2011 :

Logique :

1- La forme prenexe :

$$B_P \equiv \left(\forall x P(x) \Rightarrow \exists x Q(x)\right) \Rightarrow \exists x \exists y \left(P(x) \Rightarrow Q(y)\right)$$

$$\equiv \exists x \exists y \left(P(x) \Rightarrow Q(y)\right) \Rightarrow \exists x \exists y \left(P(x) \Rightarrow Q(y)\right)$$

$$\equiv \exists z \exists t \left(P(z) \Rightarrow Q(t)\right) \Rightarrow \exists x \exists y \left(P(x) \Rightarrow Q(y)\right)$$

$$\cancel{\equiv \exists x \exists y \forall z \forall t \left(\left(P(z) \Rightarrow Q(t)\right) \Rightarrow \left(P(x) \Rightarrow Q(y)\right)\right)}$$

$$\equiv \forall z \forall t \exists x \exists y \left(\left(P(z) \Rightarrow Q(t)\right) \Rightarrow \left(P(x) \Rightarrow Q(y)\right)\right)$$

2- Montrons que B_P valide et le même que montrons que $\neg B_P$ st contradict

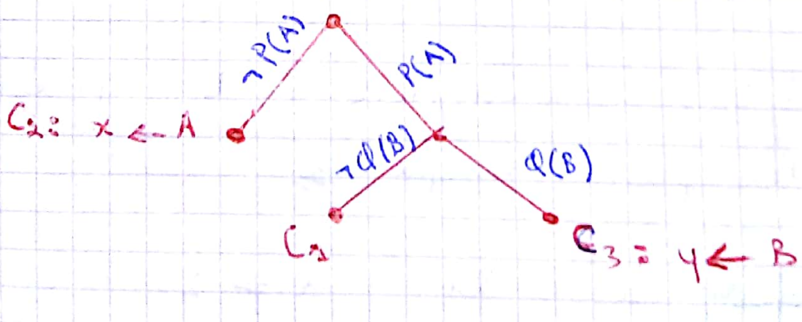
$$\neg B_P \equiv \exists z \exists t \forall x \forall y \left(\left(P(z) \Rightarrow Q(t)\right) \cdot \neg\left(P(x) \Rightarrow Q(y)\right)\right)$$

skolem

$$\neg B_{PS} \equiv \forall x \forall y \left(\left(P(A) \Rightarrow Q(B)\right) \cdot \neg\left(P(x) \Rightarrow Q(y)\right)\right)$$

$$\equiv \forall x \forall y \left(\left(\neg P(A) \vee Q(B)\right) \cdot P(x) \cdot \neg Q(y)\right)$$

$C_1 = \neg P(A) \vee Q(B)$ $\qquad C_2 = P(x)$ $\qquad C_3 = \neg Q(y)$.



alors $\neg B_P$ st contradictoire $\Rightarrow$ B_P est valide.

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