Syntax and Semantics of Propositional Logic

Week 3 Tutorial

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Prepared based off of the notes of CS245 Instrucors, past and present.

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Outline

Plan

1. Translating English Into Propositional Logic
2. Tautologies, contradictions, and more
3. Review of Axioms
4. Proofs of Equivalence
5. The End

Translation Exercises

In this question, use the following propositional variables to translate the given sentences into well-formed formul of propositional logic.

s	:	I study for exams
g	:	I get a good grades
p	:	I will pass the class
h	:	I eat healthy food

Problem

If I study for exams, then I get good grades.

Translation Exercises

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If I study for exams, then I get good grades.

$$(s \implies q)$$

Translation Exercises

$$\begin{array}{lcl} s & : & \text{I study for exams} \\ g & : & \text{I get a good grades} \\ p & : & \text{I will pass the class} \\ h & : & \text{I eat healthy food} \end{array}$$

Problem

I do not eat healthy food whether or not I study for exams.

Translation Exercises

$$\begin{array}{lcl} s & : & \text{I study for exams} \\ g & : & \text{I get a good grades} \\ p & : & \text{I will pass the class} \\ h & : & \text{I eat healthy food} \end{array}$$

Problem

I do not eat healthy food whether or not I study for exams.

$$((s \to (\neg h)) \wedge ((\neg s) \to (\neg h))), \text{ or } (\neg h)$$

Translation Exercises

s	:	I study for exams
g	:	I get a good grades
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Problem

I will pass the class only if I get a good grades.

Translation Exercises

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\end{array}
$$

Problem

I will pass the class only if I get a good grades.

$$(p \to g)$$

Translation Exercises

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If I do not study for exams, then I get good grades only if I eat healthy food.

Translation Exercises

s : I study for exams
g : I get a good grades
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h : I eat healthy food

Problem

If I do not study for exams, then I get good grades only if I eat healthy food.

$$((\neg s) \to (g \to h))$$

Translation Exercises

s : I study for exams
g : I get a good grades
p : I will pass the class
h : I eat healthy food

Problem

I will either pass the class or eat healthy food, but not both.

Translation Exercises

s	:	I study for exams
g	:	I get a good grades
p	:	I will pass the class
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Problem

I will either pass the class or eat healthy food, but not both.

$$((p \vee h) \wedge (\neg(p \wedge h)))$$

Plan

Tautologies

Consider the following formula

$$p \vee \neg(p \wedge q)$$

which has the following truth table

Tautologies

Consider the following formula

$$p \lor \neg(p \land q)$$

which has the following truth table

p	q	$(p \land q)$	$\neg(p \land q)$	$p \lor \neg(p \land q)$
T	T			
T	F			
F	T			
F	F			

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T	F	F	T	T
F	T	F	T	T
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The formula is *always* true, so it is a tautology.

Contradictions

Consider the following formula

$$(p \leftrightarrow q) \wedge (\neg p \wedge q)$$

which has the following truth table

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F	F	T		

Contradictions

Consider the following formula

$$(p \leftrightarrow q) \land (\neg p \land q)$$

which has the following truth table

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F	F	T	F	F

The formula is *never* true, so it is a contradiction.

Semantic Entailment

Do the premises semantically entail (logically imply) the conclusion? Answer this question using a truth table. Explain.

$$\{((p \implies q) \implies r)\} \models (p \implies (q \implies r))$$

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T	T	T				
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T	T	T	T	T	T	T	$\lhd$
T	F	T	F	T	T	T	$\lhd$
T	T	F	T	F	F	F	
T	F	F	F	T	T	T	$\lhd$
F	T	T	T	T	T	T	$\lhd$
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F	T	F	T	F	F	T	
F	F	F	T	T	F	T	

