Chapter 8: Continuous Probability Distributions

- Terminology:
- Take on values on the real number line.
- They are treated differently than discrete r.vs because here the $P(X = x) = 0$ for each x.
- And hence probability functions alone cannot be used to describe a distribution.
- And so with continuous r.vs we specify the probability of intervals, rather than individual points.

- Why is this the case?

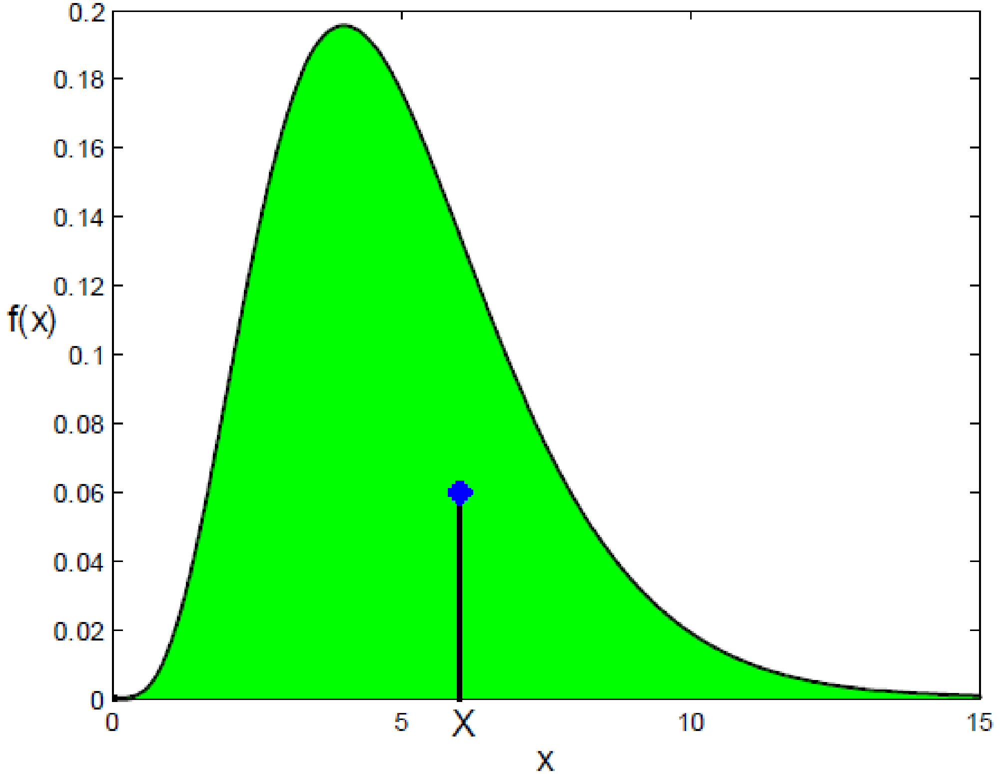

Figure 8.2: Graph of $f\ (x)$

- Again we use the cumulative distribution function, F(x), and the probability density function, $f(x)$, to describe a continuous r.v.
- **Cumulative Distribution Function (c.d.f):**
- $F(x) = P(X \leq x)$

