Tutorial 9: 3Sat and **NP**-completeness

1 3Sat and IndepSet

We know that IndepSet is **NP**-complete since in lectures we saw that

1. IndepSet $\in$ **NP**, and
2. 3Sat $\leq_{\mathbf{P}}$ Clique $\leq_{\mathbf{P}}$ IndepSet.

Show (directly) that 3Sat $\leq_{\mathbf{P}}$ IndepSet.

2 3Sat and DomSet

A *dominating set* in a graph $G = (V, E)$ is a set $S \subseteq V$ of vertices such that every vertex in V is adjacent to at least one vertex in S in the graph G.[1]

In the DomSet problem, we are given as input a graph $G = (V, E)$ and a positive integer k and we must determine whether G has a dominating set of size at most k. For this question, you will show that DomSet is **NP**-complete.

(i) Prove that DomSet $\in$ **NP**.

(ii) Prove that 3Sat $\leq_{\mathbf{P}}$ DomSet.

3 3Sat and 3Col

In the 3Col problem, we are given as input a graph $G = (V, E)$ and we must determine if there is a way to colour the vertices in V using at most 3 colours so that each edge $(u, v) \in E$ connects vertices that have different colours. For this question, you will show that 3Col is **NP**-complete.

(i) Prove that 3Col $\in$ **NP**.

(ii) Prove that 3Sat $\leq_{\mathbf{P}}$ 3Col.

[1] Is this the same as a *vertex cover* of G? You should be able to construct a small example that convinces you the two notions are quite different.