M.Sc. Courses in Electrical Power Engineering

EEEN60342: Dynamics & Quality of Electricity Supply

Lecture notes on: Dynamics

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Course Structure - Dynamics

- 15 lectures
  - pre recorded lectures A1-A10 comprising recorded parts L1-L24 and lasting approximately 481 minutes (i.e., 9.6 lectures)
  - 5 in-person –clarifications and Q&A sessions)
- 2 example & tutorial classes
- 6 hours of laboratory
- 2 additional tutorials

Questions and comments are **allowed** and **most welcome** at **any point during the lecture** to clarify required concepts.

If there are **additional questions** and clarifications required, please ask them **at the start of lectures the following day**.

All questions should be asked **in front of the class**, so that other people could potentially benefit from the answers.

Course Content - Dynamics

- Chapter 1 ***Power System Stability - Basic Concepts*** ***(58 slides)***
- Chapter 2 ***Modelling of synchronous machines*** ***(66 slides)***
- Chapter 3 ***Modelling of Network, Loads and Controls*** ***(57 slides)***
- Chapter 4 ***Small-Disturbance Stability*** ***(67 slides)***
- Chapter 5: ***Large Disturbance (Transient) Stability*** ***(98 slides)***
- Chapter 6: ***Enhancement of power system stability*** ***(104 slides)***

Some of the **450 slides** given in the notes are included as an **additional information** only and intended either for **"further reading"** or as an additional illustration of concepts and methodologies discussed during lectures. Therefore, some of the slides slides given in the notes **will not** be discussed during the lectures.

Recommended Text Books

- The following text books contain chapters devoted to power system stability and control. *(Some of them have been used in the development of the lecture notes and some of the figures presented in the notes are adopted from those books.)*

1. P. Kundur, *"Power System Stability and Control"*, McGraw-Hill Inc. 1994.
2. J. Machowski, J. W. Bialek and J. R. Bumby, "Power System Dynamics and Stability", John Wiley & Sons, 1997.
3. P. M. Anderson, A. A. Fouad, *"Power system control and stability"*, IEEE Press, Piscataway, NJ,USA, 1993.
4. T. Van Cutsem and C. Vournas, *"Voltage Stability of Electric Power Systems"*, Kluwer Academic Publishers, 1998.
5. M. Ilić and J. Zaborszky, *"Dynamics and Control of Large Electric Power Systems"*, John Wiley & Sons, 2000

Chapter 1:
Power System Stability - Basic Concepts

Classification of
Power System Dynamics

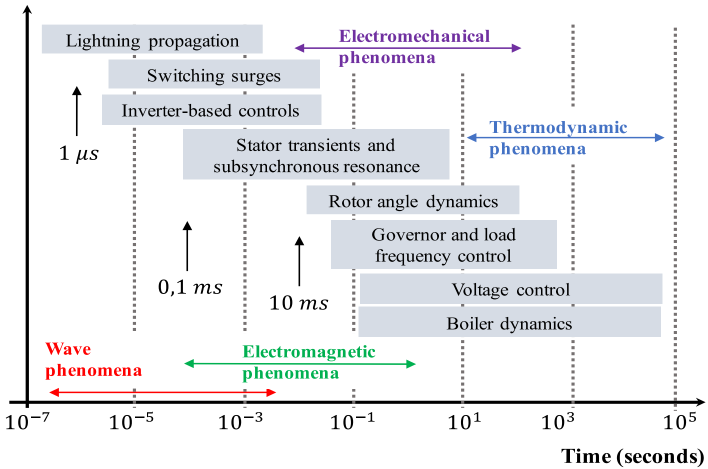
Power System Dynamics

- Power flow equations describe the steady state of a power system
  - Voltage magnitudes and angles such that the active and reactive power are in balance at each bus
- However, things are always changing in a power system:
  - Load fluctuations
  - Generation dispatch
  - Generation outages
  - Line disconnections
  - Faults

Will the system remain in equilibrium as conditions change?

Power System Dynamics - Physical Character -R

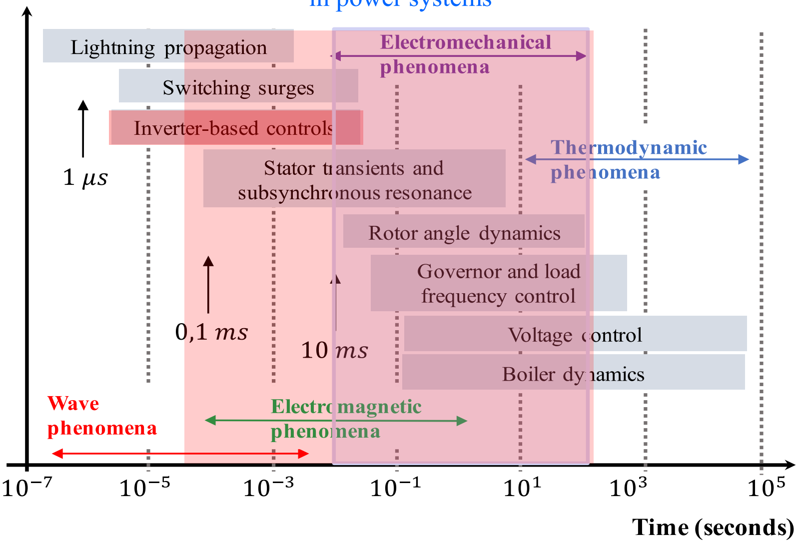
- Wave (μs - ms)
- Electromagnetic (ms - 1s)
- Electromechanical (1s - several seconds)
- Thermodynamic (several seconds - hours)



Time Scales of Power System Dynamic Phenomena

- The time scale related to the controls of CIGTs ranges from a few microseconds to several milliseconds, thus encompassing electromagnetic and wave phenomena
- With proliferation of CIGTs, faster dynamics will gain more prominence when analyzing future power system dynamic behavior compared to electromechanical phenomena

Time scales for various classes of dynamic phenomena in power systems



- Focusing on the time scale of the electromechanical transients in the past enabled several simplifications in modeling and representation, which significantly aided the characterization and analysis of the related phenomena
  - Electrical network modeled considering steady-state voltage and current phasors, → Quasi static phasor modeling approach

Dynamic Effects: Change in Power -R

- The fastest: Associated with the transfer of energy between the rotating masses in the generators and the loads. (ms- seconds)
- Slower: Due to voltage and frequency control actions needed to maintain system operating conditions. (seconds - minutes)
- Very slow: Corresponding to the way in which the generation is adjusted to meet the slow daily demand variations. (seconds - hours)

Dynamic Effects: Disturbances - 1 -R

- The fastest: Associated with the very fast wave phenomena (surges) in HV transmission lines
  - caused by lightning strikes or switching operation (μs - ms)
    - ➢ in the network, do not propagate beyond transformer windings
- Slower: Due to electromagnetic changes in electrical machines windings
  - caused by operation of the protection system or the interaction between electrical machines and the network (ms - s)
    - ➢ generator armature and damper windings and partly the network

Dynamic Effects: Disturbances - 2 -R

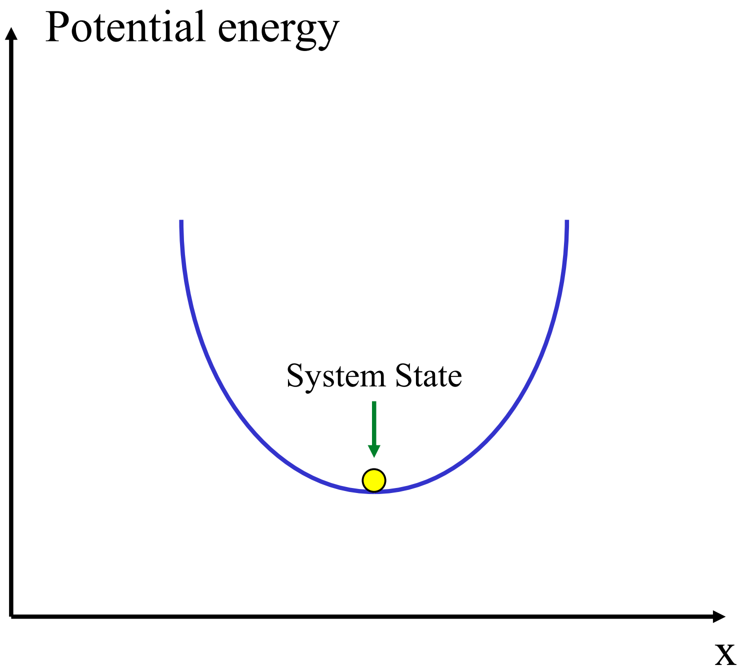
- Slow: Due to electromechanical rotor oscillations
  - Caused by oscillation of the rotating masses of generators and motors following a disturbance, operation of the protection system or voltage and prime mover control (s - several seconds)
    - rotor field and damper windings and rotor inertia (interaction between swinging generator rotors possible)
    - slightly slower are frequency oscillations with generator rotors involved but more influenced by turbine governing system and AGC
- Very Slow: Due to prime mover and automatic generation control (AGC).
  - Caused by thermodynamic changes resulting from boiler control action in steam power plants as the demands of AGC (several seconds - hours)

Power System Stability Terms and Definitions

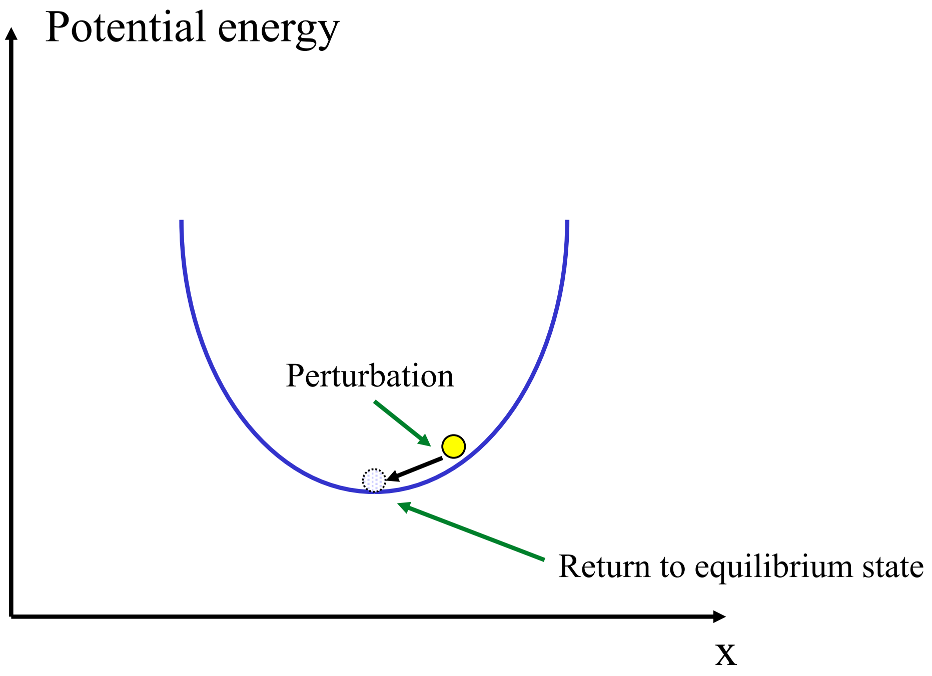
Stability

- Stability:
  - Stability is a condition of equilibrium between opposing forces!

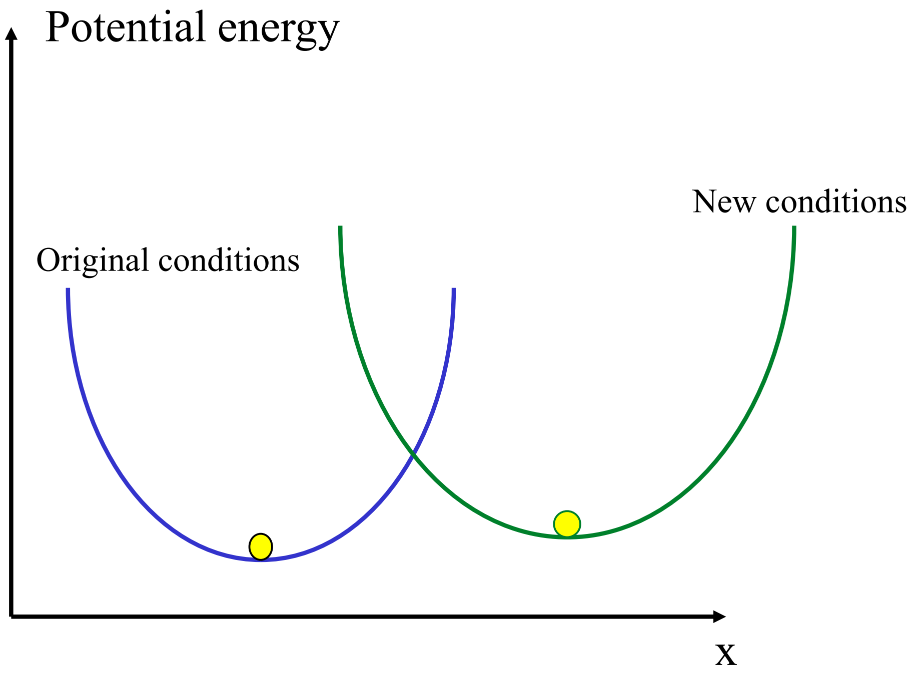
Stable System



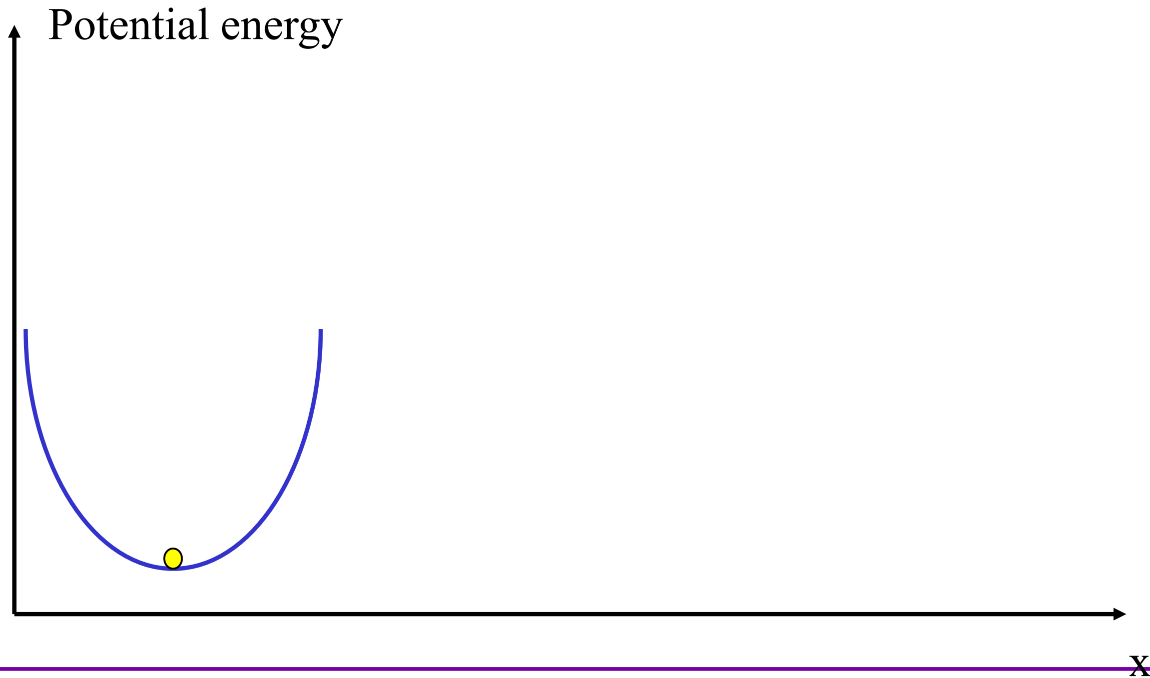
Stable System



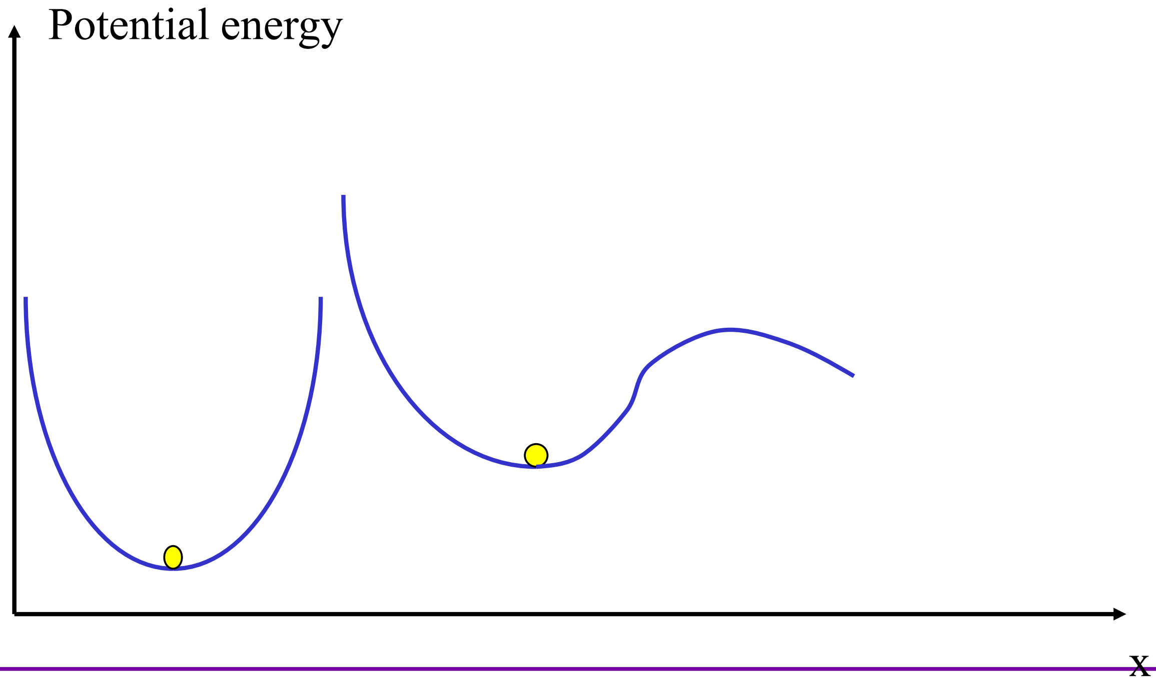
Slow Evolution of a Stable System



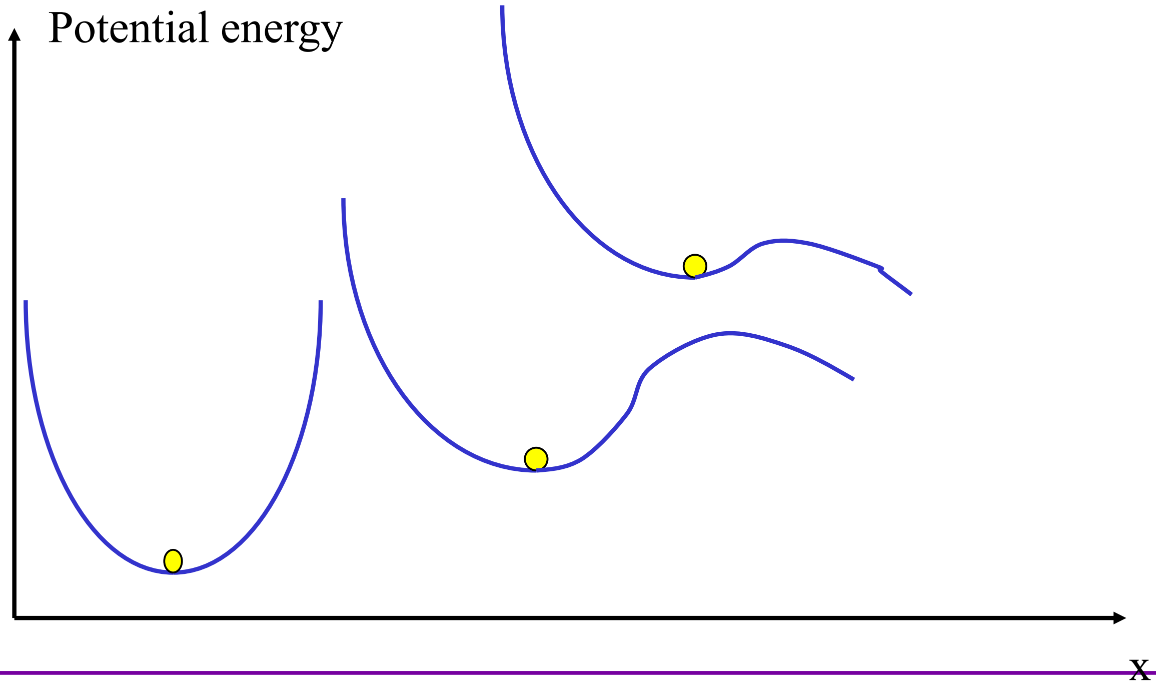
Steady State Instability



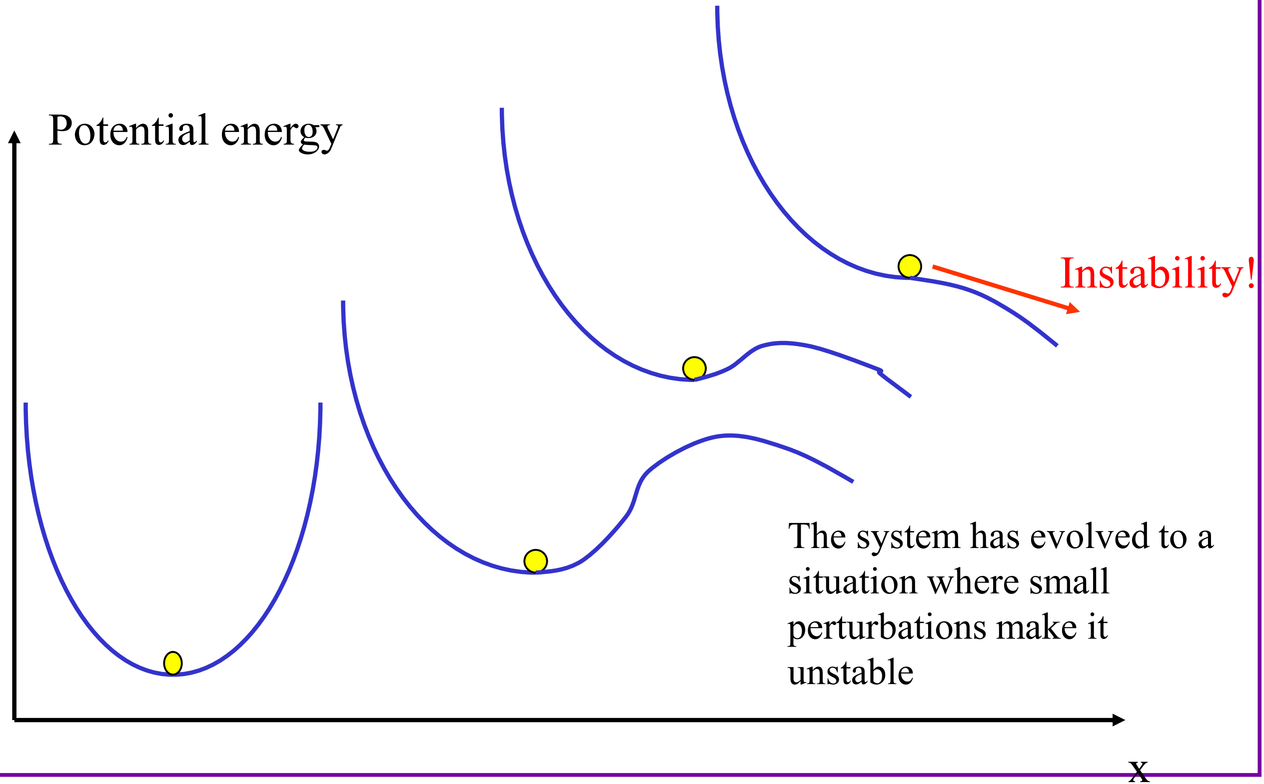
Steady State Instability



Steady State Instability

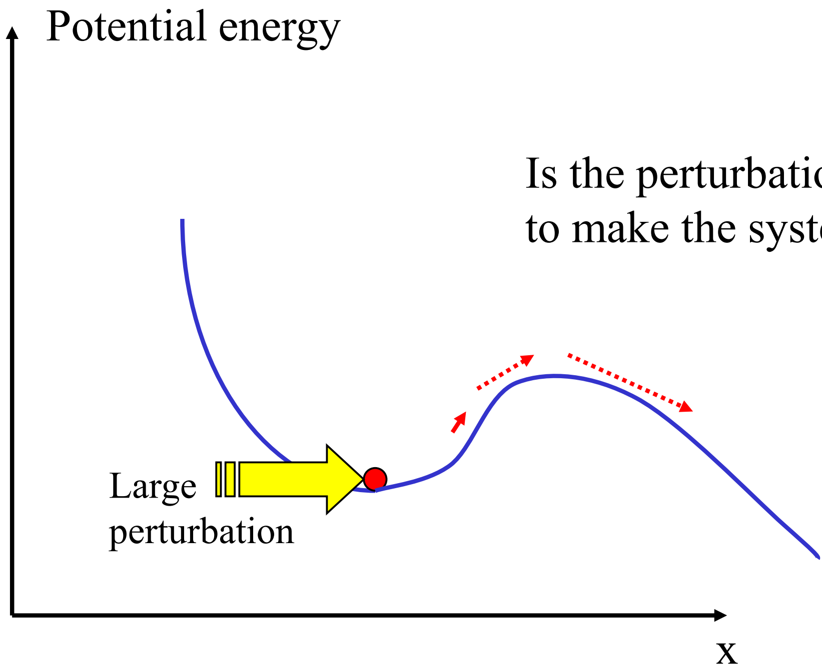


Steady State Instability



The system has evolved to a situation where small perturbations make it unstable

Transient Stability



Is the perturbation large enough to make the system unstable?

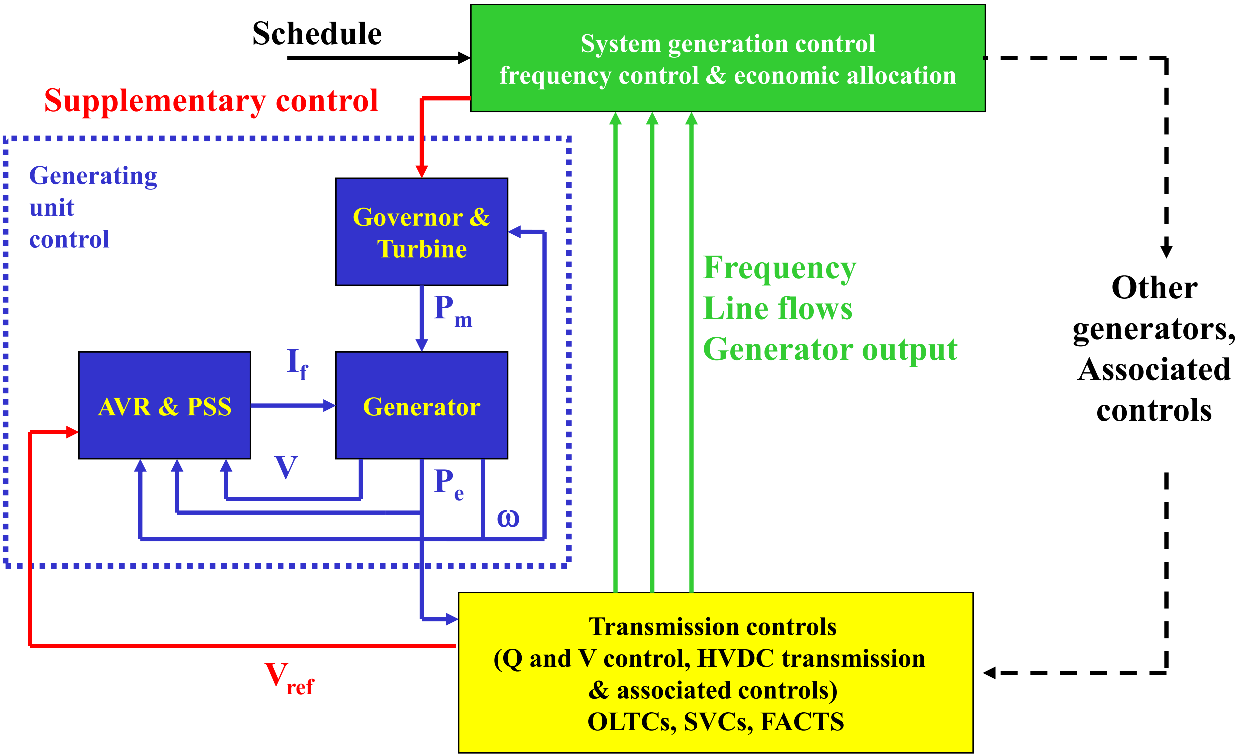
Power System Stability

- Definition:
  - Power system stability is the property of a power system that enables it to remain in a state of operating equilibrium under normal operating conditions and to regain an acceptable state of equilibrium after being subjected to a disturbance.

Power System Stability Problem

- Imbalance between mechanical power produced by the turbines and electrical power absorbed by the electrical network
  - Differential equations describing the mechanical motion of the generating units
  - Algebraic equations describing the behaviour of the network
  - Coupling between these two types of equations

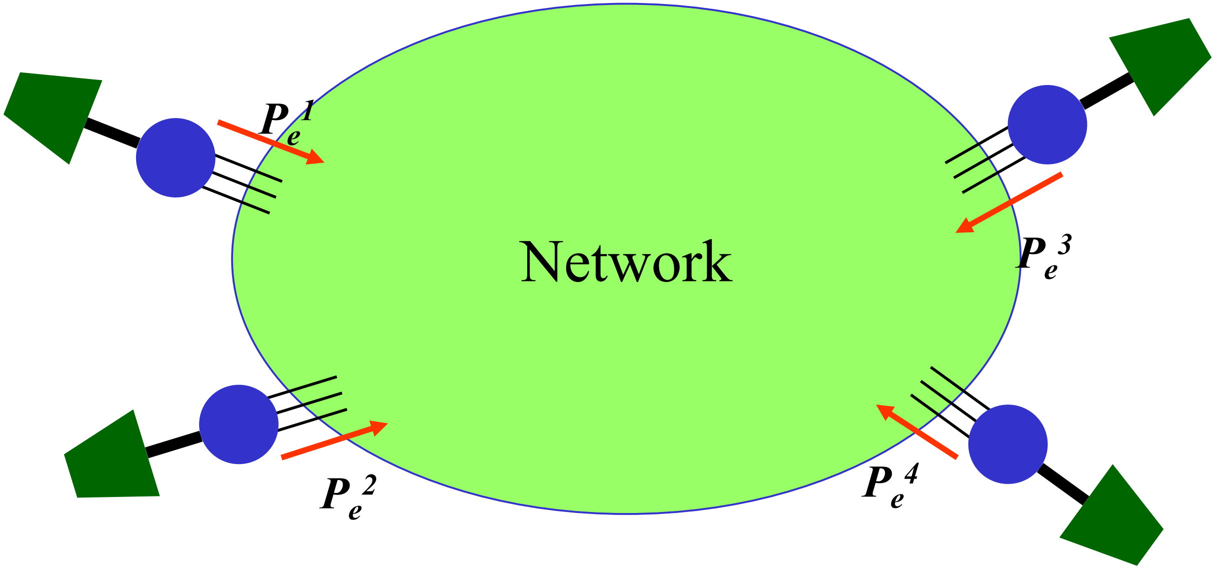
Power System and Associated Controls



Power System Control

- Requirements:
  - The system must be able to meet the continually changing load demand for active and reactive power. (Adequate "spinning reserve" of P and Q should be maintained and controlled at all times)
  - The system should supply energy at minimum cost and with minimum ecological impact.
  - The quality of power supply must meet minimum standards with regard to constancy of frequency, constancy of voltage and level of reliability.

Power System Stability Problem



- Each generator is represented by a set of equations (minimum one 2nd order differential equation)
- Each generator injects active power in the system

Types of Power System Stability -R

- Rotor angle stability:

The ability of interconnected synchronous machines of a power system to remain in synchronism. (Study of electromechanical oscillations inherent to power system is required.)

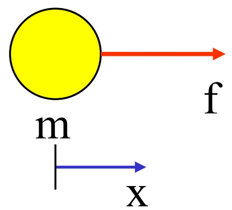
- Voltage stability:

The ability of a power system to maintain steady acceptable voltages at all buses in the system under normal operating conditions and after being subjected to a disturbance. (The main factor is inability of power system to meat the demand for reactive power.)

Equations of Motion

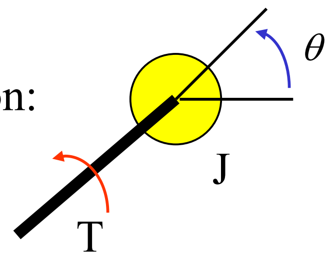
Newton's law: force = mass x acceleration

Linear motion:



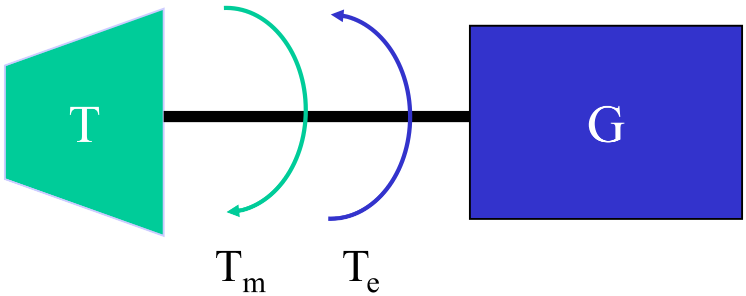
$$m\frac{d^2x}{dt^2} = f$$

Rotational motion:



$$J\frac{d^2\theta}{dt^2} = T$$

Turbine - Generator Set



T_m:	Mechanical torque applied by turbine
T_e:	Electrical reaction torque developed by generator
J:	Inertia of the turbine/generator set
Θ_m:	Position of the rotor

$$J\frac{d^2\theta_m}{dt^2} = T_m - T_e$$

Steady State

Steady state: $T_m = T_e$

$$\Rightarrow \quad J\frac{d^2\theta_m}{dt^2} = 0$$

$$\Rightarrow \quad \frac{d^2\theta_m}{dt^2} = 0$$

$$\Rightarrow \quad \frac{d\theta_m}{dt} = \text{constant} = \omega_m^{syn}$$

$$\Rightarrow \quad \theta_m = \omega_m^{syn} t + \theta_m^0$$

In the steady state, the rotor rotates at constant speed and the angle increases linearly with time.

Change of Variable

Let $\delta_m = \theta_m - \omega_m^{syn} t$

Deviation of Θ_m from rotation at synchronous speed

$$\frac{d\delta_m}{dt} = \frac{d\theta_m}{dt} - \omega_m^{syn}$$

Deviation from synchronous speed

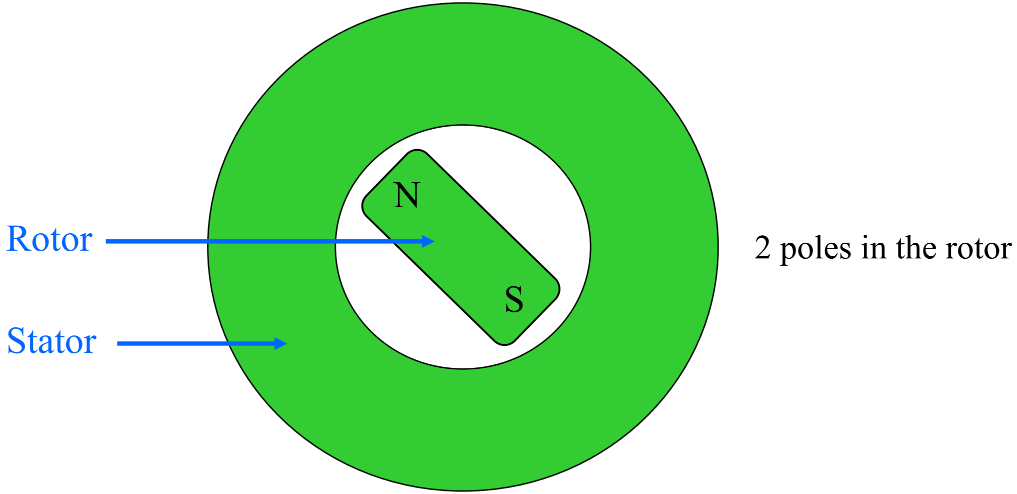
$$\frac{d^2\delta_m}{dt^2} = \frac{d^2\theta_m}{dt^2}$$

We can thus rewrite Newton's law in terms of deviation from rotation at synchronous speed:

$$J\frac{d^2\delta_m}{dt^2} = T_m - T_e$$

The swing equation

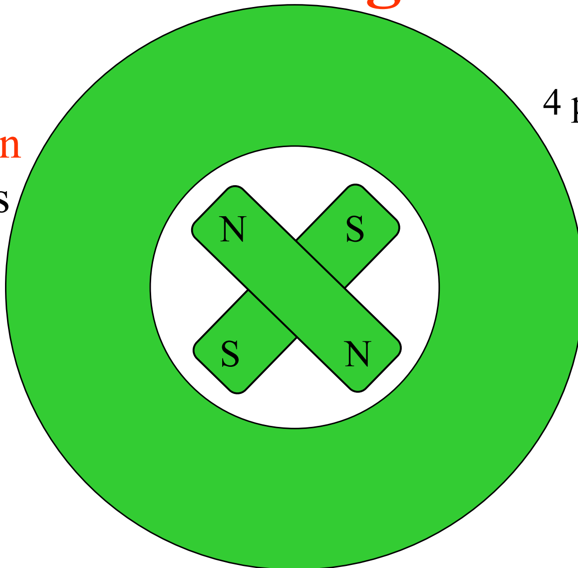
Electrical Degrees vs. Mechanical Degrees



One mechanical rotation of the rotor corresponds to one period of the voltage induced in the stator

Electrical Degrees vs. Mechanical Degrees

One mechanical rotation of the rotor corresponds to two periods of the voltage induced in the stator



$$\delta[^0 elec] = p \cdot \delta_m[^0 mech]$$

p - the number of pairs of poles in the rotor

$$J\frac{d^2\delta_m}{dt^2} = T_m - T_e \quad \Rightarrow \quad \frac{J}{p}\frac{d^2\delta}{dt^2} = T_m - T_e$$

Standard Form of the Swing Equation

$$\text{Inertia constant} = \frac{Stored\ \ kinetic\ \ energy}{Machine\ \ rating} = H = \frac{\frac{1}{2}J\left(\omega_m^{syn}\right)^2}{S_B}$$

$$\Rightarrow \quad \frac{J}{S_B} = \frac{2H}{\left(\omega_m^{syn}\right)^2}$$

$$\frac{J\omega_m}{S_B p}\frac{d^2\delta}{dt^2} = P_m^{\ pu} - P_e^{\ pu} \quad \Rightarrow \quad \frac{2H\omega_m}{(\omega_m^{\ syn})^2\, p}\frac{d^2\delta}{dt^2} = P_m^{\ pu} - P_e^{\ pu}$$

$$\Downarrow$$

$$\left.\begin{aligned} \omega^{syn} &= p \cdot \omega_m^{\ syn} \\ \frac{\omega_m}{\omega_m^{syn}} &= \omega^{\ pu} \end{aligned}\right\} \quad \Rightarrow \quad \frac{2H}{\omega^{\ syn}}\,\omega^{\ pu}\frac{d^2\delta}{dt^2} = P_m^{pu} - P_e^{pu}$$

The Swing Equation

$$\boxed{\frac{2H}{\omega^{syn}}\,\omega^{pu}\,\frac{d^2\delta}{dt^2} = P_m^{pu} - P_e^{pu}}$$

Mechanical power provided by the turbine (assumed constant)

Electrical power injected by the generator in the network

An imbalance between the electrical and mechanical power will cause the rotor angle to oscillate!

Rotor Angle Stability -R

$$\boxed{\Delta T_e = T_S \Delta\delta + T_D \Delta\omega}$$

$$(\Delta P_e = P_S \Delta\delta + P_D \Delta\omega)$$

T_e - Electrical torque

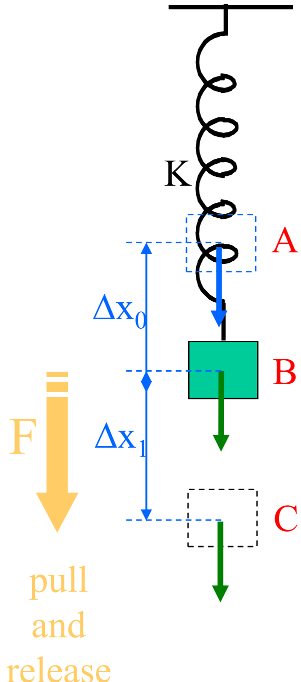
$T_S\Delta\delta$ - Synchronising torque (power) component (in phase with the rotor angle perturbation)

T_S - Synchronising torque (power) coefficient

$T_D\Delta\omega$ - Damping torque (power) component (in phase with the rotor speed deviation)

T_D - Damping torque (power) coefficient

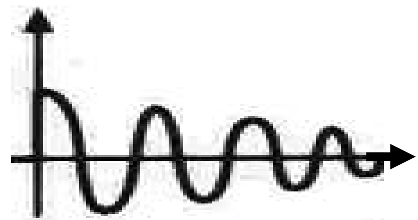
Oscillations in the air

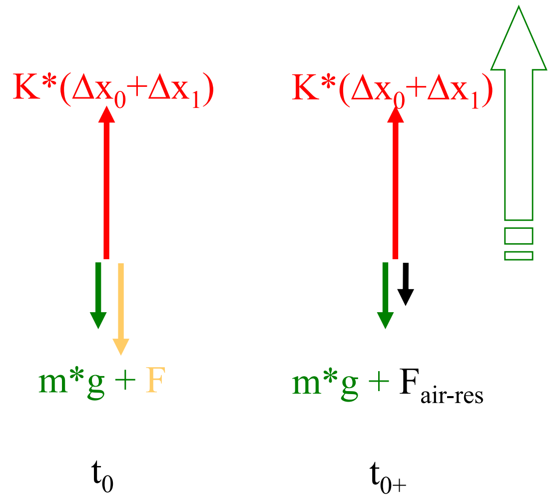


A: no mass attached

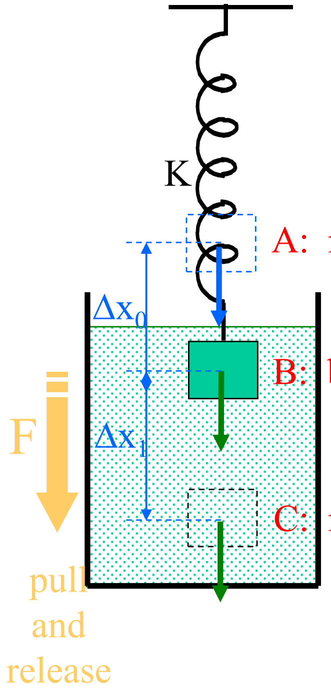
B: balance $\Rightarrow$ $m*g = K*\Delta x_0$

C: $m*g + F = K*(\Delta x_0+\Delta x_1)$





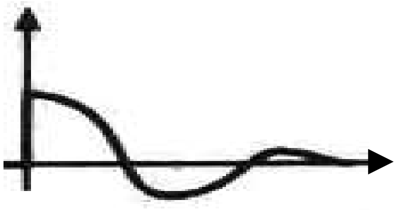
Oscillations in viscous medium

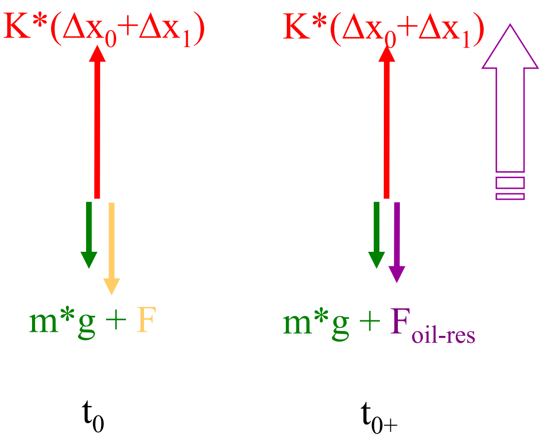


A: no mass attached

B: balance $\Rightarrow$ $m*g = K*\Delta x_0$

C: $m*g + F = K*(\Delta x_0+\Delta x_1)$



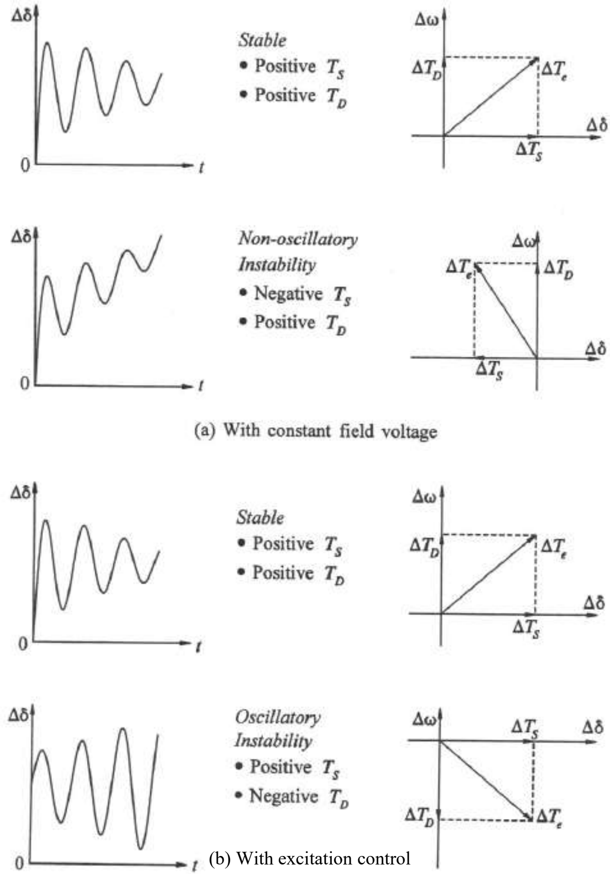


Types of Rotor Angle Instability

- Lack of synchronising torque component results in instability through an aperiodic drift in rotor angle

- Lack of damping torque component results in oscillatory instability

Types Of Rotor Angle Instability



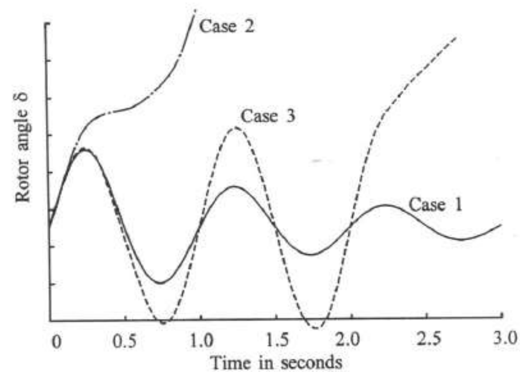
(a) With constant field voltage

(b) With excitation control

Case 1: Stable system

Case 2: Unstable system (insufficient synchronising torque).

Case 3: "First-swing stable"



Types of Rotor Angle Stability

- Small-disturbance stability

  The ability of a power system to maintain synchronism under small disturbance

- Transient stability

  The ability of a power system to maintain synchronism when subjected to a severe transient disturbance

Voltage Stability

- Definition:
  - Voltage stability is the ability of a power system to maintain steady acceptable voltages at all buses in the system under normal operating condition and after being subjected to a disturbance

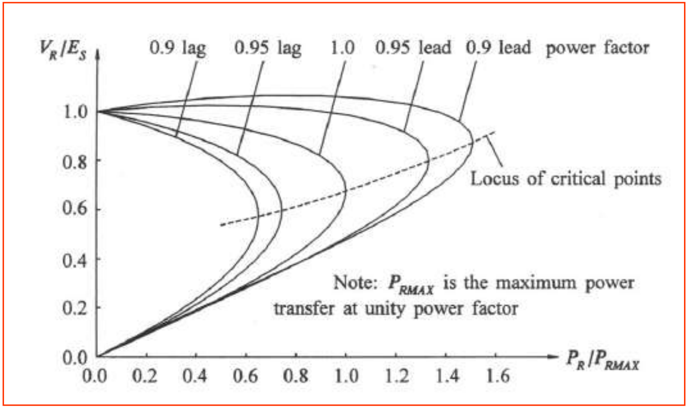
Voltage Stability - Properties

- Criterion: At a given operating condition for every bus in the system, the bus voltage magnitude increases as the reactive power injection at the same bus is increased.
- Voltage instability is generally a local phenomenon.
- Voltage collapse is the result of a sequence of events accompanying voltage instability leading to a low-voltage profile in a significant part of the system

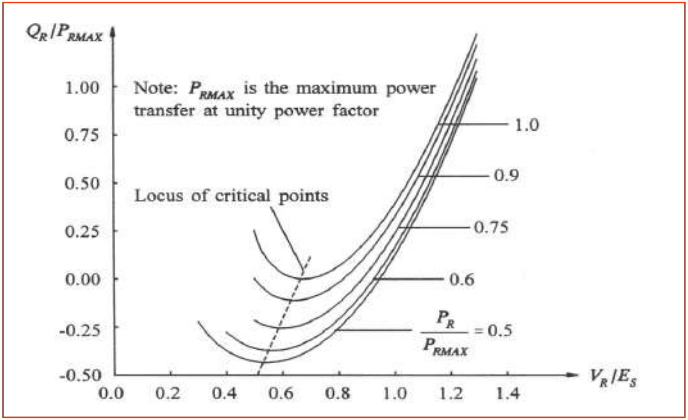
Effects of Voltage Changes

- Voltage change may cause complicated dynamic interactions inside the distribution network due to:
  - Voltage control action due to transformer tap-changing
  - Control action associated with reactive power compensation and/or embedded generators
  - Low supply voltage causing changes in power demand due to IM stalling and/or extinguishing of discharge lighting
  - Operation of protection equipment by overcurrent or undervoltage relays etc.
  - Re-ignition of discharge lighting and self-start of iIM when supply voltage recovers

P-V and Q-V Curves



V- P characteristic of the single machine isolated load system

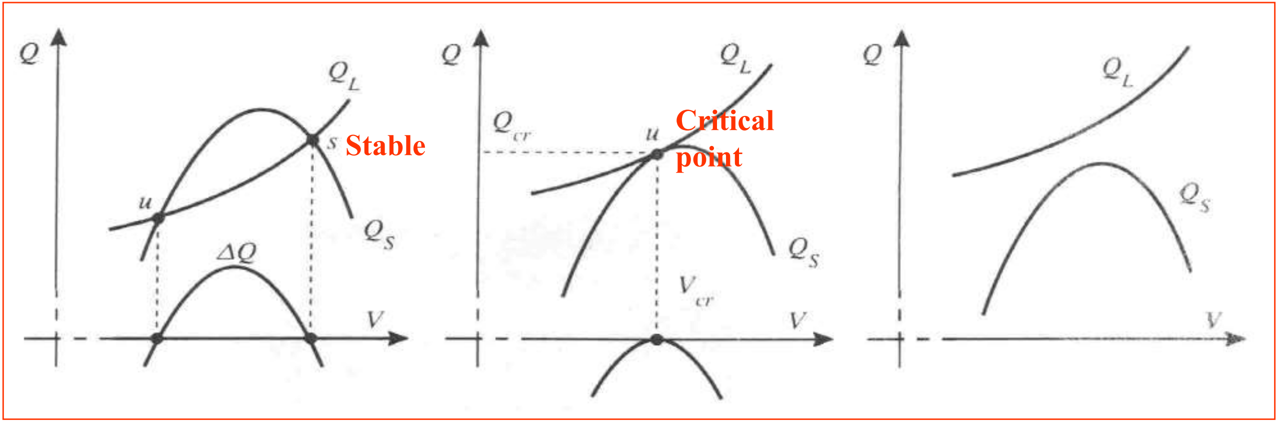


V- Q characteristic of the single machine isolated load system

Causes of Voltage Instability

- Network characteristics
- Generator characteristics
- Load characteristics
- Characteristics of reactive compensating devices
  - Shunt capacitors
  - Regulated shunt capacitors
  - Series capacitors

Critical Load Demand



Two equilibrium points

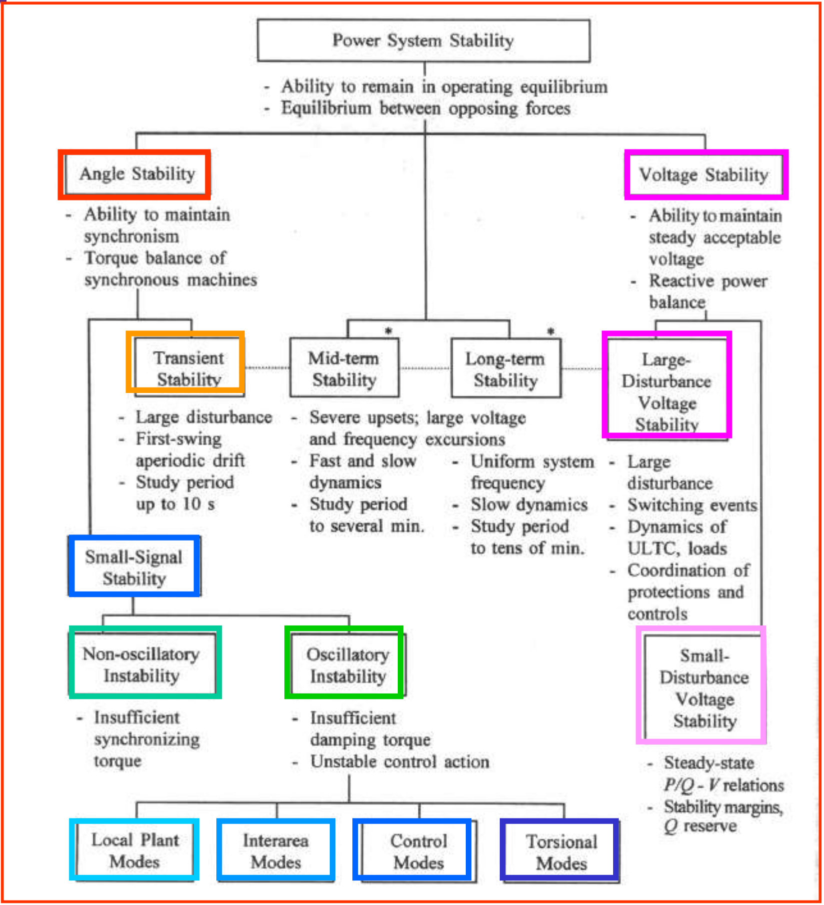
Single equilibrium point

No equilibrium points
- Voltage collapse

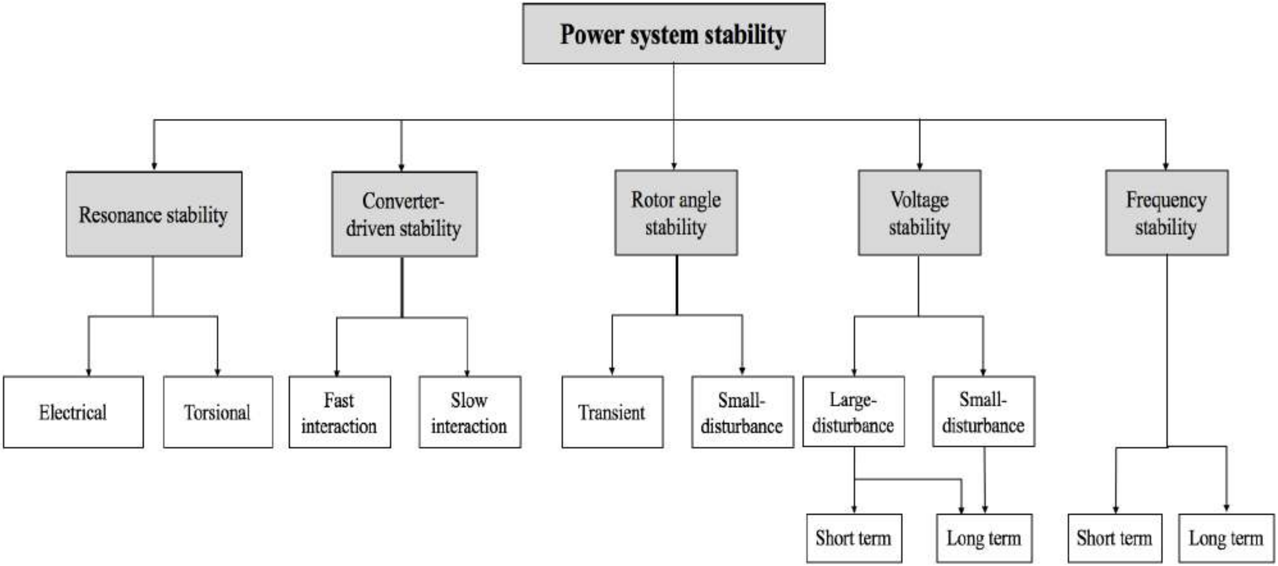
Power System Stability - Time Frame

- Short-term or transient: 0 s to 10 s
- Mid-term: 10 s to a few minutes
  - focus on synchronizing power oscillations between machines including the effects of some of the slower phenomena and large voltage and frequency excursions
- Long-term: a few minutes to 10's of minutes
  - boiler dynamics of thermal units, penstock and conduit dynamics of hydro units, AGC, plant ans transmission system protection/controls, transformer saturation, off-nominal frequency effects on loads and the network

Stability Subdivision



New Stability Subdivision



The future

The existing power systems are already, and the future ones will be even more, characterised by integration of wide range of integrated, widely distributed generation (majority of which are renewable), storage and demand technologies connected to the system through power electronics interfaces resulting in

- Reduced/variable inertia leading to different dynamic behaviour following small and large disturbances
- Faster dynamic responses associated with control circuits
- Limited short-circuit current contributions
- Increased uncertainties in system parameters and operation

Causes of reduction of “system” inertia

- Proliferation of power electronics interfaced generation technologies both, generators (e.g., wind, PV, fuel cells, micro-turbines) and storage
  - Participation of directly connected synchronous generators (SG) in power/energy production is variable and reducing (on overall annual scale)
  - SGs get disconnected or de-loaded to accommodate RES
  - SGs may continue to remain disconnected for a period of time and replaced by storage (e.g., during the night when PVs get replaced by storage)
- Proliferation of HVDC power lines which (may) decouple AC interconnected system in synchronous islands with reduced inertia
- Proliferation of power electronics interfaced load devices (variable/adjustable speed drives in particular)
  - The inertia of electric motors, though of significantly lower influence than inertia of SG, for system frequency (and dynamic response in general) becomes “invisible” to the system

What is system inertia ?

Inertia is a property or natural tendency of an object to remain at rest or in motion at a constant speed.

The rotational kinetic energy (*KE*) stored in synchronous generators (SG) provides an indication of the "inertia" of a power system. A large rotating mass of SG connected to the grid has stored *KE* given by

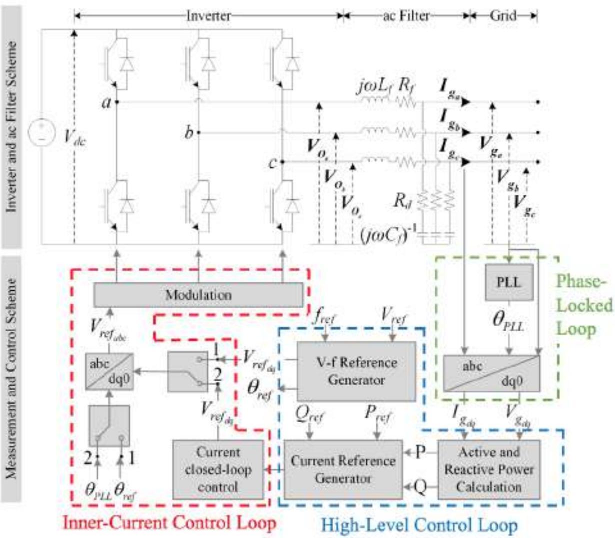
$$\boxed{KE_{gen}[Ws] = \frac{1}{2} J_{gen}[kgm^2](2\pi f_m[mech.rad/s])^2}$$

The inertia constant of a SG

$$\boxed{H_{gen} = \frac{KE_{gen}}{S_{gen}}\left[\frac{\mathrm{MW \cdot s}}{\mathrm{MVA}}\right]}$$

corresponds to the *KE* of its mass rotating at synchronous speed, and effectively represents the time in seconds the generator could continue to provide the rated power to the network if it gets disconnected from the prime mover.

Converter interface and controls associated with CIG



The main circuit includes the **direct-current bus** (e.g., fed by a BESS, PV cells or arrays, or the generator side rectifier for a type-4 wind turbine generator) and the **electronic power converter bridge** that converts the dc current to ac for injection into the network.

The inner current control loops, which **regulate the active and reactive current** to be injected into the network based on the commands issued by the high-level controls.

The phase-locked loop (PLL), which **locks onto the network fundamental frequency voltage and thus keeps the converter synchronized with the network**. (It provides the reference angle for the transformation of variables between the high-level controls and the current to be injected into the grid. The PLL is currently the dominant method used by vendors for synchronization of the CIG to the grid.)

The high-level inverter controls include all the essential control functions such as **voltage/ reactive control, active power control, ride-through functionality, frequency response,** among other functions. (Typically, these controls are significantly slower than the inner-current controls, with the exception of fault ride-through controls which tend to be fast acting controls)

Key attributes of CIG affecting system dynamics

- CIGs can provide **limited short-circuit current** contributions (often ranging from 0, as converter blocks for close in bolted 3-phase faults, to 1.5 $p.u.$ for a fully converter interfaced resource)

- The **PLL** and **inner-current control** loop play a major role in the **dynamic recovery** after a fault.
  - For connection points with low-short circuit ratio, the response of the inner current- control loop and PLL can become oscillatory due to
    - PLL not being able to quickly synchronize with the network voltage
    - high gains in the inner-current control loop and PLL.

  (This can potentially be mitigated by reducing the gains of these controllers. The exact value of the short circuit strength at which this may occur will vary depending on the equipment vendor and network configuration. A typical range of short-circuit ratios below which this may occur is 1.5 to 2.)

Key attributes of CIG affecting system dynamics

- The overall dynamic performance of CIGs is largely determined by the dynamic characteristics of the PLL, the inner-current control loop, and the high-level control loops and their design.
  - With the switching frequency of the power electronic switches typically in the kHz range, and the high-level control loops typically in the range of 1 to 10 Hz, similar to most other controllers in power systems, **CIGs can impact a wide range of dynamic phenomena**, ranging from electromagnetic transients to voltage stability, and across both small and large disturbance stability.
    - **With proper design** of both the main circuit and the converter controls, **CIGs can** contribute to power system control and **provide the vast majority** of the services traditionally provided by conventional generation
    - Due to the significant differences in the physical and electrical characteristics of CIGs compared to synchronous generation, **CIGs do not inherently provide short-circuit current nor inertial response**, and so these aspects will continue to present some challenges

(Examples of) Effects of CIGs on System Dynamic

- Changing the flows on major tie-lines, which may affect damping of inter-area modes and transient stability margins

- Displacing large synchronous generators, which may affect the mode shape, modal frequency, and damping of electromechanical modes of rotor oscillations

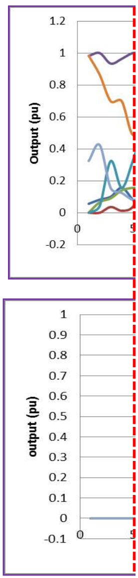
- Influencing/affecting the damping torque of nearby synchronous generators, similar to the manner in which flexible ac transmission (FACTS) devices influence damping .This is reflected in changes in the damping of modes that involve those synchronous generators.

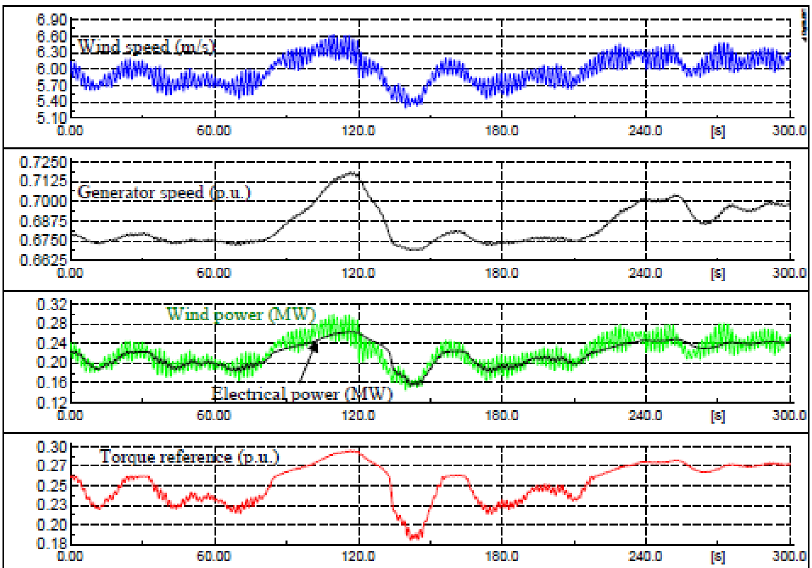
- Displacing synchronous generators that have crucial power system stabilizers.

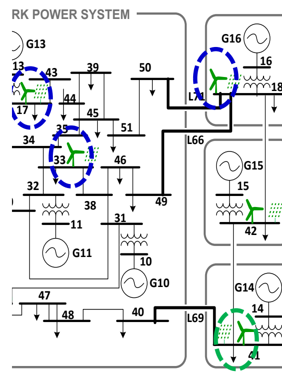
- Different dynamic behavior of RES changes the system dynamic behavior

E1: The impact of CIGs on system dynamics

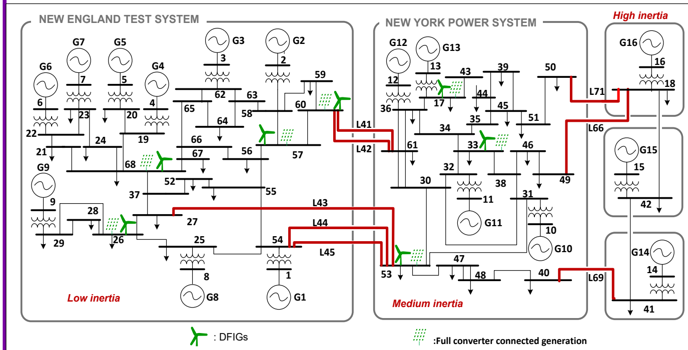
- Increased uncertainty in the pre-fault operating conditions due to the intermittent behavior of RES and their availability, both temporal and spatial







E2: The impact of CIGs on system dynamics



	NPL NET & NYPS	Average 'H' sec			
		H_{NETS}	H_{NYPS}	H_{Eq}	H_{Sys}
100% loading	0	3.9	7.9	11.1	7.95
100% loading	30%	2.7	5.5	11.1	6.8
60% loading	45%	1.64	3.32	7.8	4.1
40% loading	52%	1.28	2.26	6.6	2.86

$$H_{sys} = \frac{\sum_{i=1}^{n} S_i H_i}{\sum_{i=1}^{n} S_i}$$

$$PL_a = \frac{\sum_{n=1}^{d} P_{RES,in}^{0}}{\sum_{m=1}^{g} S_{SG,im} + \sum_{n=1}^{d} P_{RES,in}^{0}}$$

Penetration level

$$NPL_a = \frac{\sum_{n=1}^{d} P_{RES,n}^{0}}{\sum_{m=1}^{g} S_{SG,m} \cdot pf_{SG,m} + \sum_{n=1}^{d} P_{RES,n}^{0}}$$

Nominal penetration level

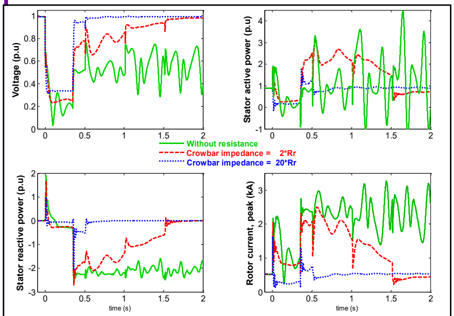
$$INPL_a = \frac{\sum_{n=1}^{d} P_{RES,in}^{0}}{\sum_{m=1}^{g} P_{SG,im} + \sum_{n=1}^{d} P_{RES,in}^{0}}$$

Instantaneous penetration level

How should be H calculated or estimated considering different definitions of "penetration level"

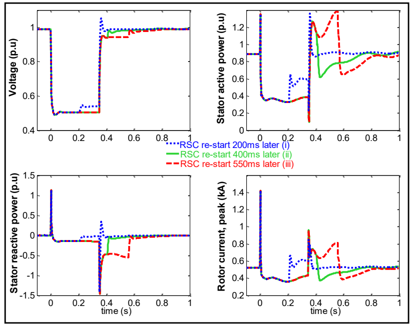
E3: The impact of CIGs on system dynamics

- Different dynamic behavior of RES changes the system dynamic behavior



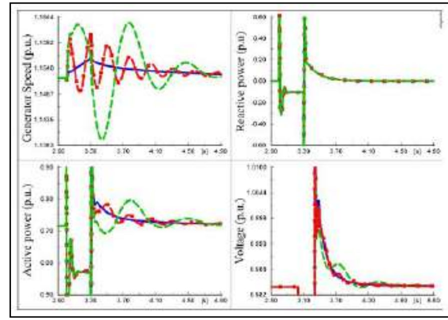
DFIG response to a short-circuit for different crowbar impedances

(Solid – no crowbar impedance, Dashed – 2Rr, Dotted – 20Rr)



DFIG response to a 350ms short-circuit for different RSC restart times.

(RSC is re-started; Dotted – 200ms, Solid – 400ms, Dashed – 550ms after the fault clearance.)



Influence of shaft stiffness on DFIG responses to a 3-phase fault.

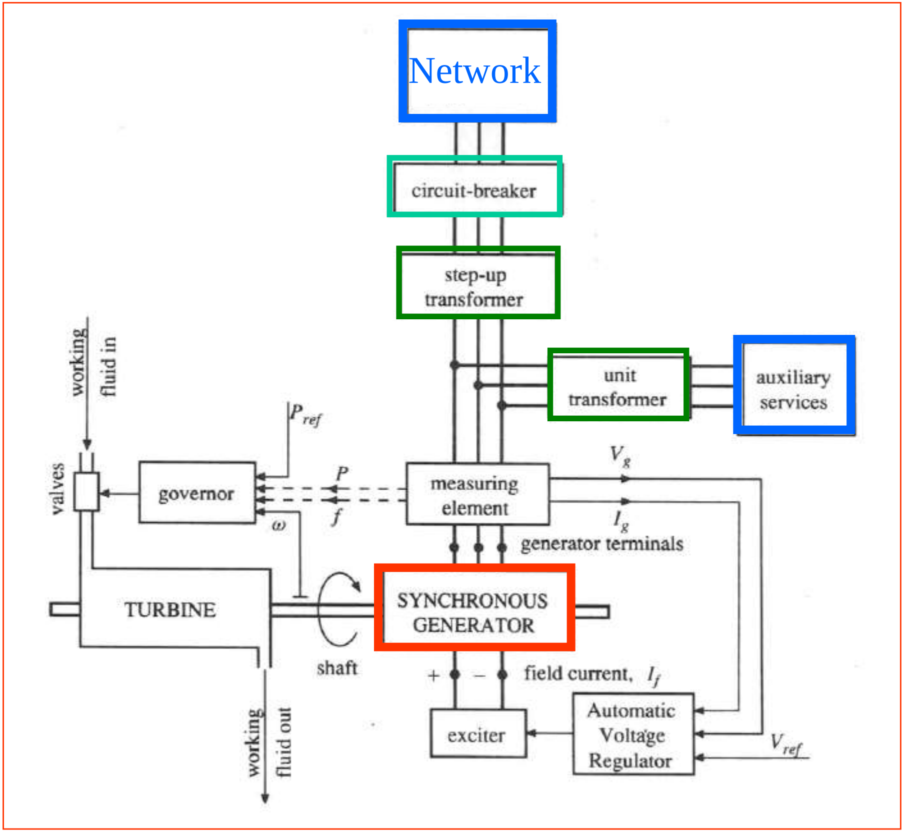
(Dash-dot – Original case (K_s = 2.1 p.u/rad), Dashed – Soft shaft (K_s = 0.3 p.u/rad), Solid – Lumped mass)

MANCHESTER 1824

The University of Manchester

Chapter 2:
Modelling Of Synchronous Machines

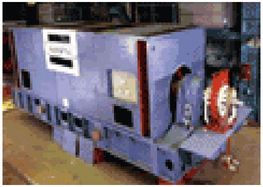
Generator in a Power System



Types of Synchronous Generators - 1

- Round rotor machines (3000 rpm or 1500rpm)
- Salient poles machines (below 500rpm)
  - Rotor (Field) winding
    - DC current
    - Placed at rotor
  - Stator (Armature) winding
    - AC current
    - Placed around inner stator surface
    - Three identical phase windings connected to form generator neutral which is normally grounded. (Each winding spans $120^0/2p$ of the stator surface - *p number of pairs of magnetic poles.)*
  - Damper windings
    - Placed at rotor (short-circuited)

Synchronous Generators



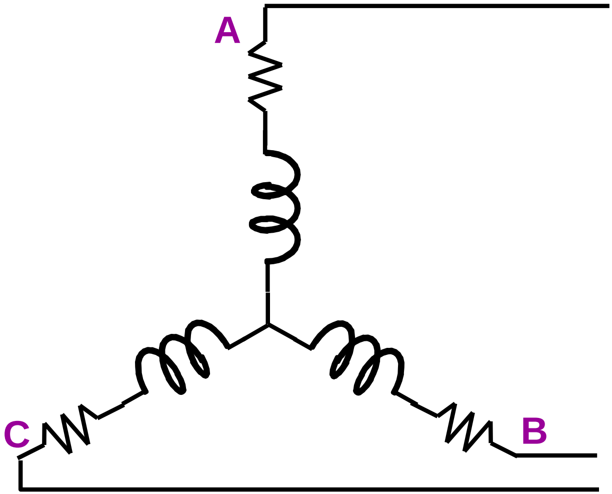
Air Cooled Generator

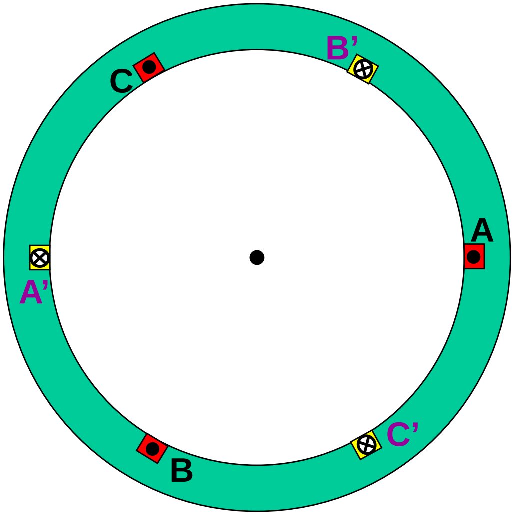
Hydrogen Cooled Generator

Hydrogen / Water Cooled Generator

Rotors of Synchronous Generators

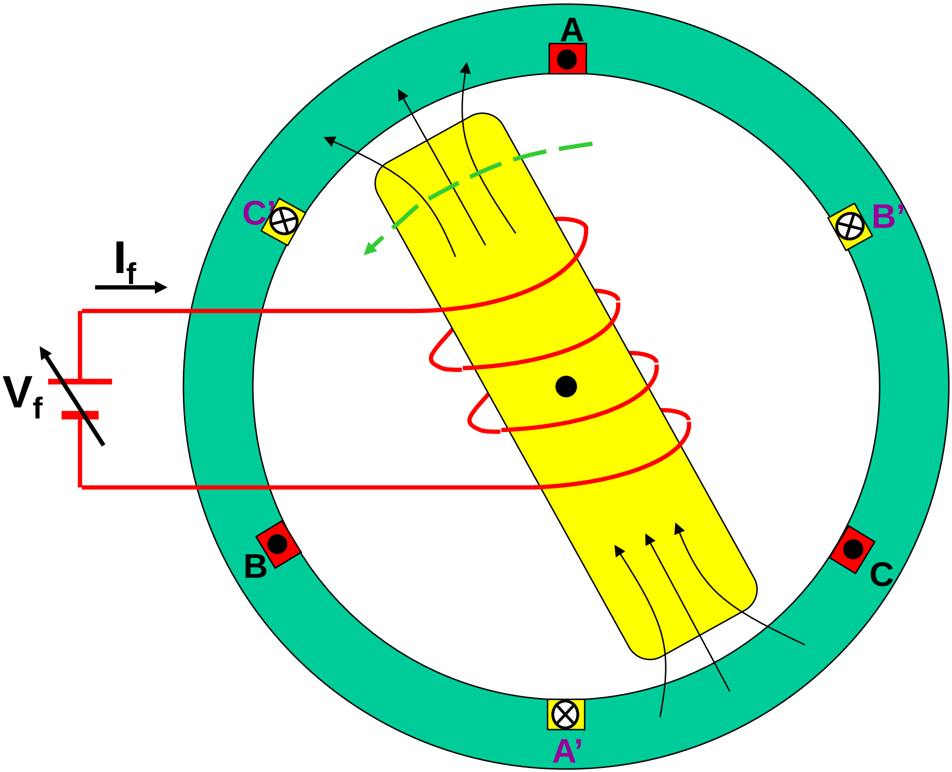
Stator (armature) windings





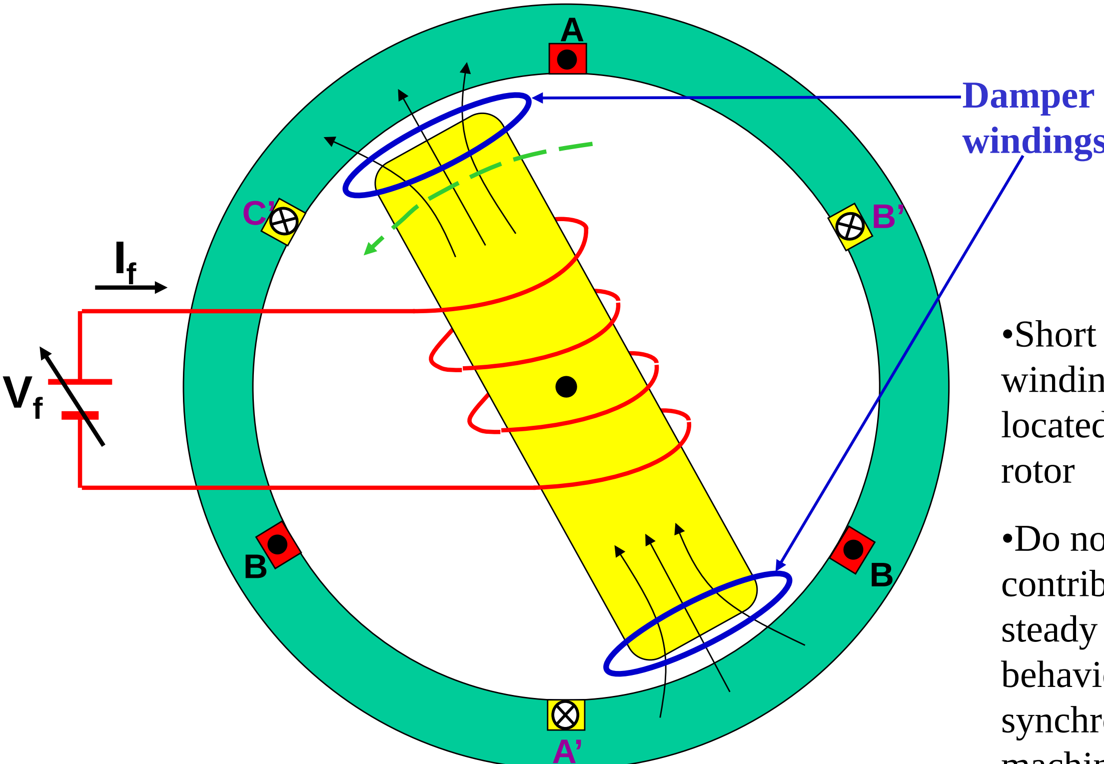
- Carries AC current
- Placed around inner stator surface
- Three identical phase windings connected to form generator neutral which is normally grounded. (Each winding spans $120^0/2p$ of the stator surface - *p number of pairs of magnetic poles.)*

Rotor (field) winding



- Carries a dc current
- Field current creates a magnetic flux tied to the rotor
- This rotor flux links the stator windings
- The rotation of this rotor flux creates an induced e.m.f. in the stator windings

Damper winding



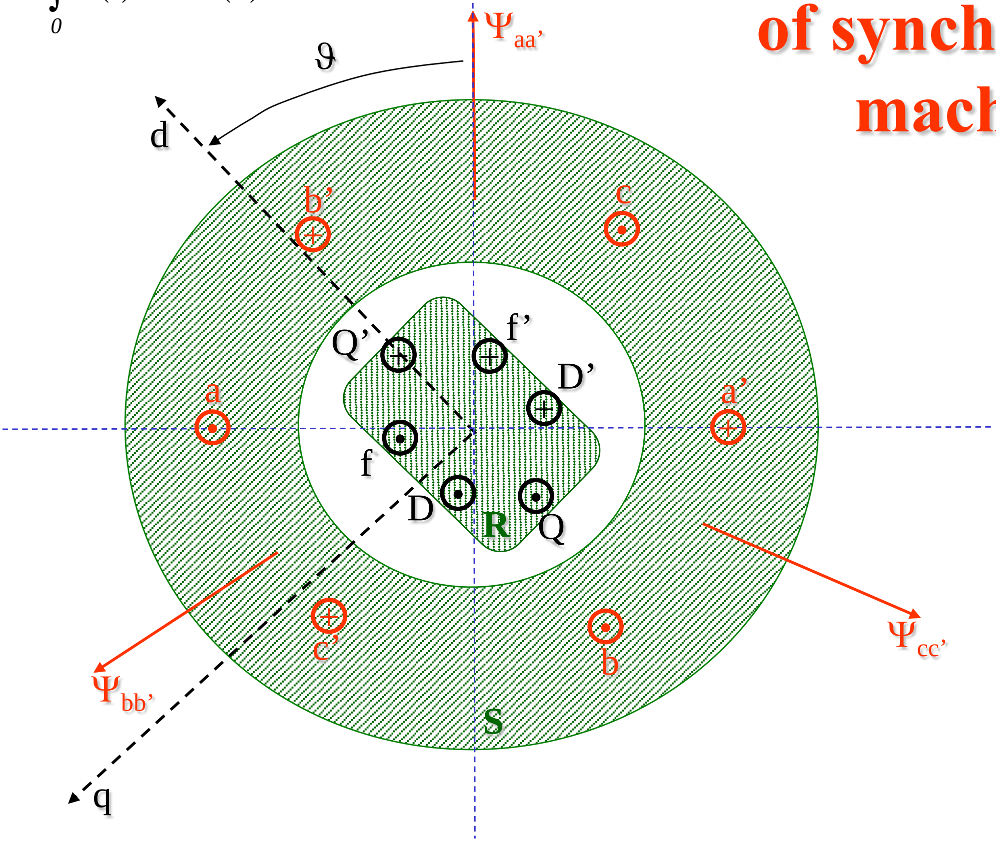
- Short circuited windings located on the rotor
- Do not contribute to the steady state behaviour of the synchronous machine

The Role of Damper Winding

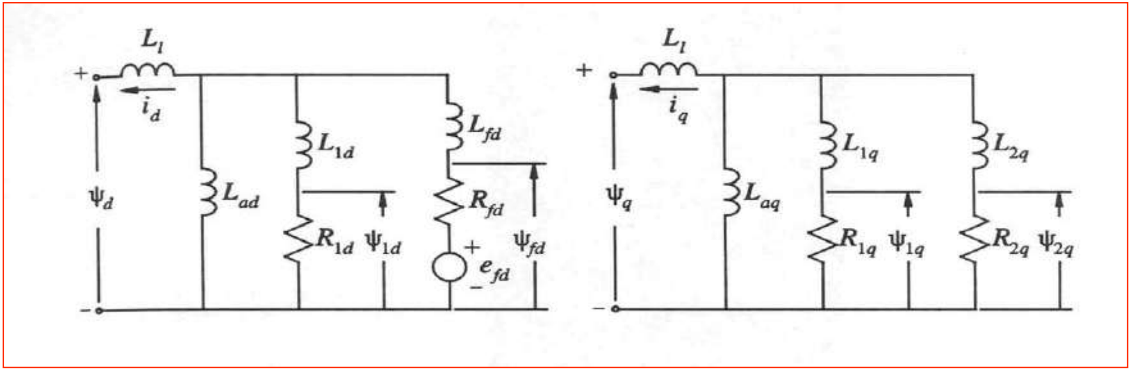
- To damp (and prevent) oscillations of synchronous machine about the mean angle
- To carry with relative safety, the current induced in the rotor as the result of unbalance between the three phases
- To offer a low impedance (and low resistance) to mmf waves of fundamental wavelength but of harmonic frequency.
- To enable a synchronous motor to be accelerated to close to synchronous speed as an induction motor. (Must give low slip for final synchronising and be able to safely absorb a lot of energy.)
- To permit a generator to continue to generate as an induction generator, asynchronously above synchronous speed, in the event of pole-slip or loss of excitation. (*Note:* Pole slip means that the generator rotor will suddenly turn as much as one complete revolution faster than it should be spinning, depending on the number of magnetic poles, and then come violently to a stop again as it tries to magnetically link up again with the stator magnetic field(s). This can cause catastrophic failure of the coupling between the prime mover and the generator)

Schematic diagram of synchronous machine

$$\theta = \int_0^\tau \omega(t)dt + \theta(0)$$



Equivalent circuits



$$L_d = L_l + L_{ad}$$

$$L_{ffd} = L_{fd} + L_{ad}$$

$$L_{11d} = L_{1d} + L_{ad}$$

$$L_q = L_l + L_{aq}$$

$$L_{11q} = L_{1q} + L_{aq}$$

$$L_{22q} = L_{2q} + L_{aq}$$

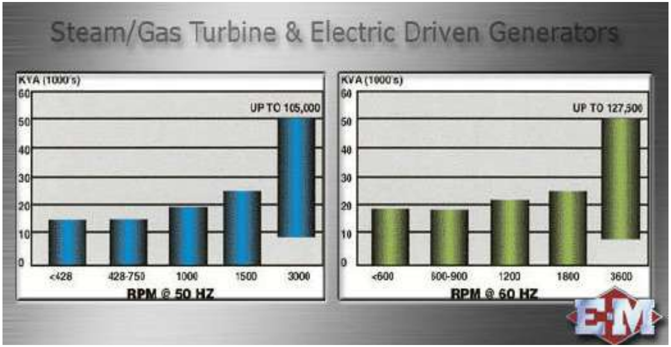
Types of Synchronous Generators - 2

- Ratings typically100MW to > 1500MW(2000MW)
- Voltages between 10kV and 32kV
- Synchronous speed

$$\omega_{sm} = \frac{\omega_e}{p} = \frac{2\pi f}{p}\left[\frac{mech.-rad}{\sec}\right]$$

or

$$n_s = \frac{60\omega_m}{2\pi} = \frac{60 f}{p}\left[\frac{rev.}{\min}\right]$$



p	1	2	3	4	5	6
n_s	3000	1500	1000	750	600	500

Model Development - 1

The phase stator and rotor voltages of a salient pole synchronous machine with one damper winding in the same axis as the field winding and two damper windings, which are displaced 90 degree ahead of the magnetic axis of the field winding, are defined as:

$$v_a = \frac{d\psi_a}{dt} - r_s i_a$$

$$v_b = \frac{d\psi_b}{dt} - r_s i_b$$

$$v_c = \frac{d\psi_c}{dt} - r_s i_c$$

$$v_f = \frac{d\psi_f}{dt} + i_f r_f$$

$$v_{d1} = \frac{d\psi_{d1}}{dt} + r_{d1} i_{d1}$$

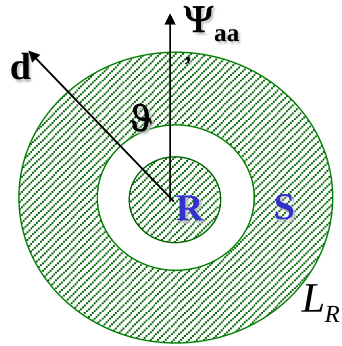
$$v_{q1} = \frac{d\psi_{q1}}{dt} + r_{q1} i_{q1}$$

$$v_{q2} = \frac{d\psi_{q2}}{dt} + r_{q2} i_{q2}$$

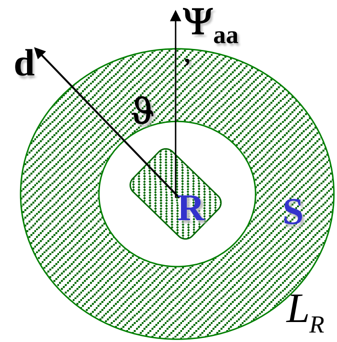
Basic assumptions:

1. The stator windings are sinusoidally distributed along the air gap as far as the mutual effects with rotor are concerned
2. The stator slots cause no appreciable variation of the rotor inductance with rotor position.
3. Magnetic hysteresis is negligible.
4. Magnetic saturation effects are negligible.

