*M.Sc. Courses in Electrical Power Engineering*

## EEEN60342: Dynamics & Quality of Electricity Supply

# Reliability Analysis of Power Systems

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- **Content**
  - Non-reparable components – systems
  - Reparable components – systems
  - Reliability modelling of (simpler) systems
  - Generation systems
  - Reliability of complex systems
  - Composite generation and transmission systems
  - Distribution networks
  - Reliability cost assessment
  - Reliability of substations
  - Reliability centered maintenance

# Literature

- **Useful textbooks**
  1. R. Billinton, R. Allan; “*Reliability Evaluation of Power Systems*”, Pitman, London
     - Chapter 2 & 3 (partly): Generation reliability
     - Chapter 6: Composite generation and transmission systems
     - Chapter 7, 8 (partly) & 9 (partly): Distribution network reliability
     - Chapter 10 (partly): Substation reliability
  2. R. Billinton, R. Allan; “*Reliability Evaluation of Engineering Systems: Concepts and Techniques*”, Pitman, London
     - (Chapter 2): Basic probability theory
     - Chapter 5 & 6: Reliability of simple and complex systems
     - Chapter 9 & 11: Reparable system theory and approximations
  3. R. Billinton, W. Li; “*Reliability Assessment of Electric Power Systems Using Monte Carlo Methods*”, Plenum Press, New York.
     - Chapter 3: Elements of Monte Carlo simulation
     - Chapter 4 (partly), 5 (partly) & 6 (partly): Generation, composite and distribution systems reliability
  4. W. Li; “Risk Assessment of Power Systems”, IEEE Press, John Wiley & Sons.
     - Chapter 2 and 4: Outage models and elements of risk evaluation methods

# What is 'Reliability'?

- Frequently asked question: *"How reliable or how safe will the system be during its future operating life?"*

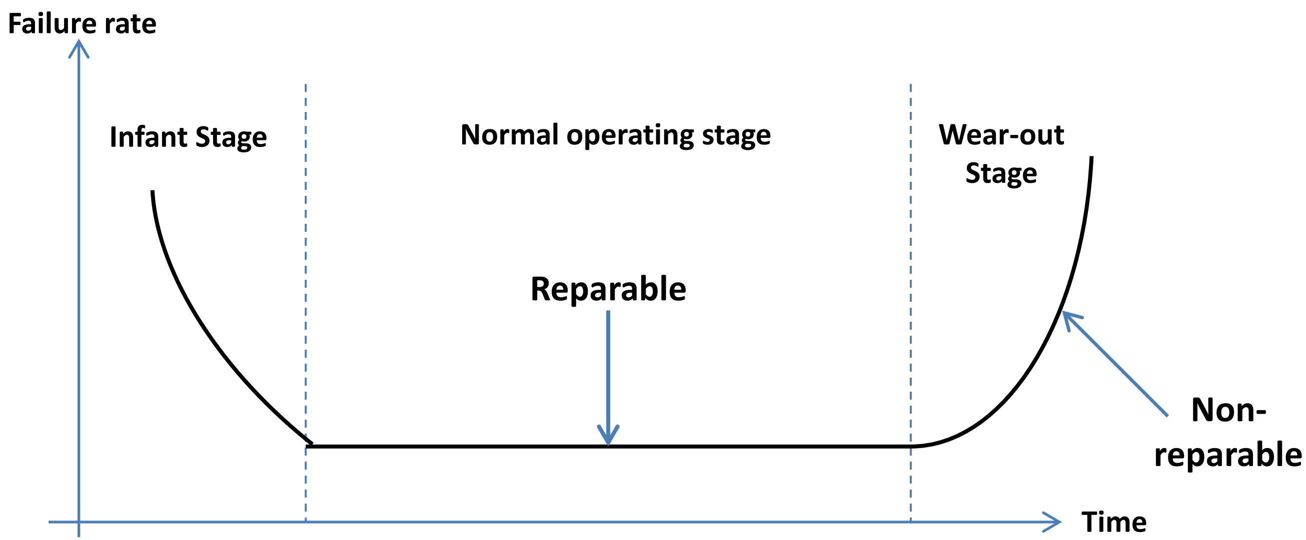
- One of definitions: *"Reliability is the probability of a device performing its purpose adequately for the period of time intended under the operating conditions encountered".*

- Four basic parts of the definition:
  - Probability = input; <u>Reliability indices</u> = output
  - Adequate performance, time & operating conditions = engineering concepts

- Reliability analysis:
  - Expansion planning and design of power systems
  - Operation planning and control of power systems

# Non-Reparable v Reparable Components

- Non-reparable: a component has experienced failure and cannot be repaired
- Reparable: a component had a failure and can be repaired




'Bathtub' Curve

- **Replacement studies**: non-reparable components
- **Reinforcement, QoS, Planning studies**: reparable components

# Non-Reparable Systems
## Reliability Functions

- Reliability (survivor) function:

  Random variable is *in-service time*!

  $R(t) \equiv P\{T > t\}$ (1) ; *T=* in-service time, *t= time started with installation*

- Unreliability (failure) function:

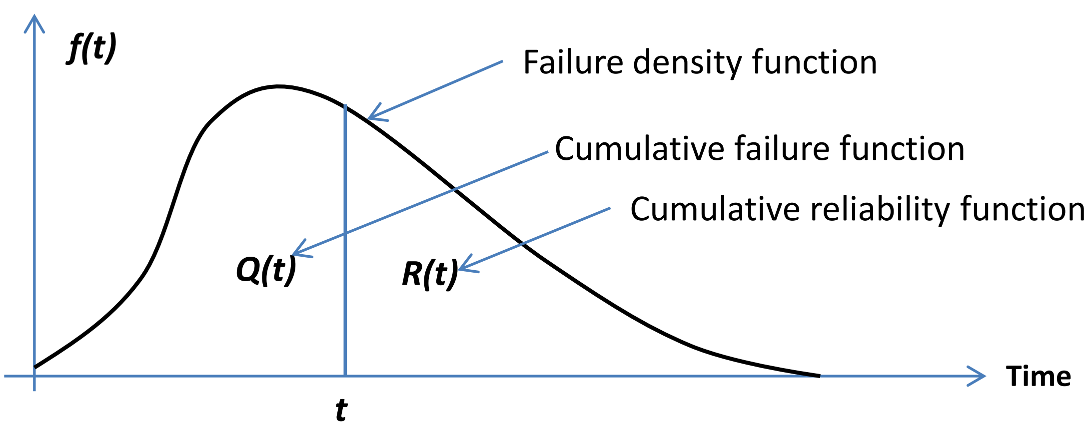
  $Q(t) \equiv P\{T \leq t\} = 1 - R(t)$ (2)

- Failure density function:

  $f(t) = dQ(t)/dt = -dR(t)/dt$ (3)

- Cumulative and density functions:

$$Q(t) = \int_0^t f(t)dt \qquad (4) \qquad\qquad R(t) = 1 - Q(t) = \int_t^\infty f(t)dt \qquad (5)$$



# Non-Reparable Systems
## General Reliability Functions

- *Hazard rate* function (*failure rate* function):

$$\lambda(t) \equiv \lim_{\Delta t \to 0} \frac{P\{(t < T \le t + \Delta t) \mid (T > t)\}}{\Delta t} \qquad (6)$$

(conditional probability P(A|B)=P(A·B)/P(B) )

$$\lambda(t) \equiv \lim_{\Delta t \to 0} \frac{P\{(t < T \le t + \Delta t) \cdot (T > t)\}}{\Delta t \cdot P\{T > t\}} = \lim_{\Delta t \to 0} \frac{P\{t < T \le t + \Delta t\}}{\Delta t \cdot R(t)} = \frac{f(t)}{R(t)} \qquad (7)$$

λ(t) = ***probability that component fails in interval (t, t+Δt) if it has not until ‘t’***

- Reliability function R(t) can be obtained by integrating eq. (7)

(first substitute *f(t)=-dR(t)/dt*):

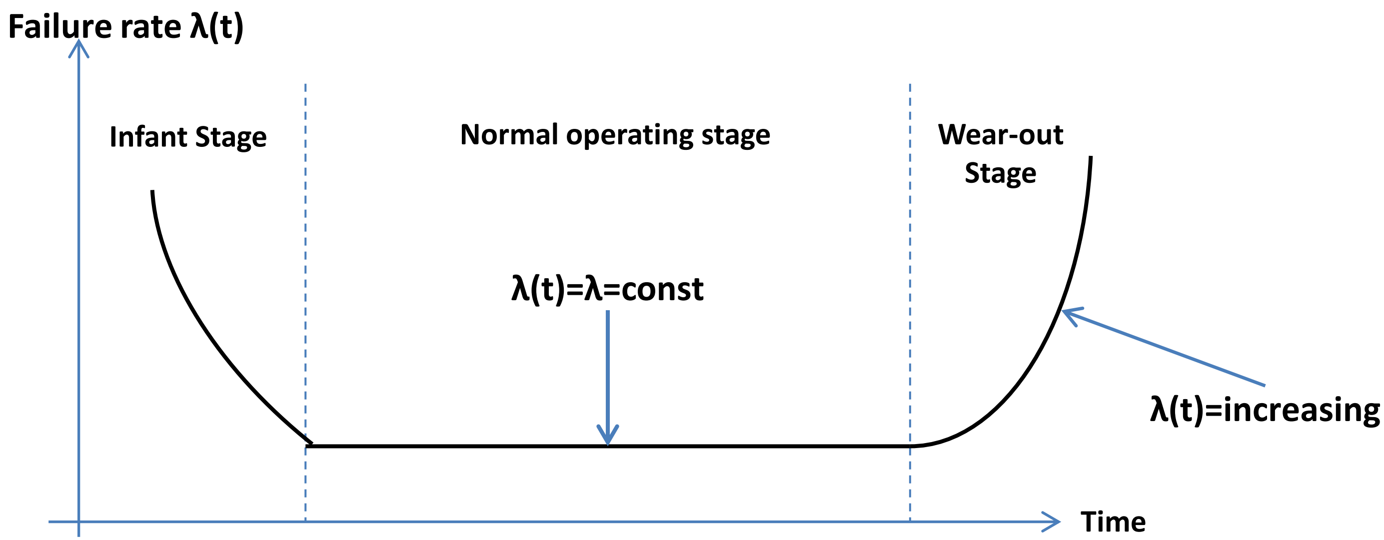
$$R(t) = \exp\{-\int_0^t \lambda(x)dx\} \qquad (8)\ ; \qquad R(t) = \exp(-\lambda \cdot t) \quad if \;\; \lambda = const \qquad (9)$$

- Mean time to failure (mathematical expectation):

$$MTTF \equiv E(T) = \int_0^\infty t \cdot f(t)dt = -\int_0^\infty t \cdot R'(t)dt = -t \cdot R(t)\Big|_0^\infty + \int_0^\infty R(t)dt = \int_0^\infty R(t)dt \qquad (10)$$

# Non-Reparable Systems
## General Reliability Functions

- ‘Bathtub’ curve:
  - Normal operating stage: hazard (failure) rate **λ(t)=λ=const** ; failure density function is **exponential** (eq. (9))
  - Wear-out stage: hazard (failure) rate λ(t) is monotonically increasing; failure density function is usually:
    - Normal distribution
    - Lognormal distribution
    - Weibull distribution, etc.




