The final answer is highlighted with color yellow answer
The work is presented with text color dark blue work

Question 7:

Solve the following questions from the Discrete Math zyBook:

a) Exercise 3.1.1, sections a-g

A = { x ∈ Z: x is an integer multiple of 3 }

B = { x ∈ Z: x is a perfect square }

C = { 4, 5, 9, 10 }

D = { 2, 4, 11, 14 }

E = { 3, 6, 9 }

F = { 4, 6, 16 }

(a) 27 ∈ A

A = { x ∈ Z: x is an integer multiple of 3 }, and 27 = 3 $\times$ 9. Therefore, 27 is an element of set A, and the answer is true.

Ans: True

(b) 27 ∈ B

B = { x ∈ Z: x is a perfect square } , and 27 is not a perfect square. Therefore, 27 is not an element of set B, and the answer is false.

Ans: False

(c) 100 ∈ B.

B = { x ∈ Z: x is a perfect square } , and 100 = 10 $\times$ 10. Therefore, 100 is an element of set B, and the answer is true.

Ans: True

(d) E ⊆ C or C ⊆ E.

If $A \subseteq B$, then $\forall x(x \in A \rightarrow x \in B)$. However, $3 \in E$, but $3 \notin C$. Therefore, $E \subseteq C$ is false. Moreover, $4 \in C$, but $4 \notin E$, so $C \subseteq E$ is false. The true value of (F or F) is false.

Ans: False

(e) $E \subseteq A$

If $A \subseteq B$, then $\forall x(x \in A \rightarrow x \in B)$.
$3 \in E$ and $3 = 3 \times 1$, so $3 \in A$.
$6 \in E$ and $6 = 3 \times 2$, so $6 \in A$.
$9 \in E$ and $9 = 3 \times 3$, so $9 \in A$.
Therefore, $E \subseteq A$ is true.

Ans: True

(f) $A \subset E$

If $A \subseteq B$, then $\forall x(x \in A \rightarrow x \in B)$. However, $12 = 3 \times 4$, so $12 \in A$. But $12 \notin E$. Therefore, A is not the subset of E, and of course, A is not a proper subset of E. $A \subset E$ is false.

Ans: False

(g) $E \in A$

E is a set with 3 elements. There is no element which is a set in the set A. Therefore, $E \in A$ is false.

Ans: False

---

b) Exercise 3.1.2, sections a-e

$A = \{ x \in Z: x \text{ is an integer multiple of } 3 \}$
$B = \{ x \in Z: x \text{ is a perfect square} \}$
$C = \{ 4, 5, 9, 10 \}$
$D = \{ 2, 4, 11, 14 \}$
$E = \{ 3, 6, 9 \}$
$F = \{ 4, 6, 16 \}$

(a) $15 \subset A$

15 is not a set. Therefore, 15 is not the subset of A, and of course, 15 is not a proper subset of A. $15 \subset A$ is false.

Ans: False

(b) $\{15\} \subset A$

If $A \subseteq B$, then $\forall x(x \in A \rightarrow x \in B)$. $\{15\}$ is a set and $15 \in \{15\}$. Moreover, $15 = 3 \times 5$, so $15 \in A$. Therefore, $\{15\} \subseteq A$. Moreover, $3 = 3 \times 1$, so $3 \in A$, but $3 \notin \{15\}$. Therefore, $\{15\} \subset A$ is true.

Ans: True

(c) $\varnothing \subset A$

$\varnothing$ is the subset of every set, so $\varnothing \subseteq A$ is true. Moreover, $3 = 3 \times 1$, so $3 \in A$, but $3 \notin \varnothing$. Therefore, $\varnothing \subset A$ is true.

Ans: True

(d) $A \subseteq A$

If $A \subseteq B$, then $\forall x(x \in A \rightarrow x \in B)$. It's obvious that $A \subseteq A$. Therefore, $A \subseteq A$ is true.

Ans: True

(e) $\varnothing \in B$

$\varnothing$ is not a perfect square of any number. Therefore, $\varnothing$ is not an element of set B. $\varnothing \in B$ is false.