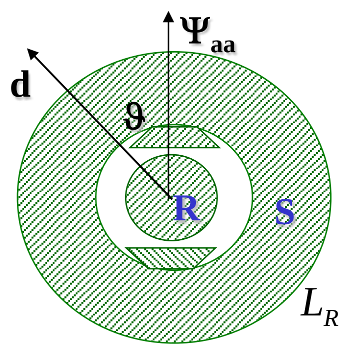
Stator and Rotor Inductances



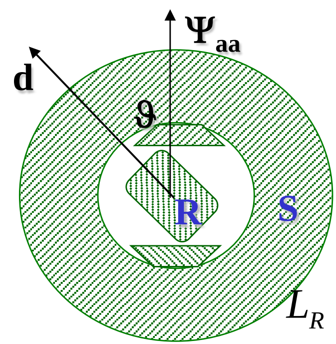
$$L_R \neq f(\vartheta) = const.$$
$$L_S \neq f(\vartheta) = const.$$



$$L_R \neq f(\vartheta) = const.$$
$$L_S = f(\vartheta) \neq const.$$



$$L_R = f(\vartheta) \neq const.$$
$$L_S \neq f(\vartheta) = const.$$



$$L_R = f(\vartheta) \neq const.$$
$$L_S = f(\vartheta) \neq const.$$

Model Development - 2

•Some of the machine inductances (mutual and self inductances of the stator circuits and mutual inductances between stator and rotor windings) are functions of (dependent on) the rotor position, consequently the coefficients of the differential equations, which describe the machine performance are time varying (i.e., non constant).

•A change of variables is introduced to reduce the complexity of these equations.

•This was done by R.H.Park in the late 1920s (1929). He refereed the variables of the stator winding of the synchronous machine to a frame of reference fixed in the rotor.

Transformation of Machine Variables - 1

- DECOUPLING – Instead of real sets of fluxes an currents a fictitious sets are used (d and q axis components) whose property is that each flux depends on its own current and some small number of the other currents (sparse matrix). This is the way to obtain matrix of inductances of predominantly diagonal structure while preserving the form of equations
- ROTATION – To exclude the effect of rotation of rotor, i.e., to remove dependence of inductances on rotor angle (θ). This transformation results in different voltage equations with one new term describing the rotation of the rotor.

$$\underline{v} = -\underline{R}\,\underline{i} - \frac{d\underline{\Psi}}{dt} = -\underline{R}\,\underline{i} - \frac{d\underline{L}(\vartheta)}{dt}\underline{i} - \underline{L}(\vartheta)\frac{d\underline{i}}{dt}$$

Differential equations with time varying coefficients

$$\underline{v}^{new} = -\underline{R}\,\underline{i}^{new} - \boxed{\frac{d\vartheta}{dt}\underline{\Psi}^{new}} - \frac{d\underline{\Psi}^{new}}{dt} = -\underline{R}\,\underline{i}^{new} - \frac{d\vartheta}{dt}\underline{L}^{new}\underline{i}^{new} - \underline{L}^{new}\frac{d\underline{i}^{new}}{dt}$$

$$\underline{L}^{new} \neq f(\vartheta)$$

new term accounting for rotation

Differential equations with constant coefficients

Transformation of Machine Variables - 2

- SCALING – This transformation is similar to scaling in the case of transformer between primary and secondary winding (the turns ratio between stator and rotor is used). This transformation is a part of per-unit system.
- LAPALCAE TRANSFORM – Transforms linear differential equations into algebraic equations. (This can be used only when rotor speed is considered to be constant.)

Park's Transformation

$$\begin{bmatrix} I_d \\ I_q \\ I_0 \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos(\theta-\frac{2\pi}{3}) & \cos(\theta+\frac{2\pi}{3}) \\ -\sin\theta & -\sin(\theta-\frac{2\pi}{3}) & -\sin(\theta+\frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \qquad \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 1 \\ \cos(\theta-\frac{2\pi}{3}) & -\sin(\theta-\frac{2\pi}{3}) & 1 \\ \cos(\theta+\frac{2\pi}{3}) & -\sin(\theta+\frac{2\pi}{3}) & 1 \end{bmatrix}\begin{bmatrix} I_d \\ I_q \\ I_0 \end{bmatrix}$$

$$\boxed{\theta = \int_0^t \omega(\xi)d\xi + \theta(0)} \qquad P = \frac{3}{2}(V_d I_d + V_q I_q + 2V_0 I_0) = \boxed{V_d I_d + V_q I_q + 2V_0 I_0}$$

(Per-unit)

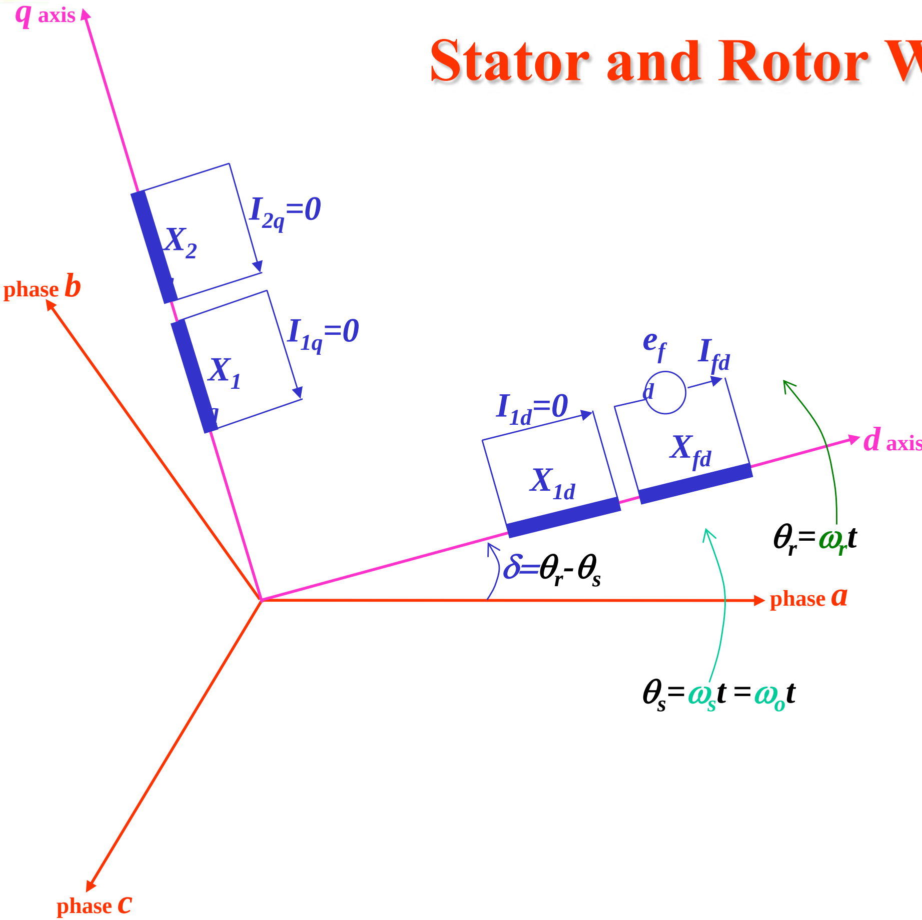
Alternative Park's transformation (Power invariant)

$$\begin{bmatrix} I_d \\ I_q \\ I_0 \end{bmatrix} = \sqrt{\frac{2}{3}}\begin{bmatrix} \cos\theta & \cos(\theta-\frac{2\pi}{3}) & \cos(\theta+\frac{2\pi}{3}) \\ -\sin\theta & -\sin(\theta-\frac{2\pi}{3}) & -\sin(\theta+\frac{2\pi}{3}) \\ \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\boxed{P = V_d I_d + V_q I_q + V_0 I_0}$$

The new coefficient however removes one to one relationship between *abc* and *dq0* variables.

Stator and Rotor Windings



Complete Mathematical Model

$$V_d = -R_a I_d + \frac{1}{\omega_0} p\Psi_d - \omega\Psi_q$$

$$V_q = -R_a I_q + \frac{1}{\omega_0} p\Psi_q + \omega\Psi_d$$

$$V_0 = -R_a I_0 + \frac{1}{\omega_0} p\Psi_0$$

$$e_{fd} = R_{fd} I_{fd} + \frac{1}{\omega_0} p\Psi_{fd}$$

$$0 = R_{1d} I_{1d} + \frac{1}{\omega_0} p\Psi_{1d}$$

$$0 = R_{1q} I_{1q} + \frac{1}{\omega_0} p\Psi_{1q}$$

$$0 = R_{2q} I_{2q} + \frac{1}{\omega_0} p\Psi_{2q}$$

$$p\omega = \frac{1}{2H}(T_m - T_e - K_D(\omega - 1))$$

$$p\delta = \omega_0(\omega - 1)$$

$$\Psi_d = -(L_{ad} + L_l) I_d + L_{ad} I_{fd} + L_{ad} I_{1d}$$

$$\Psi_q = -(L_{aq} + L_l) I_q + L_{aq} I_{1q} + L_{aq} I_{2q}$$

$$\Psi_0 = -L_0 I_0$$

$$\Psi_{fd} = -L_{ad} I_d + L_{ffd} I_{fd} + L_{f1d} I_{1d}$$

$$\Psi_{1d} = -L_{ad} I_d + L_{f1d} I_{fd} + L_{11d} I_{1d}$$

$$\Psi_{1q} = -L_{aq} I_q + L_{11q} I_{1q} + L_{aq} I_{2q}$$

$$\Psi_{2q} = -L_{aq} I_q + L_{22q} I_{2q} + L_{aq} I_{1q}$$

$$T_e = \Psi_d I_q - \Psi_q I_d$$

$$L_{fd} = L_{ffd} - L_{f1d}$$

$$L_{1d} = L_{11d} - L_{f1d}$$

$$L_{1q} = L_{11q} - L_{aq}$$

$$L_{2q} = L_{22q} - L_{aq}$$

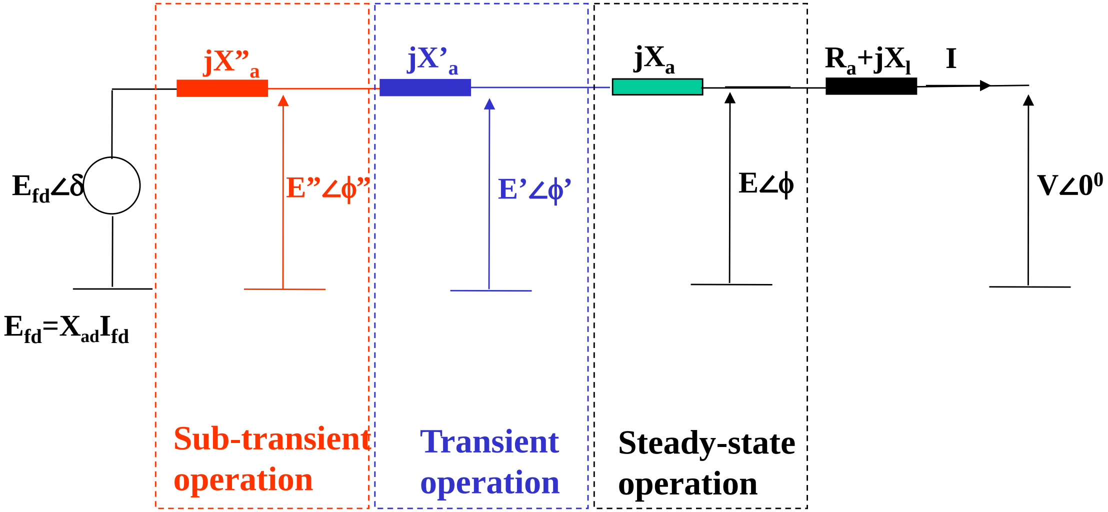
$$L_d = L_{ad} + L_l$$

$$L_q = L_{aq} + L_l$$

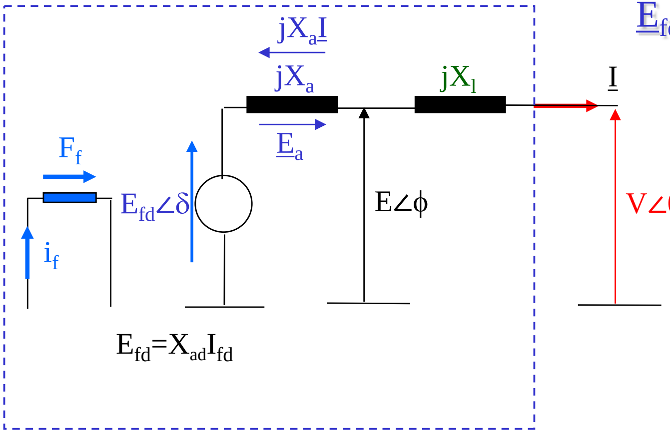
Electrical quantities and torques in p.u.;
Time in seconds;
Rotor angle in electrical radians;
$\omega_0 = 2\pi f$

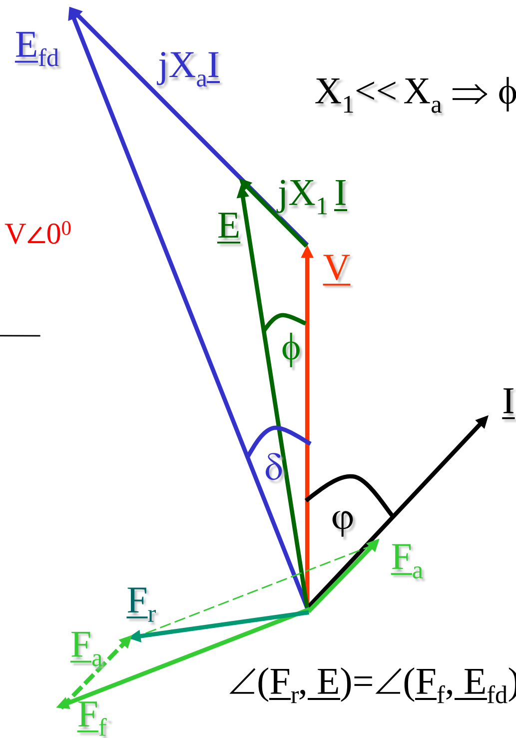
$$p = \frac{d}{dt}$$

Equivalent Circuit



Equivalent Circuit - Steady State





$X_l << X_a \Rightarrow \phi << \delta$

$\underline{E}_a = -jX_a\underline{I}$

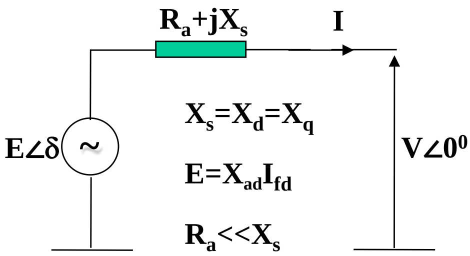
armature reaction emf

$\angle(\underline{F}_r, \underline{E}) = \angle(\underline{F}_f, \underline{E}_{fd}) = 90^o$

$$\underline{E}_{fd} = \underline{E} + jX_a\underline{I} = (\underline{V} + jX_l\underline{I}) + jX_a\underline{I} = \underline{V} + jX_s\underline{I}$$

Steady-state Model

$$V_d = -R_a I_d - \omega \Psi_q$$
$$V_q = -R_a I_q + \omega \Psi_d$$
$$e_{fd} = R_{fd} I_{fd}$$



$$I^2 = I_d^{\ 2} + I_q^{\ 2}$$

$$\Psi_d = -(L_{ad} + L_l) I_d + L_{ad} I_{fd}$$
$$\Psi_q = -(L_{aq} + L_l) I_q$$
$$\Psi_{fd} = -L_{ad} I_d + L_{ffd} I_{fd}$$
$$\Psi_{1d} = -L_{ad} I_d + L_{f1d} I_{fd}$$
$$\Psi_{1q} = -L_{aq} I_q$$
$$\Psi_{2q} = -L_{aq} I_q$$

$$T_e = \Psi_d I_q - \Psi_q I_d = P + R_a I^2$$
$$P = V_d I_d + V_q I_q$$
$$Q = V_q I_d - V_d I_q$$

Synchronous Machine as a Power Source

Round rotor machines (resistance neglected):

$$P = \frac{EV}{X_d}\sin\delta$$

$$Q = \frac{EV}{X_d}\cos\delta - \frac{V^2}{X_d}$$

$$T = \frac{P}{\omega_{sm}} = \frac{P}{\frac{2\pi n_s}{60}} = \frac{P}{\frac{2\pi}{60}\frac{60f}{p}} = \frac{P}{\frac{2\pi f}{p}} = \frac{P}{\frac{\omega_{se}}{p}}$$

If machine parameters are:
Xd=Xq=1.5, V=1, P=1, $cos\varphi = 1$
Follows: E=1.8028 and δ=56.3^{o}; which then gives:
P = 1.202$sin\delta$ = 1; Q =1.202$cos\delta$ - 0.67 = 0

ω_{sm}- mechanical angular velocity
ω_{se}- electrical angular velocity
p - number of pairs of magnetic poles

Salient-pole machines (resistance neglected):

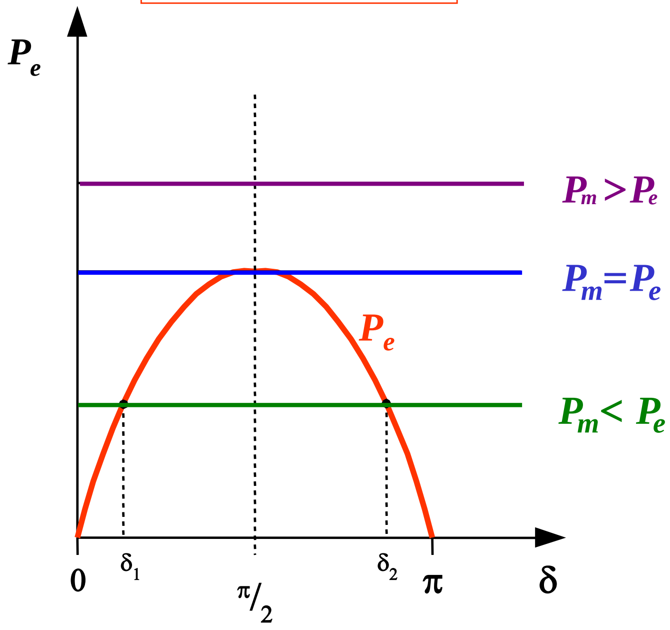
$$P = \frac{EV}{X_d}\sin\delta + \frac{V^2}{2}\frac{X_d - X_q}{X_d X_q}\sin 2\delta$$

$$Q = \frac{EV}{X_d}\cos\delta - \frac{V^2}{X_d X_q}(X_d \sin^2\delta + X_q \cos^2\delta)$$

If machine parameters are:
Xd=1.5, Xq=1, V=1, P=1, $cos\varphi = 1$
Follows: E=1.7678 and δ=45^{o}; which then gives:
P = 1.18$sin\delta$ + 0.17$sin2\delta$ = 0.83 +0.17 = 1
Q =0.78$cos\delta$ - 0.44(0.5$sin^2\delta$ +1) = 0

Power Angle Characteristic - ideal

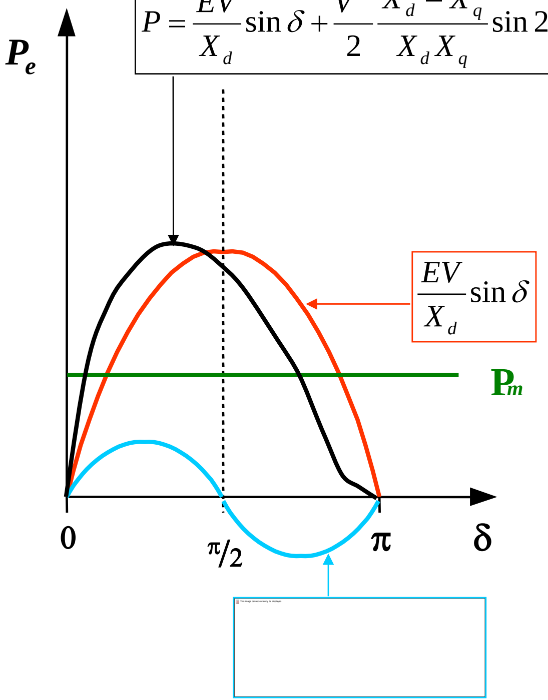
$$P_e = \frac{EV}{X_d} \sin \delta$$



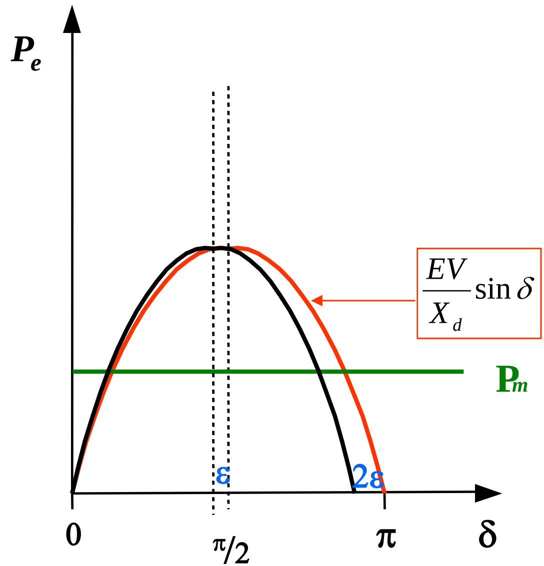
Power Angle Characteristic

With saliency and $R_s = 0$

$$P = \frac{EV}{X_d}\sin\delta + \frac{V^2}{2}\frac{X_d - X_q}{X_d X_q}\sin 2\delta$$



No saliency and $R_s \neq 0$



$$\varepsilon = \arctan\frac{R}{X_s}$$

Round Rotor - *d & q* components

$V_d = V\sin\delta$

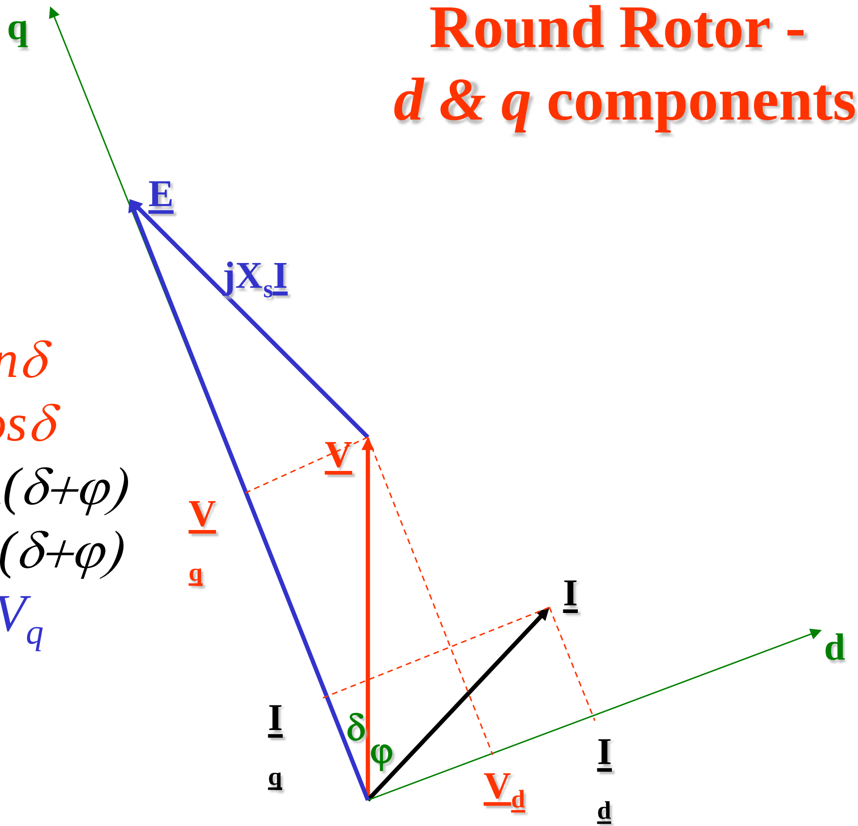
$V_q = V\cos\delta$

$I_d = I\sin(\delta+\varphi)$

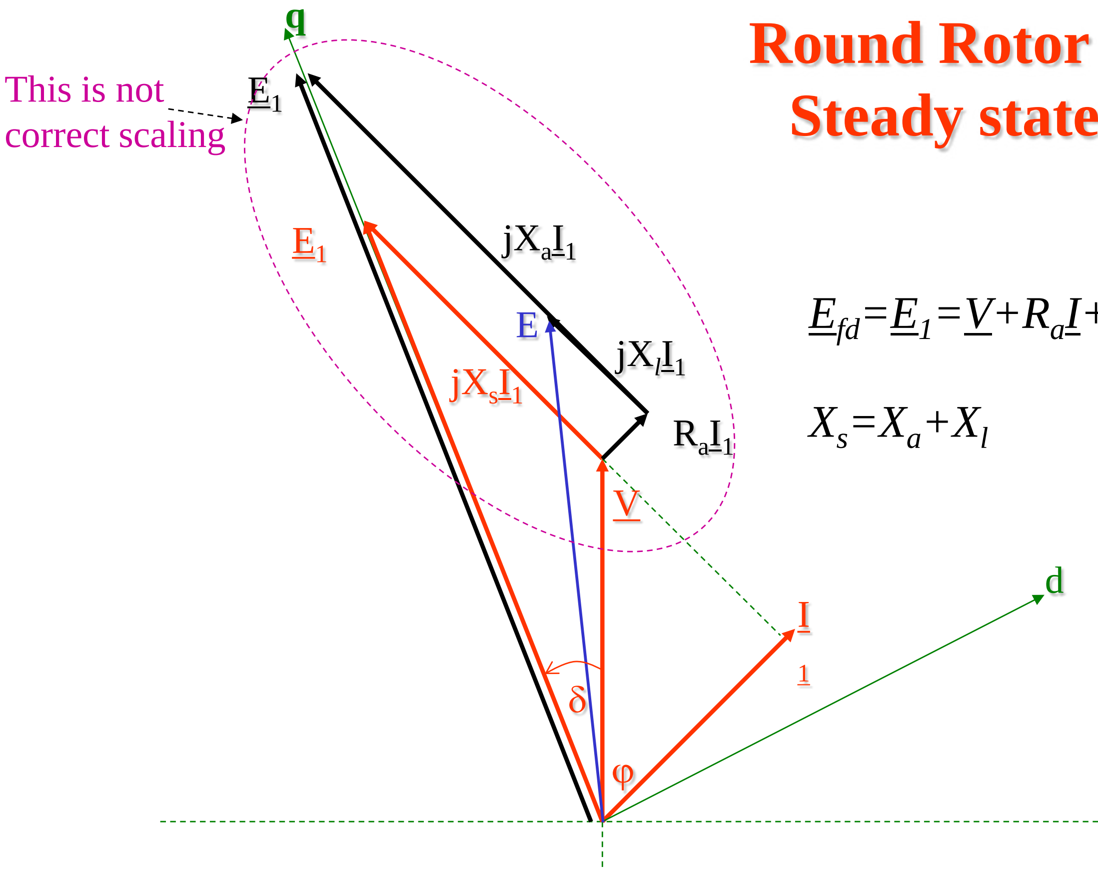
$I_q = I\cos(\delta+\varphi)$

$\underline{V} = V_d + jV_q$

$\underline{I} = I_d + jI_q$



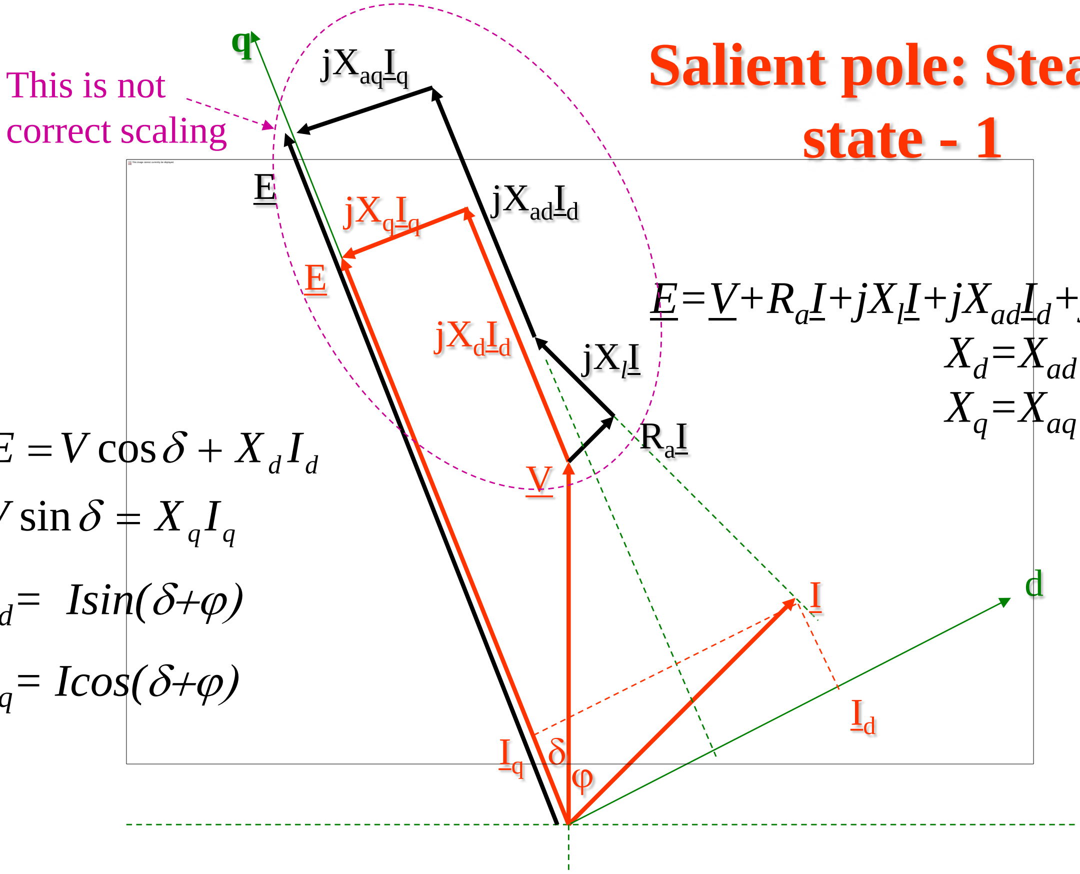
Round Rotor - Steady state



$$\underline{E}_{fd}=\underline{E}_1=\underline{V}+R_a\underline{I}+jX_s\underline{I}$$

$$X_s=X_a+X_l$$

Salient pole: Steady state - 1



$$\underline{E} = \underline{V} + R_a\underline{I} + jX_l\underline{I} + jX_{ad}\underline{I}_d + jX_{aq}\underline{I}_q$$

$$X_d = X_{ad} + X_l$$

$$X_q = X_{aq} + X_l$$

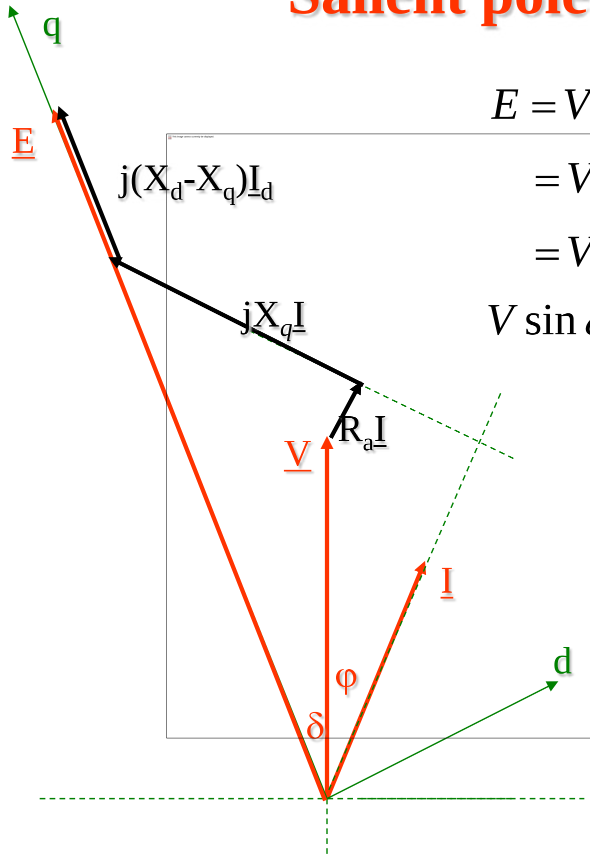
$$E = V\cos\delta + X_d I_d$$

$$V\sin\delta = X_q I_q$$

$$I_d = I\sin(\delta+\varphi)$$

$$I_q = I\cos(\delta+\varphi)$$

Salient pole: Steady state - 2



$$E = V\cos\delta + X_q I \sin(\delta+\varphi) + (X_d - X_q) I_d$$

$$= V\cos\delta + X_q I_d + X_d I_d - X_q I_d$$

$$= V\cos\delta + X_d I_d$$

$$V\sin\delta = X_q I\cos(\delta+\varphi) = X_q I_q$$

$$I_d = I\sin(\delta+\varphi)$$

$$I_q = I\cos(\delta+\varphi)$$

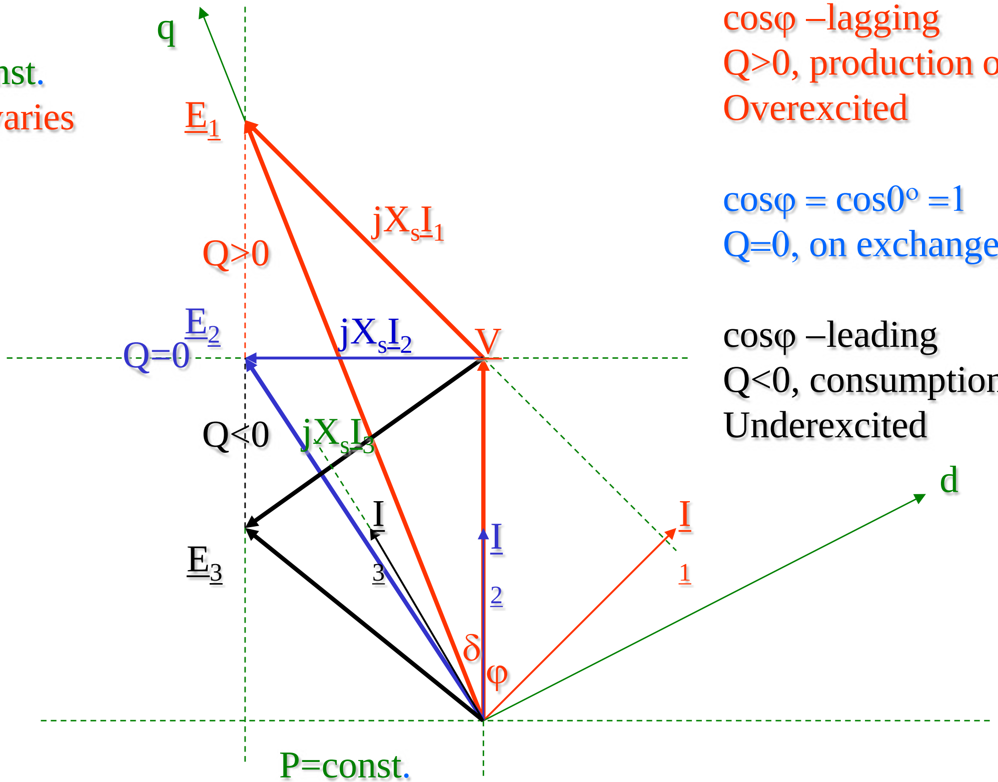
$$\underline{E} = \underline{V} + R_a\underline{I} + jX_q\underline{I} + j(X_d - X_q)\underline{I}_d$$

$$X_d = X_{ad} + X_l$$

$$X_q = X_{aq} + X_l$$

Round Rotor: Steady state - 1

P = const.
I_f (E)-varies

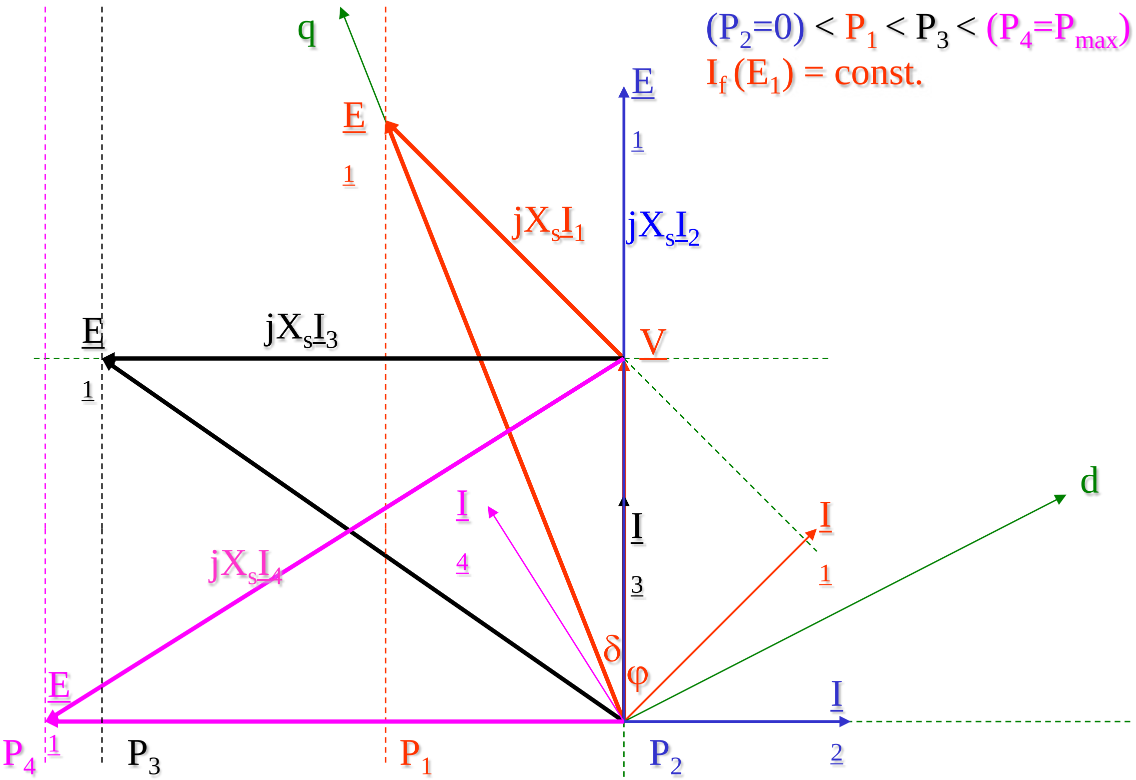


cosφ –lagging
Q>0, production of Q
Overexcited

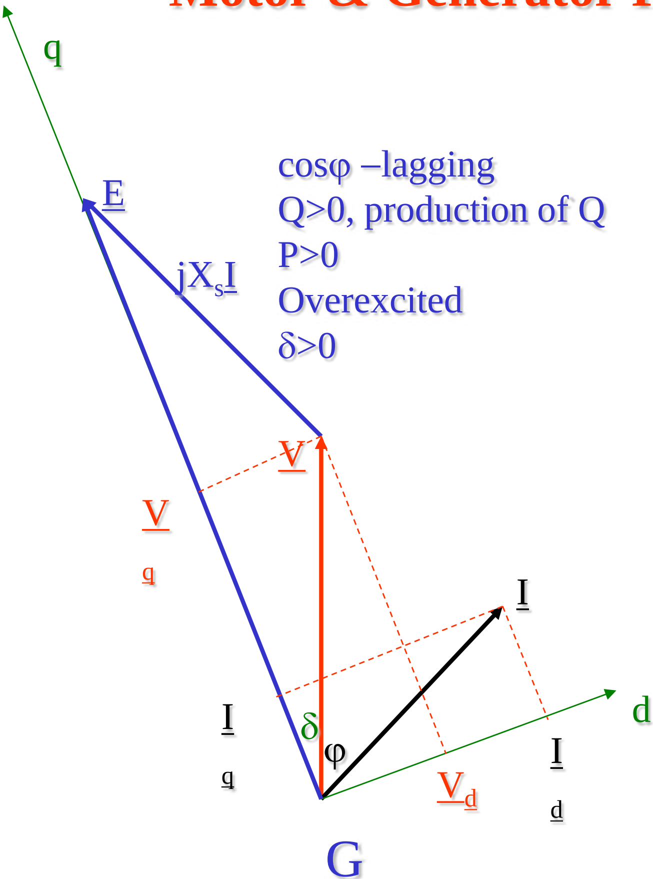
cosφ = cos0° =1
Q=0, on exchange of Q

cosφ –leading
Q<0, consumption of Q
Underexcited

Round Rotor : Steady state - 2

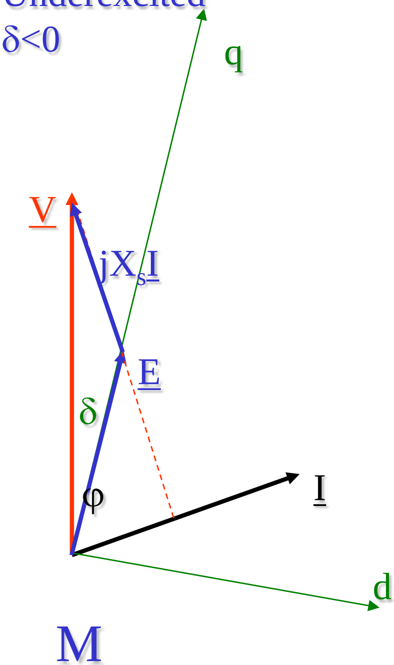


Motor & Generator Phasor Diagrams - 1



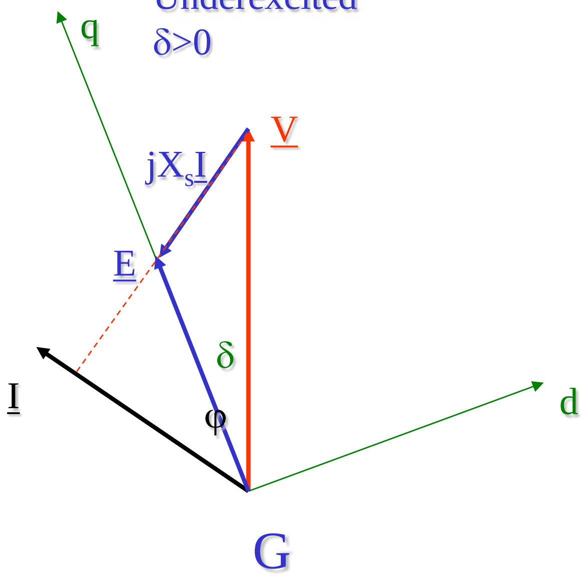
cosφ –lagging
Q>0, production of Q
P>0
Overexcited
δ>0

cosφ –lagging
Q<0, consumption of Q
P<0
Underexcited
δ<0

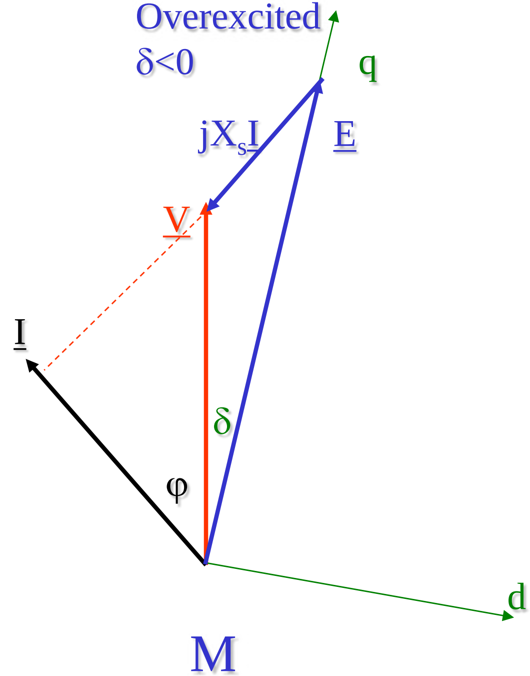


Motor & Generator Phasor Diagrams - 2

cosφ –leading
Q<0, consumption of Q
P>0
Underexcited
δ>0



cosφ –leading
Q>0, production of Q
P<0
Overexcited
δ<0

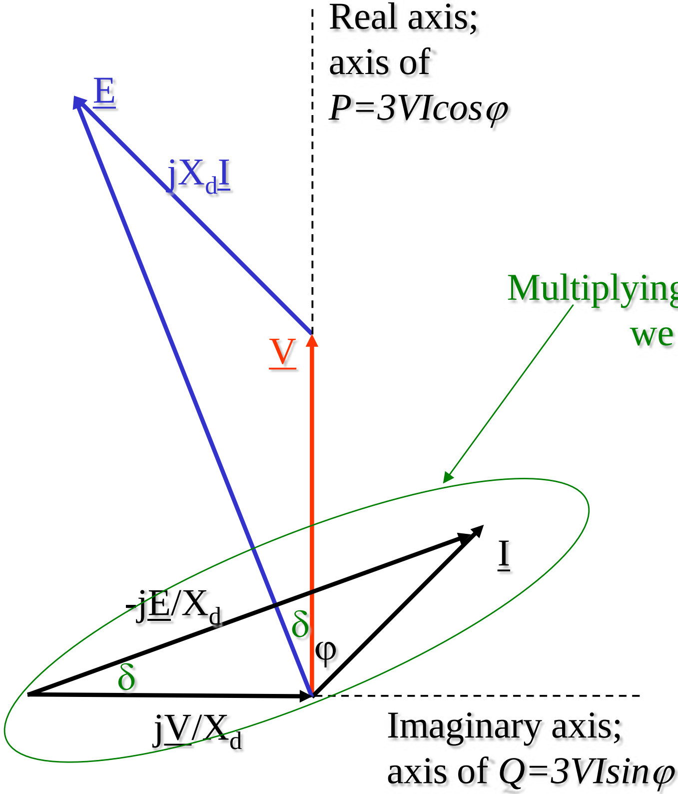


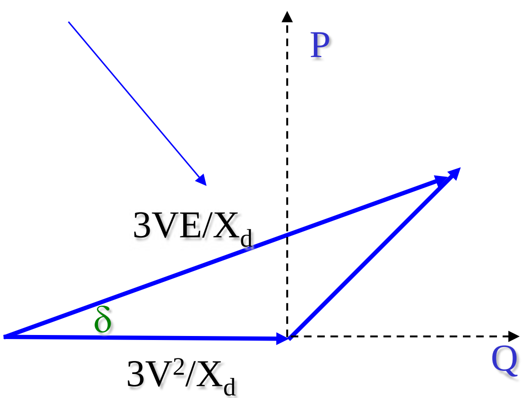
Generator Performance Chart - 1

With neglected resistance and fixed terminal voltage

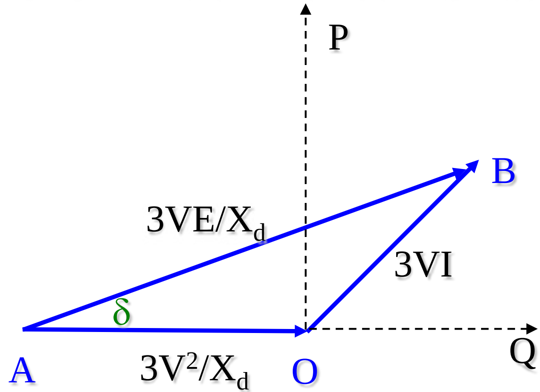
$$\underline{E} = \underline{V} + jX_d \underline{I}$$

$$\underline{I} = j\frac{\underline{V}}{X_d} - j\frac{\underline{E}}{X_d}$$





Generator Performance Chart - 2



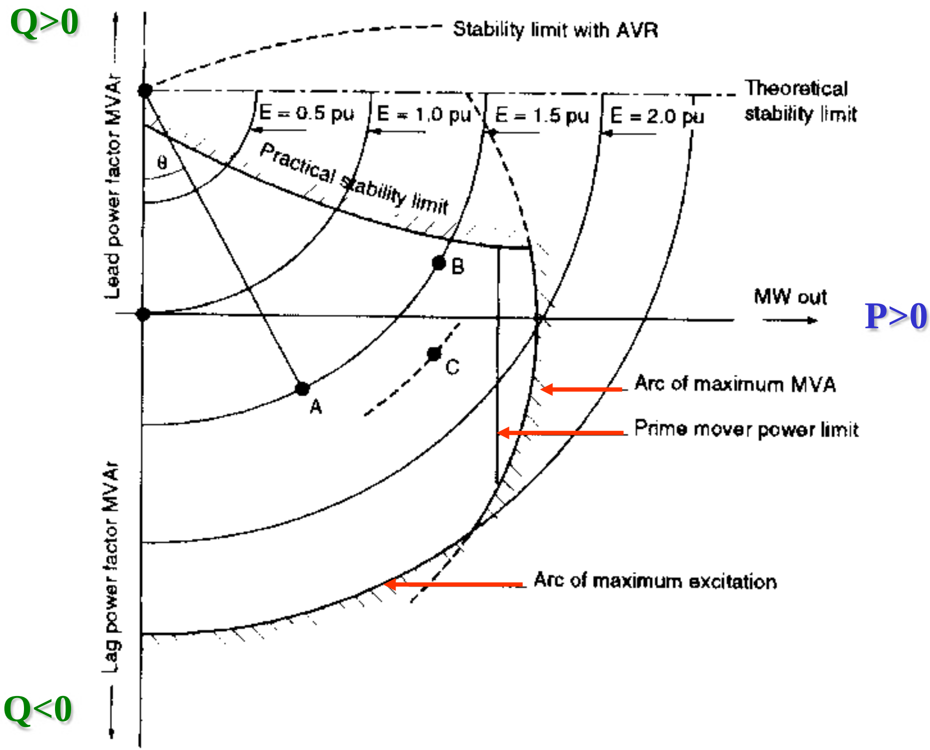
OA - reactive power which would be consumed by the machine if unexcited

OB – output apparent power (lagging); limited by maximum permissible I

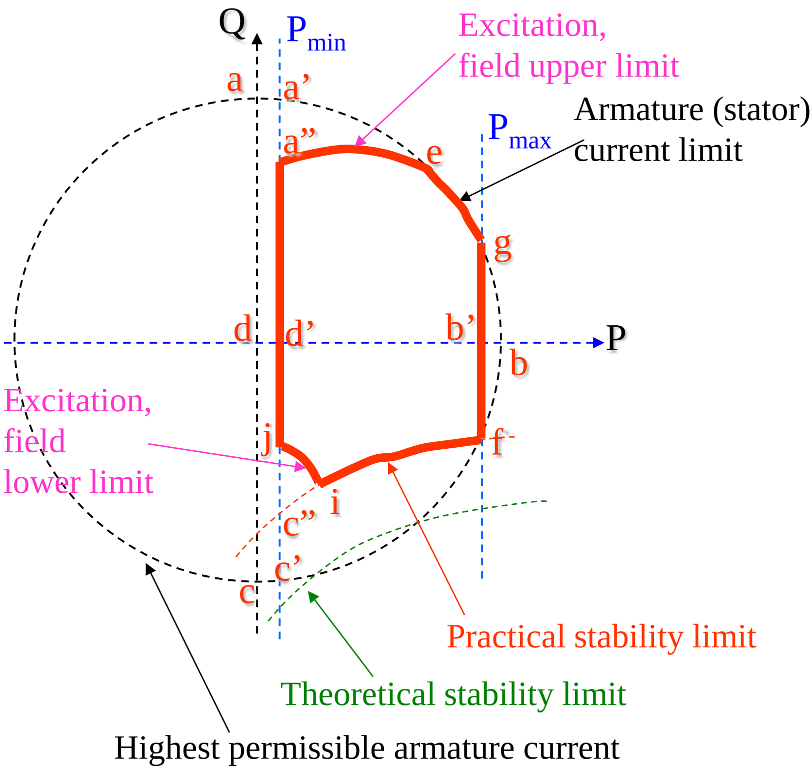
AB – measure of excitation; limited by maximum permissible field current

Point A is fixed for given V and X_d

Generator Performance Chart - 3



Generator Performance Chart - 4

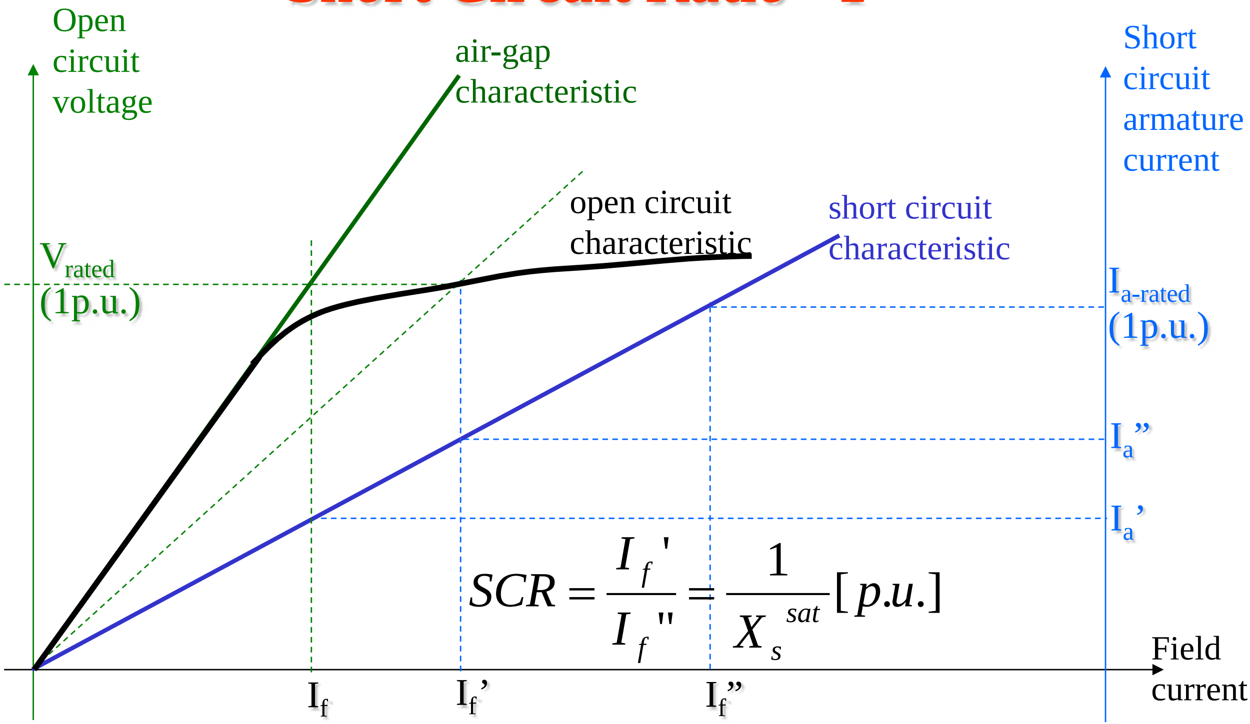


Physical limits of generator are reached when the operating temperature attains a prescribed maximum value. (Ohmic winding losses will in reality determine the loading limits.)

a-b-c-d-a – limit if the ohmic armature losses are the only losses

a"-e-g-b'-f-i-j-d'-a" – contour within which generator can be satisfactorily operated

Short Circuit Ratio - 1



$$SCR = \frac{I_f'}{I_f''} = \frac{1}{X_s^{sat}} [p.u.]$$

Ratio of the field current required for rated voltage on open circuit to the field current required for rated armature (stator) current on short circuit. (Unsaturated synchronous reactance = V_{rated}/I_a’; Saturated synchronous reactance = V_{rated}/I_a”)

Short Circuit Ratio - 2

- SCR is the ratio of the field current required for rated voltage on open circuit to the field current required for rated armature (stator) current on short circuit.

- The SCR reflects the degree of saturation of the machine as it is reciprocal of the saturated synchronous reactance of the machine.

$$SCR = \frac{I_f'}{I_f''} = \frac{1}{X_s^{sat}} [p.u.]$$

- A lower SCR is indicative of:
  - A larger change in field current required to maintain constant terminal voltage for a given change in load;
  - A requirement for an excitation system that is able to provide large changes of filed current to maintain system stability;
  - Lower size, weight, and cost of the machine

Typical Values of Machine Parameters

Parameter		Hydraulic Units	Thermal Units
Synchronous Reactance	X_d	0.6 - 1.5	1.0 - 2.3
	X_q	0.4 - 1.0	1.0 - 2.3
Transient Reactance	X'_d	0.2 - 0.5	0.15 - 0.4
	X'_q	-	0.3 - 1.0
Subtransient Reactance	X''_d	0.15 - 0.35	0.12 - 0.25
	X''_q	0.2 - 0.45	0.12 - 0.25
Transient OC Time Constant	T'_{d0}	1.5 - 9.0 s	3.0 - 10.0 s
	T'_{q0}	-	0.5 - 2.0 s
Subtransient OC Time Constant	T''_{d0}	0.01 - 0.05 s	0.02 - 0.05 s
	T''_{q0}	0.01 - 0.09 s	0.02 - 0.05 s
Stator Leakage Inductance	X_l	0.1 - 0.2	0.1 - 0.2
Stator Resistance	R_a	0.002 - 0.02	0.0015 - 0.005

H [MWs/MVA]
Thermal unit
two-pole
H=(2.5 – 6)s
four-pole
H=(4- 10)s
Hydro unit
H=(2 - 4)s

Negative sequence:
$X_2=0.5*(X_d''+X_q'')$
= 0.1-0.5 p.u.
$R_2= R_a +0.5 R_{fd}$

Zero sequence:
$X_0=(0.15-0.6)*X_d''$
= 0.02-0.45 p.u.
$R_0 \geq Ra$

T_a=0.03 – 0.35s

$X_d \geq X_q > X_q' \geq X_d' > X_q'' \geq X_d''$

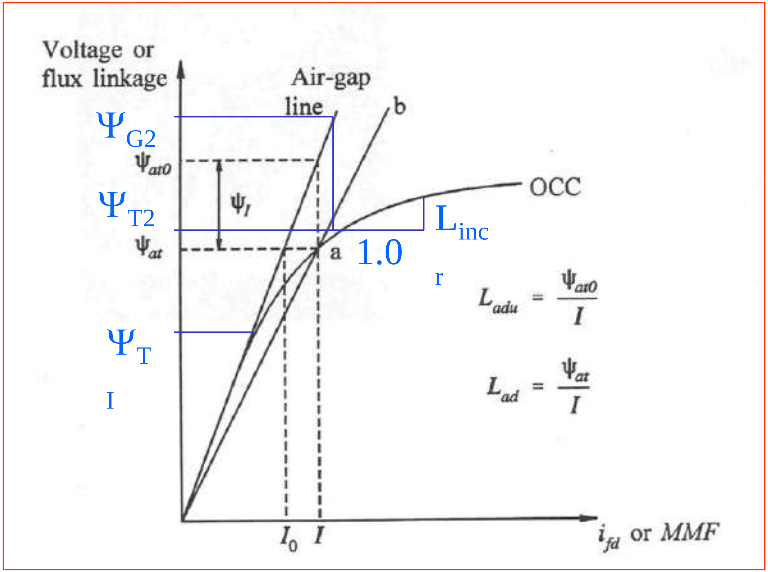
$T_{d0}' > T_d' > T_{d0}'' > T_d''$ and $T_{q0}' > T_q' > T_{q0}'' > T_q''$

Representation of Saturation

- The leakage reactances are independent on saturation. (Only L_{ad} and L_{aq} saturate.)
- The leakage fluxes do not contribute to the iron saturation. (Saturation is determined by the air-gap flux linkage.)
- The saturation relationship between the resultant air-gap flux and the mmf under loaded conditions is the same as under no-load condition. (Saturation characteristics is represented by the open circuit saturation curve.)
- There is no magnetic coupling between d and q axes as a result of nonlinearities introduced by saturation.

Generally *it is not economically possible* to operate at such low flux densities to avoid saturation, i.e., to reduce it to negligible proportions. Usually moderate degree (10%) of saturation is acceptable.

Modelling of Saturation



$$L_{ad} = \frac{\Psi_{at}}{I}$$

$$L_{adu} = \frac{\Psi_{at0}}{I}$$

$$L_{ad} = K_{sd} L_{adu}$$

$$L_{aq} = K_{sq} L_{aqu}$$

$$K_{sd} = \frac{\Psi_{at}}{\Psi_{at0}} = \frac{I_0}{I} = \frac{\Psi_{at}}{\Psi_{at} + \Psi_I}$$

$$\Psi_{at} = \sqrt{\Psi^2{}_{ad} + \Psi^2{}_{aq}}$$

$$\Psi_{ad} = V_q + R_a I_q + L_l I_d$$

$$\Psi_{aq} = -V_d - R_a I_d + L_l I_q$$

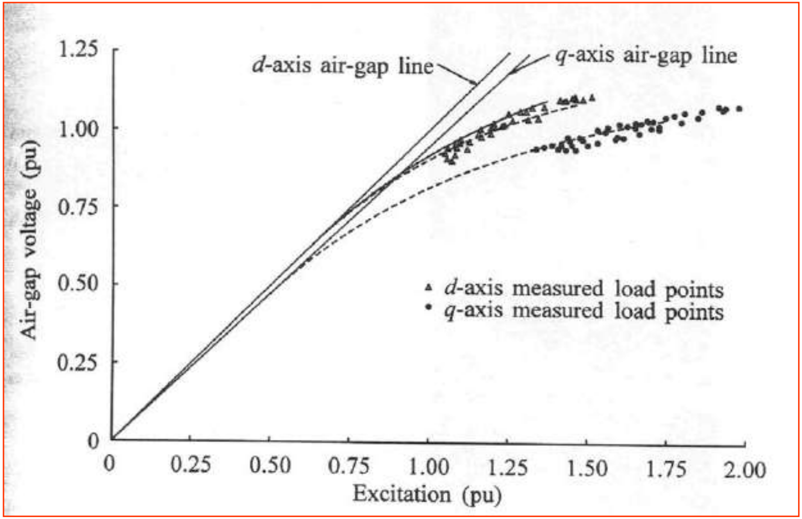
$$\Psi_{at} \le \Psi_{TI} \Rightarrow \Psi_I = 0$$

$$\Psi_{TI} < \Psi_{at} \le \Psi_{T2} \Rightarrow \Psi_I = A_{dsat} e^{B_{dsat}(\Psi_{at} - \Psi_{TI})}$$

$$\Psi_{at} > \Psi_{T2} \Rightarrow \Psi_I = \Psi_{G2} + \frac{L_{adu}}{L_{incr}}(\Psi_{at} - \Psi_{T2}) - \Psi_{at}$$

Reactance under saturated conditions is lower then when unsaturated (0.8-0.9 of unsaturated value).

Saturation in d- and q- axes



Typical sensitivity to a 10% change in mutual flux linkage:

- L_d 21%
- L'_d 1.5%
- T'_{d0} 20%
- E_{fd} 21%

For salient-pole machines K_{sq} is assumed to be 1.0 for all loading conditions.

Example: Values for saturation coefficients:

A_{dsat} = 0.03125 B_{dsat}=6.931

A_{qsat} = 0.077 B_{qsat}=3.465

Influence of Loading and K_{sq}

P_t	Q_t	E_a (pu)	K_{sd}	δ_i (deg)	i_{fd} (pu)
0	0	1.0	0.889	0	0.678
0.4	0.2	1.033	0.868	25.3	1.016
0.9	0.436	1.076	0.835	39.1	1.565
0.9	0	1.012	0.882	54.6	1.206
0.9	-0.2	0.982	0.899	64.6	1.089

Leading power factor leads to bigger rotor angle

P_t	Q_t	E_a (pu)	K_{sd}	K_{sq}	δ_i (deg)	i_{fd} (pu)
0	0	1.0	0.889	0.667	0	0.678
0.4	0.2	1.033	0.868	0.648	21.0	1.013
0.9	0.436	1.076	0.835	0.623	34.6	1.559
0.9	0	1.012	0.882	0.660	47.5	1.194
0.9	-0.2	0.982	0.899	0.676	55.9	1.074

different saturation factors lead to smaller rotor angle

Synchronous Machine in Stability Studies

- Essential simplifications:
  - Neglect transformer voltage terms: $p\Psi_d = 0$ & $p\Psi_q = 0$

$$V_d = -R_a I_d - \Psi_q \omega$$
$$V_q = -R_a I_q + \Psi_d \omega$$

  - Neglect effect of speed variations : $\omega = 1$ p.u. ($\omega = \omega_0$)

$$V_d = -R_a I_d - \Psi_q$$
$$V_q = -R_a I_q + \Psi_d$$

$$T_e = \Psi_d I_q - \Psi_q I_d$$

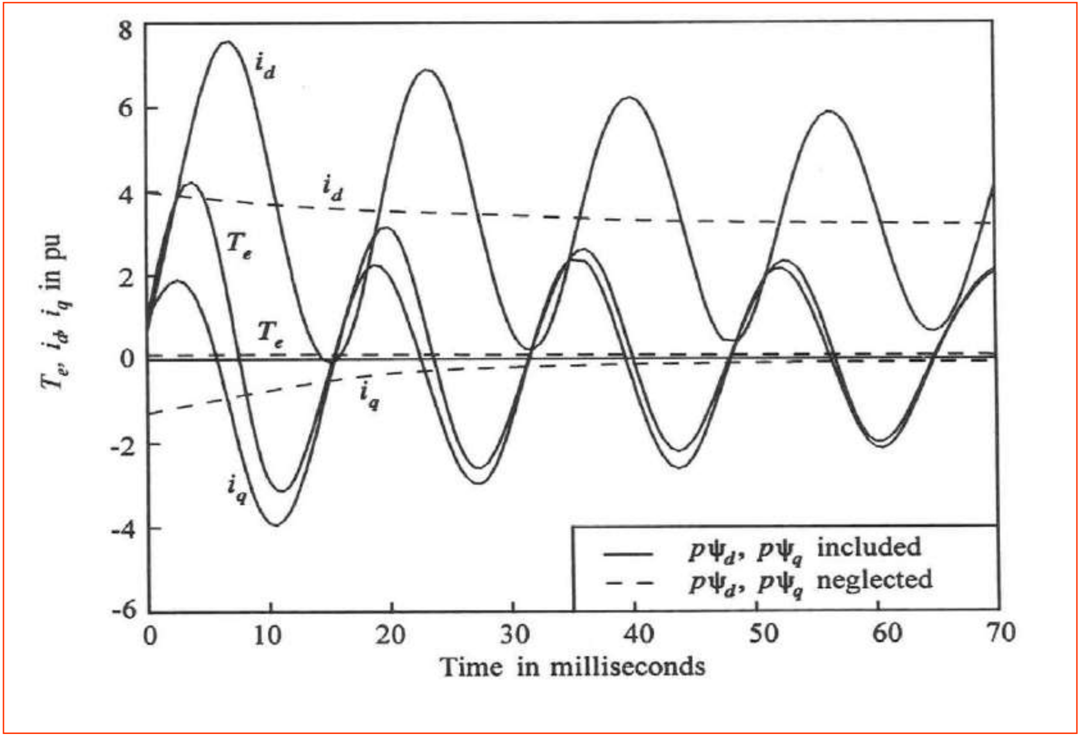
$$P = V_d I_d + V_q I_q = T_e - R_a I^2$$

$$P_e = P_t + R_a I^2$$

$$P_e = T_e$$

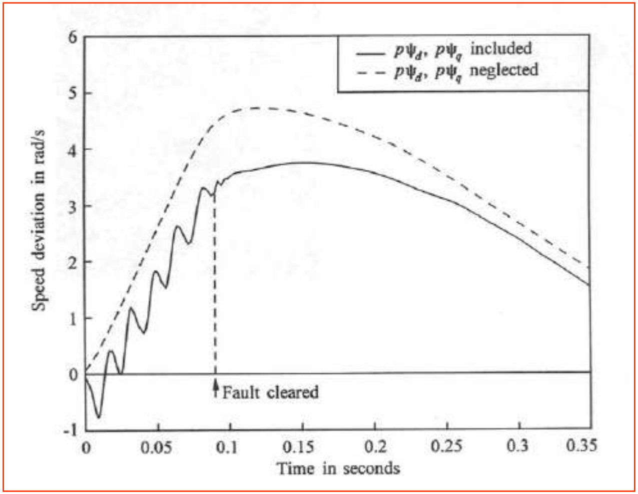
The per-unit air gap power is normally $P_e = \omega T_e$!

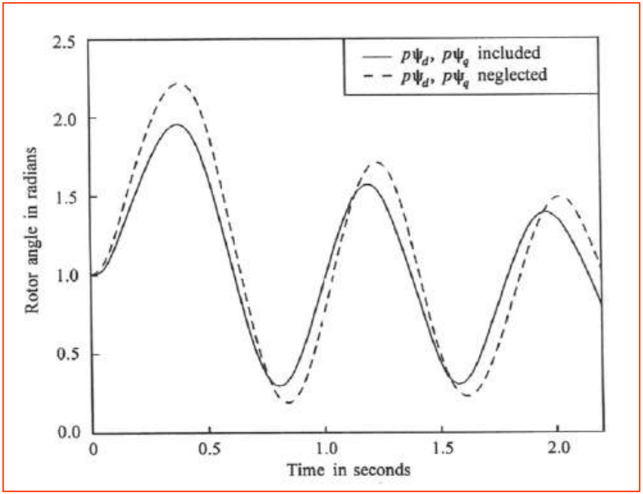
Effects of Model Simplifications - 1



Effect of neglecting stator transients on air-gap torque, and d- and q- components of stator currents

Effects of Model Simplifications - 2





Effect of neglecting stator transients on speed deviation

Effect of neglecting stator transients on rotor angle swings

Model with Neglected Stator Transients

$$V_d = -R_a I_d - \omega \Psi_q$$
$$V_q = -R_a I_q + \omega \Psi_d$$

$$\boxed{\begin{aligned} e_{fd} &= R_{fd} I_{fd} + \frac{1}{\omega_0} p\Psi_{fd} \\ 0 &= R_{1d} I_{1d} + \frac{1}{\omega_0} p\Psi_{1d} \\ 0 &= R_{1q} I_{1q} + \frac{1}{\omega_0} p\Psi_{1q} \\ 0 &= R_{2q} I_{2q} + \frac{1}{\omega_0} p\Psi_{2q} \\ p\omega &= \frac{1}{2H}(T_m - T_e - K_D(\omega - 1)) \\ p\delta &= \omega_0(\omega - 1) \end{aligned}}$$

$$\Psi_d = -(L_{ad} + L_l) I_d + L_{ad} I_{fd} + L_{ad} I_{1d}$$
$$\Psi_q = -(L_{aq} + L_l) I_q + L_{aq} I_{1q} + L_{aq} I_{2q}$$
$$\Psi_0 = -L_0 I_0$$
$$\Psi_{fd} = -L_{ad} I_d + L_{ffd} I_{fd} + L_{f1d} I_{1d}$$
$$\Psi_{1d} = -L_{ad} I_d + L_{f1d} I_{fd} + L_{11d} I_{1d}$$
$$\Psi_{1q} = -L_{aq} I_q + L_{11q} I_{1q} + L_{aq} I_{2q}$$
$$\Psi_{2q} = -L_{aq} I_q + L_{22q} I_{2q} + L_{aq} I_{1q}$$
$$T_e = \Psi_d I_q - \Psi_q I_d$$

Electrical quantities and torques in p.u.; Time in seconds; Rotor angle in electrical radians; $\omega_0 = 2\pi f$

$$p = \frac{d}{dt}$$

Model with Damper Windings Neglected

$$V_d = -R_a I_d - \Psi_q$$

$$V_q = -R_a I_q + \Psi_d$$

$$e_{fd} = R_{fd} I_{fd} + \frac{1}{\omega_0} p\Psi_{fd}$$

$$p\omega = \frac{1}{2H}(T_m - T_e - K_D(\omega - 1))$$

$$p\delta = \omega_0(\omega - 1)$$

$$\Psi_d = -(L_{ad} + L_l)I_d + L_{ad} I_{fd}$$

$$\Psi_q = -(L_{aq} + L_l)I_q$$

$$\Psi_{fd} = -L_{ad} I_d + L_{ffd} I_{fd}$$

$$T_e = \Psi_d I_q - \Psi_q I_d$$

Electrical quantities and torques in p.u.; Time in seconds; Rotor angle in electrical radians; $\omega_0=2\pi f$

$$p = \frac{d}{dt}$$

Classical Model (Constant Flux Linkage)

$$V_d = -R_a I_d - \Psi_q$$
$$V_q = -R_a I_q + \Psi_d$$

$$\boxed{\begin{aligned} p\omega &= \frac{1}{2H}(T_m - T_e - K_D(\omega - 1)) \\ p\delta &= \omega_0(\omega - 1) \end{aligned}}$$

$$\Psi_d = -(L_{ad} + L_l)I_d + L_{ad} I_{fd}$$
$$\Psi_q = -(L_{aq} + L_l)I_q$$
$$\Psi_{fd} = -L_{ad} I_d + L_{ffd} I_{fd}$$
$$T_e = \Psi_d I_q - \Psi_q I_d$$

Electrical quantities and torques in p.u.; Time in seconds; Rotor angle in electrical radians; $\omega_0 = 2\pi f$

$$p = \frac{d}{dt}$$

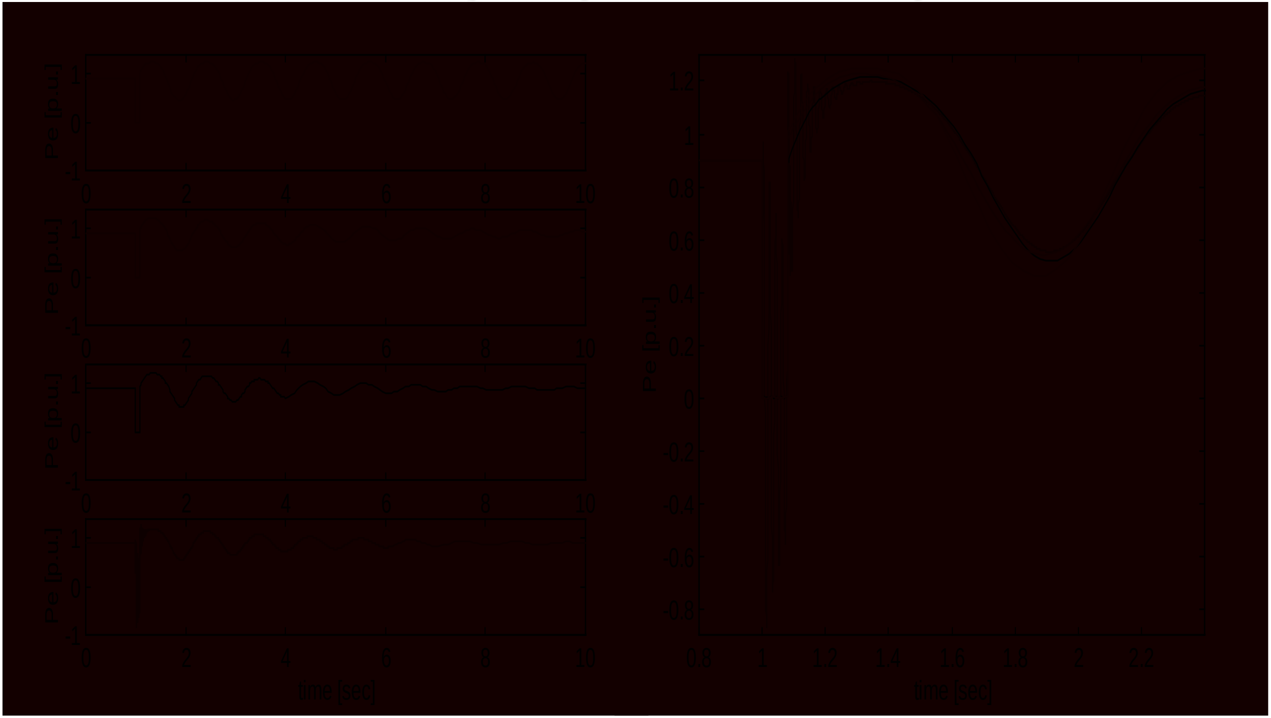
Influence of Generator Model - 1

2nd order model; Classical

3rd order model; Dampers neglctd.

6th order model; Stator trns. neglctd.

8th order model; Full model



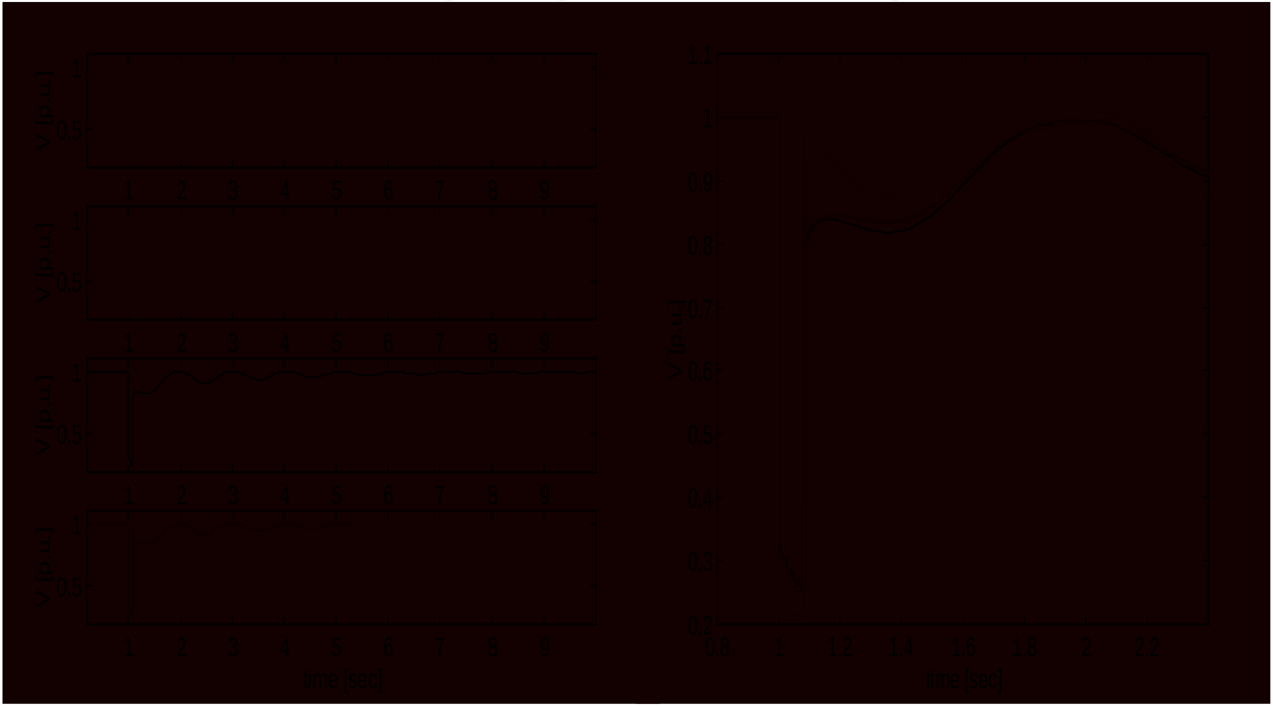
Influence of Generator Model - 2

2nd order model; Classical

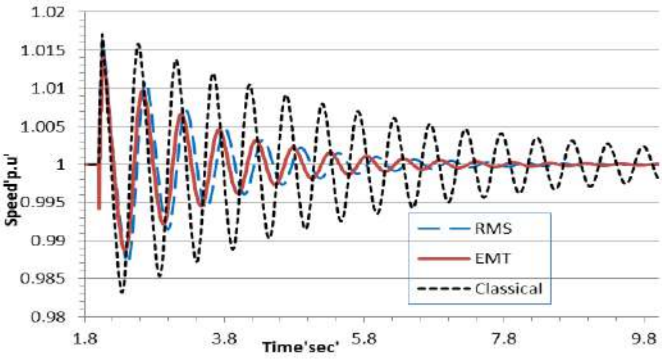
3rd order model; Dampers neglctd.

