$$A_{-2} = \begin{pmatrix} -1 & 0 & 0 \\ 1 & -2 & 0 \\ -1 & 1 & -1 \end{pmatrix}$$

ALGEBRE 2017 :

1 - Le polynome caracteristique :

$$P_{A_m}(\lambda) = \det(A_m - \lambda I_3) = \det \begin{pmatrix} -1-\lambda & 0 & m+2 \\ 1 & -2-\lambda & 0 \\ -1 & 1 & m+1-\lambda \end{pmatrix}$$

$$= (-1-\lambda) \begin{vmatrix} -2-\lambda & 0 \\ 1 & m+1-\lambda \end{vmatrix} + (m+2) \begin{vmatrix} 1 & -2-\lambda \\ -1 & 1 \end{vmatrix}$$

$$= (-1-\lambda)(-2-\lambda)(m+1-\lambda) + (m+2)(-1-\lambda)$$

3 $$= (-1-\lambda)\big((-2-\lambda)(m+1-\lambda) + (m+2)\big)$$

$$= -(\lambda+1)^2(\lambda - m)$$

2 - ~~pour~~ A_m st diagonalisable ssi $\dim(\underbrace{\ker(f+I_3)}_{E_{-1}}) = 2$

$$\begin{cases} (m+2)z = 0 \\ x - y = 0 \\ -x + y + (m+2)z = 0 \end{cases} \quad \text{si } (m \neq -2) \begin{cases} z = 0 \\ x = y \end{cases}$$

$E_{-1} = \{x(1,1,0) / x \in \mathbb{R}\}$

$\dim E_{-1} \neq 2 \Rightarrow A_m$ n'st pas diag

si $m = -2$: $E_{-1} = \{x(1,1,0) + z(0,0,1) / x, z \in \mathbb{R}\}$

$\dim E_{-1} = 2 \Rightarrow A_m$ diagonalisable

A_m diagonalisable ssi $m = -2$.

3 - cherchons le sous espace vectoriel E_2 :

$$E_{-2} = \ker(A + 2I_3) \quad \begin{cases} x = 0 \\ x = 0 \\ -x + y + z = 0 \end{cases} \quad \begin{cases} x = 0 \\ y = -z \end{cases}$$

$E_{-2} = \{y(0,1,-1) / y \in \mathbb{R}\}$

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix} \Longrightarrow P^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 1 \\ -1 & 1 & 0 \end{pmatrix}$$

4 - $A^k = P D^k P^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} (-1)^k & 0 & 0 \\ 0 & (-1)^k & 0 \\ 0 & 0 & (-2)^k \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 1 \\ -1 & 1 & 0 \end{pmatrix}$