Ans: False

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c) Exercise 3.1.5, sections b, d

(b) $\{ 3, 6, 9, 12, ... \}$

$3 = 3 \times 1$; $6 = 3 \times 2$; $9 = 3 \times 3$; $12 = 3 \times 4$.....
This set consists of the positive integers that are multiple of 3. Therefore, we can define the set as: $\{ x \in Z^+ : 3 \times x\}$. The cardinality of the set is infinite.
Ans: $\{ x \in Z^+ : x \times 3\}$, infinite
Ans: $\{ x \in Z^+ :$ x is an integer multiple of 3$\}$, infinite

(d) $\{ 0, 10, 20, 30, ..., 1000 \}$

$0 = 10 \times 0$; $10 = 10 \times 1$; $20 = 10 \times 2$; $30 = 10 \times 3$..... $1000 = 10 \times 100$
This set consists of the number from 0 to 100 which are multiple of 10. Therefore, we can define the set as: $\{ 0 \leq x \leq 100 : 10 \times x\}$. The cardinality of the set is 100 - 0 + 1= 101
Ans: $\{0 \leq x \leq 100 : x \times 10\}$, the cardinality is 101.
Ans: $\{x \in Z : 0 \leq x \leq 1000$ and x is an integer multiple of 10$\}$, the cardinality is 101.

d) Exercise 3.2.1, sections a-k

$X = \{1, \{1\}, \{1, 2\}, 2, \{3\}, 4 \}$.

(a) $2 \in X$

2 is an element of set X. Therefore, $2 \in X$ is true.

Ans: True

(b) $\{2\} \subseteq X$

If $A \subseteq B$, then $\forall x(x \in A \rightarrow x \in B)$. $\{2\}$ is a set and $2 \in \{2\}$ Moreover, $2 \in X$. Therefore, $\{2\} \subseteq X$ is true.

Ans: True

(c) $\{2\} \in X$

There is no element $\{2\}$ in the set X. Therefore, $\{2\} \in X$ is false.

Ans: False

(d) $3 \in X$

There is no element 3 in the set X. Therefore, $3 \in X$ is false.

Ans: False

(e) $\{1, 2\} \in X$

$\{1, 2\}$ is an element of set X. Therefore, $\{1, 2\} \in X$ is true.

Ans: True

(f) $\{1, 2\} \subseteq X$

If $A \subseteq B$, then $\forall x(x \in A \rightarrow x \in B)$. $\{1, 2\}$ is a set and, $1 \in \{1, 2\}$, $2 \in \{1, 2\}$ Moreover, $1 \in X$ and $2 \in X$. Therefore, $\{1, 2\} \subseteq X$ is true.

Ans: True

(g) $\{2, 4\} \subseteq X$

If $A \subseteq B$, then $\forall x(x \in A \rightarrow x \in B)$. $\{2, 4\}$ is a set and, $2 \in \{2, 4\}$, $4 \in \{2, 4\}$ Moreover, $2 \in X$ and $4 \in X$. Therefore, $\{2, 4\} \subseteq X$ is true.

Ans: True

(h) $\{2, 4\} \in X$

There is no element $\{2, 4\}$ in the set X. Therefore, $\{2, 4\} \in X$ is false.

Ans: False

(i) $\{2, 3\} \subseteq X$

If $A \subseteq B$, then $\forall x(x \in A \rightarrow x \in B)$. $\{2, 3\}$ is a set and, $2 \in \{2, 3\}$, $3 \in \{2, 3\}$ Moreover, $2 \in X$ but $3 \notin X$. Therefore, $\{2, 3\} \subseteq X$ is false.

Ans: False

(j) $\{2, 3\} \in X$

There is no element $\{2, 3\}$ in the set X. Therefore, $\{2, 3\} \in X$ is false.

Ans: False

(k) $|X| = 7$

There are 6 elements: 1, $\{1\}$, $\{1, 2\}$, 2, $\{3\}$, 4. Therefore, $|X| = 7$ is false.

Ans: False

Question 8:

Solve Exercise 3.2.4, section b from the Discrete Math zyBook.

(b) Let A = {1, 2, 3}. What is {X ∈ P(A): 2 ∈ X}?

First, we list P(A), which means the all subset of A.
The subsets of A: ∅, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}
Then, the condition of this set builder is 2 ∈ X. The subsets containing 2 are {2}, {1, 2}, {2, 3}, {1, 2, 3}.
Therefore, we have a set {{2}, {1, 2}, {2, 3}, {1, 2, 3}}.

Ans: {{2}, {1, 2}, {2, 3}, {1, 2, 3}}

Question 9:

Solve the following questions from the Discrete Math zyBook:

a) Exercise 3.3.1, sections c-e

A = {-3, 0, 1, 4, 17}
B = {-12, -5, 1, 4, 6}
C = {x ∈ Z: x is odd}
D = {x ∈ Z: x is positive}

(c) A ∩ C

A∩C = { x: x ∈ A and x ∈ C}. So x is an element in set A and x is odd. Therefore, we have -3, 1, 17, and A∩C = {-3, 1, 17}.

