & \\ & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ + & & & & & 1 & 1 & 0 & 1 \\ \hline & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}_2 \end{array}$$

3. $7A66_{16} + 45C5_{16} =$ **$C02B_{16}$**

$$\begin{array}{rcccc} & 1 & 1 & & \\ & 7 & A & 6 & 6 \\ + & 4 & 5 & C & 5 \\ \hline & \mathbf{C} & \mathbf{0} & \mathbf{2} & \mathbf{B}_{16} \end{array}$$

4. $3022_5 - 2433_5 =$ **34_5**

$$\begin{array}{rcccc} & & & 6 & \\ & 2 & 4 & \cancel{1} & 7 \\ & \cancel{3} & \cancel{0} & \cancel{2} & \cancel{2} \\ - & 2 & 4 & 3 & 3 \\ \hline & \mathbf{0} & \mathbf{0} & \mathbf{3} & \mathbf{4}_5 \end{array}$$

Question 3:

A. Convert the following numbers to their 8-bits two's complement representation. Show your work.

1. 124_{10} = $\mathbf{\underline{01111100}}_{8\ bit\ 2's\ comp}$

   124 / 2 = 62 R **0**;
   62 / 2 = 31 R **0**;
   31 / 2 = 15 R **1**;
   15 / 2 = 7 R **1**;
   7 / 2 = 3 R **1**;
   3 / 2 = 1 R **1**;
   1 / 2 = 0 R **1**;
   Because it's a positive number, we set the first bit as 0.
   Then we have $\mathbf{\underline{01111100}}_{8\ bit\ 2's\ comp}$

2. -124_{10} = $\mathbf{\underline{10000100}}_{8\ bit\ 2's\ comp}$

   According to the answer above, we know that the 8-bits two's complement representation of the positive 124_{10} is 01111100. To make the sum of the 8-bits two's complement representation of 124 and -124 equal to 2^8 , we can convert the bit value 1 to 0, 0 to 1, from the leftmost bit until the rightmost bit 1 to obtain the binary representation of -124. Therefore, we have $\mathbf{\underline{10000100}}_{8\ bit\ 2's\ comp}$

3. 109_{10} = $\mathbf{\underline{01101101}}_{8\ bit\ 2's\ comp}$

   109 / 2 = 54 R **1** ;
   54 / 2 = 27 R **0**;
   27 / 2 = 13 R **1**;
   13 / 2 = 6 R **1**;
   6 / 2 = 3 R **0**;
   3 / 2 = 1 R **1**;
   1 / 2 = 0 R **1**;
   Because it's a positive number, we set the first bit as 0.
   Then we have $\mathbf{\underline{01111100}}_{8\ bit\ 2's\ comp}$

4. -79_{10} = $\mathbf{\underline{10110001}}_{8\ bit\ 2's\ comp}$

   We first calculate the binary representation of the positive 79.
   79 / 2 = 39 R **1** ;
   39 / 2 = 19 R **1**;
   19 / 2 = 9 R **1**;
   9 / 2 = 4 R **1**;
   4 / 2 = 2 R **0**;
   2 / 2 = 1 R **0**;
   1 / 2 = 0 R **1**;
   Because it's a positive number, we set the first bit as 0. Now we have $01001111_{8\ bit\ 2's\ comp.}$ To make the sum of the 8-bits two's complement representation of 79 and -79 equal to 2^8 , we can convert the bit

value 1 to 0, 0 to 1, from the leftmost bit until the rightmost bit 1 to obtain the binary representation of -124. Therefore, we have **$\underline{10110001}_{8\text{ bit 2's comp}}$**

B. Convert the following numbers (represented as 8-bit two's complement) to their decimal representation. Show your work.