‘Bathtub’ Curve

# Non-Reparable Systems
## Evaluation of Reliability Functions

- **Example 1**: Calculation of Reliability Functions from a Data Set
  - Columns 1 and 2: Original data obtained from a real-life experiment
  - Column 3: Cumulative failures $N_f$ . Obtained by cumulating all failures in the previous time intervals.
  - Column 4: Number of survivors $N_s$ . Obtained by subtracting the cumulative number of failures from the total number of components-1000.
  - Column 5: Failure density function *f.* It is the ratio between the number of failures during a time interval and total number of components (1000).
  - Column 6: Cumulative failure distribution *Q*. It is the ratio between the cumulative number of failures and 1000.
  - Column 7: Reliability (survivor) function *R*. It is the ratio between the number of survivors and 1000.
  - Column 8: Failure (hazard) rate *λ*. It is the ratio between the number of failures in an interval and the **average** number of survivors for that period:

$$\lambda_1 = \frac{140}{(1000+860)/2} = 0.151;\ \ \lambda_2 = \frac{85}{(860+775)/2} = 0.104;...$$

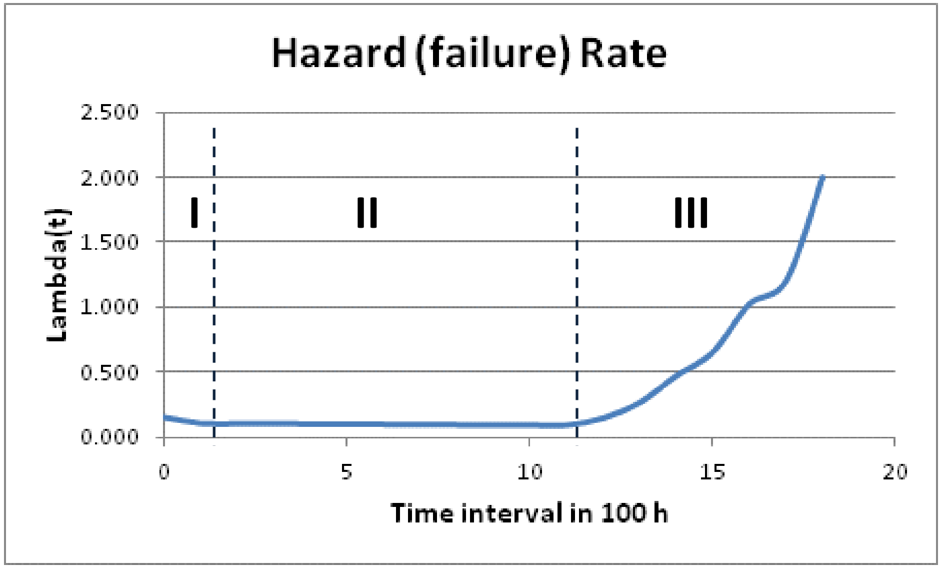
# Non-Reparable Systems

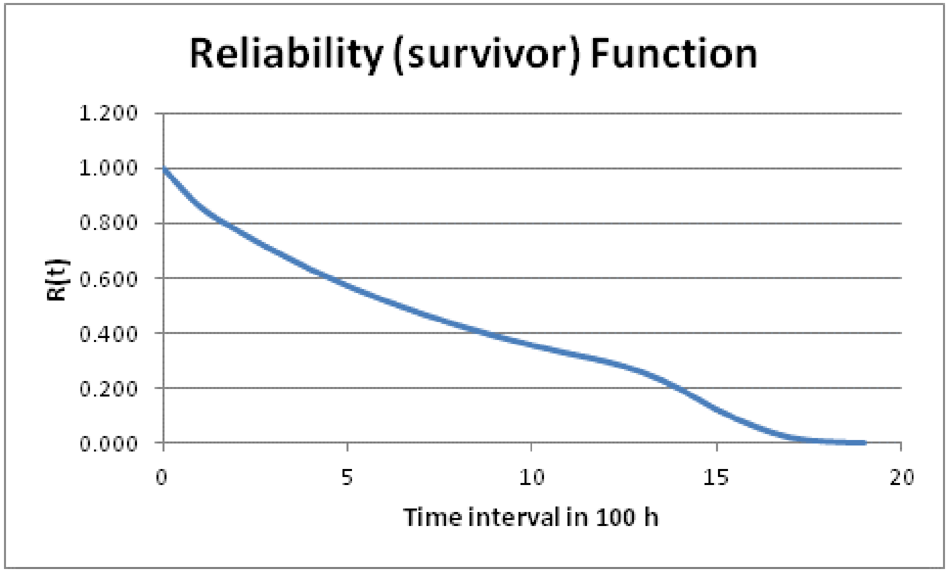
## Example1: Evaluation of Reliability Functions

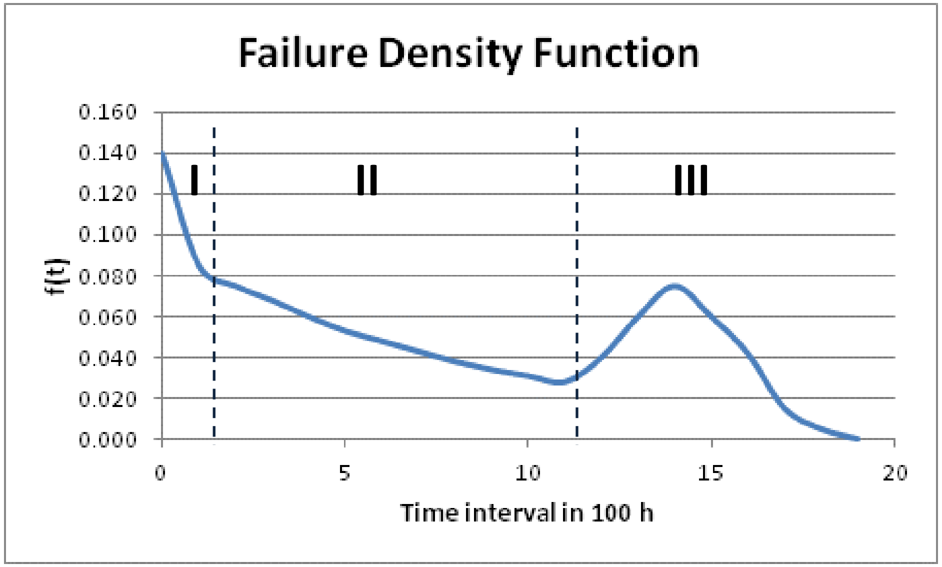
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| Time interval in 100h | Number of failures in each interval | Cumulative failures Nf | Number of survivors Ns | Failure density function f | Cumulative failure distribution Q | Survivor Function R | Hazard (failure) rate Lambda |
| 0 | 140 | 0 | 1000 | 0.140 | 0.000 | 1.000 | 0.151 |
| 1 | 85 | 140 | 860 | 0.085 | 0.140 | 0.860 | 0.104 |
| 2 | 75 | 225 | 775 | 0.075 | 0.225 | 0.775 | 0.102 |
| 3 | 68 | 300 | 700 | 0.068 | 0.300 | 0.700 | 0.102 |
| 4 | 60 | 368 | 632 | 0.060 | 0.368 | 0.632 | 0.100 |
| 5 | 53 | 428 | 572 | 0.053 | 0.428 | 0.572 | 0.097 |
| 6 | 48 | 481 | 519 | 0.048 | 0.481 | 0.519 | 0.097 |
| 7 | 43 | 529 | 471 | 0.043 | 0.529 | 0.471 | 0.096 |
| 8 | 38 | 572 | 428 | 0.038 | 0.572 | 0.428 | 0.093 |
| 9 | 34 | 610 | 390 | 0.034 | 0.610 | 0.390 | 0.091 |
| 10 | 31 | 644 | 356 | 0.031 | 0.644 | 0.356 | 0.091 |
| 11 | 28 | 675 | 325 | 0.028 | 0.675 | 0.325 | 0.090 |
| 12 | 40 | 703 | 297 | 0.040 | 0.703 | 0.297 | 0.144 |
| 13 | 60 | 743 | 257 | 0.060 | 0.743 | 0.257 | 0.264 |
| 14 | 75 | 803 | 197 | 0.075 | 0.803 | 0.197 | 0.470 |
| 15 | 60 | 878 | 122 | 0.060 | 0.878 | 0.122 | 0.652 |
| 16 | 42 | 938 | 62 | 0.042 | 0.938 | 0.062 | 1.024 |
| 17 | 15 | 980 | 20 | 0.015 | 0.980 | 0.020 | 1.200 |
| 18 | 5 | 995 | 5 | 0.005 | 0.995 | 0.005 | 2.000 |
| 19 | | 1000 | 0 | 0.000 | 1.000 | 0.000 | |

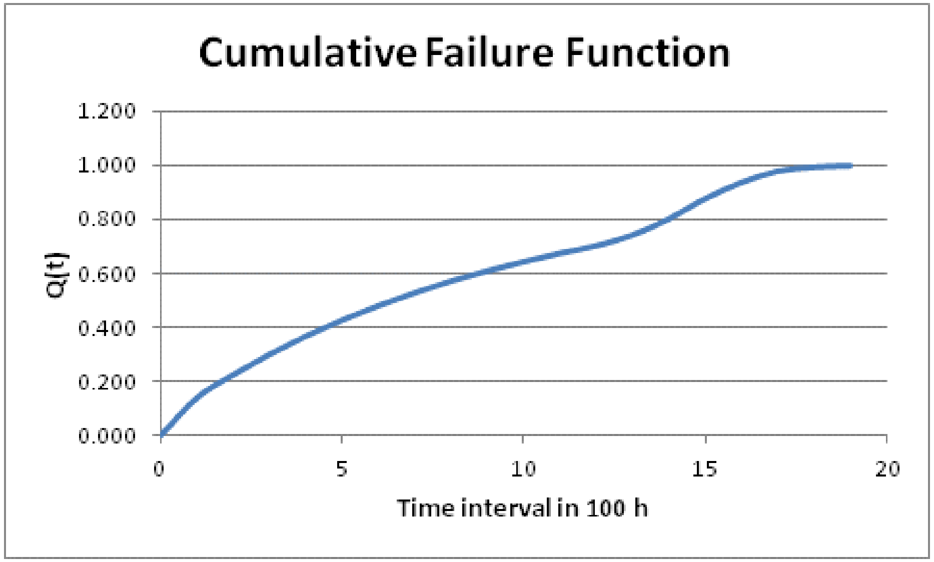
- Compare the calculations with the equations!

# Non-Reparable Systems
## Example1: Evaluation of Reliability Functions



# Non-Reparable Systems
## Normal Distribution

- Probability (failure) density function:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp[-\frac{(x-\mu)^2}{2\sigma^2}] \qquad (11)$$ $\mu$=mean; $\sigma$=standard deviation

- Cumulative probability (failure) function:

$$F(x) = \int_{-\infty}^{x} f(u)du \qquad (12)$$ ***Cannot be solved analytically!***

- 'Standard pdf':

$$z = \frac{x-\mu}{\sigma}; \quad f(z) = \frac{1}{\sqrt{2\pi}} \exp[-z^2/2] \qquad (13)$$ Probability tables!