Ans: {-3, 1, 17}

(d) A ∪ (B ∩ C)

First, B∩C = { x: x ∈ B and x ∈ C}. So x is an element in set B and x is odd. Therefore, we have -5, 1, and B∩C = {-5, 1}. Second, A∪(B∩C) = { x: x ∈ A or x ∈ (B∩C)}. Therefore, A∪(B∩C) = {-5, -3, 0, 1, 4, 17}

Ans: {-5, -3, 0, 1, 4, 17}

(e) A ∩ B ∩ C

A∩B∩C = { x: x ∈ A and x ∈ B and x ∈ C}. So x is an element in set A and B and x is odd. Therefore, we have 1, andA∩B∩C = {1}.

Ans: {1}

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b) Exercise 3.3.3, sections a, b, e, f

Ai = {i^0, i^1, i^2}
Ci = {x ∈ R: -1/i ≤ x ≤ 1/i}

(a) $\bigcap_{i=2}^{5} Ai$

$\bigcap_{i=2}^{5} Ai$ = {x: x ∈ Ai for all i such that 2 ≤ i ≤ 5}, so we list A2~A5 below.
When i = 2, A2 = {1, 2, 4}.

When i = 3, A3 = {1, 3, 9}.
When i = 4, A4 = {1, 4, 16}.
When i = 5, A5 = {1, 5, 25}.

Based on the four sets above, 1 is the only element in the all four sets. Therefore, $\bigcap_{i=2}^{5} Ai$ ={1}

Ans: {1}

(b) $\bigcup_{i=2}^{5} Ai$

$\bigcup_{i=2}^{5} Ai$ = {x: x ∈ Ai for some i such that 2 ≤ i ≤ 5}, so we list A2~A5 below.
When i = 2, A2 = {1, 2, 4}.
When i = 3, A3 = {1, 3, 9}.
When i = 4, A4 = {1, 4, 16}.
When i = 5, A5 = {1, 5, 25}.

Based on the four sets above, we have 1, 2, 3, 4, 5, 9, 16, 25. Therefore, $\bigcup_{i=2}^{5} Ai$ ={1, 2, 3, 4, 5, 9, 16, 25}

Ans: {1, 2, 3, 4, 5, 9, 16, 25}

(e) $\bigcap_{i=1}^{100} Ci$

$\bigcap_{i=1}^{100} Ci$ = {x: x ∈ Ci for all i such that 1 ≤ i ≤ 100}, so we try to list C1 and C100 below.
When i = 1, C1 = {x ∈ R: -1 ≤ x ≤ 1}
When i = 100, C100 = {x ∈ R: -1/100 ≤ x ≤ 1/100}
We can find that when i is larger, the range of x is smaller, which means C1 contains C2, C2 contains C3… C99 contains C100. When i = 100, the elements of the set C100 are in Ci for all i such that 1 ≤ i ≤ 100, which means if x ∈ $\bigcap_{i=1}^{100} Ci$, then x ∈ C1 and x ∈ C2 and x ∈ C3… and x ∈ C100. Therefore, $\bigcap_{i=1}^{100} Ci$ ={x ∈ R: -1/100 ≤ x ≤ 1/100}

Ans: {x ∈ R: -1/100 ≤ x ≤ 1/100}

(f) $\bigcup_{i=1}^{100} Ci$

$\bigcup_{i=1}^{100} Ci$ = {x: x ∈ Ci for some i such that 1 ≤ i ≤ 100}, so we try to list C1 and C100 below.
When i = 1, C1 = {x ∈ R: -1 ≤ x ≤ 1}
When i = 100, C100 = {x ∈ R: -1/100 ≤ x ≤ 1/100}

We can find that when i is smaller, the range of x is larger. When i = 1, the elements of the set C1 are in Ci for some i such that $1 \le i \le 100$, which means if $x \in \bigcup_{i=1}^{100} Ci$, then $x \in C1$ or $x \in C2$ or $x \in C3$… or x $\in C100$. Therefore, $\bigcup_{i=1}^{100} Ci = \{x \in R: -1 \le x \le 1\}$

Ans: $\{x \in R: -1 \le x \le 1\}$

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c) Exercise 3.3.4, sections b, d

A = {a, b}
B = {b, c}

(b) P(A ∪ B)

$A \cup B$ = { x: $x \in A$ or $x \in B$}. So x is an element in set A or in set B. Therefore, we have a, b, c, and $A \cup B$ = {a, b, c}. The all subsets of $A \cup B$ are ∅, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}. We have $P(A \cup B)$ = {∅, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}}.