1. $00011110_{8\text{ bit 2's comp}}$ = **$\underline{30}_{10}$**

   Because the first bit is 0, it's a positive number. The remaining 7 bit 0011110 = $(2^1) \times 1 + (2^2) \times 1 + (2^3) \times 1 + (2^4) \times 1 = 2 + 4 + 8 + 16 =$ **$\underline{30}_{10}$**

2. $11100110_{8\text{ bit 2's comp}}$ = **$\underline{-26}_{10}$**

   Because the first bit is 1, it's a negative number. To make the sum of the $11100110_{8\text{ bit 2's comp.}}$ and the 8-bits two's complement representation of the positive complement number equal to 2^8, we can convert the bit value 1 to 0, 0 to 1, from the leftmost bit until the rightmost bit 1 to obtain the 8-bits two's complement representation of the positive number . New we have 00011010 = $(2^1) \times 1 + (2^3) \times 1 + (2^4) \times 1 = 2 + 8 + 16 =$ **$\underline{26}_{10}$**

3. $00101101_{8\text{ bit 2's comp}}$ = **$\underline{45}_{10}$**

   Because the first bit is 0, it's a positive number. The remaining 7 bit 0011110 = $(2^0) \times 1 + (2^2) \times 1 + (2^3) \times 1 + (2^5) \times 1 = 1 + 4 + 8 + 32 =$ **$\underline{45}_{10}$**

4. $10011110_{8\text{ bit 2's comp}}$ = **$\underline{-98}_{10}$**

   Because the first bit is 1, it's a negative number. To make the sum of the $10011110_{8\text{ bit 2's comp.}}$ and the 8-bits two's complement representation of the positive complement number equal to 2^8, we convert the bit value 1 to 0, 0 to 1, from the leftmost bit until the rightmost bit 1 to obtain the 8-bits two's complement representation of the positive number . New we have 01100010 = $(2^1) \times 1 + (2^5) \times 1 + (2^6) \times 1 = 2 + 32 + 64 = 98_{10}$.

Question 4:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.4, sections b, c

1.2.4 - b

Write a truth table for ¬(p ∨ q)

p	q	$(p \vee q)$	$\neg(p \vee q)$
T	T	T	**F**
T	F	T	**F**
F	T	T	**F**
F	F	F	**T**

1.2.4 - c

Write a truth table for r ∨ (p ∧ ¬q)

r	p	q	¬q	p ∧ ¬q	**r ∨ (p ∧ ¬q)**
T	T	T	F	F	**T**
T	T	F	T	T	**T**
T	F	T	F	F	**T**
F	T	T	F	F	**F**
T	F	F	T	F	**T**
F	T	F	T	T	**T**
F	F	T	F	F	**F**
F	F	F	T	F	**F**

2. Exercise 1.3.4, sections b, d

1.3.4 - b

Write a truth table for (p → q) → (q → p)

p	q	p → q	q → p	**(p → q) → (q → p)**
T	T	T	T	**T**
T	F	F	T	**T**
F	T	T	F	**F**
F	F	T	T	**T**

1.3.4 - d

Write a truth table for (p ↔ q) ⊕ (p ↔ ¬q)

p	q	p ↔ q	¬q	p ↔ ¬q	**(p ↔ q) ⊕ (p ↔ ¬q)**
T	T	T	F	F	**T**
T	F	F	T	T	**T**
F	T	F	F	T	**T**
F	F	T	T	F	**T**

Question 5:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.7, sections b, c

1.2.7 - b

The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.

At least two of the following forms of identification can be presented as “(birth certificate and driver’s license) or (birth certificate and marriage license) or (driver’s license and marriage license) or (all)”. Then we have $(B \land D) \lor (B \land M) \lor (D \land M) \lor (B \land D \land M)$.

A: $\mathbf{(B \land D) \lor (B \land M) \lor (D \land M) \lor (B \land D \land M)}$

1.2.7 - c

Applicants must present either a birth certificate or both a driver's license and a marriage license.

Either a birth certificate or both a driver's license and a marriage license can be presented as “(birth certificate) or (driver’s license and marriage license)”. Then we have $B \lor (D \land M)$.

A: $\mathbf{B \lor (D \land M)}$

---

2. Exercise 1.3.7, sections b – e

1.3.7 - b

A person can park in the school parking lot if they are a senior or at least seventeen years of age.

“q if p” equals to “if p, then q”. The proposition can be organized as “if ((they are a senior) or (at least seventeen)), then (can park in the school parking lot)”. Then we have $(s \lor y) \rightarrow p$.

A: $\mathbf{(s \lor y) \rightarrow p}$

1.3.7 - c

Being 17 years of age is a necessary condition for being able to park in the school parking lot.

“q is necessary to p” equals to “if p, then q”. Being 17 years of age is a necessary condition, so we know that “if (being able to park in the school parking lot), then (being 17 years of age)”. Then we have $p \rightarrow y$.