- Numerical calculations for 'standard pdf':
  - Probability ***Q(z), z>0*** is calculated for given ***z*** (note Q(z)=Q(-z)):

$$Q(z) = y[b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5]$$

$$y = f(z) = (1/\sqrt{2\pi})\exp(-z^2/2)$$

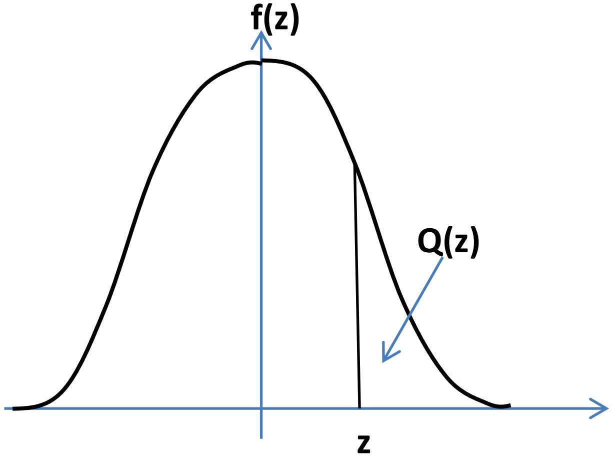
$$t = 1/(1 + r \cdot z)$$

$$r = 0.2316419;\ b_1 = 0.31938153;\ b_2 = -0.356563782;$$

$$b_3 = 1.781477937;\ b_4 = -1.821255978;\ b_5 = 1.330274429 \qquad (14)$$

# Non-Reparable Systems
## Normal Distribution

– Inverse problem ***z*** is calculated for given ***Q(z)***

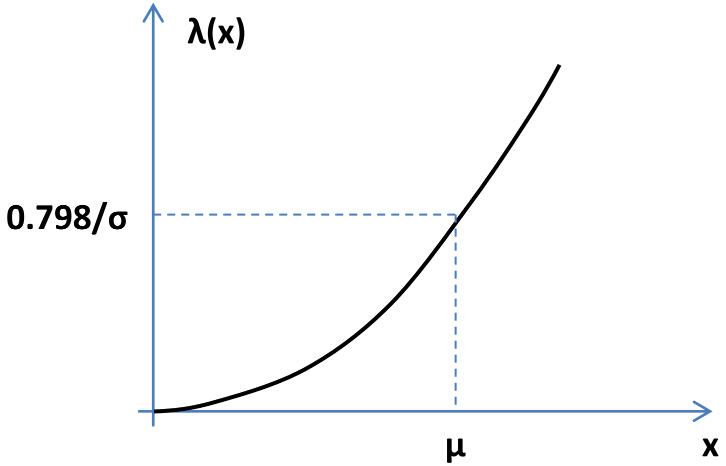



$$z = t - \frac{c_o + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3}\,; \quad t = \sqrt{\ln(1/Q^2)};$$

$$c_0 = 2.515517;\ c_1 = 0.802853;\ c_2 = 0.010328;$$

$$d_1 = 1.432788;\ d_2 = 0.189269;\ d_3 = 0.001308 \quad (15)$$

- Hazard (failure) function:
  – Numerical calculation from eq. (7) [=f(t)/R(t)]
  – 'Best' curve fitted to numerical data



# Non-Reparable Systems
## Example 2: Normal Distribution

- **Example 2**: The Lighting Department of a city installed 2000 electric lamps which have an average life of 1000 burning hours with a standard deviation of 200 hours. How many lamps might be expected to fail in the first 700 hours?
  - Solution:
  - $\mu = 1000, \sigma = 200$
  - z=(700 – 1000)/200 = -1.5
  - From eq. (14) Q(1.5) = Q(-1.5) = 0.0668
  - Expected number of failures: 2000x0.0668 = 133.6 = 134
- In the above example, after what period of burning hours would we expect 10% of lamps to have failed?
  - Solution:
  - Probability Q=0.1
  - From eq. (15) z=-1.2817
  - Inverse transform (x-1000)/200 = -1.2817 gives x=743.7=744 hours

# Non-Reparable Systems
## Weibull Distribution

- Probability (failure) density function:

$$f(t) = \frac{\beta \cdot t^{\beta-1}}{\alpha^{\beta}} \exp\left[-\left(\frac{t}{\alpha}\right)^{\beta}\right];\ t \geq 0,\ \beta > 0,\ \alpha > 0 \qquad (16)$$

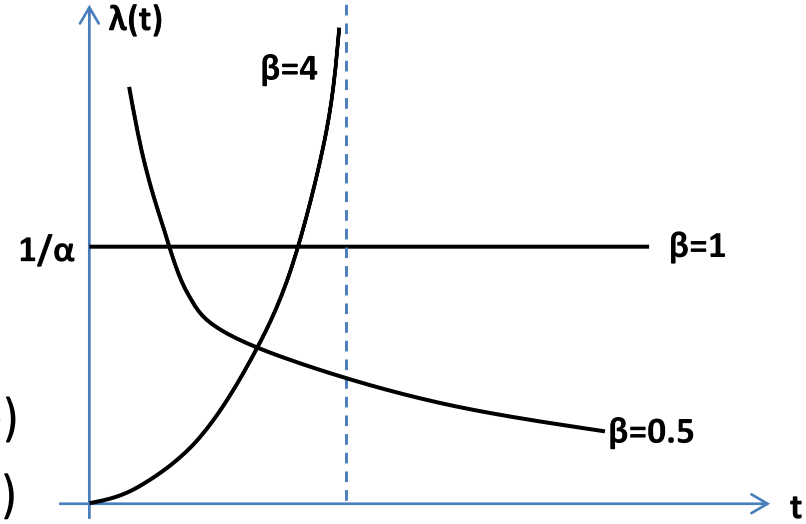
- Cumulative probability (failure) function:

$$Q(t) = 1 - \exp[-(t/\alpha)^{\beta}] \qquad (17)$$

- Hazard (failure) rate:

$$\lambda(t) = f(t)/R(t) = \frac{\beta \cdot t^{\beta-1}}{\alpha^{\beta}} \qquad (18)$$

  – β<1 => decreasing failure rate
  – β=1 => constant failure rate (normal life)
  – β>1 => increasing failure rate (wear-out)



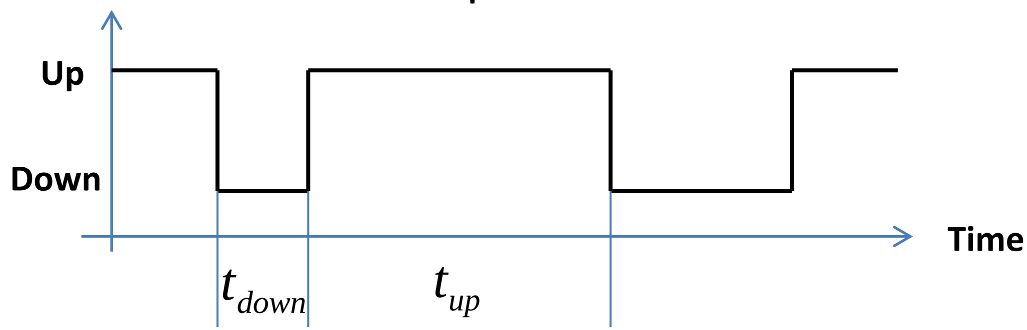

- Widely used distribution function (particularly wear-out period)

# Non-Reparable Systems
## Practical Implementation

- Objective:
  - Predict possible failure of a component and decide on component ***replacement***.
- Algorithm (reliability part):
  1. Collect and process (failure) data.
  2. Select proper probability distribution model(s).
  3. Determine the optimal (best-fitted) parameters of the distribution model(s):
     - The Least Square Estimator (LSE)
     - The Maximum Likelihood Estimator (MLE)
  4. Carry out the goodness-of-fit test to validate the presumed distribution model(s):
     - Kolmogorov-Smirnov test
  5. Return to Step No. 2 if the above test does not give satisfactory results.

# Reparable Systems
## Fundamentals

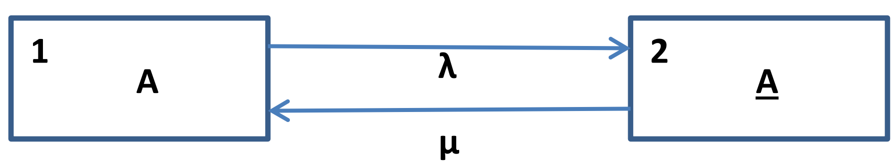
- A component can be in 2 states: up and down




- Random variables <u>up-time</u> and <u>down-time</u> have **exponential** distribution function (hazard rate $\lambda(t) = \lambda = const$ ):

$$Q(t) = 1 - \exp[-\lambda t] \; t \geq 0; \qquad f(t) = \lambda \cdot \exp[-\lambda t] \; ; \; \lambda = const \qquad (19)$$

- State space representation:




  – A - component in service
  – $\underline{A}$ - failed component
  – λ - hazard rate from state 1 to 2; called **failure rate**
  – μ - hazard rate from state 2 to 1; called **repair rate**

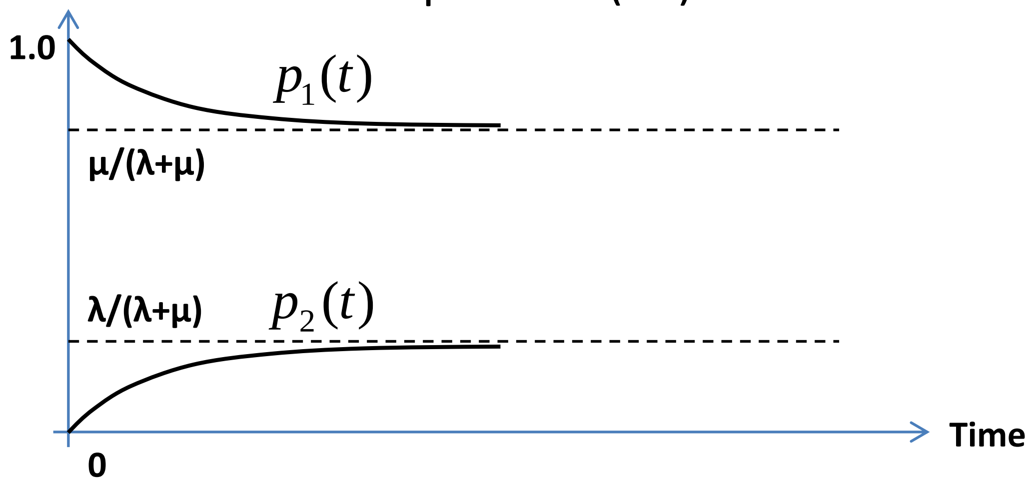
# Reparable Systems
## Fundamentals

- Probability to leave state 1 in dt is λ·dt (eq. (6)) and to leave state 2 is μ·dt:

$$p_1(t+dt) = (1-\lambda \cdot dt)\, p_1(t) + \mu \cdot dt \cdot p_2(t)$$

$$p_2(t+dt) = (1-\mu \cdot dt)\, p_2(t) + \lambda \cdot dt \cdot p_1(t); \qquad \{p_1(t) + p_2(t) = 1\} \qquad (20)$$

- Solution to the differential equations (20) – Markov model are illustrated below:




- Reliability analysis of power systems is based on **steady-state probabilities**:

