Mathematical Logic
Final Exam- Duration 2h-

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(Use only one Language in your responses)

Exercise 1 (3 points) ***(on interleaf sheet)***

Represent the following statements in first-order language:

1. Every even integer, strictly greater than 2, can be written as the sum of two primes. *(Goldbach's Conjecture)*
2. Every integer greater than 1 is divisible by a prime number.
3. There is no even prime greater than 2.

Exercise 2 (2-3 points) ***(on double sheet)***

1. Prove – using the semantic tree - that the formula:

   $\alpha : \forall x \exists y\, P(x,y) \wedge \exists x \forall y \neg P(x,y)$ is unsatisfiable

2. Prove - without using the completeness property - the following proposition:

$$\forall x \exists y \big((\neg P(x,y) \rightarrow P(x,a)) \wedge (\neg P(y,x) \rightarrow P(x,a))\big) \vdash \exists x \exists y \exists z (P(x,a) \wedge P(x,y) \wedge P(y,x))$$

Exercise 3 (2.5 points) Answer the following questions:

1. Name 3 methods to show that a set of formulas is not satisfiable.
2. Show that the following formula is satisfiable: $\forall x\, P(x) \rightarrow P(x)$
3. Show that the following formula is invalid: $\exists x\, P(x) \rightarrow P(x)$

Exercise 4 (4.5 points) Answer the following questions:

1. Find a Factor of the Clause $C : P(x,z,f(x)) \vee P(z,y,f(b))$ (specify the MGU)
2. Find a Factor of the Clause $C : P(x,z,f(x)) \vee P(z,y,f(g(y)))$ (specify the MGU)
3. Find a resolvent of the following two clauses and specify the MGU C1: $P(x,f(y)) \vee Q(g(y),x)$ C2: $\neg P(f(x), y) \vee \neg Q(f(g(y)),z) \vee R(z)$
4. Let the clause: $C : P(x,f(y)) \vee Q(g(b),x,y)$
- $C'_1 : P(h(y),f(y)) \vee Q(g(b),b,y)$: is it an instance of C (justify)
- $C'_2: P(x,f(x)) \vee Q(g(f(a)),x,x)$: is it an instance of C (justify)
5. Let the clause: $C : P(x,f(y)) \vee Q(g(b),x,y)$
- $C'_1 : P(b,f(y)) \vee Q(g(b),b,y)$ Is it a ground instance of C (justify)
- $C''_1 : P(a,f(b)) \vee Q(g(b),a,b)$ Is it a ground instance of C (justify)
- $C'''_2 : P(f(b),f(g(b))) \vee Q(g(b),f(b),b)$ Is it a ground instance of C (justify)

Exercise 5: (5 points)

For each proposal, check whether it is valid or not (Justify your answer by demonstration in case of validity and a justified example in case of invalidity)

1.	***The Herbrand base of any set of clauses is finite***
2.	***The set of ground instances of any set of clauses is infinite***
3.	Whatever the formula α $\quad \alpha \equiv \alpha_s$ *avec* α_s *the Skolem form of* α
4.	Whatever the formula α $\quad \alpha \not\equiv \alpha_s$ *avec* α_s *the Skolem form of* α
5.	Whatever the formula α $\quad \forall x \exists y\, \alpha \equiv \exists y\, \forall x\, \alpha$
6.	Whatever the formula α $\quad \forall x \exists y\, \alpha \not\equiv \exists y\, \forall x\, \alpha$