Tutorial 1

1 Big-O notation

Prove or disprove each of the following statements.

(a) For every constant $b > 1$, the functions $f : n \mapsto \log_2(n)$ and $g : n \mapsto \log_b(n)$ satisfy $f = \Theta(g)$.

(b) For every integer $k \geq 0$ and every constant $b > 1$, the functions $f : n \mapsto n^k$ and $g : n \mapsto b^n$ satisfy $f = o(g)$.

(c) The functions $f : n \mapsto \sqrt{\log n}$ and $g : n \mapsto \log(\sqrt{n})$ satisfy $f = O(g)$ but not $f = \Theta(g)$.

(d) The functions $f : n \mapsto (n+2)!$ and $g : n \mapsto n!$ satisfy $f = \Theta(g)$.

(e) For every functions $f, g : \mathbb{N} \to \mathbb{R}^{\geq 1}$, if $f = \Theta(g)$ then the functions $F : n \mapsto \log(f)$ and $G : n \mapsto \log(g)$ satisfy $F = \Theta(G)$.