**Availability:** $$A = p_1(t \to \infty) = \frac{\mu}{\lambda + \mu}$$

**Unavailability:** $$U = p_2(t \to \infty) = \frac{\lambda}{\lambda + \mu} \qquad (21)$$

# Reparable Systems
## Fundamentals

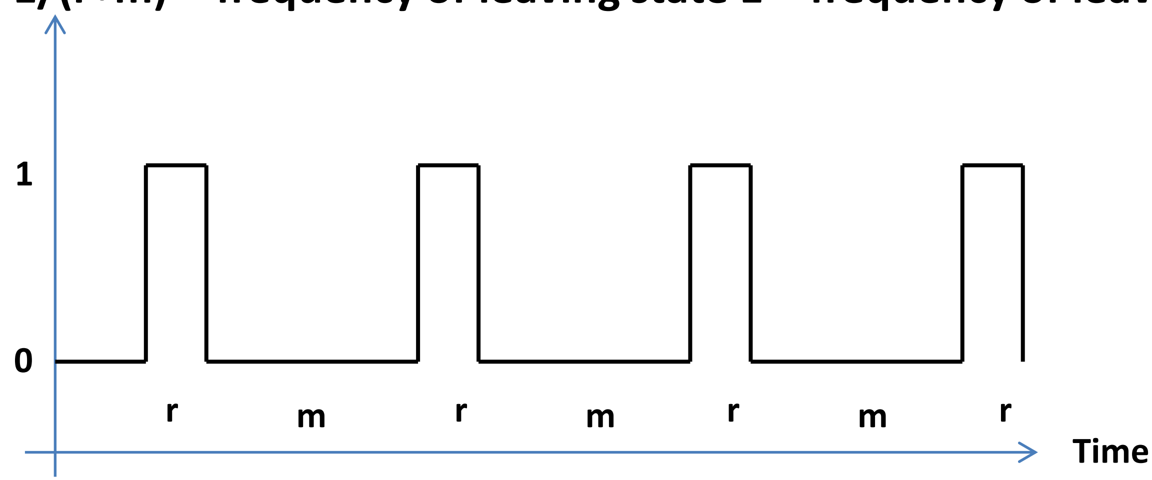
- Mean Time to Failure (eq. (10) and exponential distribution for up-time):

$$MTTF = m = \int_0^\infty p_1(t)dt = 1/\lambda \qquad (22)$$

- Mean Time to Repair (eq. (10) and exponential distribution for down-time):

$$MTTR = r = \int_0^\infty p_2(t)dt = 1/\mu \qquad (23)$$

- Averaged behaviour of the component (averaging over time):
  - **A = m/(r+m)**
  - **U = r/(r+m)**
  - **f = 1/T = 1/(r+m)** - **frequency of leaving state 1 = frequency of leaving state 2** (24)



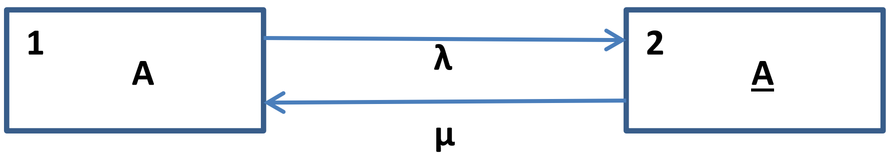
# Reparable Systems
## Example 3: Steady State Indices

- A power system component has constant failure rate of 0.001 1/h and constant repair rate of 0.02 h. Calculate steady-state indices.
- Solution:
  - $m=1/\lambda=1/0.001=1000$ h; $r=1/\mu=1/0.02=50$ h
  - $A=m/(r+m)=1000/(1000+50)=0.9524$; $U=r/(r+m)=50/1050=0.0476$
  - $T=m+r=1050$ h; $f=1/T=1/1050=0.009524$ 1/h
  - $T=U\cdot 8760=0.0476\cdot 8760=416.98$ h (***expected yearly outage duration***)

# Reparable Systems
## Steady-State Probabilities for Systems

- A system of ‘n’ components is considered; each component can be up or down
- A **state-space diagram** is drawn:
  - each state has (unknown) probability,
  - transitions between states are failure and repair rates
- A single component:
  - Frequency (failure density) of leaving state 1 is equal to frequency of entering state 2

$$A \cdot \lambda = U \cdot \mu = \lambda \cdot \mu / (\lambda + \mu) \quad (24)$$

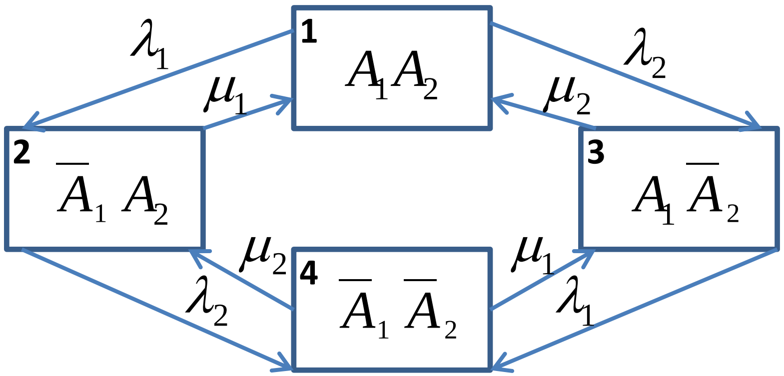



- **General methodology**:
  - Write balance of frequencies for each state i=1,2,...,n; failure/repair rates entering a state are positive, those leaving a state are negative
  - System matrix so obtained is singular => replace one of the equations with sum of state probabilities equals unity
  - Solve the system of linear algebraic equations

# Reparable Systems

## Example 4: Two Independent Components

- Calculate probabilities for two independent components which can be in 'up' and 'down' states




- Solution:
  - Four states whose (unknown) probabilities are $p_1, p_2, p_3, p_4$
  - Steady-state equations:

$$-(\lambda_1+\lambda_2)p_1+\mu_1 p_2+\mu_2 p_3=0$$

$$\lambda_1 p_1-(\mu_1+\lambda_2)p_2+\mu_2 p_4=0$$

$$\lambda_2 p_1-(\mu_2+\lambda_1)p_3+\mu_1 p_4=0$$

$$\lambda_2 p_2+\lambda_1 p_3-(\mu_1+\mu_2)p_4=0$$

$$p_1+p_2+p_3+p_4=1$$

# Reparable Systems
## Example 4: Two Independent Components

- Solution of the above system of equations:

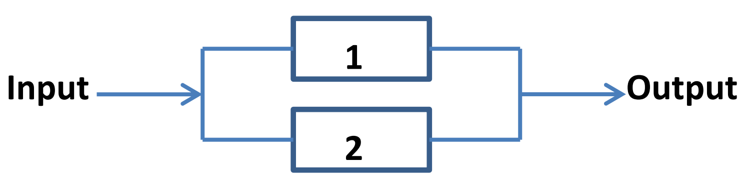
$$p_1 = \mu_1/(\lambda_1+\mu_1)\cdot\mu_2/(\lambda_2+\mu_2) = A_1\cdot A_2$$
$$p_2 = \lambda_1/(\lambda_1+\mu_1)\cdot\mu_2/(\lambda_2+\mu_2) = U_1\cdot A_2$$
$$p_3 = \mu_1/(\lambda_1+\mu_1)\cdot\lambda_2/(\lambda_2+\mu_2) = A_1\cdot U_2$$
$$p_4 = \lambda_1/(\lambda_1+\mu_1)\cdot\lambda_2/(\lambda_2+\mu_2) = U_1\cdot U_2$$

(products of individual availabilities and unavailabilities)

- Assume a series connection of the two components (eg transformer & cable):


  - Availability **A=p1**, Unavailability **U=1-A**.

- Assume a parallel connection of the two components (eg two parallel lines):




  - Unavailability **U=p4**, Availability **A=1-U**

# Reparable Systems
## Steady-State Probabilities – General Expressions

- Frequencies of leaving a state and entering a state (example 4):

$$-a_{kk} \cdot p_k = \sum_{\substack{i=1 \\ i \neq k}}^{n} a_{ki} \cdot p_i \qquad (25)$$

-Frequency of leaving a state: left-hand side

-Frequency of entering a state: right-hand side

- Expected time residing at state 'k':

$$m_k = 1/\lambda_k = 1/(-a_{kk}) \qquad (26)$$

($\lambda_k$ is rate of leaving state 'k')

- Frequency of leaving state 'k':

$$f_k = \lambda_k \cdot p_k \qquad (27)$$

- Duration of cycle at state 'k':

$$T_k = 1/f_k = m_k + r_k \qquad (28)$$

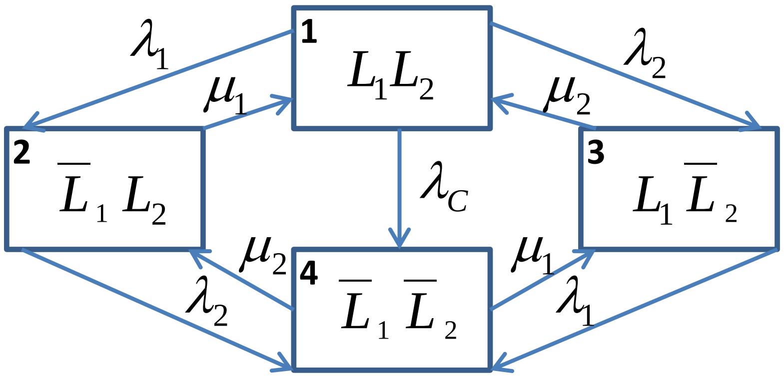
($r_k$ is expected residing outside state 'k')

- For 'n' states, system of 'n' linear algebraic equations needs to be solved. This can be done numerically.

# Reparable Systems

## Example 5: Common Mode Failures

- Two overhead lines on the same right-of-way, or two circuits on the same towers
- Simultaneous outage of two (or more components) due to a common cause; common mode failure rate is $\lambda_C$ ; no simultaneous repair of both components




- Full set of equations as before; **Approximate** solution is:

$$p_1 = A_1 \cdot A_2 - \lambda_C / (\mu_1 + \mu_2)$$

$$p_2 = U_1 \cdot A_2$$

$$p_3 = A_1 \cdot U_2$$

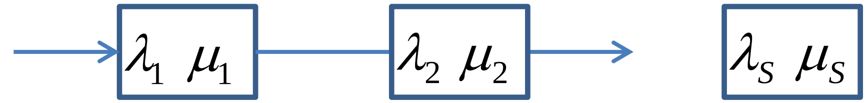
$$p_4 = U_1 \cdot U_2 + \lambda_C / (\mu_1 + \mu_2)$$

$$f_4 = (\mu_1 + \mu_2) \cdot p_4$$ (frequency of outage of both circuits)

# Reliability Modelling of Systems

## Series Systems

- Series system of 2 components:




$$P_{up} = \mu_1 \cdot \mu_2 / [(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)] = \mu_S / (\lambda_S + \mu_S) \quad (29)$$

- Transition rate from the system up state:

$$\lambda_S = \lambda_1 + \lambda_2 \qquad (30)$$

- By replacing eq. (30) in eq. (29) and noting repair rate is reciprocal of the average repair time:

$$r_S = 1/\mu_S = \frac{\lambda_1 \cdot r_1 + \lambda_2 \cdot r_2 + \lambda_1 \cdot \lambda_2 \cdot r_1 \cdot r_2}{\lambda_S} \approx \frac{\lambda_1 \cdot r_1 + \lambda_2 \cdot r_2}{\lambda_S} \qquad (31)$$

- General case of ‘n’ series components:

$$\lambda_S = \sum_{i=1}^{n} \lambda_i$$

$$r_S = \{\sum_{i=1}^{n} \lambda_i \cdot r_i\} / \lambda_S$$

**Distribution (radial) networks**

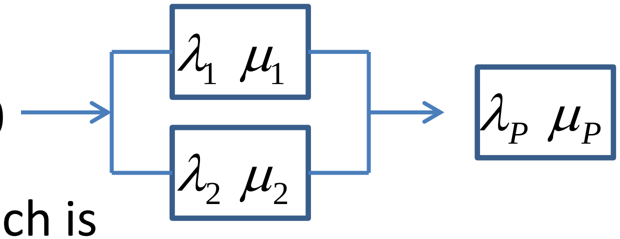
$$U_S = \lambda_S \cdot r_S = \sum_{i=1}^{n} \lambda_i \cdot r_i \qquad (32)$$

# Reliability Modelling of Systems

## Parallel Systems

- Parallel system of 2 components:

$$P_{down} = \lambda_1 \cdot \lambda_2 / [(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)] = \mu_P / (\lambda_P + \mu_P) \quad (33)$$




- The rate of transition from down state is $\mu_1 + \mu_2$ which is equivalent to $\mu_P$:

$$\mu_P = \mu_1 + \mu_2; \quad 1/r_P = 1/r_1 + 1/r_2 \Rightarrow r_P = r_1 \cdot r_2 / (r_1 + r_2) \qquad (34)$$