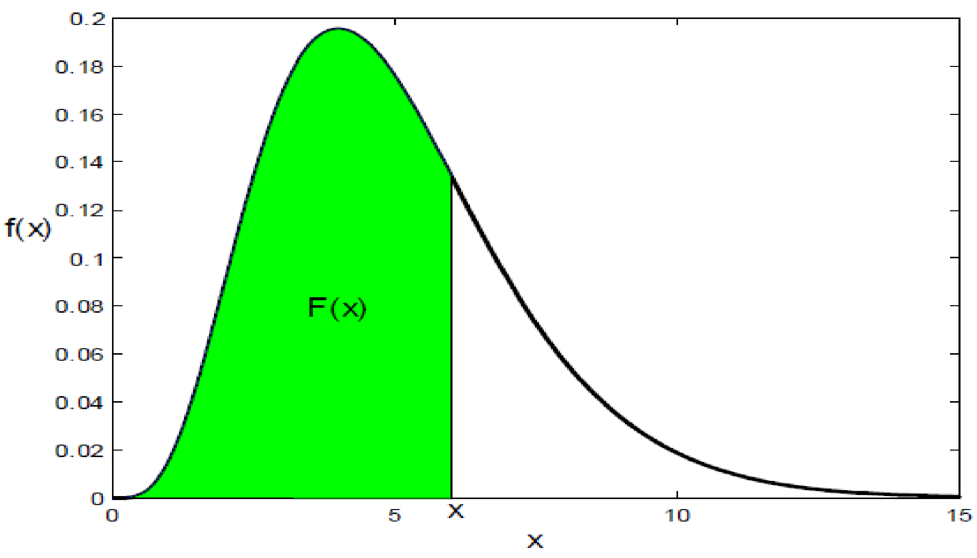

Figure 8.3: Area of shaded region equals $F(x) = P(X \leq x)$

- It satisfies the following properties:

1. $\lim_{x \to -\infty} F(x) = 0$ and $\lim_{x \to \infty} F(x) = 1$

2. $F(x)$ is a Non-decreasing function of x

3. $P(a < X \leq b) = F(b) - F(a)$

- Note: As mentioned we have
- $0 = P(X = a) = \lim_{\varepsilon \to 0} P(a - \varepsilon < X \leq a) = \lim_{\varepsilon \to 0} F(a) - F(a - \varepsilon)$
- This implies that $\lim_{\varepsilon \to 0} F(a - \varepsilon) = F(a)$
- Also because $P(X = x) = 0$ for each point x we have:

$$P(a < X < b) = P(a \leq X \leq b) = P(a \leq X < b)$$
$$= P(a < X \leq b) = F(b) - F(a)$$

- **Probability Density Function (p.d.f):**
- The c.d.f does not give an intuitive picture of which values of x are more likely, and which are less likely.
- Suppose we take a short interval of X-values, $[x, x + \Delta x]$, then the probability X lies in the interval is:

$$P(x \leq X \leq x + \Delta x) = F(x + \Delta x) - F(x)$$

- Now we divide by $\Delta x,$ and consider what happens as Δx becomes small.

- Definition: The probability density function (p.d.f) $f(x)$ for a continuous r.v X is the derivative

$$f(x) = \frac{dF(x)}{dx}$$

Where $F(x)$ is the c.d.f for X.

- Properties of a p.d.f: Assume that $f(x)$ is a continuous function of x at all points for which $0 < F(x) < 1$.

1. $P(a \leq X \leq b) = F(b) - F(a) = \int_a^b f(x)\,dx$
2. $f(x) \geq 0$,since $F(x)$ is non-decreasing, its derivative is also non-negative.
3. $\int_{-\infty}^{\infty} f(x)dx = \int_{all\ x} f(x)dx = 1$
4. $F(x) = \int_{-\infty}^{x} f(u)du$

- $f(x)$ represents the relative likelihood of different outcomes.
- To see this note that for Δx small,

$$P\left(x - \frac{\Delta x}{2} \leq X \leq x + \frac{\Delta x}{2}\right)$$

$$= F\left(x + \frac{\Delta x}{2}\right) - F\left(x - \frac{\Delta x}{2}\right) \approx f(x)\Delta x$$

- Thus, $f(x) \neq P(X = x)$ BUT $f(x)\Delta x$ is the approximate probability that X is inside an interval of length Δx centered about the value x when Δx is small.

- A plot of $f(x)$clearly shows around which values are more likely to fall.
- Example: Let

$$f(x) = \begin{cases} kx(1-x) & 0 < x < 1 \\ 0; & otherwise \end{cases}$$

Be a p.d.f for some constant k. Find

a) k

b) F(x)

c) P(0<X<1/2)

- **Defined Variables or Change of Variable**:
- Sometimes interest may lie in finding the p.d.f or c.d.f for some r.v Y that is a function of X. When the p.d.f or c.d.f of X are known.
- The procedure is as follows:

1. Write the c.d.f of Y as a function of X
2. Use $F_X(x)$ to find $F_Y(y)$. If you want the p.d.f $f_Y(y)$, you simply need to differentiate $F_Y(y)$.
3. Find the range of values for y.