Plan

Basic Rules

- Commutativity
  1. $\alpha \land \beta \equiv \beta \land \alpha$
  2. $\alpha \lor \beta \equiv \beta \lor \alpha$
  3. $\alpha \iff \beta \equiv \beta \iff \alpha$
- Associativity
  1. $\alpha \land (\beta \land \gamma) \equiv (\alpha \land \beta) \land \gamma$
  2. $\alpha \lor (\beta \lor \gamma) \equiv (\alpha \lor \beta) \lor \gamma$
- Distributivity
  1. $\alpha \lor (\beta \land \gamma) \equiv (\alpha \lor \beta) \land (\alpha \lor \gamma)$
  2. $\alpha \land (\beta \lor \gamma) \equiv (\alpha \land \beta) \lor (\alpha \land \gamma)$
- Idempotence
  1. $\alpha \lor \alpha \equiv \alpha$
  2. $\alpha \land \alpha \equiv \alpha$
- Double Negation
  1. $\neg(\neg\alpha) \equiv \alpha$
- De Morgan's Laws
  1. $\neg(\alpha \land \beta) \equiv \neg\alpha \lor \neg\beta$
  2. $\neg(\alpha \lor \beta) \equiv \neg\alpha \land \neg\beta$

More Rules

- Simplification 1 (Absorbtion)
  1. $\alpha \wedge \top \equiv \alpha$
  2. $\alpha \vee \top \equiv \top$
  3. $\alpha \wedge \bot \equiv \bot$
  4. $\alpha \vee \bot \equiv \alpha$
- Simplification 2
  1. $\alpha \vee (\alpha \wedge \beta) \equiv \alpha$
  2. $\alpha \wedge (\alpha \vee \beta) \equiv \alpha$
- Implication
  1. $\alpha \implies \beta \equiv \neg\alpha \vee \beta$
- Contrapositive
  1. $\alpha \implies \beta \equiv \neg\beta \implies \neg\alpha$
- Equivilance
  1. $\alpha \iff \beta \equiv (\alpha \implies \beta) \wedge (\beta \implies \alpha)$
- Excluded Middle
  1. $\alpha \vee \neg\alpha \equiv \top$
- Contradiction
  1. $\alpha \wedge \neg\alpha \equiv \bot$

Plan

Equivalence Examples

Show the equivalence: $(p \wedge (p \implies q)) \equiv (p \wedge q)$
Solution:

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- $(p \wedge (p \implies q))$

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Solution:

- $(p \wedge (p \implies q))$
- $\equiv (p \wedge ((\neg p) \vee q))$ Implication

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Show the equivalence: $(p \wedge (p \implies q)) \equiv (p \wedge q)$

Solution:

- $(p \wedge (p \implies q))$
- $\equiv (p \wedge ((\neg p) \vee q))$ Implication
- $\equiv ((p \wedge (\neg p)) \vee (p \wedge q))$ Distributive

Equivalence Examples

Show the equivalence: $(p \wedge (p \implies q)) \equiv (p \wedge q)$

Solution:

- $(p \wedge (p \implies q))$
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- $\equiv ((p \wedge (\neg p)) \vee (p \wedge q))$ Distributive
- $\equiv (\perp \vee (p \wedge q))$ Contradiction

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- $\equiv ((p \wedge (\neg p)) \vee (p \wedge q))$ Distributive
- $\equiv (\perp \vee (p \wedge q))$ Contradiction
- $\equiv (p \wedge q)$ Simplification I

Equivalence Examples

Show the equivalence:
$(P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R) \equiv P \wedge (\neg Q \vee R)$
Solution:

Equivalence Examples

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Solution:

- $(P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R)$

Equivalence Examples

Show the equivalence:
$(P \land Q \land R) \lor (P \land \neg Q \land R) \lor (P \land \neg Q \land \neg R) \equiv P \land (\neg Q \lor R)$
Solution:

- $(P \land Q \land R) \lor (P \land \neg Q \land R) \lor (P \land \neg Q \land \neg R)$
- $\equiv (P \land Q \land R) \lor (P \land \neg Q \land R) \lor (P \land \neg(Q \lor R))$ DeMorgan's