A: $\mathbf{p \rightarrow y}$

1.3.7 - d

A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.

If and only if proposition is straightforward, then we have $p \leftrightarrow (s \land y)$

A: **$p \leftrightarrow (s \land y)$**

1.3.7 - e

Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old.

“p implies q” equals to “if p, then q”, so the proposition can be organized as “if (being able to park in the school parking lot), then ((the person is either a senior) or (at least 17 years old))”. Then we have $p \rightarrow (s \lor y)$

A: **$p \rightarrow (s \lor y)$**

---

3. Exercise 1.3.9, sections c, d

1.3.9 - c

The applicant can enroll in the course only if the applicant has parental permission.

“p only if q” equals “if p, then q”. We have “if (the applicant can enroll in the course), then (the applicant has parental permission). Then we have $c \rightarrow p$

A: **$c \rightarrow p$**

1.3.9 - d

Having parental permission is a necessary condition for enrolling in the course.

“q is necessary to p” equals to “if p, then q”. We know “if (enrolling in the course), then (having parental permission). We have $c \rightarrow p$.

A: **$c \rightarrow p$**

Question 6:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.3.6, sections b - d

1.3.6 - b

Maintaining a B average is necessary for Joe to be eligible for the honors program.

“q is necessary to p” equals to “if p, then q”. Now we can reorganize the proposition as “ If Joe is eligible for the honor program, then he maintains a B average”.

A: If Joe is eligible for the honor program, then he maintains a B average.

1.3.6 - c

Rajiv can go on the roller coaster only if he is at least four feet tall.

“p only if q” equals to “if p, then q”. Now we can reorganize the proposition as “If Rajiv can go on the roller coaster, then he is at least four feet tall”.

A: If Rajiv can go on the roller coaster, then he is at least four feet tall.

1.3.6 - d

Rajiv can go on the roller coaster if he is at least four feet tall.

“q if p” equals to “if p, then q”. Now we can reorganize the proposition as “If Rajiv is at least four feet tall, then he can go on the roller coaster”.

A: If Rajiv is at least four feet tall, then he can go on the roller coaster.

---

2. Exercise 1.3.10, sections c – f

The variable p is true, q is false, and the truth value for variable r is unknown.

1.3.10 - c

$(p \lor r) \leftrightarrow (q \land r)$

We know $(p \lor r)$ is true because p is true. $(q \land r)$ is false because q is false. Now we know that $(p \lor r) \leftrightarrow (q \land r)$ is false.

A: False. $(p \lor r)$ is true but $(q \land r)$ is false.

1.3.10 - d

$(p \land r) \leftrightarrow (q \land r)$

We know $(q \land r)$ is false because q is false. However, we have no idea whether $(p \lor r)$ is true or false because r is unknown even p is true. If r is true, then the expression is false. If r is false, then the expression is true.

A: Unknown, If r is true, then the expression is false. If r is false, then the expression is true.

1.3.10 - e

$p \rightarrow (r \lor q)$

We know p is true. However, we have no idea whether $(r \lor q)$ is true or false because r is unknown even q is false. If r is true, then the expression is true because the hypothesis is true and the conclusion is true as well. If r is false, then the expression is false because the hypothesis is true but the conclusion is false.

A: Unknown, If r is true, then the expression is true. If r is false, then the expression is false.

1.3.10 - f

$(p \land q) \rightarrow r$

We know $p \land q$ is false because q is false. Because the hypothesis is false, the expression is always true no matter the truth value of r.

A: True, the hypothesis $(p \land q)$ is false so the expression is always true.

Question 7:

Solve Exercise 1.4.5, sections b – d, from the Discrete Math zyBook:

j: Sally got the job.

l: Sally was late for her interview

r: Sally updated her resume.

1.4.5 - b

If Sally did not get the job, then she was late for an interview or did not update her resume.
If Sally updated her resume and was not late for her interview, then she got the job.