- Example: X is a continuous r.v having

$$f(x) = \frac{1}{4}; \qquad 0 < x \leq 4$$

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Let Y=1/X. Find the p.d.f of y.

- **Expectation, Mean and Variance of Continuous Distributions:**
- Definition: If X is a continuous r.v. then we define:

$$E\left(g(X)\right) = \int_{all\ x} g(x)f(x)dx$$

Hence

$$E(X) = \int_{all\ x} xf(x)dx$$

- And

$$\sigma^2 = Var(X) = E(X^2) - [E(X)]^2$$

- Where all of the earlier properties of expected value and variance still hold:
- $E[ag(X) + b] = aE[g(X)] + b$
- $E[ag_1(X) + bg_2(X)] = aE[g_1(X)] + bE[g_2(X)]$
- $Var[ag(X) + b] = a^2Var[g(X)]$

- Example: If X is a continuous r.v having p.d.f $f(x) = \frac{1}{4}\ ; \quad 0 < x \leq 4$, find E(X) and Var(X)

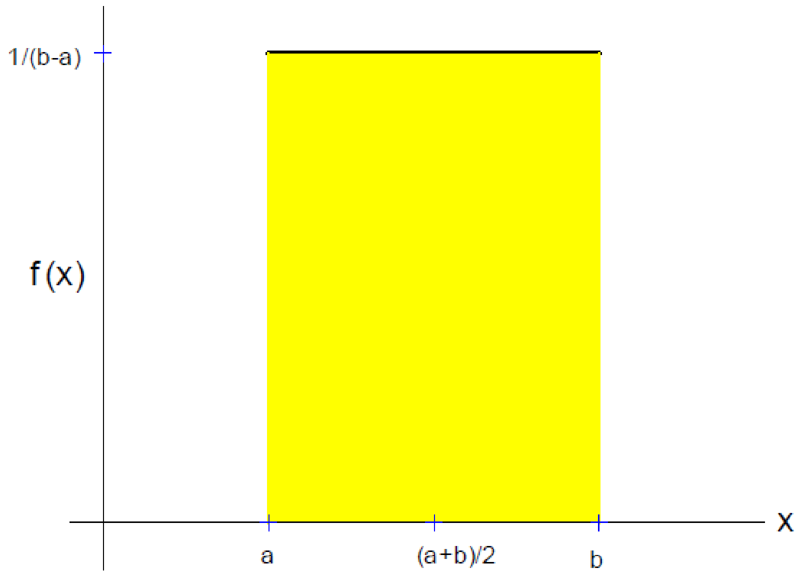
- **8.2: Continuous Uniform Distribution:**
- Physical Setup: X is a r.v taking on values in the interval [a, b] (it doesn't matter whether the interval is closed or open) with all subintervals of a fixed length being equally likely.
- Then **X~U[a, b]**

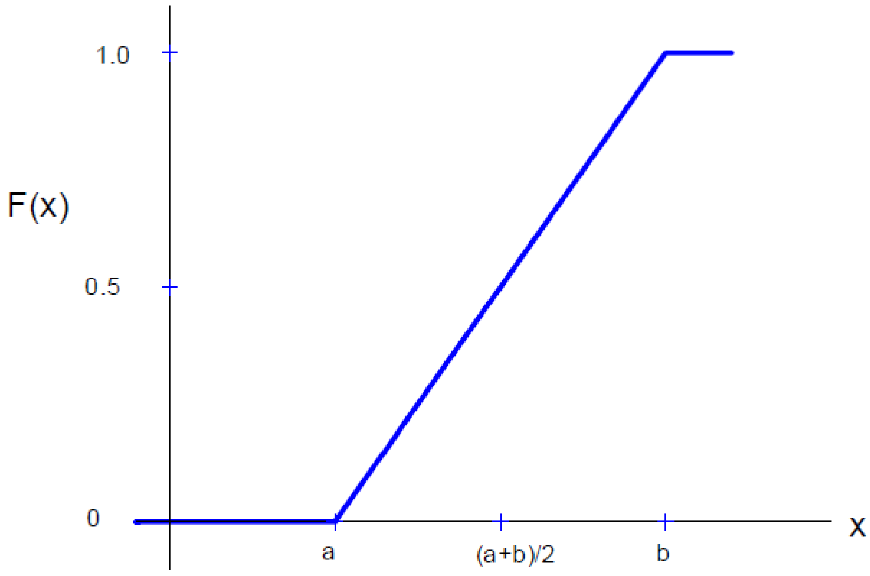
- **The Probability Density Function (p.d.f):**