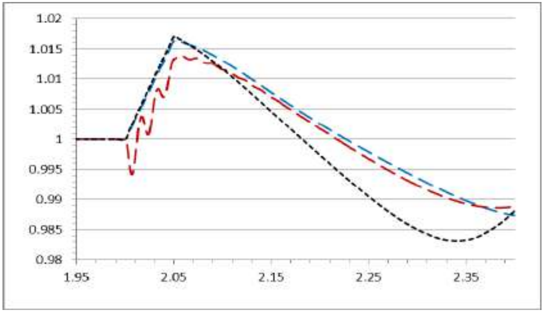
6th order model; Stator trns. neglctd.

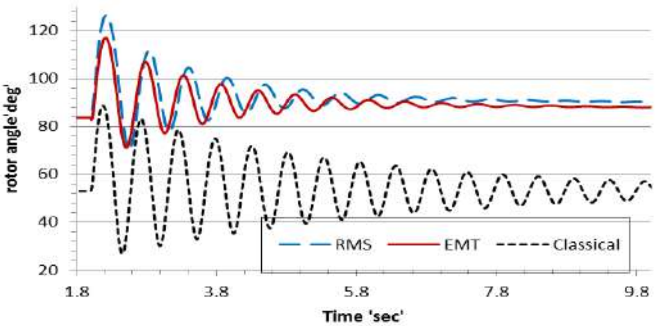
8th order model; Full model

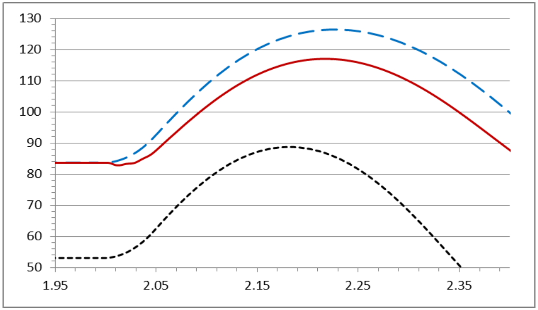


Influence of Generator Model - 3

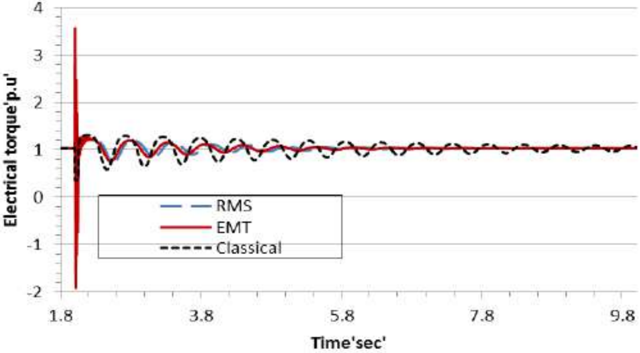


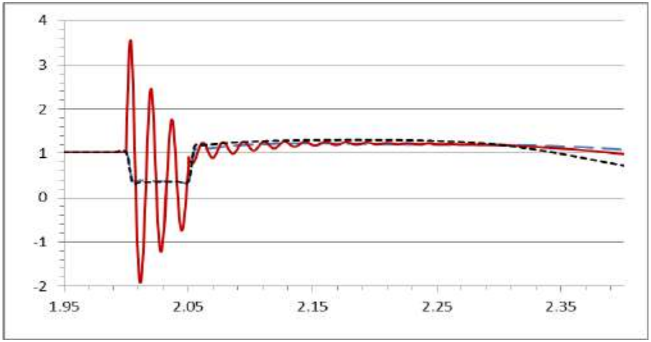


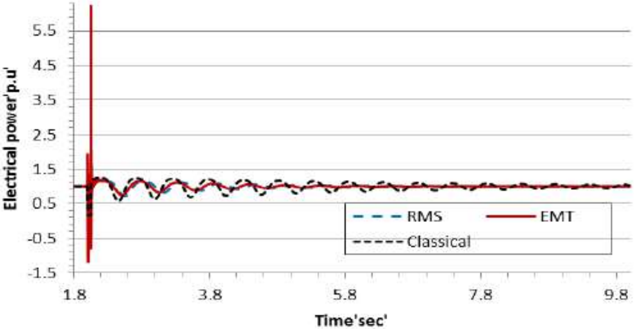


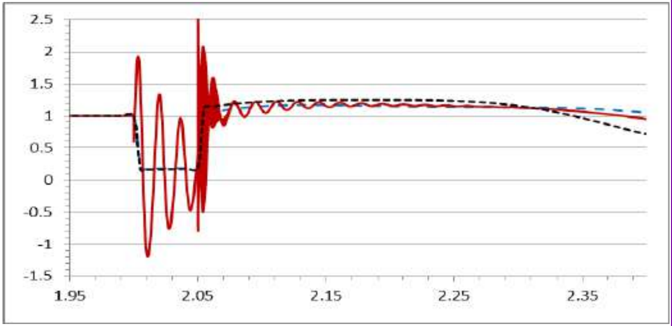


Influence of Generator Model - 4

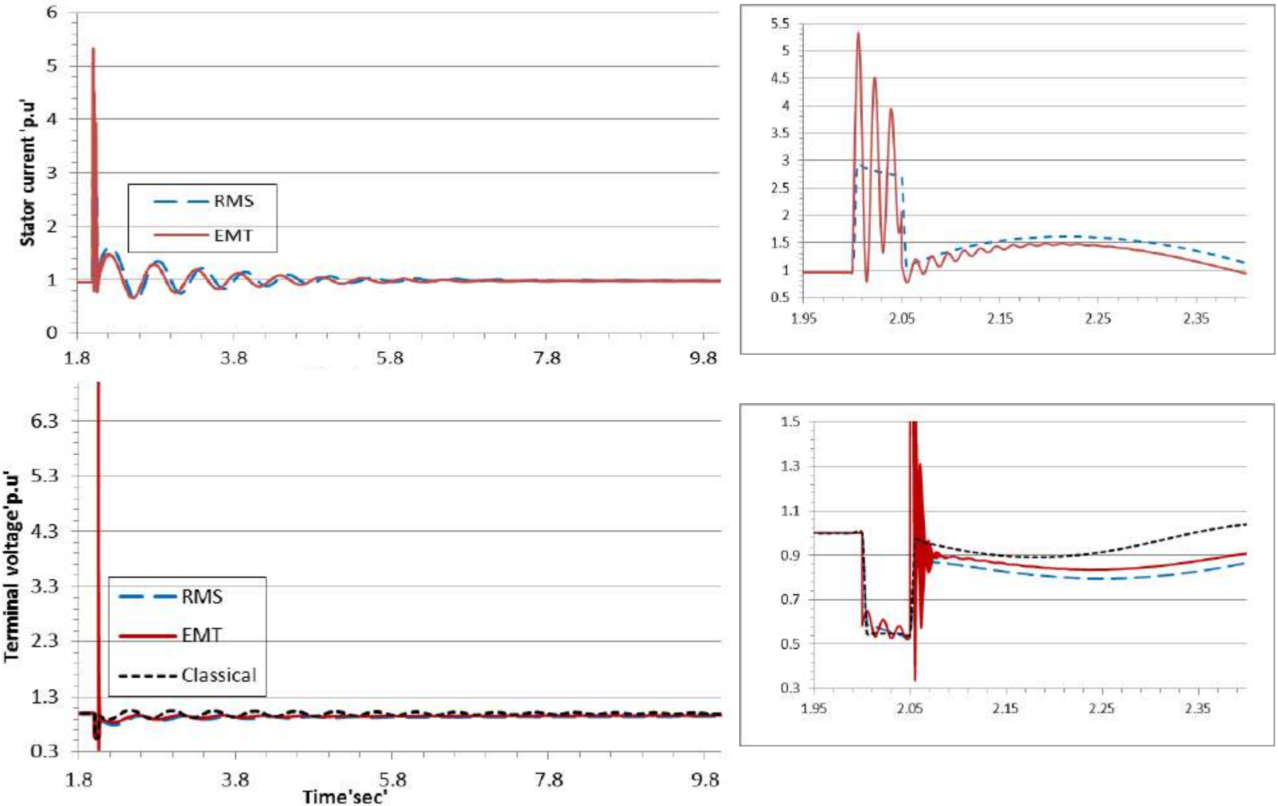




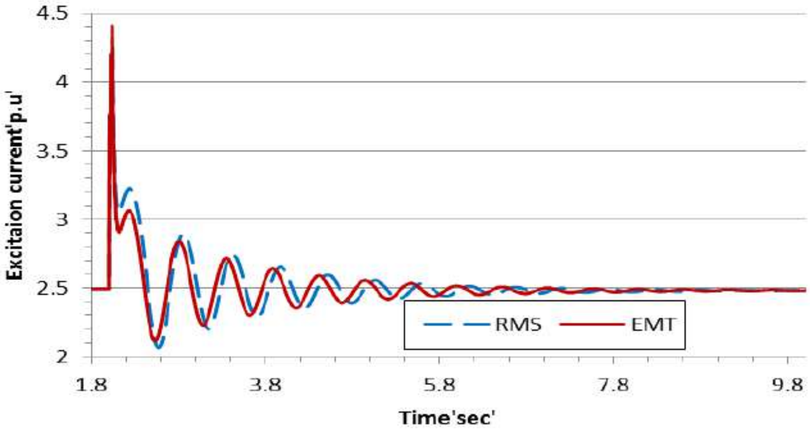


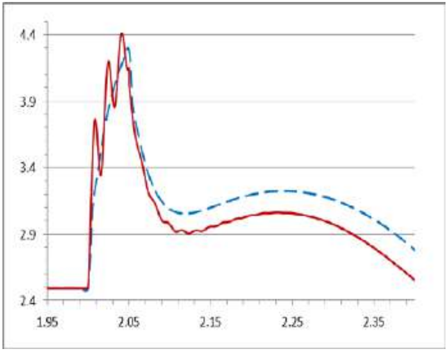


Influence of Generator Model - 5

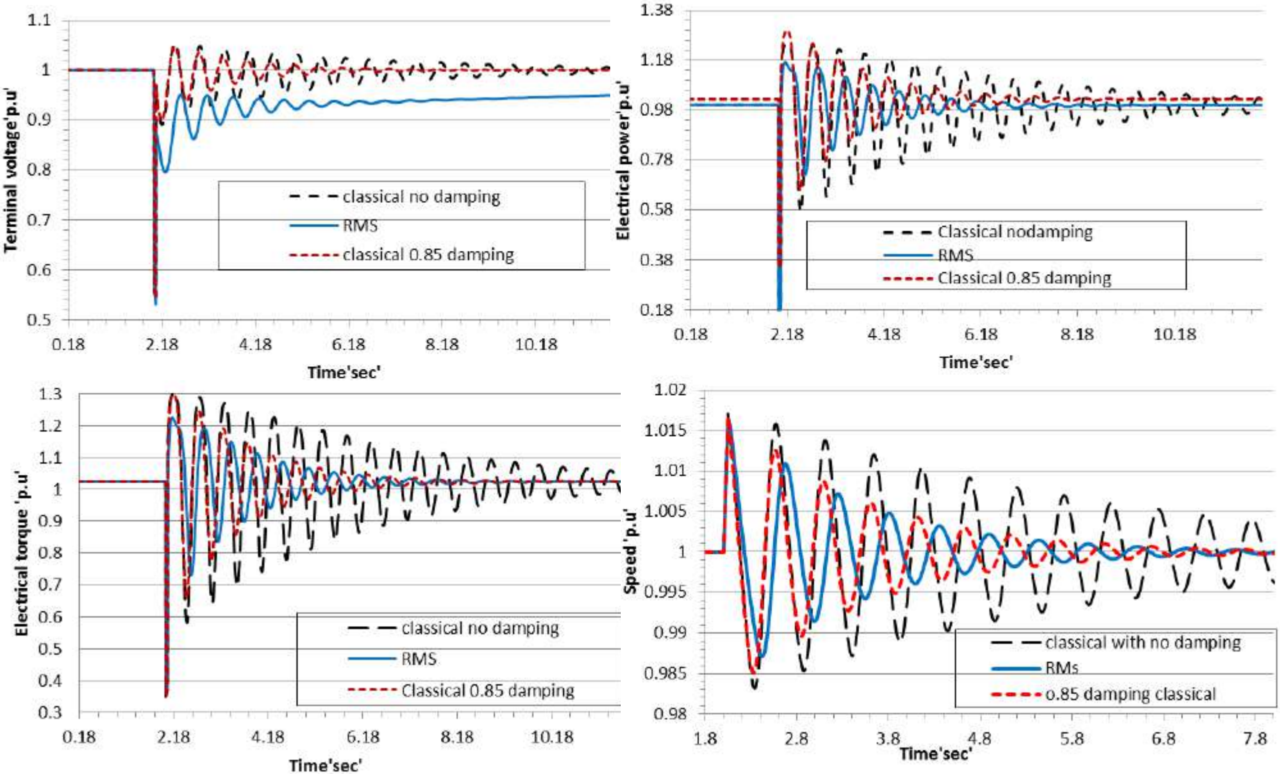


Influence of Generator Model - 6

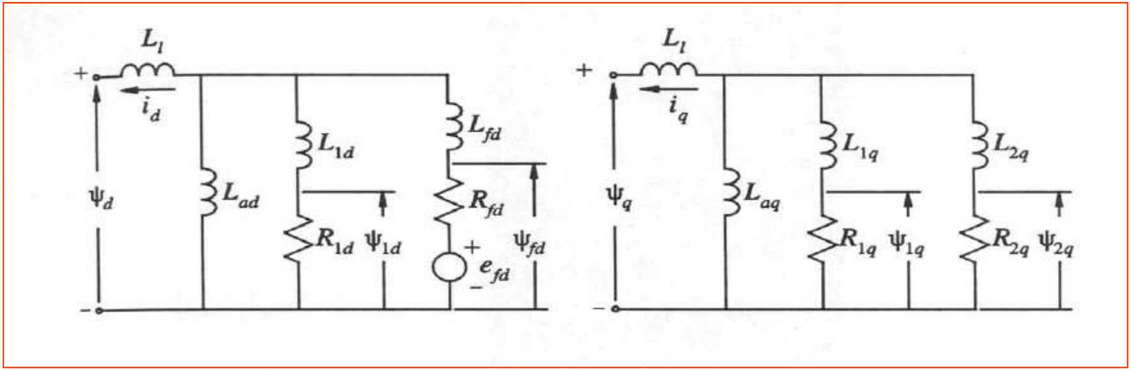




Influence of K_D (Classical Model)



Standard Machine Parameters - 1



$$L_d = L_l + L_{ad}$$

$$L_{ffd} = L_{fd} + L_{ad}$$

$$L_{11d} = L_{1d} + L_{ad}$$

$$L_q = L_l + L_{aq}$$

$$L_{11q} = L_{1q} + L_{aq}$$

$$L_{22q} = L_{2q} + L_{aq}$$

Standard Machine Parameters - 2

$$L''_d = L_d\left(\frac{T'_d T''_d}{T'_{d0} T''_{d0}}\right) = L_l + \frac{L_{ad} L_{fd} L_{1d}}{L_{ad} L_{fd} + L_{1d} L_{ad} + L_{fd} L_{1d}}$$

$$L'_d = L_d\left(\frac{T'_d}{T'_{d0}}\right) = L_l + \frac{L_{ad} L_{fd}}{L_{ad} + L_{fd}}$$

$$L''_q = L_q\left(\frac{T'_q T''_q}{T'_{q0} T''_{q0}}\right) = L_l + \frac{L_{aq} L_{1q} L_{2q}}{L_{aq} L_{1q} + L_{1q} L_{2q} + L_{2q} L_{1q}}$$

$$L'_q = L_q\left(\frac{T'_q}{T'_{q0}}\right) = L_l + \frac{L_{aq} L_{1q}}{L_{aq} + L_{1q}}$$

Since $X = \omega_0 L$ and with $\omega_0 = \omega_{base}$
$X_i\,[p.u.] = L_i\,[p.u.]$

$$L_d \geq L_q > L_q' \geq L_d' > L_q'' \geq L_d''$$

Standard Machine Parameters - 3

$$T'_{d0} = \frac{L_{ad} + L_{fd}}{R_{fd}}$$

$$T'_{d} = \frac{1}{R_{fd}}\left(L_{fd} + \frac{L_{ad}L_{l}}{L_{ad} + L_{l}}\right)$$

$$T'_{q0} = \frac{L_{aq} + L_{1q}}{R_{1q}}$$

$$T'_{q} = \frac{1}{R_{1q}}\left(L_{1q} + \frac{L_{aq}L_{l}}{L_{aq} + L_{l}}\right)$$

$$T_{a} = \frac{X_2}{R_a}$$

$$T''_{d0} = \frac{1}{R_{1d}}\left(L_{1d} + \frac{L_{ad}L_{fd}}{L_{ad} + L_{fd}}\right)$$

$$T''_{d} = \frac{1}{R_{1d}}\left(L_{1d} + \frac{L_{ad}L_{fd}L_{l}}{L_{ad}L_{fd} + L_{l}L_{ad} + L_{fd}L_{l}}\right)$$

$$T''_{q0} = \frac{1}{R_{2q}}\left(L_{2q} + \frac{L_{aq}L_{1q}}{L_{aq} + L_{1q}}\right)$$

$$T''_{q} = \frac{1}{R_{2q}}\left(L_{2q} + \frac{L_{aq}L_{1q}L_{l}}{L_{aq}L_{1q} + L_{1q}L_{l} + L_{aq}L_{l}}\right)$$

Time constants are in p.u.; To get values in seconds they should be divided by $\omega_0 = 2\pi f$

$$T_{d0}' > T_d' > T_{d0}'' > T_d''$$

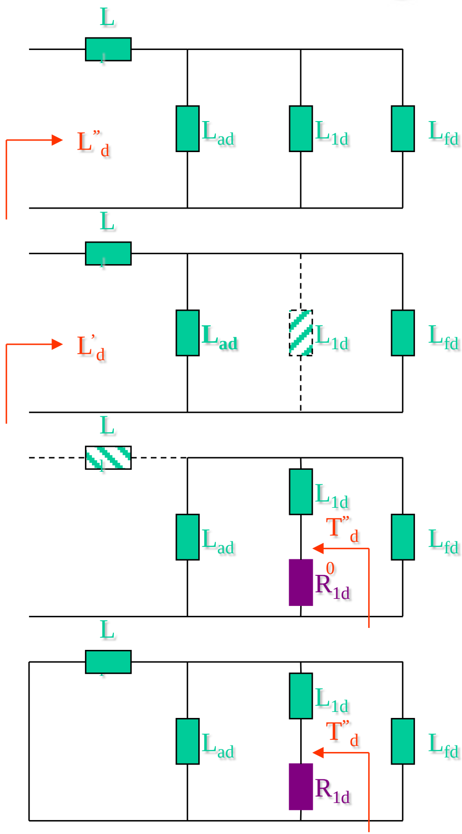
$$T_{q0}' > T_q' > T_{q0}'' > T_q''$$

$$X_2 = \frac{X''_d + X''_q}{2} \quad \text{or} \quad X_2 = \frac{X'_d + X_q}{2}$$

$$X_2 = 2\frac{X''_d\, X''_q}{X''_d + X''_q} \quad \text{or} \quad = \sqrt{X''_d\, X''_q} \quad \text{or} \quad = \sqrt{(X''_d + 0.5X_0)(X''_q + 0.5X_0)} - 0.5X_0$$

LLL fault | LL fault | SLG fault

Deriving Machine Parameters



$$L''_d = L_d\left(\frac{T'_d\,T''_d}{T'_{d0}\,T''_{d0}}\right) = L_l + \frac{L_{ad}L_{fd}L_{1d}}{L_{ad}L_{fd} + L_{1d}L_{ad} + L_{fd}L_{1d}}$$

$$L'_d = L_d\left(\frac{T'_d}{T'_{d0}}\right) = L_l + \frac{L_{ad}L_{fd}}{L_{ad} + L_{fd}}$$

$$T''_{d0} = \frac{1}{R_{1d}}\left(L_{1d} + \frac{L_{ad}L_{fd}}{L_{ad} + L_{fd}}\right)$$

$$T''_d = \frac{1}{R_{1d}}\left(L_{1d} + \frac{L_{ad}L_{fd}L_l}{L_{ad}L_{fd} + L_lL_{ad} + L_{fd}L_l}\right)$$

Machine parameters - 1

- X_d, X_q - synchronous reactance (measure of the steady state stability of the machine – the smaller its value the more stable the machine; The control of X_d is obtained almost entirely by varying X_{ad} and in most cases a reduction in X_d will tend to result in a larger and more costly machine.)
- X_l - leakage reactance (*0.1-0.25p.u.*; can be reduced by increasing the machine size (derating) and increased by artificially increasing the slot leakage. It is only about 10% of the X_d and therefore is not very influential)
- X_{ad}, X_{aq} - armature reaction reactances (*1-2.5p.u.*; can be reduced by decreasing the armature reaction, i.e., reducing the ampere conductor or electrical loading - this will often mean a physically larger machine)

Machine parameters - 2

- X_f (X_{fd}) - field leakage reactance ($X'_f \approx X_l$=0.1-0.25p.u.)
- X'_{kd} (X_{d1}; X_D) - damper winding leakage reactance (0.05-0.15p.u.; control is achieved by variation of X_l)
- $X_d' = X_l + X_f$ (X_{fd}) - transient reactance (covers the behaviour of a machine in the period 0.1-3s after a disturbance; control is achieved by variation of X_l)
- $X_d'' = X_l + X'_{kd}$ (X_{d1}; X_D) - subtransient reactance (determines the initial current peaks following a disturbance; influenced by the leakage of damper windings)
- $X_2=0.5*(X_d''+X_q'')$ - negative sequence reactance (0.1-0.5 p.u.)
- $X_0=(0.15\text{-}0.6)*X_d''$ - zero sequence reactance (0.02-0.45 p.u.)

Alternative Models - 1

Instead of fluxes (or currents) as state variables, the following substitutions are used to express the model in terms of voltages

$$E_q = L_{ad} I_{fd}$$

$$E'_q = \frac{L_{ad}}{L_{ffd}} \Psi_{fd}$$

$$E''_q = \omega L''_{ad} \left(\frac{\Psi_{fd}}{L_{fd}} + \frac{\Psi_{1d}}{L_{1d}} \right)$$

$$E''_d = -\omega L''_{aq} \left(\frac{\Psi_{1q}}{L_{1q}} + \frac{\Psi_{2q}}{L_{2q}} \right)$$

$$E_{fd} = \frac{L_{ad}}{R_{fd}} e_{fd}$$

$$T''_{d0} \frac{dE''_q}{dt} = E'_q - E''_q + I_d (X'_d - X''_d)$$

$$T''_{q0} \frac{dE''_d}{dt} = E'_d - E''_d - I_q (X'_q - X''_q)$$

$$T'_{d0} \frac{dE'_q}{dt} = E_{fd} - E'_q + I_d (X_d - X'_d)$$

$$T'_{q0} \frac{dE'_d}{dt} = -E'_d - I_q (X_q - X'_q)$$

Alternative Models - 2

Full model with saturation included:

$$E_d = E_d'' + \left(\frac{X_q'' - X_l}{k_{sq}} \right) I_q - R_a I_d$$

$$E_q = E_q'' + \left(\frac{X_d'' - X_l}{k_{sd}} \right) I_d - R_a I_q$$

$$\frac{d}{dt} E_q' = \frac{1}{T_{do}'} \left(E_f - (X_d - X_d') I_d - k_{sd} E_q' \right)$$

$$\frac{d}{dt} E_q'' = \frac{1}{T_{do}''} (k_{sd} E_q' - (X_d' - X_d'') I_d - k_{sd} E_q'')$$

$$\frac{d}{dt} E_d'' = \frac{1}{T_{qo}''} (k_{sq} E_d' + (X_q' - X_q'') I_q - k_{sq} E_d'')$$

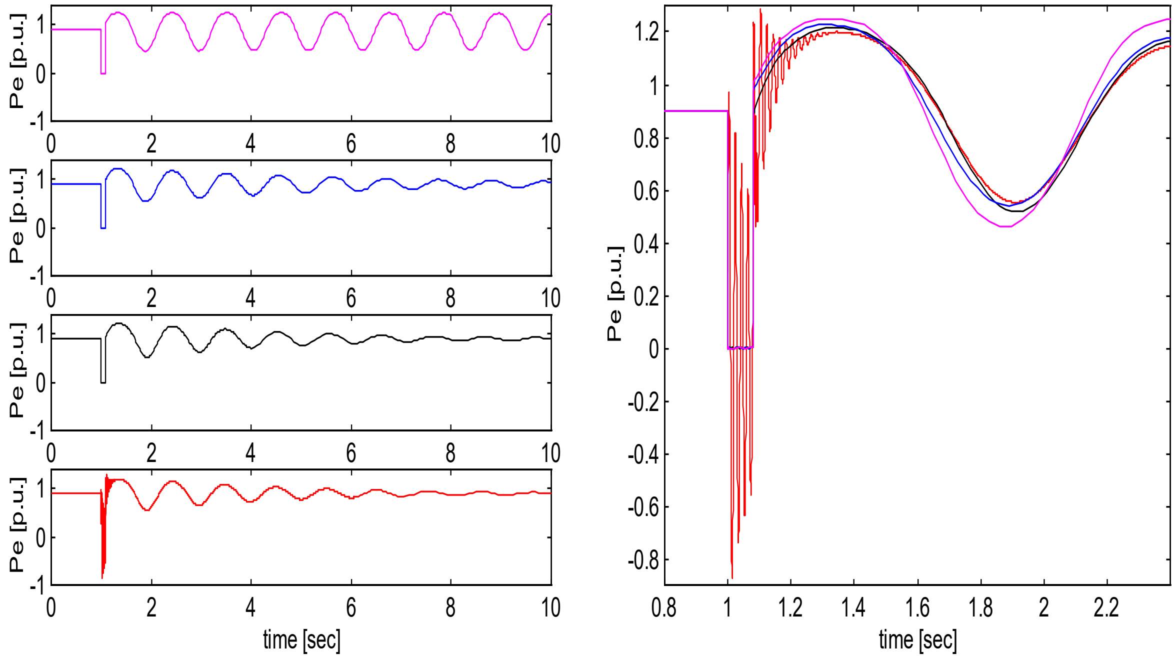
$$\frac{d}{dt} E_d' = \frac{1}{T_{qo}'} ((X_q - X_q') I_q - k_{sq} E_d')$$

$$\frac{d\delta}{dt} = \omega_0 (\omega - 1)$$

$$\frac{d\omega}{dt} = \frac{1}{2H} (T_m - T_e)$$

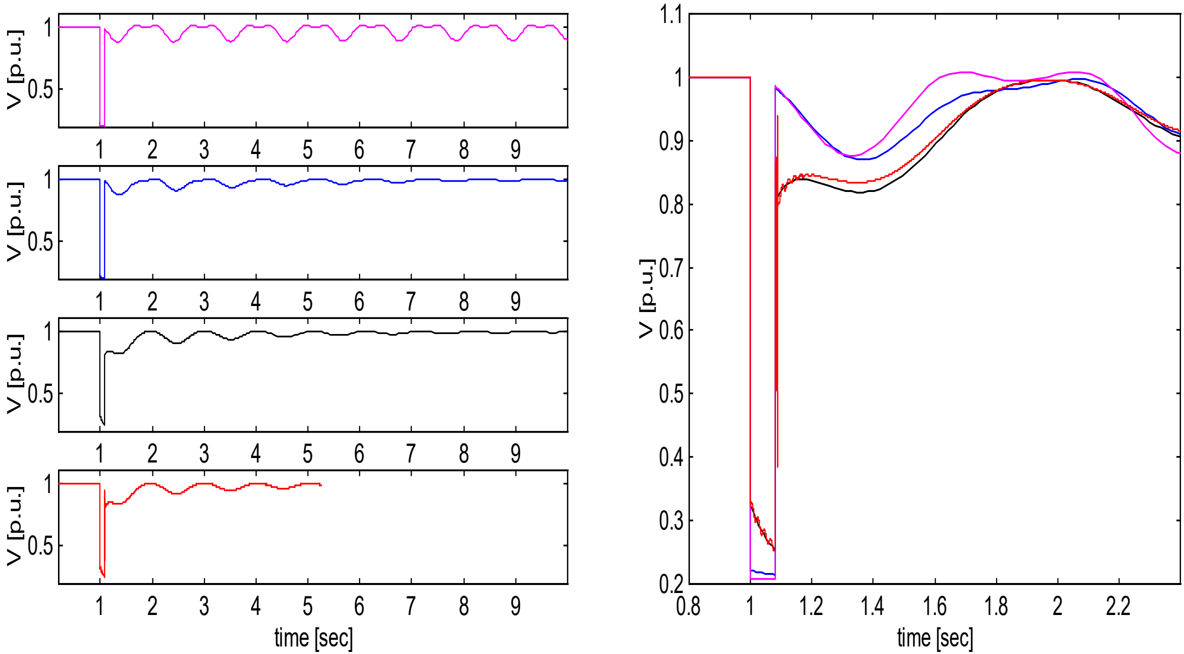
Influence of Generator Model - 1

2nd order model; Classical	3rd order model; Dampers neglctd.	6th order model; Stator trns. neglctd.	8th order model; Full model



Influence of Generator Model - 2

2nd order model; Classical	3rd order model; Dampers neglctd.	6th order model; Stator trns. neglctd.	8th order model; Full model



Chapter 3:
Modelling Of Network, Loads, Transformers and Controls

Content

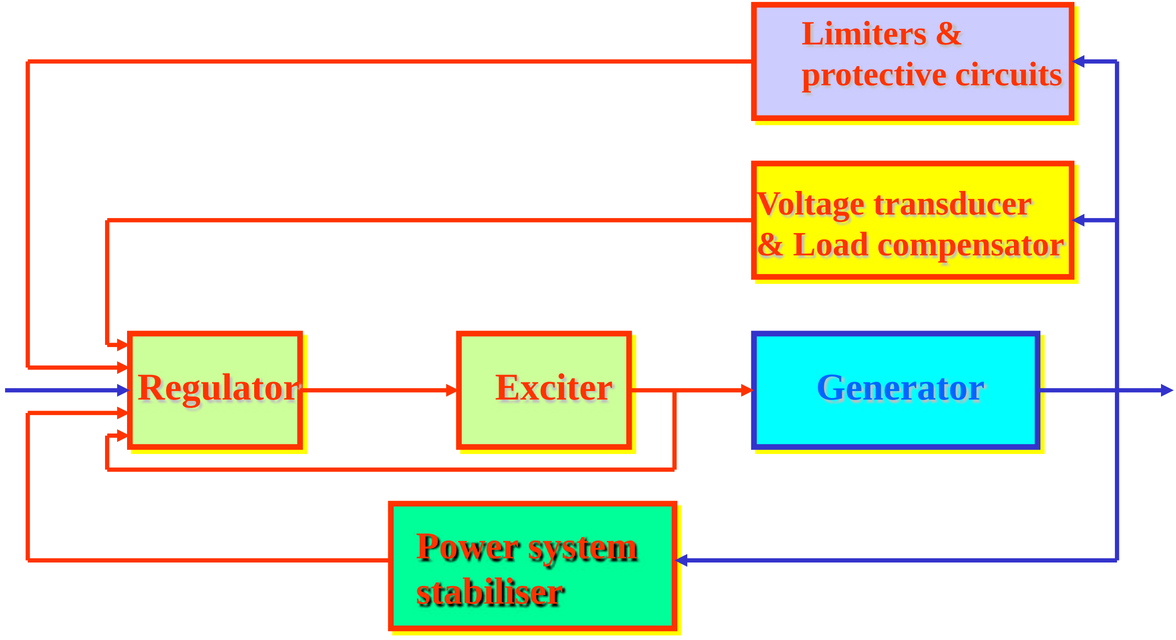
- Modelling of excitation systems
- Modelling of turbines and governors
- Modelling of transformers
- Modelling of transmission lines
- Modelling of power system loads

Modelling of Excitation Systems

Excitation System Requirements

- Excitation system should supply and automatically adjust the field current of the generator to maintain the terminal voltage as the output varies within the continuous capability of the generator.
- It must be able to respond to transient disturbances with field forcing consistent with the generator instantaneous and short-term capabilities.
- The exciter rating varies from 2.0 to 3.5 kW/MVA of generator rating.

Excitation System - Block Diagram



Excitation System Components - 1

- Exciter - provides DC power to the synchronous machine
  - DC excitation systems (DC generator as source of excitation power)
  - AC excitation systems (AC machines as source of excitation power)
  - Static excitation systems (Controlled or uncontrolled static rectifiers supply the excitation current)
- Regulator - processes and amplifies input control signals to a level and form appropriate for control of the exciter
- Voltage transducer and load compensator - senses generator terminal voltage, rectifies and filters it to DC quantity and compares it with the reference. Load compensation (account for voltage drop on connecting impedance), in addition, may be provided if it is desired to hold constant voltage at some point electrically remote from the generator terminal.

$$I_{fd} = L_{adu} i_{fd} \qquad E_{fd} = \frac{L_{adu}}{R_{fd}} e_{fd}$$

Excitation System Components - 2

- Power System Stabiliser (PSS) - provides additional input signal to the regulator to damp power system oscillations. (Rotor speed deviation, Accelerating power, frequency deviation).
- Limiters and protective circuits - provide control and protective functions which ensure that the capability limits of the exciter and generator are not exceeded. (Field current limiter, maximum excitation limiter, terminal voltage limiter, under-excitation limiter.)

Excitation System Components - 3

- High gain excitation systems tend to become unstable under open circuit conditions due to negative damping introduced by the AVR loop. The control loop must be provided with control compensation, which permits stable, satisfactory performance for all operating conditions of the generator.
  - **Forward loop compensation,** lead-lag circuit given by the transfer function of the form:

$$K_c(s) = \frac{(1 + sT_1)}{(1 + sT_2)}$$

    Under steady state conditions the gain is unity and under transient conditions, where $\omega T_1 >> 1$ and $\omega T_2 >> 1$ the gain is approximately T_1/T_2 with very little phase lag introduced. Hence a suitable choice of T_1 and T_2 enables the gain to be reduced at high frequency.

Excitation System Components - 4

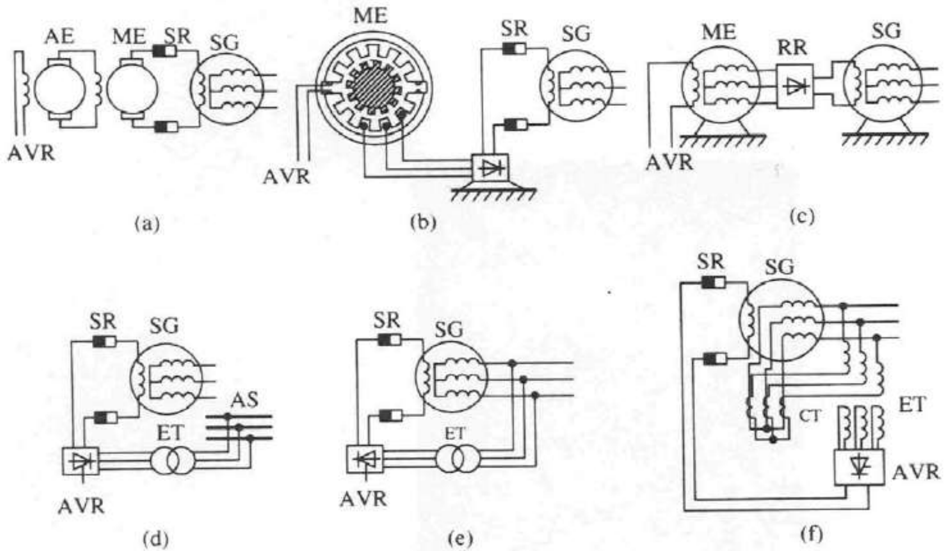
– **Transient feedback compensation** , the AVR loop stabilisation is provided by the transient feedback compensator of the form:

$$K_f(s) = \frac{sk_f}{1+sT_f} = \frac{k_f}{T_f}\frac{sT_f}{1+sT_f} = K_f \frac{sT_f}{(1+sT_f)}$$

Under steady state conditions, $K_f(s)=0$, hence preserving the high excitation gain loop and maintaining the required steady state regulation.

Under transient conditions, $\omega T_f >> 1$ and $K_f(s) \cong k_f/T_f$, so the closed loop system gain is reduced to k_f/T_f (this is in essence equivalent to a large forward loop gain.) Hence adequate choice of T_f and K_f can achieve the desired control requirement over the frequency range of interest.

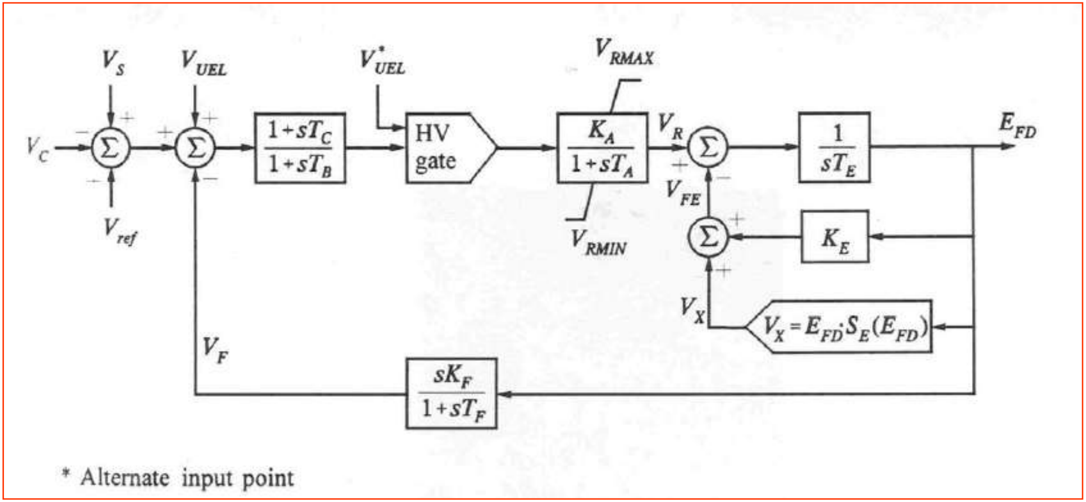
Typical Exciter Systems



SG - synchronous generator; SR - slip rings; ME - main exciter; AE - auxiliary exciter; RR - rotating rectifier; ET - excitation transformer; AS - auxiliary service busbars; CT - current transformer; AVR - automatic voltage regulator

a) cascaded DC generator b) reluctance machine with rectifier
c) inside-out synchronous generator with rotating rectifier d) controlled rectifier fed from the auxiliary supply e) controlled rectifier fed from the generator terminals f) controlled rectifier fed by the generator's voltage and current

IEEE DC1A Excitation System Model



Field controlled DC commutator with continuously acting voltage regulator

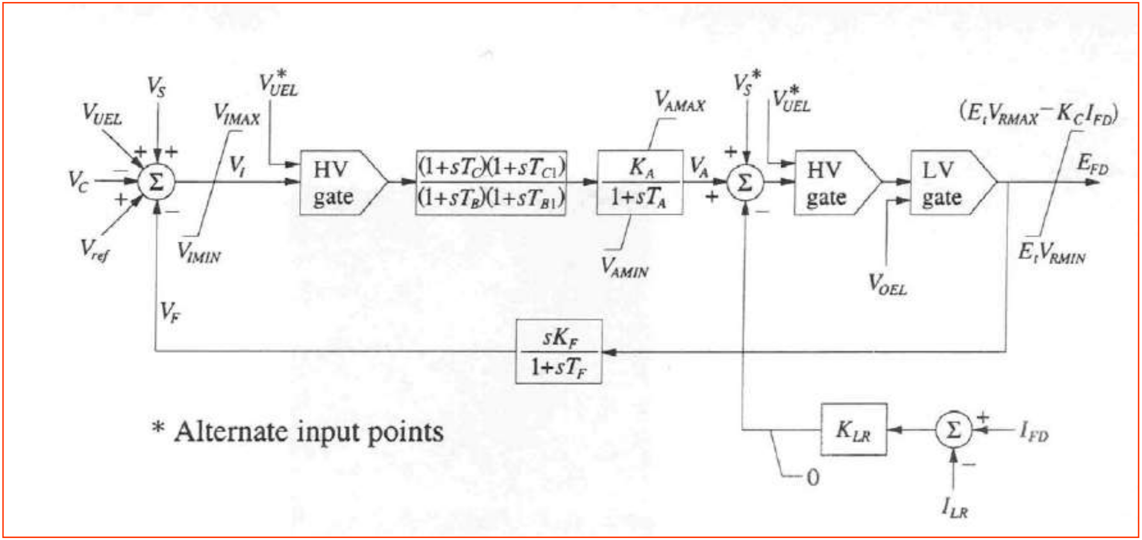
Sample data (Self-excited DC exciter):
K_A=187, T_A=0.89, T_E=1.15, A_{EX}=0.014, B_{EX}=1.55, K_F=0.058, T_F=0.62, T_B=0.06, T_C=0.173, T_R=0.05, V_{RMAX}=1.7, V_{RMIN}= - 1.7 , K_E - computed so that initially V_R=0.

IEEE DC1A Typical Data

T_R - transducer time constant, very small, usually neglected
K_A=20 - 400
T_A=0.05s - 0.2s
K_E=0.8 - 0.95 (approx. 1) (if self excited K_E= - 0.05 to - 0.2)
T_E< 1s (approx. 0.5s)
K_F=0.01 - 0.1
T_F=0.35s - 1s

A typical gain of 200, infers that the terminal voltage change of 0.005 p.u. will produce an excitation voltage change of 1.0 p.u. (1.0 p.u. excitation voltage is the field required to provide a 1 p.u. generator terminal voltage under open circuit conditions).

IEEE ST1A Excitation System Model



Potential source controlled rectifier system

Sample data:

K_A=200, T_A=0, K_F=0, T_F=0, T_{B1}=0, T_{C1}=0,
T_B=0, T_C=0, K_{LR}=4.54, I_{LR}=4.4, V_{RMAX}=7, V_{RMIN}= - 6.4, K_C=0.04,
V_{AMAX}, V_{AMIN}, V_{IMAX}, V_{IMIN}, - not represented

Excitation Control Design

- Requirements:
  - Maximisation of the damping of the local plant mode as well as inter-area oscillations without compromising the stability of other modes.
  - Enhancement of system transient stability.
  - Prevention of adverse effects on system performance during major system upsets that cause large frequency excursions.
  - Minimisation of the consequences of excitation system malfunction because of component failures.

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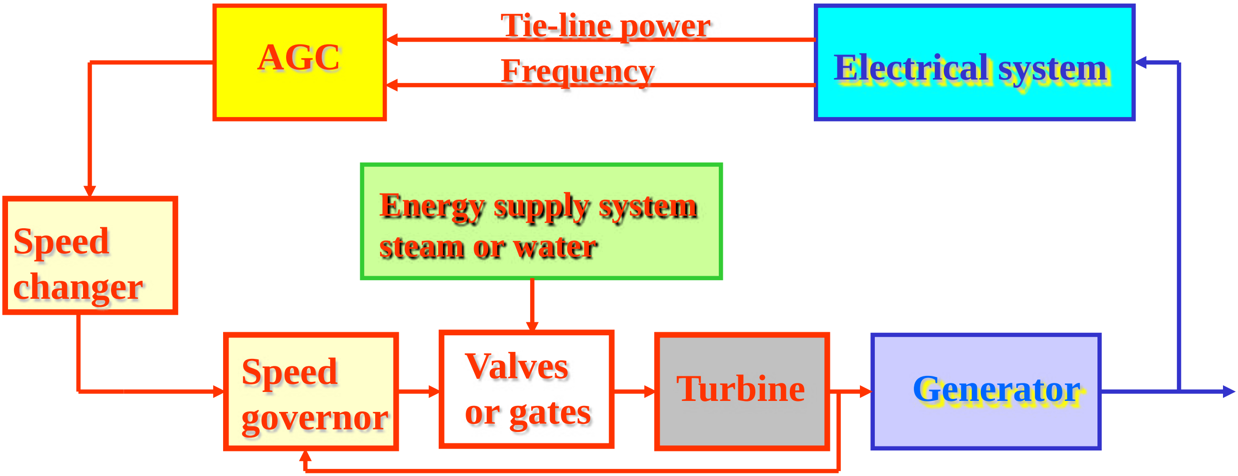
Modelling of
Turbines and Governors

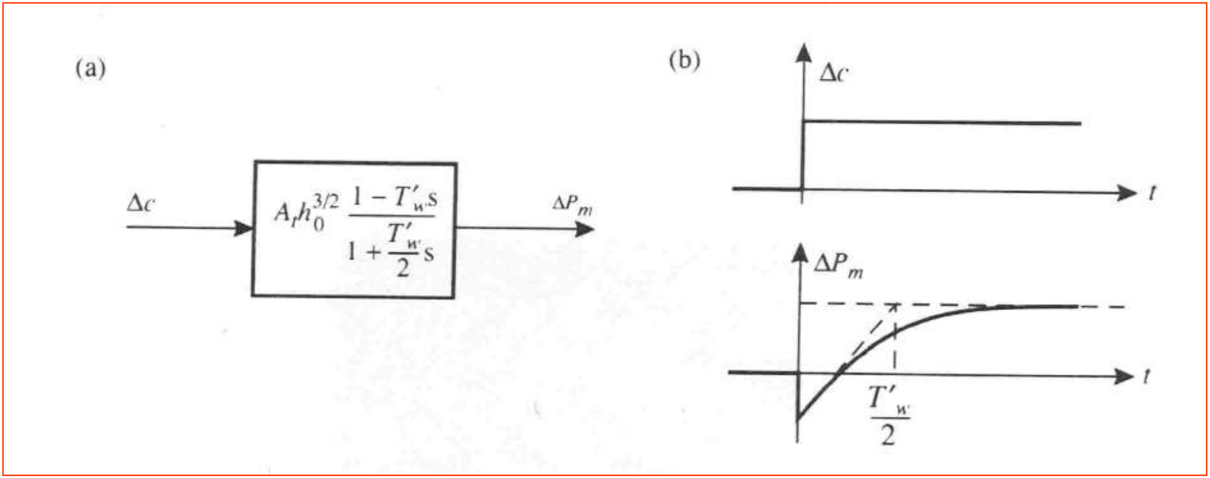
Power Generation and Control System - Block Diagram



Hydraulic Turbines

- Impulse type turbine (the runner is at atmospheric pressure)
  - Pelton wheel: for high heads of 300 m or more
- Reaction turbine (the pressure within the turbine is above atmospheric)
  - Francis turbine: for high heads up to 360 m
  - Propeller turbine (Kaplan wheel): for low heads, up to 45m

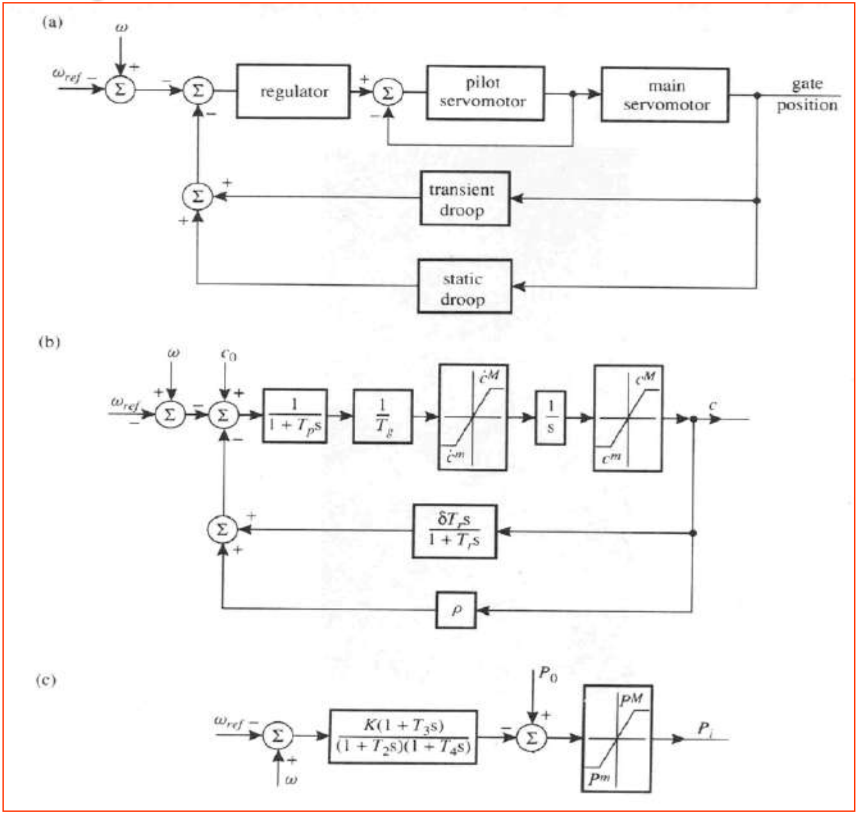
Hydraulic Turbines - Linear Model



Linear model
(T'_w = 0.5s - 5s)
h_0 - initial value of the normalised pressure head
A_t - turbine power (MW)/ generator MVA rating

Response to a step change in gate position

Hydraulic Turbine Governing System

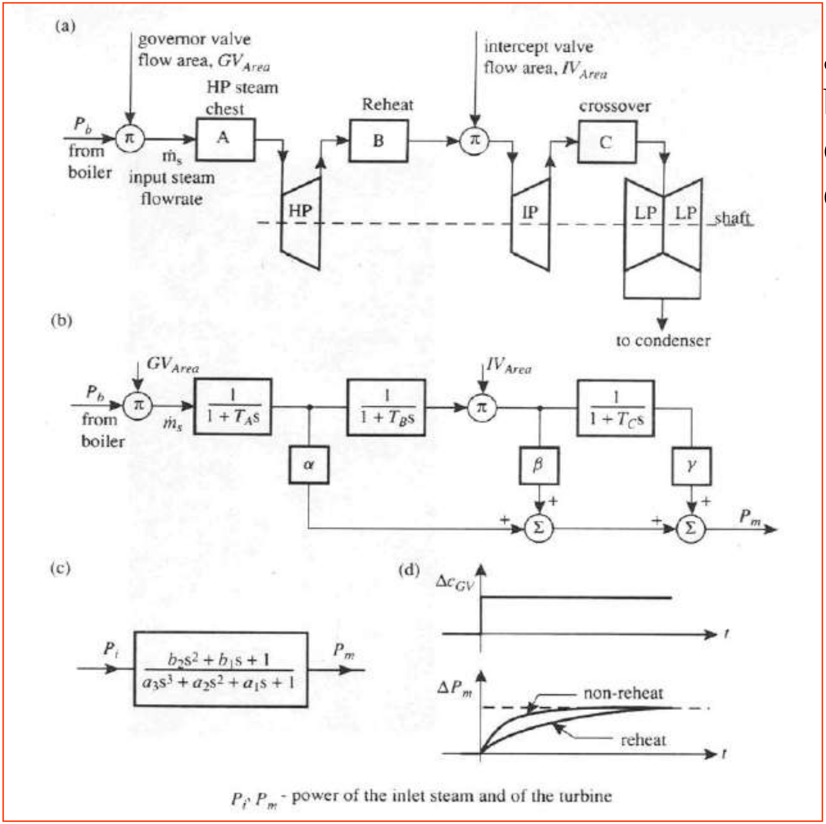


Functional diagram

Block diagram

Simplified block diagram

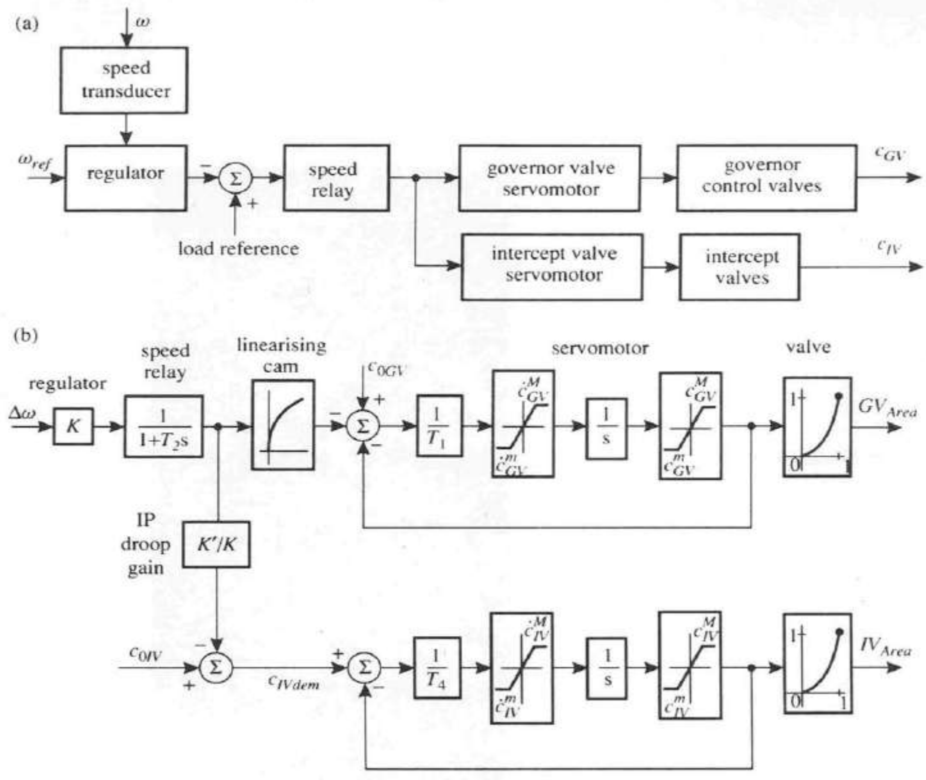
Steam Turbines - Block Diagram



a) Schematic diagram
b) Block diagram
c) Transformed block diagram
d) Response to a step change in valve position

$$a_1 = T_A + T_B + T_C$$
$$a_2 = T_A T_B + T_A T_C + T_C T_B$$
$$a_3 = T_A T_B T_C$$
$$b_1 = \alpha(T_B + T_C) + \beta T_C$$
$$b_2 = \alpha T_B T_C$$
$$0.1s \le T_A \le 0.4s$$
$$4s \le T_B \le 11s$$
$$0.3s \le T_C \le 0.5s$$
$$\alpha = 0.3$$
$$\beta = 0.4$$

Steam Turbine Governing System

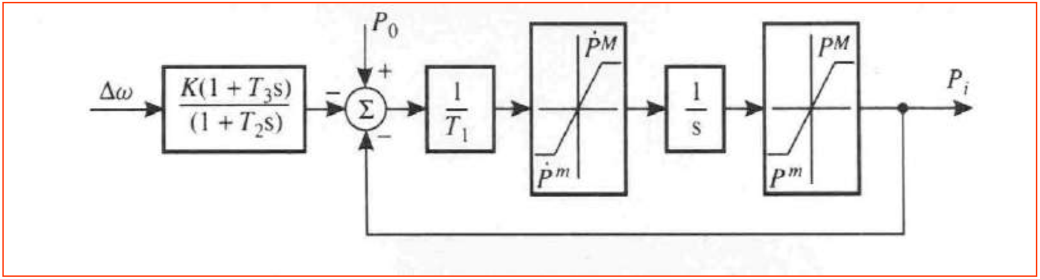


c - steam valve position, GV - governor valves, IV - intercept valves

Functional diagram

Block diagram

Steam Turbine Simplified Governing System



Typical values of parameters of turbine governing systems

Turbine type	$\rho=1/K$ [p.u.]	T_1 [s]	T_2 [s]	T_3 [s]	T_4 (slide 22) [s]
Steam	0.02-0.07	0.1	0.2-0.3	0	-
Hydro	0.02-0.04	-	0.5	5	50

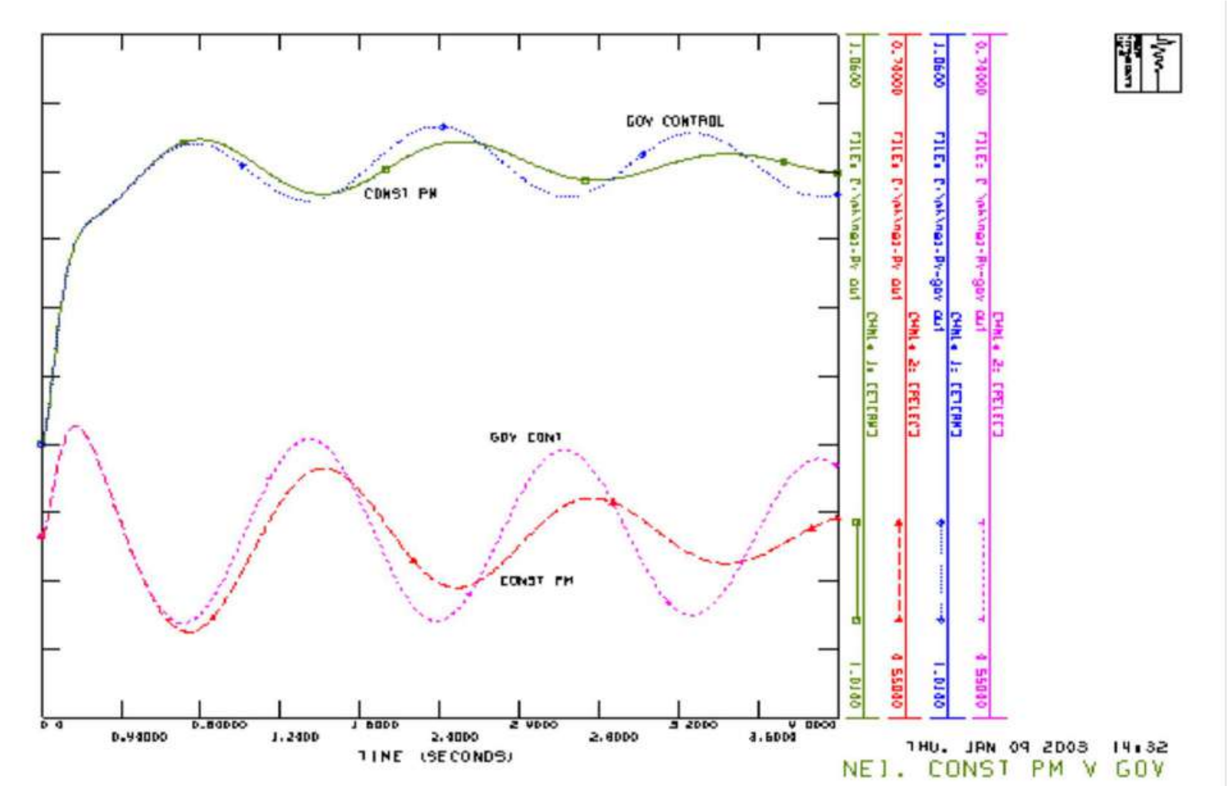
Review of Turbine Governor Control

- Mechanical governor
  - provides frequency regulation and limits over-speed
  - restricted flexibility and capability
  - detrimental to network damping
  - dynamic influence can be limited by dead zone and slugged response
- Electronic governor
  - very flexible with wide control capability
  - turbine control requirements remain essentially those for mech. governor, hence control capability is untapped
  - no grid code requirement for turbine governing to contribute to network dynamics and stability

Gas Turbine Control Potential

- A Gas turbine has good control capability
- It offers the potential to contribute to system dynamics and stability
- Current trends in distributed generation will impose increased stability demands on network control
- It is highly desirable that the capabilities of gas turbine governing to contribute to network operation are explored

Influence Of Governor Control



Voltage reference step – Comparison of standard governor with constant power case

Modelling Of Transformers

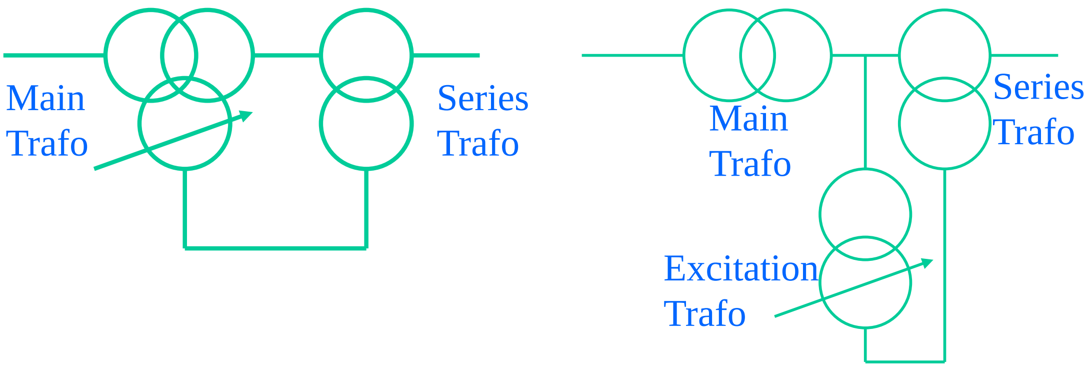
Transformers - Classification

- Generator step-up transformers (connection to the transmission network) (usually Δ-Y with neutral grounded)
- Unit transformers (auxiliary services)
- Transmission transformers (usually Y-Y with neutral grounded, may have low power, medium voltage Δ connected tertiary - for circulating currents when HV winding is asymmetrically loaded)
- Distribution transformers (usually Y-Δ to minimise load asymmetry)

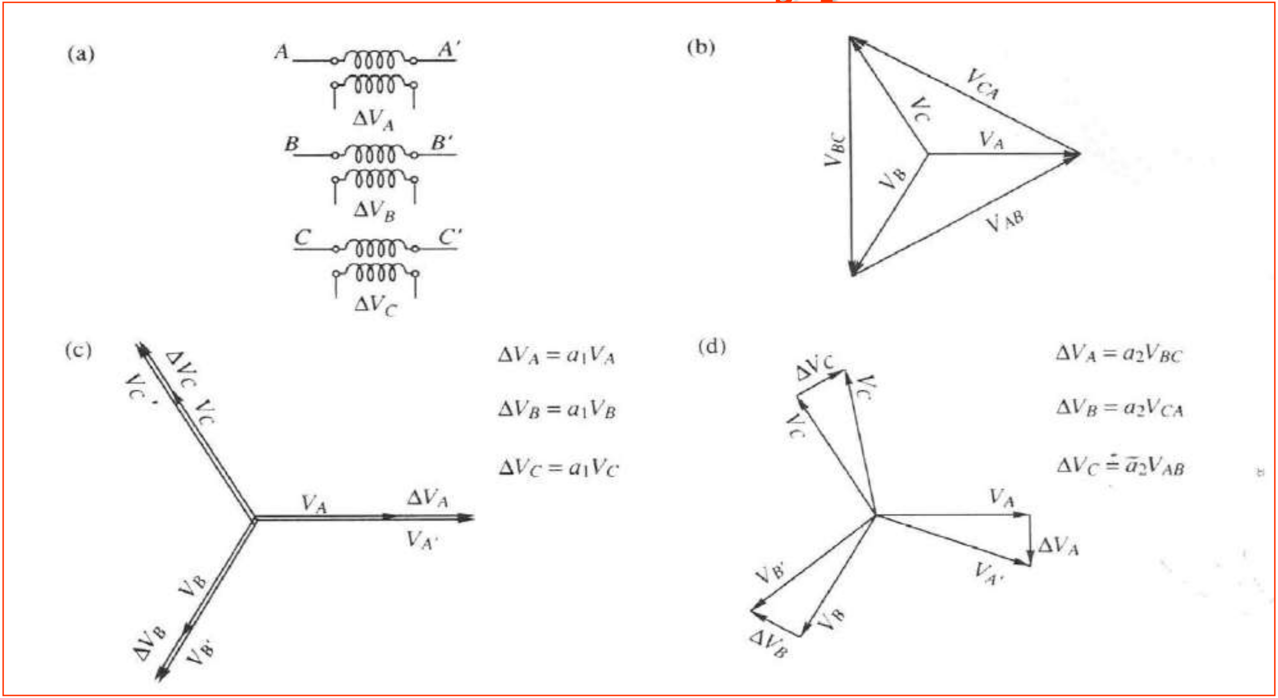
Total MVA rating of all the transformers in a power system is about *five times* the total MVA rating of all the generators!

Regulating Transformer

- On load tap changing transformers (OLTC or ULTC or LTC) (Change in turns ratio between ±10% and ±15%)
- Phase shifting transformers



Transformer Types - 5



a) Windings of the series transformer b) Phase and line voltages
c) In-phase booster voltages d) Quadrature booster voltages

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Dynamic OLTC Models

- OLTC respond with a time delay of 10-120s, typically 20-60 s.
- Step sizes quite small (usually ±10% range consisting of up to 32 steps)

Discrete model:

$$n(t+1)=\begin{cases} n(t)+\Delta n & (V^0{}_k - V_k) > \varepsilon \\ n(t) & |V^0{}_k - V_k| = \varepsilon \\ n(t)-\Delta n & (V^0{}_k - V_k) < \varepsilon \end{cases}$$

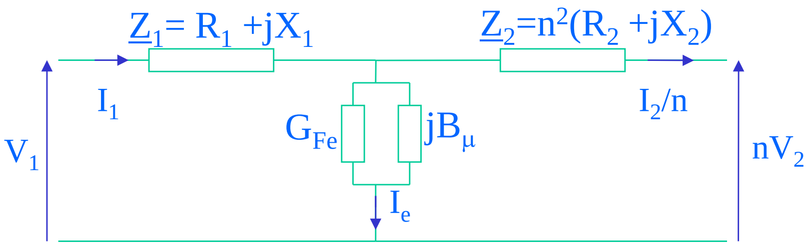
V_k - Voltage to be controlled at $V^0{}_k$
Δn - Tap ratio step size
ε – Dead band

Continuous model:

$$\frac{dn}{dt}=\frac{1}{T}(V^0{}_k - V_k)$$

T - Time constant reflecting the response speed of tap changer

Equivalent Circuit



$$n = \frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$$

$$V_1 = n\,V_2$$

$$I_1 = \frac{I_2}{n}$$

Equivalent circuit with secondary referred to primary

$$\frac{1}{G_{Fe} + jB_\mu} = \frac{1}{\underline{Y}_E} >> \underline{Z}_1 + \underline{Z}_2 = \underline{Z}_T$$

$G_{Fe} = P_{Fe}(p.u.)$

$Y_E = I_E(p.u.)$

$B_\mu = \sqrt{Y^2{}_E - G^2{}_{Fe}}$

No-load test

$Z_T = V_{SC}(p.u.)$

$R_T = P_{Cu}(p.u.)$

$X_T = \sqrt{Z^2{}_T - R^2{}_T}$

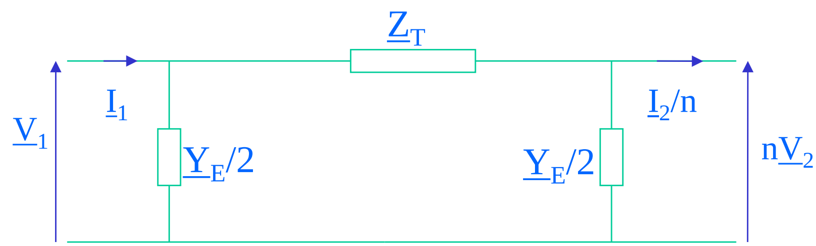
Short-circuit test

Typical parameters

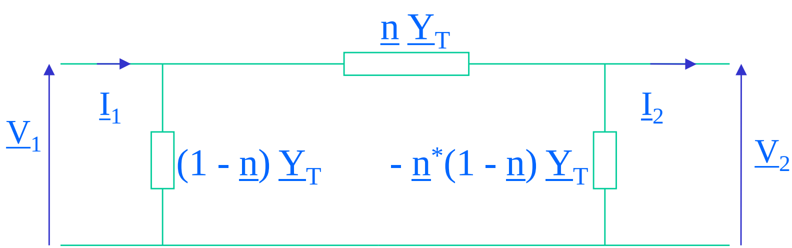
S [MVA]	V_{sc} [p.u.]	P_{Cu} [p.u.]	I_E [p.u.]	P_{Fe} [p.u.]
150	0.110	0.0031	0.0030	0.0010
240	0.150	0.0030	0.0025	0.0006
426	0.145	0.0029	0.0020	0.0006
630	0.143	0.0028	0.0040	0.0007

The order of Z_T and shunt (excitation) reactance are 10% and 2000% respectively

Π – Equivalent Circuit

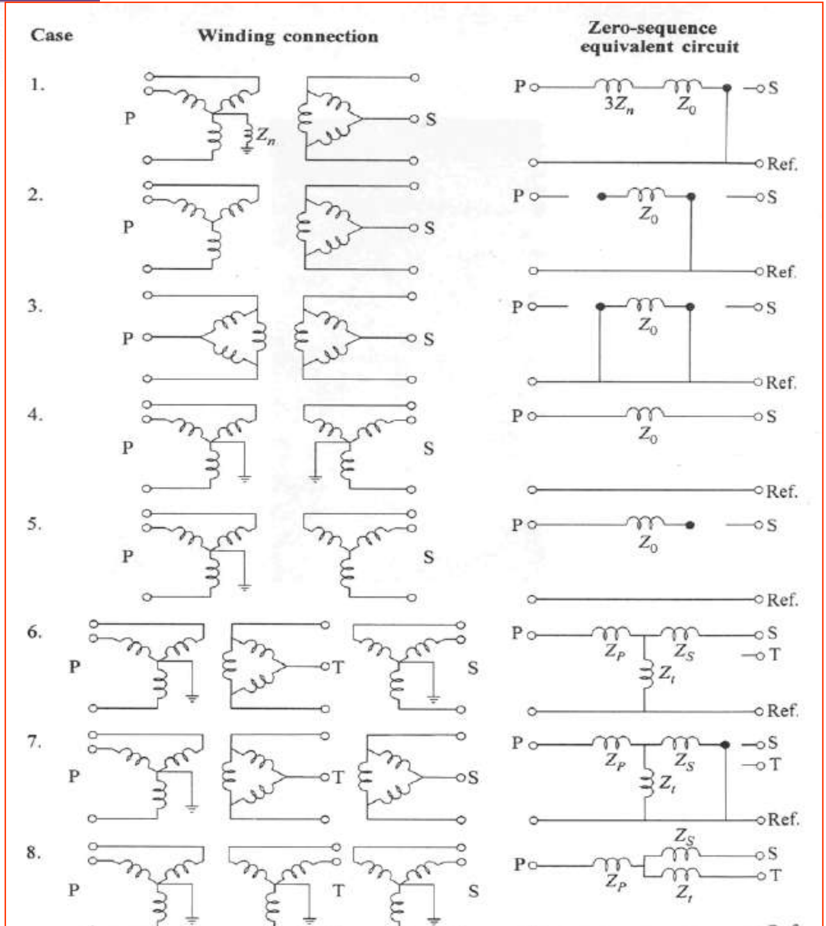


π-circuit of a two winding transformer



π-circuit of a two winding transformer with off-nominal turns ratio

The order of Z_T and shunt (excitation) reactance are 10% and 2000% respectively. For zero sequence equivalent circuit the order of the excitation impedance is much lower then for the positive sequence circuit (100% - 400%) but still high enough to be neglected in most fault studies.

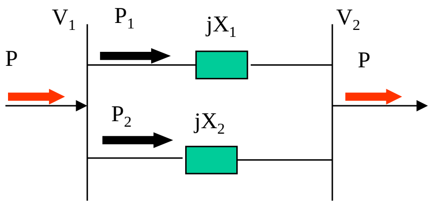


Transformer Zero-sequence Networks

The order of Z_T and shunt (excitation) reactance are 10% and 2000% respectively.

For zero sequence equivalent circuit the order of the excitation impedance is much lower then for the positive sequence circuit (100% - 400%) but still high enough to be neglected in most fault studies.

Transformers for voltage phase angle control - 1



$$P_1 = \frac{V_1 V_2}{X_1}\sin\delta \cdots (1) \qquad P = P_1 + P_2 \cdots (3)$$

$$P_2 = \frac{V_1 V_2}{X_2}\sin\delta \cdots (2) \qquad \delta = \theta_1 - \theta_2$$

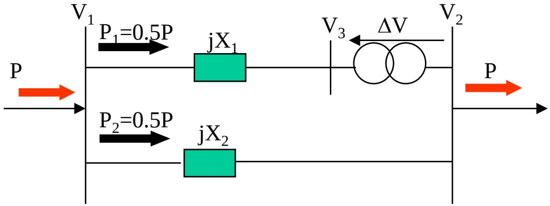
$P_1 \neq P_2$ because line reactances are different

$$\text{for}: V_1 = V_2 = 1 p.u.$$
$$X_1 = 0.4 p.u.$$
$$X_2 = 0.6 p.u.$$
$$P = 1 p.u.$$

we obtain from (1), (2) & (3) $\delta = 15.796°$, $P_1 = 0.6$p.u., $P_2 = 0.4$p.u.

Power transfer is inversely proportional to line reactances!

Transformers for voltage phase angle control - 2



Calculate the value of the required angle shift in order to have equal power distribution between the two lines.

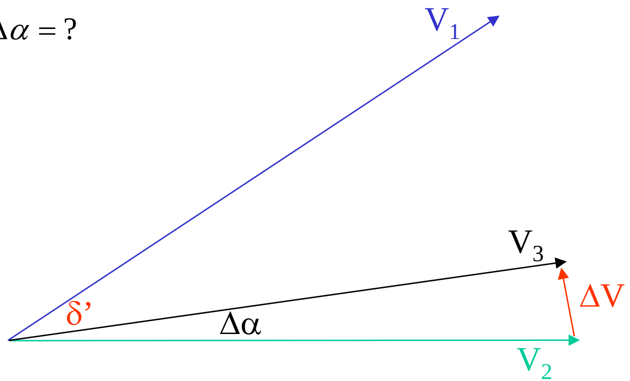
$$P_1 = \frac{P}{2} = 0.5 = \frac{V_1 V_2}{X_1} \sin\left(\delta^1 - \Delta\alpha\right) \cdots (4) \qquad \Delta\alpha = ?$$

$$P_2 = \frac{P}{2} = 0.5 = \frac{V_1 V_2}{X_1} \sin \delta^1 \cdots (5)$$

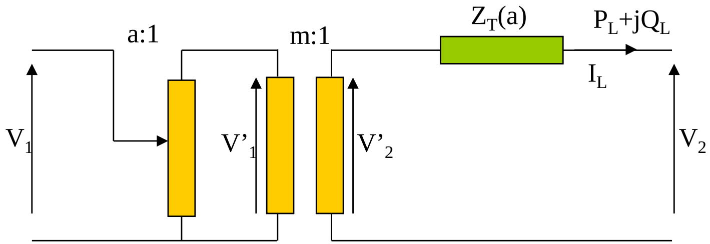
$$\mathit{from}\ (4)\ \&\ (5) \Rightarrow \delta^1 = 19.89°$$

$$\Delta\alpha = 6.75° = 0.118\ rad$$

$$\Delta V \approx V_2 \,.\, \Delta\alpha = 15.6 \frac{kV}{ph}$$



Tap changing transformers - 1



$$V_1' = \frac{V_1}{a} \text{ and } V_2' = \frac{1}{m} V_1' \Rightarrow V_2' = \frac{1}{m\,a} V_1$$

$$\text{for } V_1 = const. \text{ and } a < 1 \Rightarrow V_2' \text{ increases}$$

$$\text{for } V_1 = const. \text{ and } a > 1 \Rightarrow V_2' \text{ decreases}$$

$$\text{for } V_1 = const;\ Z_T = j\,X_T \neq f(a);\ I_L \neq f(a);\ \delta V_2 << \Delta V_2;$$

The change (ΔV_2) in secondary voltage V_2 due to the change in tap position from ($a_1=1$), i.e., $a_{1*}m = 1*m$ to ($a_2=a<1$), i.e., $a_{2*}m = a*m < m$ is given by:

$$\Delta V_2 = V_2(a_1) - V_2(a_2) = \frac{V_1}{m}\,\frac{a-1}{a} - \frac{m}{V_1}(1-a)\,X_T\,Q_L$$

Tap changing transformers - 2

Example 1: for $m=1$, $a_1=1$, $a_2=0.9$, $V_1=1$, $X_T=0.1$, $Q_L=0.4$

Decrease in a is followed by the increase in secondary voltage $(V_2(a_2)>V_2(a_1))$ so the voltage change $(\Delta V_2=V_2(a_1)-V_2(a_2))$ is negative.

$$\Delta V_2 = \frac{1}{1}\frac{0.9-1}{0.9} - \frac{1}{1}(1-0.9)\,X_T\;Q_L = -0.111 - 0.1X_T\;Q_L = -0.111 - 0.004 = -0.115$$

Example 2: for $m=1$, $a_1=1$, $a_2=1.1$, $V_1=1$, $X_T=0.1$, $Q_L=0.4$

Increase in a is followed by the decrease in secondary voltage $(V_2(a_2)<V_2(a_1))$ so the voltage change $(\Delta V_2=V_2(a_1)-V_2(a_2))$ is positive.

$$\Delta V_2 = \frac{1}{1}\frac{1.1-1}{1.1} - \frac{1}{1}(1-1.1)\,X_T\;Q_L = 0.091 + 0.1X_T\;Q_L = 0.091 + 0.004 = 0.095$$

Modelling Of Transmission Lines

Transmission Lines

- Overhead transmission lines
- Cables

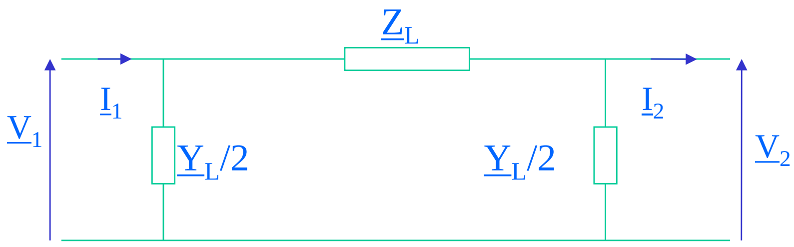
Classification:

- Short lines (lumped parameters)
  - less than 80km(100km)
- Medium length lines (lumped parameters)
  - 80km(100km) - 250km(300km)
- Long lines (distributed parameters)
  - more than 250km(300km)

60Hz(50Hz)

Long Lines

Equivalent π circuit



$$Z_L = Z_C \sinh(\gamma l)$$

$$\frac{Y_L}{2} = \frac{1}{Z_C}\tanh\left(\frac{\gamma l}{2}\right)$$

Characteristic impedance

$$Z_C = \sqrt{\frac{z}{y}}$$

$$\gamma = \sqrt{zy} = \alpha + j\beta$$

$$P_{SIL} = \frac{V^2{}_0}{Z_C}$$

Surge Impedance Loading

$$\lambda = \frac{2\pi}{\beta}$$

Wavelength

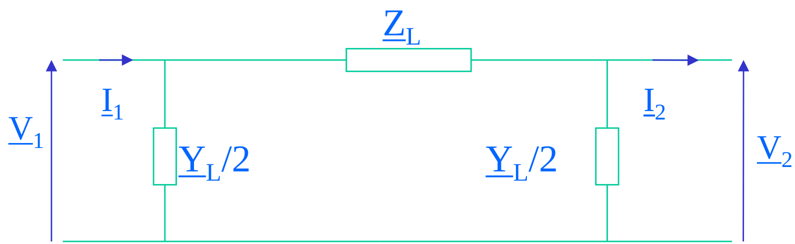
Propagation constant

Phase constant

Attenuation constant

Short & Medium Lines

Equivalent π circuit



$$Z_L = R_L + jX_L$$

$$Y_L = G_L + jB_L$$

$$Z_L = z[\frac{\Omega}{km}]L[km]$$

$$Y_L = y[\frac{S}{km}]L[km]$$

For typical power lines
G = 0
$R_L << X_L$

This approximation is good for:
- Overhead lines shorter than 170km (60Hz) or 200km(50Hz)
- Underground cables shorter than 50km (60Hz) or 60km (50Hz)

Transmission Lines - Capability

- Power transmission capability of transmission line is determined by:
  - Thermal limits
    - lines up to 80 (100) km
  - Voltage regulation
    - lines between 80 (100) km and 320 (380) km
  - System stability
    - lines longer than 320 (380) km

60Hz(50Hz)

Modelling Of Power System Loads

Power System Loads

Bulk power delivery point

Bus load = P+ jQ

Industrial | Commercial | Residential | Agricultural ….

Load class mix

Space heater | Water heater | Air Cond. | Lighting | Refrigeration

Load components

Power factor | P(f,V) | Q(f,V) | Motor parameters

Individual components characteristics

Static Load Models - 1

Exponential (classical) load model

$$P = P_0 \left(\frac{V}{V_0} \right)^{n_p}$$

n_p corresponds to the slope dP/dV at V_0 of P=f(V) curve

$$Q = Q_0 \left(\frac{V}{V_0} \right)^{n_q}$$

n_q corresponds to the slope dQ/dV at V_0 of Q=g(V) curve

$n_p = n_q = 2$ - Constant impedance load
$n_p = n_q = 1$ - Constant current load
$n_p = n_q = 0$ - Constant power load

Generally:

n_p can have any value between 0 and 3

n_q can have any value between 0 and 7 (even 11 for individual loads)

Static Load Models - 2

Polynomial load models:

$$P = P_0[p_1\left(\frac{V}{V_0}\right)^2 + p_2\left(\frac{V}{V_0}\right) + p_3]$$

$$Q = Q_0[q_1\left(\frac{V}{V_0}\right)^2 + q_2\left(\frac{V}{V_0}\right) + q_3]$$

$$p_1 + p_2 + p_3 = 1$$

$$q_1 + q_2 + q_3 = 1$$

ZIP load model

ZIP load model including frequency dependency

K_{pf} = 0 - 3; K_{qf} = -2 - 0

$$P = P_0[p_1\left(\frac{V}{V_0}\right)^2 + p_2\left(\frac{V}{V_0}\right) + p_3][1 + K_{pf}(f - f_0)]$$

$$Q = Q_0[q_1\left(\frac{V}{V_0}\right)^2 + q_2\left(\frac{V}{V_0}\right) + q_3][1 + K_{qf}(f - f_0)]$$

Static Load Models - 3

$$P = P_0\left(\frac{V}{V_0}\right)^{n_p}[1+K_{pf}(f-f_0)]$$

$$Q = Q_0\left(\frac{V}{V_0}\right)^{n_q}[1+K_{qf}(f-f_0)]$$

Exponential load model including frequency dependency

Comprehensive static load models:

$$P = P_0\{p_1\left(\frac{V}{V_0}\right)^2 + p_2\left(\frac{V}{V_0}\right) + p_3 + p_4\left(\frac{V}{V_0}\right)^{n_{p1}}[1+K_{pf1}(f-f_0)] + p_5\left(\frac{V}{V_0}\right)^{n_{p2}}[1+K_{pf2}(f-f_0)]\}$$

$$Q = Q_0\{q_1\left(\frac{V}{V_0}\right)^2 + q_2\left(\frac{V}{V_0}\right) + q_3 + q_4\left(\frac{V}{V_0}\right)^{n_{q1}}[1+K_{qf1}(f-f_0)] + q_5\left(\frac{V}{V_0}\right)^{n_{q2}}[1+K_{qf2}(f-f_0)]\}$$

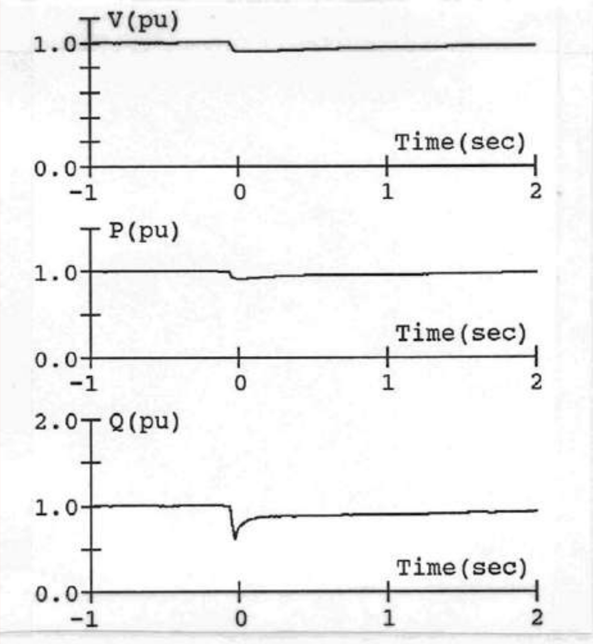
Dynamic Load Models

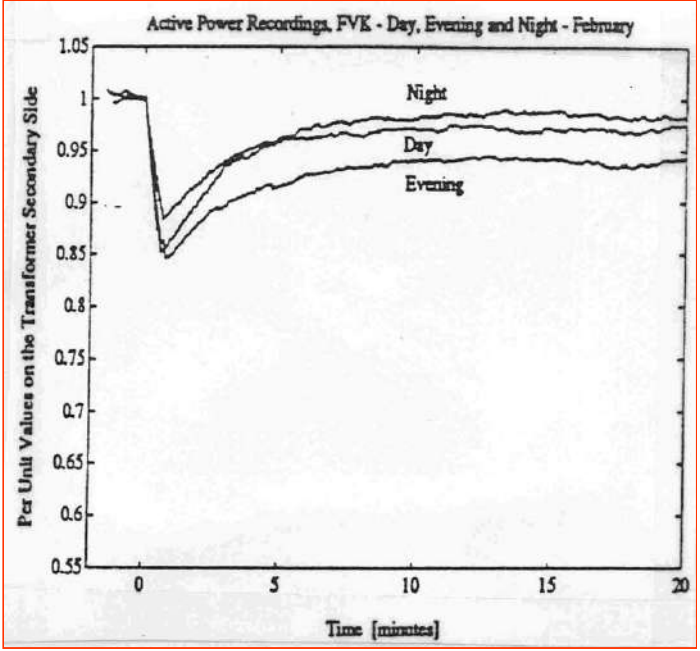
- Responses of many composite loads to small changes in voltage and frequency is very fast and a new steady state is reached quickly - static load models are justified for such cases!
- Studies of inter-area oscillations, voltage stability and long term stability (or systems with large concentration of induction motors) require load dynamics to be modelled.