- Substituting eq. (34) in eq. (33):

$$\lambda_P = \frac{\lambda_1 \cdot \lambda_2 (r_1 + r_2)}{1 + \lambda_1 \cdot r_1 + \lambda_2 \cdot r_2} \approx \lambda_1 \cdot \lambda_2 (r_1 + r_2)$$

$$U_P = \lambda_P \cdot r_P = \lambda_1 \cdot \lambda_2 \cdot r_1 \cdot r_2 \qquad (35)$$

- Parallel system of 3 components:

$$\lambda_P \approx \lambda_1 \cdot \lambda_2 \cdot \lambda_3 (r_1 \cdot r_2 + r_1 \cdot r_3 + r_2 \cdot r_3)$$

$$1/r_P = 1/r_1 + 1/r_2 + 1/r_3$$

$$U_P = \lambda_P \cdot r_P = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot r_1 \cdot r_2 \cdot r_3 \qquad (36)$$

- Very useful for simplifying systems with series-parallel structure

# Reliability Modelling of Systems

## Example 6: Series and Parallel Systems

- The failure rates of three *series* components are 0.05 f/yr, 0.01 f/yr and 0.02 f/yr and their repair times are 20 hr, 15 hr and 25 hr respectively. Calculate the system failure rate, average repair time and unavailability of the system.

$$\lambda_S = 0.05 + 0.01 + 0.02 = 0.08\ f\,/\,yr$$

$$U_S = 0.05 \cdot 20 + 0.01 \cdot 15 + 0.02 \cdot 25 = 1.65\,hr\,/\,yr$$

$$r_S = 1.65 / 0.08 = 20.6\,hr$$

- The failure rates of two parallel components are 0.05 f/yr and 0.02 f/yr and the average repair times are 20 hr and 25 hr respectively. Calculate the system failure rate, average repair time and unavailability.

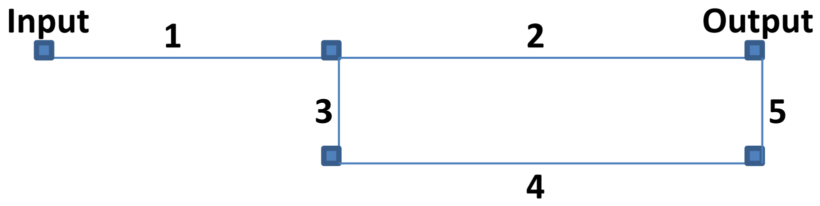
$$\lambda_P = 0.05 \cdot 0.02 \cdot (20 + 25) / 8760 = 5.14 \cdot 10^{-6}\ f\,/\,yr \quad NOTE\ 8760\,hr!$$

$$r_P = 20 \cdot 25 / (20 + 25) = 11.1\ hr$$

$$U_P = \lambda_P \cdot r_P = 5.71 \cdot 10^{-5}\ hr\,/\,yr$$

# Reliability Modelling of Systems
## Example 7: Series-Parallel Structure

- Calculate reliability indices (U, A, failure rate, average time ‘r’) for the series-parallel system shown below.



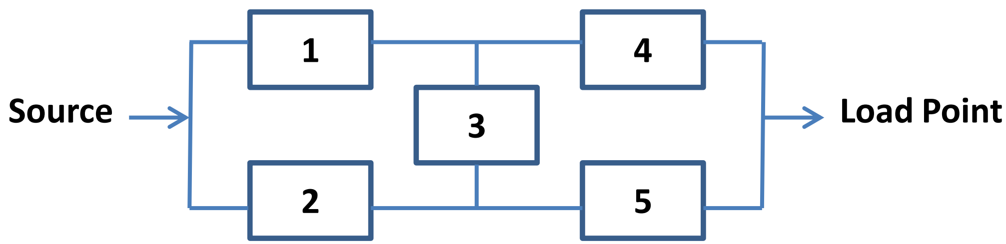

- Solution:
  - Series components 3, 4 and 5 give a ‘new component’ 6 (equations (32))
  - Parallel components 2 and 6 give a ‘new component’ 7 (equations (34) and (35))
  - Series components 1 and 7 give a ‘new component’ 8 (equations (32))
  - ***Solve the problem with assumed figures similar to the previous example***

# Reliability Modelling of Systems
## Minimal Tie Set

- A **path** is a set of components that form a connection between input and output.
- A **minimal path** is one in which no node is traversed more than once.
- The i-th minimal path is denoted $T_i$, $i = 1, \ldots, n$. System reliability is:

$$R_S = P[\bigcup_{i=1}^{n} T_i] \qquad (37)$$ P[] = probability, U = union (logical 'OR' operation)

- All minimal paths are connected in **parallel**.
- Calculations are explained on the 'Bridge Network':




  - Visual inspection shows there are 4 minimal paths:
    T1=[1,4]-path 1; T2=[2,5]-path 2; T3=[1,3,5]-path 3; T4=[2,3,4]-path 4;
  - **Parallel** connection of 4 minimal paths T1, T2, T3 & T4.
  - Minimal paths are **NOT independent** because one component is in several paths.

# Reliability Modelling of Systems
## Minimal Tie Set

- Reliability of the bridge network:

  $$R_S = P(T1 \cup T2 \cup T3 \cup T4) \qquad (38)$$

- If R(Ti) is probability that tie set 'i' is reliable, then:

$$\begin{aligned} R_S = {} & R(T1) + R(T2) + R(T3) + R(T4) - R(T1 \cap T2) - R(T1 \cap T3) - R(T1 \cap T4) \\ & - R(T2 \cap T3) - R(T2 \cap T4) - R(T3 \cap T4) \\ & + R(T1 \cap T2 \cap T3) + R(T1 \cap T2 \cap T4) + R(T1 \cap T3 \cap T4) + R(T2 \cap T3 \cap T4) \\ & - R(T1 \cap T2 \cap T3 \cap T4) \qquad (39) \end{aligned}$$

(proof of eq. (39) – draw 3 sets which intersect each other & find their union)

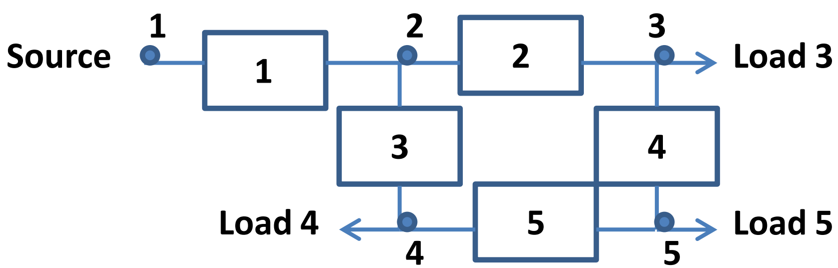
**Example 8:** (Homework)

- Assume availability and unavailability for each component 1,2,3,4 & 5 and calculate system reliability using eq. (39)
- Note that we did NOT calculate frequencies and durations, just probabilities.

# Reliability Modelling of Systems
## Example 9: Minimal Tie Sets

- For the given network find minimal paths, write expression for reliability Rs and calculate Rs if reliability of each component is 0.95.

  Note: Reliability means that all load points are supplied




- Solution:
  - T1=[1,2,4,5] – path 1: component 3 failed
  - T2=[1,3,4,5] – path 2: component 2 failed
  - T3=[1,2,3,4] – path 3: component 5 failed
  - T4=[1,2,3,5] – path 4: component 4 failed
  - Rs = P(T1 U T2 U T3 U T4) = ...    (homework)

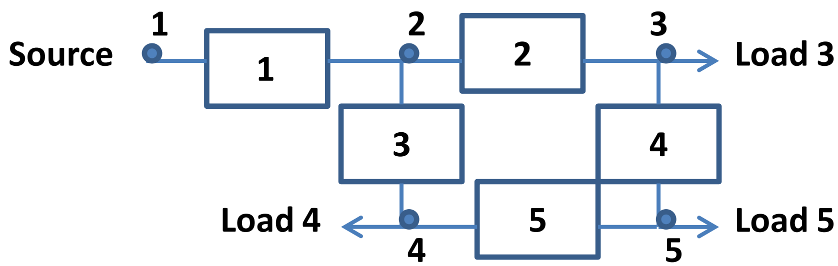
# Reliability Modelling of Systems
## Minimum Cut Set

- A **cut** is a ‘line’ that disconnects inputs from the outputs
- A **cut set** is a set of elements that, if fails, causes the system to fail
- A **minimum cut set** is one in which there is no proper subset of elements whose failure alone will cause system to fail
- The i-th minimum cut set is denoted $C_i \ \ i = 1,2,\ldots,n$ . System unavailability is
  $U_S = P[\bigcup_{i=1}^{n} C_i]$ (40) P[] = probability; U = union
- Reliability of a system is then:
  $R_S = 1 - P[\bigcup_{i=1}^{n} C_i]$ (41)
- Minimum cut sets are not independent, equation (40) is expanded in a similar way as equation (39)
- All minimum cut sets are connected in **series** because each one leads to the system failure

# Reliability Modelling of Systems

## Example 10: Minimal Cut Sets

- **Example 10:** Find minimum cut sets and unavailability for the system given below.




- Minimum cut sets are:
  - C1=[1] – component 1 failed;
  - C2=[2,3] – components 2 & 3 failed
  - C3=[2,4] – components 2 & 4 failed;
  - C4=[2,5] – components 2 & 5 failed
  - C5=[3,4] – components 3 & 4 failed;
  - C6=[3,5] – components 3 & 5 failed
  - C7=[4,5] – components 4 & 5 failed
- Part of the expression for system unavailability:

$$
\begin{aligned}
U_S = & U(C1)+U(C2)+U(C3)+U(C4)+U(C5)+U(C6)+U(C7) \\
& -U(C1\cap C2)-U(C1\cap C3)-U(C1\cap C4)-U(C1\cap C5)-U(C1\cap C6)-U(C1\cap C7) \\
& -U(C2\cap C3)-U(C2\cap C4)-U(C2\cap C5)-U(C2\cap C6)-U(C2\cap C7) \\
& -U(C3\cap C4)-U(C3\cap C5)-U(C3\cap C6)-U(C3\cap C7) \\
& -U(C4\cap C5)-U(C4\cap C6)-U(C4\cap C7) \\
& -U(C5\cap C6)-U(C5\cap C7) \\
& -U(C6\cap C7)+\ldots \qquad \{U(C1)=U_1; U(C2)=U_2\cdot U_3;\ldots\} \qquad (42)
\end{aligned}
$$

- For highly reliable systems, first-order approximation can be used:

$$U_S \approx U(C1)+U(C2)+U(C3)+U(C4)+U(C5)+U(C6)+U(C7) \qquad (43)$$

# Reliability Modelling of Systems
## Bayes' Theorem (Total Probability)

- If an event *A* depends on mutually exclusive events $B_1, B_2, \ldots$ then probability of the occurrence of event *A* is:

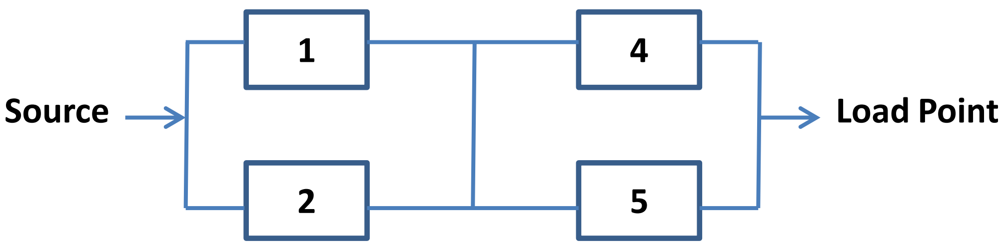
$$P(A) = \sum_i P(A \mid B_i) \qquad (43)$$