Since all points are equally likely, the p.d.f must be a constant $f(x) = k; \quad a \leq x \leq b$ for some constant k.

But to make $\int_a^b f(x)dx = 1$, k must equal:

- It follows that the **Cumulative distribution function** is:


1/(b-a)
f(x)
a
(a+b)/2
b
x


1.0
F(x)
0.5
0
a
(a+b)/2
b
x

- **Mean:**

- **Variance:** $Var(X) = \frac{(b-a)^2}{12}$

- Example: Transforming a r.v. with a different continuous distribution to obtain a uniform.
- Let X have continuous p.d.f:

$$f(x) = 0.1e^{-0.1x} \text{ for x>0}$$

- Then the new random variable

$$Y = e^{-0.1X}$$

Will have a uniform distribution, U[0,1]

- Example: Buses arrive at a specified stop at 15-min intervals starting at 7am. That is, they arrive at 7, 7:15, 7:30, 7:45 and so on. If a passenger arrives at the stop at a time that is uniformly distributed between 7 and 7:30, find the probability that he waits

a) Less than 5 mins for a bus;

b) More than 10 mins for a bus.

a) Less than 5 mins for a bus;

b) More than 10 mins for a bus.

- **8.3: Exponential Distribution:**
- Physical setup: In a Poisson Process for events in time let X: length of time we wait until the first occurrence. Then X has an exponential distribution.

- Example:

If phone calls to a fire station follows a Poisson process, then the ***length of time between*** phone calls follows an exponential distribution.

- **The probability Density function (p.d.f) and the c.d.f:**

- Alternate Form:

Figure 8.9: Graph of the probability density function of a $Exponential\,(\theta)$ random variable

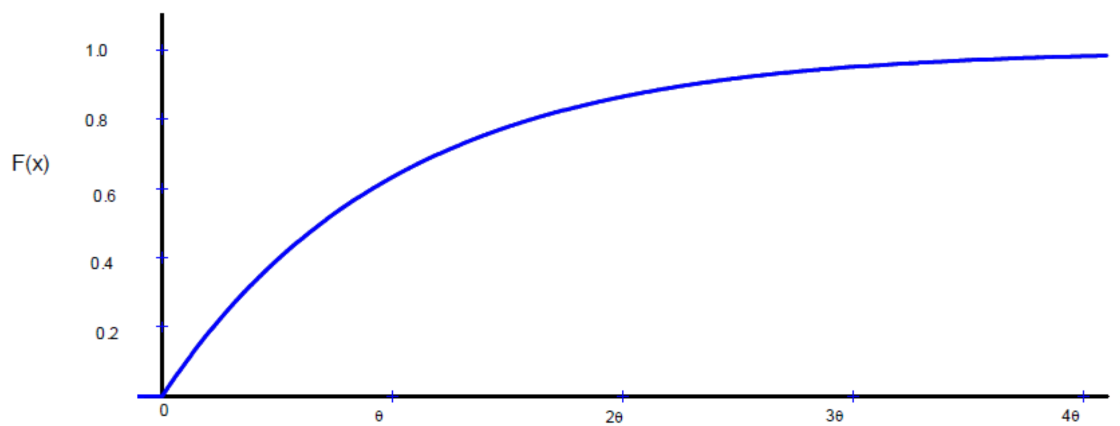


Figure 8.10: Cumulative distribution function for a $Exponential\,(\theta)$ random variable

- **Mean and Variance**: Finding the mean and variance directly involves integration by parts. To avoid this integration we use the properties of the Gamma Function.