Contributors to Load Dynamics

- Induction motors - time response up to a few seconds
- Discharge lamps (mercury vapour, sodium vapour and fluorescent lamps) - extinguish at voltages of 0.7 p.u. - 0.8 p.u. and restart with 1 s to 2 s delay.
- Thermal and overcurrent relays
- Thermostatic control of loads (space heaters/coolers, water coolers, refrigerators)
- On Load Tap Changing transformers - control starts about 1 min after the disturbances and voltages are restored within 2 min - 3 min.

Measured (field) Load Responses





Measured load responses of mixed commercial/domestic loads

Dynamic Model of IM for Stability Studies

$$V'_d = -\frac{\omega_0 L_m}{L_{rr}}\Psi_{qr}$$

$$V'_q = \frac{\omega_0 L_m}{L_{rr}}\Psi_{dr}$$

$$\frac{1}{\omega_0}pV'_d = -\frac{1}{T'_0}[V'_d + (X_s - X'_s)I_{qs}] + \frac{1}{\omega_0}p(1-\omega_r)V'_q$$

$$\frac{1}{\omega_0}pV'_q = -\frac{1}{T'_0}[V'_q - (X_s - X'_s)I_{ds}] - \frac{1}{\omega_0}p(1-\omega_r)V'_d$$

$$p\omega_r = \frac{1}{2H}(T_e - T_m)$$

$$T_e = \Psi_{qr}I_{dr} - \Psi_{dr}I_{qr}$$

$$X'_s = \omega_0(L_{ss} - \frac{L^2{}_m}{L_{rr}})$$

$$X_s = \omega_0 L_{ss}$$

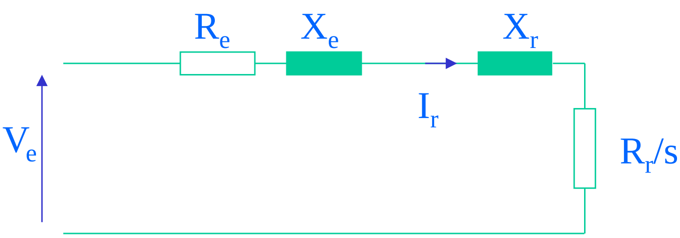
$$T'_0 = \frac{L_{rr}}{R_r}$$

Electrical quantities and torques in p.u.;
Time in seconds;
$\omega_0 = 2\pi f$

$$p = \frac{d}{dt}$$

Simplified Model of IM for Stability Studies

- For small motors dynamics of the rotor electrical circuit can be neglected (time constant very small)!
- Only electromechanical differential equation is left!



$$\boxed{\frac{d\omega_r}{dt} = -\frac{\omega_e}{p}\frac{ds}{dt} = \frac{1}{2H}(T_e - T_m)}$$

$$V_e = \frac{jX_m V_s}{R_s + j(X_s + X_m)}$$

$$R_e + jX_e = \frac{jX_m(R_s + jX_s)}{R_s + j(X_s + X_m)}$$

$$T_e = 3p\frac{R_r}{s\omega_e}\frac{V^2_e}{(R_e + \frac{R_r}{s})^2 + (X_e + X_r)^2}$$

$$s = \frac{n_0 - n_r}{n_0}; \quad n_0 = \frac{60f}{p}; \quad \omega_e = 2\pi f$$

X_m -magnetising reactance
X_s -stator leakage reactance
X_r - rotor leakage reactance
p - number of pairs of magnetic poles

Generic Dynamic Load Models

Model 1

$$pX_p = \frac{1}{T_p}(-X_p + P_s(V) - P_t(V))$$

$$P = X_p + P_t(V)$$

Model 2

$$pX_p = \frac{1}{T_p}(-X_p + P_s(V) - P_t(V))$$

$$P = X_p P_t(V)$$

Static and transient power characteristics can be conveniently defined as:

$$P_s(V) = P_0\left(\frac{V}{V_0}\right)^{n_{ps}}$$

$$P_t(V) = P_0\left(\frac{V}{V_0}\right)^{n_{pt}}$$

$$0 < n_{ps} < 3 \qquad 0 < n_{qs} < 7$$

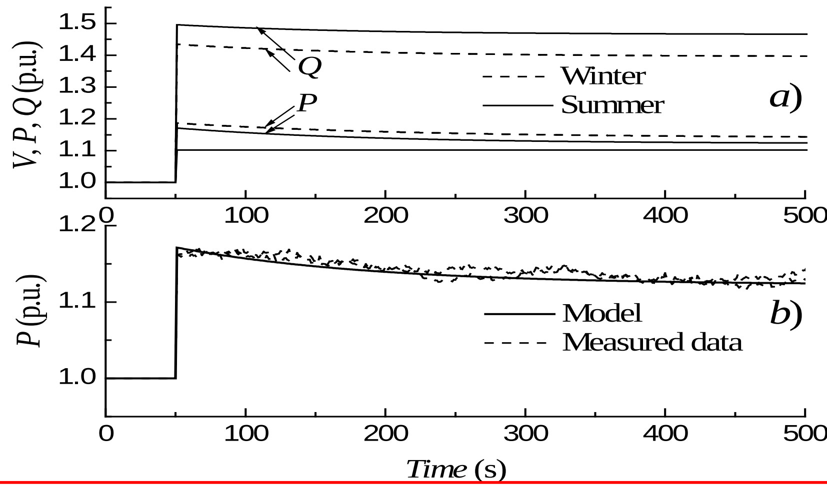
$$1.5 < n_{pt} < 2.5 \qquad 4 < n_{qt} < 7$$

(Note: Similar equations apply for reactive power)

Mean Parameter Values for Summer, Winter and the Year

SEASON	n_{ps}	n_{pt}	T_p	n_{qs}	n_{qt}	T_q
SUMMER	1.19	1.63	142	3.93	4.15	127
WINTER	1.35	1.76	169	3.43	3.71	138
YEAR	1.24	1.67	150	3.74	3.98	131
WINTER/ SUMMER	1.14	1.08	1.19	0.87	0.90	1.08

Load Parameter Identification



Load responses to 10% voltage step increase: (a) simulated for winter and summer, (b) simulated and measured P responses for summer

From the total installed power of all 10/0.4kV substations fed through 31.5 MVA, 110/10kV transformer, 48.01% deliver electrical power to residential load without central heating, 14.39% to residential load with central heating, 15.27% to suburban domestic loads, 15.58% to villages, 1.6% to commercial load and 5.15% to hospitals.

Mean Parameter Values for Summer, Winter and the Year

SEASON	n_{ps}	n_{pt}	T_p	n_{qs}	n_{qt}	T_q
SUMMER	**1.19**	**1.63**	**142**	**3.93**	**4.15**	**127**
WINTER	**1.35**	**1.76**	**169**	**3.43**	**3.71**	**138**
YEAR	**1.24**	**1.67**	**150**	**3.74**	**3.98**	**131**
WINTER/ SUMMER	**1.14**	**1.08**	**1.19**	**0.87**	**0.90**	**1.08**

Past experience - CIGRE WG C4 605 (1)

- International survey on industrial practice on load modelling identified:
  - **constant power PQ load model** is by far the most dominant type of load model used in **steady state power system studies** (84% of all responses).
  - **constant power PQ load model** is still the most widely used in a majority of **power system stability studies** (as it is the most conservative approach), together with the **constant current** for **real power** and **constant impedance** for **reactive power** model.

Past experience - CIGRE WG C4 605 (2)

- Load models used in **dynamic power system studies** are very different and there is no dominant dynamic load modelling practice, however **static load models** are again dominant.
  - **Constant power** and **constant current** load models account for about 42% of all models used **for real power.**
  - **Constant power** and **constant impedance** load models account for 45% of all models used **for reactive power**.
  - For modelling both real and reactive power demand, about 30% of the used models represent dynamic load by some form of induction motor model (IM with ZIP or exponential, or composite load model).

Key question

- In spite of these relatively "crude" load models that have been and are being used, there haven't been many problems and blackouts around the world, at least not those directly attributed to inadequate load modelling.

- So the key question is

How accurate load model at different buses of the network does actually need to be for different types of system studies?

Chapter 4:
Small-Disturbance Stability

Content

- Small system applications
- Modelling requirements
- Mathematical tools for the analysis
- Large system applications

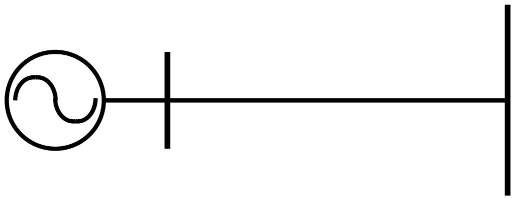
Stability of Dynamic System

- Stability of linear system is entirely independent of the input. The state of the stable system with zero input will always return to the origin of the state space, independent of the finite initial state.
- Stability of a non-linear system depends of the type and magnitude of input and the initial state.

MANCHESTER 1824

The University of Manchester

One machine vs. an infinite bus



- Infinite bus is so big that:
  - Nothing affects its voltage magnitude
  - Nothing affects its voltage phase or frequency
- Simplified representation of a network with many generators connected by lots of lines with small impedances and where the capacity of the independent generator is very small compared to the capacity of the rest of the system.

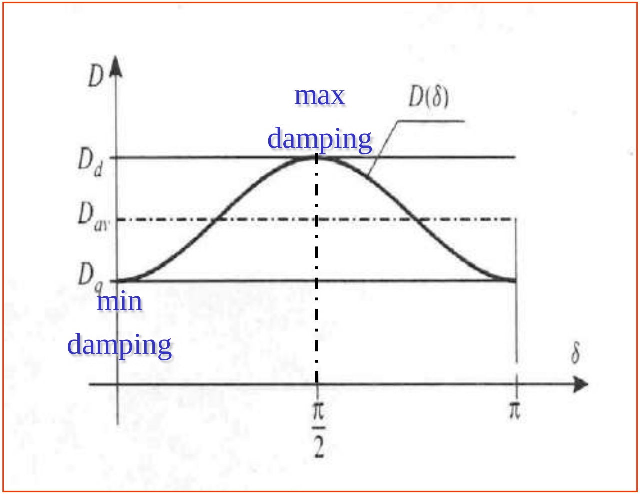
Rotor angle/Power angle/ Internal machine angle/Angle δ

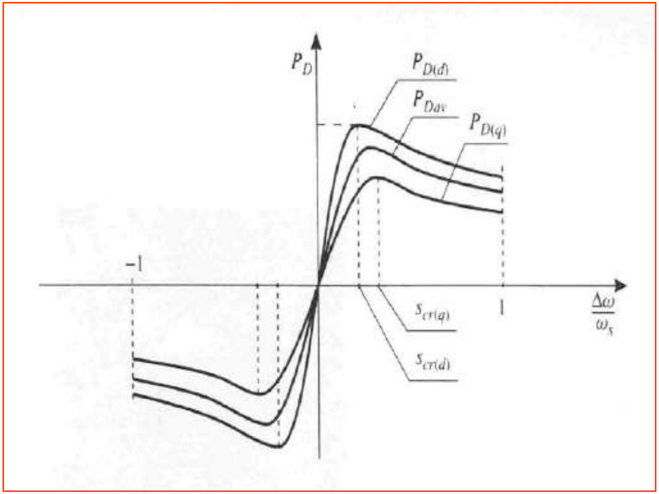
- P_e represents the power transferred between the internal e.m.f. (*E*) of the generator and the infinite bus
- The internal e.m.f. of the generator is the voltage induced in the stator due to the rotation of the field winding on the rotor and it is thus linked to the position of the rotor
- The phase angle δ of the internal e.m.f. is equal to the mechanical position of the rotor (expressed in *electrical* degrees or radians)

Damping Power Variation

$$P_D = \left[D_d \sin^2\delta + D_q \cos^2\delta\right]\Delta\omega$$
$$= \left[K_{D_d} \sin^2\delta + K_{D_q} \cos^2\delta\right]\Delta\omega$$

When δ is large damping is strongest in the d-axis, when δ is small the q-axis damper winding produces the stronger damping.





Effect of Damper Windings

$$p\omega = \frac{1}{2H}(P_m - (P_E + P_D))$$

$$P_D = K_D \Delta\omega$$

- For $\Delta\omega<0$ damping power is negative and it is opposing electrical power, i.e., increase in acceleration
- For $\Delta\omega>0$ damping power is positive and it assists electrical power, i.e., decrease in acceleration
- Generally, when damping coefficient (torque) is in phase with speed deviation that results in decrease in acceleration.

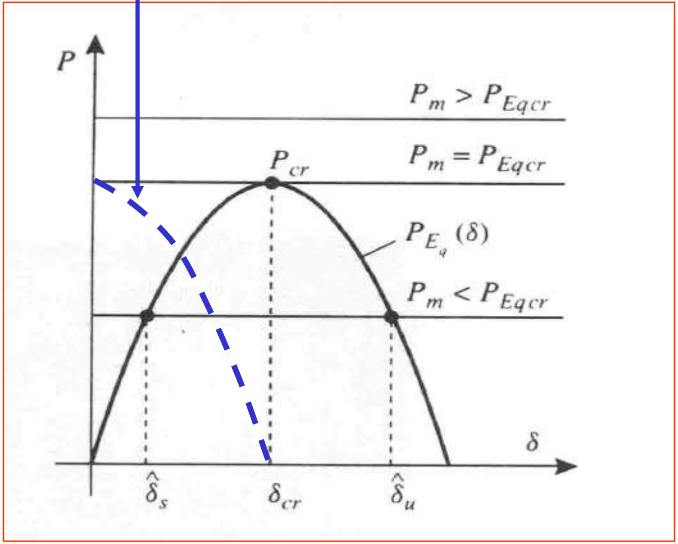
Synchronizing Power Variation

$$P_{Eq} = \frac{E_q V_s}{X_d} \sin\delta$$

$$K_{Eq} = \frac{\partial P_{Eq}}{\partial \delta} = \frac{E_q V_s}{X_d} \cos\delta$$ ← Synchronising coefficient

$$K_{Eq}(\delta = 0) = \max$$

$$K_{Eq}(\delta = 90^0) = \min$$



Typical Rotor Swings

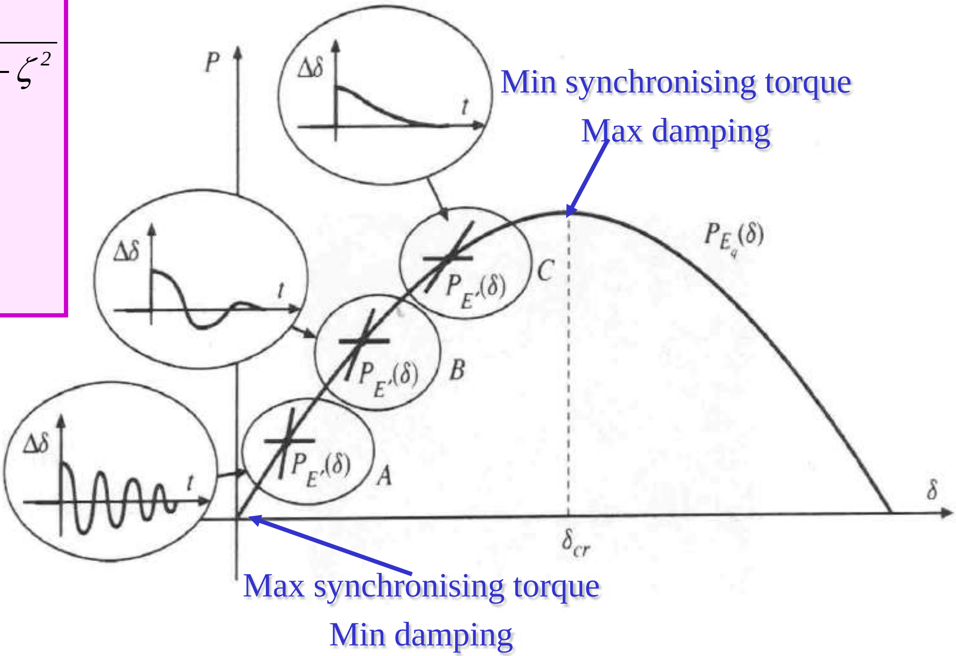
$$\lambda = \sigma \pm j\omega$$

$$\zeta = \frac{-\sigma}{\sqrt{\sigma^2 + \omega^2}}$$

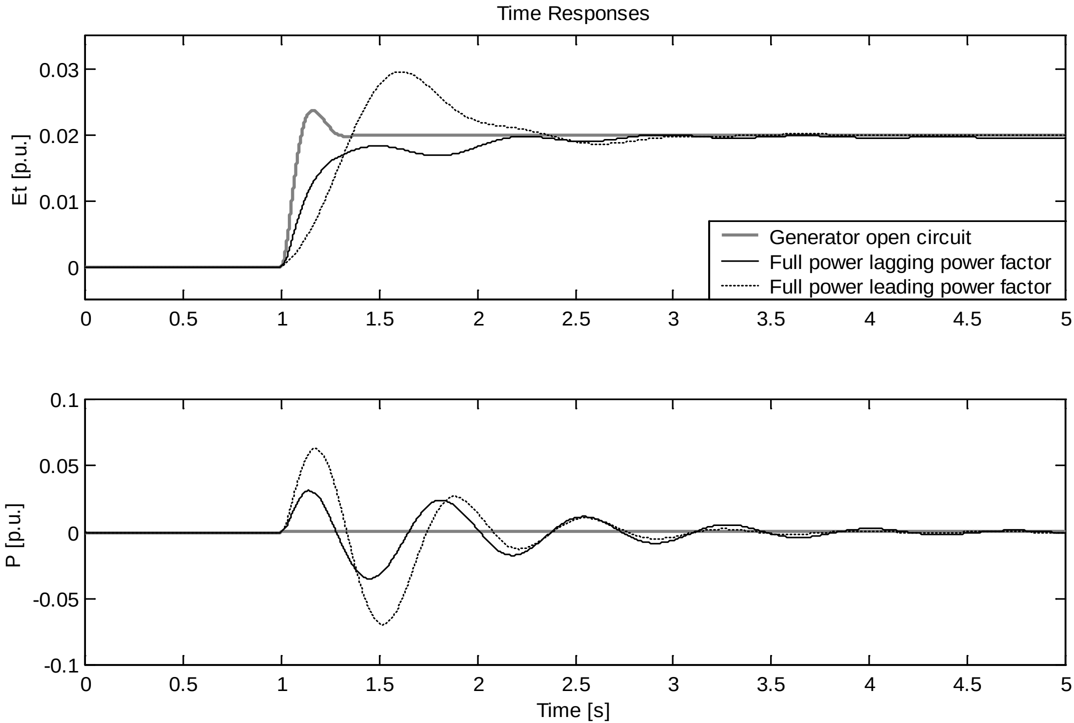
$$\lambda = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

$$\omega_n = \sqrt{K_s \frac{\omega_0}{2H}}$$

$$\zeta = \frac{1}{2}\frac{K_D}{\sqrt{2K_s H\omega_0}}$$

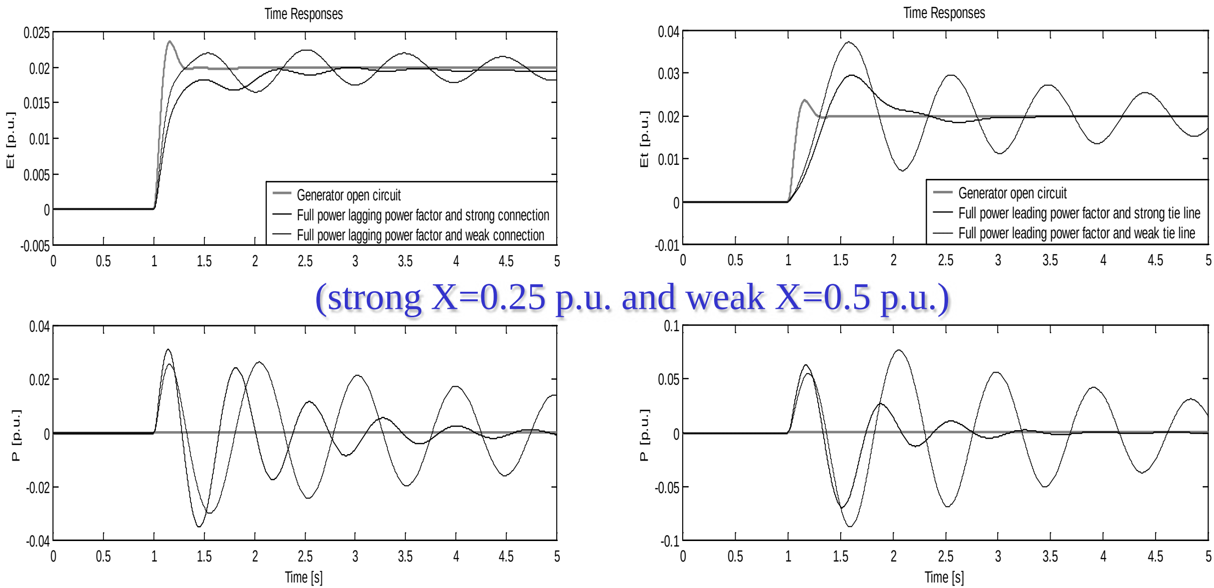


Leading vs. Lagging operation



Response to step change ΔE_{tref}=0.02, NEI AVR, leading and lagging PF

Leading vs. Lagging operation – 2



(strong X=0.25 p.u. and weak X=0.5 p.u.)

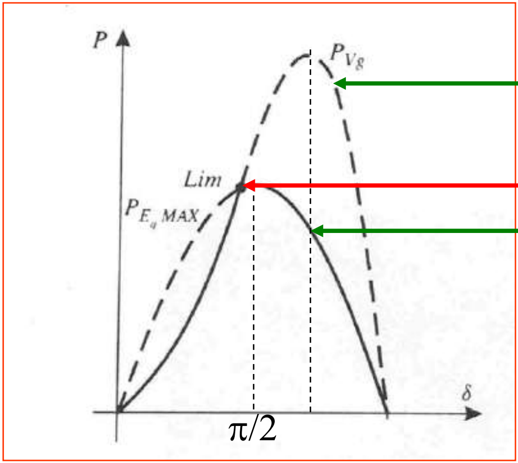
lagging PF

leading PF

Response to step change ΔE_{tref}=0.02, with NEI AVR

Effects of AVR - 1

- If the time constant of the Automatic Voltage Regulator (AVR) is large, it will not react during the transient state (it will be to slow).
- If the time constant of the AVR is small (an excitation system having a voltage response time of 0.1s or less represents a high response and fast-acting system), it may increase or decrease stability limit depending on system and AVR parameters.



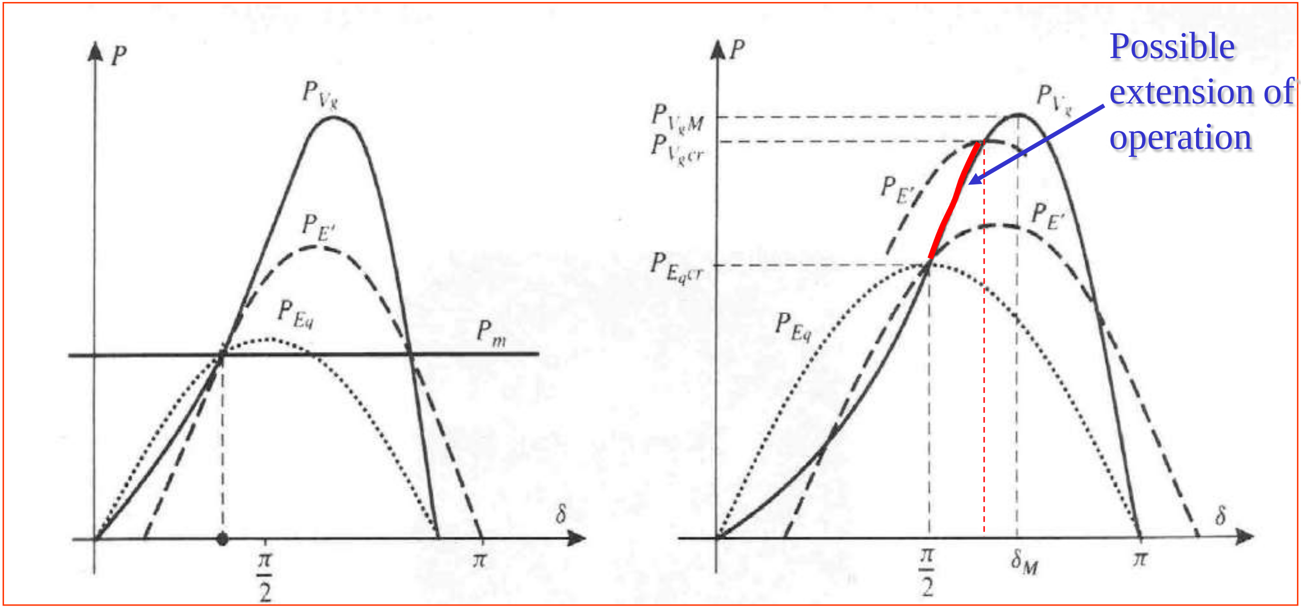
Power characteristic with AVR

Field current limit reached

Power characteristic with AVR and field current limiter

Typical requirement is voltage response time of 0.5s since following a severe disturbance the generator rotor angle swing normally peaks between 0.4 and 0.75s

Transient Characteristic with AVR



All synchronising power coefficients are positive

Points above the limit of natural stability

Effects of AVR - 2

1 Voltage regulation in general weakens the damping introduced by field winding and net damping contribution may become negative

2 This negative damping is enhanced by:
   - large generator load
   - large gain of AVR controller
   - large network reactance

3 With low values of external impedance and low generator output AVR introduces negative synchronising torque *(this is of no particular concern as this is very small decrease)* and positive damping torque.

4 In cases described in 2 *(common case in power systems)* AVR introduces negative damping torque and positive synchronising torque. This effect is more pronounced as the exciter response increases!

Modelling Requirements

Power System Dynamic Modeling

Linearization of the Differential and Algebraic Set of Equations

$$\dot{x} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{r}, u)$$
$$\boldsymbol{0} = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{r}, u)$$
$$y = h(\boldsymbol{x}, \boldsymbol{r}, u)$$

$\Rightarrow$

$$\begin{bmatrix} \Delta\dot{\boldsymbol{x}} \\ \boldsymbol{0} \end{bmatrix} = \left[\begin{array}{c|c} \boldsymbol{J}_1 & \boldsymbol{J}_2 \\ \hline \boldsymbol{J}_3 & \boldsymbol{J}_4 \end{array}\right] \cdot \begin{bmatrix} \Delta\boldsymbol{x} \\ \Delta\boldsymbol{r} \end{bmatrix} + \left[\begin{array}{c} \boldsymbol{b}_x \\ \hline \boldsymbol{b}_r \end{array}\right] \cdot \Delta u$$

$$\Delta y = \left[\mathbf{c}_{\mathbf{x}}^{\mathrm{t}} \mid \mathbf{c}_{\mathbf{r}}^{\mathrm{t}}\right] \cdot \left[\begin{array}{c} \Delta\boldsymbol{x} \\ \hline \Delta\boldsymbol{r} \end{array}\right]$$

Transfer Function

$$G(s) = \frac{\Delta y(s)}{\Delta u(s)} = \left[\mathbf{c}_{\mathbf{x}}^{\mathrm{t}} \mid \mathbf{c}_{\mathbf{r}}^{\mathrm{t}}\right] \cdot \left[\begin{array}{c|c} s\mathbf{I} - \boldsymbol{J}_1 & -\boldsymbol{J}_2 \\ \hline -\boldsymbol{J}_3 & -\boldsymbol{J}_4 \end{array}\right]^{-1} \cdot \left[\begin{array}{c} \boldsymbol{b}_x \\ \hline \boldsymbol{b}_r \end{array}\right]$$

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State-space Representation - 1

- The behaviour of power system may be described by a set of n first-order non-linear ordinary differential equations:

$$px_i = f_i(x_1, x_2, ..., x_n, u_1, u_2, ..., u_r, t) \qquad i = 1, ..., n$$

or

$$\boxed{p\mathbf{x} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t)}$$

Vector of system inputs

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{bmatrix} \qquad \mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ . \\ . \\ u_r \end{bmatrix} \qquad \mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \\ . \\ . \\ f_n \end{bmatrix}$$

Vector of state variables

State-space Representation - 2

- and by a set of *m* non-linear algebraic equations

$$\boxed{\mathbf{y} = \mathbf{g}(\mathbf{x},\mathbf{u},t)}$$

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ y_m \end{bmatrix} \qquad \mathbf{g} = \begin{bmatrix} g_1 \\ g_2 \\ . \\ . \\ g_m \end{bmatrix}$$

Vector of system outputs

- If the derivatives of the state variables are not explicit function of time the system is said to be autonomous (and t may be omitted in previous expressions).

Equilibrium Points

- Equilibrium points are those where all the derivatives of state variables are simultaneously zero.

$$p\mathbf{x} = \mathbf{f}(\mathbf{x}_0, \mathbf{u}) = 0$$

State vector **x** at the equilibrium point

Small-disturbance Stability

- Definition:

The ability of a power system to maintain synchronism under small disturbance.

- Requirement

Linearised equations about given equilibrium point!

Linearised Form of State Equations

$$p\mathbf{x} = \mathbf{f}(\mathbf{x},\mathbf{u},t)$$
$$\mathbf{y} = \mathbf{g}(\mathbf{x},\mathbf{u},t)$$

$$\xrightarrow{\textit{linearisation}}$$

$$p\Delta\mathbf{x} = \mathbf{A}\,\Delta\mathbf{x} + \mathbf{B}\,\Delta\mathbf{u}$$
$$\Delta\mathbf{y} = \mathbf{C}\,\Delta\mathbf{x} + \mathbf{D}\,\Delta\mathbf{u}$$

$$\mathbf{A} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \cdots & \frac{\partial f_1}{\partial u_r} \\ \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial u_1} & \cdots & \frac{\partial f_n}{\partial u_r} \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \cdots & \frac{\partial g_1}{\partial x_n} \\ \cdots & \cdots & \cdots \\ \frac{\partial g_m}{\partial x_1} & \cdots & \frac{\partial g_m}{\partial x_n} \end{bmatrix} \qquad \mathbf{D} = \begin{bmatrix} \frac{\partial g_1}{\partial u_1} & \cdots & \frac{\partial g_1}{\partial u_r} \\ \cdots & \cdots & \cdots \\ \frac{\partial g_m}{\partial u_1} & \cdots & \frac{\partial g_m}{\partial u_r} \end{bmatrix}$$

A- State matrix - *nxn*
B - Input matrix - *nxr*
C - Output matrix - *mxn*
D - Feedforward matrix - *mxr*

Example:Linearisation

$$\left.\begin{aligned} f_1 &= x_1^{\,2} + 2x_1x_2 + u_1u_2 \\ f_2 &= -x_1x_2 + 3x_2^{\,2} - u_2^{\,2} \\ g_1 &= x_2^{\,2} + u_1u_2 \\ g_2 &= x_1^{\,2} - u_1u_2 \end{aligned}\right\} \xrightarrow{\textit{linearisation}} \quad A = \begin{bmatrix} 2x_1 & 2x_1 \\ -x_2 & 6x_2 \end{bmatrix} \quad B = \begin{bmatrix} u_2 & u_1 \\ 0 & -2u_2 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 2x_2 \\ 2x_1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} u_2 & u_1 \\ -u_2 & -u_1 \end{bmatrix}$$

For $u_1 = 0$ and $u_2 = 5$

$$\begin{aligned} 0 &= x_1^{\,2} + 2x_1x_2 \\ 0 &= -x_1x_2 + 3x_2^{\,2} - 25 \end{aligned} \xrightarrow{\textit{Solving those gives}} \begin{aligned} x_1 &= \mp 2\sqrt{5} \\ x_2 &= \pm\sqrt{5} \end{aligned}$$

$$A = \begin{bmatrix} -4\sqrt{5} & -4\sqrt{5} \\ -\sqrt{5} & 6\sqrt{5} \end{bmatrix} \quad B = \begin{bmatrix} 5 & 0 \\ 0 & -10 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 2\sqrt{5} \\ -4\sqrt{5} & 0 \end{bmatrix} \quad D = \begin{bmatrix} 5 & 0 \\ -5 & 0 \end{bmatrix}$$

Mathematical Tools For The Analysis

Eigenvalues

$$\mathbf{A}v_i = \lambda_i v_i$$

$$(\mathbf{A} - \lambda_i \mathbf{I})v_i = 0$$

$$\det(\mathbf{A} - \lambda_i \mathbf{I}) = 0$$

Characteristic equation whose solutions λ_i are the eigenvalues of **A**.

Identity matrix of dimension *n*x*n*

n eigenvalues λ_i ($i = 1,\ldots,n$) of **A** of dimension *n*x*n*

Eigenvectors

$$\mathbf{A}\mathbf{v}_i = \lambda_i \mathbf{v}_i$$

Right eigenvector of **A** associated with λ_i, *n*-dimensional column vector

$$\mathbf{w}_i \mathbf{A} = \lambda_i \mathbf{w}_i$$

Left eigenvector of **A** associated with λ_i, *n*-dimensional row vector

$$\mathbf{w}_j \mathbf{v}_i = 0$$

$$\mathbf{w}_i \mathbf{v}_i = C_i \neq 0$$

After normalisation

$$\mathbf{w}_i \mathbf{v}_i = 1$$

Modal Matrices

$$\mathbf{V} = \begin{array}{c} \begin{array}{cccc} \phantom{v_{11}} & \mathbf{v}_2 & & \phantom{v_{1n}} \end{array} \\ \begin{bmatrix} v_{11} & v_{12} & \dots & v_{1n} \\ v_{21} & v_{22} & \dots & v_{2n} \\ \dots & \dots & \dots & \dots \\ v_{n1} & v_{n2} & \dots & v_{nn} \end{bmatrix} \end{array}$$

Right modal matrix

$$\boldsymbol{\Lambda} = \begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \lambda_n \end{bmatrix}$$

Matrix of eigenvalues

$$\mathbf{W} = \begin{bmatrix} w_{11} & w_{21} & \dots & w_{n1} \\ w_{12} & w_{22} & \dots & w_{n2} \\ \dots & \dots & \dots & \dots \\ w_{1n} & w_{2n} & \dots & w_{nn} \end{bmatrix} \begin{array}{c} \\ \mathbf{w}_2 \\ \\ \\ \end{array}$$

Left modal matrix

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

Identity matrix

$$\mathbf{WV} = \mathbf{I} \qquad \mathbf{W} = \mathbf{V}^{-1} \qquad \mathbf{WAV} = \boldsymbol{\Lambda}$$

Eigenvalues and Time Response

Time response of the *i*-th state variable depends on eigenvalues:

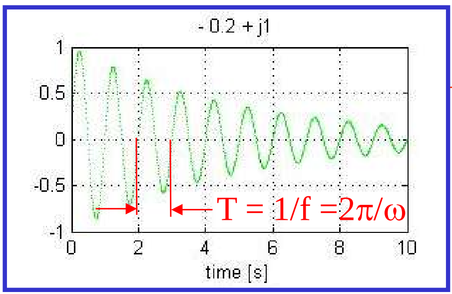
$$\Delta x_i(t) = v_{i1}\mathbf{w}_1\Delta\mathbf{x}(0)e^{\lambda_1 t} + v_{i2}\mathbf{w}_2\Delta\mathbf{x}(0)e^{\lambda_2 t} + \ldots + v_{in}\mathbf{w}_n\Delta\mathbf{x}(0)e^{\lambda_n t}$$

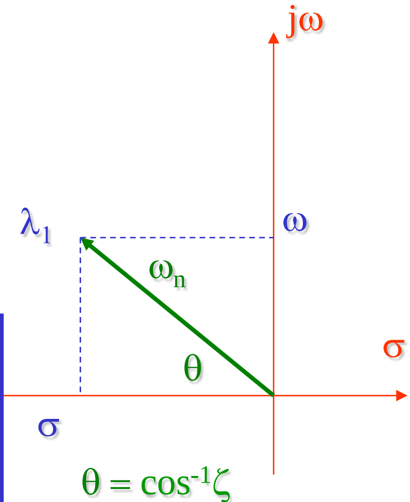
$\lambda = \sigma \pm j\omega$ Complex eigenvalue

$f = \dfrac{\omega}{2\pi}$ Actual or damped frequency of oscillation [Hz]

$\zeta = \dfrac{-\sigma}{\sqrt{\sigma^2 + \omega^2}}$ Damping ratio

$\lambda = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$





Eigenvalues and Time Response - 2

Time response of the *i*-th state variable depends on eigenvalues:

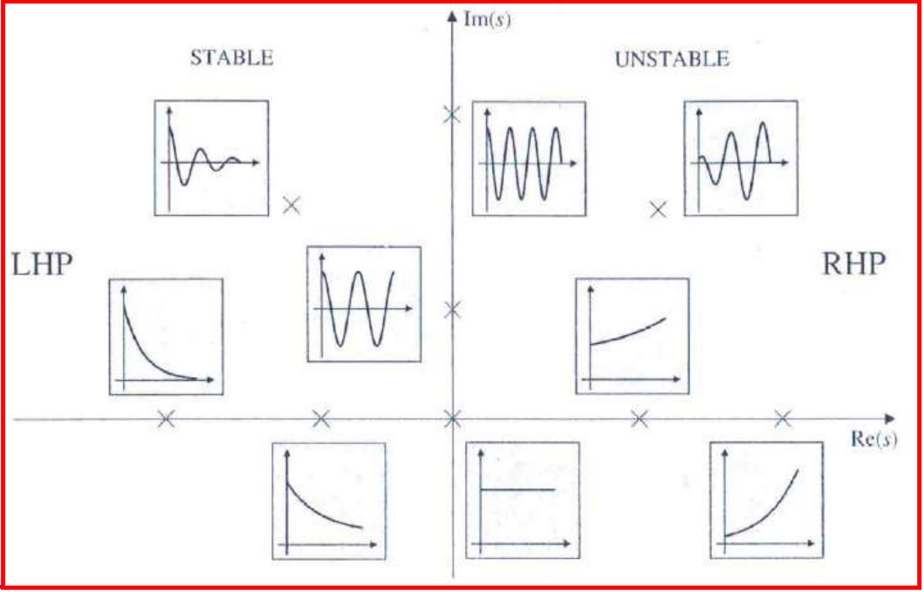
$$\Delta x_i(t) = v_{i1}\mathbf{w}_1\Delta\mathbf{x}(0)e^{\lambda_1 t} + v_{i2}\mathbf{w}_2\Delta\mathbf{x}(0)e^{\lambda_2 t} + \ldots + v_{in}\mathbf{w}_n\Delta\mathbf{x}(0)e^{\lambda_n t}$$

$$\lambda = \sigma \pm j\omega$$

$$f = \frac{\omega}{2\pi}$$

$$\zeta = \frac{-\sigma}{\sqrt{\sigma^2 + \omega^2}}$$

$$\lambda = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

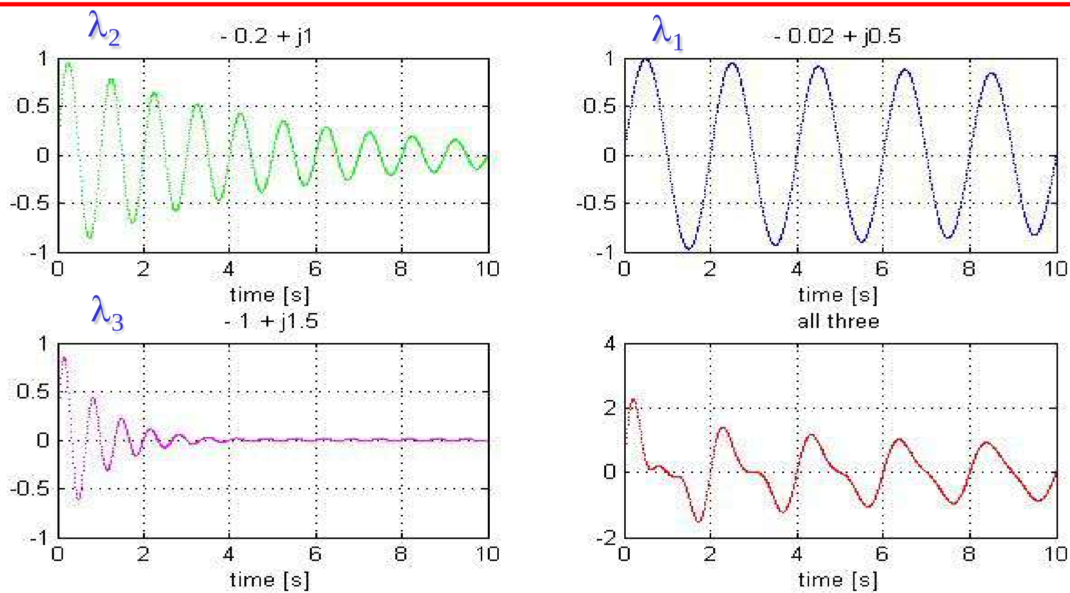


Example 1: Eigenvalues & Time Response

Assume: Time response of the *i*-th state variable to be:

$$\Delta x_i(t) = v_{i1} w_1 \Delta x(0) e^{\lambda_1 t} + v_{i2} w_2 \Delta x(0) e^{\lambda_2 t} + v_{i3} w_3 \Delta x(0) e^{\lambda_3 t}$$

$$= 1e^{\lambda_1 t} + 1e^{\lambda_2 t} + 1e^{\lambda_3 t}$$

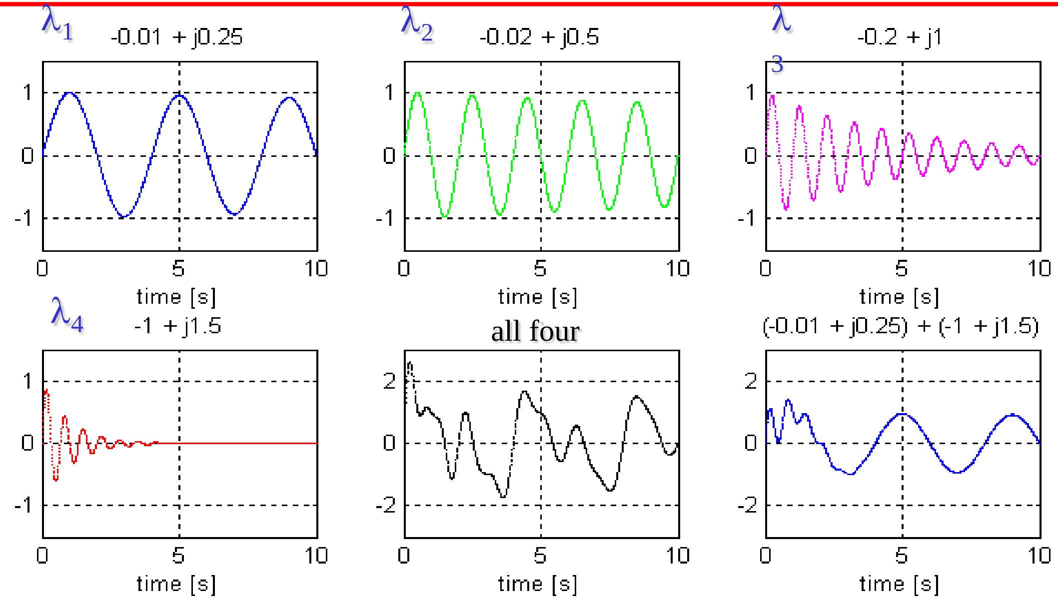


Example 2: Eigenvalues & Time Response

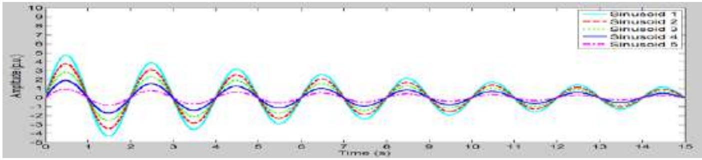
Assume: Time response of the *i*-th state variable to be:

$$\Delta x_i(t) = v_{i1} w_1 \Delta x(0) e^{\lambda_1 t} + v_{i2} w_2 \Delta x(0) e^{\lambda_2 t} + \ldots + v_{i4} w_4 \Delta x(0) e^{\lambda_4 t}$$

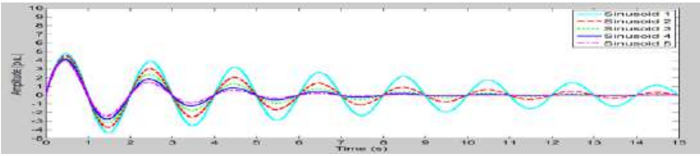
$$= 1e^{\lambda_1 t} + 1e^{\lambda_2 t} + 1e^{\lambda_3 t} + 1e^{\lambda_4 t}$$



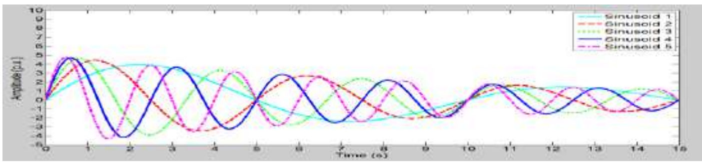
Example 3: Eigenvalues & Time Response



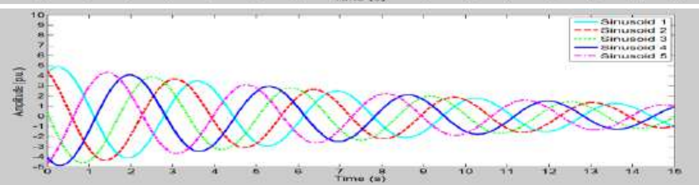
Five sinusoids with the same frequency (0.5Hz), damping (-0.1s-1) and phase (0 deg). Sinusoid 1 has an amplitude of 5p.u., Sinusoid 2 an amplitude of 4p.u., Sinusoid 3 an amplitude of 3p.u., Sinusoid 4 an amplitude of 2p.u. and Sinusoid 5 has an amplitude of 1p.u..



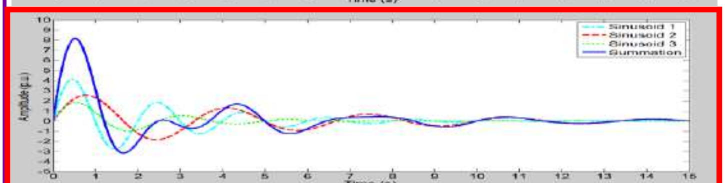
Five sinusoids with the same frequency (0.5Hz), amplitude (5p.u.) and phase (0 deg). Sinusoid 1 has damping of 0.1s-1, Sinusoid 2 has damping of 0.2s-1, Sinusoid 3 has damping of 0.3s-1, Sinusoid 4 has damping of 0.4s-1 and Sinusoid 5 has damping of 0.5s-1.



Five sinusoids with the same amplitude (5p.u.), damping (-0.1s-1) and phase (0 deg). Sinusoid 1 has a frequency of 0.1Hz, Sinusoid 2 has a frequency of 0.2Hz, Sinusoid 3 has a frequency of 0.3Hz, Sinusoid 4 has a frequency of 0.4Hz and Sinusoid 5 has a frequency of 0.5Hz.

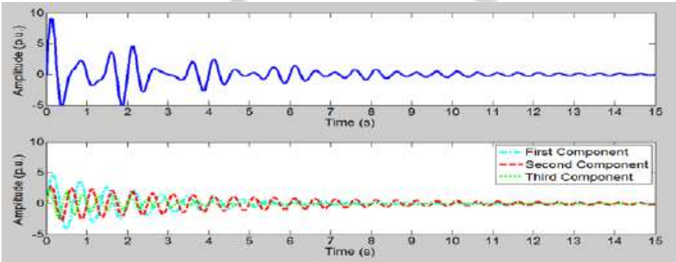


Five sinusoids with the same amplitude (5p.u.), damping (-0.1s-1) and frequency (0.3Hz). Sinusoid 1 has a phase of 45 degrees, Sinusoid 2 has a phase of 90 degrees, Sinusoid 3 has a phase of 135 degrees, Sinusoid 4 has a phase of 180 degrees and Sinusoid 5 has a phase of 225 degrees).

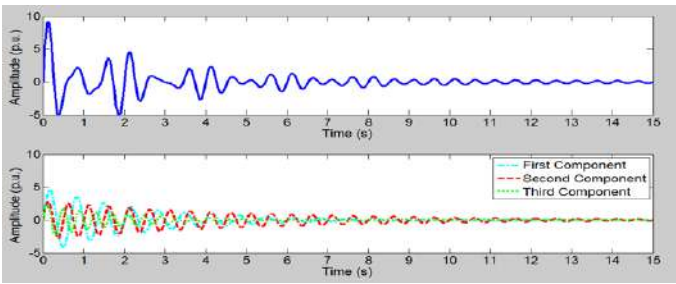


A plot of three damped sinusoids, which are then summed to form a more complex signal. Sinusoid 1: Amplitude 5 p.u., Damping -0.4s^{-1}, Frequency 0.5Hz, Phase 0 deg. Sinusoid 2: Amplitude 3 p.u., Damping -0.2s^{-1}, Frequency 0.3Hz, Phase 0 deg.

Example 4: Eigenvalues & Time Response

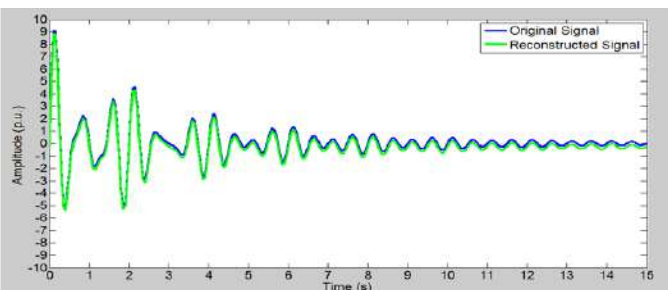


A damped sinusoid (top window) formed by summing the three sinusoids in the bottom window. The first component is a sinusoid with an amplitude of 5 p.u., damping of -0.4s-1, a frequency of 1.5Hz and phase of 0 degrees. The second component is a sinusoid with an amplitude of 3 p.u., a frequency of 2Hz, damping of -0.2s-1 and phase of 0 degrees. The third component is a sinusoid with an amplitude of 2.5 p.u., damping of -0.5s-1, a frequency of 2.5Hz and phase of 0 degrees.



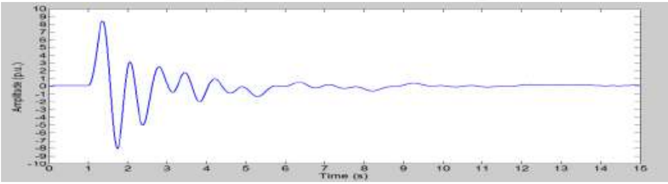
The same input signal (top window) and the three sinusoids extracted by the Prony algorithm (bottom window) using a 10th order model

Mode No.	Real (1/s)	Imag (rad/s)	Amp (p.u.)	Freq (Hz)	Damp (1/s)	Phase (deg)
1	-0.4	-9.4248	5	1.5	-0.4	87.57
2	-0.4	9.4248	5	1.5	-0.4	-87.57
3	-0.2	-12.566	3	2	-0.2	89.088
4	-0.2	12.566	3	2	-0.2	-89.088
5	-0.5	-15.708	2.5	2.5	-0.5	88.177
6	-0.5	15.708	2.5	2.5	-0.5	-88.177
7	2.4564e-06	0	0.48142	0	2.4564e-06	0
8	-2.4564e-06	0	0.002404	0	-2.4564e-06	0
9	-3.5273	-56.938	4.1848e-15	9.0619	-3.5273	86.455
10	-3.5273	56.938	4.1848e-15	9.0619	-3.5273	-86.455

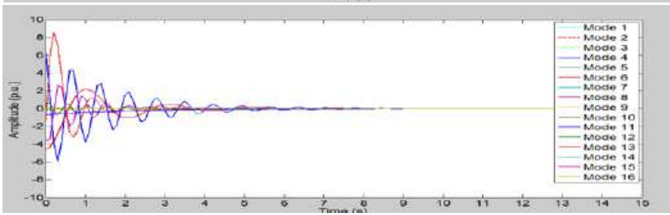


Comparison of the original input signal and the reconstructed signal, formed by summing the ten extracted modes together

Example 5: Eigenvalues & Time Response

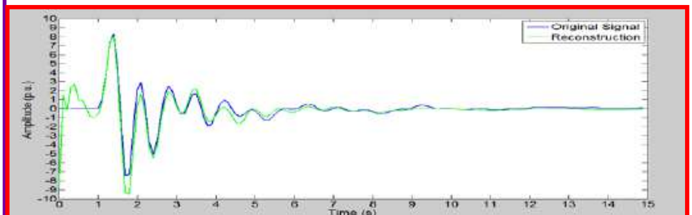


Simulated response of a generator from the system

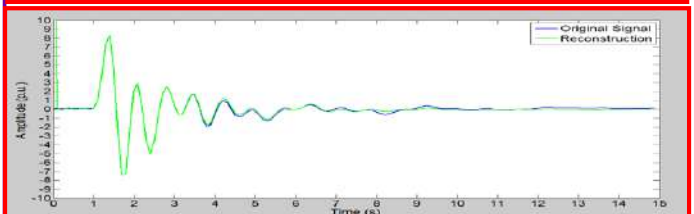


Extracted 16 modes of the system

Mode No.	Real(1/s)	Imag(rad/s)	Amp(p.u.)	Freq(Hz)	Damp(1/s)	Phase(deg)
1	-2.016	6.6011	27.06	1.0506	-2.016	-73.017
2	-2.016	-6.6011	27.06	1.0506	-2.016	73.017
3	-0.63507	-8.8173	14.328	1.4033	-0.63507	85.88
4	-0.63507	8.8173	14.328	1.4033	-0.63507	-85.88
5	-0.71158	-3.0129	9.2721	0.47952	-0.71158	76.712
6	-0.71158	3.0129	9.2721	0.47952	-0.71158	-76.712
7	-1.1491	-10.76	8.6696	1.7124	-1.1491	83.904
8	-1.1491	10.76	8.6696	1.7124	-1.1491	-83.904
9	-0.62222	12.708	1.6627	2.0226	-0.62222	-87.197
10	-0.62222	-12.708	1.6627	2.0226	-0.62222	87.197
11	-0.45059	0	1.3846	0	-0.45059	0
12	-3.7218	31.416	0.51471	5	-3.7218	-83.244
13	-2.6623	25.567	0.24426	4.0692	-2.6623	-84.055
14	-2.6623	-25.567	0.24426	4.0692	-2.6623	84.055
15	-1.4636	-20.554	0.16643	3.2713	-1.4636	85.927
16	-1.4636	20.554	0.16643	3.2713	-1.4636	-85.927



Comparison of the original input signal and the reconstructed signal using Prony algorithm (16th order model)



Comparison of the original input signal and the reconstructed signal using Prony algorithm (26th order model)

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Eigenvalues and Stability

- Real eigenvalue corresponds to non-oscillatory mode
  - negative real eigenvalue represents a decaying mode
  - positive real eigenvalue represents aperiodic instability
- Complex eigenvalues occur in conjugate pairs and each pair corresponds to an oscillatory mode
  - negative real part represents a decaying oscillation
  - positive real part represents oscillation of increasing amplitude

$$e^{\lambda t} = e^{(\sigma + j\omega)t} = e^{\sigma t} e^{j\omega t} = \boxed{e^{\sigma t}}(\cos\omega t + j\sin\omega t)$$

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Eigenvalue Sensitivity

$$\frac{\partial \lambda_i}{\partial a_{kj}} = \frac{\mathbf{w}_i \dfrac{\partial \mathbf{A}}{\partial a_{kj}} \mathbf{v}_i}{\mathbf{w}_i \mathbf{v}_i} = \mathbf{w}_i \frac{\partial \mathbf{A}}{\partial a_{kj}} \mathbf{v}_i = w_{ik} v_{ji}$$

having in mind that $\mathbf{w}_i \mathbf{v}_i = 1$ and that all elements of $\dfrac{\partial \mathbf{A}}{\partial a_{kj}}$ are zero except

the element in k-th row and j-th column which is equal to 1.

The sensitivity of the eigenvalue to the element a_{ij} of the state matrix is equal to the product of the left eigenvector element w_{ik} and the right eigenvector element v_{ji}.

Participation Factors - 1

The elements of the eigenvectors are dependent on units and scaling associated with the state variables.

$$\mathbf{P} = [\mathbf{p}_1 \ \ \mathbf{p}_2 \ \ \ldots \ \ \mathbf{p}_n] \quad \text{Participation matrix}$$

$$\mathbf{p}_i = \begin{bmatrix} p_{1i} \\ p_{2i} \\ \vdots \\ p_{ni} \end{bmatrix} = \begin{bmatrix} v_{1i} w_{i1} \\ v_{2i} w_{i2} \\ \vdots \\ v_{ni} w_{in} \end{bmatrix}$$

$$\boxed{p_{ki} = v_{ki} w_{ik} = \frac{\partial \lambda_i}{\partial a_{kk}}}$$

Participation factor, measure of the relative participation of the *k-th* state variable in the *i-th* mode, and vice versa.

Participation Factors - 2

v_{ki} The element of the *k-th* row and *i-th* column of the modal matrix **V** (*k-th* entry of the eigenvector $\mathbf{v}_i$)

w_{ik} The element of the *i-th* row and *k-th* column of the modal matrix **W** (*k-th* entry of the eigenvector $\mathbf{w}_i$)

Since v_{ki} measures the activity of x_k in the *i-th* mode and w_{ik} weighs the contribution of this activity to the mode, participation factor measures the net participation.

The sum of of the participation factors associated with any mode or with any state variable is equal to 1.

Transfer Function and Residues

$$p\Delta\mathbf{x} = \mathbf{A}\,\Delta\mathbf{x} + \mathbf{b}\,\Delta u$$
$$\Delta y = \mathbf{c}\,\Delta\mathbf{x}$$

State space representation of a single input single output system

$$G(s) = \frac{\Delta y(s)}{\Delta u(s)} = \mathbf{c}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}$$

Transfer function

$$G(s) = \sum_{i=1}^{n} \frac{R_i}{s - \lambda_i}$$

Partial fraction expansion of transfer function

$$R_i = \mathbf{c}\mathbf{v}_i\mathbf{w}_i\mathbf{b}$$

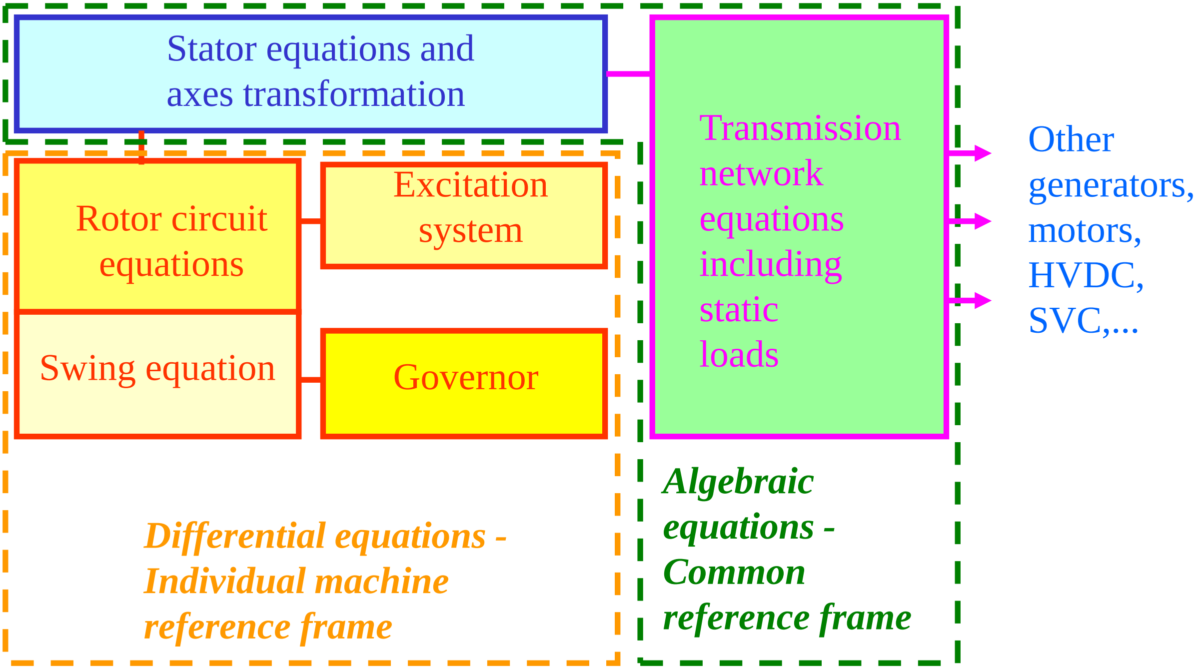
Residue of transfer function at pole λ_i

Large System Applications

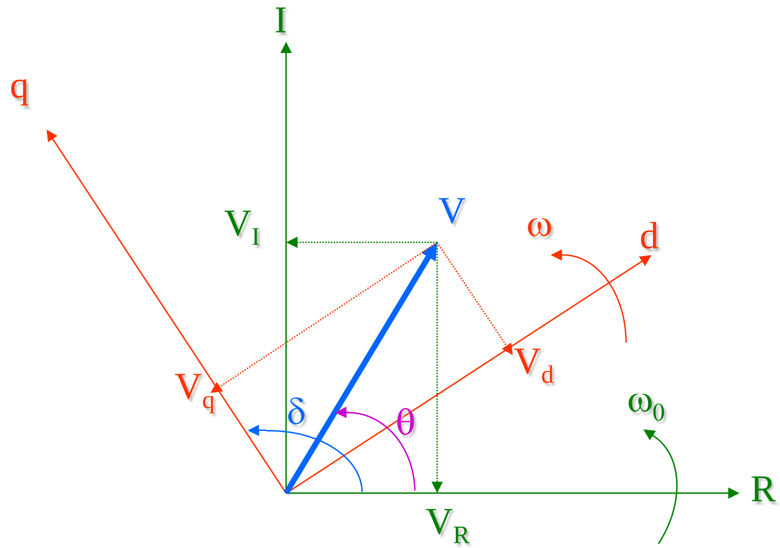
Modelling Requirements

- Simultaneous solution of equations representing
  - Synchronous machines
  - Excitation systems and prime movers
  - Interconnecting transmission network
  - Static and dynamic system loads
  - Other transmission system devices (HVDC, SVC, etc.)
- All equations are linearised about an operating point and all variables except the state variables are eliminated.

Structure of the Model



Reference Frame Transformations



$$V_d = V_R \sin\delta - V_I \cos\delta$$
$$V_q = V_I \sin\delta + V_R \cos\delta$$

$$V_R = V_d \sin\delta + V_q \cos\delta$$
$$V_I = V_q \sin\delta - V_d \cos\delta$$

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Formulation of State Equations - 1

$$p\mathbf{x}_i = \mathbf{A}_i\,\mathbf{x}_i + \mathbf{B}_i\,\Delta\mathbf{v}_i$$
$$\Delta\mathbf{i}_i = \mathbf{C}_i\,\mathbf{x}_i - \mathbf{Y}_i\,\Delta\mathbf{v}_i$$

Linearised model of each dynamic device.

$$p\mathbf{x} = \mathbf{A}_D\,\mathbf{x} + \mathbf{B}_D\,\Delta\mathbf{v}$$
$$\Delta\mathbf{i} = \mathbf{C}_D\,\mathbf{x} - \mathbf{Y}_D c$$

Combined state equations for all the dynamic devices in the system.

$$\Delta\mathbf{i} = \mathbf{Y}_N\ \Delta\mathbf{v}$$

Interconnecting transmission network.

$$p\mathbf{x} = \mathbf{A}_D\,\mathbf{x} + \mathbf{B}_D(\mathbf{Y}_N + \mathbf{Y}_D)^{-1}\mathbf{C}_D\,\mathbf{x}$$

$$\boxed{\mathbf{A} = \mathbf{A}_D + \mathbf{B}_D(\mathbf{Y}_N + \mathbf{Y}_D)^{-1}\mathbf{C}_D}$$

System state matrix (complete system)

Formulation of State Equations - 2

$\mathbf{x}_i$ - perturbed values of the individual device state variables

i_i - current injection into the network from the device (has real and imaginary component)

$\mathbf{v}_i$ - the vector of the network bus voltages (has two elements per bus associated with the device)

$\mathbf{B}_i$ & $\mathbf{Y}_i$ have non-zero elements corresponding only to the terminal voltage of the device and any remote bus voltages used to control the device.

$\mathbf{x}$ - state vector of the complete system

$\mathbf{A}_D$ & $\mathbf{C}_D$ are block diagonal matrices composed of A_i and C_i associated with the individual devices.

$\mathbf{Y}_N$ - network admittance matrix that includes the effects of non-linear static loads

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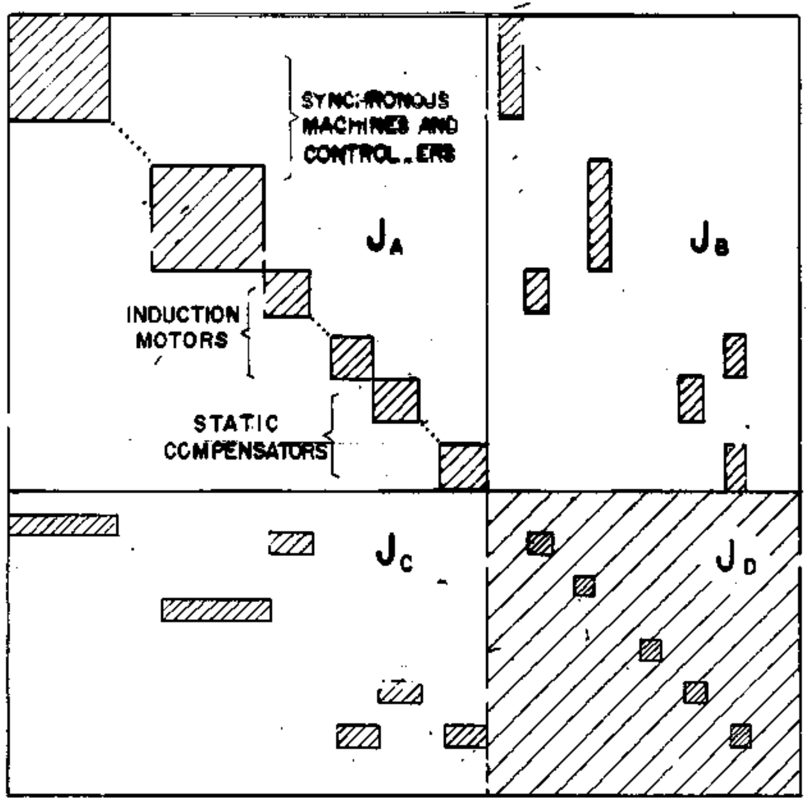
Matrix Formulations for Linear Systems

	Time Domain	*s*-Domain
State Space:	$\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u}$ $\mathbf{y} = \mathbf{C}\,\mathbf{x} + \mathbf{D}\,\mathbf{u}$	$(s\mathbf{I} - \mathbf{A})\,\mathbf{x} = \mathbf{B}\,\mathbf{u}$ $\mathbf{y} = \mathbf{C}\,\mathbf{x} + \mathbf{D}\,\mathbf{u}$
Descriptor System:	$\mathbf{T}\cdot\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u}$ $\mathbf{y} = \mathbf{C}\,\mathbf{x} + \mathbf{D}\,\mathbf{u}$	$(s\mathbf{T} - \mathbf{A})\cdot\mathbf{x} = \mathbf{B}\,\mathbf{u}$ $\mathbf{y} = \mathbf{C}\,\mathbf{x} + \mathbf{D}\,\mathbf{u}$
s-Domain Model:	Closed-form equations not available for infinite systems	$\mathbf{Y}(s)\cdot\mathbf{x} = \mathbf{B}(s)\,\mathbf{u}$ $\mathbf{y} = \mathbf{C}(s)\,\mathbf{x} + \mathbf{D}(s)\,\mathbf{u}$

Characteristics of Power System Models

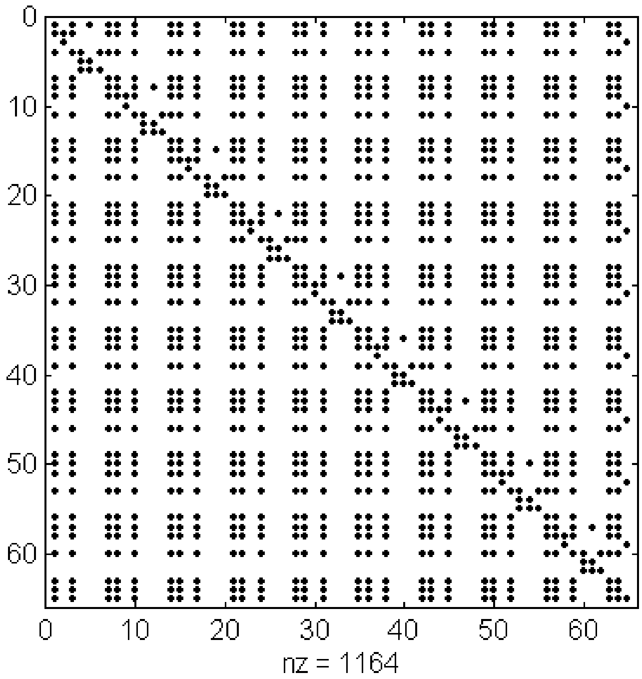
- State space modelling produces dense matrices in some applications (e.g., electromechanical stability)
- Descriptor system modeling (retains states and algebraic variables) produces very sparse matrices
- *s*-Domain models (they allow the modeling of infinite systems, such as long lines)
- Partial eigensolution algorithms for large descriptor system and *s*-Domain models are now very powerful

Power System Jacobian Matrix



State Matrix for the for the 39-bus, 10-generators system

Non-Sparse



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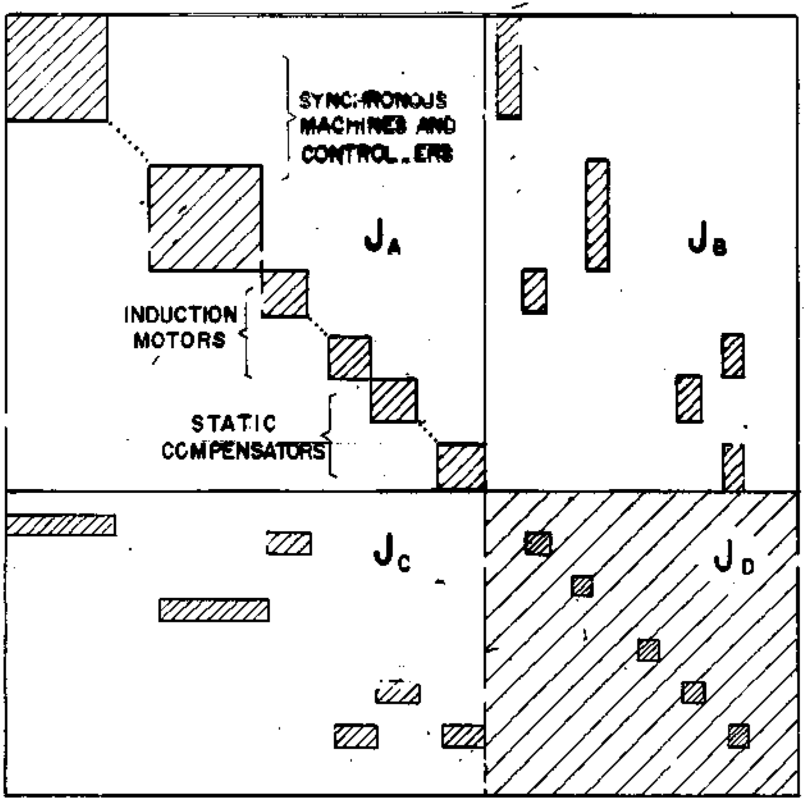

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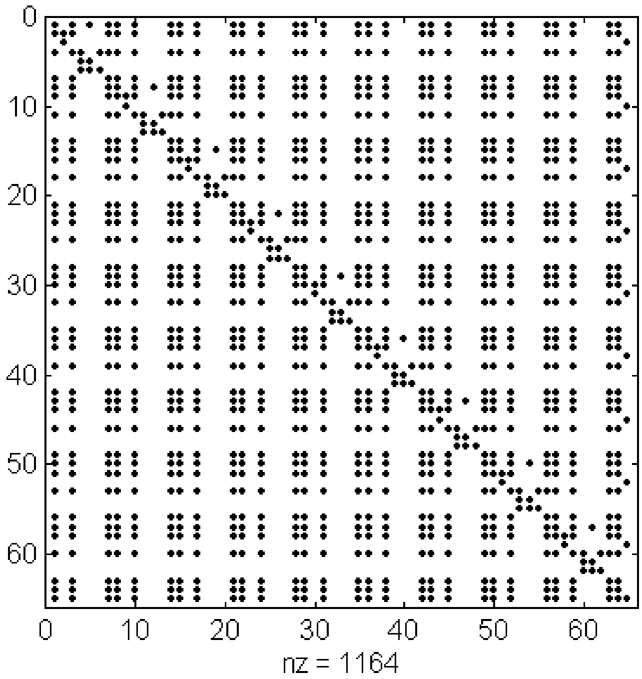
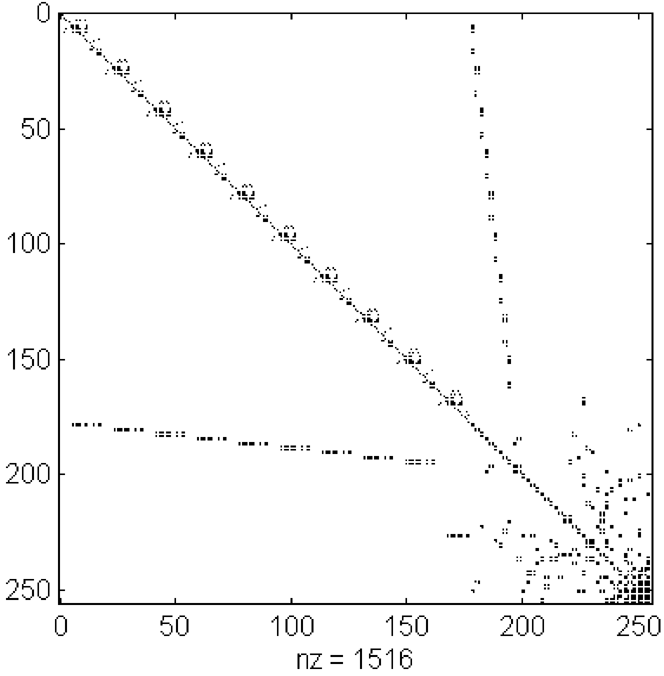


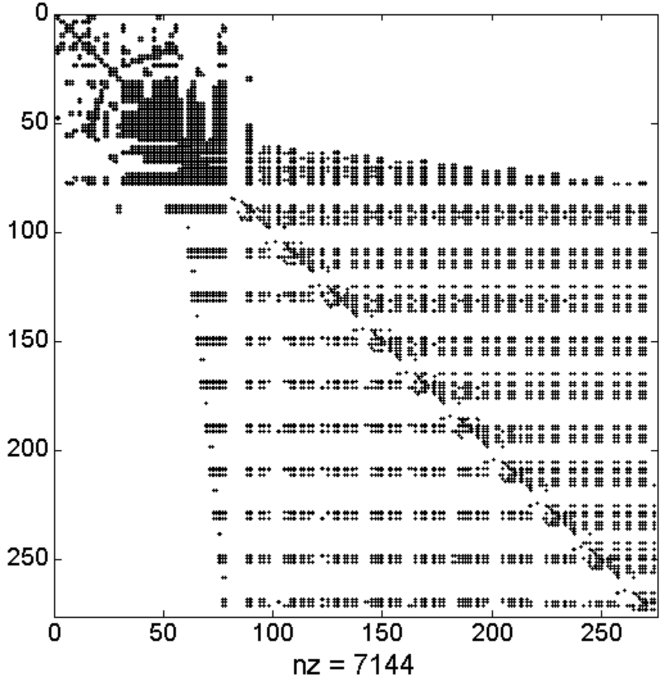
LU Factors Topology Map for the 39-bus, 10-generators system Jacobian Matrix

Very sparse for good ordering



LU Factors Topology Map for the 39-bus, 10-generators system Jacobian Matrix

Not sparse for bad ordering



Computational Techniques

- Systems with as many as 2000+ dynamic devices (15 states per device) and 12000 buses are not uncommon for studies of system (inter-area) electromechanical oscillations.
- Special techniques have been developed for dealing with such big systems
  - AESOPS algorithm (frequency response approach to calculates eigenvalues associated with the rotor angle modes)
  - Sparsity based eigenvalue techniques
    - ➢ Simultaneous iterrations
    - ➢ Modified Arnoldi method
    - ➢ Selective Modal Analysis (identifies variables that are relevant to the selected modes, and then constructs a reduced-order model that involves only the relevant variables.)

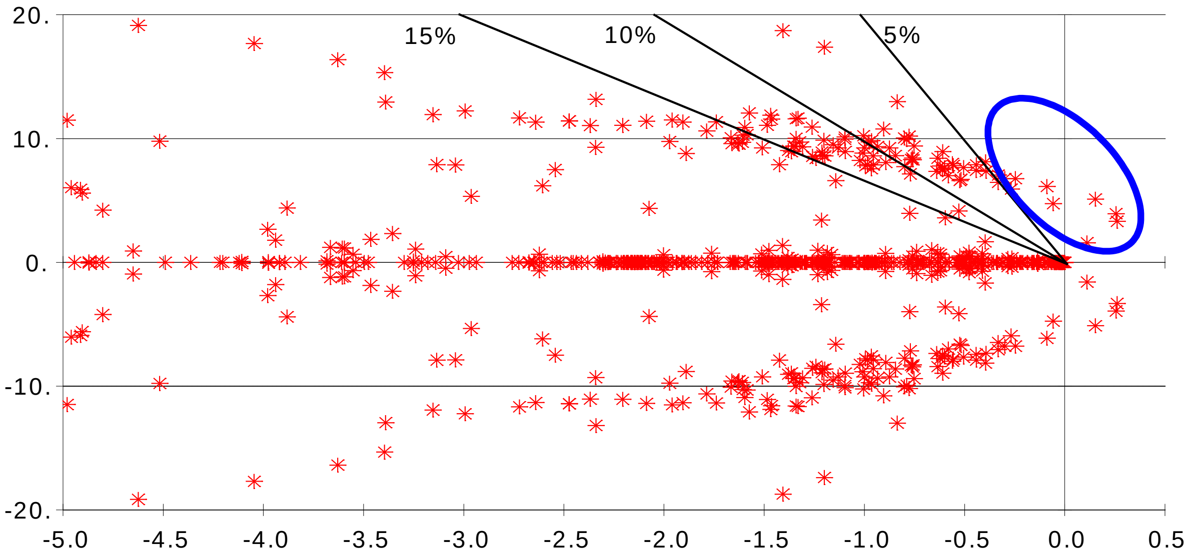
Characteristic Studies - Local

- Study of local modes (associated with rotor angle oscillations of a single generator or a single plant against the rest of the system) f ≈ 1Hz (0.7(1.0)Hz - 2.5(4)Hz)
- Study of intermachine or interplant modes (oscillations between the rotors of a few generators close to each other) f ≈ 1Hz (0.7(1.0)Hz - 2.5(4)Hz)
- Study of control modes (associated with AVR, HVDC converters, SVCs) f ≈ <0.1(0.3)Hz
- Study of torsional modes (interaction between controls and turbine-generator shaft system) f ≈ 5Hz - 50Hz

Characteristic Studies - Global

- Study of inter-area modes
  - associated with rotor angle oscillations of groups of generators swinging against each other, $f \approx 0.4Hz - 0.8Hz)$
  - associated with rotor angle oscillations of all generators in the system when one half of the system generators is swinging against the other half, $f \approx 0.1Hz - 0.3Hz$

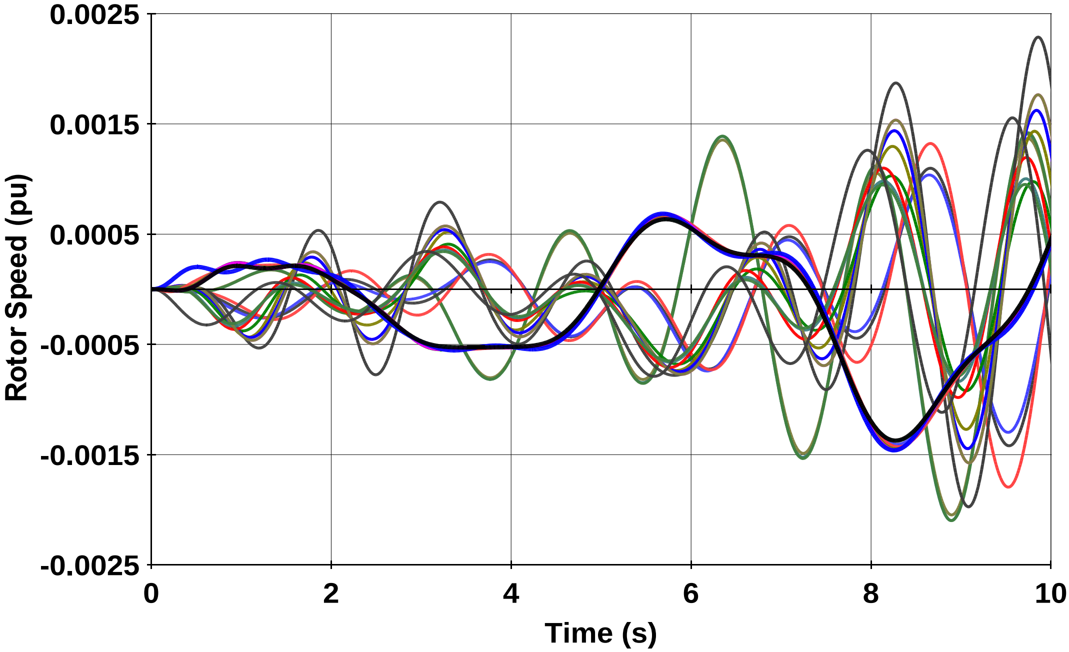
Eigenvalue spectrum for large system without controllers



Jacobian Matrix
13,063 lines
48,626 non-zeros
1,676 states

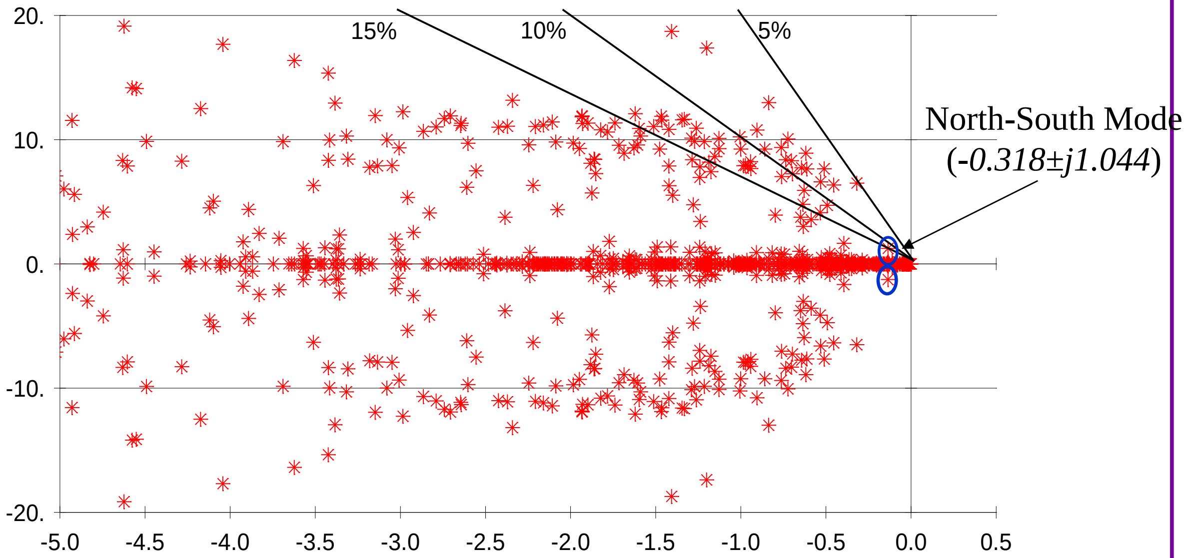
Various unstable or poorly damped modes!

