Algebre 2016 :

1- $P_{A_\alpha}(\lambda) = \det\begin{pmatrix} 1-\lambda & 0 & 0 \\ \alpha & -1-\lambda & -2 \\ 0 & 0 & 1-\lambda \end{pmatrix} = (1-\lambda)^{\boxed{2}}(-1-\lambda)$

2- pour A_α diagonalisable il suffit que $\overset{\text{dim}}{\mathrm{Ker}}(A-I_3)=2$

$$\begin{cases} (1-1)x+0y+0z=0 \\ \alpha x+(-1-1)y - 2z=0 \\ 0x+0y+(1-1)z=0 \end{cases} \Rightarrow \left\{ \alpha x-2y-2z=0 \Rightarrow z=\frac{\alpha}{2}x-y \right.$$

$$E_1=\left\{x\left(1,0,\frac{\alpha}{2}\right)+y(0,1,-1) \,/\, x,y\in\mathbb{R}\right\}$$

pour tout $\alpha\in\mathbb{R}$, A_α st diagonalisable.

3- $A_1=\begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$

a- $E_1=\left\{x\left(1,0,\frac{1}{2}\right)+y(0,1,-1) \,/\, x,y\in\mathbb{R}\right\}$

$E_{-1}=\mathrm{Ker}(A+I_3)\begin{cases} 2x = 0 \\ x - 2z=0 \\ 2z=0 \end{cases} \Rightarrow \begin{cases} x=0 \\ z=0 \end{cases}$

$$E_{-1}=\{y(0,1,0)\,/\,y\in\mathbb{R}\}$$

b- $P=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1/2 & -1 & 0 \end{pmatrix}$ $\quad P^{-1}=\begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 0 & -1 \\ -1/2 & 1 & 1 \end{pmatrix}$

$D=P^{-1}A_1P=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

c- $A_1^n = P D^n P^{-1}$.