- Definition: The **Gamma Function:**

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$$

Is called the gamma function of α, where $\alpha > 0$

- Example:

- Properties we will use:

1. $\Gamma(\alpha) = (\alpha - 1)\Gamma(\alpha - 1)$ for $\alpha > 1$

2. $\Gamma(\alpha) = (\alpha - 1)!$ If α is positive

3. $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

- Going back to the exponential distribution we have:
- Mean

- Hence **Mean**: $E(X) = \theta = \frac{1}{\lambda}$

- **Variance**: $Var(X) = \theta^2$

- Note: Read questions carefully.
- $\boldsymbol{\lambda}$: average **rate** of occurrence in a Poisson process
- $\boldsymbol{\theta}$: average waiting **time** for an occurrence.

- Example: Suppose the #7 bus arrives at the bus stop according to a Poisson process with an average of 5 buses per hour. Find

a) The probability that you have to wait longer than 15mins for a bus

b) The probability you have to wait more than 15mins longer, having already been waiting for 6mins.

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- Part b) illustrates the "**memoryless property**" of the exponential distribution:

$$P(X > c + b | X > b) = P(X > c)$$

- Example:
- Suppose that the length of a phone call in minutes is an exponential random variable with parameter $\lambda = \frac{1}{10}$. If someone arrives immediately ahead of you at a public telephone booth, find the probability that you will have to wait

a) More than 10 minutes;

b) Between 10 and 20 minutes.

a) More than 10 minutes;

b) Between 10 and 20 minutes

8.5: The Normal Distribution:

- Physical setup: A r.v. X defined on $(-\infty, \infty)$ has a normal distribution if it has probability density function of the form

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}, \qquad -\infty < x < \infty$$

Where $-\infty < \mu < \infty$ and $\sigma > 0$ are paramters.

- It turns out that $\mu = E(X)$ and $\sigma^2 = Var(X)$
- We write $X \sim N(\mu, \sigma^2)$