Time response for large system without controllers



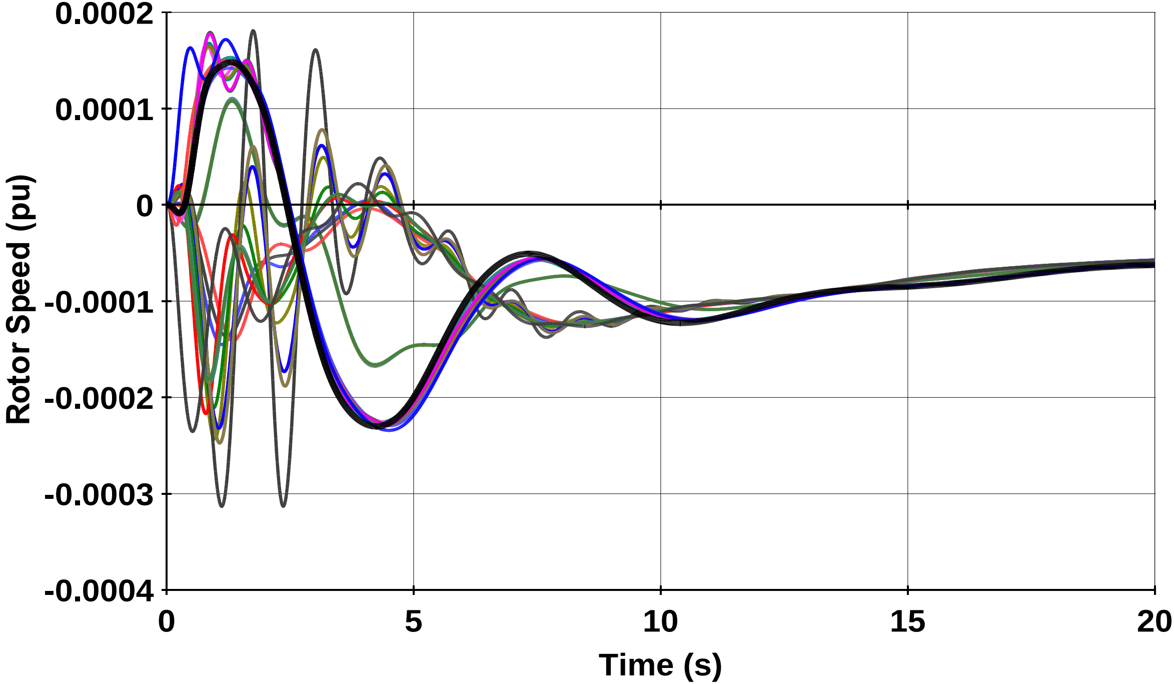
Unstable Oscillations

Eigenvalue spectrum for large system with controllers



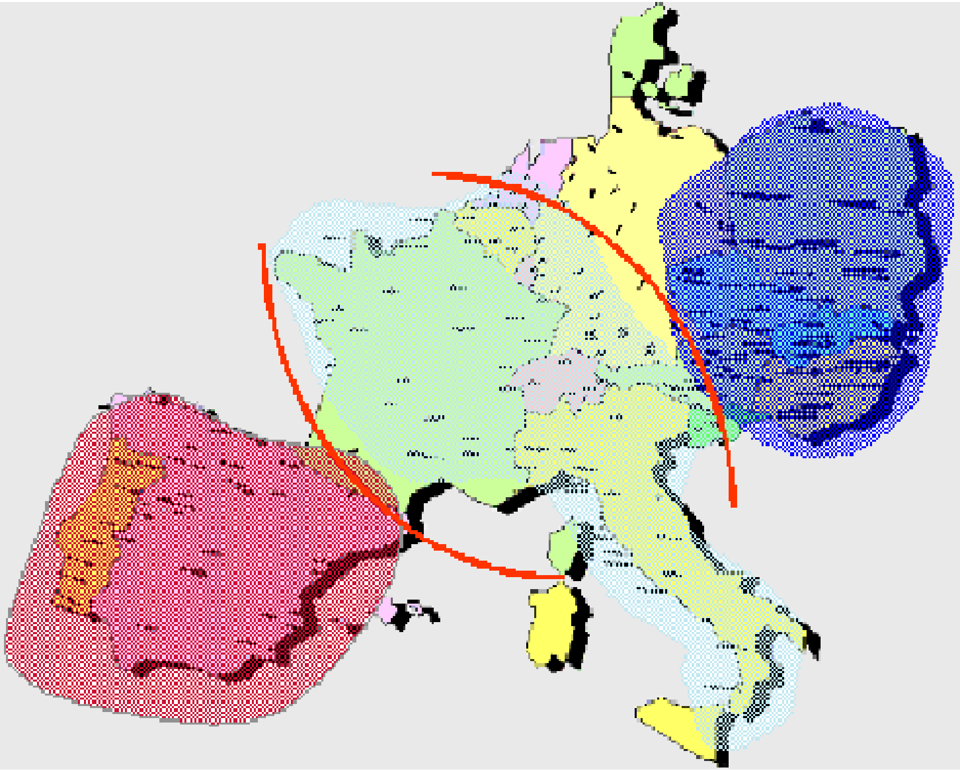
The two TCSCs of the North-South Intertie have PODs to confer damping to the N-S mode. System has 1,676 states and shows well- damped response.

Time response for large system with controllers



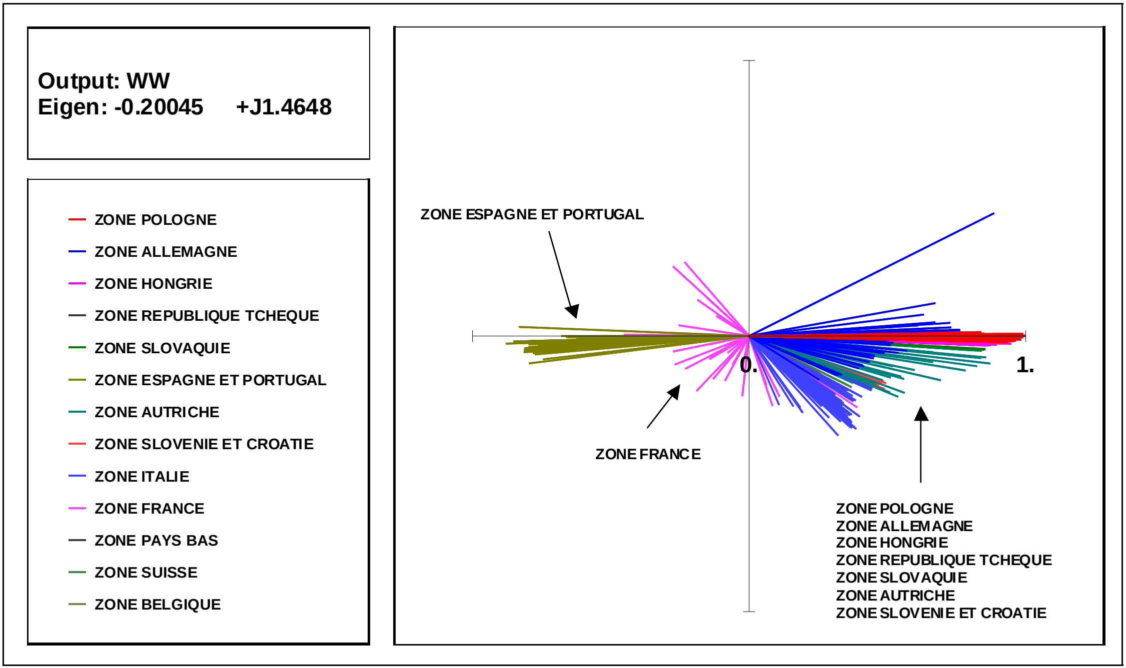
System with 1,676 states, well-damped oscillations

UCPTE-CENTREL Study (1/4)



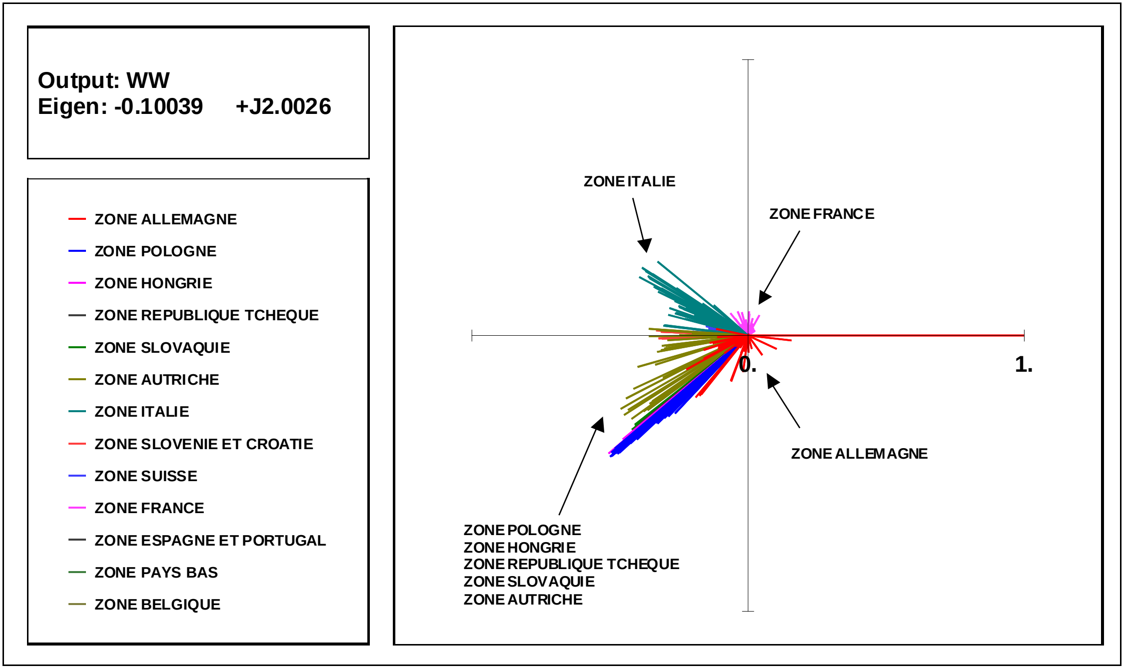
Geo Mode-Shape from NETOMAC (Siemens)

UCPTE-CENTREL Study (2/4)



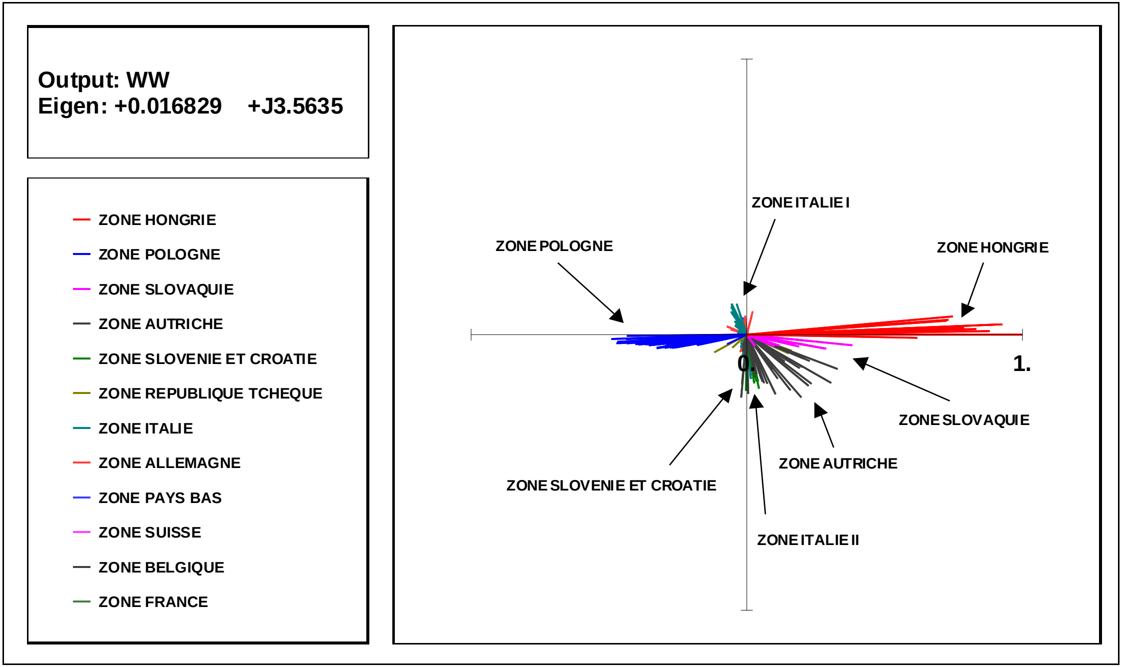
Rotor Speed Shape for Mode 1 ($-0.200 \pm 1.465j$) = 0.23 Hz

UCPTE-CENTREL Study (3/4)



Rotor Speed Shape for Mode 2 ($-0.100 \pm 2.002j$) = 0.32 Hz

UCPTE-CENTREL Study (4/4)



Rotor Speed Shape for Mode 3 ($+0.017 \pm 3.563j$) = 0.57Hz

Inter-area Oscillations

- Influenced strongly by *load* characteristics.
- Influence of excitation system dependent on type and location of the exciters and *load* characteristics.
- Speed-governors do not influence them generally however, they may reduce damping slightly if not properly tuned. (In extreme situations blocking off the governor may provide some relief.)
- Controllability of these modes with PSS is not straightforward. It depends on location of PSS, location and characteristics of *loads*, types of exciters.
- HVDC converter controls and SVC controls are effective in stabilising these modes

Currently run large system studies

System	Load MW	Generation MW	Buses	Generators	State Variables	Jacobian Matrix Dimension
NORDEL	61,000	62,000	4,780	1,100	**11,800**	**66,000**
CAMMESA	12,800	24,000	1,200	160	**1,800**	
UCPTE	160,000		1,900	380	**2,800**	**13,300**
East-Coast			12000	1,500	**8,200**	**71,000**
Peru	3,120	3,300		77 plants	**1,000**	
Latin American South-Cone	22,000	23,500	2,200	230	**2,800**	**17,000**
Brazilian System	50,000	60,000	2,400	123	**1,700**	**13,000**

Chapter 5:
Large-Disturbance (Transient) Stability

Content

- Modelling requirements
- Mathematical tools for the analysis
- Small system applications
- Large system applications
- Sub-synchronous resonance in power systems

Modelling Requirements

Characteristics of Transient Stability

- Generator rotor angle change significantly.
- Non-linear equations are used.
- Post-disturbance operating state is usually very different from the pre-disturbance operating state.
- First swing stability (1-2s) may be sufficient in some cases whereas in other cases, damper and stabiliser responses may be more important

(10-15s).

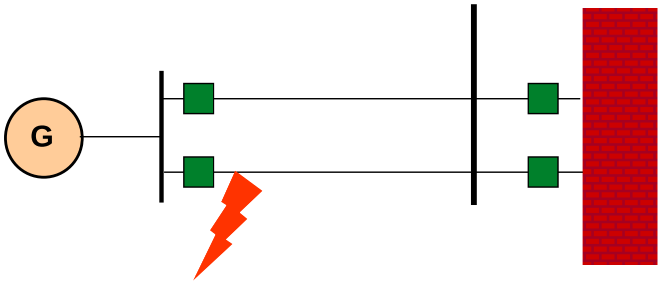
Causes of Transient Stability Problems

- Power system faults
- Synchronisation
- Asynchronous operation and resynchronisation

Essential Requirements

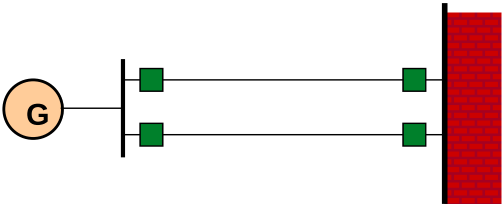
- How much detail is required?
- What time period is of interest?
  - Equations with small time constants may be neglected as the effects disappear quickly.
  - Equations with long time constants may be neglected as changes over that period of time are not of interest.
  - The final model should be verified for sufficient accuracy by comparison with real data or more detailed model.
- How much computer power is available?
- How quickly should a result be found?

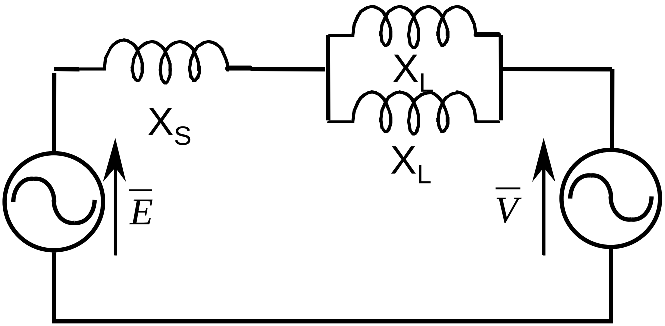
Transient Stability – Example: One Machine/Infinite Bus System



Fault close to the generator bus

Before the Fault

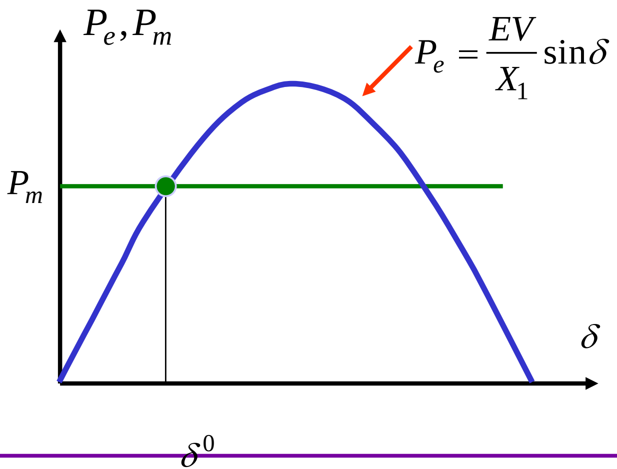




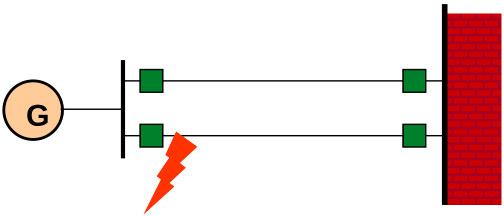
Steady state operation with:

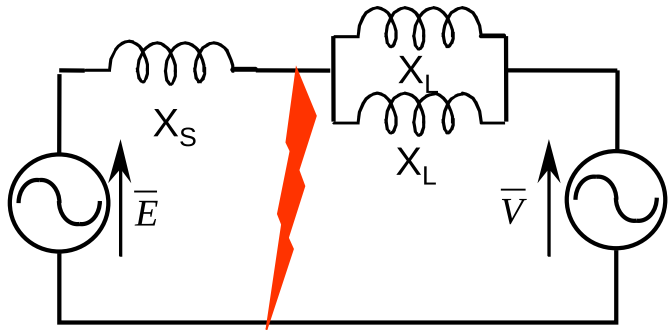
$$P_e = \frac{EV}{X_1}\sin\delta$$

$$X_1 = X_S + \tfrac{1}{2} X_L$$



During the Fault

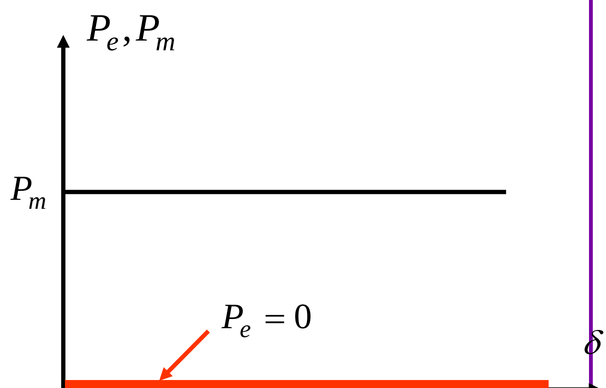




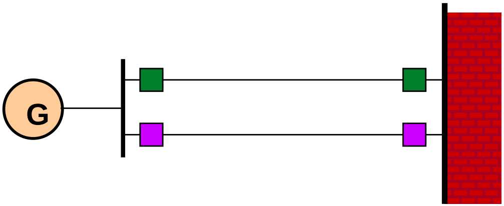
Fault at terminal of the generator

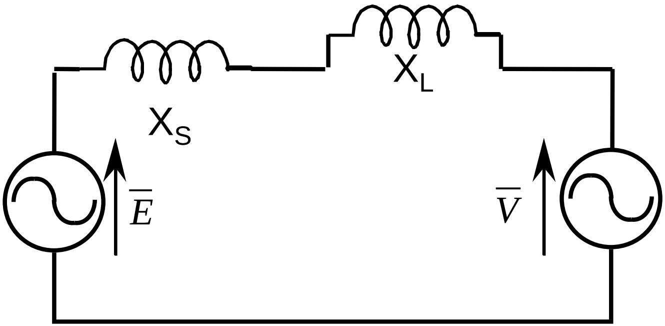
- Terminal voltage of generator = 0
- $P_e = 0$

(because $P_e = \dfrac{EV}{X}\sin\delta$ applies between any two voltage sources)



After the Fault



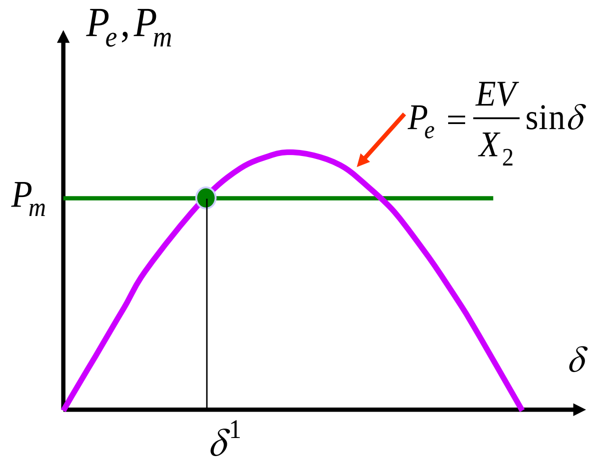


Line disconnected to clear the fault

- Assume that E is constant
- δ^1 the steady state operating point
- Can we reach that steady state?

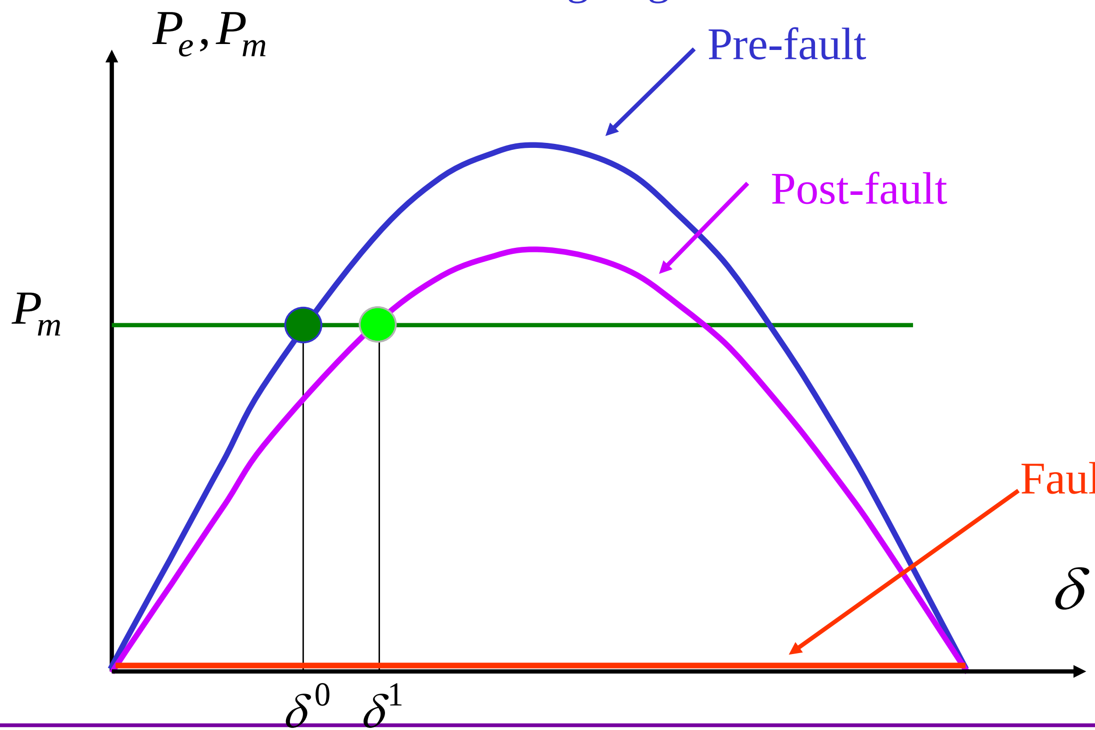
$$P_e = \frac{EV}{X_2}\sin\delta$$

$$X_2 = X_S + X_L > X_1$$



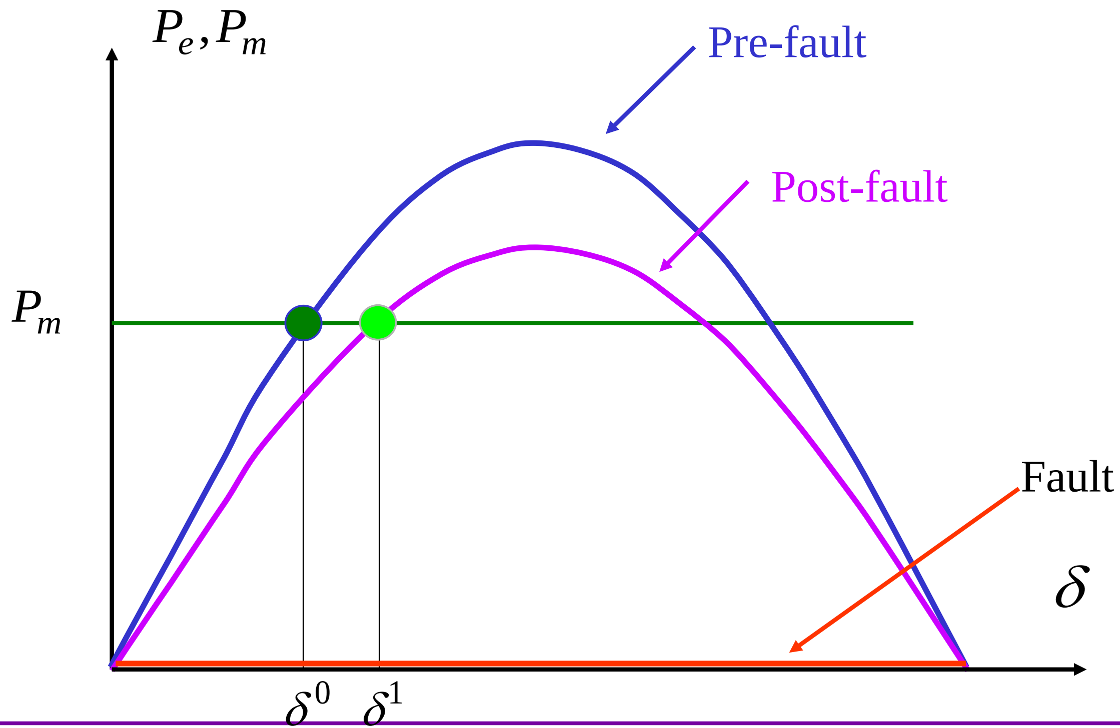
Evolution of the State of the System

Can we get from one steady state operating point to another without going unstable?



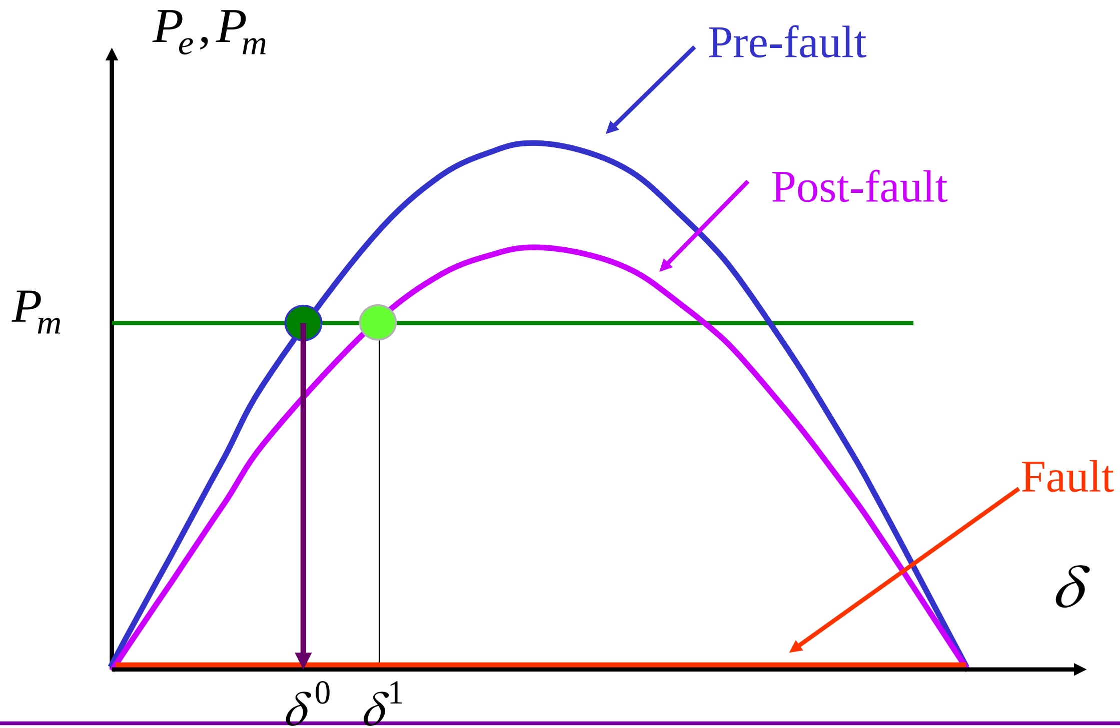
Step 1: Pre-fault steady state operating point

System operates at $P_e = P_m$ and $\delta = \delta^0$



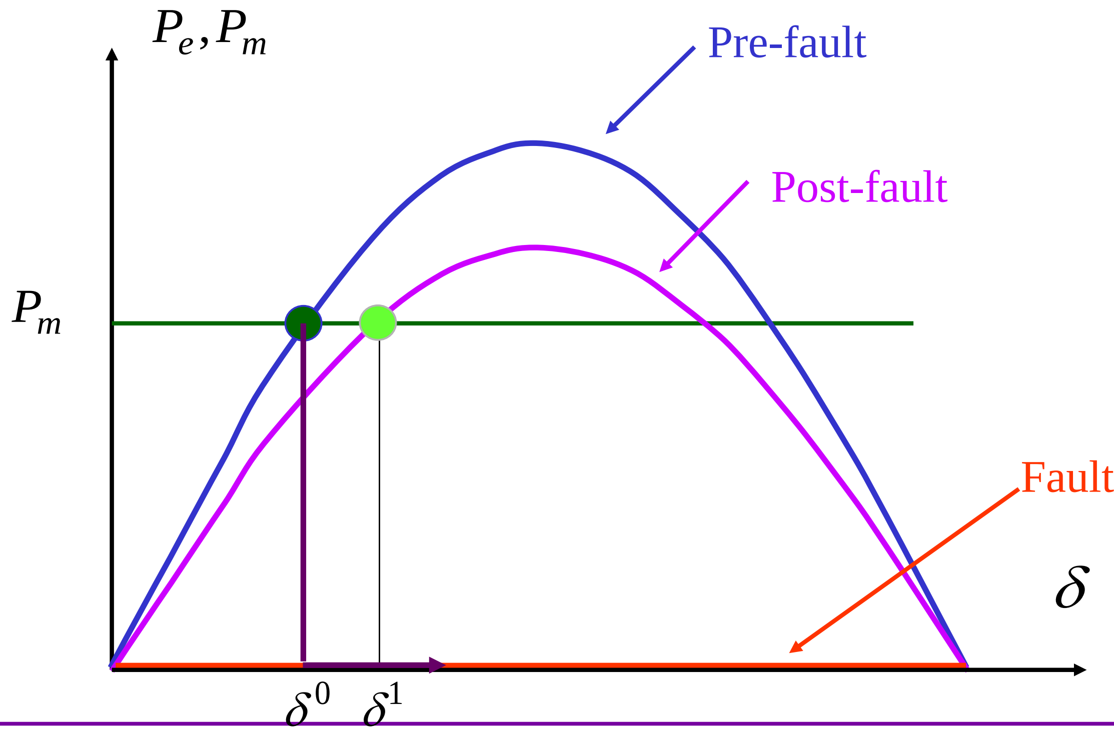
Step 2: Occurrence of the Fault

P_e drops to zero because voltage is zero



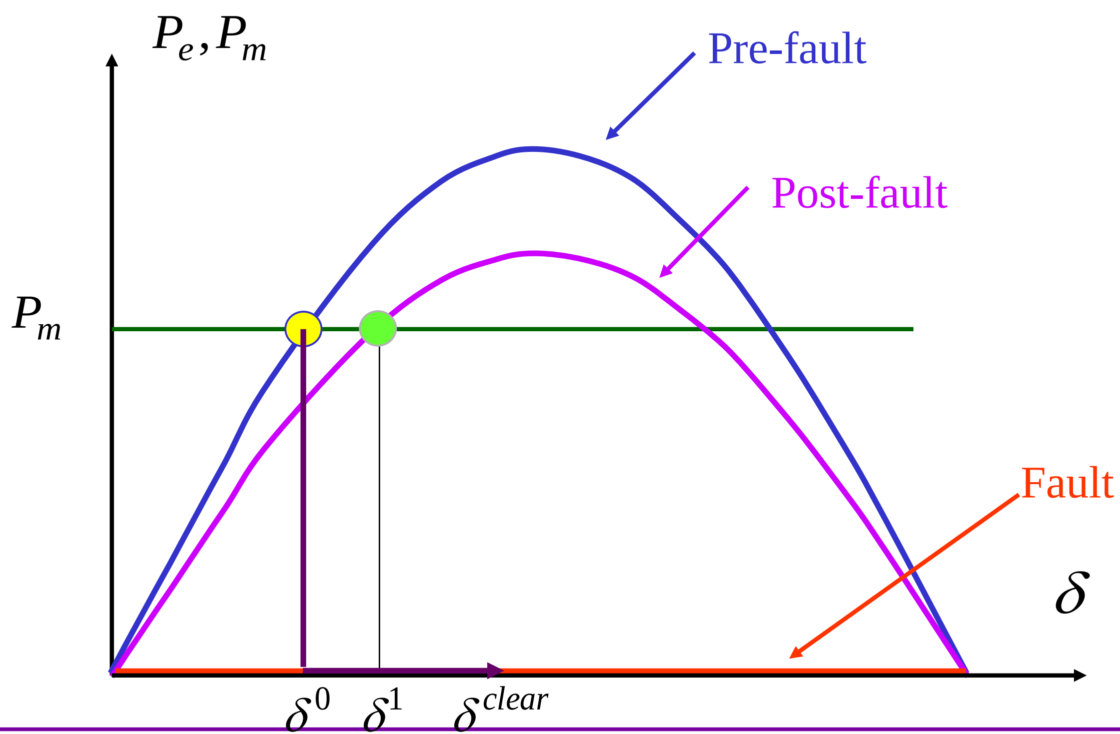
Step 3: During the fault

δ increases because $P_m > P_e = 0$



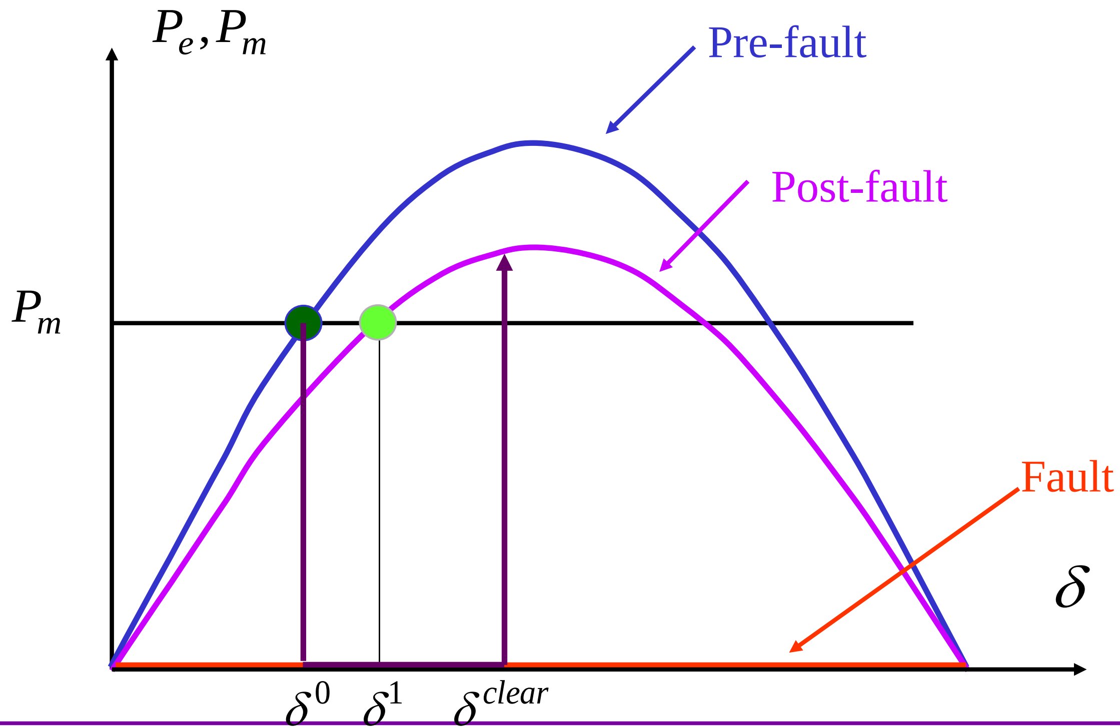
Step 4: Protection System Operates

Let δ^{clear} be the value of δ when this happens



Step 5: Fault Cleared

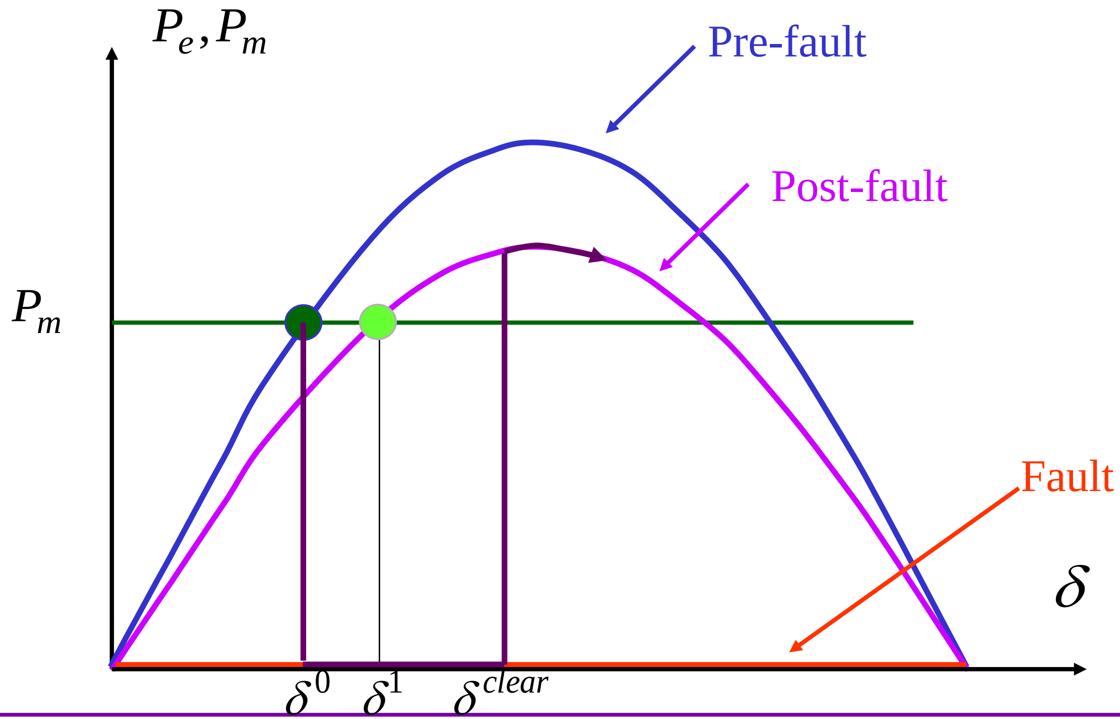
Value of P_e is given by post-fault power angle curve



Step 6: δ continues to increase

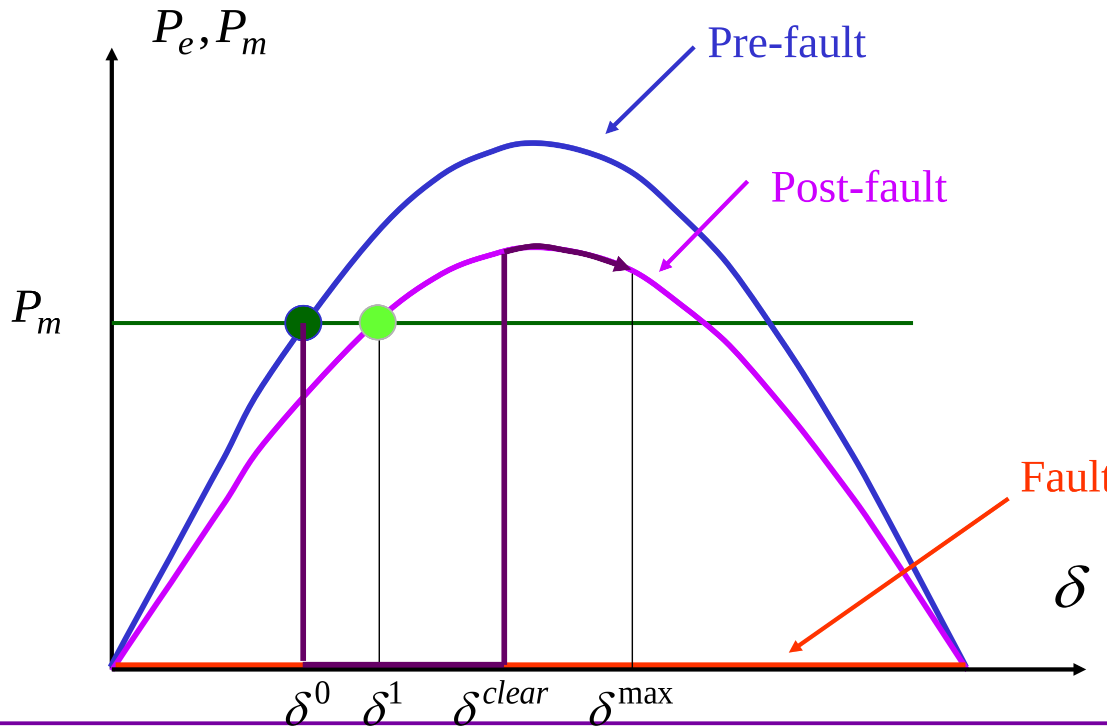
δ increases because of momentum gained during the fault.

Since $P_e > P_m$ the rate of increase progressively slows down



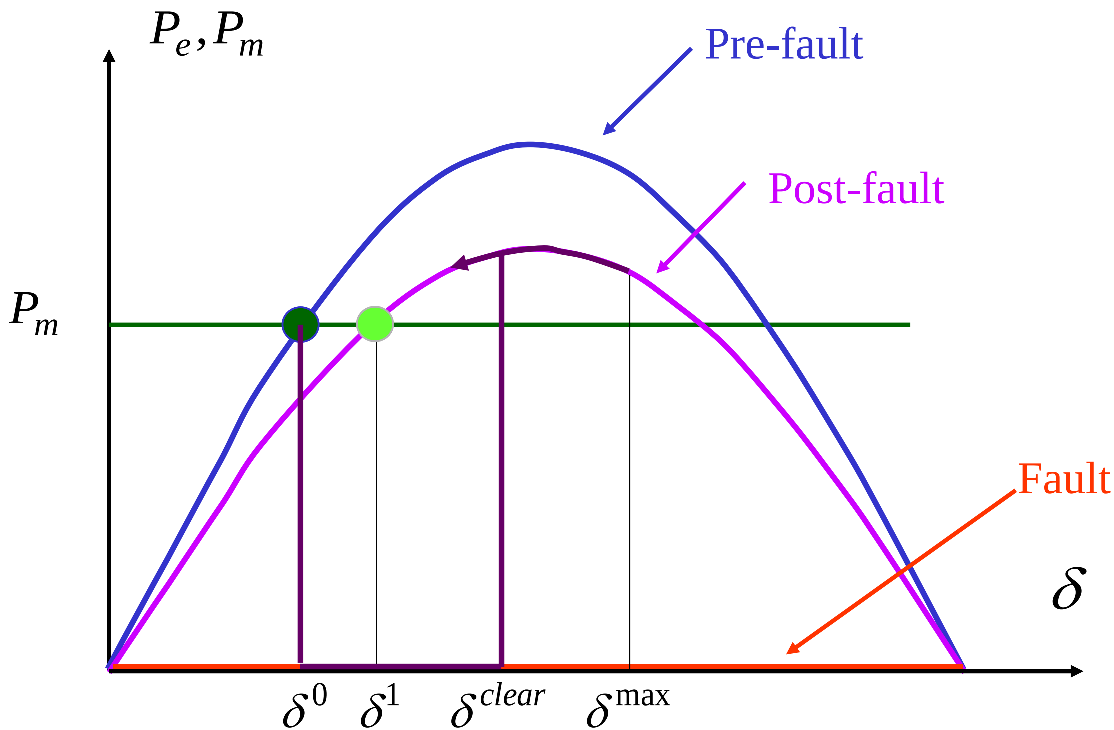
Step 7: δ reaches its maximum value

δ reaches its maximum value when the momentum gained during the fault is exhausted



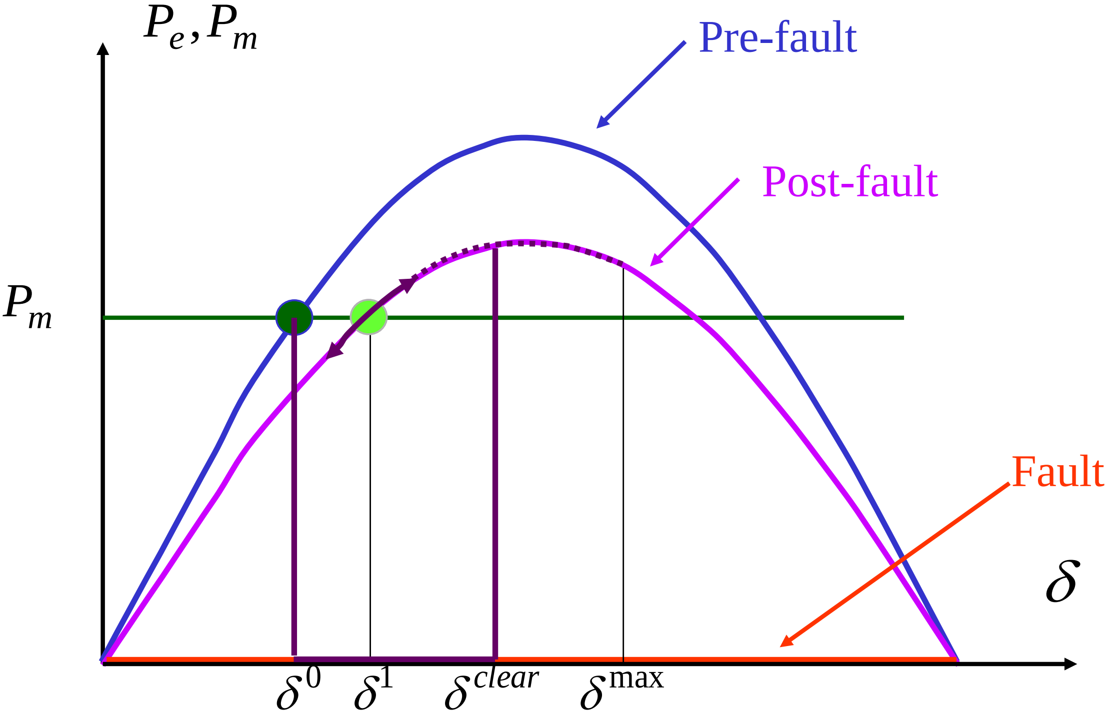
Step 8: δ begins to decrease

Since $P_e > P_m$ δ begins to decrease

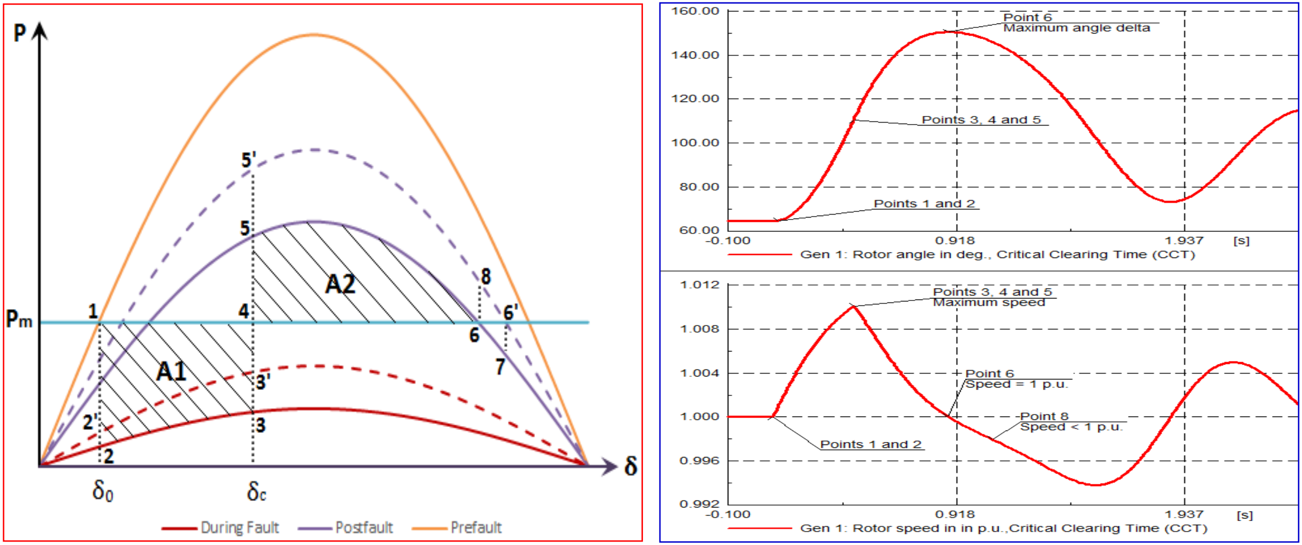


Step 9: δ converges towards δ^1

δ oscillates around δ^1 until damping makes it settle at that value
(note: for simplicity's sake, our model ignores damping)



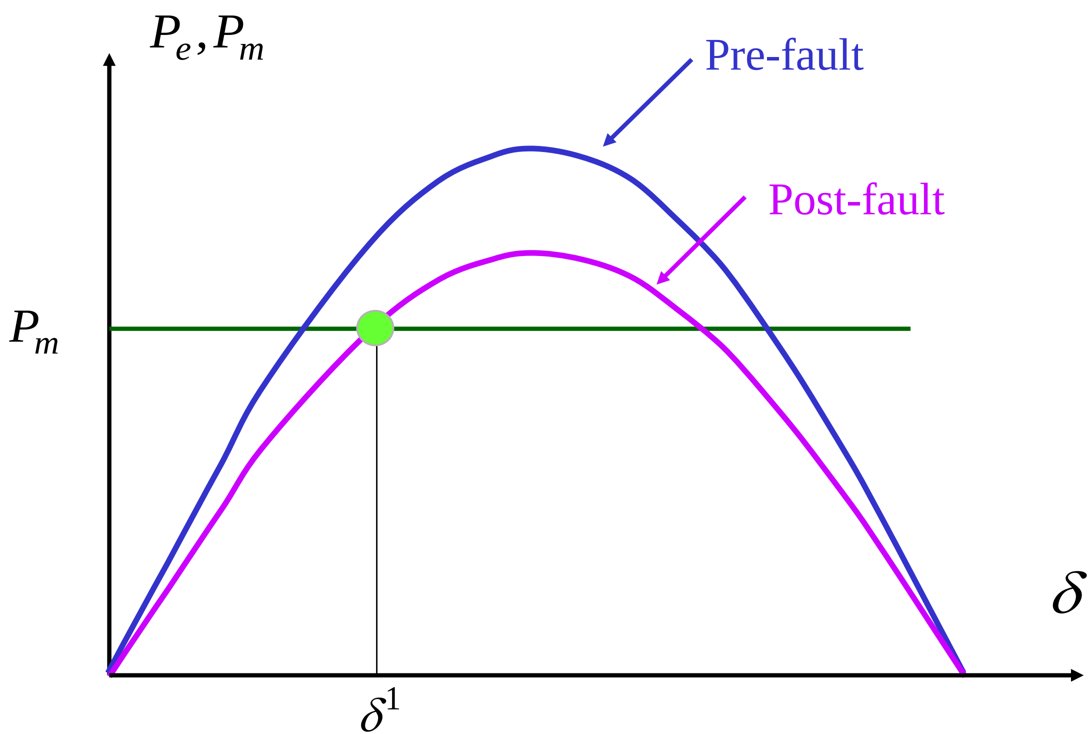
Rotor angle and speed responses



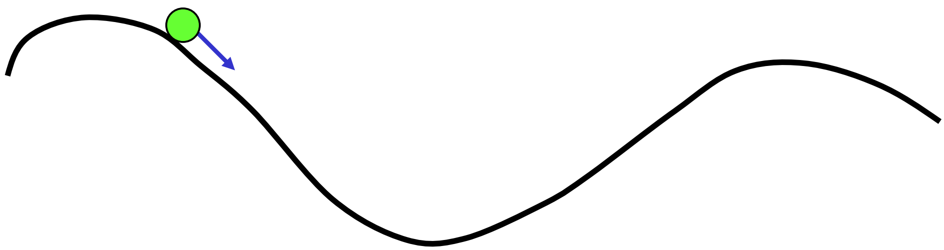
Transient stability assessment: (a) power-angle curve ; (b) rotor angle and rotor speed versus time

Step 10: New Steady State Operating Point

The system is stable

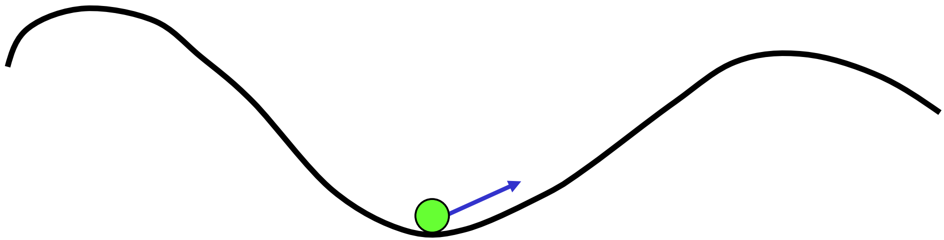


Marble Analogy - 1



Marble released. Accelerates because subject to gravity.

Marble Analogy

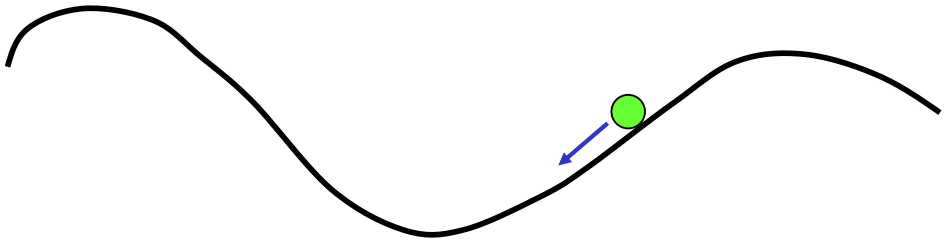


Marble reaches bottom

Acceleration becomes negative because it goes uphill

Marble does not stop because it has acquired momentum

Marble Analogy - 3

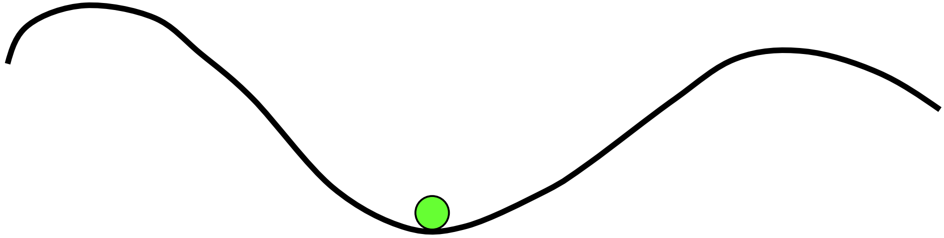


Marble has exhausted its momentum

Speed is zero

Marble starts accelerating downwards because of gravity

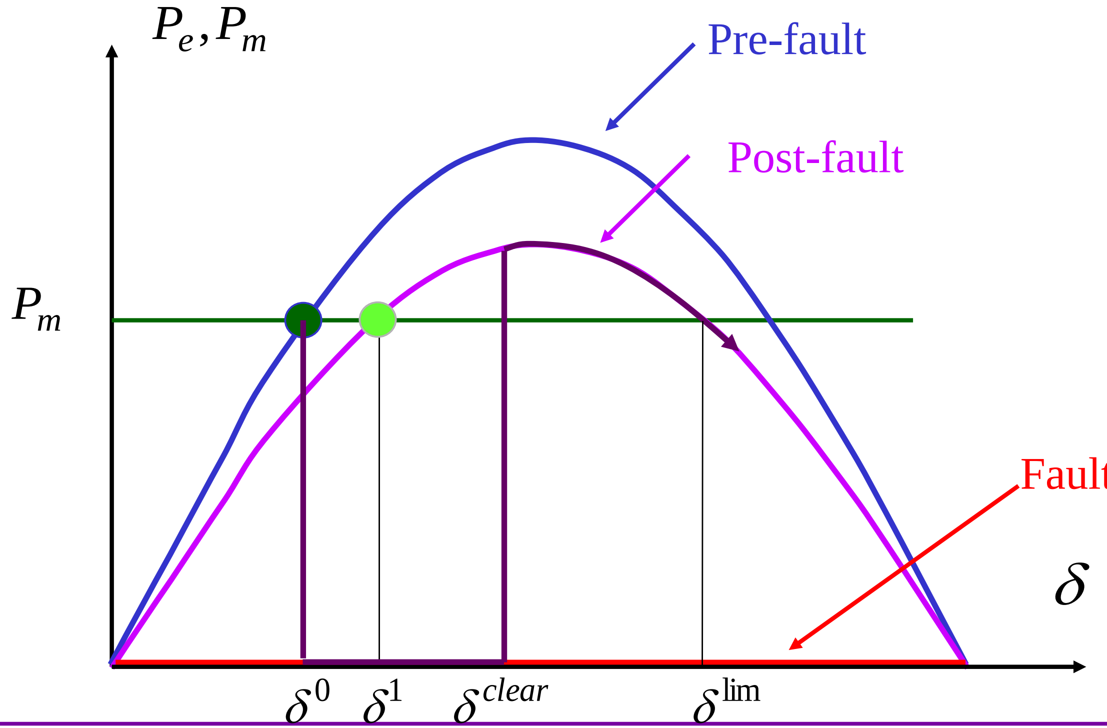
Marble Analogy - 4



Marble settles at the bottom when friction (damping) has consumed all the kinetic energy acquired during the initial downhill

Step 7.1: δ increase beyond $\delta^{\lim}$

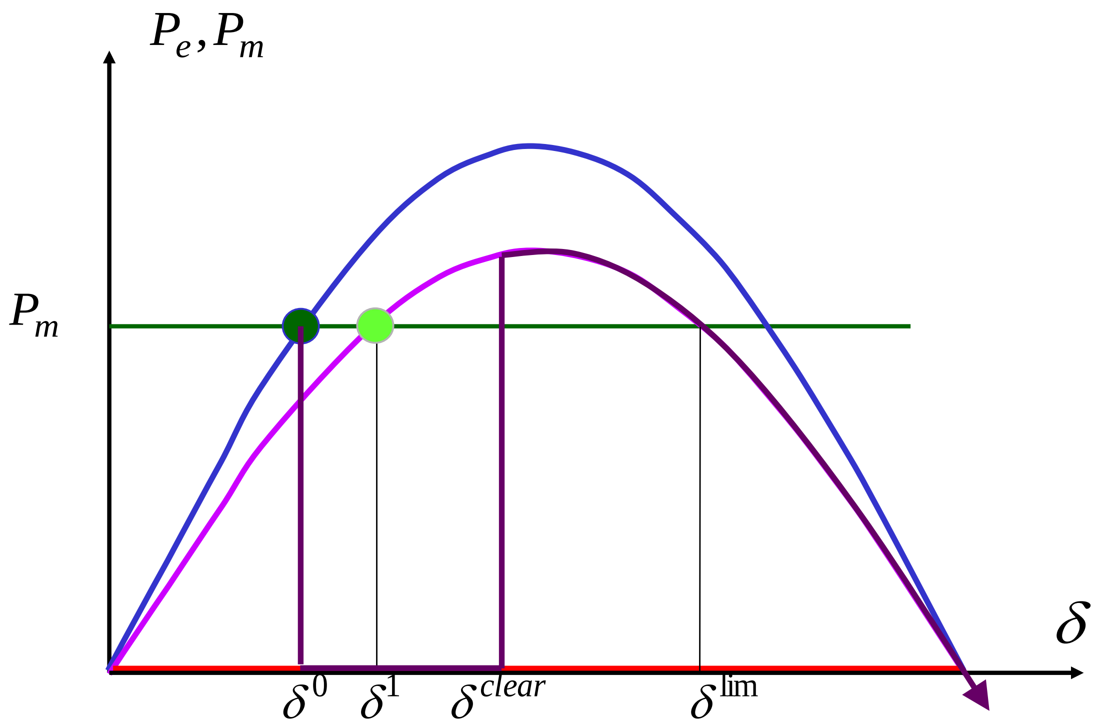
As soon as δ goes beyond δ^{lim}, we have $P_m > P_e$
and δ starts accelerating again



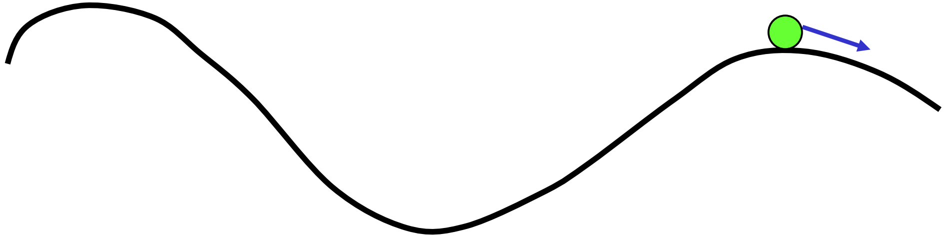
Step 8.1: The System Goes Unstable

Since $P_m > P_e$, δ increase without limit: synchronism is lost

(In practice, the generator protection system operates to prevent damage)



Marble Analogy



Marble still has some momentum when it reaches the top of the next hill
Marble starts accelerating again and disappears into never-never land...

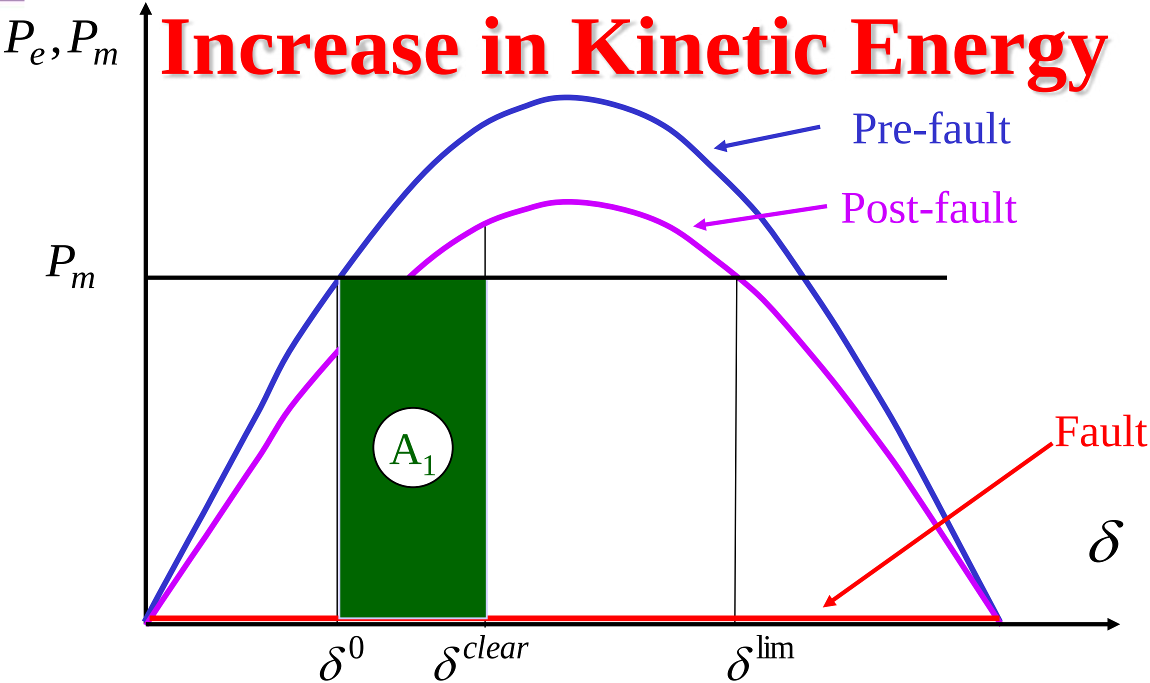
Transient Power System Stability?

- How can we check beforehand whether the power system will remain stable following a fault?
- System is stable if δ never exceeds δ^{lim}, i.e.:

$$\boxed{\delta^{max} < \delta^{lim}}$$

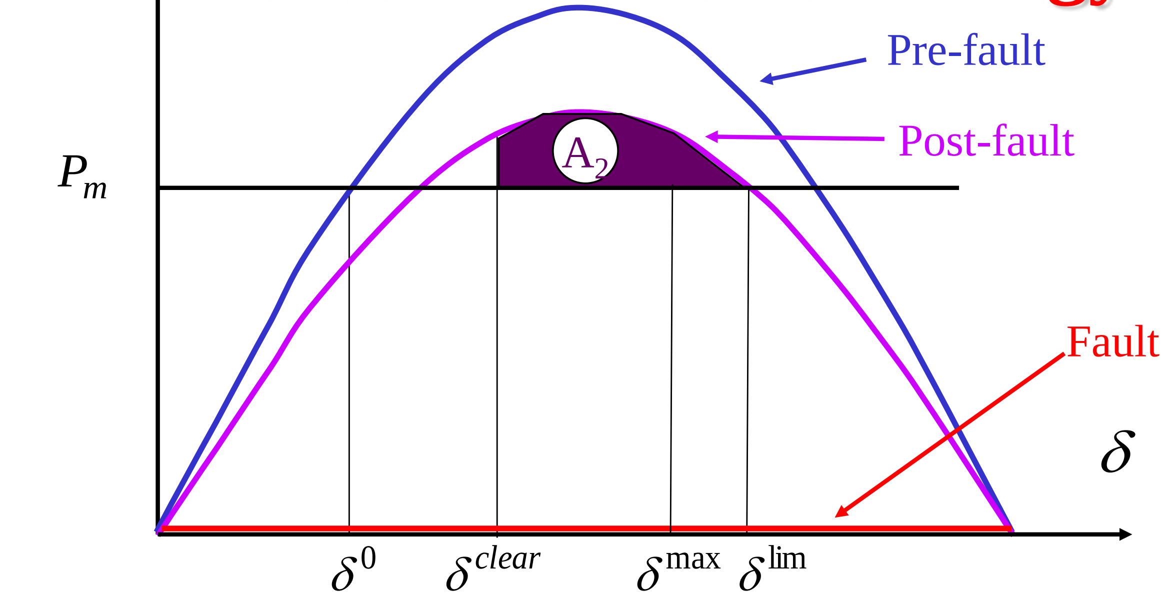
- How can we translate that into a practical stability criterion? (i.e. relating to something we can control)
- Equal area criterion is a simple criterion based on the concept of stored kinetic energy

Increase in Kinetic Energy



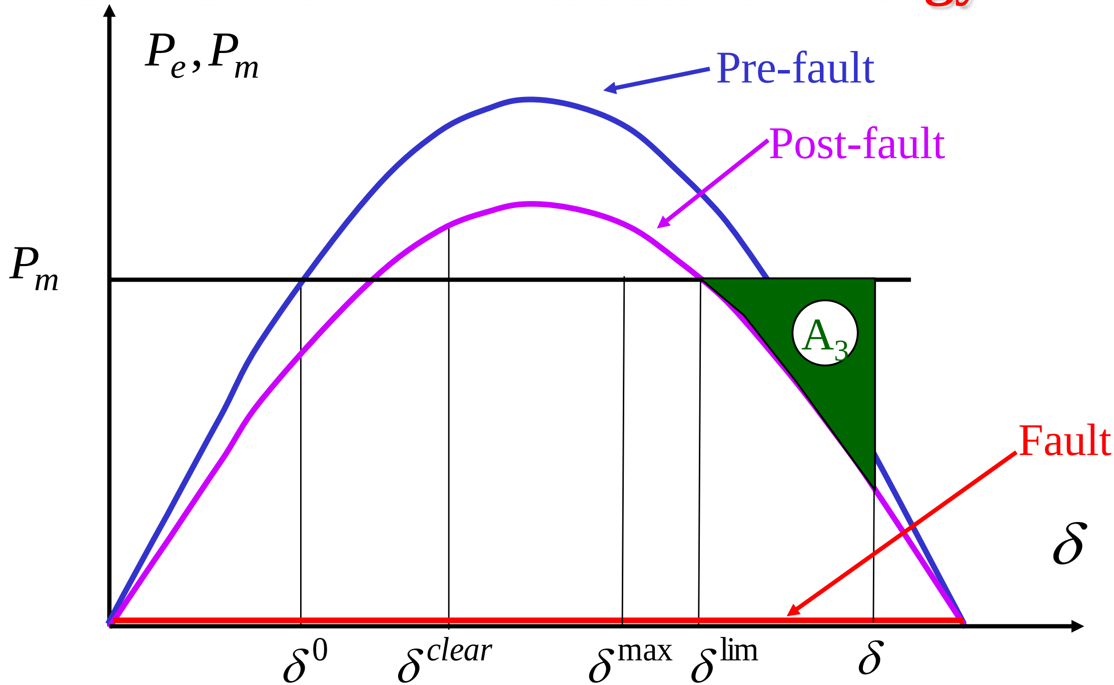
- For $\delta^0 < \delta < \delta^{\text{clear}}$, we have $P_m > P_e$
  - Turbine-generator set accelerates
  - Increase in the stored kinetic energy - area A_1
  - If δ^{clear} increases, the stored kinetic energy increases

Decrease in Kinetic Energy



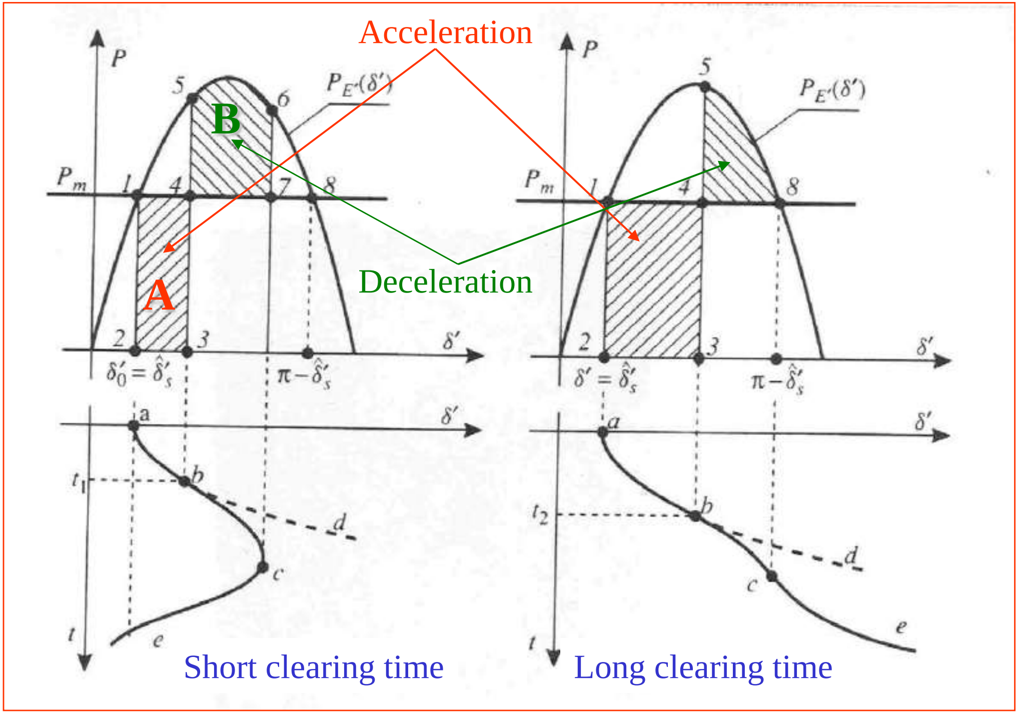
- For $\delta^{\text{clear}} < \delta < \delta^{\lim}$, we have $P_e > P_m$
  - Turbine-generator set decelerates
  - Decrease in the stored kinetic energy - area A_2
  - If δ^{clear} increases, the decrease in kinetic energy is smaller

Further Increase in Kinetic Energy



- For δ increases beyond $\delta^{\lim}$, we again have $P_m > P_e$
  – Turbine-generator set re-accelerates
  – Increase in the stored kinetic energy - area A_3

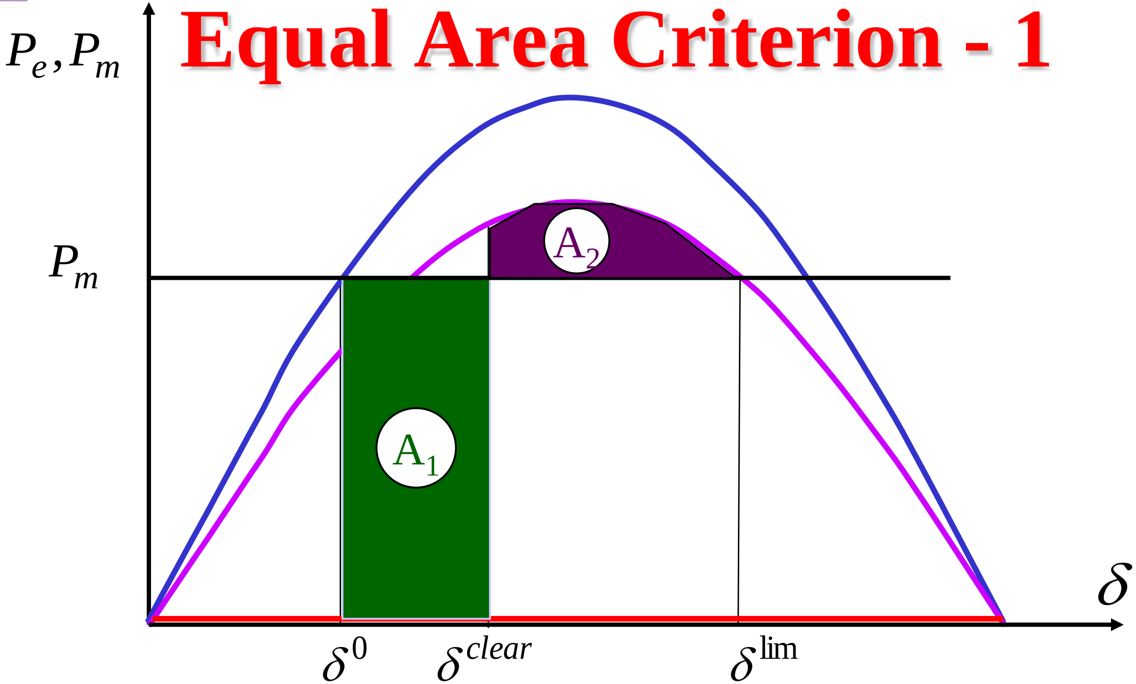
Three-phase Faults



For stability

A ≤ B

Equal Area Criterion - 1



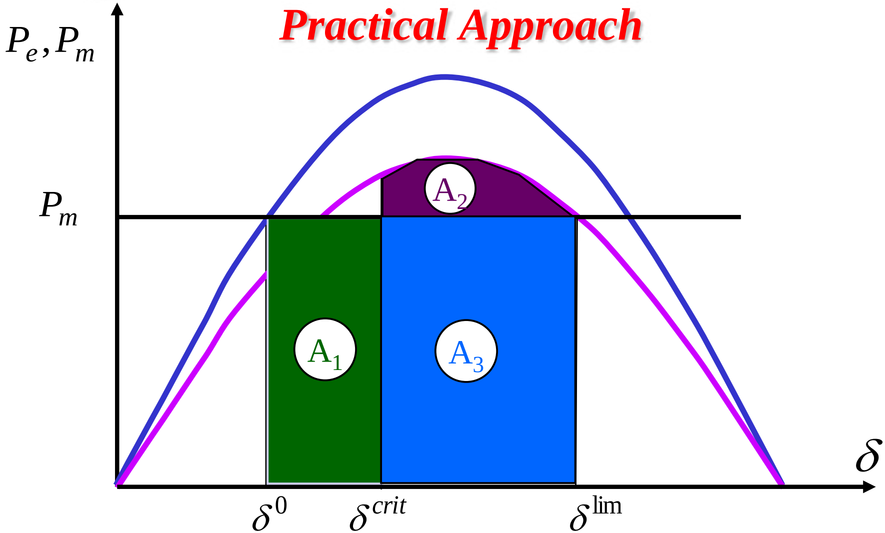
- To maintain stability, the increase in kinetic energy (A_1) must have been injected in the network (A_2) before δ reaches $\delta^{\lim}$
- The largest (critical) value of δ^{clear} is thus such that $A_1 = A_2$

Equal Area Criterion - 2

- Find the critical value of the clearing angle δ^{crit} such that $A_1 = A_2$
- Compare the actual clearing angle δ^{clear} with δ^{crit}
- If $\delta^{clear} < \delta^{crit}$ the system is stable
- If $\delta^{clear} > \delta^{crit}$ the system is unstable

Equal Area Criterion – 3

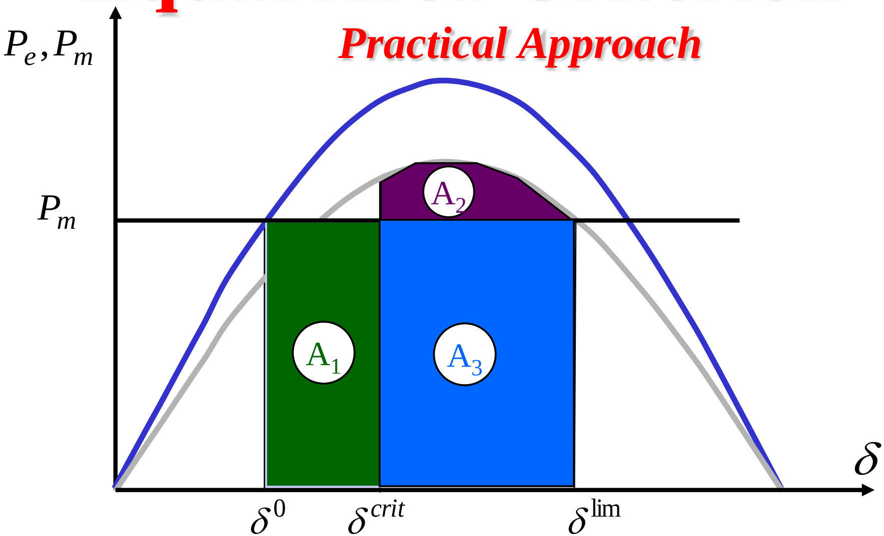
Practical Approach



$$A_1 = A_2 \Leftrightarrow A_1 + A_3 = A_2 + A_3$$

Equal Area Criterion – 4

Practical Approach

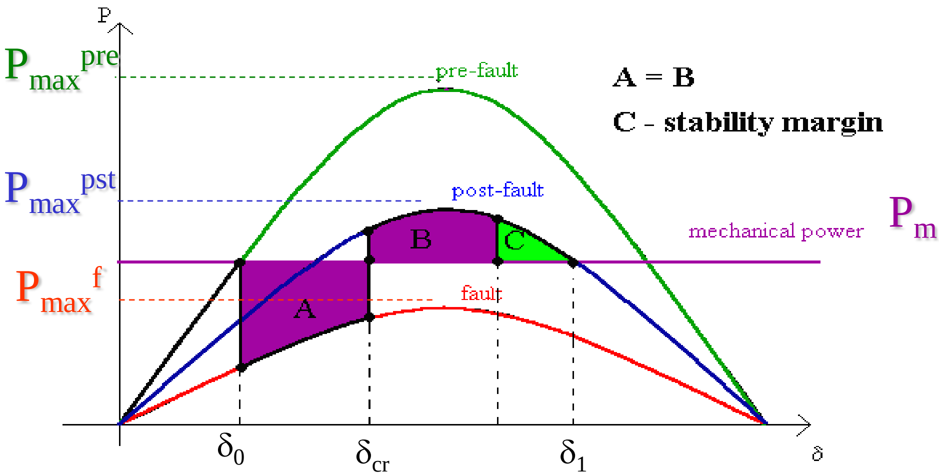


$$\underbrace{A_1 + A_3} = \underbrace{A_2 + A_3}$$

$$P_m(\delta^{\lim} - \delta^0) = \int_{\delta^{crit}}^{\delta^{\lim}} P_e d\delta$$

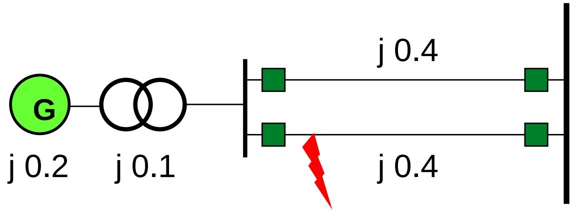
Use this expression to calculate δ^{crit}