(conditional probabilities)

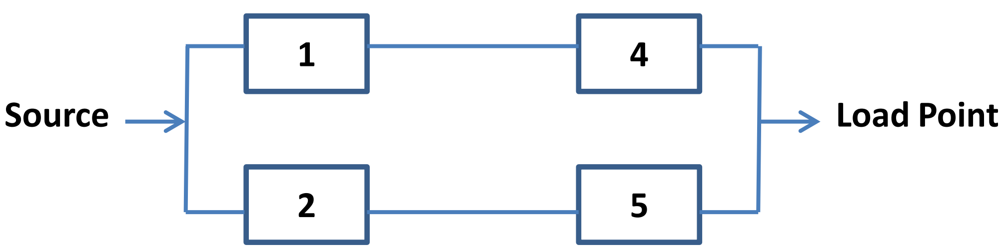
- Bayes' theorem applied to system reliability Rs:

$$R_S = P(system\,sucess \mid component\; i\; is\; up) \cdot R_i + P(system\,sucess \mid component\; i\; is\; down) \cdot U_i \qquad (44)$$

- Calculation on Bridge Network from Example 8 :




Component 3 fully reliable




Component 3 failed

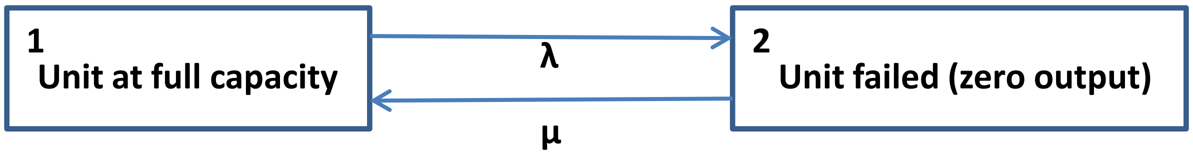
$$R_S = R_S(3{=}healthy) \cdot A_3 + R_S(3{=}\,failed) \cdot U_3$$

(finish the example at home)

# Generation

## Modelling of Generating Units

- Two-State Model




$$U = FOR = \lambda/(\lambda+\mu) = r/(m+r) = \sum downtime/\{\sum downtime + \sum uptime\},$$
$$A = \mu/(\lambda+\mu) = m/(m+r) = \sum uptime/\{\sum downtime + \sum uptime\},$$
$$f = 1/T = \lambda \cdot A = \mu \cdot U \qquad (45)$$

U, A = unavailability, availability

FOR = forced outage rate

λ, μ = expected failure rate, expected repair rate

m, r = mean time to failure MTTF (=1/λ), mean time to repair MTTR (=1/μ)

m+r = mean time between failures MTBF (=1/f)

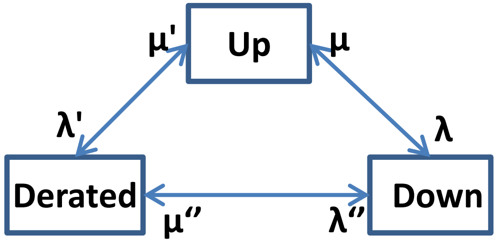
f, T = cycle frequency (=1/T), cycle time (=1/T)

  - Practical calculations are done using down and up times (FOR)
  - Generating units often operate in derated states with reduced output

# Generation

## Modelling of Generating Units

- Equivalent Two-State Models
  - Full models with derated states are sometimes used in ops planning




(full set of equations based on frequency balances needs to be solved!)

  - Equivalent two-state model is used more frequently

Idea: calculate equivalent time with full output and equivalent time with zero output

$$T_{Upe} = T_{Up} + \sum_{k=1}^{n-1} T_k \frac{C_k}{C_0},$$

$$T_{Downe} = T_{down} + \sum_{k=1}^{n-1} T_k \frac{C_0 - C_k}{C_0} \qquad (46)$$

$T_{upe}$, $T_{downe}$ = equivalent up (full output) and down time (zero output)

$T_k$ = duration of k-th derated state

$C_0$, $C_k$ = full capacity and derated capacity of state ‘k’

# Generation

## Modelling of Generating Units

– Equivalent availability and unavailability (= equivalent FOR = EFOR)

$$A_e = T_{Upe} / (T_{Upe} + T_{Downe}),$$
$$U_e = T_{Downe} / (T_{Upe} + T_{Downe}) \qquad (47)$$

– It is possible to define equivalent failure and repair rates (we are not doing this!)

– **Example 12:** A 500MW unit has a derated state of 300MW. Full, derated and zero capacity times are 5000 h, 100 h and 50 h respectively. Find equivalent availability and unavailability.

$T_{upe}$ = 5000 + 100(300/500) = 5060 h

$T_{downe}$ = 50 + 100((500-300)/500) = 90 h

$A_e$ = 5060/(5060+90) = 0.9825

$U_e$ = 90/(5060+90) = 0.0175

# Generation
## Modelling of Generating Units

- Identical units
  - Identical units shall be grouped to reduce the number of states. For 'n' identical two-state units, $2^n$ states is reduced to (n+1) states
  - State space model is set and states with the same probability are grouped
  - Figure: all units available; 1 failed unit; 2 failed units;...
  - Probability that 'k' units are unavailable (Binomial distribution)

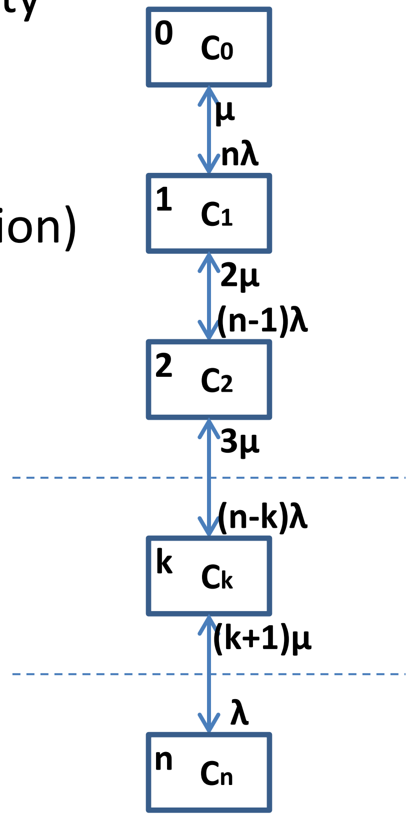
$$P_k = \binom{n}{k} U^k \cdot A^{n-k} = \frac{n!}{(n-k)!\cdot k!} U^k \cdot A^{n-k} \qquad (48)$$

  - Failure rates

$$\lambda_{k\to k+1} = (n-k)\cdot\lambda,$$
$$\lambda_{k\to k-1} = k\cdot\mu \qquad (49)$$

  - Available capacity in state 'k'

$$C_k = (n-k)\cdot C \qquad (50)$$



# Generation

## Modelling of Generating Units

- **Example 13**: For a system of 4 identical 50MW units with λ=0.4  1/yr and μ=9.6  1/yr calculate capacity outage probability table and equivalent state space diagram.
  - Solution

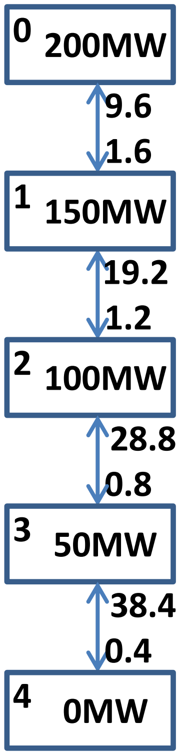
    U = 0.4/(9.6+0.4) = 0.04

    A = 1-U = 0.96

    ... $P_1$ = 4!/(3!1!)x0.96x0.96x0.96x0.04 = 0.1415578

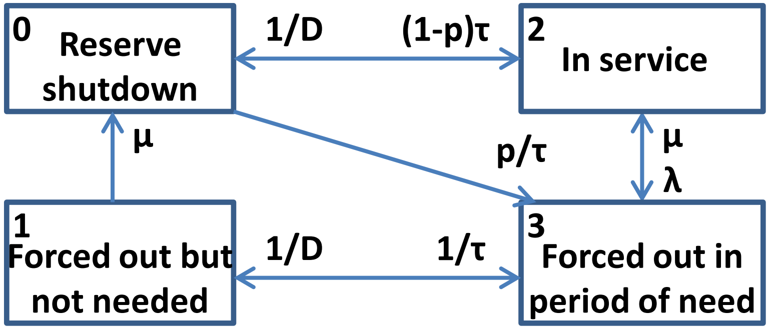
    $P_2$ = 4!/(2!2!)x0.96x0.96x0.04x0.04 = 0.0088474

| Number of units out k | Capacity loss (MW) | Capacity in service (MW) | Probability of state 'k' |
|---|---|---|---|
| 0 | 0 | 200 | 0.8493466 |
| 1 | 50 | 150 | 0.1415578 |
| 2 | 100 | 100 | 0.0088474 |
| 3 | 150 | 50 | 0.0002458 |
| 4 | 200 | 0 | 0.0000026 |



# Generation
## Modelling of Generating Units

- Peaking Units
  - Intermittent operation
  - Unit can fail when it is not needed
  - Four-state model shown bellow
  - Unavailability is conditional probability that the unit fails when it is needed ($U=P_2/(P_2+P_3)$) in the figure below




  - τ = average shut-down time between periods of need

    D = average in-service time per occasion of need

    p = probability of starting failure
  - Unavailability when p=0:

$$U = \lambda \cdot k / (\mu + \lambda \cdot k),$$

$$k = \frac{1 + \mu \cdot \tau}{1 + \mu \cdot \tau + \tau / D} \qquad (51)$$

# Generation

## Modelling of Generating Units

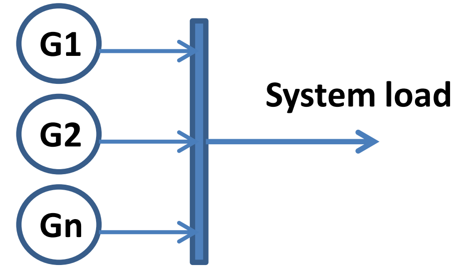
- **Example 14:** A peaking unit has failure rate of λ=0.001 1/h and repair rate of μ=0.01 1/h. The peaking unit operates on average for D=10 h when it is needed. Find the unit unavailability and availability for different shut-down times.
  - **Solution**

| Shut-down time h | Correction factor k | Unavailability U | Availability A |
|---:|---:|---:|---:|
| 0 | 1 | 0.091 | 0.909 |
| 1 | 0.991 | 0.09 | 0.91 |
| 5 | 0.808 | 0.075 | 0.925 |
| 10 | 0.687 | 0.064 | 0.936 |
| 50 | 0.375 | 0.036 | 0.964 |
| 100 | 0.286 | 0.028 | 0.972 |

- The longer the shut-down time τ, the smaller the unavailability.

# Generation

## Generating Capacity in a System




- 'One=point' system is always studied (networks are ignored)

- Capacity Outage (Probability) Tables
  - All units are independent, modelled with two-state models (derated states can also be accounted for)
  - The system has 'many' states; probability of each state is a product of individual unit availabilities and unavailabilities – see example below

- A recursive algorithm for capacity outage table (computer program)

The cumulative probability of a capacity outage state of X MW after a unit of capacity C MW and unavailability of U is added (no derated states):

$$P(X) = (1 - U) \cdot P'(X) + U \cdot P'(X - C)$$

$$P'(X) = 1.0 \;\; for \;\; X \leq 0, \qquad P'(X) = 0 \; otherwise \qquad (52)$$

(P'(X) = cumulative probability before adding unit; P(X) = after adding unit)

Generalised equation (52) when units have derated states (multi-state modelling):

$$P(X) = \sum_{i=1}^{n} p_i \cdot P'(X - C_i) \qquad (53)$$

n=number of unit states; $C_i$=capacity of state 'i'; $p_i$=probability of state 'i'

(the same initialisation as before); Derivation of (52) & (53) based on convolution!!

