suite

d'où: $\mathcal{L}(f'(t))(s) = s\mathcal{L}(f(t))(s). \leftarrow$ $\boxed{0.5point}$

2) a) $\mathcal{L}(e^{at}f(t))(s) = \int_0^{+\infty} e^{-st}e^{at}f(t)dt = \int_0^{+\infty} e^{-(s-a)t}f(t)dt = F(s-a) \quad \forall s \succ a+\gamma_f. \leftarrow$ $\boxed{1point}$

b) $Q(s) = (s-\alpha_1)(s-\alpha_2)\dots(s-\alpha_n)$ où: $\alpha_1,\alpha_2,\dots,\alpha_n \in \mathbb{R}$ avec $\alpha_i \neq \alpha_j \ \forall i \neq j$.

$$Q'(\alpha_i) = \lim_{s\to\alpha_i} \frac{Q(s)-Q(\alpha_i)}{s-\alpha_i} = \lim_{s\to\alpha_i} \frac{Q(s)}{s-\alpha_i} = \lim_{s\to\alpha_i} \frac{(s-\alpha_1)(s-\alpha_2)\dots(s-\alpha_n)}{s-\alpha_i}$$
$$= \lim_{s\to\alpha_i} \prod_{\substack{j=1\\ i\neq j}}^{n} (s-\alpha_j) = \prod_{\substack{j=1\\ i\neq j}}^{n} (\alpha_i-\alpha_j). \leftarrow \boxed{1point}$$

c) Décomposons $\frac{P}{Q}$ en éléments simples:

$$\frac{P}{Q}(s) = \frac{\lambda_1}{s-\alpha_1} + \frac{\lambda_2}{s-\alpha_2} + \dots + \frac{\lambda_n}{s-\alpha_n}; \quad \lambda_1,\lambda_2,\dots,\lambda_n \in \mathbb{R}. \leftarrow \boxed{0.5point}$$

$$\lambda_i = (s-\alpha_1).\frac{P}{Q}(s)\Big|_{s=\alpha_1} = \frac{P(\alpha_1)}{\prod_{\substack{j=1\\ i\neq j}}^{n} (\alpha_i-\alpha_j)} = \frac{P(\alpha_1)}{Q'(\alpha_i)} \leftarrow \boxed{0.5point}$$

donc $\frac{P}{Q}(s) = \sum_{i=1}^{n} \frac{P(\alpha_i)}{Q'(\alpha_i)} \frac{1}{s-\alpha_i}$.

$$\mathcal{L}^{-1}\left(\frac{P}{Q}(s)\right)(t) = \mathcal{L}^{-1}\left(\sum_{i=1}^{n} \frac{P(\alpha_i)}{Q'(\alpha_i)} \frac{1}{s-\alpha_i}\right)(t) = \sum_{i=1}^{n} \mathcal{L}^{-1}\left(\frac{P(\alpha_i)}{Q'(\alpha_i)} \frac{1}{s-\alpha_i}\right)(t)$$
$$= \sum_{i=1}^{n} \frac{P(\alpha_i)}{Q'(\alpha_i)} \mathcal{L}^{-1}\left(\frac{1}{s-\alpha_i}\right)(t) \leftarrow \boxed{0.5point}$$

or $\mathcal{L}^{-1}\left(\frac{1}{s-\alpha_i}\right)(t) = e^{\alpha_i t}\mathcal{L}^{-1}\left(\frac{1}{s'}\right)(t) = e^{\alpha_i t}$ d'où $\mathcal{L}^{-1}\left(\frac{P}{Q}(s)\right)(t) = \sum_{i=1}^{n} \frac{P(\alpha_i)}{Q'(\alpha_i)} e^{\alpha_i t}$.

d) En déduire $\mathcal{L}^{-1}\left(\frac{2s+3}{(s-1)(s-2)(s-3)}\right)(t)$.

$$\mathcal{L}^{-1}\left(\frac{2s+3}{(s-1)(s-2)(s-3)}\right)(t) = \frac{5}{2}e^{t} - 7e^{2t} + \frac{9}{2}e^{3t}. \leftarrow \boxed{1point}$$

3) $\mathcal{L}^{-1}(Arctgs)$ n'existe pas, pourquoi?

Supposons que $\mathcal{L}^{-1}(Arctgs) = f(t)$ donc $\mathcal{L}(f(t))(s) = Arctgs$ or $\lim_{s\to+\infty} \mathcal{L}(f(t))(s) = 0$ donc $0 = \lim_{s\to+\infty} Arctgs = \frac{\pi}{2}$.

d'où l'absurdité. $\leftarrow$ $\boxed{1point}$

Reception des étudiants ce

mardi 12-06 :

$12^h \to 13^h$ $G_1 + G_2 + G_5$ } salle CP1

$13^h \to 14^h$ $G_3 + G_4$

AP