Assignment 0 is designed to help you familiarize with the Crowdmark assignment submission process. It is an optional assignment, but it is highly recommended that you complete it. This assignment will not count in your grade, but it will be graded as if it is a regular assignment. You do not need any knowledge from Math 239 to complete this assignment, however it previews some of the concepts you will see in the course such as graphs, binary strings, counting, and colouring.

1. {4 marks}

   A *graph* is a set of elements called *vertices* and a set of pairs of distinct vertices called *edges*.

   (a) What is the largest number of edges that a graph on n vertices can have? Explain.

   (b) Suppose that the vertices can be partitioned into two sets A and B so that all edges have one end in A and the other in B. What is the largest number of edges the graph can have in terms of $|A|$ and $|B|$? Explain.

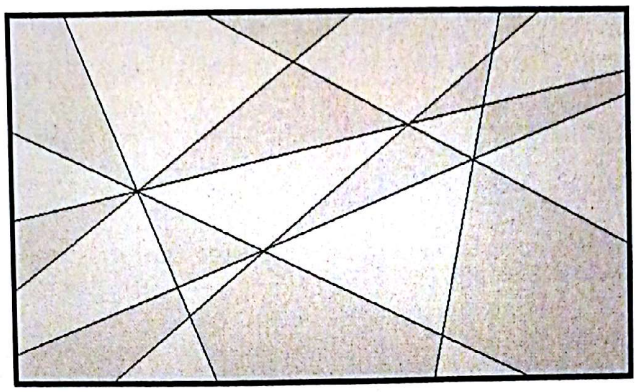
2. {4 marks} A *binary string* is a sequence of digits where each digit is either 0 or 1. The *length* of a binary string is the number of digits in the sequence.

   (a) How many binary strings are there of length n? Explain.

   (b) How many binary strings are there of length n where the first two digits are distinct? Explain.

   (c) How many binary strings are there of length n where the first three digits are all distinct? Explain.

3. {4 marks} We are given a rectangle with several line segments, each joining border to border inside of the rectangle. This divides the rectangle into several regions. We wish to colour these regions with red and blue so that any two regions that border each other through some line segment receive different colours (two regions that touch each other at only a point may have the same colour). An example is given below.



Prove (using induction on the number of line segments) that regardless of how we divide the rectangle, such a colouring is always possible.