Question 5:

Use the definition of Θ in order to show the following:

a. $5n^3 + 2n^2 + 3n = \Theta(n^3)$

Ans:
To show that $5n^3 + 2n^2 + 3n = \Theta(n^3)$, we need to find positive real constants c_1, c_2 and a positive integer n_0 such that $c_1n^3 \le 5n^3 + 2n^2 + 3n \le c_2n^3$ for every $n \ge n_0$.

We can assume $n_0 = 1$ and $n \ge n_0$ to find c_1 and c_2.
First, it's obvious that $5n^3 \le 5n^3 + 2n^2 + 3n$, so $c_1 = 5$
Second, $5n^3 + 2n^2 + 3n < 5n^3 + 2n^3 + 3n^3 = 10n^3$, so $c_2 = 10$.

Therefore, we have $c_1n^3 \le 5n^3 + 2n^2 + 3n \le c_2n^3$ for every $n \ge n_0$, when $c_1 = 5$, $c_2 = 10$, $n_0 = 1$, and $5n^3 + 2n^2 + 3n = \Theta(n^3)$.

b. $\sqrt{7n^2 + 2n - 8} = \Theta(n)$

Ans:
To show that $\sqrt{7n^2 + 2n - 8} = \Theta(n)$, we need to find positive real constants c_1, c_2 and a positive integer n_0 such that $c_1n \le \sqrt{7n^2 + 2n - 8} \le c_2n$ for every $n \ge n_0$.

We assume $2n - 8 \ge 0$ to find the n_0. $2n \ge 8$, then $n \ge 4$. So we can assume $n_0 = 4$.
Then we let $n = n_0 = 4$ and $2n - 8 = 0$ to find c_1 and c_2.
First, $\sqrt{4n^2} \le \sqrt{7n^2} \Longrightarrow \sqrt{(2n)^2} \le \sqrt{7n^2} \Longrightarrow 2n \le \sqrt{7n^2}$, so $c_1 = 2$.
Second, $\sqrt{7n^2} \le \sqrt{9n^2} \Longrightarrow \sqrt{7n^2} \le \sqrt{(3n)^2} \Longrightarrow \sqrt{7n^2} \le 3n$, so $c_2 = 3$.

Therefore, we have $c_1n \le \sqrt{7n^2 + 2n - 8} \le c_2n$ for every $n \ge n_0$, when $c_1 = 2$, $c_2 = 3$, $n_0 = 4$, and $\sqrt{7n^2 + 2n - 8} = \Theta(n)$.