STAT 230, S16 **Tutorial #2** **Additional Questions and Solutions**
Week of June 6th 2016

Question (4)

(Ques 5.17) Assume power failures occur independently of each other at a uniform rate through the months of the year, with little chance of 2 or more occurring simultaneously. Suppose that 80% of months have no power failures.

a) Seven months are picked at random. What is the probability that 5 of these months have no power failures?
b) Months are picked at random until 5 months without power failures have been found. What is the probability that 7 months will have to be picked?
c) What is the probability a month has more than one power failure?

Question (5)

Computer system "crashes" at a large financial firm occur according to a Poisson Process with an average of λ per week.

a) Let X be the number of weeks with no "crashes" during a period of n weeks. Find the probability function of X.
b) The company wants at least 80% of weeks to have no "crashes". Find the largest value of λ.
c) What is the probability that the company experiences two "crashes" in a month?
d) Let Y be the number of weeks observed until the 10^{th} week with no "crashes" is observed. Find the probability function of Y.

Solutions

Question (4)

a) Let Y=# of months among 7 with no power failures. Y~Bin(n=7, p=0.8)
P(Y=5)=$\binom{7}{5}(0.8)^5(0.2)^2$

b) Let W=# of months with power failures until the 5^{th} one with no power failures.
W~Negative Binomial (k=5, p=0.8)
P(W=2)=$\binom{2+5-1}{2}(0.8)^5(0.2)^2 = 0.1966 = 0.20$

c) Let X=# of power failures in a month. X~Poisson(λ)

$$P(X > 1) = 1 - P(X \leq 1) = 1 - P(X = 0) - P(X = 1)$$
$$= 1 - 0.8 - \frac{e^{-\lambda}\lambda^1}{1!} = 1 - 0.8 - e^{-0.22} \times 0.22 = 0.024$$

Now to solve for λ note

P(X=0)=0.8=$e^{-\lambda}$ hence we find $\lambda = -\ln(0.8) = 0.22$

Question (5)

a) X=# of weeks with no crashes in n weeks. $x = 0,1,2,\ldots,n$

Note: X is such that it has Two outcomes:

- Success= week with "no crashes"
- Failure= week with "at least 1 Crash"

X has independent trials: trail=a week, were since we are considering non-overlapping weeks we have independence through the properties of the Poisson Process.

X is defined over multiple trials, n>1

X is defined such that we have the same P(Success) in each trial.

Where P(Success)=P(A week has 0 crashes)=$\frac{e^{-\lambda}\lambda^0}{0!}$ For each week

Therefore X~Bin(n, p=$e^{-\lambda}$)

b) Y=# of crashes observed in a week, Y~Poi(λ)

$$P(Y = 0) \geq 0.8$$

$$\frac{e^{-\lambda}\lambda^0}{0!} \geq 0.8$$

Solving for λ we find that $0 < \lambda \leq -\ln(0.8)$ per week

c) W=# of crashes observed in a month

W~Poisson($\mu = \lambda t = 4\lambda$) assuming 4 weeks in a month

P(W=2)=$\frac{e^{-4\lambda}(4\lambda)^2}{2!}$

d) Notice $y = 10,11,12,\ldots$

To find P(Y=y) note that we can define Y=Z+10 where Z=# of weeks with at "least 1 crash" (failure) until the 10th Success.

Then Z~Negative Binomial(k=10, p=P(A week has no crashes)= $e^{-\lambda}$) from (a)

$$P(Y = y) = P(Z + 10 = y) = P(Z = y - 10)$$

$$= \binom{y - 10 + 10 - 1}{y - 10} \left(e^{-\lambda}\right)^{10} \left(1 - e^{-\lambda}\right)^{y-10}$$

$$= \binom{y - 1}{y - 10} \left(e^{-\lambda}\right)^{10} \left(1 - e^{-\lambda}\right)^{y-10} \quad y = 10,11,12,\ldots$$