2017

Analyse :

Exercice 01 :

- $\frac{\partial f}{\partial x}(x,y) = 3x^2y^2(6-x-y) - y^2x^3$

$= x^2y^2(18-3x-3y-x)$

$= x^2y^2(-4x-3y+18)$

- $\frac{\partial f}{\partial y}(x,y) = 2yx^3(6-x-y) - x^3y^2$

$= x^3y(12-2x-2y-y)$

$= x^3y(-2x-3y+12)$

x et $y \neq 0$

$$\begin{cases} \frac{\partial f}{\partial x}(x,y)=0 \\ \frac{\partial f}{\partial y}(x,y)=0 \end{cases} \Rightarrow \begin{cases} x^2y^2(-4x-3y+18)=0 \\ x^3y(-2x-3y+12)=0 \end{cases} \Rightarrow \begin{cases} -4x-3y+18=0 \\ -2x-3y+12=0 \end{cases}$$

$$\Rightarrow \begin{cases} x=+3 \\ y=+2 \end{cases}$$

$M(3,2)$ st un point critique de f

La nature de M :

$r = \frac{\partial^2 f}{\partial x^2}(x,y) = 2xy^2(-4x-3y+18) - 4x^2y^2$.

$s = \frac{\partial^2 f}{\partial x \partial y}(x,y) = 2yx^2(-4x-3y+18) - 3x^2y^2$.

$t = \frac{\partial^2 f}{\partial y^2} = x^3(-2x-3y+12) - 3x^3y$.

$r = -144 < 0$ $s = -108$ $t = 162$

$\Delta = rt - s^2 = (-144)(-162) - (-108)^2 = 11664 > 0$

M : max ~~local~~.

dans une direction

et min dans une autre direction.

Exercice 2:

1- Les coefficients de Fourrier:

f st paire $\Rightarrow b_n = 0$

* $\cos\left(\frac{t}{2}\right)\cos(nt) = \frac{1}{2}\left(\cos\left(\frac{t}{2}+nt\right) + \cos\left(\frac{t}{2}-nt\right)\right)$

$$a_n = \frac{2}{\pi}\int_0^{\pi} \cos\left(\frac{t}{2}\right)\cos(nt)\,dt = \frac{2}{\pi}\cdot\frac{1}{2}\left[\frac{2\sin\left(\left(\frac{1}{2}+n\right)t\right)}{1+2n} + \frac{2\sin\left(\left(\frac{1}{2}-n\right)t\right)}{1-2n}\right]_0^{\pi}$$

$$a_n = \frac{2}{\pi}\left[\frac{\sin\left(\frac{\pi}{2}+n\pi\right)}{1+2n} + \frac{\sin\left(\frac{\pi}{2}-n\pi\right)}{1-2n}\right]$$

$$a_n = \frac{2}{\pi}\left(\frac{(-1)^n}{1+2n} + \frac{(-1)^n}{1-2n}\right)$$

$$\boxed{a_n = \frac{4}{\pi}\,\frac{(-1)^n}{1-4n^2}}$$

$$\boxed{a_0 = \frac{4}{\pi}}$$

La série de Fourier de F : $S(f)(x) = a_0 + \sum_{n=1}^{\infty} a_n\cos(nx) + b_n\sin(nx)$

$$S(f)(x) = \frac{2}{\pi} + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{(1-4n^2)}\cos(nx)$$

$$\boxed{S(f)(x) = \frac{2}{\pi} + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{1-4n^2}\cos(nx)}$$

2- f st continue $[-\pi,\pi]$ alors f st contin en $x=\pi$

$\Rightarrow S(f)(\pi) = f(\pi) \ldots\ldots \alpha$

$\cos(n\pi) = (-1)^n \Rightarrow (-1)^n\cos(n\pi) = 1 \ldots\ldots \beta$

d'après α et β : $\cos\left(\frac{\pi}{2}\right) = \frac{2}{\pi} + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{1-4n^2}$

$$\Rightarrow \boxed{\sum_{n=1}^{\infty}\frac{1}{1-4n^2} = \frac{-1}{2}}$$