Equal Area Criterion - 5

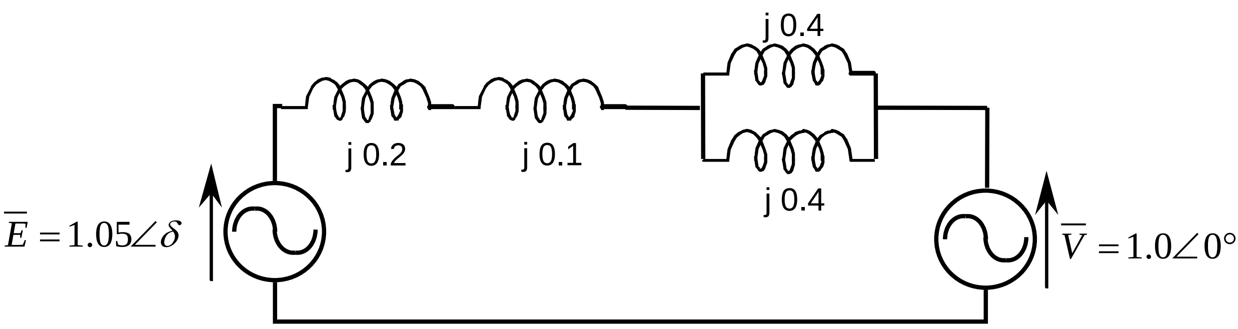


$$\int_{\delta_0}^{\delta_{cr}} (P_m - P_e^{\,f})d\delta = \int_{\delta_{cr}}^{\delta_1} (P_e^{\,pst} - P_m)d\delta$$

Example 1



$$P_e^0 = 1.0pu$$
$$E = 1.05pu$$
$$V_\infty = 1.0pu$$



Step 1: Calculate the Initial Conditions

Before the fault:

$$P_e^0 = P_m = \frac{EV}{X_1}\sin\delta^0$$

Known: P_m, E, V, X_1

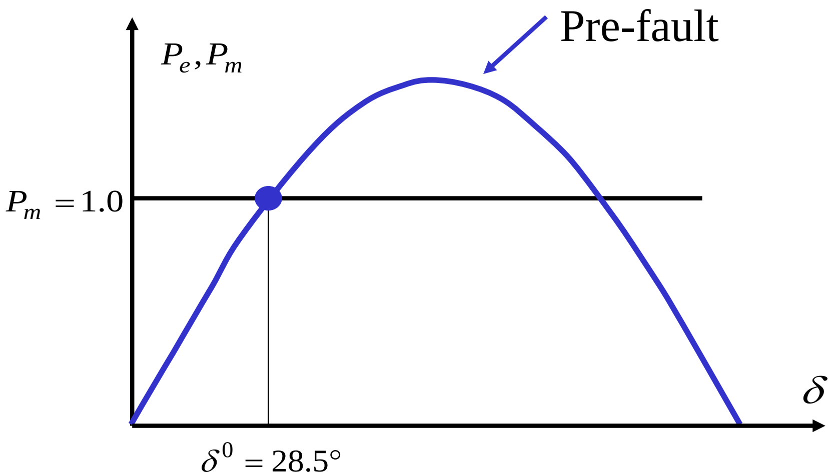
Unknown: δ^0

$$X_1 = X_S + X_T + \tfrac{1}{2}X_L = 0.5pu$$

$$1.0 = \frac{1.05 \times 1.0}{0.5}\sin\delta^0$$

$$\Rightarrow \sin\delta^0 = \frac{0.5}{1.05}$$

$$\Rightarrow \delta^0 = 28.5°$$



Step 2: Calculate the Post-Fault Conditions

After the fault:

$$P_e^0 = P_m = \frac{EV}{X_2}\sin\delta^1$$

$$X_2 = X_S + X_T + X_L = 0.7\,pu$$

Known: P_m, E, V, X_2

Unknown: δ^1

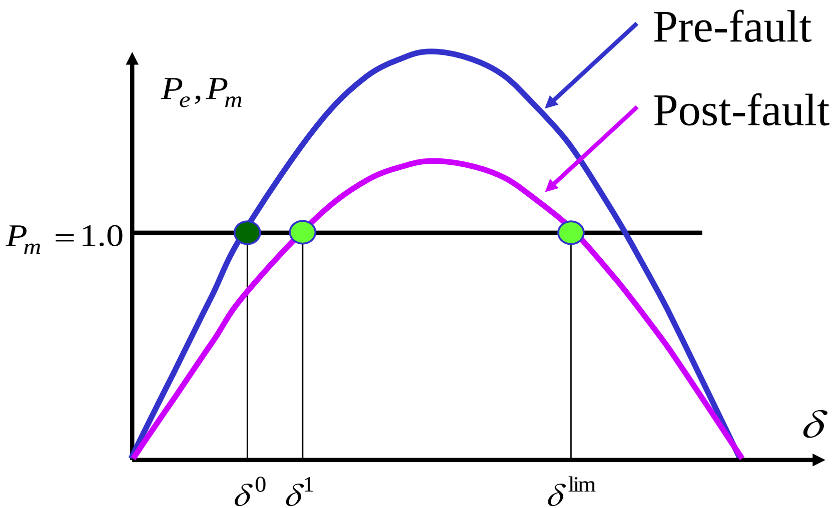
$$1.0 = \frac{1.05 \times 1.0}{0.7}\sin\delta_1$$

$$\Rightarrow \sin\delta^1 = \frac{0.7}{1.05}$$

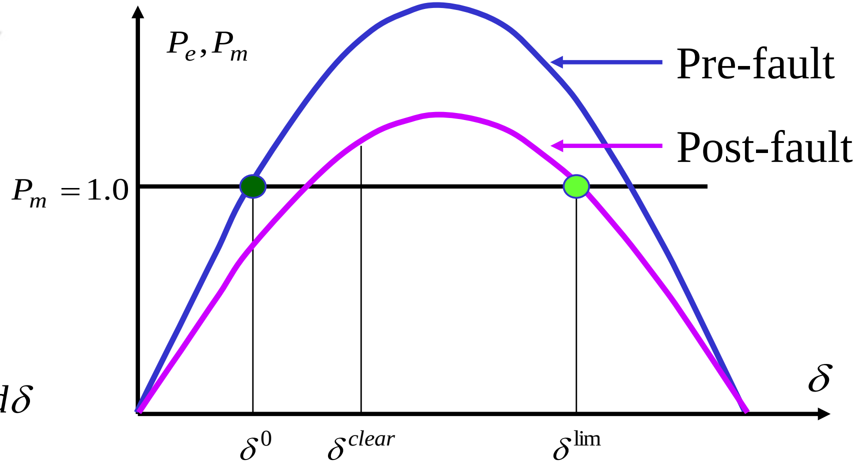
$$\Rightarrow \delta^1 = 41.8°$$

$$\Rightarrow \delta^{\lim} = 180° - \delta^1 = 138.2°$$

The goal of this step is to find $\delta^{\lim}$



Step 3: Apply Equal Area Criterion



$$P_m(\delta^{\lim} - \delta^0) = \int_{\delta^{crit}}^{\delta^{\lim}} P_e \, d\delta$$

$$1.0 \times (138.2° - 28.5°) \times \frac{\pi}{180°} = \int_{\delta^{crit}}^{\delta^{\lim}} \frac{EV}{X_2} \sin\delta \;\; d\delta = \int_{\delta^{crit}}^{\delta^{\lim}} 1.5 \;\; \sin\delta \;\; d\delta$$

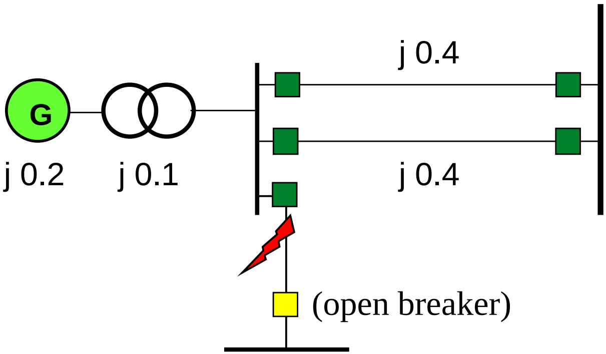
$$1.92 = [-1.5 \;\; \cos\delta]_{\delta^{crit}}^{\delta^{\lim}}$$

$$1.92 = -1.5 \;\; \cos(138.2°) + 1.5 \;\; \cos\delta^{crit}$$

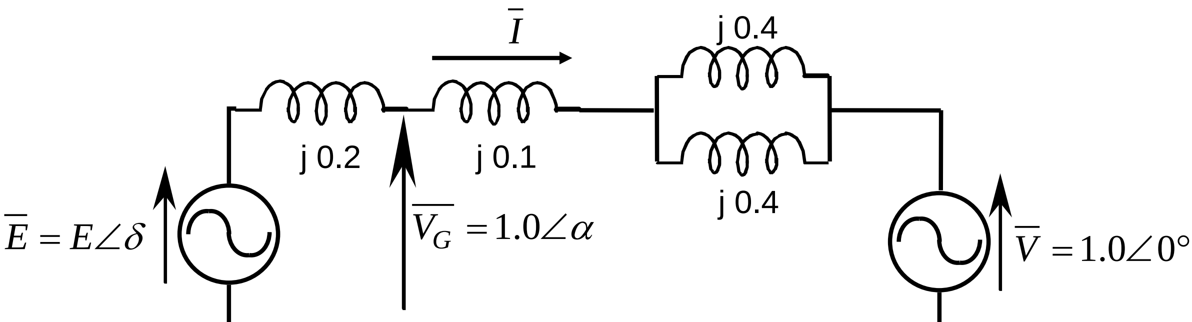
$$\cos\delta^{crit} = \frac{1.92 - 1.12}{1.5} \Rightarrow \delta^{crit} = 57.9°$$

To maintain stability, the fault must be cleared before δ reaches 57.9°

Example 2



$$P_e^0 = 1.0pu$$
$$V_G = 1.0pu$$
$$V_\infty = 1.0pu$$



Step 1: Calculate the Initial Conditions

Before the fault:

Known: $P_e^0 = P_m, V_G, V, X_1$

Unknown: E, δ^0, α

Power transfer between V_G and V:

$$P_e^0 = P_m = \frac{V_G V}{X_3}\sin\alpha \quad \Rightarrow \quad 1.0 = \frac{1.0 \times 1.0}{0.3}\sin\alpha$$

$$\Rightarrow \sin\alpha = 0.3 \quad \Rightarrow \alpha = 17.46^\circ$$

$$X_3 = X_T + \tfrac{1}{2}X_L = 0.3pu \qquad \Rightarrow \overline{V_G} = 1.0\angle 17.46^\circ$$

$$\bar{I} = \frac{\overline{V_G} - \overline{V}}{X_3} = \frac{1.0\angle 17.46^\circ - 1.0\angle 0^\circ}{j0.3} = 1.012\angle 8.7^\circ \quad pu$$

$$\overline{E} = \overline{V_G} + jX_S\bar{I} = 1.0\angle 17.46^\circ + j0.2 \times 1.012\angle 8.7^\circ = 1.05\angle 28.44^\circ$$

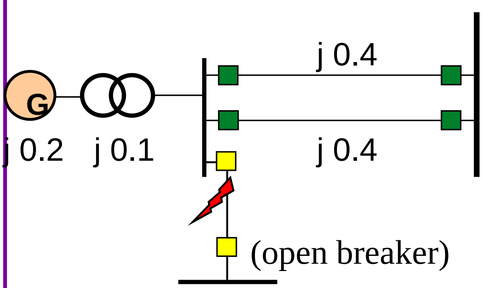
$$\boxed{\text{Check: } P_e = \frac{EV}{X_1}\sin\delta = \frac{1.05 \times 1.0}{0.5}\sin(28.44^\circ) = 1.0pu \quad \Rightarrow OK!}$$

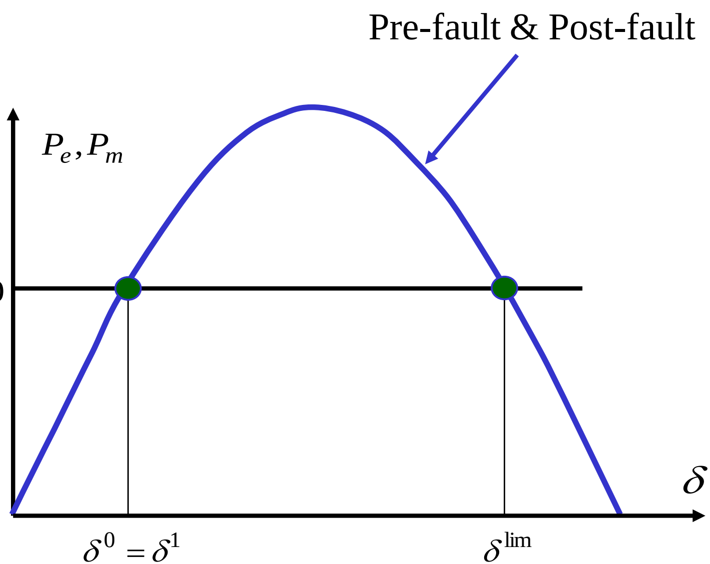
Step 2: Calculate the Post-Fault Conditions

In this case, the post-fault steady state operating point is the same as the pre-fault operating point because the disconnected line does not carry power.

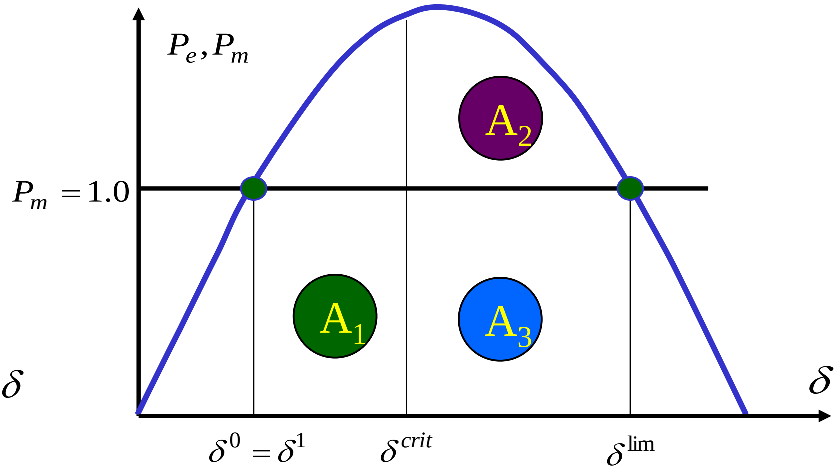
$$\delta^1 = \delta^0 = 28.44°$$

$$\delta^{\lim} = 180° - \delta^1 = 151.56°$$





Step 3:Apply Equal Area Criterion



$$A_1 + A_3 = A_2 + A_3$$

$$P_m(\delta^{\lim} - \delta^0) = \int_{\delta^{crit}}^{\delta^{\lim}} P_e d\delta$$

$$1.0 \times (151.56° - 28.44°) \times \frac{\pi}{180°} = \int_{\delta^{crit}}^{\delta^{\lim}} \frac{EV}{X_1} \sin\delta \;\; d\delta = \int_{\delta^{crit}}^{\delta^{\lim}} 2.1 \;\; \sin\delta \;\; d\delta$$

$$2.15 = [-2.1 \;\; \cos\delta]_{\delta^{crit}}^{\delta^{\lim}}$$

$$2.15 = -2.1 \;\; \cos(151.56°) + 2.1 \;\; \cos\delta^{crit}$$

$$\cos\delta^{crit} = 0.144 \qquad \Rightarrow \qquad \delta^{crit} = 81.7°$$

Comparison of the Two Examples

First example: $\delta^{crit} = 57.9°$

Second example: $\delta^{crit} = 81.7°$

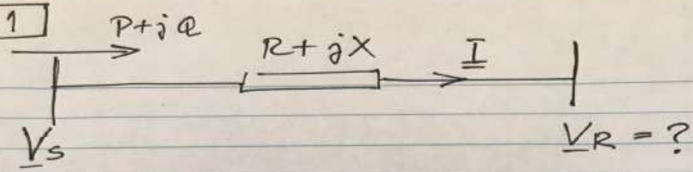
What is the difference between the two examples that results in such a large change in the critical clearing time?

Case 1:

Voltage ($\underline{V}$) and power ($\underline{S}$=P+jQ) given at the **same** end of the line.

Calculate the voltage at the other end of the line.

CASE 1



$$\underline{V}_s = \underline{V}_R + \underline{Z}\,\underline{I} = \underline{V}_R + \underline{Z}\frac{\underline{S}^*}{\underline{V}_s^*}$$

$$\underline{V}_s = V_s\angle 0^\circ \leftarrow \text{assume}$$

$$V_s = \underline{V}_R + (R+jX)\frac{P-jQ}{V_s}$$

$$\underline{V}_R = V_s - \frac{(R+jX)(P-jQ)}{V_s}$$

$$\underline{V}_R = V_s - \frac{RP - jRQ + jXP + XQ}{V_s}$$

$$\underline{V}_R = V_s - \frac{RP+XQ}{V_s} - j\frac{XP-RQ}{V_s}$$

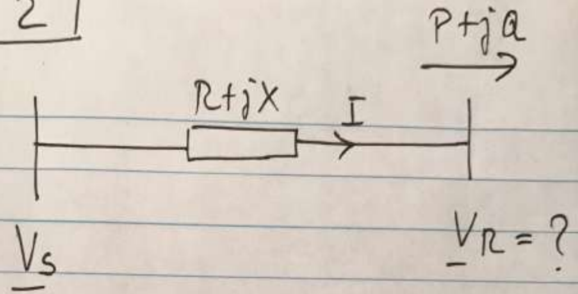
$$\underline{V}_R = \left[V_s - \frac{RP+XQ}{V_s}\right] - j\frac{XP-RQ}{V_s}$$

Case 2:

Voltage ($\underline{V}$) and power ($\underline{S}$=P+jQ) given at **different** ends of the line.

Calculate the voltage at the other end of the line.

CASE 2



$$\underline{V}_S = \underline{V}_R + \underline{Z}\,\underline{I} = \underline{V}_R + \underline{Z}\,\frac{P - jQ}{\underline{V}_R^*}$$

$$\underline{V}_S = V_S \angle 0^\circ \leftarrow \text{assume}$$

$$\underline{V}_S = \underline{V}_R + (R+jX)\,\frac{P - jQ}{\underline{V}_R^*} \quad \Big/ \; \underline{V}_R^*$$

$$V_S V_R^* = |V_R|^2 + (R+jX)(P-jQ)$$

$$\underline{V}_R = a + jb \leftarrow \text{assume}$$

$$V_s(a - jb) = a^2 + b^2 + (RP - jRQ + jXP + XQ)$$

$$V_s a - V_s jb = a^2 + b^2 + RP + XQ + j(XP - RQ)$$

$$-jV_s b = j(XP - RQ) \quad \text{.....(1)}$$

$$V_s a = a^2 + b^2 + RP + XQ \quad \text{...(2)}$$

$$(1) \Rightarrow \quad b = \frac{XP - RQ}{-V_s} = \frac{RQ - XP}{V_s} \quad \text{...(3)}$$

substitute (3) into (2) and solve for "a":

$$(2) \Rightarrow \quad a^2 - V_s a + (b^2 + RP + XQ) = 0$$

$$a_{1/2} = \frac{V_s \pm \sqrt{V_s^2 - 4(b^2 + RP + XQ)}}{2}$$

select larger of two solutions

Critical Clearing Time

- The equal area criterion provides a method to calculate the critical clearing angle
- However, we don't have direct control over the clearing angle: it depends on the loading of the system and the characteristics of the fault
- A more useful parameter is the critical clearing time:
  - How fast should the breaker clear the fault to maintain stability?
- To relate the critical clearing time and the critical clearing angle, we need to know how the machine angle changes with time
- Use the swing equation

Critical Clearing Time

Acceleration

$$\frac{d^2\delta}{dt^2} = \frac{\omega_0}{2H}(P_m - P_e)$$

$$\frac{d\delta}{dt} = C_1 + \frac{\omega_0}{2H}(P_m - P_e)t$$

Initial machine rotor angle

$$\delta = \delta_0 + \omega_0 t + \frac{\omega_0}{4H}(P_m - P_e)t^2$$

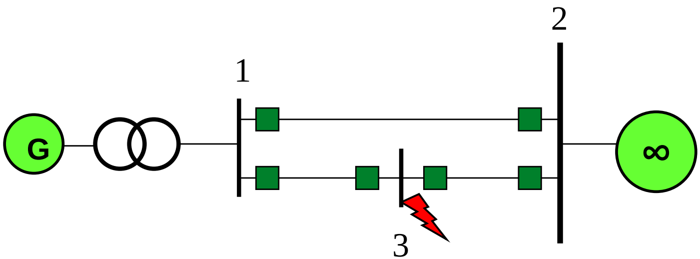
Transient change in angle

Change in angle due to system frequency – this component is 0

For $P_a = (P_m - P_e) = \text{const}$

$$\boxed{\Delta t = t_{cr} = \sqrt{\frac{4H\Delta\delta_{cr}}{\omega_0(P_m - P_e)}} = \sqrt{\frac{4H[\text{sec}](\delta_{cr} - \delta_0)[^0]}{18000(P_m - P_e)[p.u.]}}}$$

Example 3



$X_S = 0.3\,pu$

$X_T = 0.1\,pu$

$X_{12} = 0.2\,pu$

$X_{13} = 0.1\,pu$

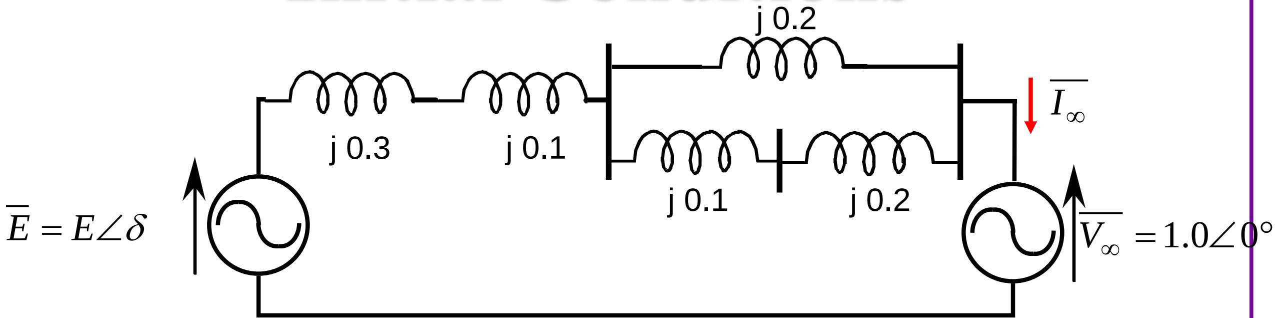
$X_{23} = 0.2\,pu$

$H = 3.0$ seconds

$V_\infty = 1.0\,pu$

$P_\infty = 1.0\,pu$ at 0.95 pf lagging

Initial Conditions



Power into the infinite bus: 1.0 pu at 0.95 pf lagging

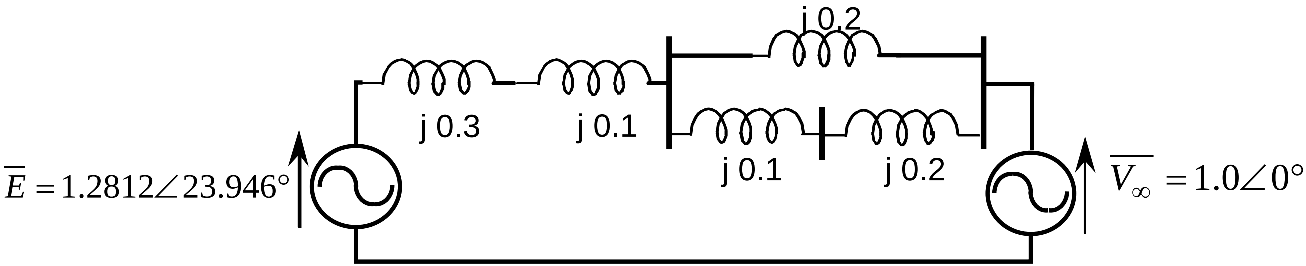
$$S_\infty = P_\infty + jQ_\infty \text{ with } P_\infty = 1.0\,pu \text{ and } Q_\infty = 1.0 \times \tan\left(\cos^{-1}(0.95)\right) = 0.329\,pu$$

$$S_\infty = \overline{V_\infty}\,\overline{I_\infty}^* \quad \Rightarrow \quad \overline{I_\infty} = \frac{1.0 - j0.329}{1.0\angle 0°} = 1.053\angle -18.195°$$

$$X_1 = 0.3 + 0.1 + 0.2 \| (0.1 + 0.2) = 0.52\,pu$$

$$\overline{E} = \overline{V_\infty} + jX_1\overline{I_\infty} = 1.0 + j0.52 \cdot 1.053\angle -18.195° = 1.2812\angle 23.946°$$

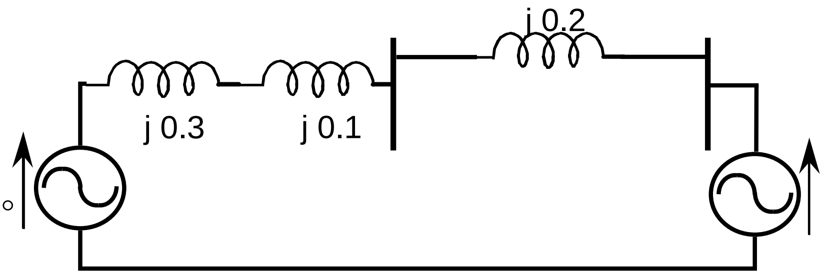
Pre-Fault Power Transfer



$$X_1 = 0.3 + 0.1 + 0.2 \| (0.1 + 0.2) = 0.52\, pu$$

$$P_e^{pre} = \frac{EV_\infty}{X_1}\sin\delta = \frac{1.2812 \times 1.0}{0.52}\sin\delta = 2.4368\sin\delta$$

Post-Fault Power Transfer



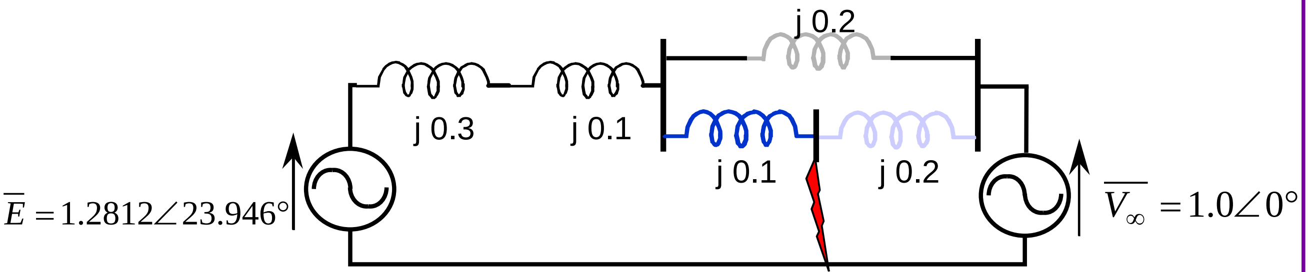
$\overline{E} = 1.2812\angle 23.946°$

$\overline{V}_{\infty} = 1.0\angle 0°$

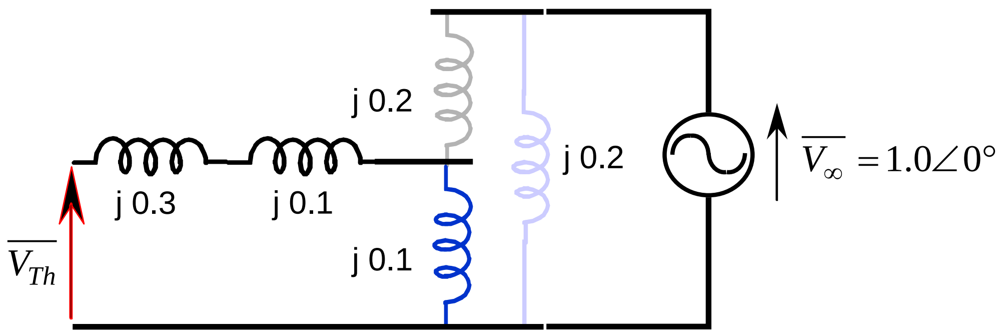
$$X_2 = 0.3 + 0.1 + 0.2 = 0.6\,pu$$

$$P_e^{post} = \frac{EV_{\infty}}{X_2}\sin\delta = \frac{1.2812 \times 1.0}{0.6}\sin\delta = 2.1353\sin\delta$$

Power Transfer During the Fault - 1

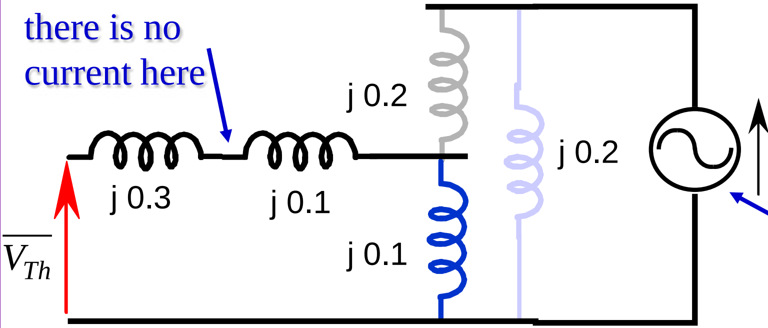


In this case, the generator does not feed into a zero voltage source! Thevenin equivalent circuit as seen from generator's internal emf:



Power Transfer During the Fault - 2

there is no current here



this is the only voltage source in the circuit

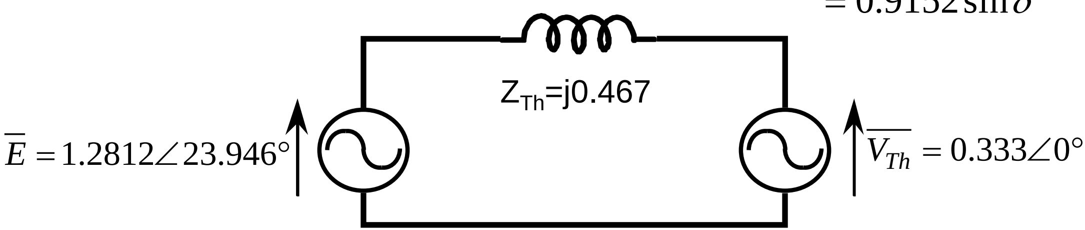
$$\overline{V_{Th}} = \frac{0.1}{0.1+0.2}\overline{V_\infty} = 0.333\angle 0°\, pu$$

$$Z_{Th} = j(0.3+0.1+0.1\|0.2) = j\,0.4667\, pu$$

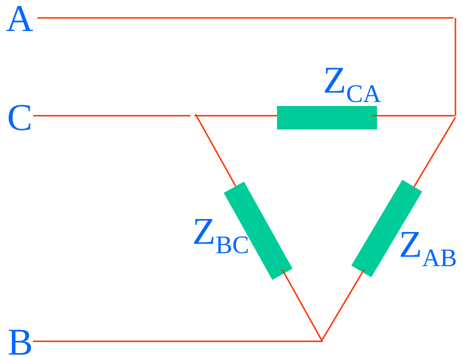
$$P_e^{fault} = \frac{EV_{Th}}{X_{Th}}\sin\delta$$

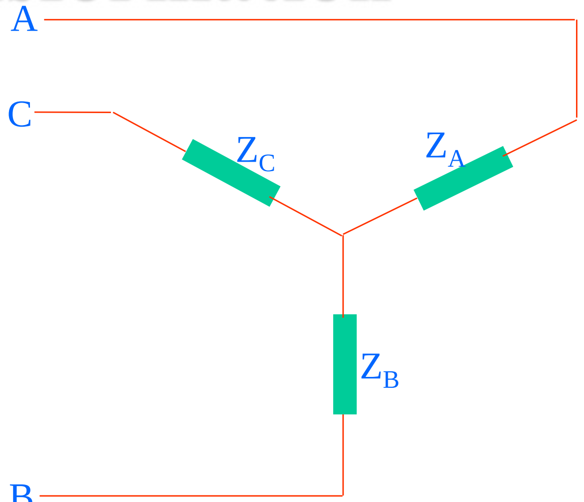
$$= \frac{1.2812\times 0.333}{0.4667}\sin\delta$$

$$= 0.9152\sin\delta$$



$\Delta \longleftrightarrow$ Y transformation





Y → Δ

$$Z_{AB} = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_C}$$

$$Z_{BC} = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_A}$$

$$Z_{CA} = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_B}$$

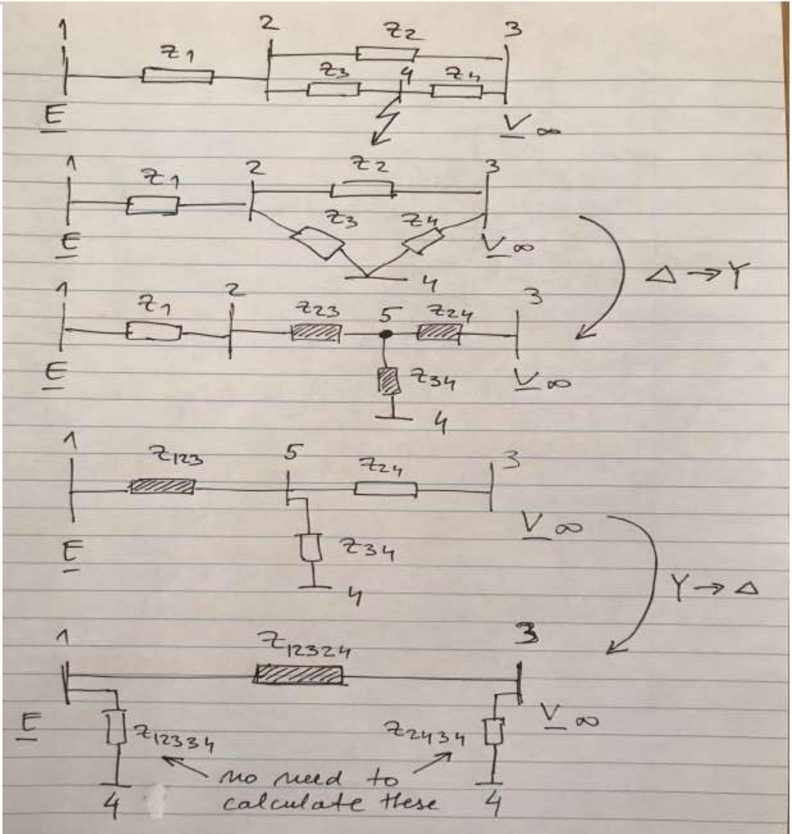
Δ → Y

$$Z_A = \frac{Z_{AB} Z_{CA}}{Z_{AB} + Z_{BC} + Z_{CA}}$$

$$Z_B = \frac{Z_{AB} Z_{BC}}{Z_{AB} + Z_{BC} + Z_{CA}}$$

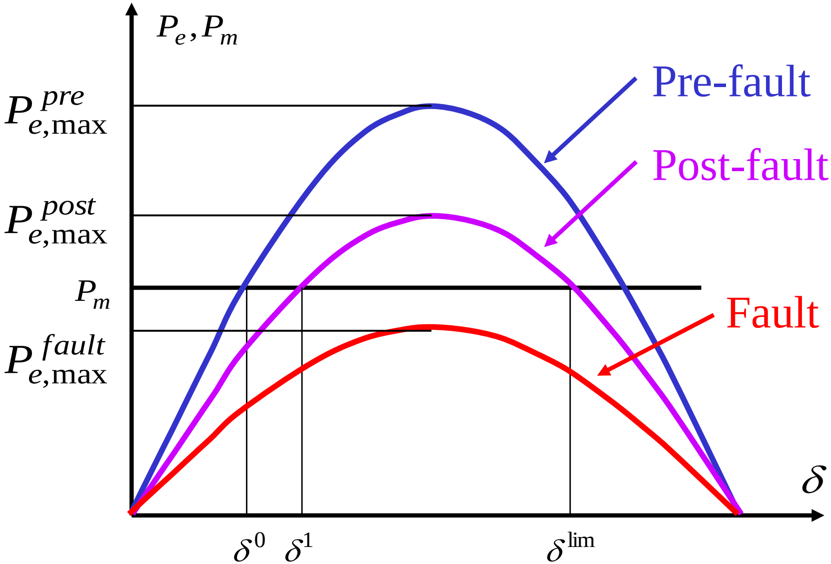
$$Z_C = \frac{Z_{CA} Z_{BC}}{Z_{AB} + Z_{BC} + Z_{CA}}$$

Power Transfer During the Fault: using Δ ⟷ Y transformation



$$P_e^{fault} = \frac{E V_\infty}{z_{12324}} \sin\delta$$

Power Transfer before, during and after the Fault

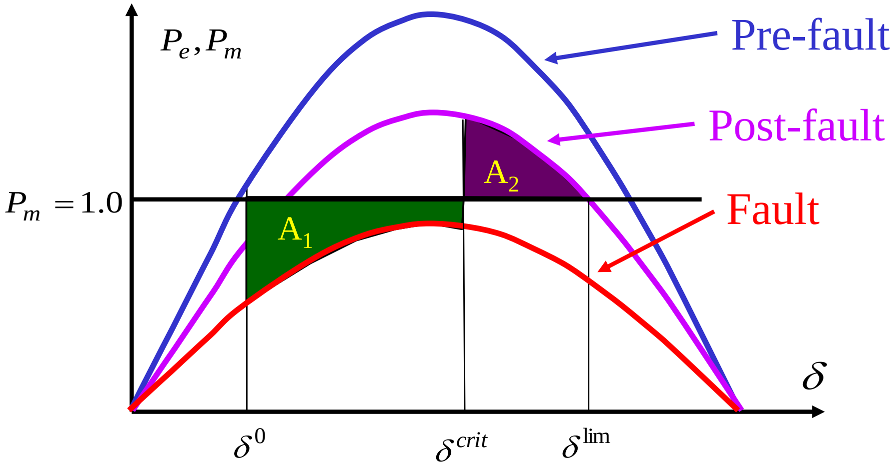


$$P_m = P_{e,\max}^{pre} \sin\delta^0 \Rightarrow \delta^0 = 0.4179 \quad radians$$

$$P_m = P_{e,\max}^{post} \sin\delta^1 \Rightarrow \delta^1 = 0.4874 \quad radians$$

$$\delta^{\lim} = \pi - \delta^1 = 2.6542 \quad radians$$

Equal Area Criterion



$$A_1 = A_2$$

$$\int_{\delta^0}^{\delta^{crit}} (P_m - P_{e,\max}^{fault} \sin\delta) \ d\delta = \int_{\delta^{crit}}^{\delta^{\lim}} (P_{e,\max}^{post} \sin\delta - P_m) \ d\delta$$

$$\int_{0.4179}^{\delta^{crit}} (1.0 - 0.9152 \sin\delta) \ d\delta = \int_{\delta^{crit}}^{2.6542} (2.1353 \sin\delta - 1.0) \ d\delta$$

Critical Clearing Angle

$$\int_{0.4179}^{\delta^{crit}} (1.0 - 0.9152\sin\delta)\ d\delta = \int_{\delta^{crit}}^{2.6542} (2.1353\sin\delta - 1.0)\ d\delta$$

$$(\delta^{crit} - 0.4179) + 0.9152(\cos\delta^{crit} - \cos 0.4179)$$
$$= 2.1353(\cos\delta^{crit} - \cos 2.6542) - (2.6542 - \delta^{crit})$$

$$-1.2201\cos\delta^{crit} = 0.4868$$

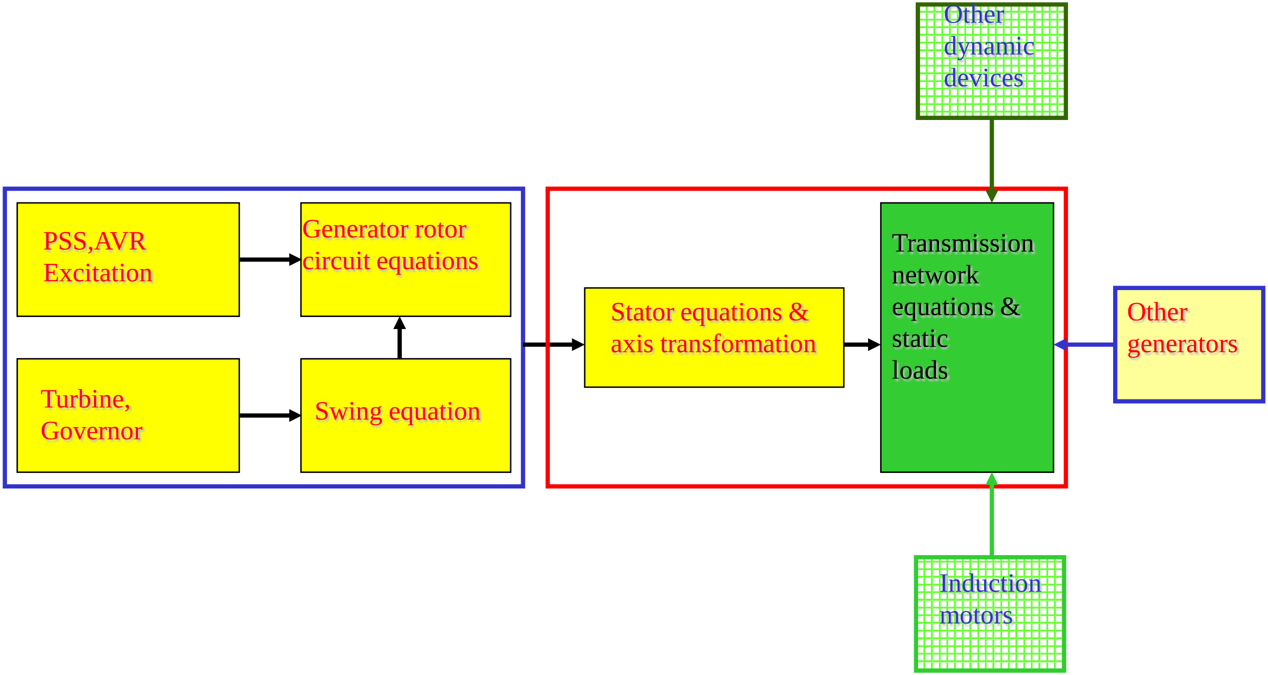
$$\boxed{\delta^{crit} = 1.9812\, radians = 113.5^\circ}$$

Influences on Transient Stability

Transient stability depends on:

- Generator(s) loading
- Generator(s) excitation
- Generator(s) parameters (X, H)
- Fault location
- Type of fault (L-L-L, L-L-G, L-L, L-G)
- Fault clearing time
- Post-fault transmission system impedance

Model for Transient Stability Analysis



Essential Requirements

- How much detail is required?
- What time period is of interest?
  - Equations with small time constants may be neglected as the effects disappear quickly.
  - Equations with long time constants may be neglected as changes over that period of time are not of interest.
  - The final model should be verified for sufficient accuracy by comparison with real data or more detailed model.
- How much computer power is available?
- How quickly should a result be found?

Types of Models

- Non-linear machine models
- Controller limits are important
- Transformer winding connections are important
- Symmetrical components are used for representation of unsymmetrical faults

$$\begin{bmatrix} V_a \\ V_b \\ Vc \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix}$$

$$a = e^{j120^0} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

Synchronous Machine Model - 1

$$V_d = -R_a I_d + \omega L_q'' I_q + E_d''$$

$$V_q = -R_a I_q - \omega L_d'' I_d + E_q''$$

$$p\Psi_{fd} = \omega_0 [e_{fd} + \frac{(\Psi_{ad} - \Psi_{fd}) R_{fd}}{L_{fd}}]$$

$$p\Psi_{1d} = \omega_0 \frac{(\Psi_{ad} - \Psi_{1d}) R_{1d}}{L_{1d}}$$

$$p\Psi_{1q} = \omega_0 \frac{(\Psi_{aq} - \Psi_{1q}) R_{1q}}{L_{1q}}$$

$$p\Psi_{2q} = \omega_0 \frac{(\Psi_{aq} - \Psi_{2q}) R_{2q}}{L_{2q}}$$

$$p\omega = \frac{1}{2H}(T_m - T_e - K_D(\omega - 1))$$

$$p\delta = \omega_0 (\omega - 1)$$

Generator field voltage

$$e_{fd} = R_{fd} I_{fd} = \frac{R_{fd}}{L_{adu}} E_{fd}$$

Exciter output voltage

$$I_{fd} = \frac{\Psi_{fd} - \Psi_{ad}}{L_{fd}}$$

$$i_{fd} = L_{adu} I_{fd}$$

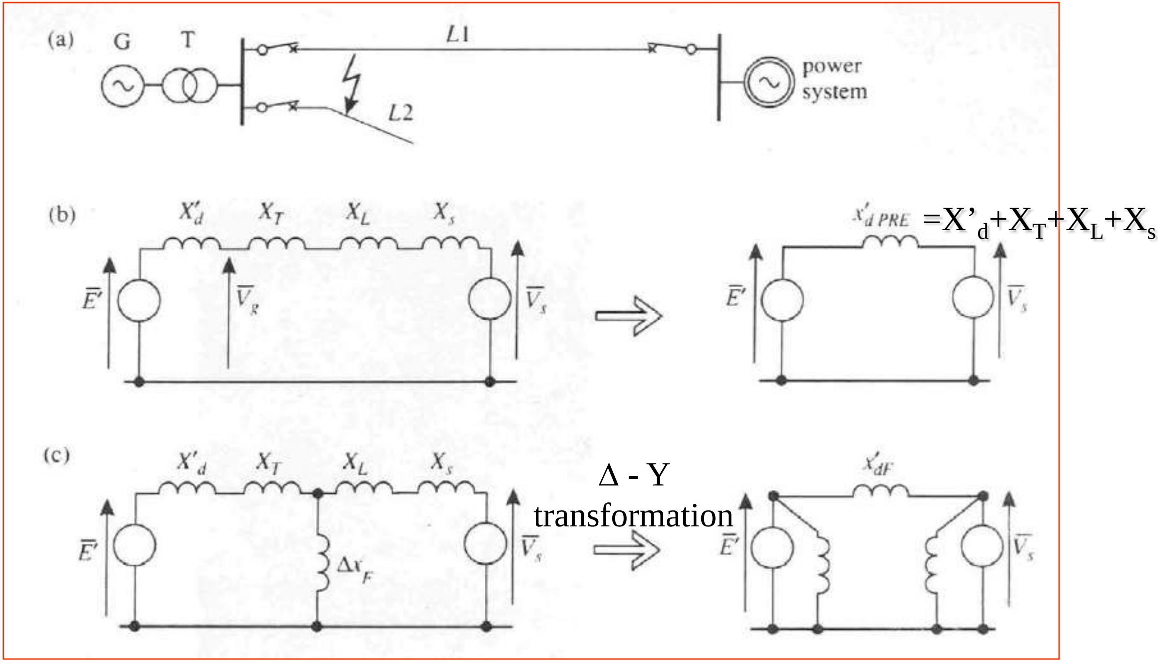
Generator field current

Exciter output current

Electrical quantities and torques in p.u.; Time in seconds; Rotor angle in electrical radians; $\omega_0 = 2\pi f$

$$p = \frac{d}{dt}$$

Faults in the System



Fault without a change in the the equivalent network impedance

Representation of Faults

Three-phase fault (LLL) $\Delta x_F = 0$

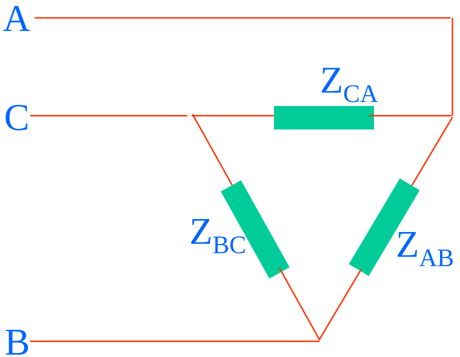
Double-phase to ground fault (LLG) $\Delta x_F = X_2 X_0 / (X_2 + X_0)$

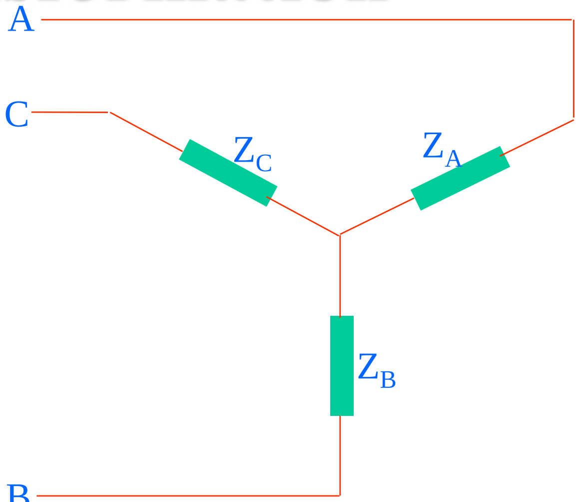
Phase to phase fault (LL) $\Delta x_F = X_2$

Single-phase fault (LG) $\Delta x_F = X_2 + X_0$

X_2 and X_0 negative and zero sequence Thevenin equivalent reactances as seen from the fault terminals

$\Delta \longleftrightarrow Y$ transformation





$Y \rightarrow \Delta$

$$Z_{AB} = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_C}$$

$$Z_{BC} = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_A}$$

$$Z_{CA} = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_B}$$

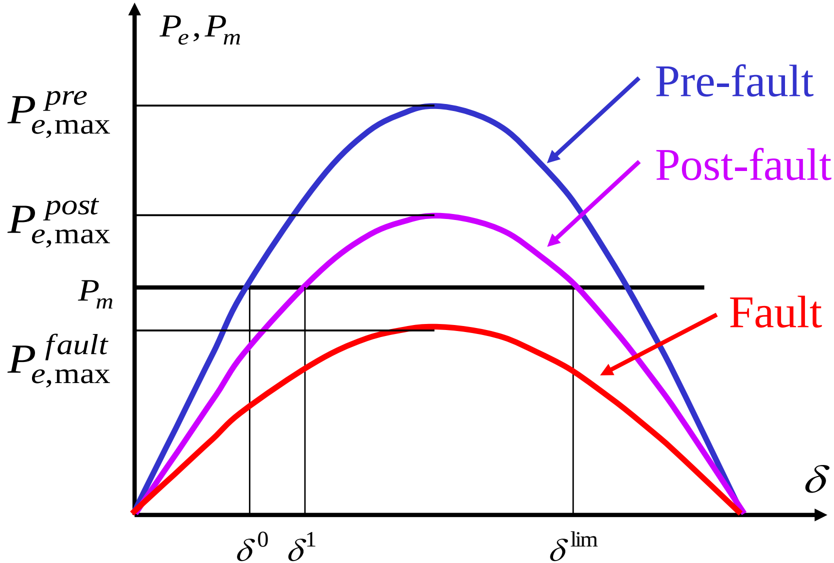
$\Delta \rightarrow Y$

$$Z_A = \frac{Z_{AB} Z_{CA}}{Z_{AB} + Z_{BC} + Z_{CA}}$$

$$Z_B = \frac{Z_{AB} Z_{BC}}{Z_{AB} + Z_{BC} + Z_{CA}}$$

$$Z_C = \frac{Z_{CA} Z_{BC}}{Z_{AB} + Z_{BC} + Z_{CA}}$$

Power transfer before, during and after the fault



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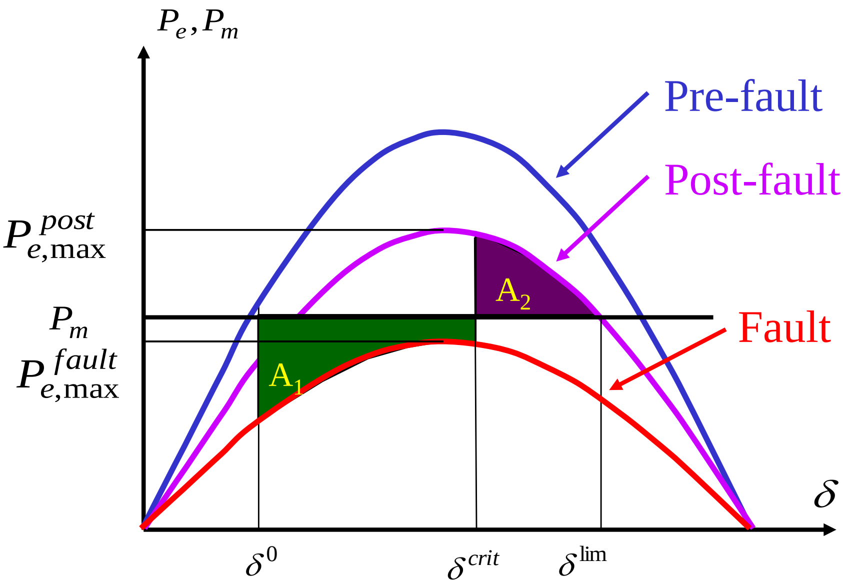
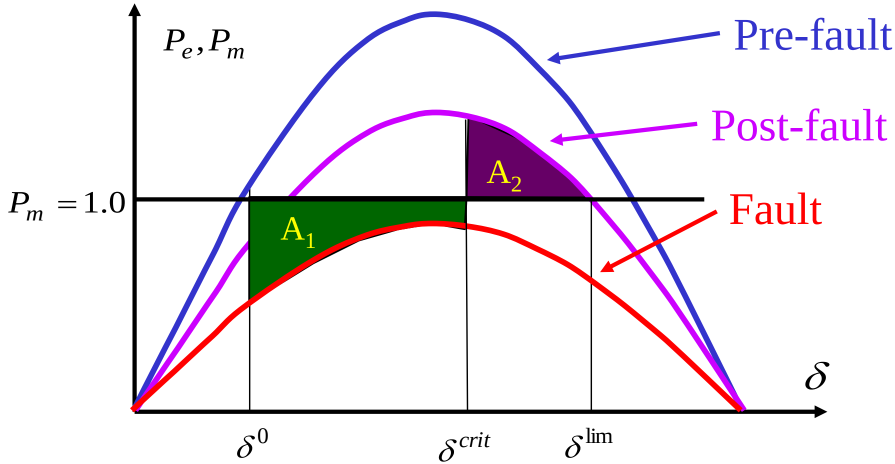
$$P_m = P_{e,\max}^{post} \sin\delta^1 \Rightarrow \delta^1 = 0.4874 \quad radians$$

$$\delta^{\lim} = \pi - \delta^1 = 2.6542 \quad radians$$

Equal Area Criterion - 1



Equal Area Criterion - 2



$$A_1 = A_2$$

$$\int_{\delta^0}^{\delta^{crit}} (P_m - P_{e,\max}^{fault} \sin\delta)\ d\delta = \int_{\delta^{crit}}^{\delta^{\lim}} (P_{e,\max}^{post} \sin\delta - P_m)\ d\delta$$

$$\int_{0.4179}^{\delta^{crit}} (1.0 - 0.9152 \sin\delta)\ d\delta = \int_{\delta^{crit}}^{2.6542} (2.1353 \sin\delta - 1.0)\ d\delta$$

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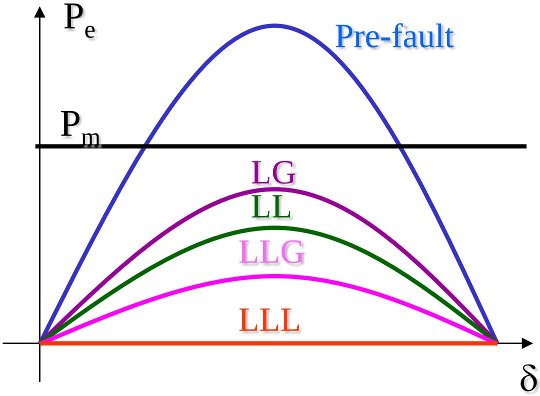
$$\int_{0.4179}^{\delta^{crit}} (1.0 - 0.9152\sin\delta)\ \ d\delta = \int_{\delta^{crit}}^{2.6542} (2.1353\sin\delta - 1.0)\ \ d\delta$$

$$(\delta^{crit} - 0.4179) + 0.9152(\cos\delta^{crit} - \cos 0.4179)$$
$$= 2.1353(\cos\delta^{crit} - \cos 2.6542) - (2.6542 - \delta^{crit})$$

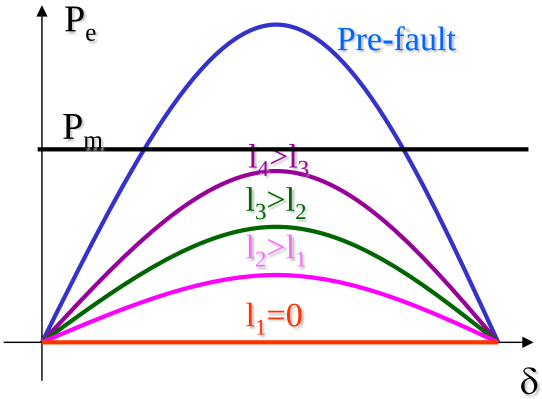
$$-1.2201\cos\delta^{crit} = 0.4868$$

$$\boxed{\delta^{crit} = 1.9812\, radians = 113.5°}$$

The Influence of Type and Distance of Fault

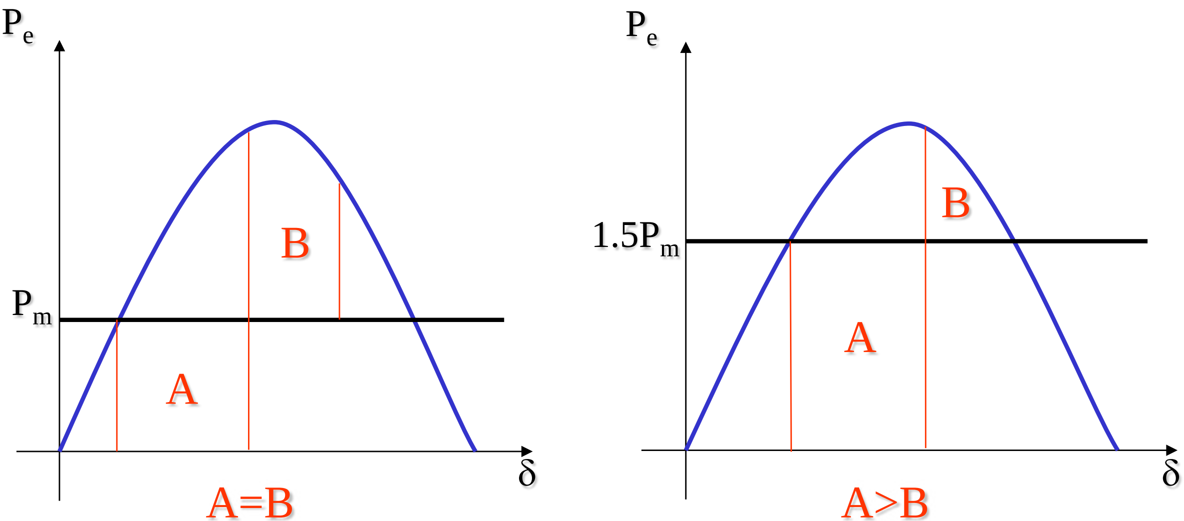


The influence of type of fault



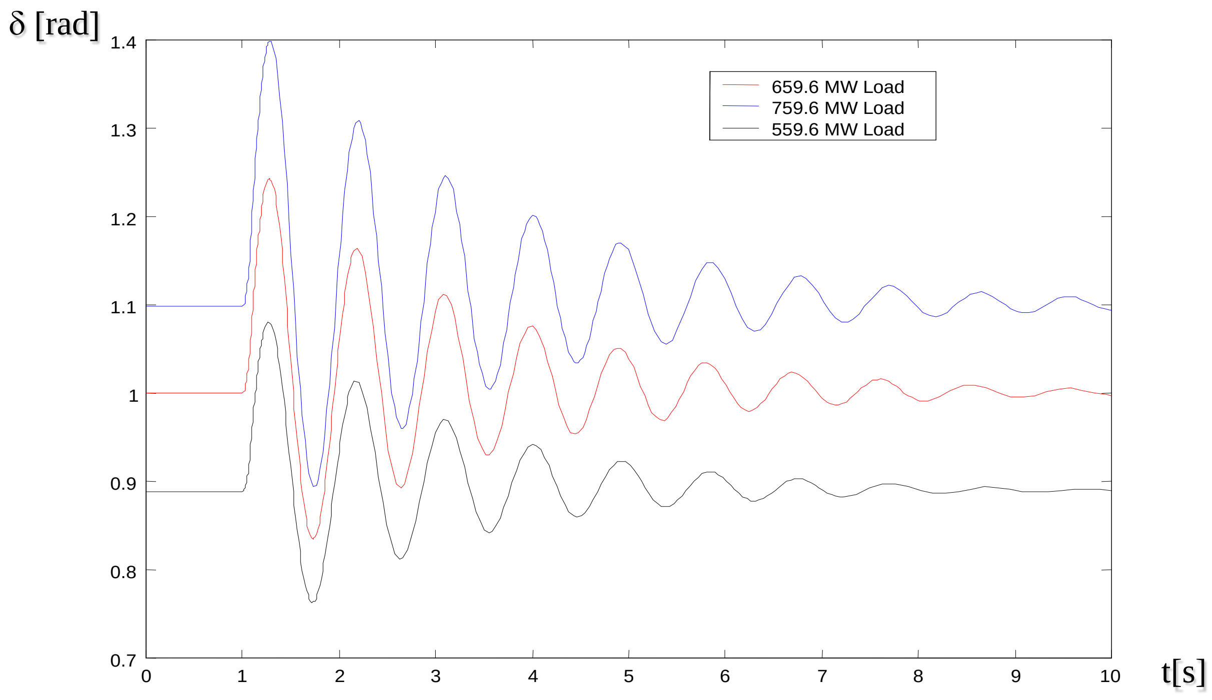
The influence of the fault distance

The Influence of the Pre-fault Load

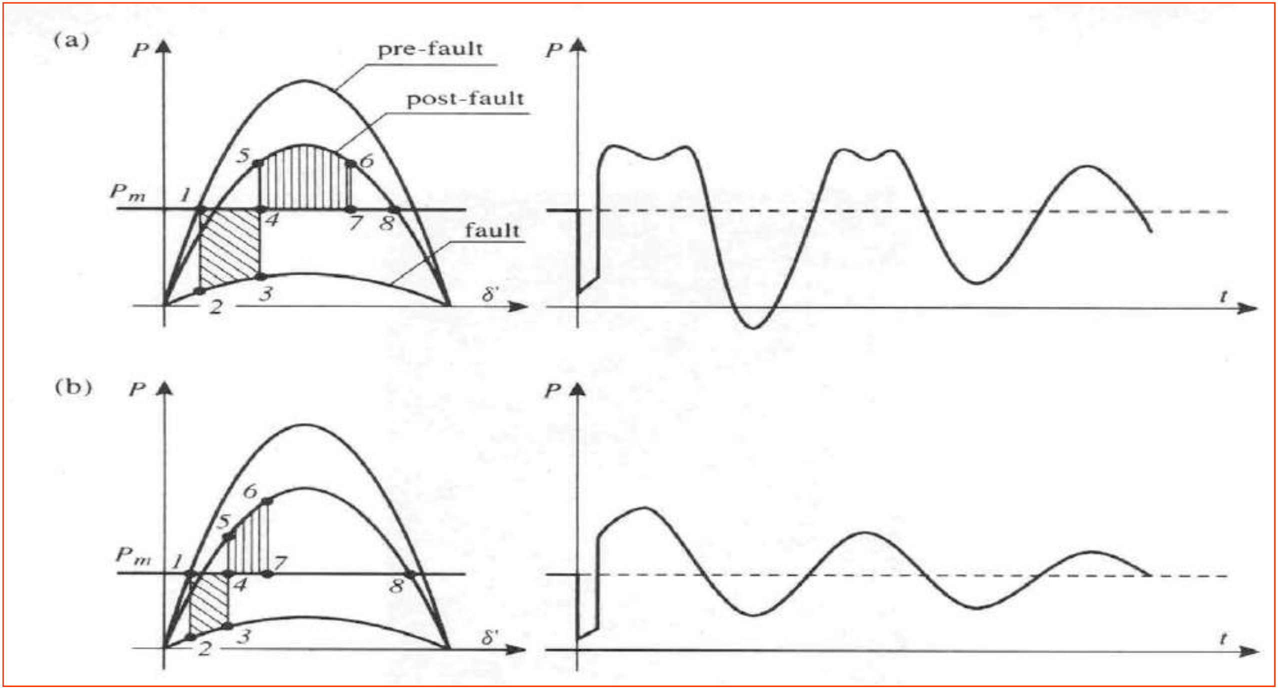


The same clearing time

The Influence of Generator Loading

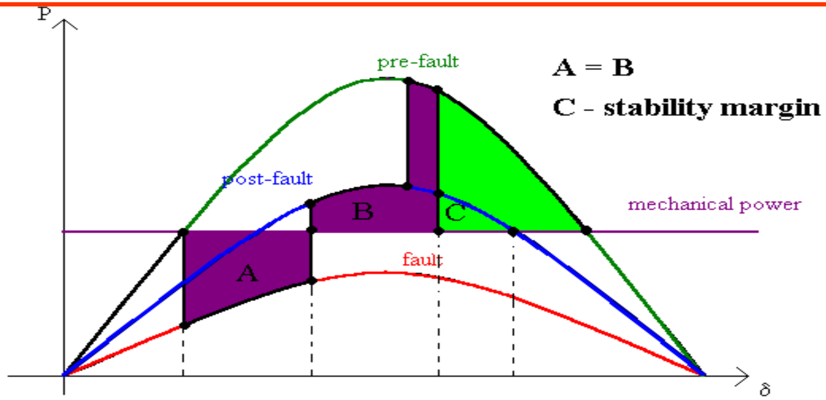


The Influence of Stability Margin

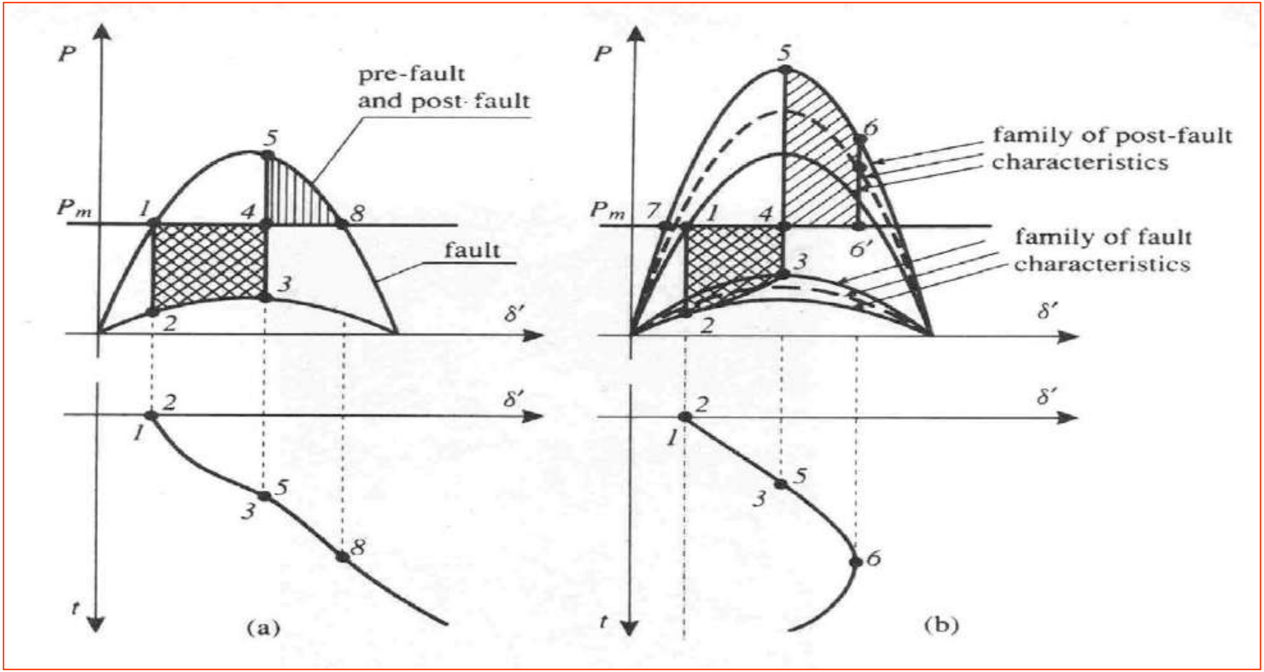


a) Low stability margin
b) Large stability margin

The Influence of Auto Re-closing -2

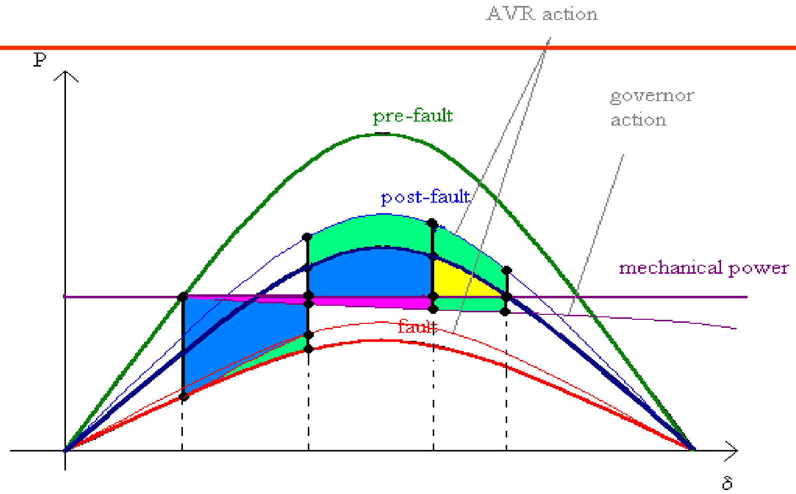


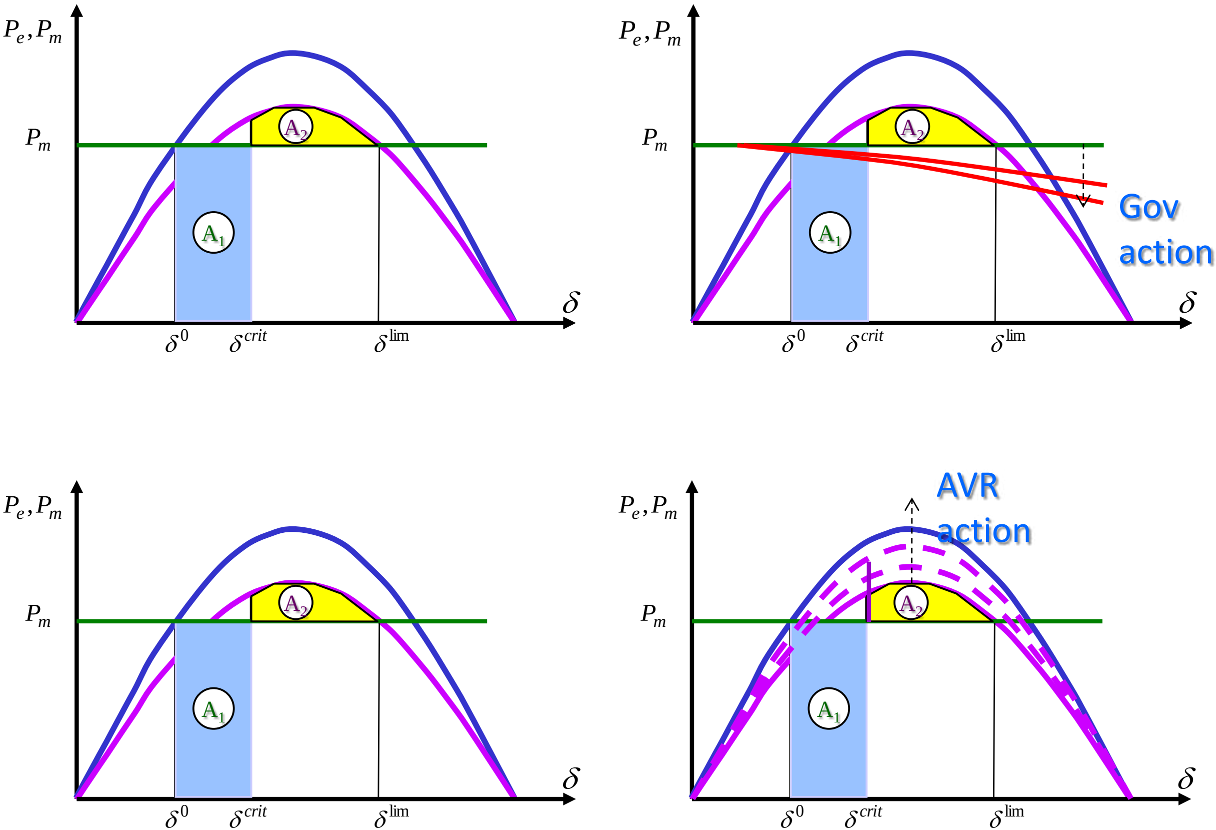
The Influence of AVR



a) The influence of AVR neglected
b) The Influence of AVR included

The Influence of AVR and Governor




P_e, P_m
P_m
A_2
A_1
δ
δ^0
δ^{crit}
$\delta^{\lim}$
Gov action
AVR action

Large System Applications

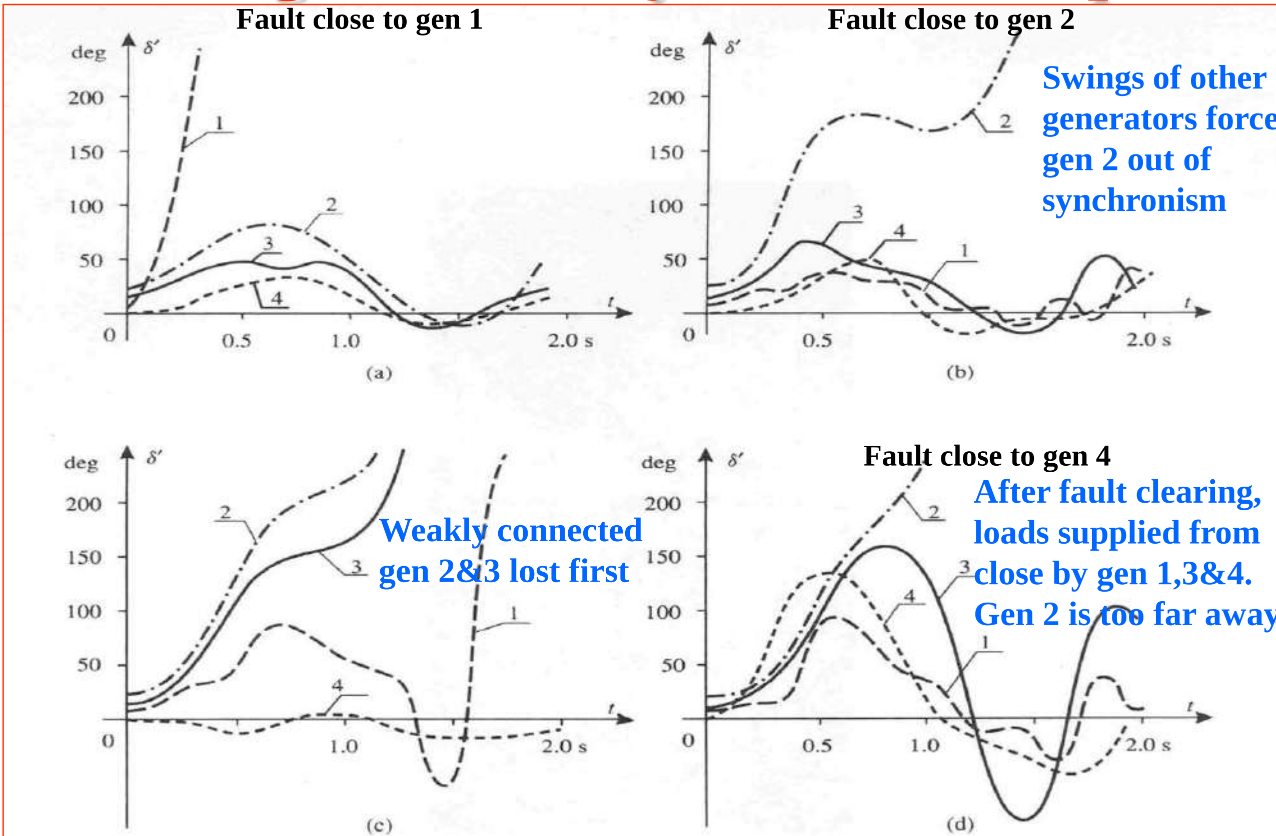
Transient Stability Problems in the System -1

- The generator (or generators) nearest to the fault may lose synchronism without exhibiting any synchronous swings, while other generators in the system undergo a period of synchronous oscillations and return to synchronous operation.
- The generator (or generators) nearest to the fault is the first to lose synchronism and is then followed by other generators in the system.

Transient Stability Problems in the System - 2

- The generator (or generators) nearest to the fault lose synchronism after exhibiting synchronous swings.
- The generator (or generators) nearest to the fault exhibit synchronous swings without losing synchronism, but one or more other generators remote from the fault loses synchronism with the system.

Five-generator System - Example



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Chapter 6:
Enhancement of Power System Stability

Content

- General considerations
- Small disturbance stability
- Large disturbance (Transient) stability
- Voltage stability

General Considerations

Improving Stability

- The use of protection equipment and circuit breakers that ensure the fastest possible fault clearing.
- The use of single-pole circuit breakers so that only the faulted phase is cleared.
- The use of appropriate system configuration (avoiding long, heavily loaded lines).
- Ensuring an appropriate reserve in transmission capability.
- Avoiding operating the system at low frequency and/or voltage.
- Avoiding weakening the network by the simultaneous outage of a large number of lines and transformers.

Trade Off

- Often (if not always) financial considerations determine which of the available options will be implemented and there must be a compromise between operating a system near to its stability limit and operating system with an excessive reserve of generation and transmission.

Control Options

- Available control options are to use:
  - Power System Stabilisers
    - Applied to the excitation system (usual)
      - » speed, electrical power or frequency as an auxiliary signal
    - Applied to the turbine governor (less common)
  - AVRs
  - Fast valving
  - Generator tripping
  - Braking resistors
  - Shunt elements
  - Series elements

Small Disturbance Stability

Excitation Control Design

- Requirements:
  - Maximisation of the damping of the local plant mode as well as inter-area oscillations without compromising the stability of other modes.
  - Enhancement of system transient stability.
  - Prevention of adverse effects on system performance during major system upsets that cause large frequency excursions.
  - Minimisation of the consequences of excitation system malfunction because of component failures.

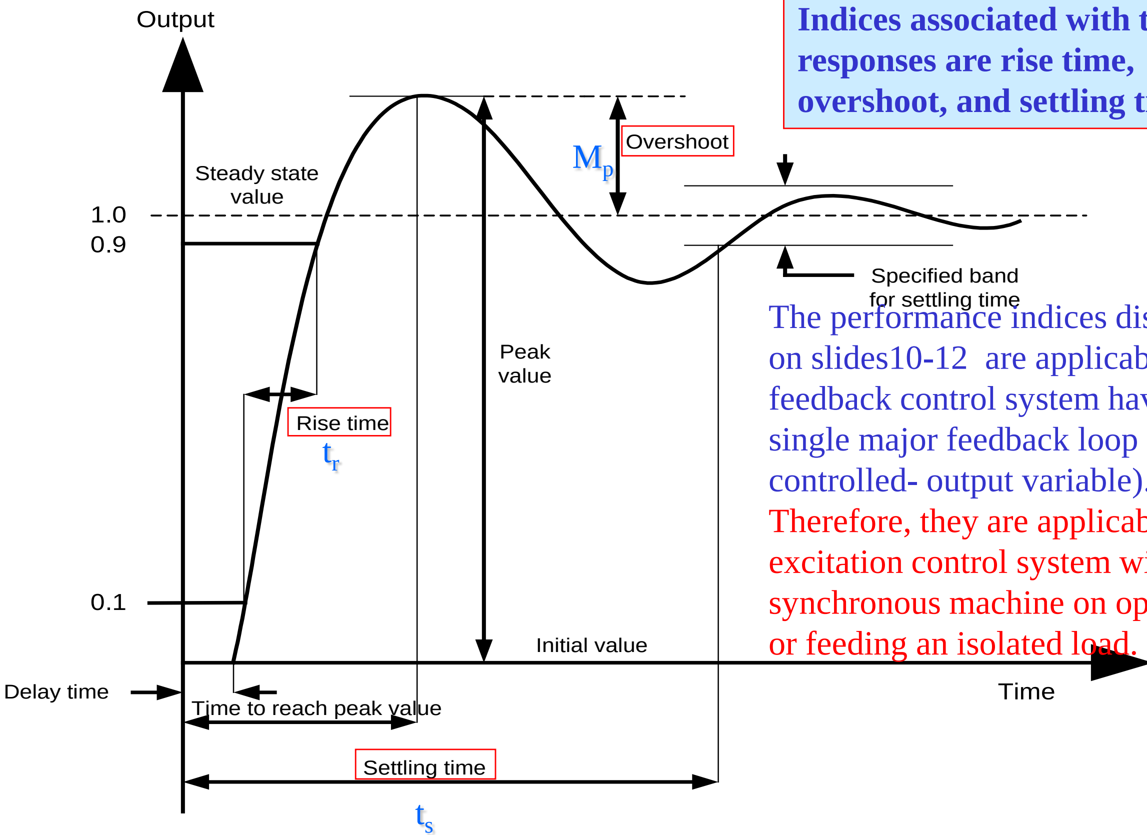
AVR - improves voltage control and transient stability but reduces generator damping

PSS - improves damping at the expense of AVR voltage control performance and does not improve transient stability

Excitation Control Design

- Recommendations:
  - Terminal voltage transducer circuitry can be modelled by a single time constant in the range of 0.01s to 0.02s.
  - A high value of AVR gain is desirable from the point of view of transient stability. Typical value of 200 (this infers that the terminal voltage change of 0.005 p.u. will produce an excitation voltage change of 1.0 p.u. (1.0 p.u. excitation voltage is the field required to provide a 1 p.u. generator terminal voltage under open circuit conditions)) should not require transient gain reduction (TGR) at high frequencies.
  - If needed a lead-lag block is added with time constants of 1s and 10s for numerator and denominator, respectively. (Good voltage regulation is a basic requirement, so the steady state gain of the loop cannot be reduced. Any supplementary compensation must therefore maintain the steady state gain of this loop but reduce the transient (or high frequency) gain to provide a stable loop.)

Excitation System Performance - 1

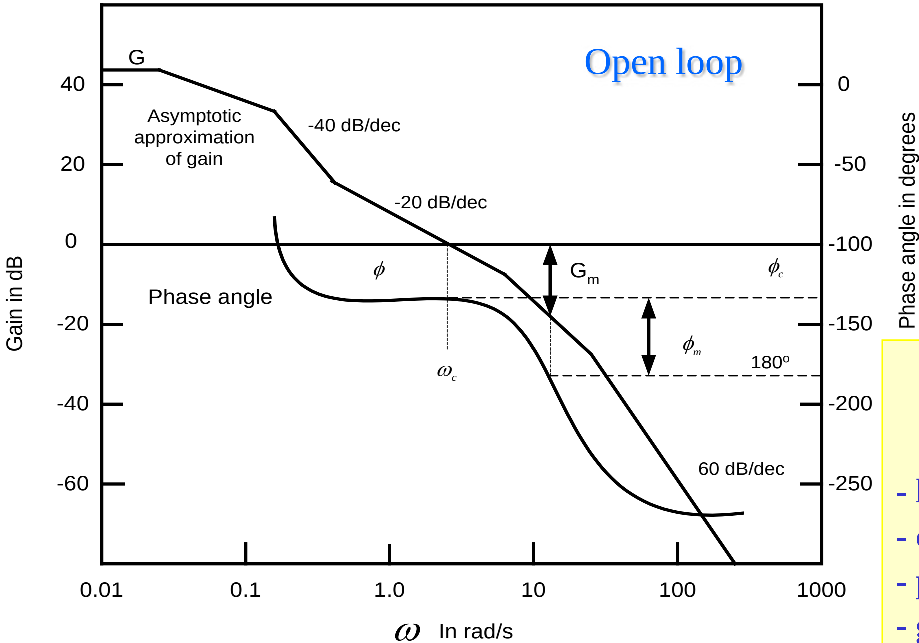


Indices associated with time responses are rise time, overshoot, and settling time.