The first expression is $\neg j \rightarrow (l \lor \neg r)$
The second expression is $(r \land \neg l) \rightarrow j$

j	l	r	$\neg j$	$l \lor \neg r$	$\neg j \rightarrow (l \lor \neg r)$	$r \land \neg l$	$(r \land \neg l) \rightarrow j$
T	T	T	F	T	**T**	F	**T**
T	T	F	F	T	**T**	F	**T**
T	F	T	F	F	**T**	T	**T**
F	T	T	T	T	**T**	F	**T**
T	F	F	F	T	**T**	F	**T**
F	T	F	T	T	**T**	F	**T**
F	F	T	T	F	**F**	T	**F**
F	F	F	T	T	**T**	F	**T**

A: Logically equivalent.

1.4.5 - c

If Sally got the job then she was not late for her interview.
If Sally did not get the job, then she was late for her interview.

The first expression is $j \rightarrow \neg l$
The second expression is $\neg j \rightarrow l$

j	l	$\neg l$	$j \rightarrow \neg l$	$\neg j$	$\neg j \rightarrow l$
T	T	F	**F**	F	**T**
T	F	T	**T**	F	**T**
F	T	F	**T**	T	**T**
F	F	T	**T**	T	**F**

A: Not logically equivalent.

1.4.5 - d

If Sally updated her resume or she was not late for her interview, then she got the job.
If Sally got the job, then she updated her resume and was not late for her interview.

The first expression is $(r \vee \neg l) \rightarrow j$

The second expression is $j \rightarrow (r \wedge \neg l)$

j	l	r	$\neg l$	$r \vee \neg l$	$(r \vee \neg l) \rightarrow j$	$(r \wedge \neg l)$	$j \rightarrow (r \wedge \neg l)$
T	T	T	F	T	**T**	F	**F**
T	T	F	F	F	**T**	F	**F**
T	F	T	T	T	**T**	T	**T**
F	T	T	F	T	**F**	F	**T**
T	F	F	T	T	**T**	F	**F**
F	T	F	F	F	**T**	F	**T**
F	F	T	T	T	**F**	T	**T**
F	F	F	T	T	**F**	F	**T**

A:Not logically equivalent.

Question 8:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.5.2, sections c, f, i

1.5.2 - c

$(p \to q) \land (p \to r) \equiv p \to (q \land r)$

A:

$(p \to q) \land (p \to r)$	
$(\neg p \lor q) \land (p \to r)$	Conditional identity
$(\neg p \lor q) \land (\neg p \lor r)$	Conditional identity
$\neg p \lor (q \land r)$	Distributed laws
$p \to (q \land r)$	Conditional identity

1.5.2 - f

$\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$

A:

$\neg(p \lor (\neg p \land q))$	
$\neg p \land \neg(\neg p \land q)$	De Morgan's laws
$\neg p \land (\neg\neg p \lor \neg q)$	De Morgan's laws
$\neg p \land (p \lor \neg q)$	Double negation
$(\neg p \land p) \lor (\neg p \land \neg q)$	Distributed laws
$F \lor (\neg p \land \neg q)$	Complement laws
$(\neg p \land \neg q) \lor F$	Commutative laws
$\neg p \land \neg q$	Identity laws

1.5.2 - i

$(p \land q) \to r \equiv (p \land \neg r) \to \neg q$

A:

$(p \land q) \to r$	
$\neg(p \land q) \lor r$	Conditional identity
$(\neg p \lor \neg q) \lor r$	De Morgan's laws
$\neg p \lor (\neg q \lor r)$	Associative laws
$\neg p \lor (r \lor \neg q)$	Commutative laws
$(\neg p \lor r) \lor \neg q$	Associative laws
$(\neg p \lor \neg\neg r) \lor \neg q$	Double negation
$\neg(p \land \neg r) \lor \neg q$	De Morgan's laws
$(p \land \neg r) \to \neg q$	Conditional identity

---

2. Exercise 1.5.3, sections c, d

1.5.3 - c

$\neg r \lor (\neg r \to p)$

A:

$\neg r \lor (\neg r \to p)$	
$\neg r \lor (\neg\neg r \lor p)$	Conditional identity
$\neg r \lor (r \lor p)$	Double negation
$(\neg r \lor r) \lor p$	Associative laws
$T \lor p$	Complement laws
$p \lor T$	Commutative laws
T	Domination laws

1.5.3 - d

$\neg(p \rightarrow q) \rightarrow \neg q$

A:

$\neg(p \rightarrow q) \rightarrow \neg q$	
$\neg\neg(p \rightarrow q) \lor \neg q$	Conditional identity
$(p \rightarrow q) \lor \neg q$	Double negation
$(\neg p \lor q) \lor \neg q$	Conditional identity
$\neg p \lor (q \lor \neg q)$	Associative laws
$\neg p \lor T$	Complement laws
T	Domination laws

Question 9:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.6.3, sections c, d

1.6.3 - c

There is a number that is equal to its square.