Ans: {∅, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}}

(d) P(A) ∪ P(B)

The all subsets of A are ∅, {a}, {b}, {a, b}. We have P(A) = {∅, {a}, {b}, {a, b}}. The all subsets of B are ∅, {b}, {c}, {b, c}. We have P(B) = {∅, {b}, {c}, {b, c}}. $P(A) \cup P(B) = \{x \in A$ or $x \in B\}$. Therefore, we have $P(A) \cup P(B)$ = ~~{∅, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}}.~~

Ans: {∅, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}}

Ans: {∅, {a}, {b}, {c}, {a, b}, {b, c}}

Question 10:

Solve the following questions from the Discrete Math zyBook:

a) Exercise 3.5.1, sections b, c

A = {tall, grande, venti}
B = {foam, no-foam}
C = {non-fat, whole}

(b) Write an element from the set B × A × C.

A1 x A2 x ... x An = { (a1, a2, ... , an) : ai ∈ Ai for all i such that 1 ≤ i ≤ n }. To find an element from the set B x A x C, we need to select an element in the order from set B, setA, setC. Therefore, (foam, tall, non-fat) is one of elements from B x A x C.

Ans: (foam, tall, non-fat)

(c) Write the set B × C using roster notation.

A x B = { (a, b) : a ∈ A and b ∈ B }. We list all combinations of B x C: (foam, non-fat), (foam, whole), (no-foam, non-fat), (no-foam, whole). Therefore, B x C = {(foam, non-fat), (foam, whole), (no-foam, non-fat), (no-foam, whole)}

Ans: {(foam, non-fat), (foam, whole), (no-foam, non-fat), (no-foam, whole)}

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b) Exercise 3.5.3, sections b, c, e

(b) $Z^2 \subseteq R^2$

Ans: True

If A ⊆ B, then ∀ x(x ∈ A → x ∈ B). We know that every integer is also a real number, so Z ⊆ R. The elements of Z2 are the pairs of integers. If (a, b) ∈ Z2, then a ∈ Z and b ∈ Z. Since Z ⊆ R, a ∈ R and b ∈ R. Therefore, (a, b) ∈ R2 and Z2 ⊆ R2 is true.

(c) $Z^2 \cap Z^3 = \varnothing$

Ans: True

The elements of Z2 are the pairs of integers. The elements of Z3 are the triples of integers. There is no common element in these two sets. Therefore, Z2 ∩ Z3 = ∅ is true.

(e) For any three sets, A, B, and C, if $A \subseteq B$, then $A \times C \subseteq B \times C$.

Ans: True

If $A \subseteq B$, then $\forall x(x \in A \rightarrow x \in B)$. So every element in A is also in B. If $(x, c) \in A \times C$, then $x \in A$ and $c \in C$. Since $A \subseteq B$, $x \in B$. Therefore, $(x, c) \in B \times C$ as well. As a result, $\forall x \forall y((x \in A \wedge y \in C) \rightarrow (x \in B \wedge y \in C))$. Therefore, $A \times C \subseteq B \times C$.

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c) Exercise 3.5.6, sections d, e

(d) $\{xy$: where $x \in \{0\} \cup \{0\}^2$ and $y \in \{1\} \cup \{1\}^2\}$

First, we list all elements of $\{0\} \cup \{0\}^2$ and $\{1\} \cup \{1\}^2$. $\{0\} \cup \{0\}^2 = \{0, 00\}$, $\{1\} \cup \{1\}^2 = \{1, 11\}$. The combination of xy = 01, 011, 001, 0011. Therefore, the roster notation is {01, 011, 001, 0011}.

Ans: {01, 011, 001, 0011}

(e) $\{xy$: $x \in \{aa, ab\}$ and $y \in \{a\} \cup \{a\}^2\}$

First, we list all elements of $\{a\} \cup \{a\}^2$. $\{a\} \cup \{a\}^2 = \{a, aa\}$. The combination of xy = aaa, aaaa, aba, abaa. Therefore, the roster notation is {aaa, aaaa, aba, abaa}.