The performance indices discussed on slides10-12 are applicable to any feedback control system having a single major feedback loop (single controlled- output variable).

Therefore, they are applicable to an excitation control system with the synchronous machine on open circuit or feeding an isolated load.

Excitation System Performance - 2

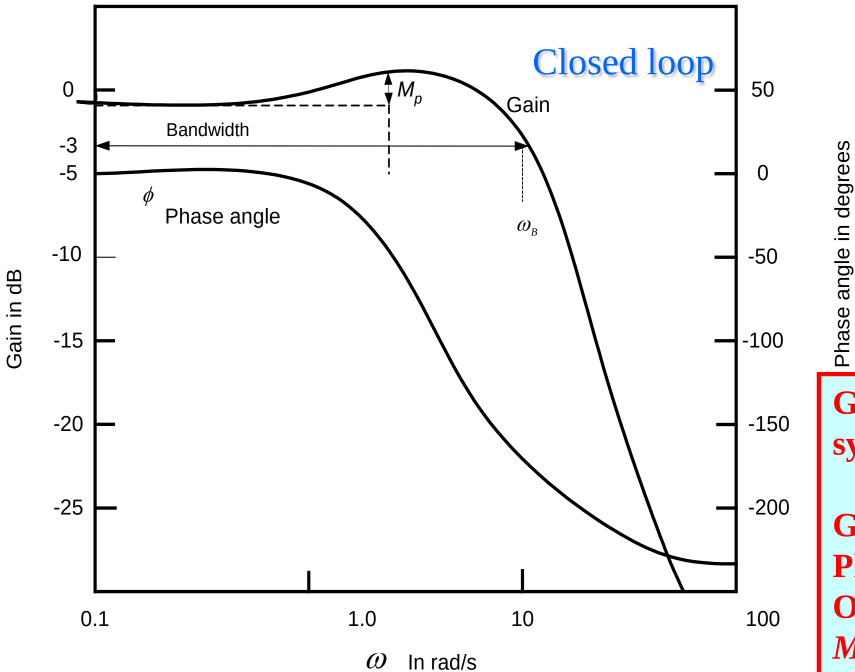


Open loop performance indices:

- **low frequency gain G**
- **crossover frequency ω_c**
- **phase margin ϕ_m**
- **gain margin G_m**

Larger values of G provide better steady state voltage regulation.
Larger crossover frequency indicates faster response.
Larger values of phase margin ϕ_m and gain margin G_m provide a more stable excitation control loop.

Excitation System Performance - 3



Closed-loop performance indices:

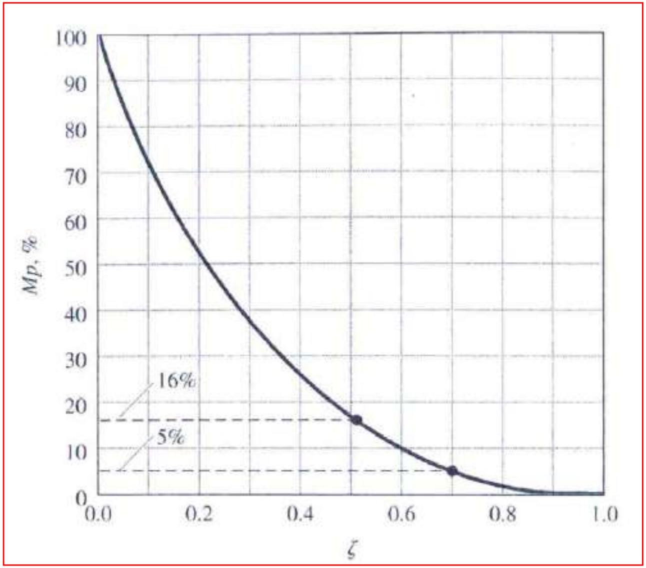
bandwidth ω_B
peak value M_P.

Good feedback control system performance:

Gain margin ≥ 6dB
Phase margin ≥ 40º
Overshoot = 5-15 %
M_p = 1.1-1.6

High value of M_p (>1.6) is indicative of an oscillatory system exhibiting large overshoot in its transient response. Bandwidth is an important closed-loop frequency response index. Larger values of bandwidth indicate faster response. It approximately describes filtering or noise-rejection characteristics of a system.

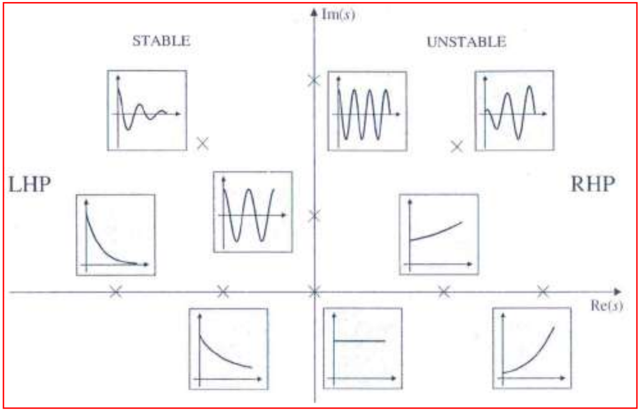
Excitation System Design Criteria



$$\omega_n \geq \frac{1.8}{t_r}$$

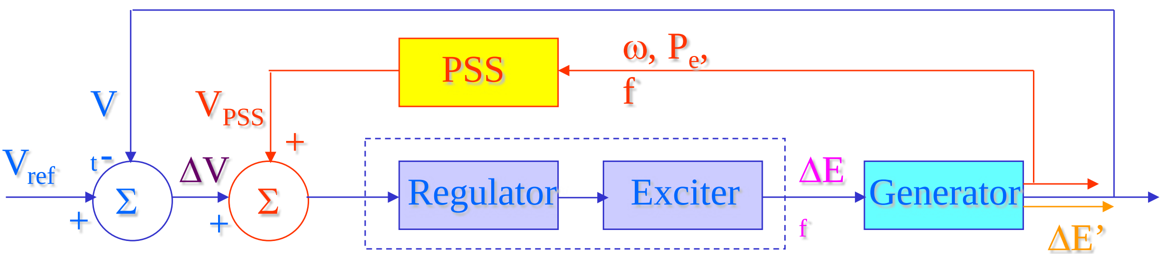
$$\xi \geq \xi(M_p)$$

$$\sigma \geq \frac{4.6}{t_s}$$

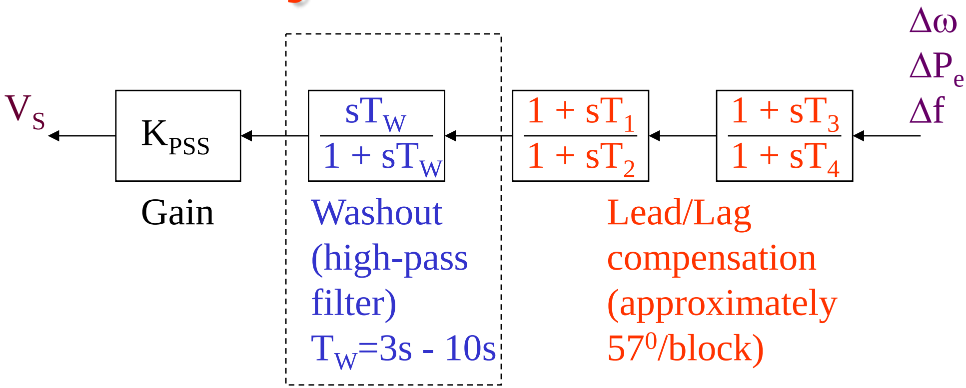


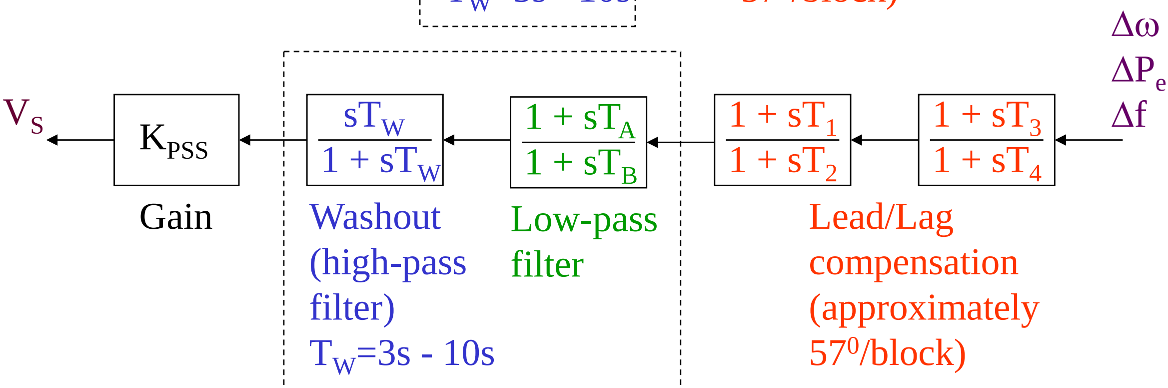
Power System Stabiliser - 1

- First introduced in 1960s to countermand the adverse effects on small signal stability due to the introduction of the automatic voltage regulator (AVR)
- In the steady state ($\Delta\omega=0$) voltage controller should be driven by the voltage error ($\Delta V=0$) only!
- In the transient sate ($\Delta\omega\neq0$) and voltage ΔV oscillates due to rotor swings as well!
- The task of the PSS is to add an additional signal which compensates for ΔV oscillations and provides a damping component that is in phase with $\Delta\omega$!



Power System Stabilisers





57^0/block – compromise between acceptable phase margin and noise sensitivity at higher frequencies

High & Low-pass Filters

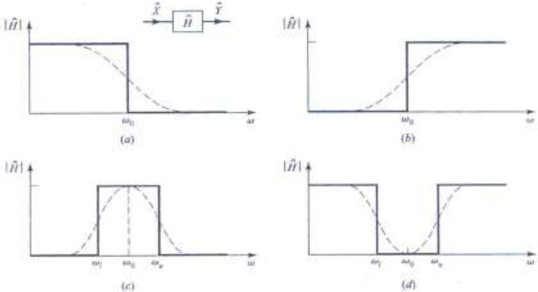
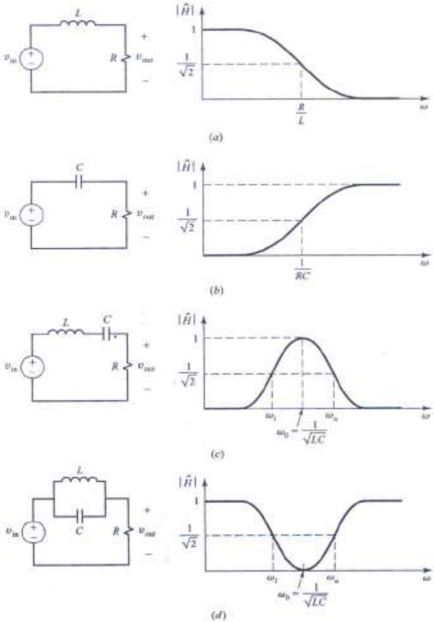
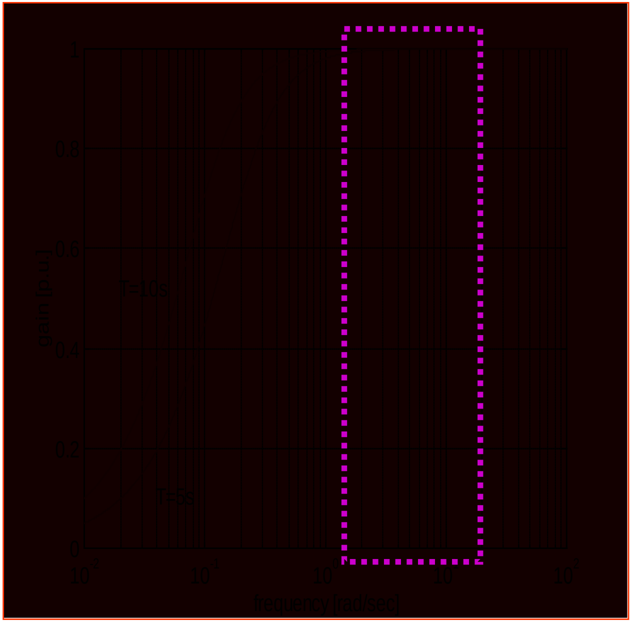


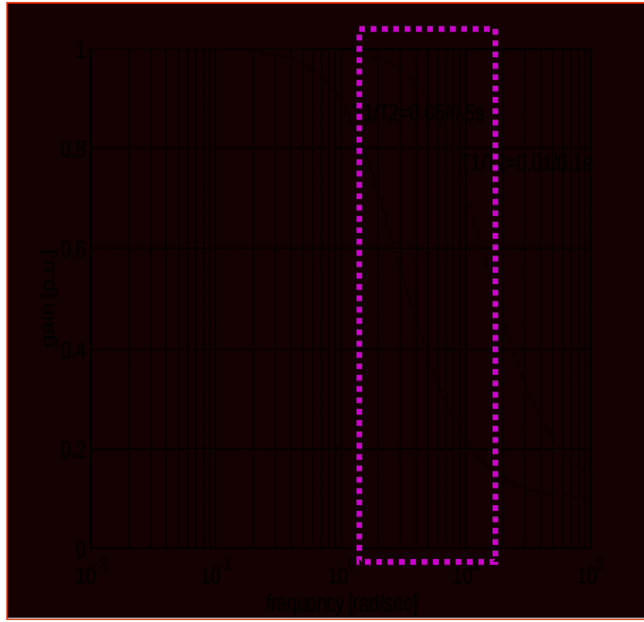
Illustration of the magnitude of the transfer function for (*a*) a lowpass filter, (*b*) a highpass filter, (*c*) a bandpass filter, and (*d*) a bandreject filter.



Circuit implementation of (*a*) a lowpass filter, (*b*) a highpass filter, (*c*) a bandpass filter, and (*d*) a bandreject filter.

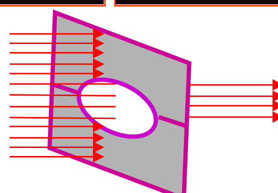
Washout & Low-pass Block - 1





Washout (high-pass filter)

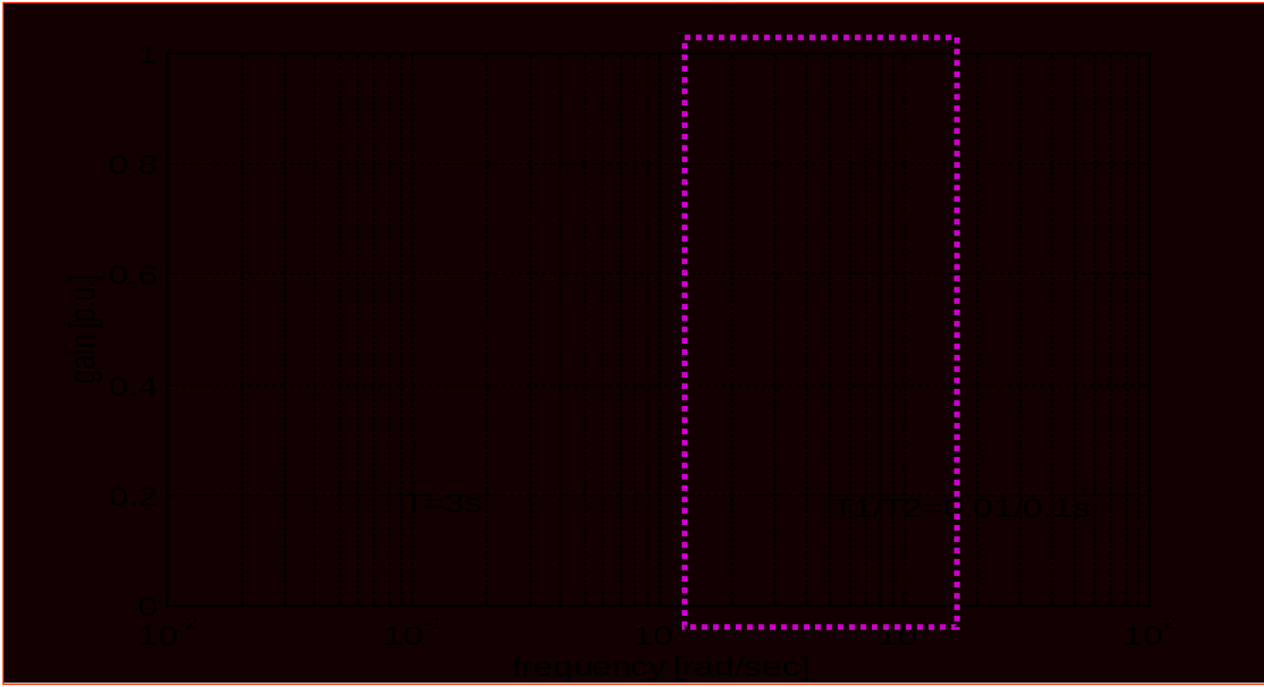
$$W(s) = \left(\frac{Ts}{1+Ts} \right)$$



Low-pass filter

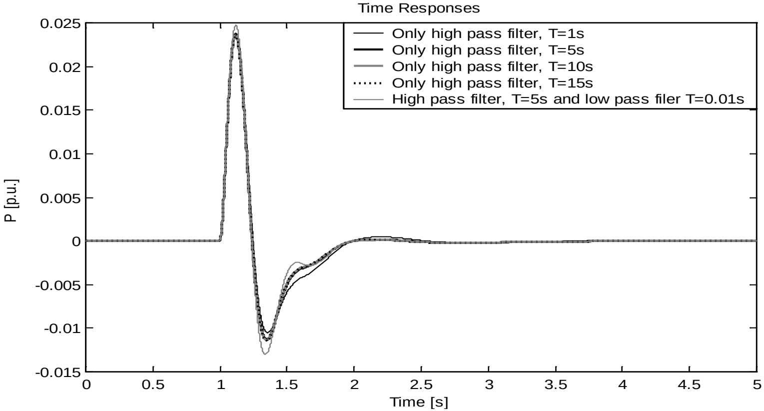
$$F(s) = \left(\frac{1+T_1 s}{1+T_2 s} \right)$$

Washout & Low-pass Block - 2

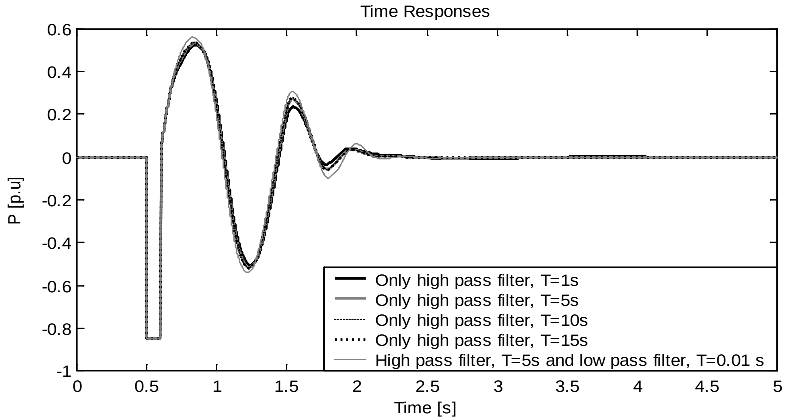


$$W(s)F(s) = \left(\frac{3s}{1+3s}\right)\left(\frac{1+0.01s}{1+0.1s}\right)$$

Washout & Low-pass Block - 3



small disturbance performance of the PSS with high and low pass filters



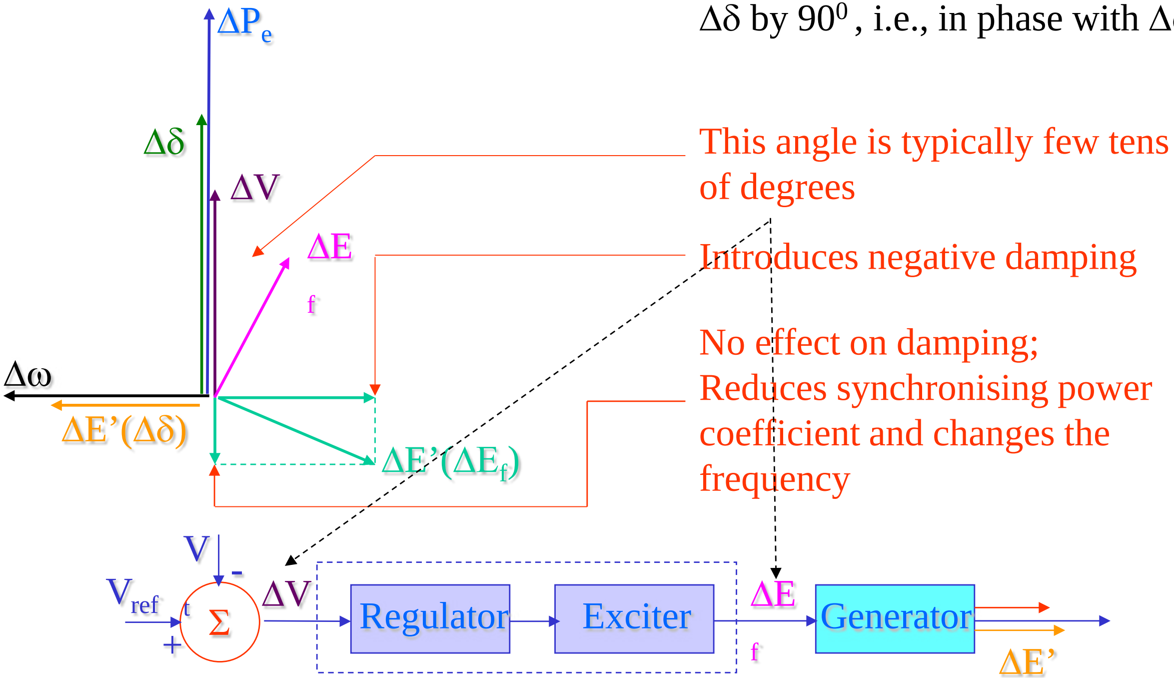
large disturbance performance of the PSS with high and low pass filters

Effects of rotor flux linkage variation

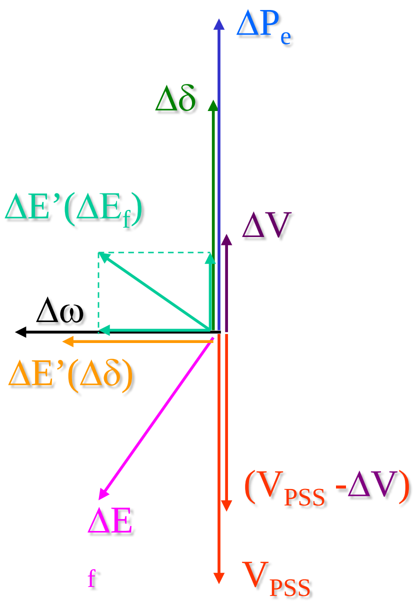
$$\Delta E' = \Delta E'(\Delta\delta) + \Delta E'(\Delta E_f)$$

Effect of voltage regulation (lags ΔE_f by 90^0)

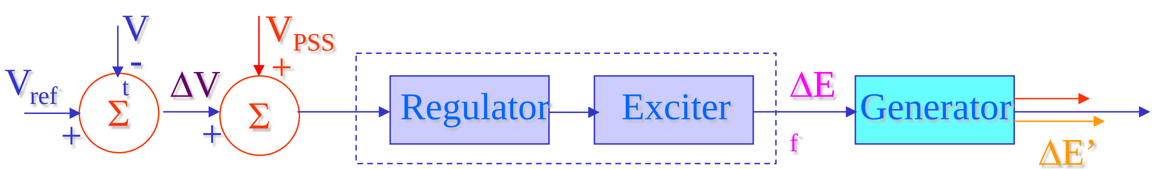
Effect of rotor swings (leads $\Delta\delta$ by 90^0, i.e., in phase with $\Delta\omega$)



Power System Stabiliser - 2



- In steady state additional signal produced by the PSS (V_{PSS}) must be equal to zero so that it does not distort the voltage regulation process!



Types of PSSs - 1

- Speed as an auxiliary signal
  - The problem with speed based PSS is the selection of measurement point along the long shaft (when applied to turbogenerators).
  - Speed should be measured at various points, and average value should be used.
  - The stabiliser gain is constrained by the influence the PSS has on torsional oscillations (when applied to turbogenerators).
- Electrical power as an auxiliary signal
  - Electrical power as input can be used if mechanical power doesn’t change. Otherwise, transient oscillations are produced in the voltage and reactive power.

Types of PSSs - 2

- Frequency as an auxiliary signal
  - The frequency of the generator terminal voltage is generally used.
  - The problem appears when large industrial loads introduce noise in the voltage waveform.
  - The solution can be improved by using transient emf and its frequency. (By adding voltage drop across X' to the terminal voltage).
  - This is generally good input signal for damping inter-area modes.

Effects of PSS

- Though PSS is tuned with respect to the damping of small-disturbance oscillations (using linearised power system model) it also improves system damping under large-disturbances (if properly tuned).
- Additional control loop can be added to the PSS to enhance first-swing stability. (Similar to forced excitation in old electromechanical AVRs when E_f is increased to its ceiling value for about 0.5s.)

Efficiency of PSS

- Generators are linked by the transmission system so voltage control of one of them influences the dynamic responses of the others!
- PSS that improves the damping of one generator does not necessarily improves the damping of the other generators.
- The local design of PSS may not provide the global optimal solution, in particular the inter-area modes may not be damped sufficiently.
- The coordinated robust tuning of PSSs is desirable.
- This may be very computationally demanding.

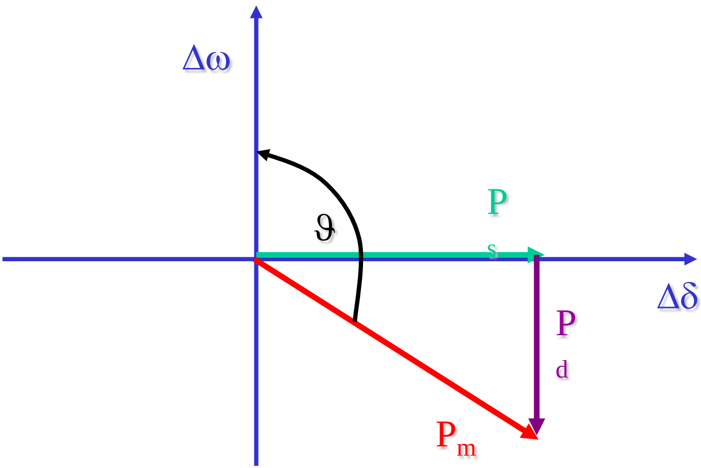
Shortcomings of PSS

- Although PSS usage can considerably benefit damping and consequently extend dynamic stability margins it does not improve transient stability margins.
- Under post fault conditions it is the AVR that improves transient stability by forcing the generator excitation voltage to its upper limit. This serves to maximise generator load torque (T_e) and minimise rotor angle swing. Over this crucial post fault period the PSS acts in opposition to the AVR, so that when a PSS is employed the field is brought off the upper limit value earlier than is the case for the AVR alone.
- Hence, a PSS signal needs to be suitably limited to ensure the maximum field voltage is maintained until after the peak rotor angle swing is reached to avoid reducing transient stability limits.

Governor contribution to damping

- A basic speed governor introduces negative damping. This is caused by the phase lag introduced by the governor and turbine in the power control loop at mechanical oscillation frequencies.

with a phase lag of $\vartheta = 0^0$ the governor loop would provide pure damping, and with a phase lag of $\vartheta = 90^0$ pure synchronising power.



Unfortunately, the phase lag introduced by a basic speed governor and turbine at mechanical oscillation frequencies is > 90^0, so that a negative damping power component is produced and system damping is reduced.

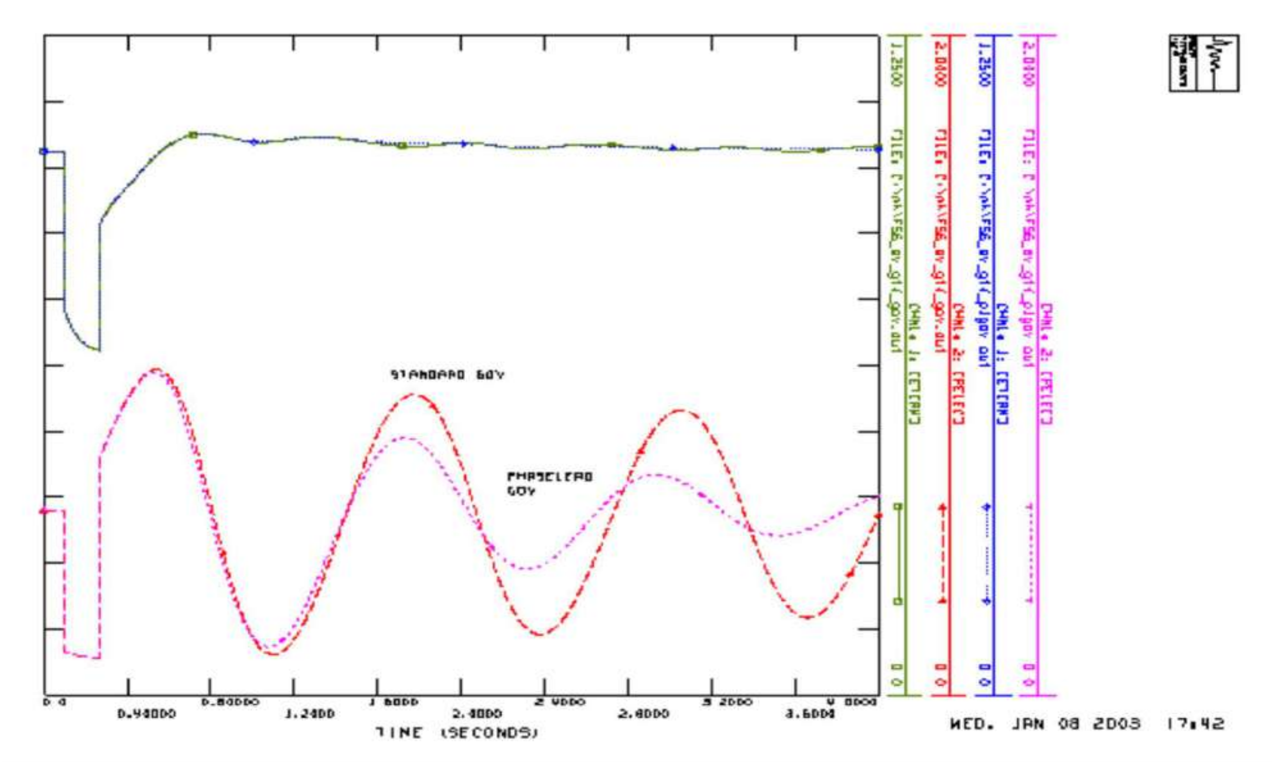
Solutions to this…so far

- One practice that is adopted to restrict the adverse influence of this negative damping characteristic is to introduce a dead band into the control loop and to slug (restrict/slow down) governor loop response.
- The introduced dead band
  - prevents the governor from responding to small signals
  - for signal magnitudes outside the dead-band the slugged response attenuates control action, thereby reducing mechanical power variation and the attendant negative damping it introduces.

PSS Applied to the Governor

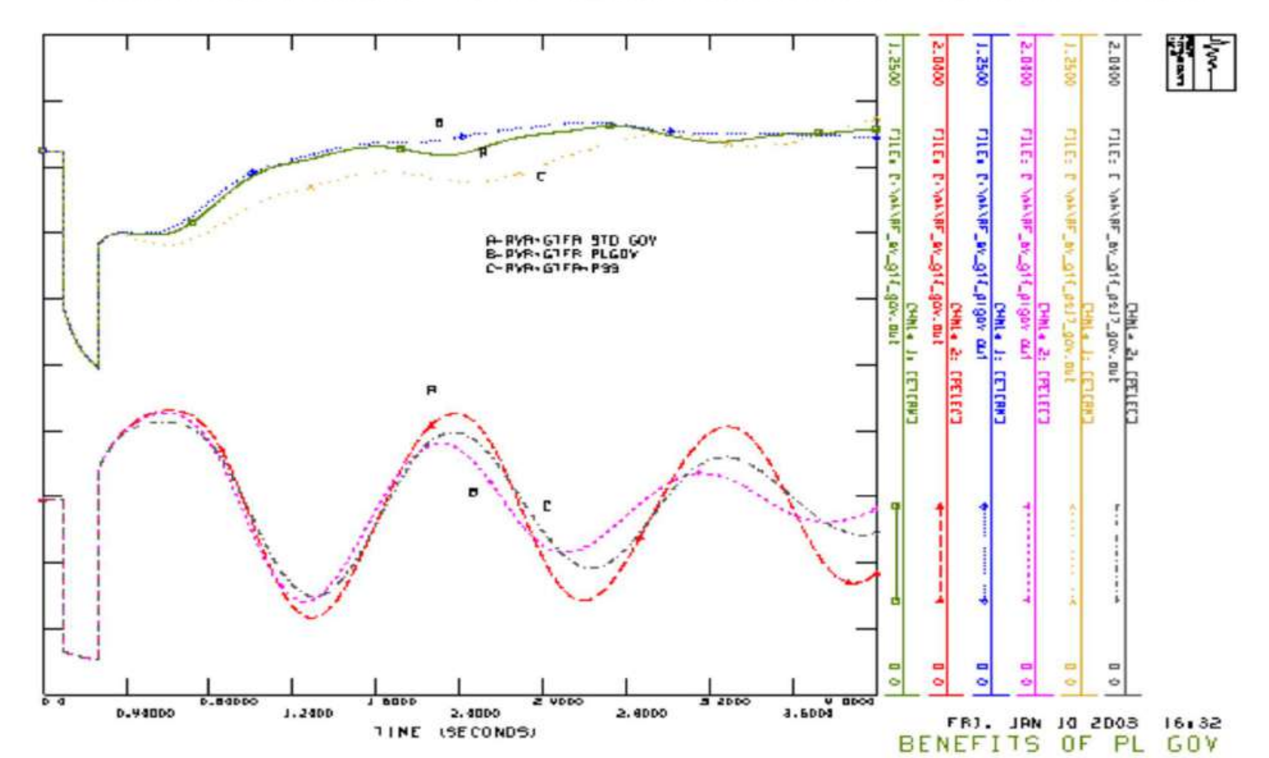
- The same principle is used as for the application in excitation loop. (First solutions proposed in early 70's for hydro turbines.)
- PSS should provide compensation of the phase shift introduced by the turbine and governor and such introduce changes in mechanical power that are in phase with speed change.
- The introduction of governor compensation to reduce the phase lag of the power loop to less than 90º, enables the turbine to contribute positively to system damping and produce a widespread improvement in the system dynamic characteristics.
- Phase lead compensation can achieve this and not only improve damping, but also, by enhancing the reduction in mechanical power following system faults, can reduce the magnitude of first rotor angle swing and thereby extend transient stability margins.
- Turbine-governor dynamics is weakly coupled with the rest of the system consequently the parameters of designed PSS do not depend on the network parameters.

Influence Of Governor Control



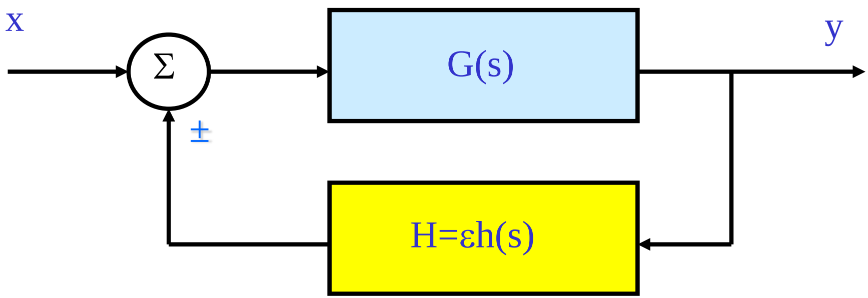
Fault study – Comparison of basic governor with phase lead governor

Influence Of Governor Control



Fault Study – (A) standard governor - AVR control; (B) Phase lead governor – AVR control; (C) Standard Governor – AVR + PSS

Background theory - 1



The closure of the feedback transfer function with very small gain $(\varepsilon \to 0)$ will have a very small effect on the open loop eigenvalues.

It can be shown that for $\varepsilon \to 0$:

$$|\Delta\lambda_i| = \pm |R_i| |H(\lambda_i)|$$

+ for positive feedback
– for negative feedback

Residue corresponding to open-loop eigenvalue λ_i

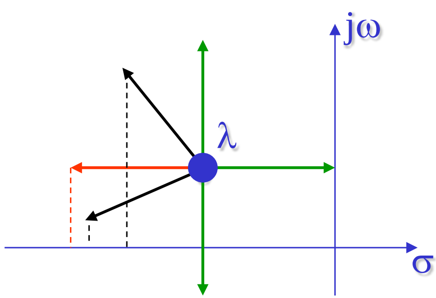
Tuning of Controllers - 2

Larger residue will cause a larger change in the eigenvalue.

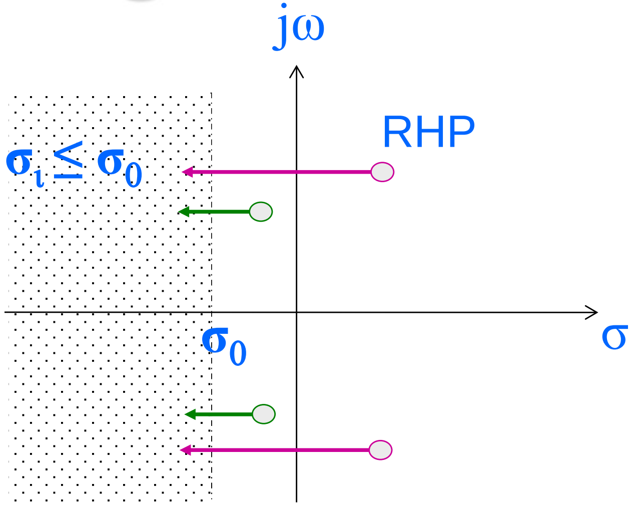
$$\boxed{\left|\Delta\lambda_i\right| = \pm\left|R_i\right|\left|H(\lambda_i)\right|}$$

For all points of the circumference whose centre is at λ_i and whose radius is $|\Delta\lambda|$, corresponds the same gain value $|H(\lambda_i)|$ of the additional feedback.

Therefore, for a given gain of the feedback $|H(\lambda_i)|$, the maximum increase in damping corresponds to the shift of eigenvalue in the direction -180^0.



Shifting critical modes –1

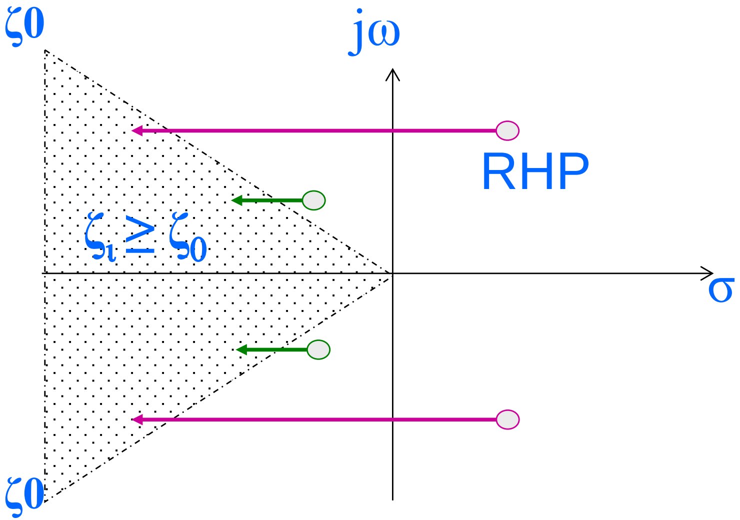


$$\lambda = \sigma \pm j\omega$$

$$\zeta = \frac{-\sigma}{\sqrt{\sigma^2 + \omega^2}}$$

Shifting of real part of Eigenvalues

Shifting critical modes – 2

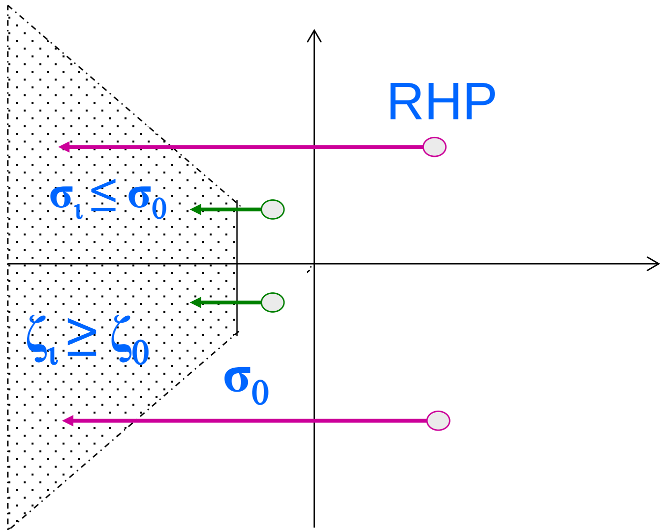


$$\lambda = \sigma \pm j\omega$$

$$\zeta = \frac{-\sigma}{\sqrt{\sigma^2 + \omega^2}}$$

Shifting damping factor of Eigenvalues

Shifting critical modes – 3

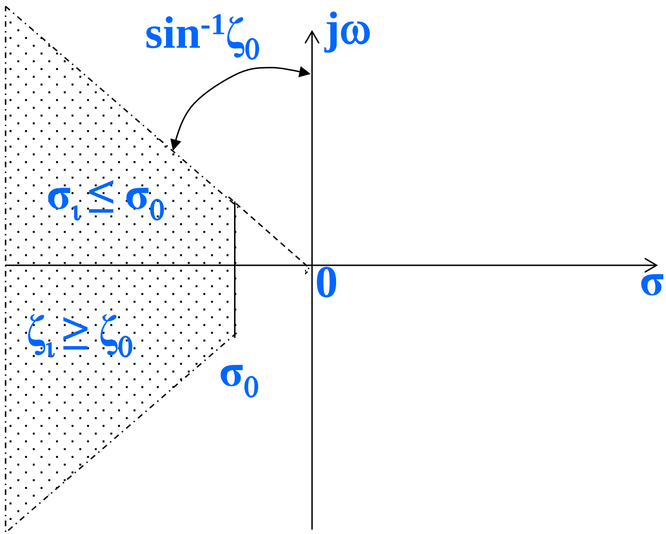


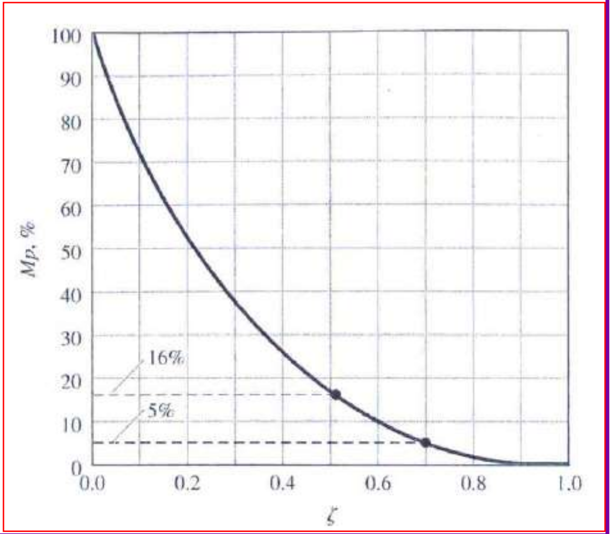
$$\lambda = \sigma \pm j\omega$$

$$\zeta = \frac{-\sigma}{\sqrt{\sigma^2 + \omega^2}}$$

Shifting both real part and damping factor of Eigenvalues

Shifting critical modes – 4





Tuning of Controllers - 3

The phase shift through the feedback controller H(s) at the frequency of the mode (eigenvalue) whose damping should be increased, should be:

$$\boxed{\theta_{H(j\omega_i)} = \pm 180^0 - \theta_{R_i}}$$

PSS in a positive feedback loop

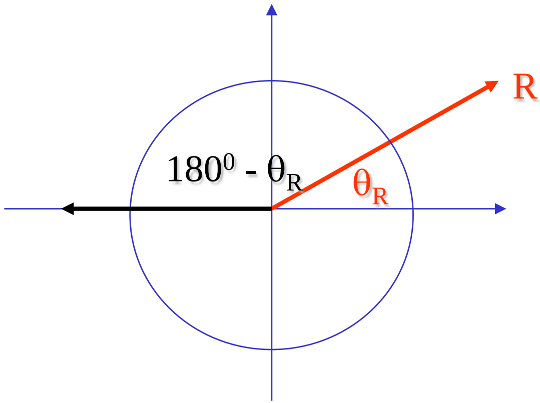
$$\boxed{\theta_{H(j\omega_i)} = \pm 180^0 - (\theta_{R_i} + 180^0)}$$

PSS in a negative feedback loop

With such phase shift the increment of the eigenvalue defined by

$$|\Delta\lambda_i| = |R_i|\,|H(\lambda_i)|$$

will be in -180^0 direction.



Tuning of PSS - 1

PSS Transfer function

(Time constants of wash-out and low pass filter are pre-set to T_W=1s-20s (typically 3s-10s) and T_B/T_A=0.1126/0.0563<10).

$$H_{PSS}(s) = \mathrm{K}_{\mathrm{PSS}}\left(\frac{1+\mathrm{T}\mathrm{s}}{1+\alpha T\ \mathrm{s}}\right)^{N} W(s)F(s)$$

$$N = \frac{\vartheta_{PSS}}{57^{o}} = \begin{cases} 1 & \vartheta_{PSS} \le 60^{o} \\ 2 & \vartheta_{PSS} \le 120^{o} \\ 3 & 120^{o} < \vartheta_{PSS} \le 180^{o} \end{cases}$$

To ensure the acceptable phase margin and acceptable noise sensitivity at high frequencies, (to limit the value of α to α <10 to avoid excessive noise interference)

$$\alpha = \frac{1-\sin\dfrac{\vartheta_{\mathrm{PSS}}}{N}}{1+\sin\dfrac{\vartheta_{\mathrm{PSS}}}{N}}$$

$$\mathrm{T} = \frac{1}{\omega_{\mathrm{i}}\sqrt{\alpha}}$$

General formula for either lag or lead compensator. In the case when a phase leg compensator ($\theta_{PSS}<0$) is needed to stabilise the system, θ_{PSS} should be substituted as negative

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Tuning of PSS – 1a

$$H_{PSS} = K_{PSS}\left(\frac{1+aTs}{1+Ts}\right)^{n} W(s)F(s)$$ PSS Transfer function

$$\alpha = \frac{\pi - \theta_R}{n} \qquad a = \frac{1+\sin\alpha}{1-\sin\alpha} > 1 \qquad T = \frac{1}{\omega_i\sqrt{a}}$$

$W(s)$ - washout (high - pass filter) block

$F(s)$ - low - pass filter block

n - number of required lead/lag blocks

Tuning of PSS - 2

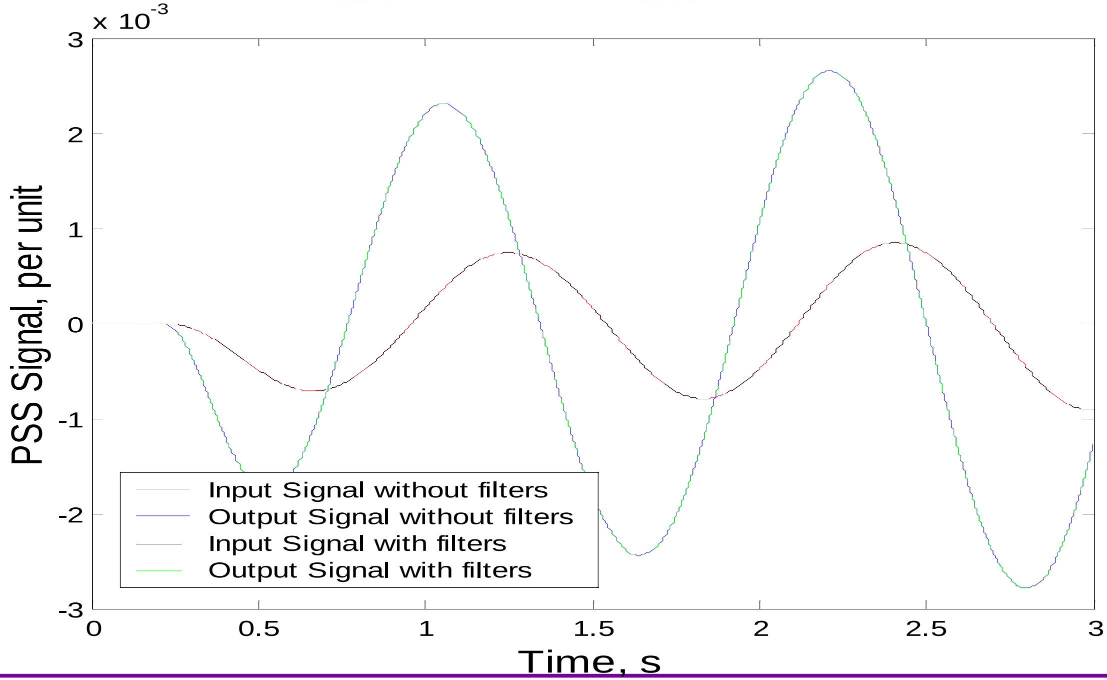
- In tuning of lead/lag blocks, phase compensation between the exciter input and the generator output (speed or electrical power), slight under-compensation (about 10^0 over the frequency range of interest) is better so that the PSS does not contribute to the negative synchronising torque component.
- Usually two to four lead/lag blocks are needed, each of them should not provide more than about 57^0 compensation (not necessarily true with modern digital PSSs). (To ensure an acceptable phase margin and an acceptable noise sensitivity at high frequencies. There is also a physical limitation of the lead-lag circuit, which is only practical to achieve 60 degrees and below.)
- When the stabiliser limits are included they should be at the lower side in the range of -0.05 p.u. to -0.1 p.u. and at the upper side in the range of 0.1 p.u. to 0.2 p.u.

Tuning of PSS - 3

- Washout block (high-pass filter) prevents steady state changes in speed from modifying the field voltage.
- It should not interfere with the stabilising signals at the frequencies of interest (0.1Hz - 2Hz).
- Time constants is usually between 1s and 20s. For local modes lower values (1.5s) are appropriate and for inter-are modes higher values (10s) are recommended.
- Low-pass filter provides high attenuation at torsional frequencies.

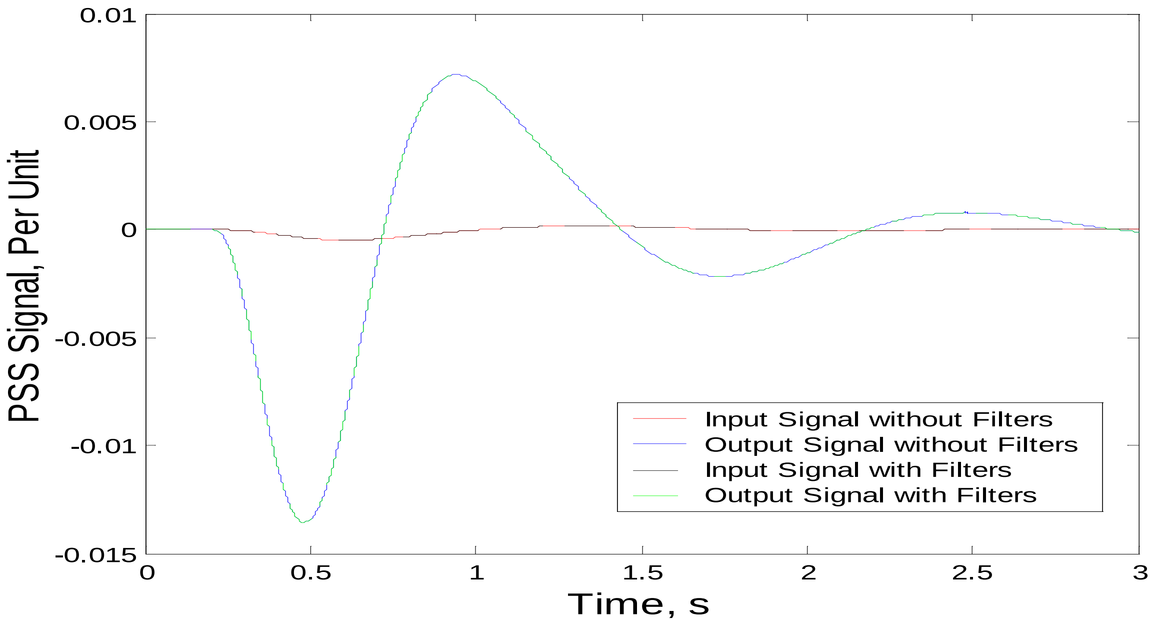
Effects of the PSS

- Inclusion of a 60 degree phase lead with gain of 1



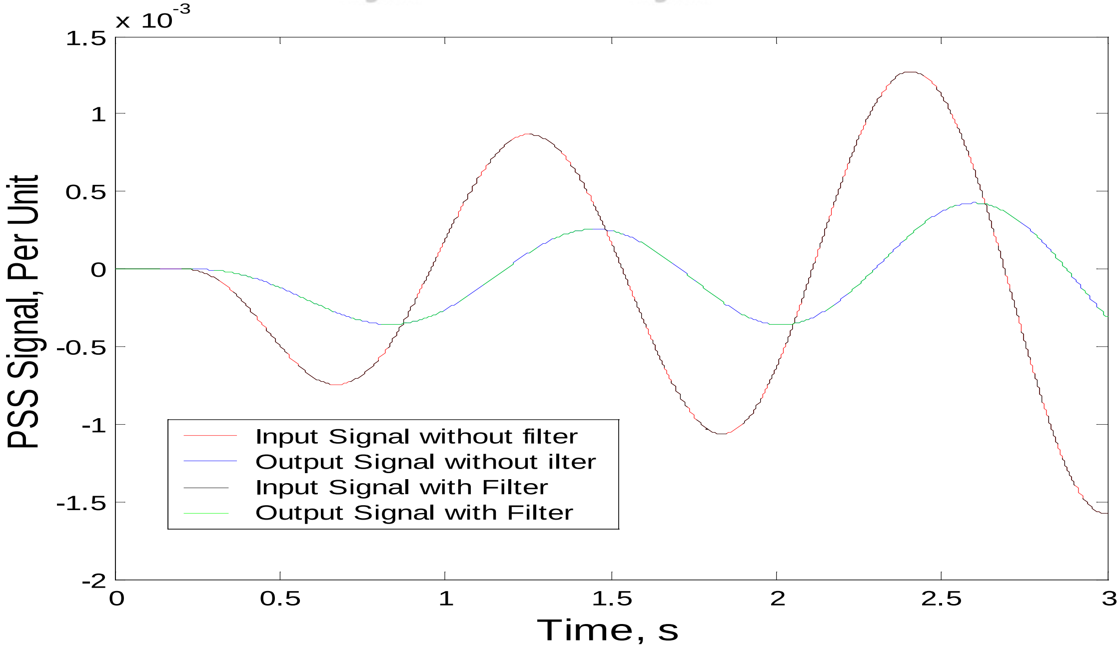
Effects of the PSS

- Increasing gain by 10 for case of phase lead

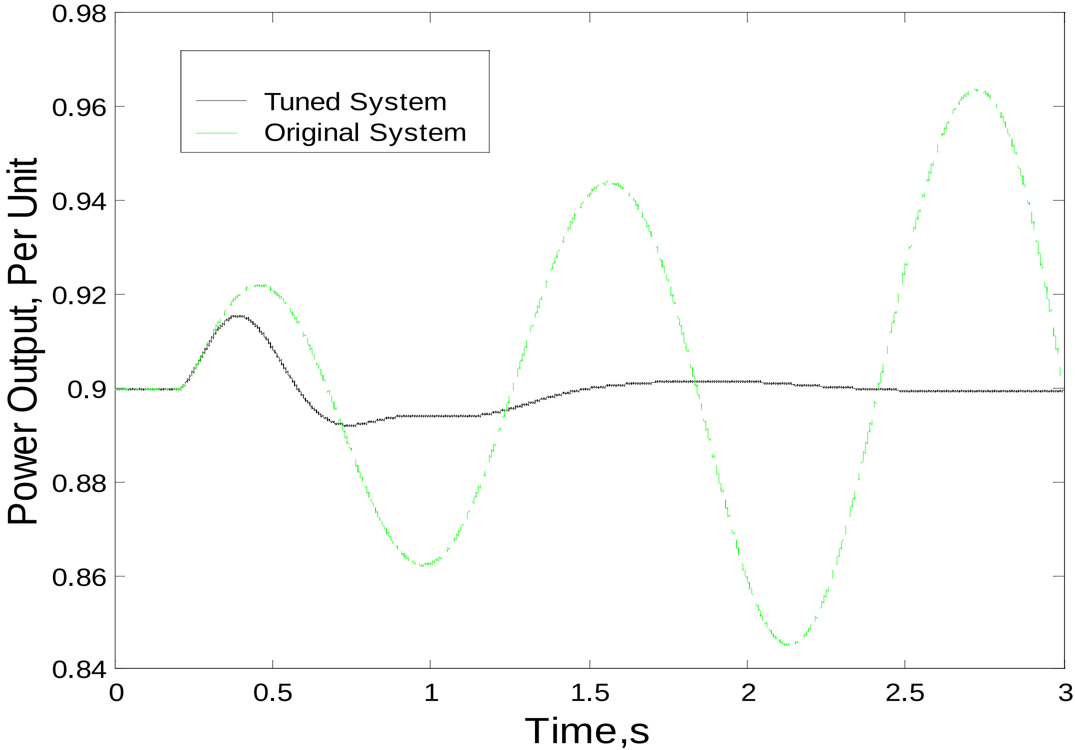


Effects of the PSS

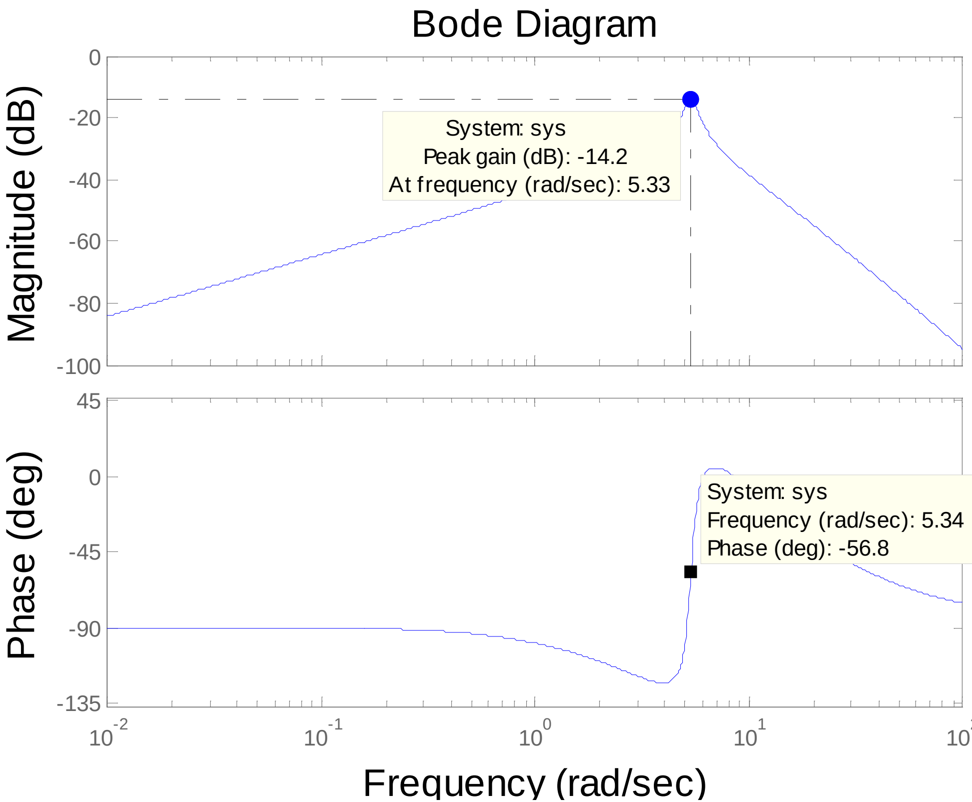
- Inclusion of a 60 degree phase lag with gain of 1



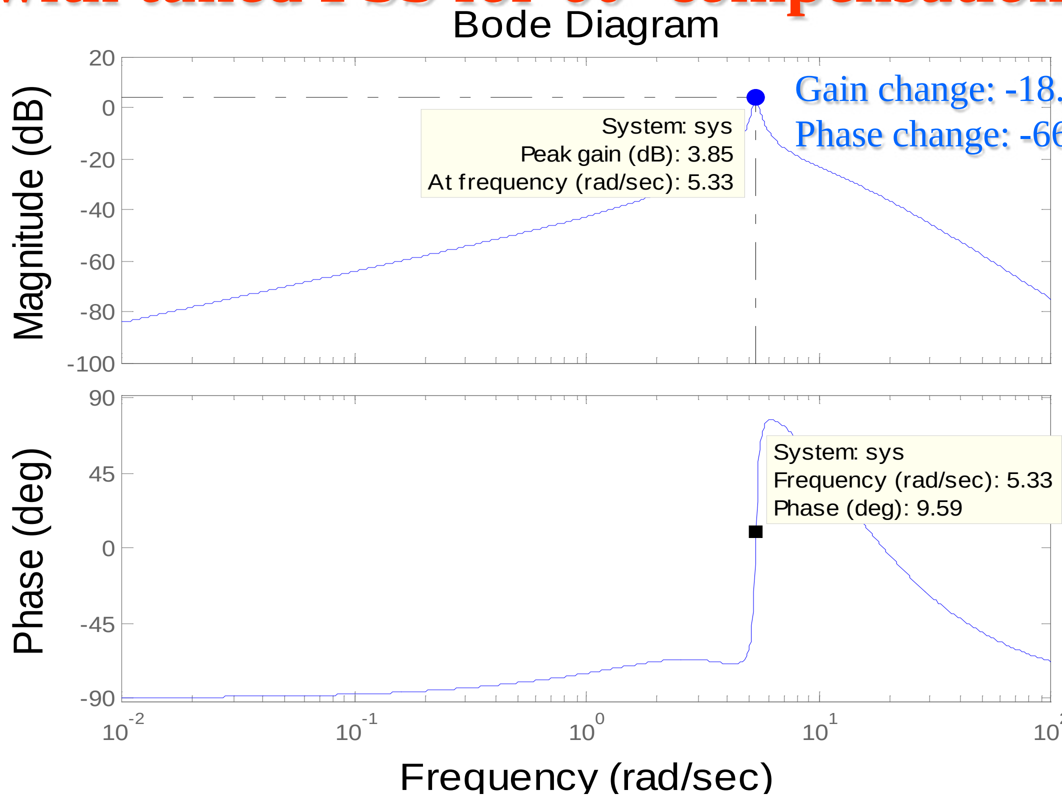
System response with tuned PSS



Open loop TF


Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)
System: sys
Peak gain (dB): -14.2
At frequency (rad/sec): 5.33
System: sys
Frequency (rad/sec): 5.34
Phase (deg): -56.8
0
-20
-40
-60
-80
-100
45
0
-45
-90
-135
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}

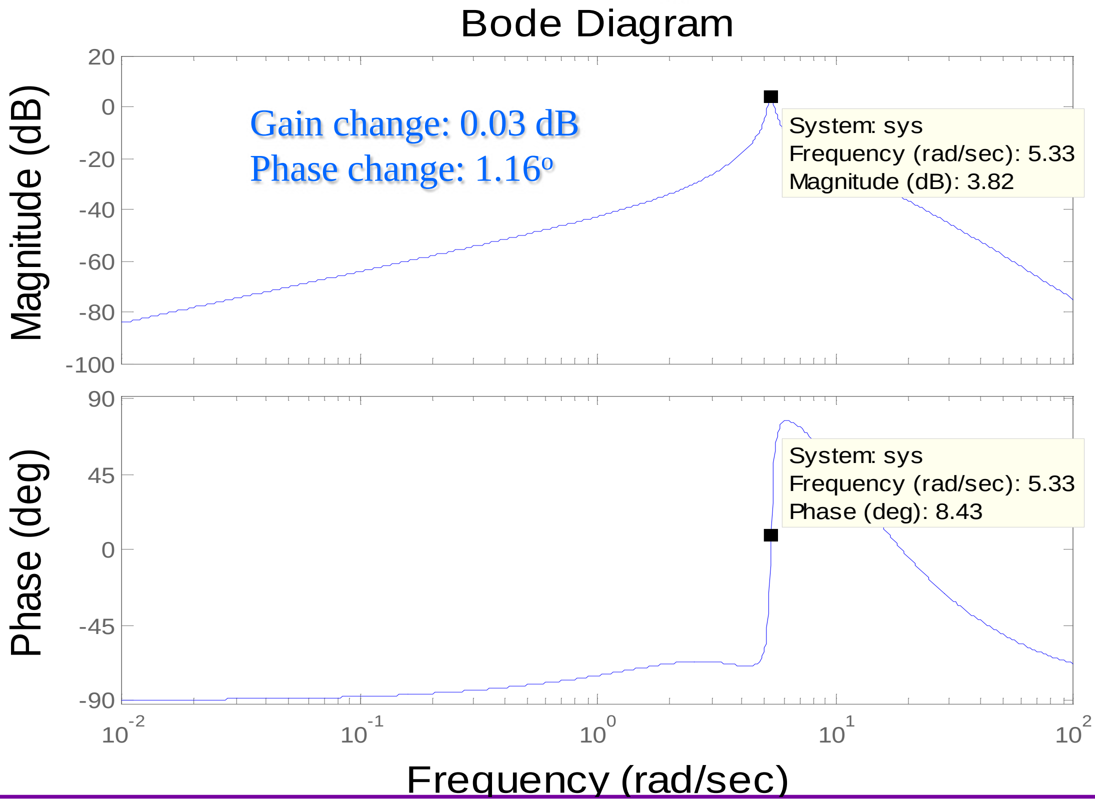
Closed loop TF without filters and with tuned PSS for 60^0 compensation



Gain change: -18.05 dB

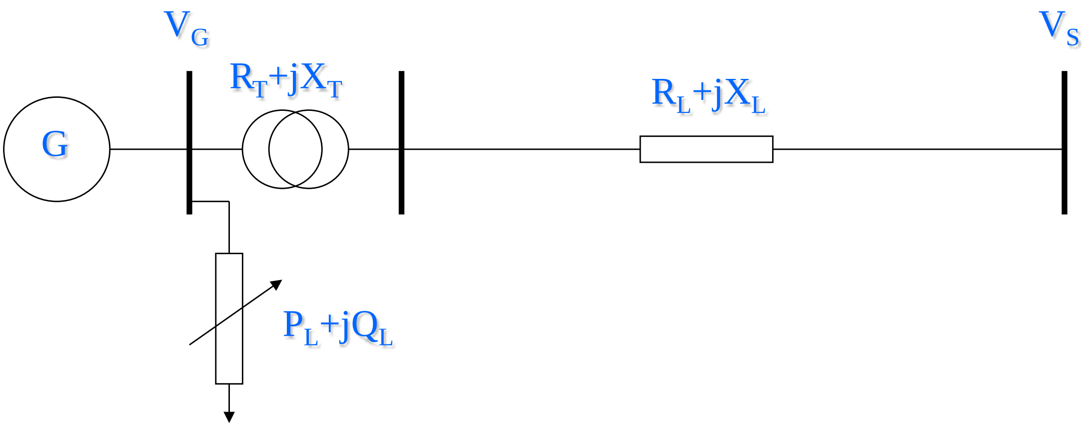
Phase change: -66.39^o

Closed loop TF with filters and with tuned PSS for 60^0 compensation



Tuning of PSS: Case Study - 1

- Single machine (3rd order model) connected to the infinite bus through transformer and transmission line with a local dynamic load and equipped with AVR and PSS.



Tuning of PSS: Case Study - 1

Original system

-0.1180 + 6.1786i

-0.1180 - 6.1786i

-0.1371

With slow acting AVR

-0.1098 + 6.1335i

-0.1098 - 6.1335i

-2.0392 + 1.6511i

-2.0392 - 1.6511i

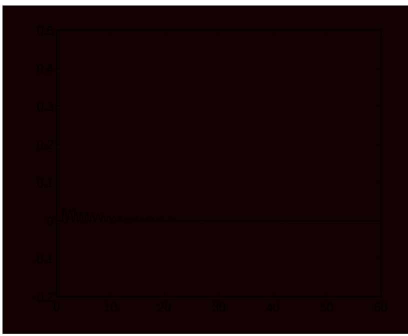
With fast acting AVR

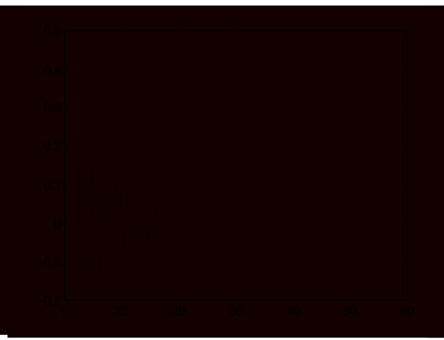
+0.1163 + 6.3885i

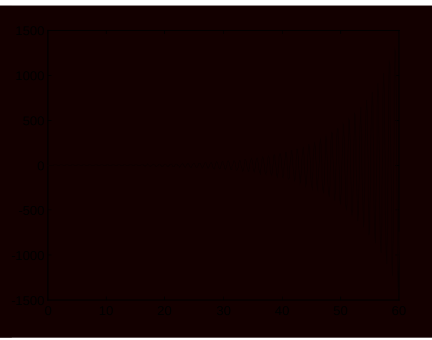
+0.1163 - 6.3885i

-10.2653 +14.2354i

-10.2653 -14.2354i







Tuning of PSS: Case Study - 1

With fast acting AVR

+0.1163 + 6.3885i
+0.1163 - 6.3885i
-10.2653 +14.2354i
-10.2653 -14.2354i

Critical eigenvalues

$$R_1 = \mathbf{c}\mathbf{v}_1\mathbf{w}_1\mathbf{b} = \mathbf{w}_1\mathbf{b}\mathbf{c}\mathbf{v}_1$$

Residue of the open loop transfer function between voltage reference and speed of the generator corresponding to 1st eigenvalue

$\mathbf{v}_1$ - Right eigenvector corresponding to 1st eigenvalue

$\mathbf{w}_1$ - Left eigenvector corresponding to 1st eigenvalue

b - Input vector corresponding to V_{ref}

c - Output vector corresponding to ω

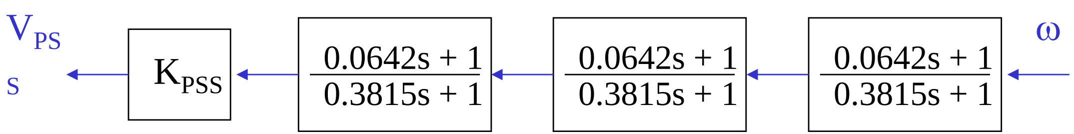
Tuning of PSS: Case Study - 1

$R_1 = 41.3030 - j39.6637$
$|R_1| = 57.2638$
$\theta_{R1} = -43.8401^0$

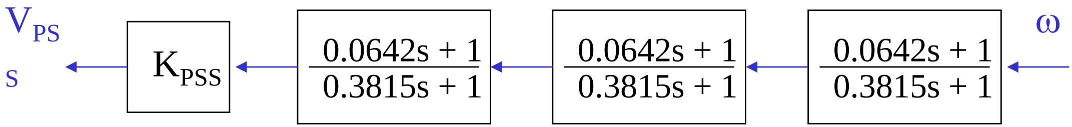
Required phase compensation by the PSS:
$\theta_{PSS} = -180^0 - \theta_R = -136.1599$

$\theta_{PSS} > 2 \times 57^0 \Rightarrow$ PSS with three lead-lag blocks is needed

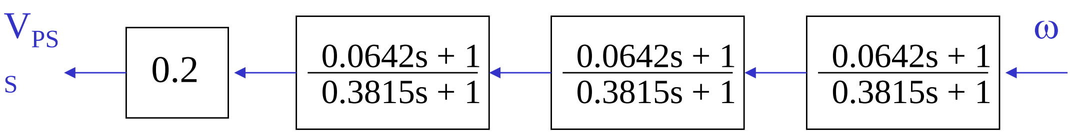
Parameters of each of three blocs are: $a = 0.1683$ and $T=0.3815$, i.e., $T_1 = 0.0642$ and $T_2 = 0.3815$.



Tuning of PSS: Case Study - 1



With K_{PSS}=0.41 system becomes unstable through a control mode 0.0297 ± j1.7950, so gain was set to 50% of that (critical) value to K_{PSS}= 0.2

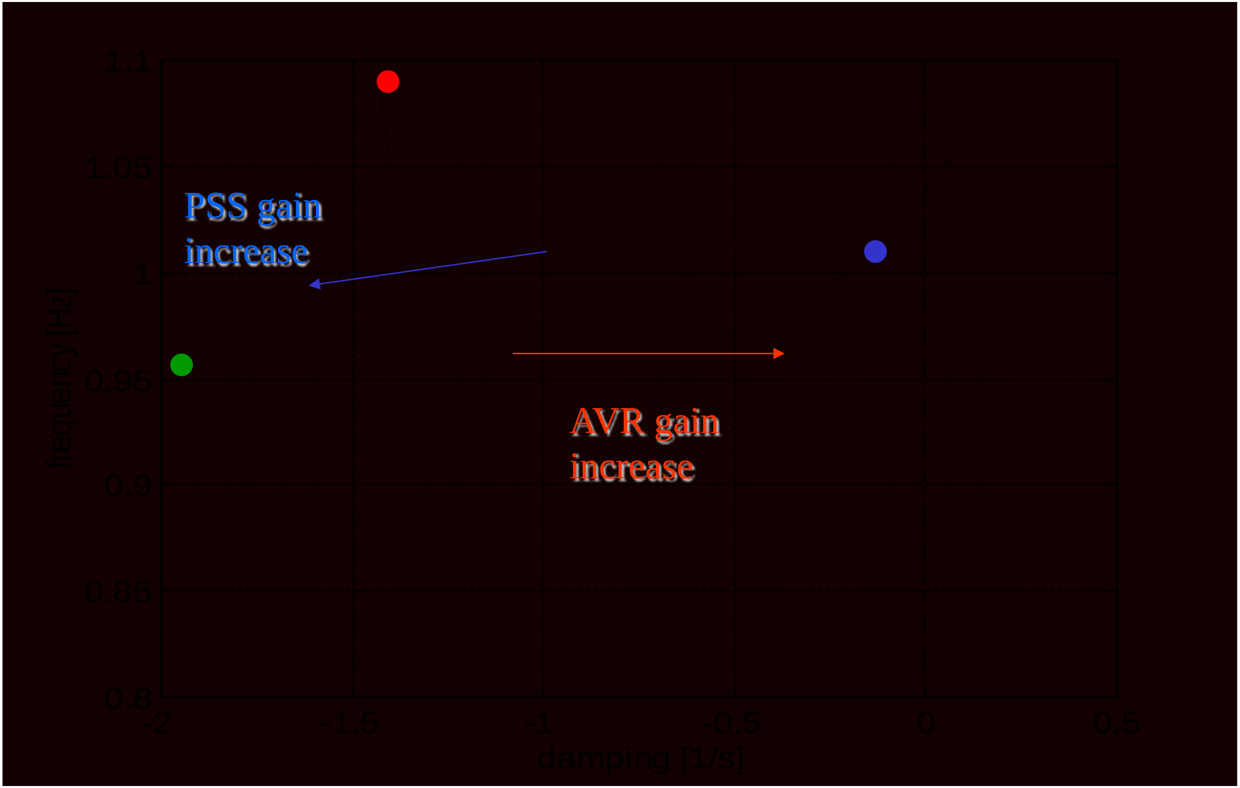


Tuning of PSS: Case Study - 1

With fast acting AVR
+0.1163 + 6.3885i
+0.1163 - 6.3885i
-10.2653 +14.2354i
-10.2653 -14.2354i

With fast acting AVR and PSS
-10.2202 +14.2275i
-10.2202 -14.2275i
-0.5689 + 6.2779i
-0.5689 - 6.2779i
-4.8145
-0.8846 + 1.7537i
-0.8846 - 1.7537i

Influence of AVR and PSS Gain



Choosing PSS Location

- PSS contributes to damping of inter-area modes largely by modulating system loads, while its influence on local modes is only slightly affected by load characteristics.
- In order to enhance damping of inter-area modes participation factors corresponding to speed deviations of generators and residues (associated with those modes) should be used to identify the best generator for the installation of PSS.
- High values of open-loop residues associated with the inter-area modes indicate the best generators for installing PSS.

Tuning of PSS: Case Study - 2

Large system (10 generators, 39 buses) application:

Unstable

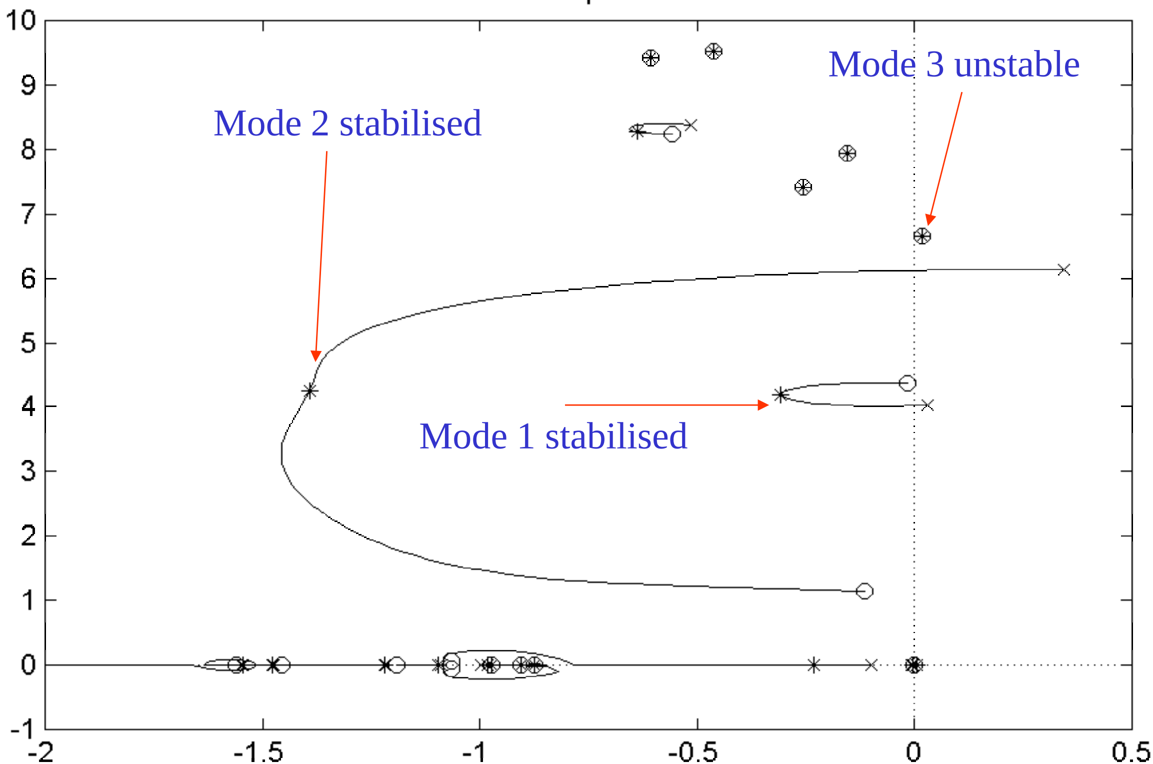
	λ [p.u.]	F (Hz)	Damping Factor	Generator Involvement
1	0.029 + j4.04	0.64	-0.007	**2** vs. the rest
2	0.34 + j6.13	0.98	-0.056	**9**
3	0.016 +j 6.66	1.06	-0.002	**3** & 1 vs. the rest
4	-0.26 + j7.42	1.18	0.034	**4** vs. 6
5	-0.15 + j7.94	1.26	0.019	**1** vs. 3
6	-0.51 + j8.37	1.33	0.061	**8**
7	-0.61 + j9.41	1.50	0.064	**7** vs. 6
8	-0.46 + j9.53	1.52	0.048	**5** vs. 4 & 7
9	-3.17 + j9.59	1.53	0.331	**10**

Based on participation actors

First mode to be stabilised

Mode	Generators ranked in order of involvement		
	Part. factors	Speed Residue	Power Residue
1	2	5, 4, 6	2, 9
2	9	**9**	9
3	3, 1	3, 1	3, 1
4	4, 6	4, 6	4, 6
5	1, 3	1, 3	3, 1
6	8	8	8
7	7, 6	7, 6	7, 6
8	5	5	5
9	10	10	10

Tuning of PSS: Case Study - 2



Tuning of PSS: Case Study - 2

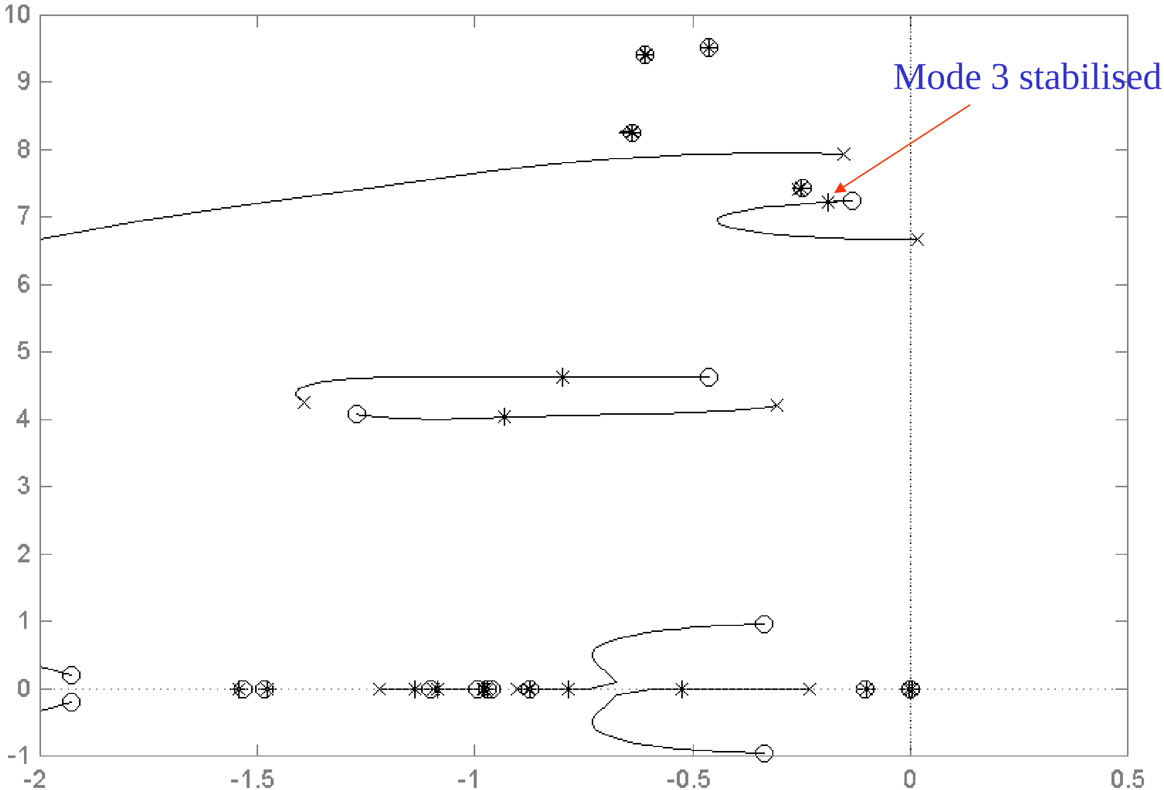
The PSS at generator 9 successfully stabilised mode 2 and also mode 1.
The mode 3 remained unstable as it is not controllable by PSS at generator 9.
New set of residues for mode 3 is calculated with the PSS at generator 9:

Second PSS
at generator 3

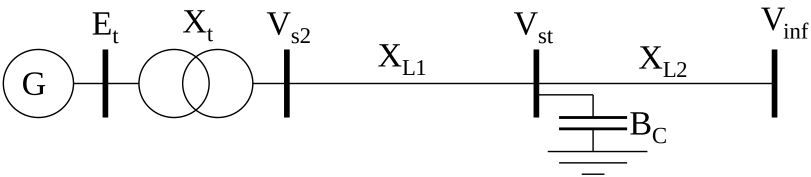
gen. #	ω participation factors	ω-vr residues	Pe-vr residues
1	**0.1633**	**0.0117**	**4.7434**
2	0.0044	0.0001	0.8167
3	**0.1952**	**0.0162**	**7.7129**
4	0.0599	0.0057	1.9735
5	0.0264	0.0029	1.1196
6	0.0293	0.0018	0.8460
7	0.0162	0.0015	0.5170
8	0.0009	0.0001	0.0238
9	0.0001	0.0000	0.0028
10	0.0007	0.0001	0.0471
rank	3, 1	3,1	3,1

With second PSS at generator 3 the system was successfully stabilised.