# Generation

## Generating Capacity in a System

- **Example 15:** A system consists of two 3MW units and one 5MW unit whose FOR is 0.02. Construct a capacity outage table.
  - All units available (0MW out of service): 0.98x0.98x0.98 = 0.941992
  - One 3MW unit unavailable (3MW out of service): **2**x0.02x0.98x0.98 = 0.038416
  - 5MW unit unavailable (5MW out of service): 0.98X0.98x0.02 = 0.019208
  - One 3MW and 5MW units unavailable (8MW out of service): **2**x0.02x0.98x0.02=0.000784

| Capacity out of service (MW) | In-service capacity (MW) | Individual probability | Cumulative probability |
|---:|---:|---:|---:|
| 0 | 11 | 0.941192 | 1 |
| 3 | 8 | 0.038416 | 0.058808 |
| 5 | 6 | 0.019208 | 0.020392 |
| 6 | 5 | 0.000392 | 0.001184 |
| 8 | 3 | 0.000784 | 0.000792 |
| 11 | 0 | 0.000008 | 0.000008 |

# Generation

## Generating Capacity in a System

- **Example 16:** A system consists of two 25MW units whose unavailability is 0.02, and one 50MW unit whose unavailability (full outage) is 0.007 and partial unavailability (20MW is out) is 0.033. Apply recursive algorithm.
  - **Solution**
    - Step 1: Add the first unit (25MW)

      $P(0)=(1-0.02)(1.0) + (0.02)(1.0) = 1.0$

      $P(25)=(1-0.02)(0) + (0.02)(1.0) = 0.02$
    - Step 2: Add the second unit (25MW)

      $P(0)=(1-0.02)(1.0) + (0.02)(1.0) = 1.0$

      $P(25)=(1-0.02)(0.02) + (0.02)(1.0) = 0.0396$

      $P(50)=(1-0.02)(0) + (0.02)(0.02) = 0.0004$
    - Step 3: add the third (multi-state) unit (50MW)

$$
\begin{array}{llll}
P(0) = (0.96)(1.0) & + (0.033)(1.0) & + (0.007)(1.0) & = 1.0 \\
P(20) = (0.96)(0.0396) & + (0.033)(1.0) & + (0.007)(1.0) & = 0.078016 \\
P(25) = (0.96)(0.0396) & + (0.033)(0.0396) & + (0.007)(1.0) & = 0.0463228 \\
P(45) = (0.96)(0.0004) & + (0.033)(0.0396) & + (0.007)(1.0) & = 0.0086908 \\
P(50) = (0.96)(0.0004) & + (0.033)(0.0004) & + (0.007)(1.0) & = 0.0073972 \\
P(70) = (0.96)(0) & + (0.033)(0.0004) & + (0.007)(0.0396) & = 0.0002904 \\
P(75) = (0.96)(0) & + (0.033)(0) & + (0.007)(0.0396) & = 0.0002772 \\
P(100) = (0.96)(0) & + (0.033)(0) & + (0.007)(0.0004) & = 0.0000028
\end{array}
$$

# Generation

## Generating Capacity in a System

- Capacity outage table – frequency and duration approach
  - Entire state-space diagram with transition rates (ie failure and repair) is modelled
  - Identical capacity states are combined into a new state; subscript 'i' refers to the identical capacity states, and 'k' refers to the new merged state

    Capacity outage of state 'k' $C_k = C_1 = C_2 = ... = C_i$

    Probability of state 'k' $p_k = p_1 + p_2 + ... + p_i$

    Frequency of state 'k' $f_k = f_1 + f_2 + ... + f_i$

    Rates of departure from state 'k' - $\lambda_{\pm k}$ (+k – higher available capacity;
    -k – lower available capacity)

$$\lambda_{\pm k} = (\sum p_j \cdot \lambda_{\pm j}) / p_k \qquad (54)$$

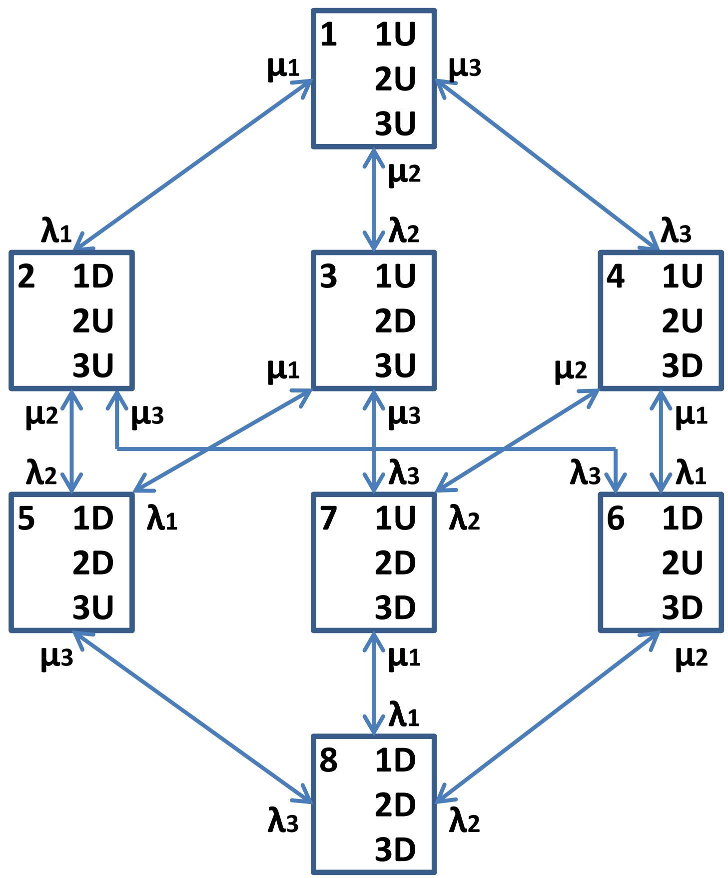
  - Equations (54) were used in Example 13 to obtain equivalent diagram

# Generation

## Generating Capacity in a System

- **Example 17:** A system consists of two 25MW units and one 50MW unit. All units have failure rate of 0.01 f/day and repair rate of 0.49 r/day. Develop the state space diagram, aggregate identical capacity states and calculate probability and frequency of each state.

**Solution**



# Generation

## Generating Capacity in a System

- State probabilities and frequencies

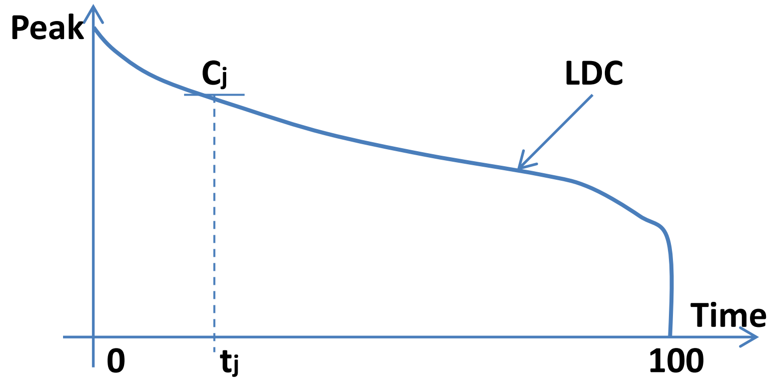
| State No. | Capacity out Ci (MW) | State probability pi | State frequency fi |
|---|---|---|---|
| 1 | 0 | 0.98x0.98x0.98 = 0.941192 | 0.941192x0.03 = 0.02823576 |
| 2 | 25 | 0.02x0.98x0.98 = 0.019208 | 0.019208x0.51 = 0.00979608 |
| 3 | 25 | 0.98x0.02x0.98 = 0.019208 | 0.019208x0.51 = 0.00979608 |
| 4 | 50 | 0.98x0.98x0.02 = 0.019208 | 0.019208x0.51 = 0.00979608 |
| 5 | 50 | 0.02x0.02x0.98 = 0.000392 | 0.000392x0.99 = 0.00038808 |
| 6 | 75 | 0.02x0.98x0.02 = 0.000392 | 0.000392x0.99 = 0.00038808 |
| 7 | 75 | 0.98x0.02x0.02 = 0.000392 | 0.000392x0.99 = 0.00038808 |
| 8 | 100 | 0.02x0.02x0.02 = 0.000008 | 0.000008x1.47 = 0.00001176 |

- Reduced generation model

| State No. | Capacity out (MW) | Capacity in (MW) | Probability pk | Frequency (occ/day) fk |
|---|---|---|---|---|
| 1 | 0 | 100 | 0.941192 | 0.02823576 |
| 2 | 25 | 75 | 0.038416 | 0.01959216 |
| 3 | 50 | 50 | 0.0196 | 0.01018416 |
| 4 | 75 | 25 | 0.000784 | 0.00077616 |
| 5 | 100 | 0 | 0.000008 | 0.00001176 |

# Generation
## Reliability (Loss of Load) Indices

- Generation-demand system is studied ('one-point' system)
- Reliability indices are calculated by comparing available generation capacity against (system) load curve




- Similar but different definitions can be found in literature
- **Loss of Load Expectation** (**LOLE**)
  - Simplest load model: each day is represented by its daily peak
  - The individual daily peaks are arranged in descending order and form 'daily peak load variation curve'
  - Shape of the curve is similar to the above figure, there are 365 discrete values in a year

# Generation
## Reliability (Loss of Load) Indices

- **LOLE** is expected number of days in the specified period in which the daily peak load will exceed the available generation capacity
- Simple calculation from the **capacity outage cumulative probability table**

$$LOLE = \sum_{i=1}^{n} P_i(C_i - L_i) \cdot 1 \quad days / period \qquad (55)$$

$C_i$ = available generation capacity on day 'i'

$L_i$ = forecast peak load on day 'i'

$P_i(C_i - L_i)$ = probability of loss of load on day 'i'; obtained directly from the capacity outage probability table

n = number of days in the period

- Load is modelled as a load duration curve (daily peak loads)
- LOLE (or LOLP) is one of the basic indices used in the generation expansion planning

# Generation
## Reliability (Loss of Load) Indices

- **Example 18:** The generation system is the same as in the previous example (two 25MW units, one 50MW unit, all failure rates 0.01 f/day and repair rates 0.49 r/day). Load data for a period of 365 days are shown below. Calculate the LOLE index.

| | | | | | |
|---|---|---|---|---|---|
| **Daily Peak Load (MW)** | **57** | **52** | **46** | **41** | **34** |
| **No. of occurences** | **12** | **83** | **107** | **116** | **47** |

- **Solution**
  - Capacity outage cumulative probability table was calculated in the previous example

| State No. | Capacity out (MW) | Capacity in (MW) | Probability pk | Cumulative probability Pk |
|---|---|---|---|---|
| 1 | 0 | 100 | 0.941192 | 1 |
| 2 | 25 | 75 | 0.038416 | 0.058808 |
| 3 | 50 | 50 | 0.0196 | 0.020392 |
| 4 | 75 | 25 | 0.000784 | 0.000792 |
| 5 | 100 | 0 | 0.000008 | 0.000008 |

  - Direct application of the above formula (number of days were plugged in; watch out which cumulative probabilities are replaced)

    LOLE = 12$P$(100-57) + 83$P$(100-52) + 107$P$(100-46) + 116$P$(100-41) + 47$P$(100-34) =

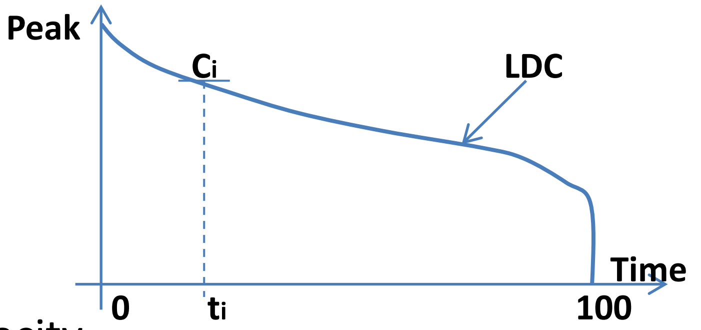
    =12x0.020392 + 83x0.020392 + 107x0.000792 + 116x0.000792

    + 47x0.000792 = 2.15108 days/year
  - The problem can be solved with individual state probabilities $p_k$. Do it at home!