Ans: {aaa, aaaa, aba, abaa}

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d) Exercise 3.5.7, sections c, f, g

A = {a}
B = {b, c}
C = {a, b, d}

(c) $(A \times B) \cup (A \times C)$

First, we list all elements of A x B : ab, ac. Then we list all elements of A x C : aa, ab, ad. $A \cup B = \{ x: x \in A \text{ or } x \in B\}$. So $(A \times B) \cup (A \times C)$ = {aa, ab, ac, ad}

Ans: {(a, a), (a, b), (a, c), (a, d)}

Ans: {aa, ab, ac, ad}

(f) $P(A \times B)$

First, we list all elements of A x B : ab, ac. So A x B = {ab, ac}. The power set of A x B contains its all subsets. All subsets are: ∅, {ab}, {ac}, {ab, ac}. Therefore, P(A × B) = {∅, {ab}, {ac}, {ab, ac}.

Ans: {∅, {(a, b)}, {(a, c)}, {(a, b), (a, c)}}.

Ans: {∅, {ab}, {ac}, {ab, ac}}.

(g) P(A) × P(B). Use ordered pair notation for elements of the Cartesian product.

First, we list all subsets of A : ∅ and {a}. So P(A) = {∅, {a}}. Then we list all subsets of B: ∅, {b}, {c} and {b, c}. So P(B) = {∅, {b}, {c}, {b, c}}. The ordered pair notation of P(A) × P(B) are : {(∅, ∅), (∅, {b}), (∅, {c}), (∅, {b, c}), ({a}, ∅), ({a}, {b}), ({a}, {c}), ({a}, {b, c})}

Ans: {(∅, ∅), (∅, {b}), (∅, {c}), (∅, {b, c}), ({a}, ∅), ({a}, {b}), ({a}, {c}), ({a}, {b, c})}

Question 11:

Solve the following questions from the Discrete Math zyBook:

a) Exercise 3.6.2, sections b, c

(b) $(B \cup A) \cap (\overline{B} \cup A) = A$

Ans:

$(B \cup A) \cap (\overline{B} \cup A)$	
$(A \cup B) \cap (\overline{B} \cup A)$	Commutative laws
$(A \cup B) \cap (A \cup \overline{B})$	Commutative laws
$A \cup (B \cap \overline{B})$	Distributive laws
$A \cup \varnothing$	Complement laws
A	Identity laws

(c) $\overline{A \cap \overline{B}} = \overline{A} \cup B$

Ans:

$\overline{A \cap \overline{B}}$	
$\overline{A} \cup \overline{\overline{B}}$	De Morgan's laws
$\overline{A} \cup B$	Double Complement law

---

b) Exercise 3.6.3, sections b, d

(b) $A - (B \cap A) = A$

To show that this set equation is not an identity, we need to make the right side not equal to the left side. One possible solution is making the left side be an empty set and the right side is not an empty set. So we make A = {a}, B = {a}. Then $B \cap A$ = {a}. Now $A - (B \cap A)$ = {a} - {a} = $\varnothing$. $\varnothing$ is definitely not equal to {a}, which is the set A.

Ans: If A= {a}, and B={a}, then B∩A = {a}. Now A - (B∩A) = {a} - {a} = ∅. ∅ is definitely not equal to {a}. Therefore, A - (B ∩ A) ≠ A.

(d) (B - A) ∪ A = A

To show that this set equation is not an identity, we need to make the right side not equal to the left side. One possible solution is making the right side be an empty set and the left side is not an empty set. So we make A = ∅, B = {a}. Then B - A = {a}. Now (B - A) ∪ A = {a} ∪ ∅ = {a}. {a} is definitely not equal to ∅, which is set A.

Ans: If A = ∅, B = {a}. Then B - A = {a}. Now (B - A) ∪ A = {a} - ∅ = {a}. {a} is definitely not equal to ∅, which is set A. Therefore, (B - A) ∪ A ≠ A.

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c) Exercise 3.6.4, sections b, c

(b) A ∩ (B - A) = ∅

Ans:

$A \cap (B - A)$	
$A \cap (B \cap \overline{A})$	Set subtraction law
$A \cap (\overline{A} \cap B)$	Commutative laws
$(A \cap \overline{A}) \cap B$	Associative laws
$\varnothing \cap B$	Complement laws
$B \cap \varnothing$	Commutative laws
$\varnothing$	Domination laws

(c) A ∪ (B - A) = A ∪ B

Ans:

$A \cup (B - A)$	
$A \cup (B \cap \overline{A})$	Set subtraction law
$(A \cup B) \cap (A \cup \overline{A})$	Distributive laws
$(A \cup B) \cap U$	Complement laws
$A \cup B$	Identity laws