“There is .. “ implies that we should use the Existential quantifier $\exists$. “A number x equals to its square” can be presented as $x = x^2$

A: $\exists x (x = x^2)$

1.6.3 - d

Every number is less than or equal to its square.

“There is .. “ implies that we should use the Universal quantifier $\forall$. “A number x is less than or equals to its square” can be presented as $x \leq x^2$

A: $\forall x (x \leq x^2)$

---

2. Exercise 1.7.4, sections b - d

1.7.4 - b

Everyone was well and went to work yesterday.

“Everyone“ implies that we should use the Universal quantifier $\forall$. “A person x was well and went to work” can be presented as $\neg S(x) \land W(x)$.

A: $\forall x (\neg S(x) \land W(x))$

1.7.4 - c

Everyone who was sick yesterday did not go to work.

“Everyone“ implies that we should use the Universal quantifier $\forall$. “A person x was sick did not go to work” means “if a person x was sick, then he did not go to work”, and it can be presented as $S(x) \rightarrow \neg W(x)$.

A: $\forall x (S(x) \rightarrow \neg W(x))$

1.7.4 - d

Yesterday someone was sick and went to work.

“Someone“ implies that we should use the Existential quantifier $\exists$. “A person x was sickl and went to work” can be presented as $S(x) \wedge W(x)$.

A: $\exists x\, (S(x) \wedge W(x))$

Question 10:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.7.9, sections c - i

1.7.9 - c

$\exists x((x = c) \rightarrow P(x))$

When $x \neq c$ (means x = a or b or d or e), the expression $((x = c) \rightarrow P(x))$ is always true because the hypothesis is false. Therefore, the quantified expression is true.

A: True.

1.7.9 - d

$\exists x(Q(x) \land R(x))$

When x= e, the expression $Q(x) \land R(x)$ is true. Therefore, the quantified expression is true.

A: True.

1.7.9 - e

$Q(a) \land P(d)$

Q(a) is true and P(d) is true as well. Therefore, the expression is true.

A: True.

1.7.9 - f

$\forall x ((x \neq b) \rightarrow Q(x))$

When x = a, c, d, e, all of the Q(x) are true. So the expression $(x \neq b) \rightarrow Q(x)$ is true. If x = b, then the hypothesis is false, so the expression $(x \neq b) \rightarrow Q(x)$ is also true. Therefore, the quantified expression is true.

A: True.

1.7.9 - g

$\forall x (P(x) \lor R(x))$

When x = c, P(c) and R(c) are both false. So the expression $P(x) \lor R(x)$ is false. Therefore, the quantified expression is false due to the counter-example: c.

A: False, counter-example: c.

1.7.9 - h

$\forall x\ (R(x) \rightarrow P(x))$

When x = e, R(e) is true, and P(e) is true. So the expression $P(x) \rightarrow R(x)$ is true. When x = a, b, c, d, all of their R(x) are false, so the expression $P(x) \rightarrow R(x)$ is true due to the false hypothesis. Therefore, the quantified expression is true.

A: True.

1.7.9 - i

$\exists x(Q(x) \lor R(x))$

When x = a or c or d or e, $Q(x) \lor R(x)$ is true. Therefore, the quantified expression is true.

A: True.

---

2. Exercise 1.9.2, sections b - i

1.9.2 - b

$\exists x\ \forall y\ Q(x, y)$

When x = 2, such that Q(2,1), Q(2,2) and Q(2,3) are all true. Therefore, the quantified expression is true.

A: True, when x = 2.

1.9.2 - c

$\exists x\ \forall y\ P(y, x)$

When x = 1, such that P(1,1), P(2,1) and P(3,1) are all true. Therefore, the quantified expression is true.

A: True.

1.9.2 - d

$\exists x\ \exists y\ S(x, y)$

There is no x and y such that for any S(x,y) is true. Therefore, the quantified expression is false.

A: False.