Tuning of PSS: Case Study - 2

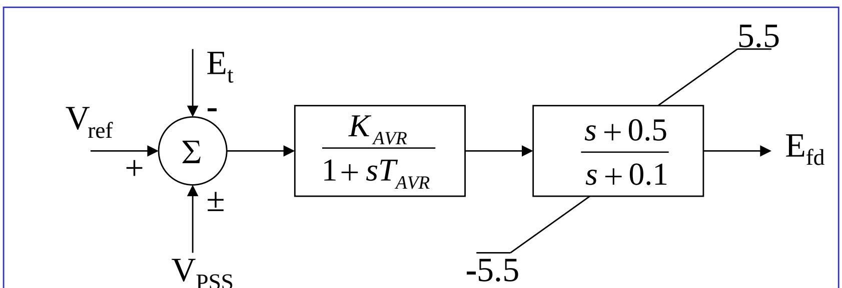


System, PSS and AVR model: Case study -3



$$H(s) = k\left(\frac{1+Ts}{1+\alpha T\,s}\right)^N W(s)F(s) = k\left(\frac{1+Ts}{1+\alpha Ts}\right)^N \left(\frac{sT_w}{1+T_w s}\right)\left(\frac{1+T_A s}{1+T_B s}\right)$$

PSS



AVR

Original set of eigenvalues: Case study -3

-33000

-31200

-3110

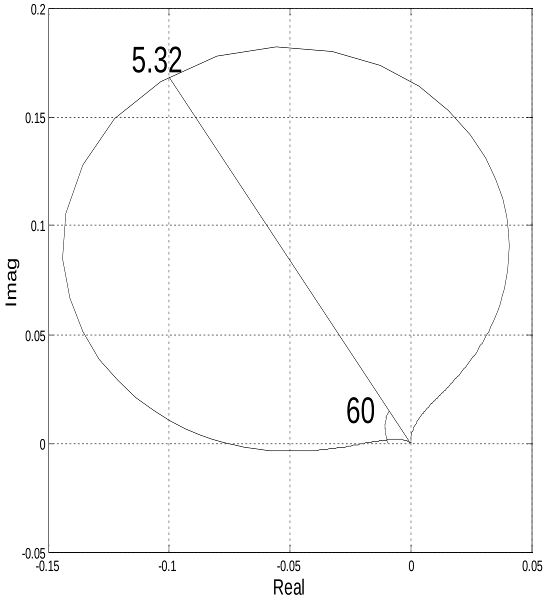
-40.3

-12.2±j2.14 Control mode

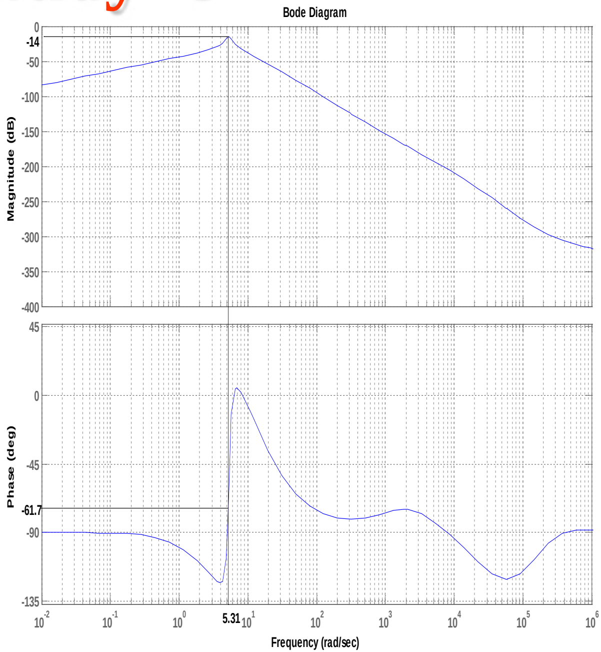
0.349±j5.32 Electromechanical mode

-0.533

Bode and Nyquist plots: Case study -3



Nyquist Plot of the open loop system based on speed with positive feedback (required compensation 60^0)



Bode plots of the speed transfer Function (required compensation 62.2^0)

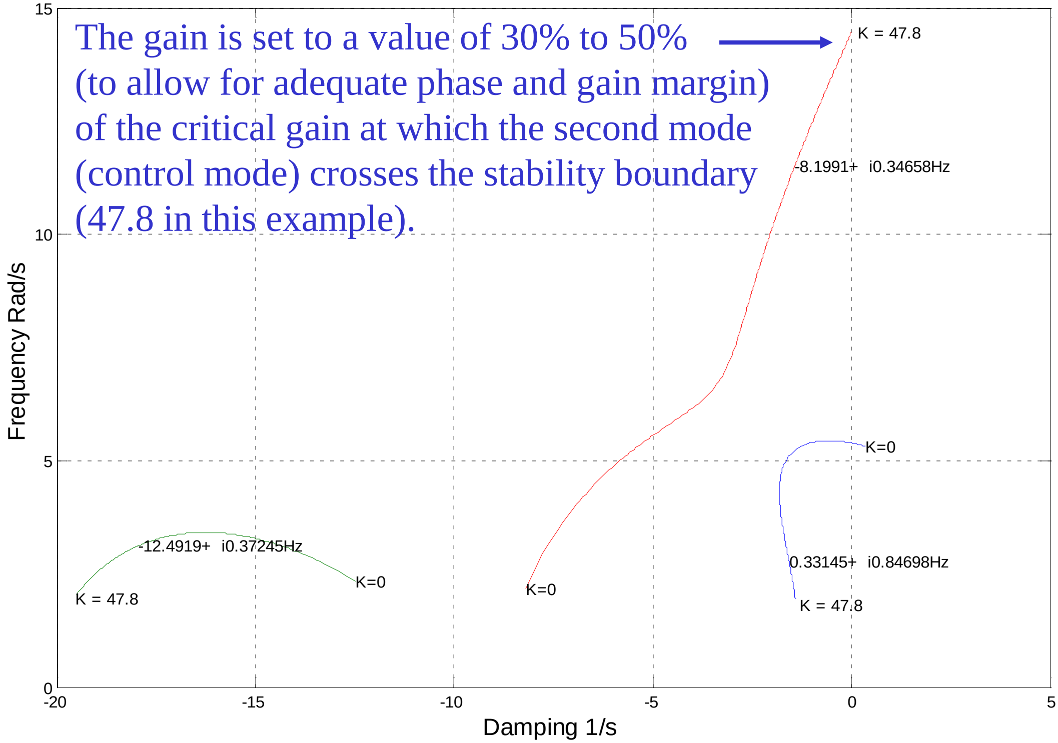
Required phase shifts: Case study -3

The phase angle of the residue of the open loop TF $\Delta\omega/\Delta Vref$ was: 119.84^0

The phase angle of the residue of the open loop TF $\Delta P_e/\Delta Vref$ was: 26.09^0

Case	PSS input signal & feedback $\frac{\Delta\omega}{\Delta V_{ref}}$	Required Phase Compensation
1	$\Delta\omega$ (Positive feedback)	60.16°
2	$\Delta\omega$ (Negative feedback)	-119.84°
3	ΔP_e (Positive feedback)	153.91°
4	ΔP_e (Negative feedback)	-26.09°

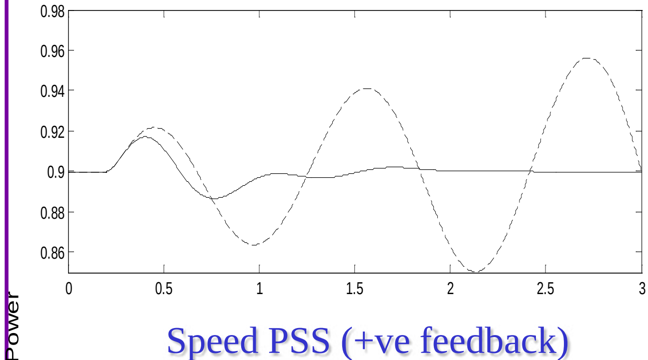
Setting the gain of the PSS: Case study -3



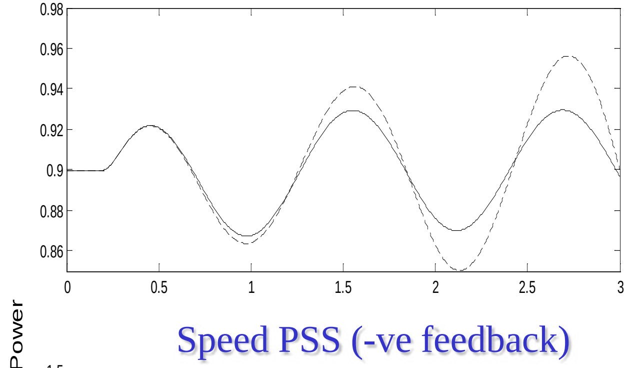
Parameters of the PSS: Case study - 3

PSS	θ_{PSS}	N	$T_1 = T$	$T_2 = \alpha T$	k
$\Delta\omega$ (+ve fdbk)	60.16°	2	0.3318	0.1067	15.93
$\Delta\omega$ (-ve fdbk)	-119.84°	3	0.0878	0.4029	50.5
ΔP_e (+ve fdbk)	153.91°	3	0.549	0.06886	0.005
ΔP_e (-ve fdbk)	-26.09°	1	0.1195	0.3183	1.2

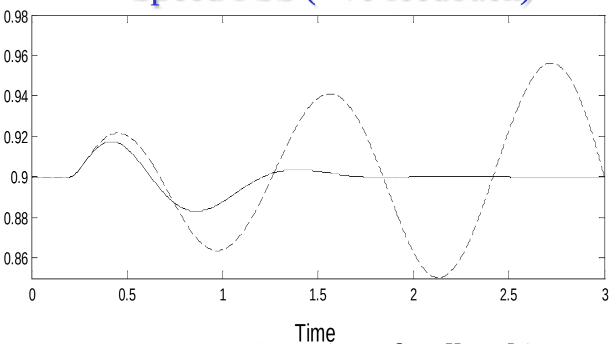
Time responses: Case study -3



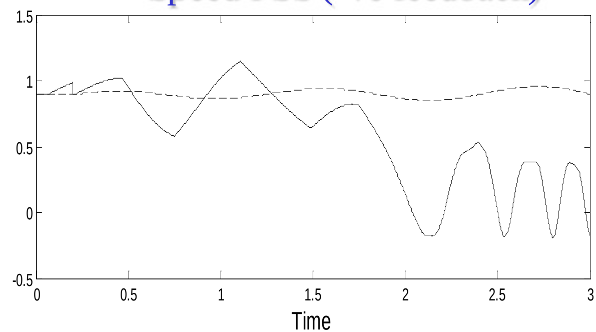
Speed PSS (+ve feedback)



Speed PSS (-ve feedback)



Power PSS (-ve feedback)



Power PSS (+ve feedback)

Dotted line – response of the system without PSS

Case study – 3: Comments - 1

- In the case of electrical power based PSS with positive feedback and speed based PSS with negative feedback the system cannot be stabilized if the "standard" tuning procedure is used (followed).
- The use of an excessive phase lead and lag respectively, in these two cases caused the system to have unsatisfactory (unstable, oscillatory) response.

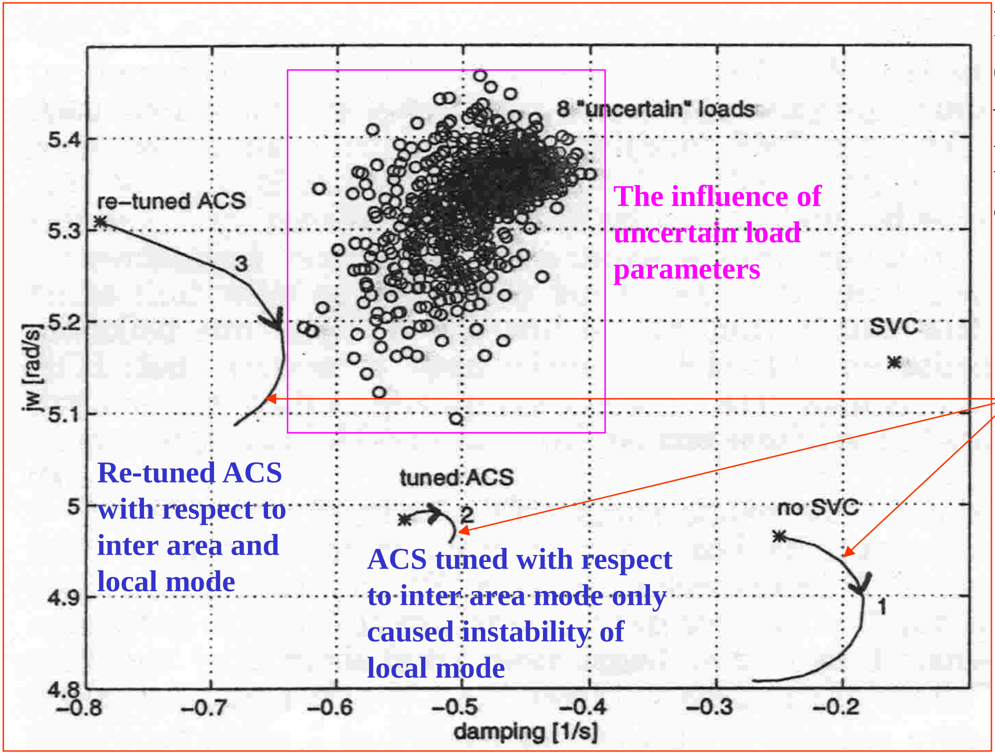
Case study – 3: Comments - 2

- Excessive phase lead tends to emphasize the gain, resulting in the system being unable to maintain stability at very small gains.
- Excessive phase lag on the other hand, forces the cross over frequency of the system to a low value and hence reduces the gain margin.
- Therefore these combinations of PSS input signal and the type of the feedback should not be utilized to tune controllers and the phase compensation should ideally be kept to a minimum.

Additional Controllers - 1

- SVC with an additional controller
  - By rapidly controlling voltage and reactive power an SVC can contribute to the enhancement of damping of system oscillations. The additional controllers are needed for this task.
  - The effectiveness of the control action depends on the type of controller, location of SVC and the type of input signal used (speed, electrical power or line current).
  - Usually they are placed at electrical midpoint between two areas for controlling voltage. An additional controller with electrical power as the input signal provides good enhancement in damping of the inter area mode.

Example: SVC With Additional Controller

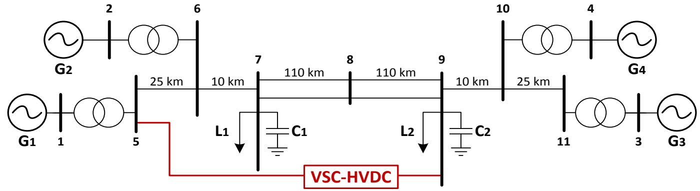


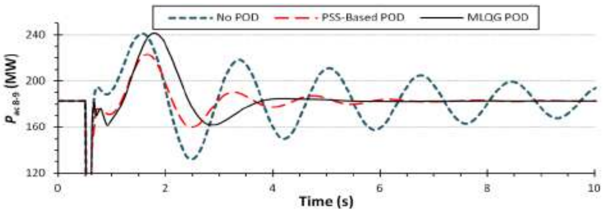
Four generator eight bus system with eight dynamic loads and SVC with additional controller

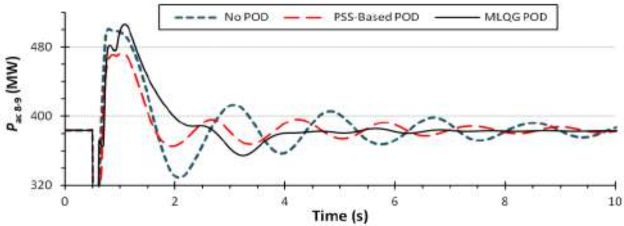
The influence of the dynamics of local (at SVC bus) load

Additional Controllers - 2

- Supplementary control of HVDC links
  - Normally the DC current is controlled by the rectifier and the DC voltage is maintained by the inverter control.
  - Damping can be improved by modulating the current order at the rectifier or by modulating the current order at the rectifier and the voltage order at the inverter.







Large Disturbance (Transient) Stability

Options Available - 1

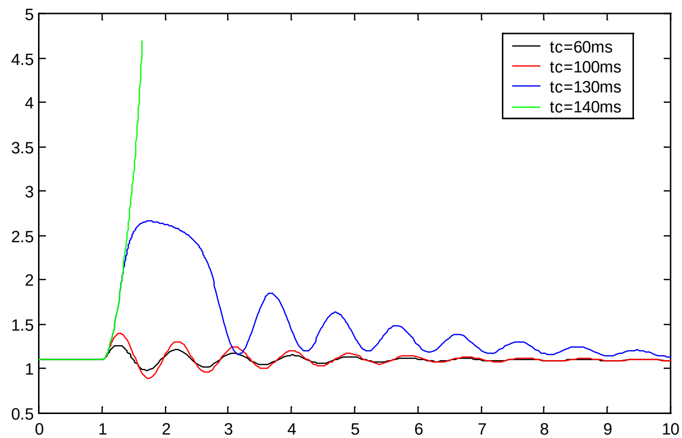
- High speed fault clearing
- Reduction of transmission system reactance
- Regulated shunt compensation
- Dynamic braking
- Reactor switching
- Independent pole operation of circuit breakers
- Single pole switching

Options Available - 2

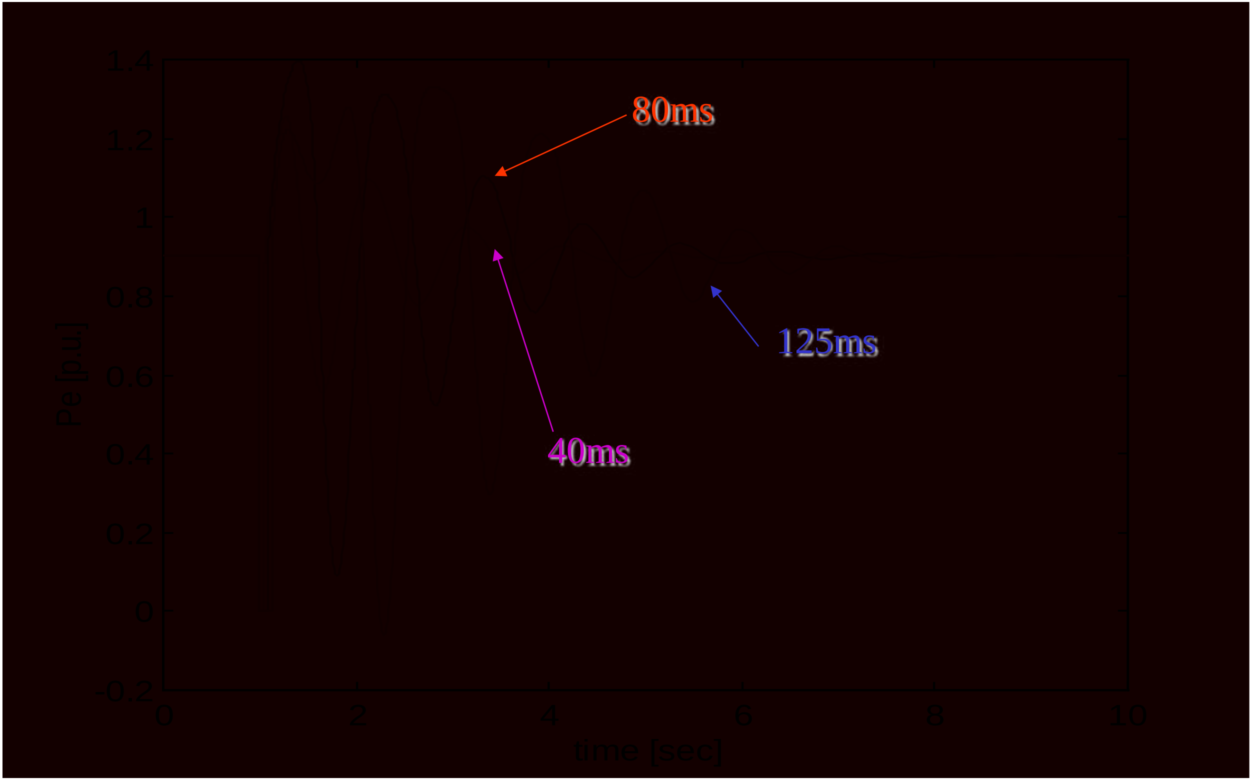
- Steam turbine fast valving
- Generator tripping
- Load shedding and controlled system separation
- High speed excitation systems
- Control of HVDC links
- Control of series elements

Fast Fault Clearing

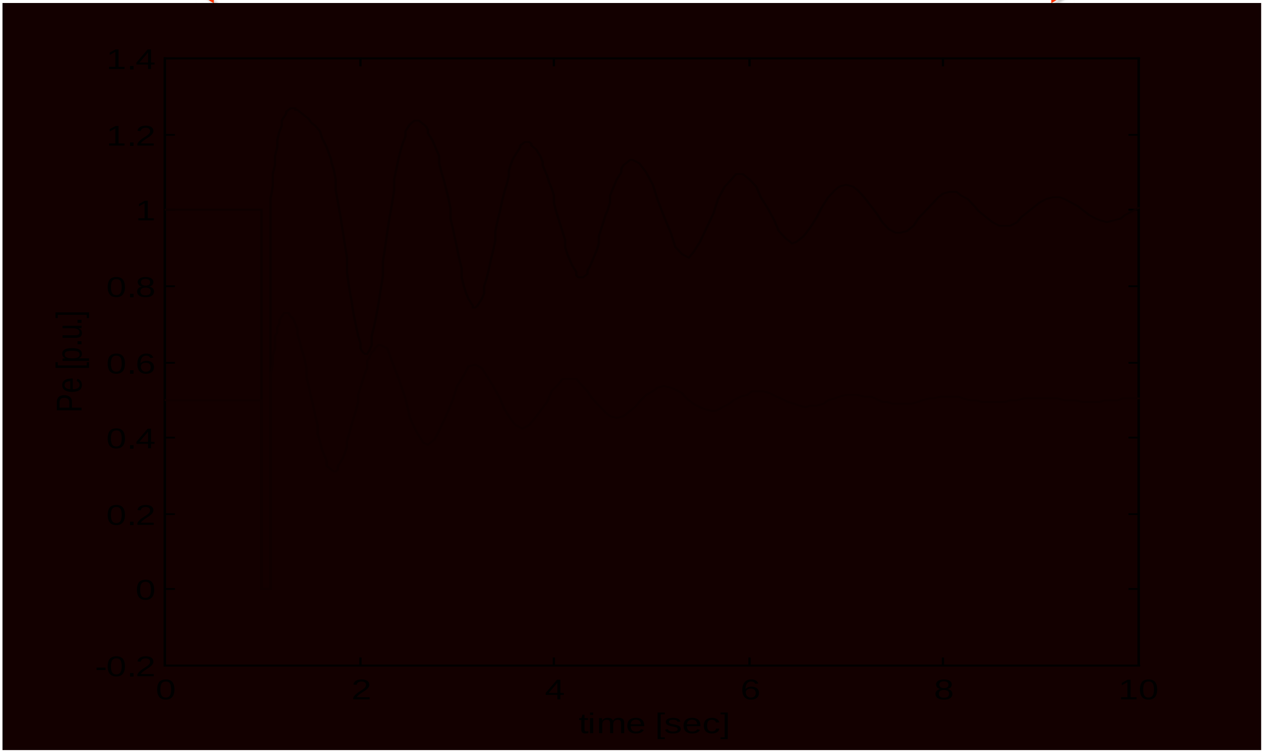
- The quicker the fault is cleared the less disturbance it causes
  - Two cycle breakers with high speed relays and communication are now used in where needed
  - One cycle breakers are not yet in wide-spread use



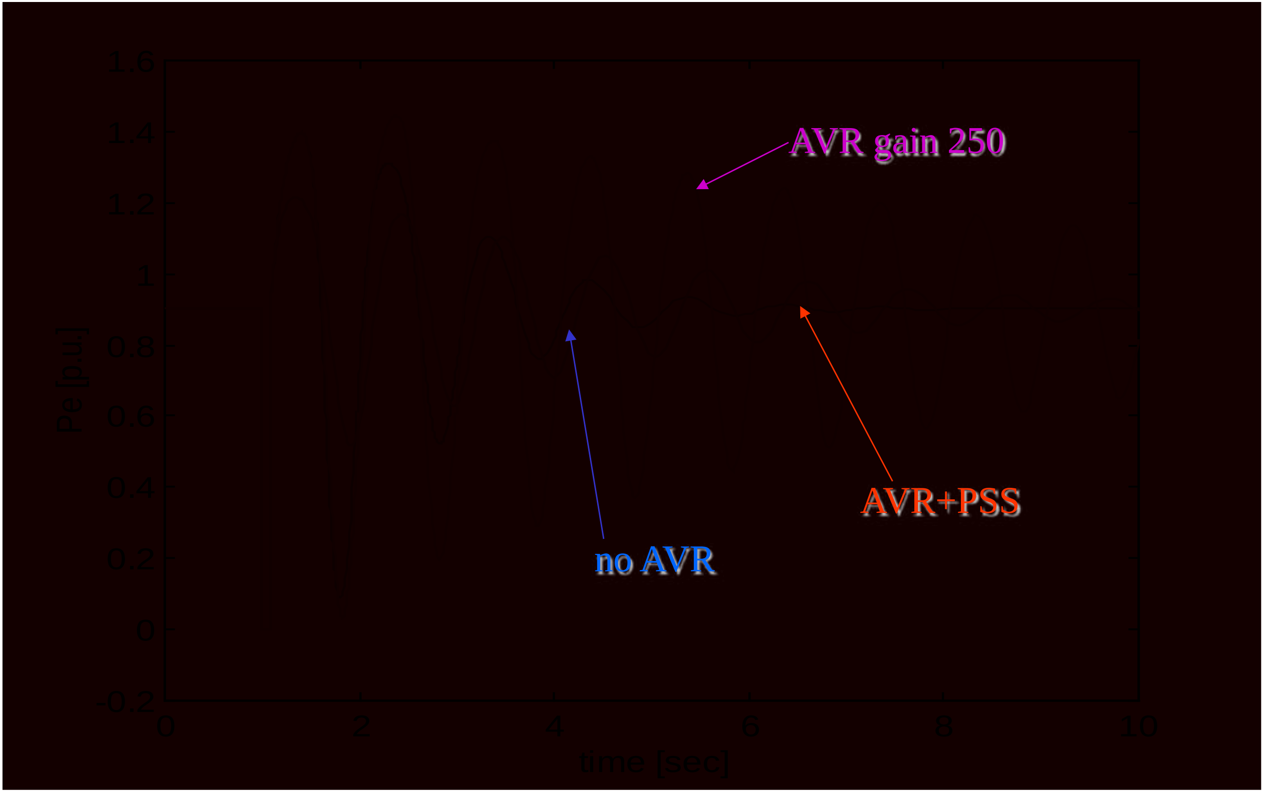
The Influence of Fault Clearing Time



The Influence of Generator Loading (without excitation control)

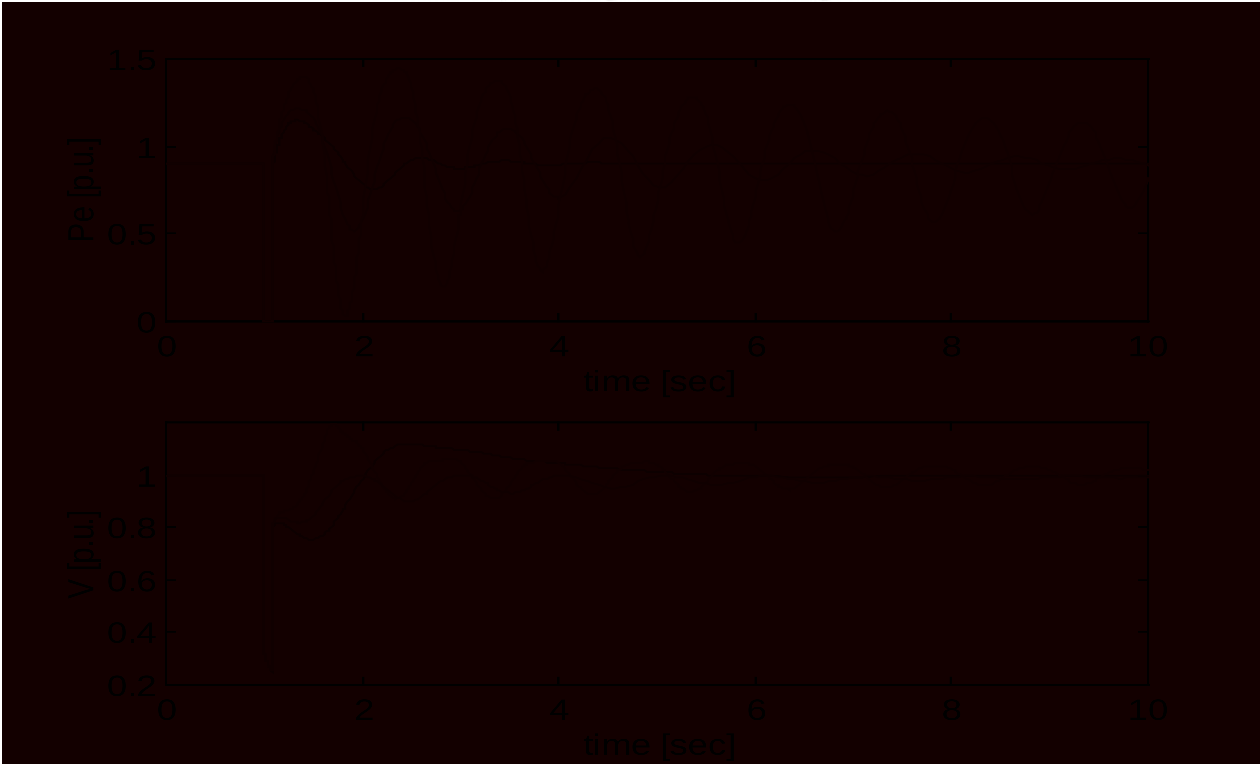


The Influence of AVR and PSS



The Influence of AVR Gain

No AVR; AVR gain 20; AVR gain 250;



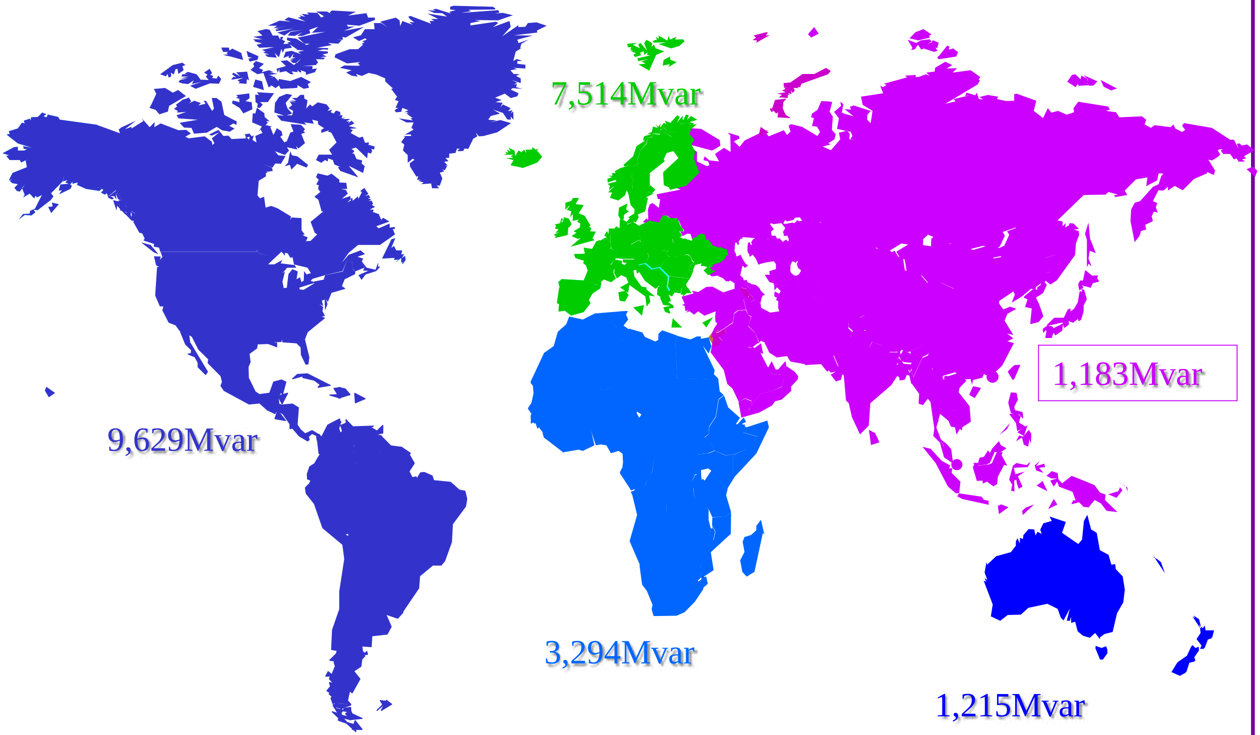
Transmission System Reactance

- Reduction of transmission system series impedance improves stability by increasing post-fault synchronising power transfers.
  - Reduce the reactance of transmission circuit
    - line and conductor configurations
    - number of parallel circuits
  - Use transformers with lower leakage reactances (typical 0.1 - 0.15p.u.)
  - Series capacitor compensation of transmission lines (increase of power transfer, speed of re-insertion after fault clearing is critical)

Control of Series Elements

- Series capacitor directly offset the line series reactance and increases the power transfer.
  - Speed of re-insertion after fault clearing is critical (use of non-linear resistors of zinc oxide enables practically instantaneous insertion)
  - There might be problems with subsynchronous resonance
  - Switched series capacitors can be used (switched it in after the detection of power swing and switch it out 0.5s later). Bang-bang control mode.
  - Advanced Series Capacitors or Thyristor Controlled Series Compensators (TCSC) can be used for continuous control

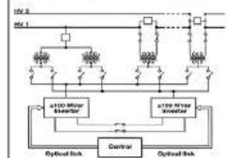
22.9 GVAr FACTS Installed (2008)



EPRI-Sponsored FACTS Installations in the U.S.

Unified Power Flow Controller (UPFC):
"All Transmission Parameters Controller"
± 160 MVA Shunt and ± 160 MVA Series at Inez Substation (AEP) 1998

UPFC



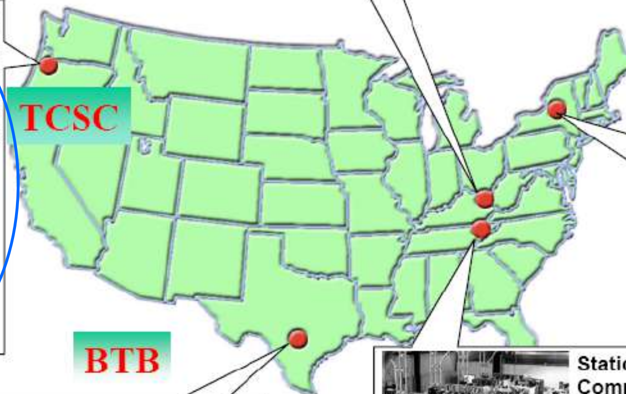
Convertible Static Compensator (CSC):
"Flexible Multi-functional Compensator"
± 200 MVA at Marcy Substation (NYPA)
2000 & 2002

CSC

AREVA Power Electronics

Thyristor Controlled Series Capacitor (TCSC):
"Line Impedance Controller"
208 Mvar TCSC at Slatt Substation (BPA) 1993

TCSC



BTB

FACTS Controller
"Back-To-Back HVDC Tie"
36 MW at Eagle Pass (CSW)
2000

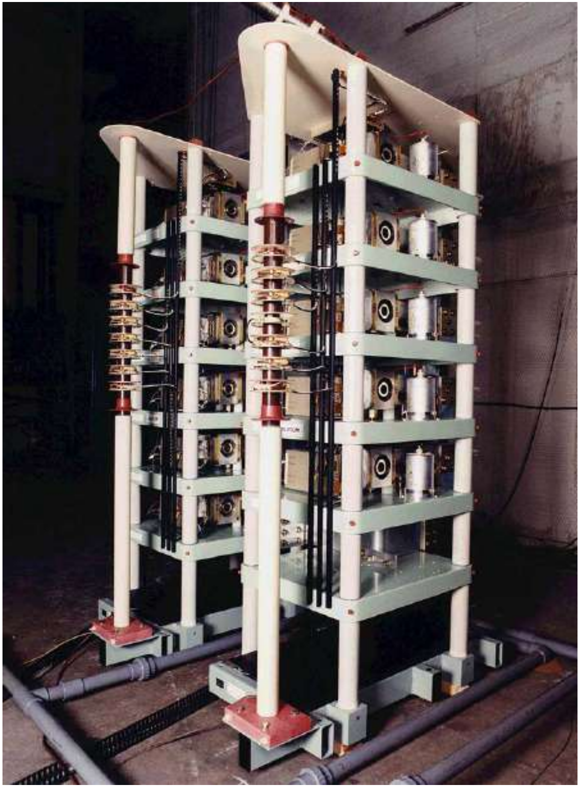
Static Synchronous Compensator (STATCOM) :
"Voltage Controller"
± 100 Mvar STATCOM at Sullivan Substation (TVA)
1995

STATCOM

Source: EPRI

What are SVCs?

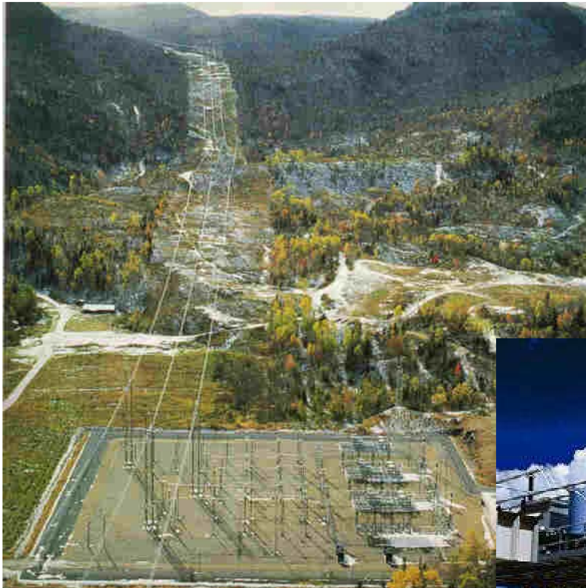
- SVCs are continuously adjustable impedances which quickly respond to network changes,
  - Counterbalances the variations of the active load or fault
  - Capacitive (+ve) through to inductive (-ve)
- SVCs are shunt compensation systems
  - i.e. independent devices connected at appropriate points in the transmission system
  - Topology is based on a current source converter

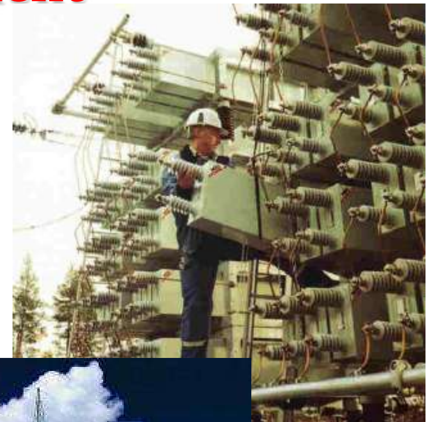


Installed SVC

STATCOM Valve (one phase)

TCSC Arrangement





TCSC Serra da Mesa, Brazil

Siemens/Westinghouse Inez UPFC

UPFC Building

(Inverters and Controls

Main Shunt Transformer

Shunt & Series Intermediate Transformers

AC Bus

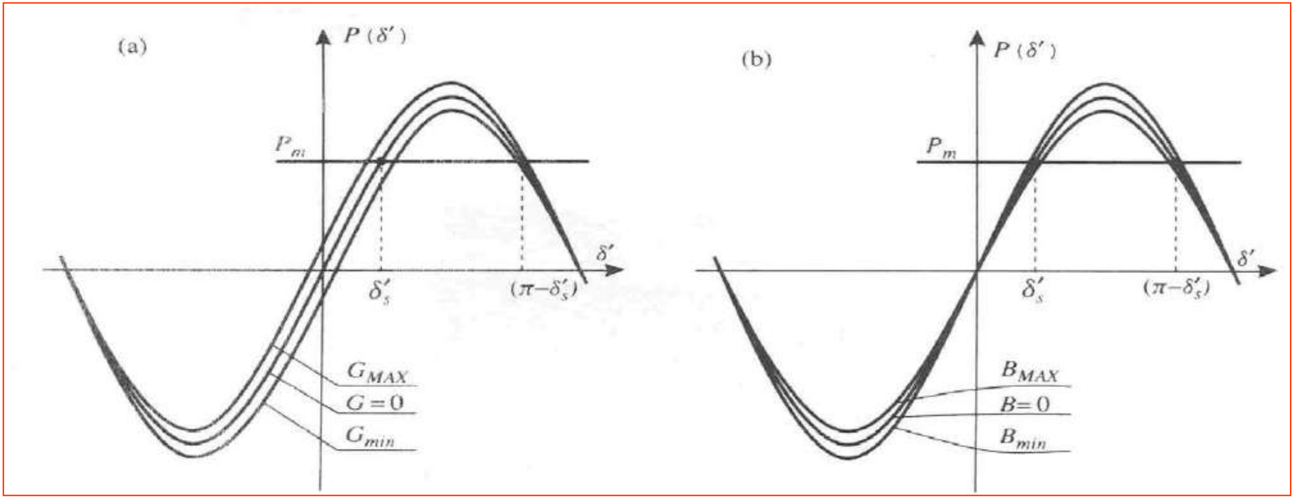
Inverter Poles

DC Clamp

Back to Back HVDC Station

Shunt Compensation - 1

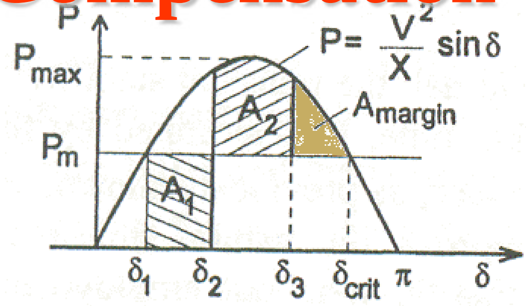
- Shunt compensation maintains voltages at selected points of the transmission system and improves system stability by increasing the flow of synchronising power among interconnected generators.
  - Switching on and off shunt capacitors or reactors



Shunt Compensation - 2

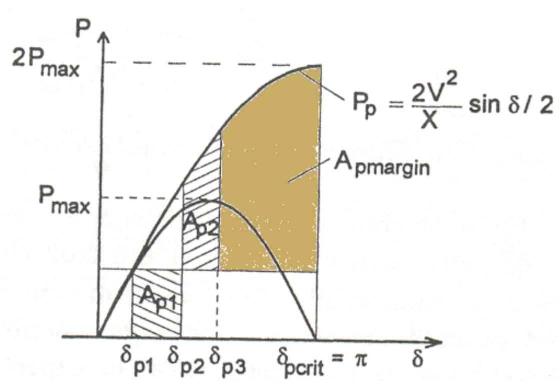
- Controllable shunt elements
  - Modulation controller of Superconducting Magnetic Energy Storage (SMES)
  - Supplementary control of SVC
  - Braking resistors
  - Combination of SVC and braking resistors
- Shunt reactors (near the generator)
  - Normally connected, they increase the generator internal voltage as the result of reactive load increase and such contribute to stability
  - Switched out following the fault

Shunt/Series Compensation

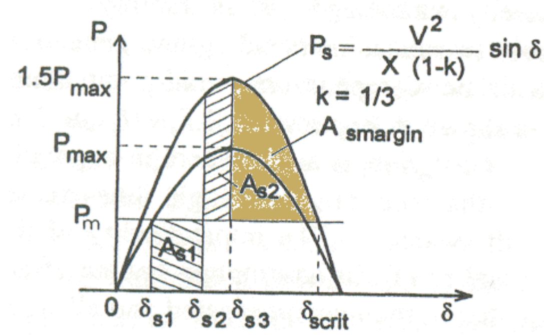


Assuming fault cleared without disconnecting any line

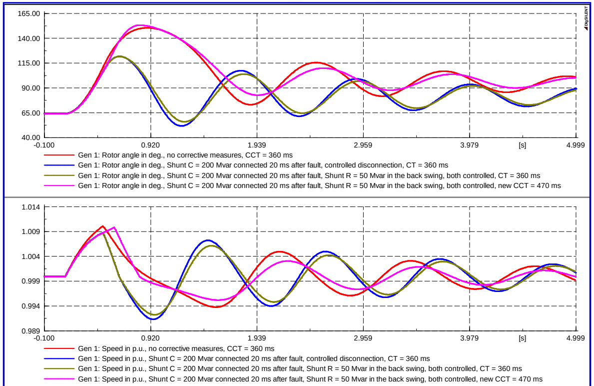
With SVC

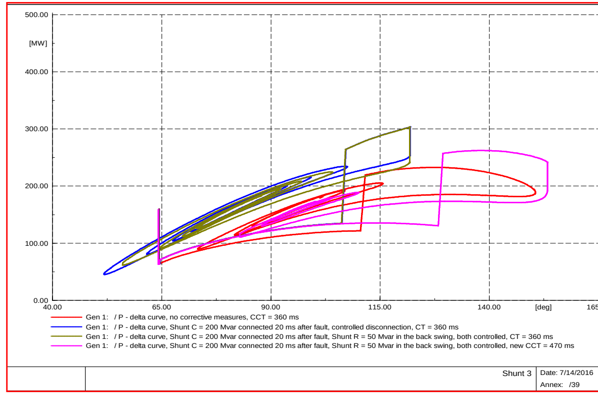


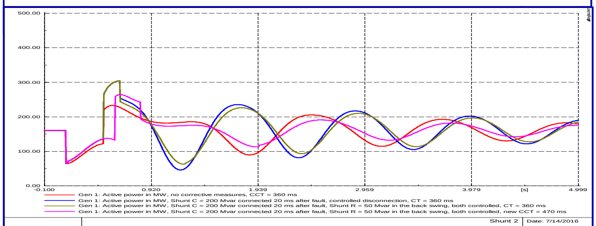
With TCSC

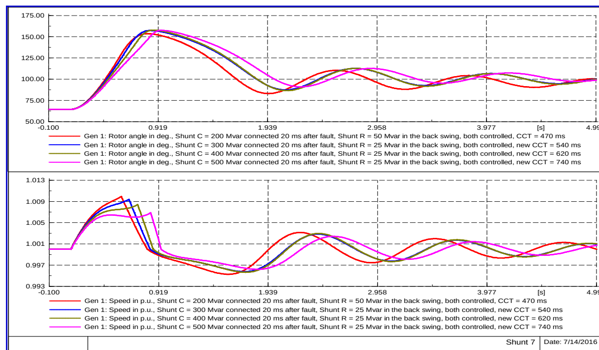


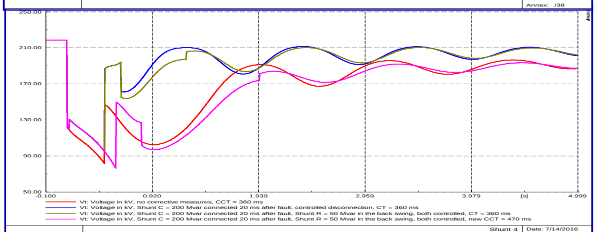
Shunt Compensation



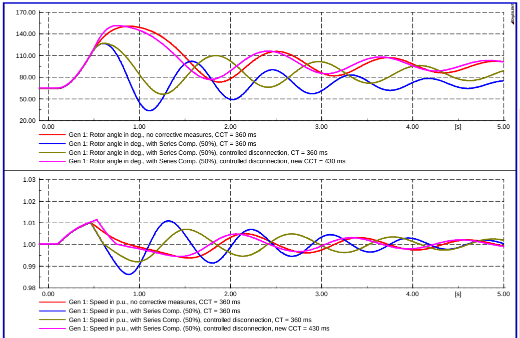


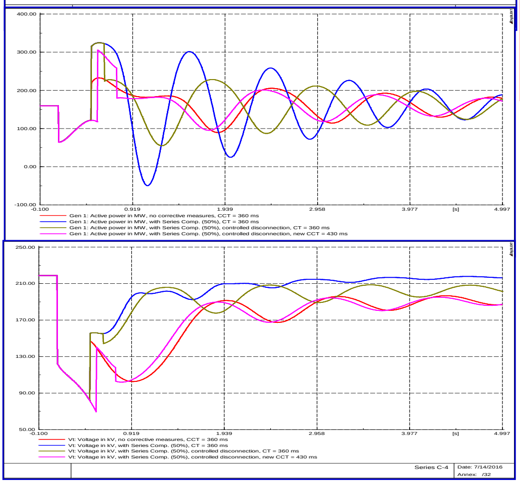


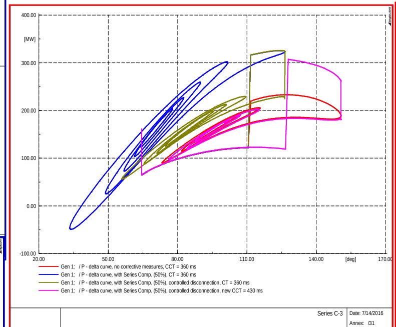


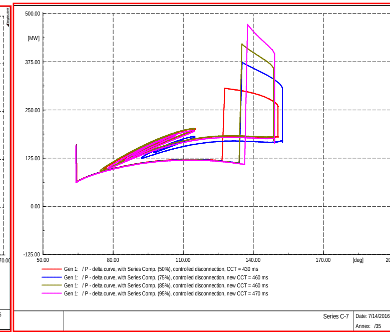


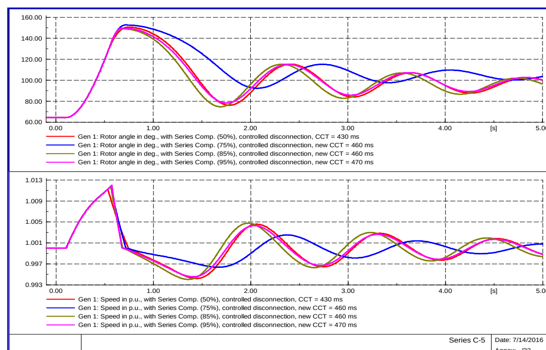
Series Compensation







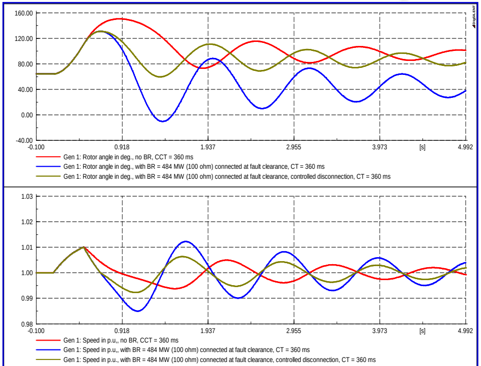


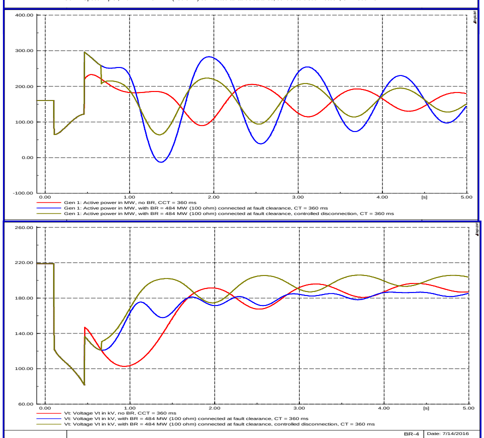


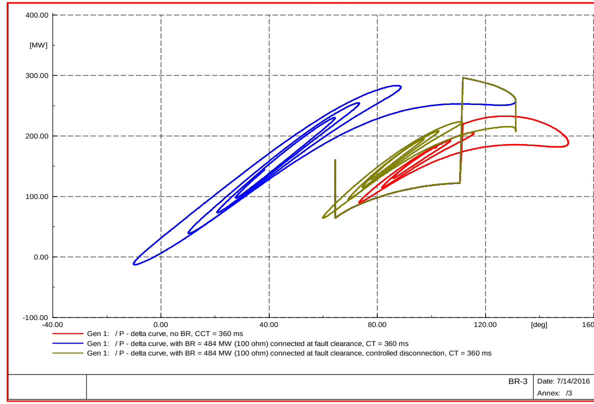
Dynamic Braking

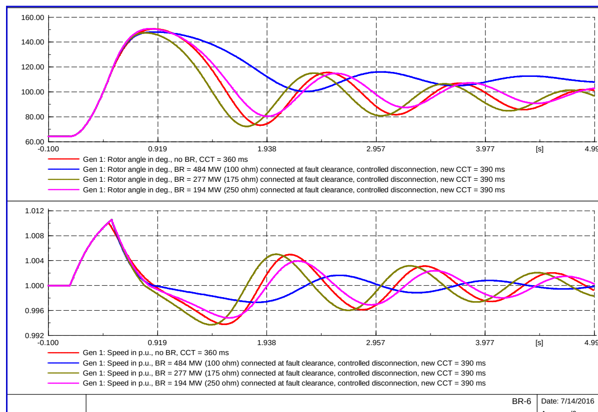
- Applying an artificial electrical load during a transient disturbance to increase the electrical power output of generators and reduce rotor acceleration.
  - Switch in shunt resistors for about 0.5s following a fault (dissipated energy proportional to voltage)
    - So far applied only to hydraulic units remote from load centers (they may cause unacceptable shaft fatigue if applied to thermal units)
  - Switch in series resistors (dissipated energy proportional to current)
  - Resistors connected permanently between the ground and the neutral of the Y-connected HV winding of the generator transformer. (For unbalanced ground faults only)

Dynamic Braking









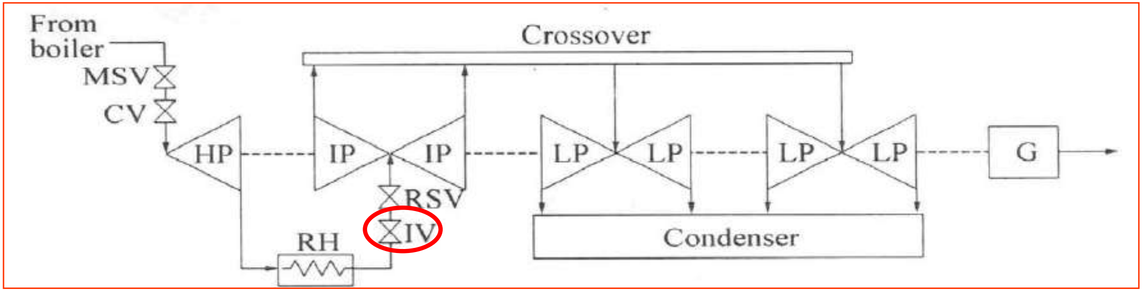
Circuit Breaker Operation

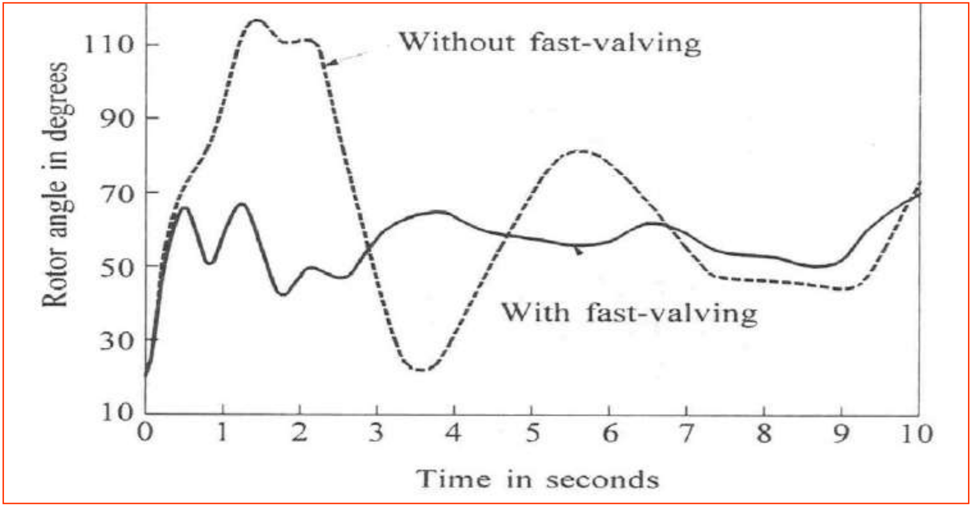
- Independent pole operation
  - Each phase is opened and closed independently.
  - The relaying system is normally arranged to trip all three poles for any type of fault.
  - The severity of a three phase fault with a stuck breaker is significantly reduced.
- Single-pole switching
  - Separate operating mechanisms on each phase (following fast re-closure within 0.5s to 1.5s).
  - When one phase is open, power is transferred over the remaining two phases.

Steam Turbine Fast Valving -1

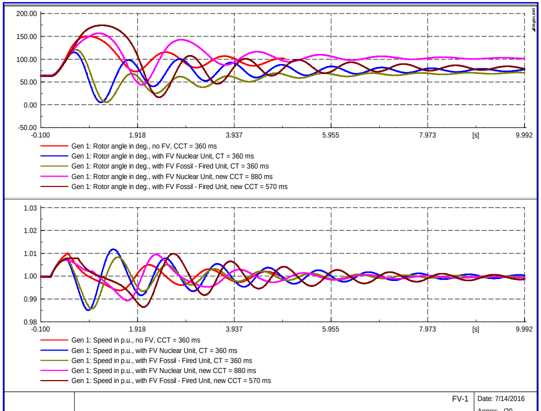
- Rapid closing and opening of steam valves to reduce the generator accelerating power.
- Main inlet control valves (CV) and the reheat intercept valves (IV) provide a convenient means of controlling mechanical power.
- Usually only the IV (they control nearly 70% of the total unit power in comparison to CV which control 30% of the power) are rapidly closed and then fully reopened after a short time delay.
  - Complete closure within 0.08s to 0.4s, reopening is delayed for 0.3s to 1s, full reopening 3s to 10s (USA) less than 1s (Europe).

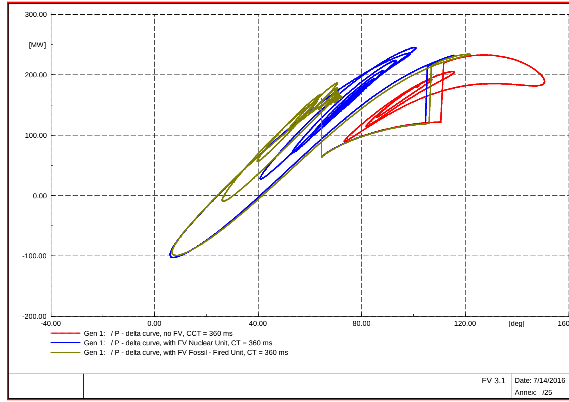
Steam Turbine Fast Valving - 2

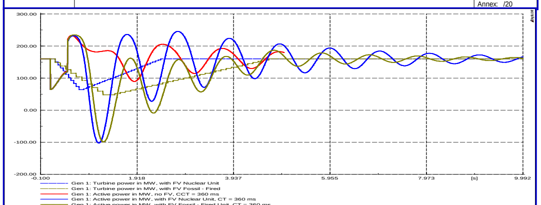


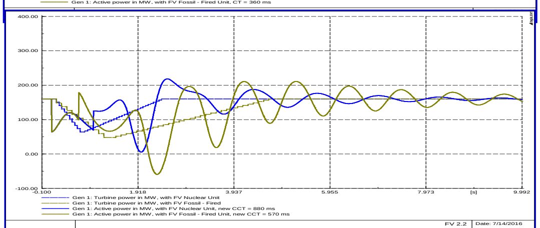


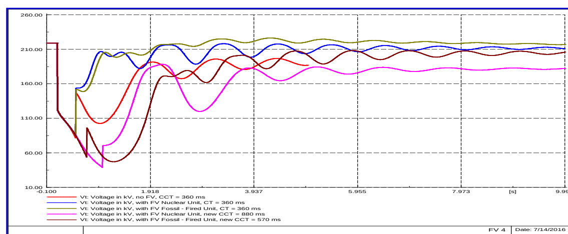
Steam Turbine Fast Valving -1



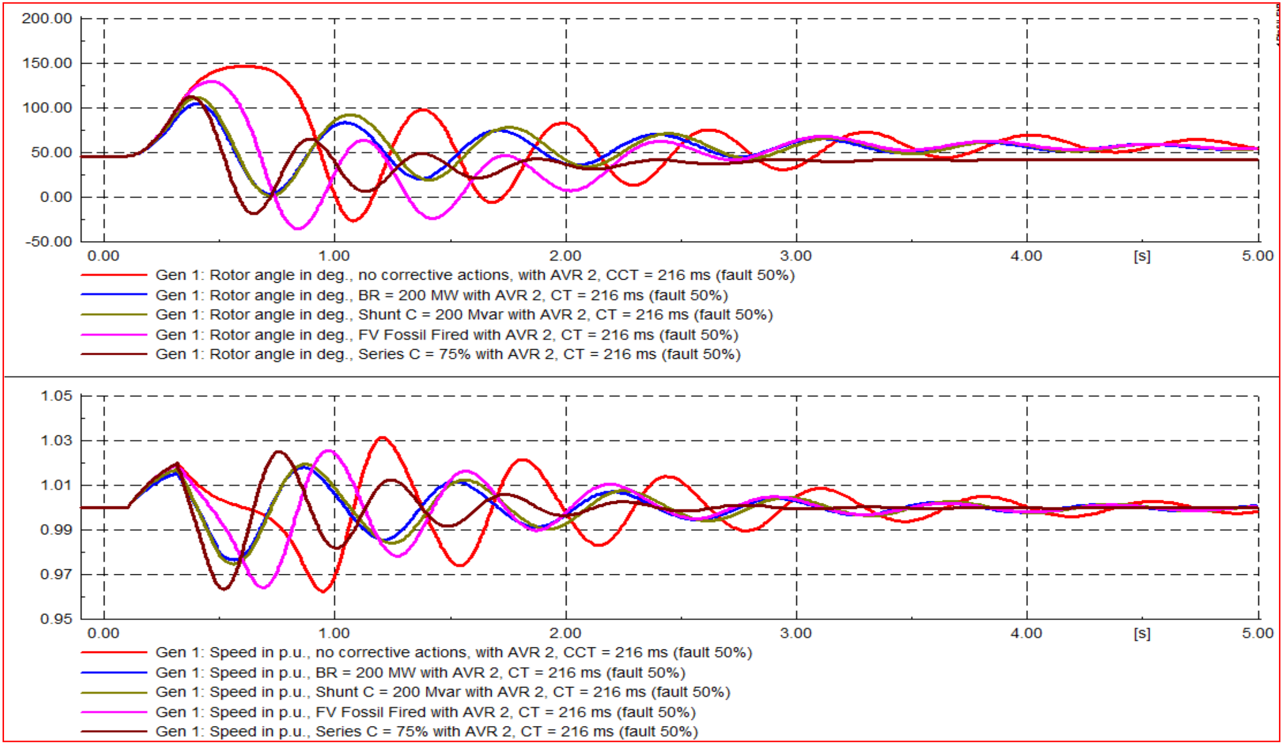








Comparison of corrective control

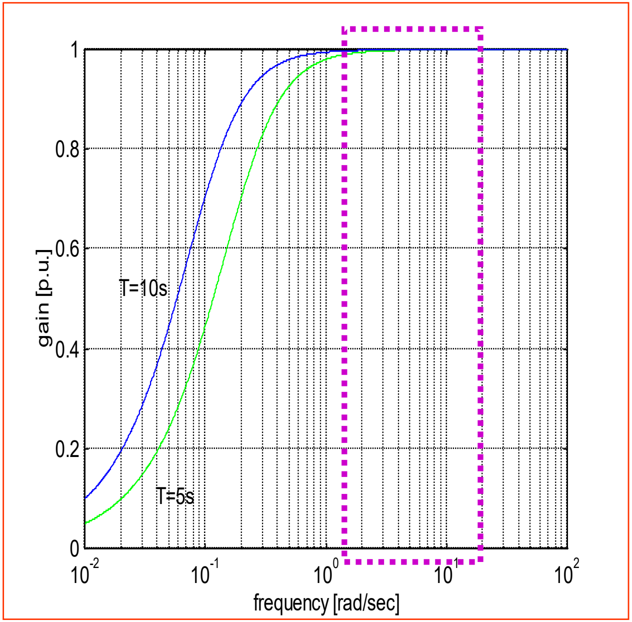


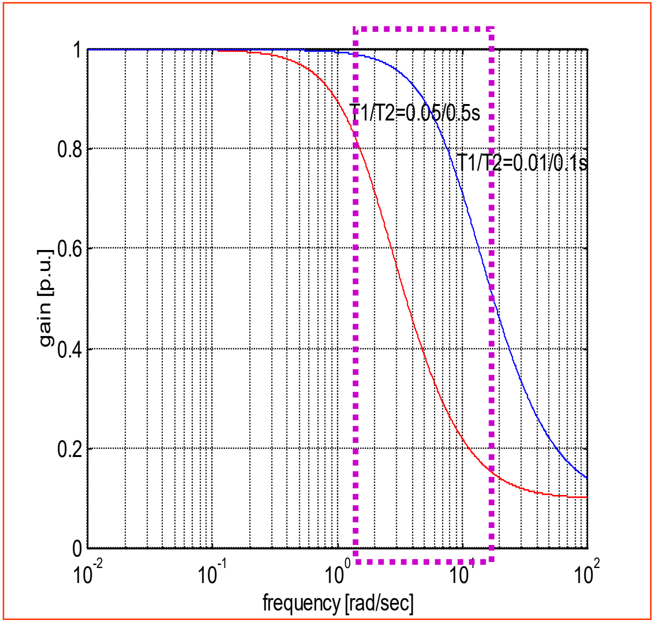
Summary of Stabilising Measures

Subprocess of interest	Typical rate of response	Typical cause	Control means
Subsynchronous resonance (SSR)	Very fast (m sec to sec)	Interplay between rotor shafts and transmission	Excitation systems, power system stabilizers, FACTS
Angle instability	Fast (10^{-2} sec to sec)	Deviations in p and $x(0)$	Excitations systems, power system stabilizers, FACTS, fast valving
Voltage instability	Midrange (1–10 sec)	Inadequate network design and/or excitation control	Excitation systems
Frequency deviations	Slow (>10 sec)	Persistent deviations in real power load	Governor control
Voltage deviations	Slow (>10 sec)	Persistent deviations in reactive power load	On-load tag changing transformers, capacitor banks
Frequency drift	Very slow (10^{1} 10^{4} sec)*	Slow persistent real power load deviations	Secondary level frequency regulation, AGC
Load voltage drift	Very slow (10^{1} 10^{4} sec)*	Slow persistent reactive power load deviations	Secondary level voltage regulation, AVC

*System-dependent designs.

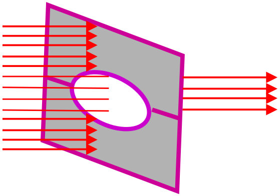
Washout & Low-pass Block - 1





Washout (high-pass filter)

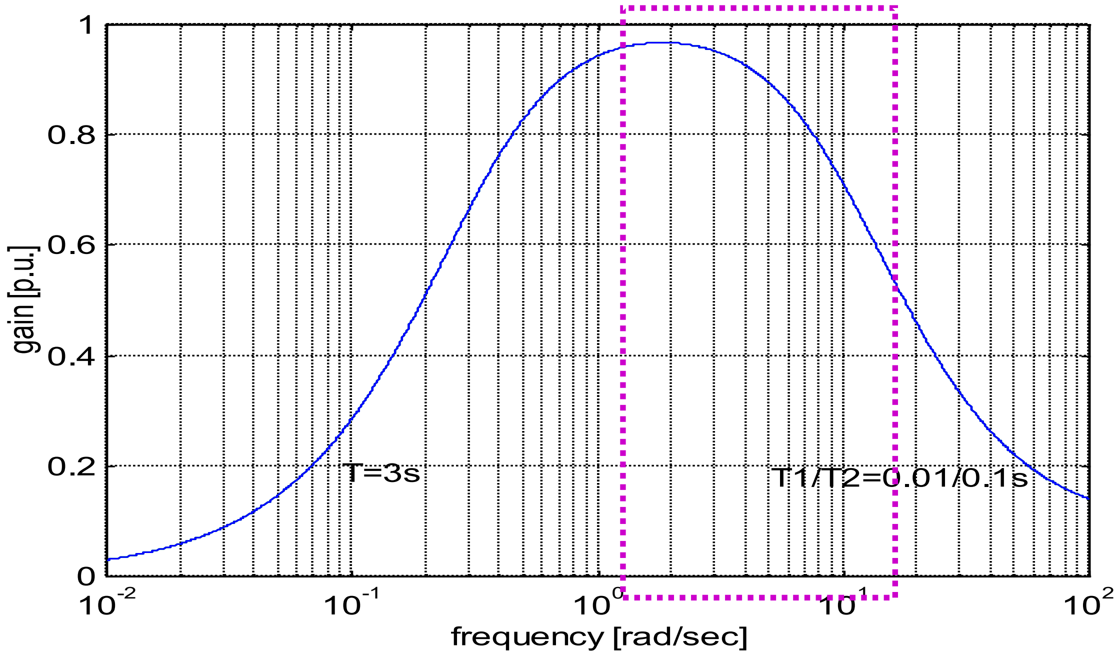
$$W(s)=\left(\frac{Ts}{1+Ts}\right)$$



Low-pass filter

$$F(s)=\left(\frac{1+T_1 s}{1+T_2 s}\right)$$

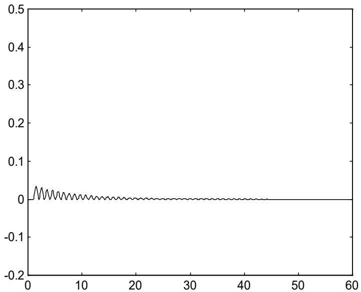
Washout & Low-pass Block - 2

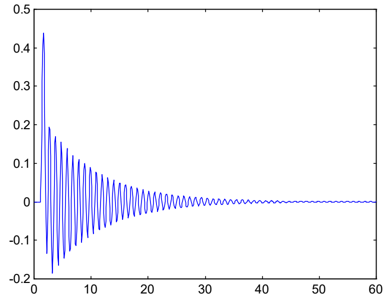


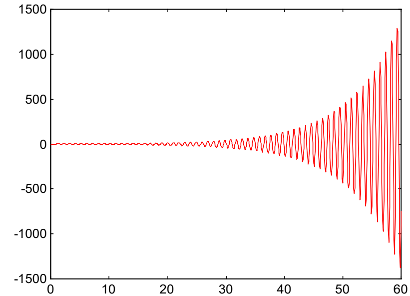
$$W(s)F(s) = \left(\frac{3s}{1+3s}\right)\left(\frac{1+0.01s}{1+0.1s}\right)$$

Tuning of PSS: Case Study - 1

Original system	With slow acting AVR	With fast acting AVR
-0.1180 + 6.1786i	-0.1098 + 6.1335i	+0.1163 + 6.3885i
-0.1180 - 6.1786i	-0.1098 - 6.1335i	+0.1163 - 6.3885i
-0.1371	-2.0392 + 1.6511i	-10.2653 +14.2354i
	-2.0392 - 1.6511i	-10.2653 -14.2354i



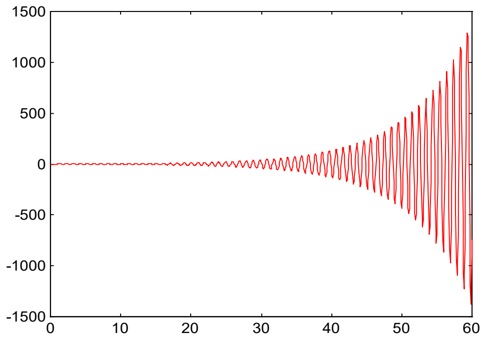


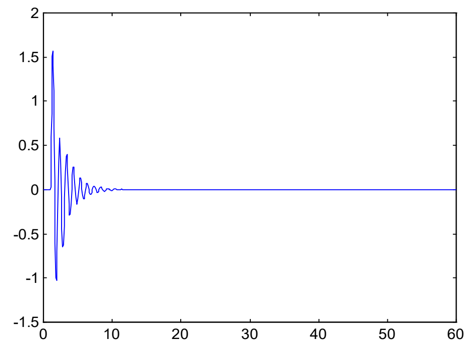


Tuning of PSS: Case Study - 1

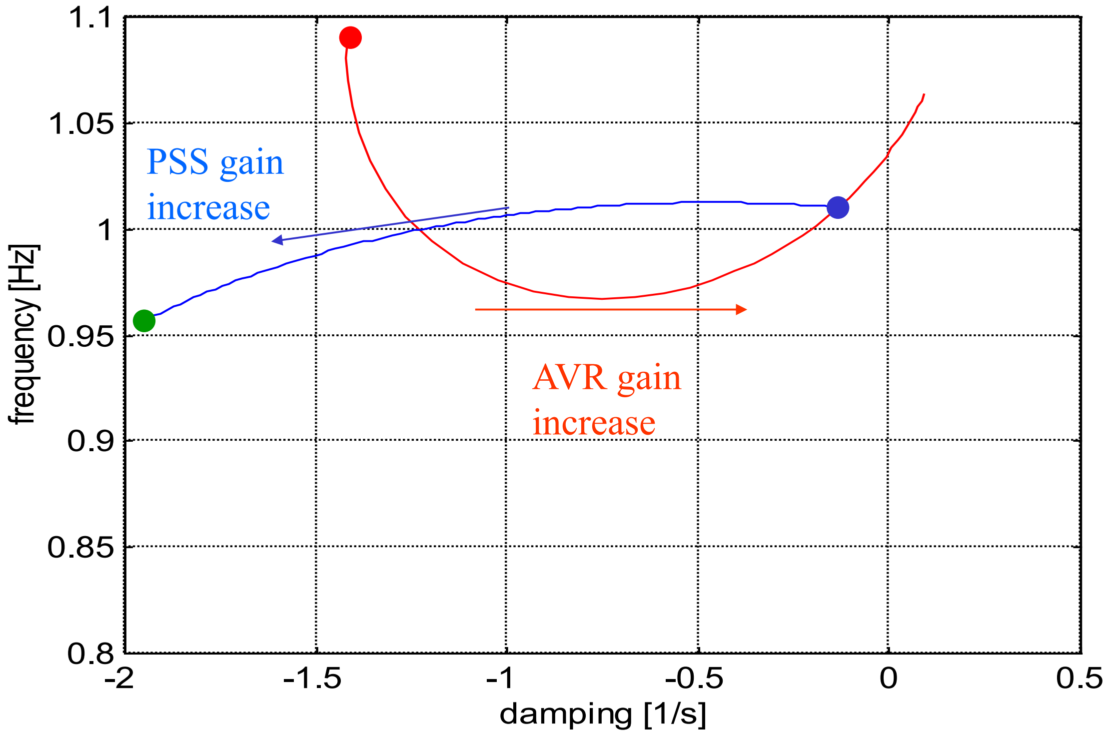
With fast acting AVR
+0.1163 + 6.3885i
+0.1163 - 6.3885i
-10.2653 +14.2354i
-10.2653 -14.2354i

With fast acting AVR and PSS
-10.2202 +14.2275i
-10.2202 -14.2275i
-0.5689 + 6.2779i
-0.5689 - 6.2779i
-4.8145
-0.8846 + 1.7537i
-0.8846 - 1.7537i

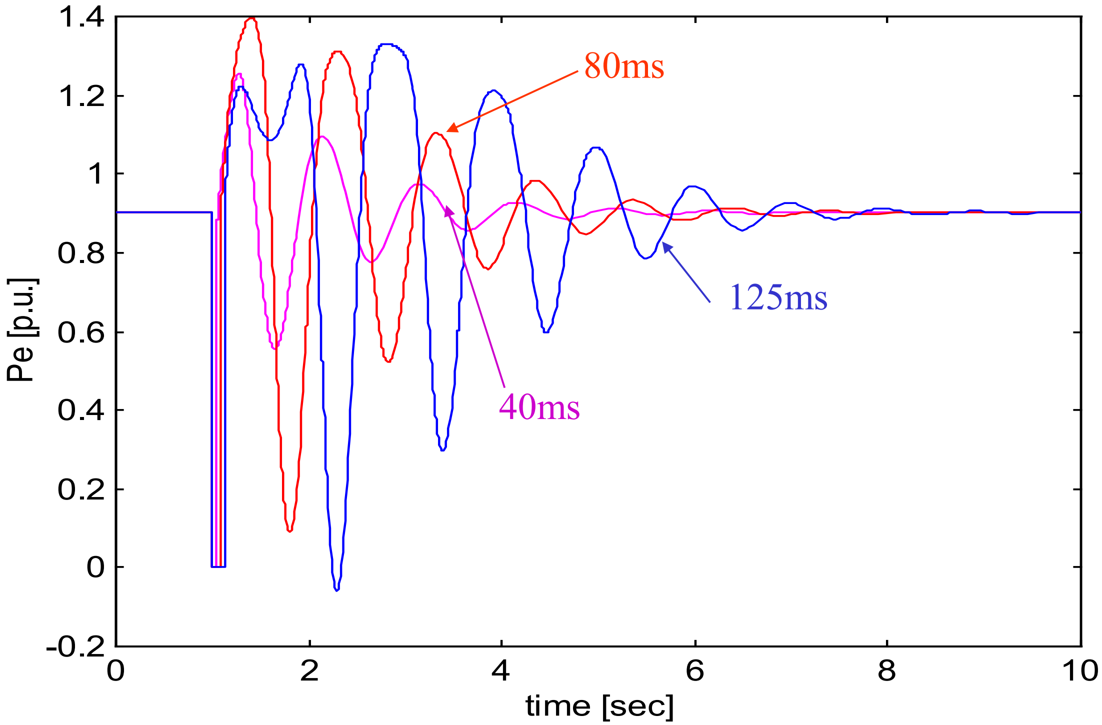




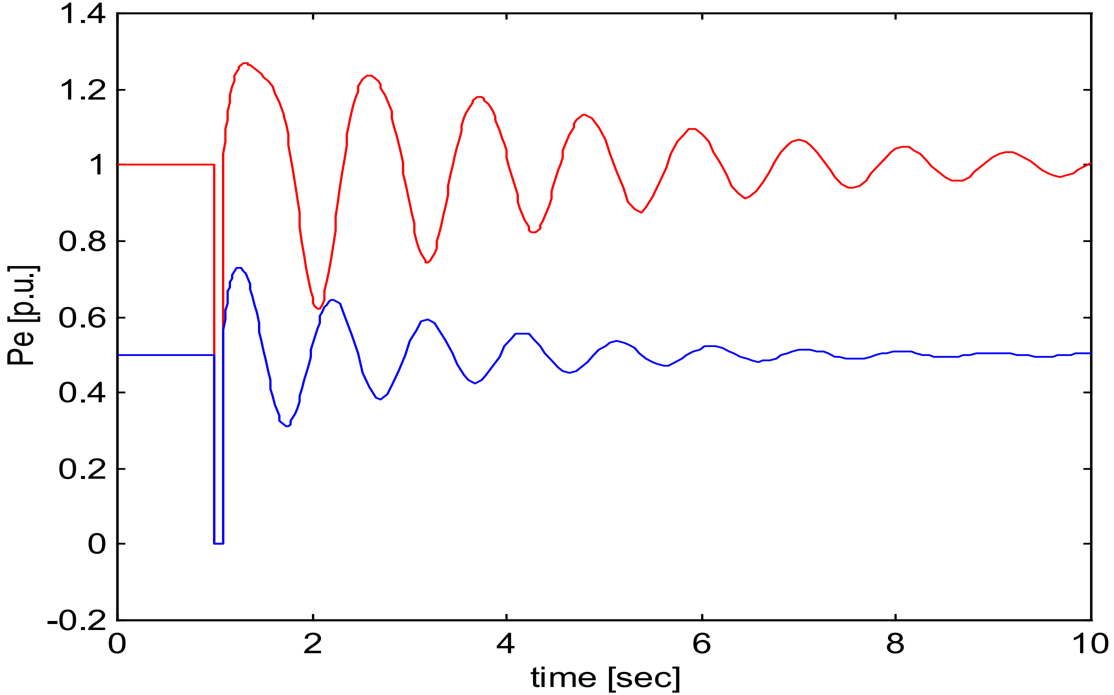
Influence of AVR and PSS Gain



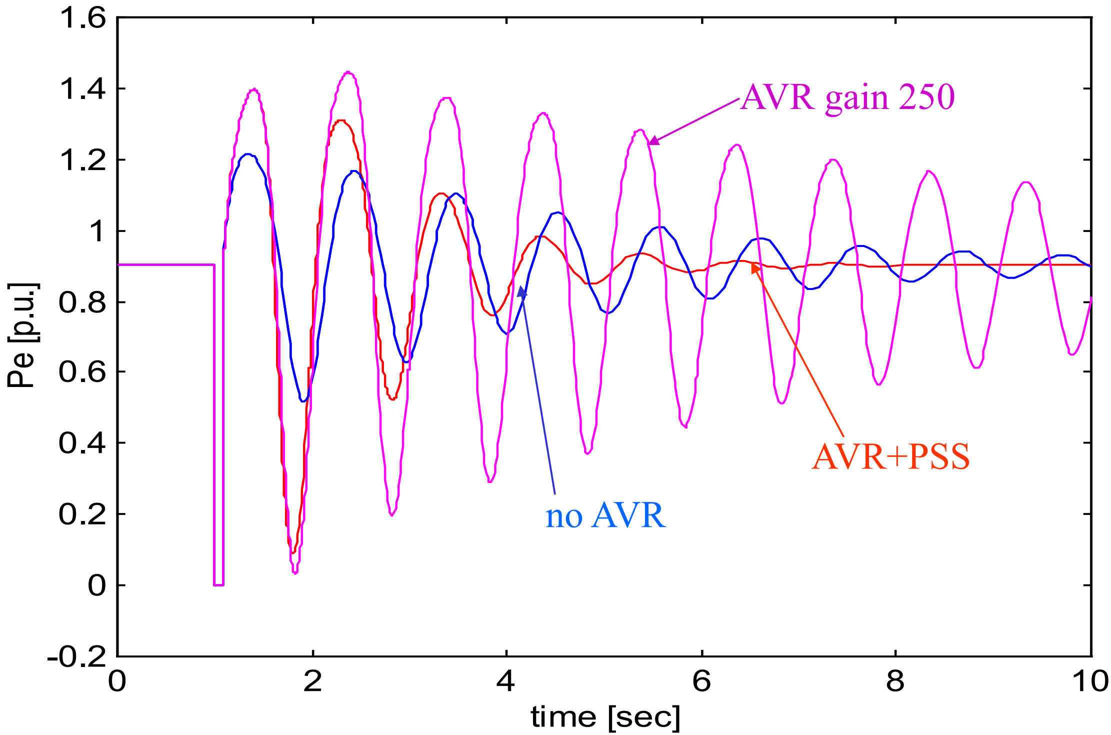
The Influence of Fault Clearing Time



The Influence of Generator Loading (without excitation control)



The Influence of AVR and PSS



The Influence of AVR Gain

No AVR; AVR gain 20; AVR gain 250;