Tutorial 3: Finding the median

Definition 1 (Median problem)**.** An instance of the *median problem* is an array $A \in \mathbb{Z}^n$ of n integers. The valid solution to such an instance is the value $y = A'\big[n/2\big]$ when A' is obtained by sorting A.[1]

The simplest solution to the median problem sorts A and outputs the $\frac{n}{2}$-th value, an algorithm with time complexity $\Theta(n \log n)$. Your goal is to design a more efficient algorithm.

1 Median using approximate median

The approximate median problem is a relaxed version of the median problem defined as follows.

Definition 2 (Approximate median problem)**.** An instance of the *approximate median problem* is an array $A \in \mathbb{Z}^n$ of n integers. A valid solution to such an instance is any value $y \in A'\big[\frac{3n}{10}\big], \ldots, A'\big[\frac{7n}{10}\big]$ when A' is obtained by sorting A.

Assume you have an ApproxMedian algorithm that solves the approximate median problem and has time complexity $\Theta(n)$. Using this algorithm and the Divide & Conquer approach, design an algorithm that solves the median problem and has time complexity $\Theta(n)$.

2 Approximate median using median

Assume that we have a Median algorithm that runs in time $\Theta(n)$, but can only find the median in sets of size at most $\frac{n}{5}$. Use this algorithm to solve the approximate median problem on instances of size n with and has time complexity $\Theta(n)$.

3 Linear-time median algorithm

Use the ideas developed above to design an algorithm that solves the median problem and has time complexity $\Theta(n)$.

[1] Technically, the median can be either $A'\big[\lfloor n/2 \rfloor\big]$ or $A'\big[\lceil n/2 \rceil\big]$ when n is even, but we'll ignore floors and ceilings for this tutorial.