1.9.2 - e

$\forall x\ \exists y\ Q(x, y)$

When x = 1, there is no y such that Q(1,y) is true. Therefore, the quantified expression is false.

A: False, the counter-example: x = 1.

1.9.2 - f

$\forall x\ \exists y\ P(x, y)$

When x = 1, P(1,1) and P(1,3) are true. When x = 2, P(2,1) and P(2,3) are true. When x = 3, P(3,1) and P(3,2) are true.Therefore, the quantified expression is true.

A: True.

1.9.2 - g

$\forall x\ \forall y\ P(x, y)$

When x = 1 and y = 2, P(1,2) is false. When x = 2 and y = 2, P(2,2) is false. When x = 3 and y = 3, P(3,3) is false.Therefore, the quantified expression is false.

A: False.

1.9.2 - h

$\exists x\ \exists y\ Q(x, y)$

When x = 2 and y = 1, Q(2,1) is true. Therefore, the quantified expression is true.

A: True.

1.9.2 - i

$\forall x\ \forall y\ \neg S(x, y)$

For any x and y, S(x,y) is always false. So, ¬S(x, y) is always true. Therefore, the quantified expression is true.

A: True.

Question 11:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.10.4, sections c - g

1.10.4 - c

There are two numbers whose sum is equal to their product.

"There are two" implies that we should use two Existential quantifier $\exists$ x and $\exists$ y. For number x and number y, $x + y = x \times y$.

A: $\exists x \exists y(x + y = x \times y)$.

1.10.4 - d

The ratio of every two positive numbers is also positive.

"Every two" implies that we should use the Universal quantifier $\forall$ x and $\forall$ y. If both the numbers are positive, then their ratio is also positive. It can be presented as $(x > 0) \wedge (y > 0) \rightarrow (x / y > 0)$.

A: $\forall x \forall y((x > 0) \wedge (y > 0) \rightarrow (x / y > 0))$.

1.10.4 - e

The reciprocal of every positive number less than one is greater than one.

"Every" implies that we should use the Universal quantifier $\forall$ x. If the number is positive and less than 1, then its reciprocal is larger than 1. It can be presented as $(x > 0) \wedge (x < 1) \rightarrow (1 / x > 1)$.

A: $\forall x(((x > 0) \wedge (x < 1)) \rightarrow (1 / x > 1))$.

1.10.4 - f

There is no smallest number.

"There is" implies that we should use the Existential quantifier $\exists$ x. "Smallest" means that the number is smaller than every number, so we need to use the second quantifier $\forall$ y to express the statement. "There is a number x is smallest number" can be presented as $\exists x \forall y (x < y)$. Finally, we add the negation to express that "There is no smallest number" as $\neg \exists x \forall y (x < y)$.

A: $\neg \exists x \forall y(x < y)$ or $\forall x \exists y(x \geq y)$.

1.10.4 - g

Every number besides 0 has a multiplicative inverse.

“Every” implies that we should use the Universal quantifier $\forall x$. If the number x is not 0, then it has a multiplicative inverse. So there is a multiplicative inverse for the number, and we can use the Existential quantifier $\exists y$ to express it. $\forall x \exists y ((x \neq 0) \rightarrow (x \times y = 1))$.

A: $\forall x \exists y((x \neq 0) \rightarrow (x \times y = 1))$.

---

2. Exercise 1.10.7, sections c - f

1.10.7 - c

There is at least one new employee who missed the deadline.

“There is” implies that we should use the Existential quantifier $\exists x$. “A new employee and he missed the deadline” can be expressed as $N(x) \land D(x)$.

A: $\exists x(N(x) \land D(x))$.

1.10.7 - d

Sam knows the phone number of everyone who missed the deadline.

“everyone” implies that we should use the Universal quantifier $\forall x$. The statement can be reorganized as “if a person x missed the deadline, then Sam knows the phone number of x”. And it can be expressed as $D(x) \rightarrow P(Sam, x)$.

A: $\forall x(D(x) \rightarrow P(Sam, x))$.

1.10.7 - e

There is a new employee who knows everyone's phone number.

“There is” implies that we should use the Existential quantifier $\exists x$, and “everyone” implies that we should use a second quantifier $\forall$. The statement can be expressed as $N(x) \land P(x, y)$.