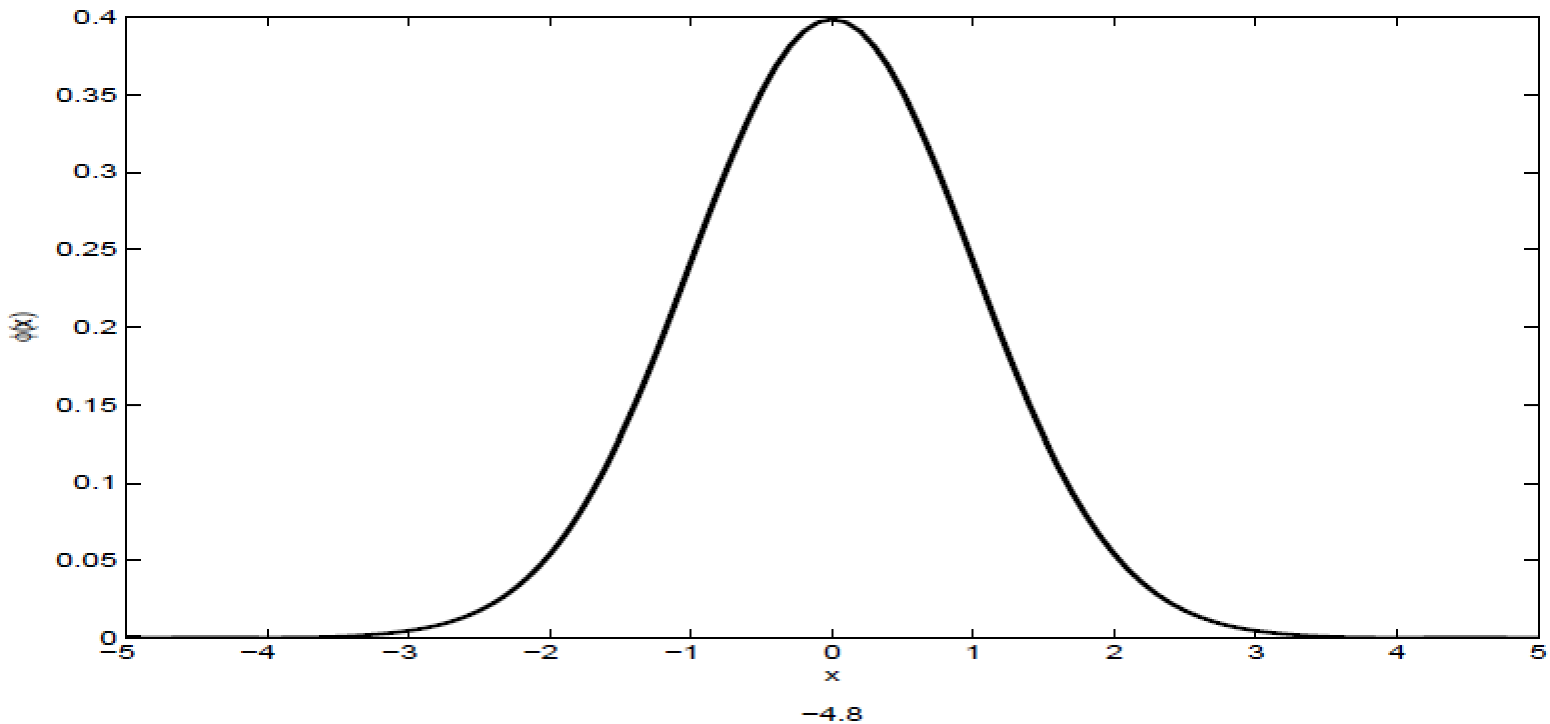


Figure 9.7: The Standard Normal ($N(0,1)$) probability density function

- Example:
- The heights or weights of males (or females) in a large populations tend to follow normal distributions.
- **The cumulative distribution function (c.d.f):**
- The c.d.f of the normal distribution $N(\mu, \sigma^2)$ is

$$\boldsymbol{F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2} dy}$$

This integral does not give a simple mathematical expression so numerical methods are used.

