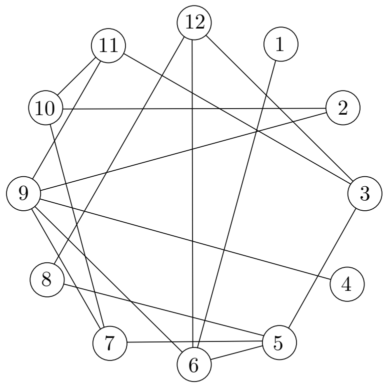
1. {6 marks} Let G be a bipartite graph with bipartition (A, B).

   (a) {2 marks} Prove that
   $$\sum_{v \in A} \deg(v) = \sum_{v \in B} \deg(v).$$

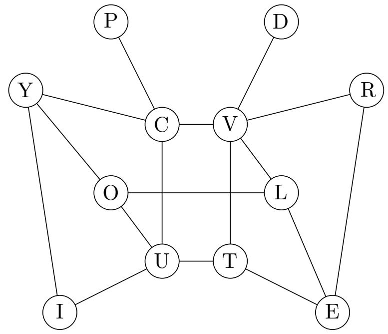
   (b) {2 marks} Let a, b be the number of odd-degree vertices in A, B respectively. Prove that $a \equiv b \pmod 2$.

   (c) {2 marks} Let $k \geq 1$ be an integer. Prove that if G is k-regular, then $|A| = |B|$.

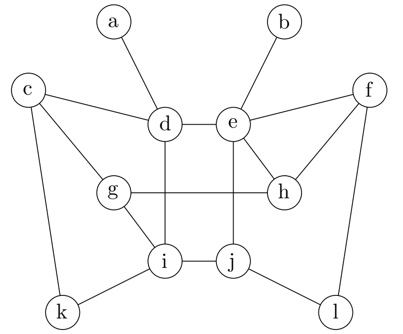
2. {6 marks} Consider the following three graphs.



Graph G



Graph H



Graph J

(a) {3 marks} Graphs G and H are isomorphic. Provide an isomorphism. (You do not need to prove that your mapping is an isomorphism.)

(b) {3 marks} Graphs H and J are not isomorphic. Explain why.

3. {4 marks} Prove that any graph with at least 2 vertices contains two vertices of the same degree. (Hint: Prove by contradiction.)

4. {5 marks} Let G be a graph where the degrees of the vertices are either 1 or 3.

   (a) {2 marks} Prove that G has even number of vertices.

   (b) {3 marks} Prove that if the number of vertices is equal to the number of edges in G, then the number of vertices of degree 1 is equal to the number of vertices of degree 3 in G.