A: $\exists x \forall y(N(x) \land P(x, y))$.

1.10.7 - f

Exactly one new employee missed the deadline.

“Exactly one” implies that we should use the Existential quantifier $\exists x$, and the statement can be expressed as $N(x) \land D(x)$. To express ”exactly “, we need to use a second quantifier $\forall y$. The statement means “for all other new employees, all of them didn’t miss the deadline”, and the expression can be expressed as $\forall y(((y \neq x) \land (N(y)) \rightarrow \neg D(y))$.

A: $\exists x \forall y(N(x) \land D(x) \land (((y \neq x) \land N(y)) \rightarrow \neg D(y))$.

3. Exercise 1.10.10, sections c – f

1.10.10 - c

Every student has taken at least one class besides Math 101.

“Every” implies that we should use the Universal quantifier $\forall x$. “at least one” implies that we should use a second quantifier $\exists y$, and “besides” implies $y \neq$ Math 101. The statement can be expressed as ~~T(x, Math 101)~~ $\wedge (y \neq \text{Math 101}) \wedge T(x, y)$.

T(x, Math 101) is wrong because the statement doesn’t mention that they take Math 101, simply that they took one or more class besides Math 101

A: $\forall x \exists y(\cancel{T(x, \text{Math 101})} \wedge (y \neq \text{Math 101}) \wedge T(x, y))$.

A: $\forall x \exists y((y \neq \text{Math 101}) \wedge T(x, y))$.

1.10.10 - d

There is a student who has taken every math class besides Math 101.

“There is” implies that we should use the Existential quantifier $\exists x$, and “every” implies that we should use a second quantifier $\forall y$ and $y \neq$ Math 101. The statement means “For every math course, if the course is not Math 101, then the student takes it”. The statement can be expressed as $(y \neq \text{Math 101}) \rightarrow T(x, y)$.

A: $\exists x \forall y((y \neq \text{Math 101}) \rightarrow T(x, y))$.

1.10.10 - e

Everyone besides Sam has taken at least two different math classes.

“everyone” implies that we should use the Universal quantifier $\forall x$, and “every two” implies that we should use a second quantifier $\exists y$ and $\exists z$. The statement means “For every math course, if the person is not Sam, then he has taken at least two different math classes”. The statement can be expressed as $(x \neq \text{Sam}) \rightarrow ((y \neq z) \wedge T(x, y) \wedge T(x, z))$.

A: $\forall x \exists y \exists z((x \neq \text{Sam}) \rightarrow ((y \neq z) \wedge T(x, y) \wedge T(x, z)))$.

1.10.10 - f

Sam has taken exactly two math classes.

“Exactly two” implies that we should use two Existential quantifier $\exists x \exists y$, and Sam has taken x and y. Moreover, “exactly” implies that x and y are different and we need to use the third quantifier $\forall z$. The statement means “For other math courses z, if z is not x and z is not y, then Sam didn’t take z”. The statement can be expressed as $((z \neq x) \wedge (z \neq y) \rightarrow (\neg T(\text{Sam}, z))$.

A: $\exists x \exists y \forall z((x \neq y) \wedge T(Sam, x) \wedge T(Sam, y) \wedge (((z \neq x) \wedge (z \neq y)) \rightarrow (\neg T(Sam, z)))$.

Question 12:

Solve the following questions from the Discrete Math zyBook:

P(x): x was given the placebo

D(x): x was given the medication

M(x): x had migraines

1. Exercise 1.8.2, sections b – e

1.8.2 - b

Every patient was given the medication or the placebo or both.

A:

- $\forall x(D(x) \lor P(x))$
- Negation: $\neg \forall x(D(x) \lor P(x))$
- Applying De Morgan's law: $\exists x(\neg D(x) \land \neg P(x))$

  $\neg \forall x(D(x) \lor P(x)) \Rightarrow \exists x \neg(D(x) \lor P(x)) \Rightarrow \exists x\ (\neg D(x) \land \neg P(x))$
- English: Some patients were not given the medication and the placebo.
- English: Some patients were not given the medication and **not given** the placebo.

1.8.2 - c

There is a patient who took the medication and had migraines.