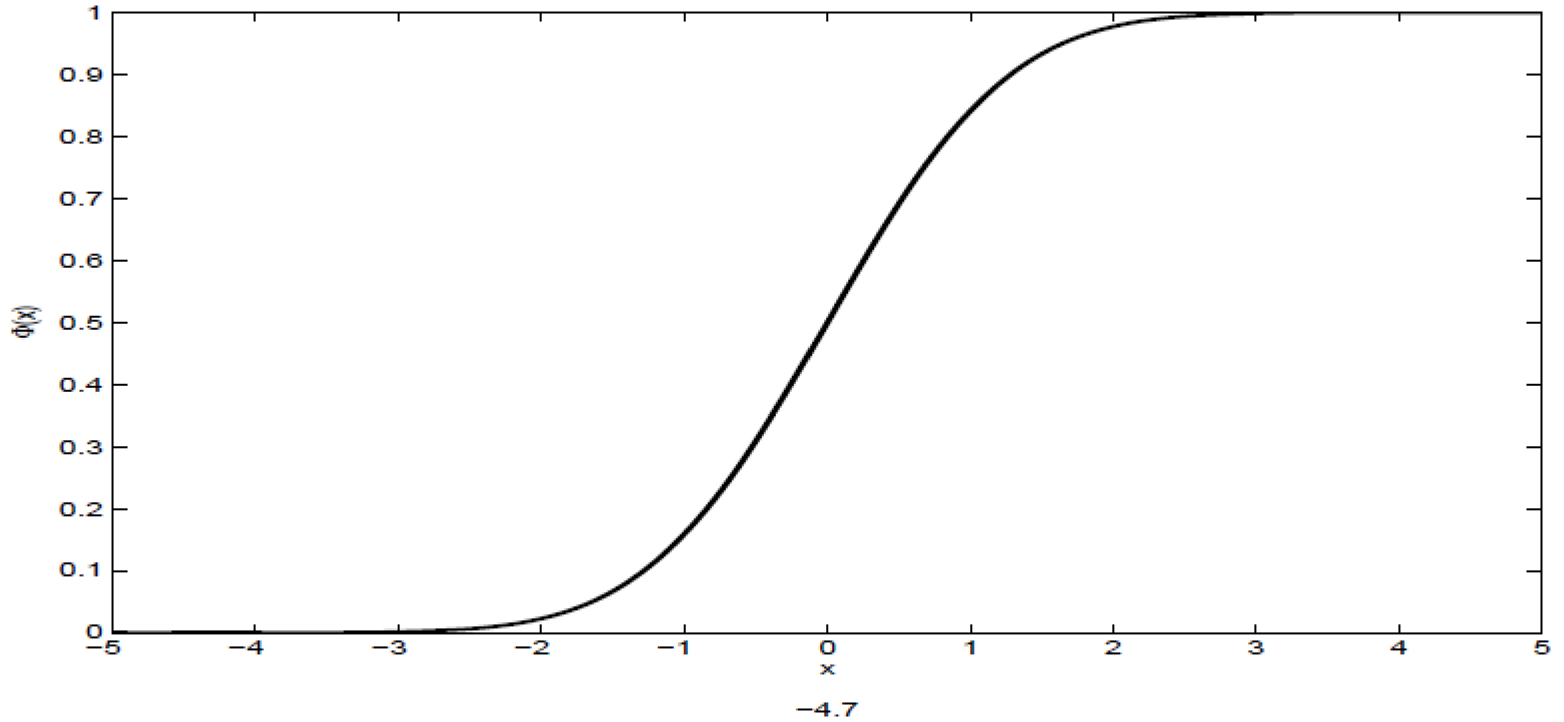


Figure 9.8: The standard normal c.d.f.

- Since numerical methods need to be used, before computers people produced tables of probabilities of F(x) by numerical integration, using mechanical calculators.

- Fortunately only one table is needed and is based on the normal distribution with $\boldsymbol{\mu = 0}$ and $\boldsymbol{\sigma = 1}$. This is called the **"standard"** normal distribution, N(0,1).
- This is because any normal r.v can be converted to a "standard" normal.
- If $X \sim N(\mu, \sigma^2)$ then $\boldsymbol{Z = \frac{(X-\mu)}{\sigma} \sim N(0,1)}$
- We will use this to compute probabilities for X.

- **Finding Normal Probabilities via N(0,1) Tables**:
- Theorem: Let $X \sim N(\mu, \sigma^2)$ and define

$$Z = \frac{(X - \mu)}{\sigma}$$

- Then $Z \sim N(0,1)$ and

$$F_X(x) = P(X \leq x) \equiv P\left(Z \leq \frac{x-\mu}{\sigma}\right) = F_Z\left(\frac{x-\mu}{\sigma}\right)$$

- Note:

- $f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}, -\infty < z < \infty$

