# STAT 230 Tutorial #3

Week of June $20^{th}$, 2016

# Question (1)

(I) A box contains 5 marbles (3 green and 2 white). Three marbles are selected without replacement.

(a) Find the probability that exactly 2 out of the 3 selected are green marbles. (Answer: 0.6)

(b) Calculate the probability in (a) using the Binomial approximation.to the Hypergeometric: (Answer: 0.432. As expected, this is a poor approximation.)

(II) A box contains 5000 marbles (3500 green and 1500 white). Fifteen marbles are selected without replacement.

(a) Find the probability that exactly 10 out of the 15 selected are green marbles. (Answer: 0.2064)

(b) Calculate the probability in (a) using the Binomial approximation to the Hypergeometric. (Answer: 0.2061. As expected, the approximation here is much better.)

# Question (2)

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Friends add to you to Facebook according to a Poisson process with rate $\lambda$ per day.

(1) On any given day, the probability that nobody adds you is $0.1353$. Find $\lambda$.

$$P(X = 0) = 0.1353$$

$$e^{-\lambda} = 0.1353$$

$$\lambda = 2 \textit{ per day}$$

(2) Given that 5 friends added you in 3 days, what is the probability that 2 of them were on the first day?

$$P(2\ on\ 1^{st}\ day|\ 5\ in\ 3\ days) = \frac{P(2 on 1^{st}\ day\ and\ 3\ on\ next\ 2\ days)}{P(5\ in\ 3\ days)}$$

$$= \frac{\frac{e^{-2}2^2}{2!} \times \frac{e^{-4}4^3}{3!}}{\frac{e^{-6}6^5}{5!}}$$

$$= 0.329$$

(3) A “bad" day is when 1 or fewer friends add you. Show that the probability of a “bad" day is = 0.41. Calculate to at least 3 decimal places. Use the rounded value in the rest of this question.

$$
\begin{aligned}
P(bad) = P(X \leq 1) &= P(X = 0) + P(X = 1) \\
&= e^{-2}(\frac{2^0}{0!} + \frac{2^1}{1!}) \\
&= 0.406
\end{aligned}
$$

(4) What is the probability of having 2 “bad” days in a week (7days)?

X$\sim$Binomial$(n = 7, p = 0.41)$

$$P(X = x) = \binom{n}{x} (p)^x (1 - p)^{n-x} \quad for \;\; x = 0, 1, 2, .., n$$

$$P(X = 2) = \binom{7}{2} (0.41)^2 (1 - 0.41)^5 = 0.252$$

(5) What is the probability of having to wait at least 5 days (total) to have one “bad" day?

$$X \sim Geometric(p = 0.41)$$

$$f(x) = P(X = x) = (1 - p)^x p \quad for \;\; x = 0, 1, 2, ...$$

$$P(X \geq 4) = 1 - P(X \leq 3)$$

$$= 1 - [P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)]$$

$$= 1 - [p + p(1 - p) + p(1 - p)^2 + p(1 - p)^3]$$

$$= 1 - 0.8788 = 0.121$$

# Question (3)

Suppose the random variable X has a Geometric ($p = 0.3$) distribution.

(a) Prove that E (X) $= 2.333$. Be sure to show all your work.

$$E(X) = \sum_{x=0}^{\infty} xp(1-p)^x$$

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By the Geometric series we have

$$a\sum_{i=0}^{\infty} r^i = \frac{a}{1-r}, |r| < 1$$

By differentiating with respect to r we obtain

$$a\sum_{i=1}^{\infty} ir^{i-1} = \frac{a}{(1-r)^2}, |r| < 1$$

Therefore,

$$E(X) = \sum_{x=1}^{\infty} xp(1-p)^{x}$$

$$= p(1-p)\sum_{x=1}^{\infty} x(1-p)^{x-1}$$

$$= \frac{p(1-p)}{[1-(1-p)]^{2}}$$

$$= \frac{p(1-p)}{p^{2}}$$

$$= \frac{1-p}{p} = \frac{0.7}{0.3} = 2.333$$

(b) Prove that the probability function of the random variable Y, where Y is the remainder when X is divided by 4 is

$$f_Y(y) = 0.395(0.7)^y \quad \textit{for} \;\; y = 0, 1, 2, 3.$$

Let Y $=$ remainder of $\frac{X}{4}$, which could be

$$
Y = \begin{array}{lll} 0 & \textit{where} & x = 4n \\ 1 & \textit{where} & x = 4n+1 \\ 2 & \textit{where} & x = 4n+2 \\ 3 & \textit{where} & x = 4n+3 \end{array}
$$

For $n = 0, 1, 2, 3, ....$

The probability function then will be also defined pieccewise.

$$P(Y = 0) = \sum_{n=0}^{\infty} P(X = 4n)$$

$$= \sum_{n=0}^{\infty} p(1-p)^{4n}$$

$$= p \sum_{n=0}^{\infty} (1-p)^{4n}$$

By the Geometric series we have

$$a\sum_{n=0}^{\infty} r^n = \frac{a}{1-r}, |r| < 1$$

$$P(Y=0) = p\sum_{n=0}^{\infty} \left((1-p)^4\right)^n$$

Where $a = p$, $r = (1-p)^4$.

$$P(Y=0) = \frac{p}{1-(1-p)^4}$$

For Y=1

$$P(Y = 1) = \sum_{n=0}^{\infty} P(X = 4n + 1)$$

$$= \sum_{n=0}^{\infty} p(1 - p)^{4n+1}$$

$$P(Y = 1) = p(1 - p) \sum_{n=0}^{\infty} \left((1 - p)^4\right)^n$$

Where $a = p(1 - p)$, $r = (1 - p)^4$.

$$P(Y=1) = \frac{p(1-p)}{1-((1-p)^4)}$$

$$P(Y=2) = \frac{p(1-p)^2}{1-(1-p)^4}$$

$$P(Y=3) = \frac{p(1-p)^3}{1-(1-p)^4}$$

So,

$$f_Y(y) = \frac{p(1-p)^y}{1-(1-p)^4}$$

$$f_Y(y) = \frac{0.3(1-0.3)^y}{1-(1-0.3)^4}$$

$$f_Y(y) = \frac{0.3(0.7)^y}{1-(0.7)^4} = 0.395(0.7)^y \quad \mathit{for} \;\; y = 0, 1, 2, 3.$$