Tutorial 8: Polynomial-time reductions

1 Travelling Salesman

In the HamCycle problem, we are given a graph $G = (V, E)$ and we must determine whether there is a cycle in G that goes through every vertex in V.

In the decision version of the travelling salesman problem, that we will denote by TSP, we are given a weighted graph $G = (V, E)$ with positive integer weights $w : E \to \{1, 2, \dots, L\}$ and a positive integer t; we need to determine whether there is a cycle that goes through every vertex in V and has total weight (= sum of the weights of the edges in the cycle) at most t.

Prove that HamCycle $\leq_{\mathbf{P}}$ TSP.

2 Longest Path

In the HamPath problem, we are given a graph $G = (V, E)$ and we must determine whether there is a path in G that goes through every vertex in V exactly once.

In the LongestPath problem, we are given a graph $G = (V, E)$ and a positive integer k, we must determine whether there is a simple path in G that goes through k edges.

Prove that HamPath $\leq_{\mathbf{P}}$ LongestPath.

3 Hamiltonian paths and cycles

Prove that HamCycle $\leq_{\mathbf{P}}$ HamPath.

4 Set packing

In the IndepSet problem we are given a graph $G = (V, E)$ and a positive integer k and we must determine whether there is an independent set (=set of vertices that are all disconnected from each other) of size k in G.

In the SetPacking problem, we are given a collection of subsets of $\{1, 2, \dots, m\}$ and we must determine whether there are k sets $S_1, \dots, S_k$ in the collection that are all disjoint from each other.

Prove that IndepSet $\leq_{\mathbf{P}}$ SetPacking.