- It can be shown that $\int_{-\infty}^{\infty} f_Z(z)dz = 1$

- **Probability Tables:**

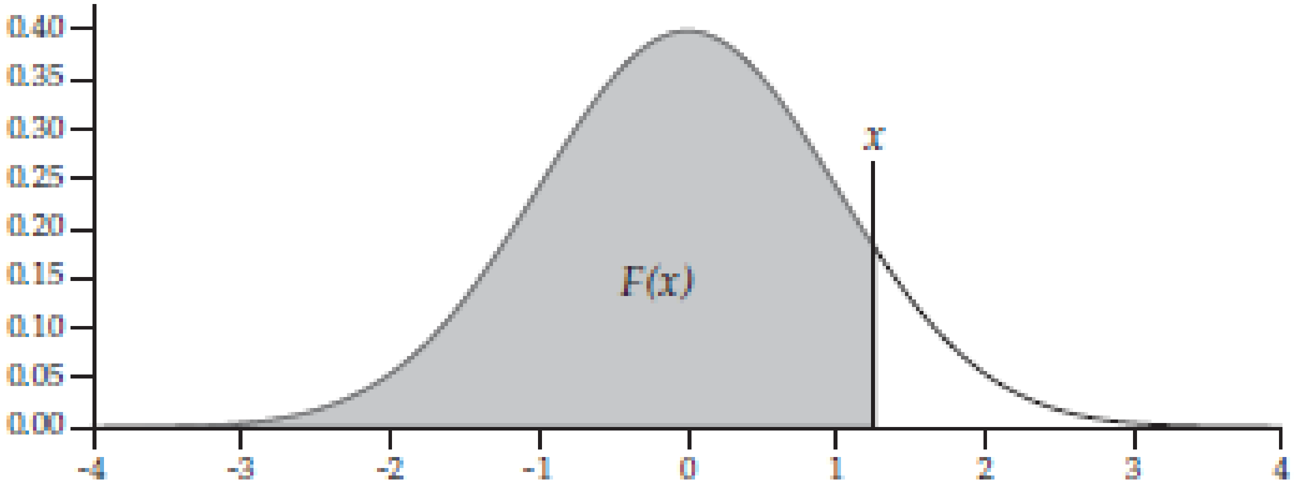
This table gives the values of $F(x)$ for $x \geq 0$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56750	0.57142	0.57534
0.2	0.57926	0.58317	0.58706	0.59095	0.59484	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899

- Example: Find the following probabilities, where Z~N(0,1).

1. $P(Z \leq 2.11)$
2. $P(Z > 1.06)$
3. $P(Z < -1.06)$
4. $P(-1.06 < Z < 2.11)$

Probabilities for Standard Normal *N(0,1)* Distribution



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- $P(Z \leq 2.11)$

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0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
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1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
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1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899

- $P(Z > 1.06)$

- $P(Z < -1.06)$

This table gives the values of $F(x)$ for $x \geq 0$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56750	0.57142	0.57534
0.2	0.57926	0.58317	0.58706	0.59095	0.59484	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670

- $P(-1.06 < Z \leq 2.11)$

- Example:

1. Find a number c such that $P(Z < c) = 0.85$
2. Find a number d such that $P(Z > d) = 0.90$
3. Find a number b such that $P(-b < Z < b) = 0.95$

Ans:

1.

This table gives the values of $F(x)$ for $x \geq 0$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56750	0.57142	0.57534
0.2	0.57926	0.58317	0.58706	0.59095	0.59484	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
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1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
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1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670

2.

3.

- **Gaussian Distribution**: This is another name for the Normal distribution. Where the notation $X \sim G(\mu, \sigma)$ means that X has a Gaussian (normal) distribution with **mean μ** and **standard deviation σ**.
- Example: if X~N(1, 4) then

- Example: A machine producing vitamin E capsules operates so that the actual amount of vitamin E in each capsule is normally distributed with mean 5mg and standard deviation 0.05.

a) What is the probability that a randomly selected capsule contains less than 4.9?
b) What is the minimum amount of Vitamin E found in 90% of the capsules?

a) What is the probability that a randomly selected capsule contains less than 4.9?

This table gives the values of $F(x)$ for $x \geq 0$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56750	0.57142	0.57534
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1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
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1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670

b) What is the minimum amount of Vitamin E found in 90% of the capsules?

This table gives the values of $F(x)$ for $x \geq 0$

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1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670

- Example:
- The intelligence quotient (IQ) score, as measured by the Stanford-Binet IQ test, is normally distributed in a certain population of children. The mean IQ score is 100 points, and the standard deviation is 16 points. What percentage of children in the population have IQ scores

a) 140 or more?

b) Between 80 and 120?

c) 80 or less?

a) 140 or more?

This table gives the values of $F(x)$ for $x \geq 0$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
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1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
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0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56750	0.57142	0.57534
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c) 80 or less?

This table gives the values of $F(x)$ for $x \geq 0$

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0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670