Equivalence Examples

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Solution:

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- $\equiv (P \wedge Q \wedge R) \vee (P \wedge \neg(Q \vee \neg R)) \vee (P \wedge \neg(Q \vee R))$ DeMorgan's

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Show the equivalence:
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- $\equiv (P \land Q \land R) \lor (P \land \neg(Q \lor \neg R)) \lor (P \land \neg(Q \lor R))$ DeMorgan's
- $\equiv (P \land Q \land R) \lor (P \land (\neg(Q \lor \neg R) \lor \neg(Q \lor R)))$ Distributivity

Equivalence Examples

Show the equivalence:
$(P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R) \equiv P \wedge (\neg Q \vee R)$
Solution:

- $(P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R)$
- $\equiv (P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg(Q \vee R))$ DeMorgan's
- $\equiv (P \wedge Q \wedge R) \vee (P \wedge \neg(Q \vee \neg R)) \vee (P \wedge \neg(Q \vee R))$ DeMorgan's
- $\equiv (P \wedge Q \wedge R) \vee (P \wedge (\neg(Q \vee \neg R) \vee \neg(Q \vee R)))$ Distributivity
- $\equiv (P \wedge (Q \wedge R)) \vee (P \wedge (\neg(Q \vee \neg R) \vee \neg(Q \vee R)))$ Add parenthesis

Equivalence Examples

Show the equivalence:
$(P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R) \equiv P \wedge (\neg Q \vee R)$
Solution:

- $(P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R)$
- $\equiv (P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg(Q \vee R))$ DeMorgan's
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- $\equiv (P \wedge Q \wedge R) \vee (P \wedge (\neg(Q \vee \neg R) \vee \neg(Q \vee R)))$ Distributivity
- $\equiv (P \wedge (Q \wedge R)) \vee (P \wedge (\neg(Q \vee \neg R) \vee \neg(Q \vee R)))$ Add parenthesis
- $\equiv P \wedge ((Q \wedge R) \vee (\neg(Q \vee \neg R) \vee \neg(Q \vee R)))$ Distributivity

Equivalence Examples

Show the equivalence:
$(P \land Q \land R) \lor (P \land \neg Q \land R) \lor (P \land \neg Q \land \neg R) \equiv P \land (\neg Q \lor R)$
Solution:

- $(P \land Q \land R) \lor (P \land \neg Q \land R) \lor (P \land \neg Q \land \neg R)$
- $\equiv (P \land Q \land R) \lor (P \land \neg Q \land R) \lor (P \land \neg(Q \lor R))$ DeMorgan's
- $\equiv (P \land Q \land R) \lor (P \land \neg(Q \lor \neg R)) \lor (P \land \neg(Q \lor R))$ DeMorgan's
- $\equiv (P \land Q \land R) \lor (P \land (\neg(Q \lor \neg R) \lor \neg(Q \lor R)))$ Distributivity
- $\equiv (P \land (Q \land R)) \lor (P \land (\neg(Q \lor \neg R) \lor \neg(Q \lor R)))$ Add parenthesis
- $\equiv P \land ((Q \land R) \lor (\neg(Q \lor \neg R) \lor \neg(Q \lor R)))$ Distributivity
- $\equiv P \land ((Q \land R) \lor \neg((Q \lor \neg R) \land (Q \lor R)))$ DeMorgan's

Equivalence Examples

Show the equivalence:
$(P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R) \equiv P \wedge (\neg Q \vee R)$
Solution:

- $(P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R)$
- $\equiv (P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg(Q \vee R))$ DeMorgan's
- $\equiv (P \wedge Q \wedge R) \vee (P \wedge \neg(Q \vee \neg R)) \vee (P \wedge \neg(Q \vee R))$ DeMorgan's
- $\equiv (P \wedge Q \wedge R) \vee (P \wedge (\neg(Q \vee \neg R) \vee \neg(Q \vee R)))$ Distributivity
- $\equiv (P \wedge (Q \wedge R)) \vee (P \wedge (\neg(Q \vee \neg R) \vee \neg(Q \vee R)))$ Add parenthesis
- $\equiv P \wedge ((Q \wedge R) \vee (\neg(Q \vee \neg R) \vee \neg(Q \vee R)))$ Distributivity
- $\equiv P \wedge ((Q \wedge R) \vee \neg((Q \vee \neg R) \wedge (Q \vee R)))$ DeMorgan's
- $\equiv P \wedge ((Q \wedge R) \vee \neg(Q \vee (\neg R \wedge R)))$ Distributivity

Equivalence Examples

Show the equivalence:
$(P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R) \equiv P \wedge (\neg Q \vee R)$
Solution:

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- $\equiv P \wedge ((Q \wedge R) \vee \neg((Q \vee \neg R) \wedge (Q \vee R)))$ DeMorgan's
- $\equiv P \wedge ((Q \wedge R) \vee \neg(Q \vee (\neg R \wedge R)))$ Distributivity
- $\equiv P \wedge ((Q \wedge R) \vee \neg(Q \vee (\perp)))$ Contradiction

Equivalence Examples

Show the equivalence:
$(P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R) \equiv P \wedge (\neg Q \vee R)$
Solution:

- $(P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R)$
- $\equiv (P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg(Q \vee R))$ DeMorgan's
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- $\equiv (P \wedge Q \wedge R) \vee (P \wedge (\neg(Q \vee \neg R) \vee \neg(Q \vee R)))$ Distributivity
- $\equiv (P \wedge (Q \wedge R)) \vee (P \wedge (\neg(Q \vee \neg R) \vee \neg(Q \vee R)))$ Add parenthesis
- $\equiv P \wedge ((Q \wedge R) \vee (\neg(Q \vee \neg R) \vee \neg(Q \vee R)))$ Distributivity
- $\equiv P \wedge ((Q \wedge R) \vee \neg((Q \vee \neg R) \wedge (Q \vee R)))$ DeMorgan's
- $\equiv P \wedge ((Q \wedge R) \vee \neg(Q \vee (\neg R \wedge R)))$ Distributivity
- $\equiv P \wedge ((Q \wedge R) \vee \neg(Q \vee (\bot)))$ Contradiction
- $\equiv P \wedge ((Q \wedge R) \vee \neg(Q))$ Simplification I

Equivalence Examples

Show the equivalence:
$(P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R) \equiv P \wedge (\neg Q \vee R)$

Solution (continued):

- $\equiv P \wedge ((Q \wedge R) \vee \neg Q)$ Simplification I

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Show the equivalence:
$(P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R) \equiv P \wedge (\neg Q \vee R)$
Solution (continued):

- $\equiv P \wedge ((Q \wedge R) \vee \neg Q)$ Simplification I
- $\equiv P \wedge ((\neg Q \vee Q) \wedge (\neg Q \vee R))$ Distributivity

Equivalence Examples

Show the equivalence:
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Solution (continued):

- $\equiv P \wedge ((Q \wedge R) \vee \neg Q)$ Simplification I
- $\equiv P \wedge ((\neg Q \vee Q) \wedge (\neg Q \vee R))$ Distributivity
- $\equiv P \wedge ((\top) \wedge (\neg Q \vee R))$ Excluded Middle

Equivalence Examples

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- $\equiv P \wedge ((\neg Q \vee Q) \wedge (\neg Q \vee R))$ Distributivity
- $\equiv P \wedge ((\top) \wedge (\neg Q \vee R))$ Excluded Middle
- $\equiv P \wedge (\neg Q \vee R)$ Simplification I

Plan

The end

Thats it folks. Feel free to hang out and ask questions.

These slides are based off of the tutorial notes and lecture slides provided to you online.
If you want a copy feel free to email me. The are also available on my personal website joe-scott.net

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