A:

- $\exists x(D(x) \land M(x))$
- Negation: $\neg \exists x(D(x) \land M(x))$
- Applying De Morgan's law: $\forall x(\neg D(x) \lor \neg M(x))$

  $\neg \exists x(D(x) \land M(x)) \Rightarrow \forall x \neg(D(x) \land M(x)) \Rightarrow \forall x\ (\neg D(x) \lor \neg M(x))$
- English: Every patient was not given the medication or the placebo.

1.8.2 - d

Every patient who took the placebo had migraines.

A:

- $\forall x(P(x) \rightarrow M(x))$
- Negation: $\neg \forall x(P(x) \rightarrow M(x))$
- Applying De Morgan's law: $\exists x(P(x) \land \neg M(x))$

  $\neg \forall x(\neg P(x) \lor M(x)) \Rightarrow \exists x \neg(\neg P(x) \lor \neg M(x)) \Rightarrow \exists x(\neg\neg P(x) \land \neg M(x))$

  $\Rightarrow \exists x(P(x) \land \neg M(x))$
- English: Some patients took the placebo ~~but~~ didn't have migraines.
- English: Some patients took the placebo and didn't have migraines.

1.8.2 - e

There is a patient who had migraines and was given the placebo.

A:

- $\exists x(M(x) \wedge P(x))$
- Negation: $\neg \exists x(M(x) \wedge P(x))$
- Applying De Morgan's law: $\forall x(\neg M(x) \vee \neg P(x))$

  $\neg \exists x(M(x) \wedge P(x)) \Rightarrow \forall x \neg(M(x) \wedge P(x)) \Rightarrow \forall x(\neg M(x) \vee \neg P(x))$
- English: Every patient didn't have migraines or was not given the placebo.

---

2. Exercise 1.9.4, sections c - e

1.9.4 - c

$\exists x \ \forall y \ (P(x, y) \rightarrow Q(x, y))$

Negation: $\neg \exists x \ \forall y \ (P(x, y) \rightarrow Q(x, y)) \Rightarrow \forall x \ \exists y \neg \ (P(x, y) \rightarrow Q(x, y))$
$\Rightarrow \forall x \ \exists y \neg(\neg P(x, y) \vee Q(x, y)) \Rightarrow \forall x \ \exists y \ (\neg\neg P(x, y) \wedge \neg Q(x, y))$
$\Rightarrow$ $\boldsymbol{\forall x \ \exists y \ (P(x, y) \wedge \neg Q(x, y))}$

A: $\forall x \ \exists y(P(x, y) \wedge \neg Q(x, y))$

1.9.4 - d

$\exists x \ \forall y \ (P(x, y) \leftrightarrow P(y, x))$

Negation: $\neg \exists x \ \forall y \ (P(x, y) \leftrightarrow P(y, x)) \Rightarrow \forall x \ \exists y \neg \ (P(x, y) \leftrightarrow P(y, x))$
$\Rightarrow \forall x \ \exists y \neg((P(x, y) \rightarrow P(y, x)) \wedge (P(y, x) \rightarrow P(x, y))$
$\Rightarrow \forall x \ \exists y \neg((\neg P(x, y) \vee P(y, x)) \wedge (\neg P(y, x) \vee P(x, y))$
$\Rightarrow \forall x \ \exists y(\neg(\neg P(x, y) \vee P(y, x)) \vee \neg(\neg P(y, x) \vee P(x, y))$
$\Rightarrow$ $\boldsymbol{\forall x \ \exists y(P(x, y) \wedge \neg P(y, x)) \vee (P(y, x) \wedge \neg P(x, y))}$

A: $\forall x \ \exists y(P(x, y) \wedge \neg P(y, x)) \vee (P(y, x) \wedge \neg P(x, y))$

1.9.4 - e

$\exists x \ \exists y \ P(x, y) \wedge \forall x \ \forall y \ Q(x, y)$

Negation: $\neg(\exists x \ \exists y \ P(x, y) \wedge \forall x \ \forall y \ Q(x, y)) \Rightarrow \neg \exists x \ \exists y \ P(x, y) \vee \neg \forall x \ \forall y \ Q(x, y)$
$\Rightarrow$ $\boldsymbol{\forall x \ \forall y \ \neg P(x, y) \vee \exists x \ \exists y \ \neg Q(x, y)}$

A: $\forall x \ \forall y \neg P(x, y) \vee \exists x \ \exists y \neg Q(x, y)$