# Generation
# Reliability (Loss of Load) Indices

- **Loss of Load Probability (LOLP)**
  - In vertically integrated power systems this was the main index for generation expansion planning (typical constraint LOLP<0.5 h/year)
  - Load Duration Curve (LDC) with half-hourly data is often used
  - Expected probability of generation deficiency:

$$LOLP = \sum_i P\{X = x_i\} \cdot t_i = \sum_i p_i \cdot t_i \qquad (56)$$




X = available generation capacity

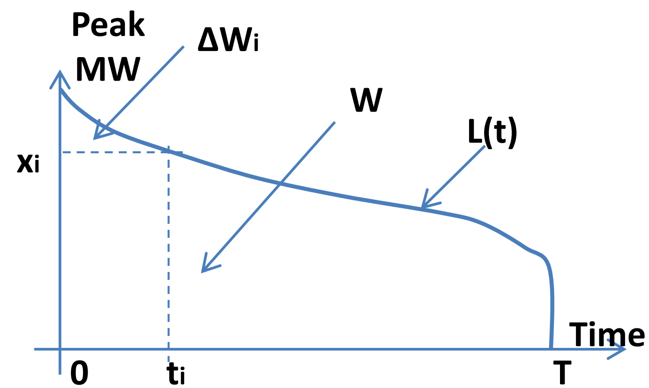
$x_i$ = state 'i' generation capacity

$t_i$ = duration of load loss for i-th state of gen capacity

  - Duration $t_i$ can be either percentage value or hour/period value
  - Example will be given after energy indices

# Generation
## Reliability (Loss of Energy) Indices

- **Loss of Energy Expectation**
  - Similar approach as LOLP
  - Expected energy curtailment:




$$LOEE = \sum_i p_i \cdot \Delta W_i = \sum_i p_i \cdot \int_0^{t_i} (L(t) - x_i)dt \quad (57)$$

$p_i$ = probability of generation capacity state ‘i’

$\Delta W_i$ = curtailed (non-delivered) energy at state ‘i’

$L(t)$ = load duration curve

$x_i$ = generation capacity at state ‘i’

Summation goes over all generation states ‘i’

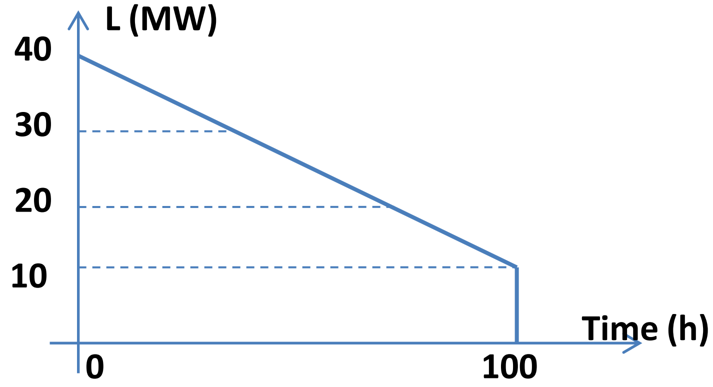
  - Energy Index of Unreliability (EIU) & Energy Index of Reliability (EIR)

$$EIU = LOEE / W$$

$$EIR = 1 - EIU \quad (58)$$

# Generation
## Reliability Indices

- **Example 19:** A system consists of a 30MW and a 20MW units. Probability of the full output of the first unit (=30MW) is 0.85, probability of partial output (=15MW) is 0.1 and probability of complete shutdown 0.05. Availability of the 20MW unit is 0.95 and unavailability 0.05. Load curve over an interval of 100 h is shown below. Calculate LOLP, EIU and EIR.




  – Solution
    - Capacity outage probability table is done as before (homework!)

| State No. | Capacity out (MW) | Capacity in xi (MW) | Probability p(xi) |
|---|---|---|---|
| 1 | 0 | 50 | 0.8075 |
| 2 | 15 | 35 | 0.095 |
| 3 | 20 | 30 | 0.0425 |
| 4 | 30 | 20 | 0.0475 |
| 5 | 35 | 15 | 0.005 |
| 6 | 50 | 0 | 0.0025 |

# Generation
## Reliability Indices

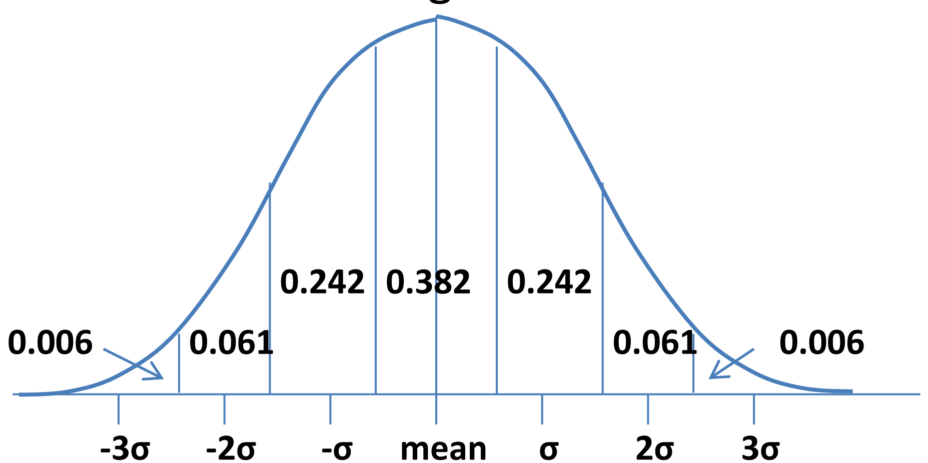
- Individual components of LOLP and LOEE

| State No. | Capacity in xi (MW) | Probability of state i pi | Deficit duration ti (h) | pi x ti (h) | Curtailed energy ΔWi (MWh) | pi x Wi (MWh) |
|---|---|---|---|---|---|---|
| 1 | 50 | 0.8075 | 0 | 0 | 0 | 0 |
| 2 | 35 | 0.095 | 16.7 | 1.59 | 42 | 4 |
| 3 | 30 | 0.0425 | 33.3 | 1.42 | 167 | 7.1 |
| 4 | 20 | 0.0475 | 66.7 | 3.17 | 667 | 31.7 |
| 5 | 15 | 0.005 | 83.3 | 0.42 | 1042 | 5.2 |
| 6 | 0 | 0.0025 | 100 | 0.25 | 2500 | 6.3 |

- LOLP = 0 + 1.59 + 1.42 + 3.17 + 0.42 + 0.25 = 6.85 h/100h = 6.85%
- LOEE = 0 + 4 + 7.1 + 31.7 + 5.2 + 6.3 = 54.3 MWh
- EIU = 54.3/2500 = 0.0217
- EIR = 1 – EIU = 0.9783

# Generation
## Reliability Indices and Load Forecast Uncertainty

- Load Forecast Uncertainty
  - The uncertainty in load forecasting can be included into computation of reliability indices
  - The load forecast p.d.f. is usually normal distribution; seven-step approximation is shown in the figure below



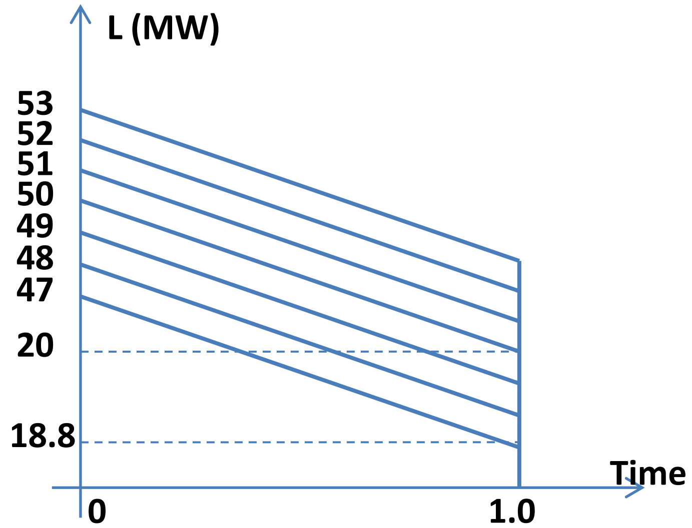

  - Calculation of reliability indices is done for **each** load level individually; result(s) shall be multiplied by load probabilities (7 values given in figure)
  - Assume a straight-line per unit LDC that connects points (0;1.0) and (1.0;0.4). The forecast peak load is 50MW and standard deviation 2% (=1MW). Calculate discrete peak loads and their probabilities.

# Generation
## Reliability Indices and Load Forecast Uncertainty

- Peak probabilities:

| Load (MW) | 47 | 48 | 49 | 50 | 51 | 52 | 53 |
|---|---|---|---|---|---|---|---|
| Probability | 0.006 | 0.061 | 0.242 | 0.382 | 0.242 | 0.061 | 0.006 |

- Load duration curves:




– Do Example 19 with load forecast uncertainty!

# Generation

## Reliability Indices and Load Forecast Uncertainty

- Load duration curve modelled as multiple-level step model (discrete points)
- Load levels and probabilities – uncertainties not included

| *Load level* | Pr*obability* |
|---|---|
| $L_1$ | $P_1 = T_1 / T$ |
| $L_2$ | $P_2 = T_2 / T$ |
| $L_k$ | $P_k = T_k / T$ |
| $L_n$ | $P_n = T_n / T$ |

- Load levels and probabilities – uncertainties included (level 'k')

| *Load level* | Pr*obability* |
|---|---|
| $L_k - 3\sigma_k$ | $P_{k1} = 0.006 T_k / T$ |
| $L_k - 2\sigma_k$ | $P_{k2} = 0.061 T_k / T$ |
| $L_k - 1\sigma_k$ | $P_{k3} = 0.242 T_k / T$ |
| $L_k$ | $P_{k4} = 0.382 T_k / T$ |
| $L_k + 1\sigma_k$ | $P_{k5} = 0.242 T_k / T$ |
| $L_k + 2\sigma_k$ | $P_{k6} = 0.061 T_k / T$ |
| $L_k + 3\sigma_k$ | $P_{k7} = 0.006 T_k / T$ |

# Generation
## Capacity Expansion Planning

- Generation expansion planning based on reliability indices: most frequently used are LOLP and LOLE
- Expansion planning is done over the planning period (say 20 yr) in which load grows
- Units are added when the index LOLP (or LOLE) goes above the prespecified threshold; after adding a generating unit reliability index goes down.
- The actual future load is uncertain and it should be considered as a random variable; we'll use deterministic model - simplified approach

- **Example 20:** A system consists of five 40MW units whose unavailability (FOR) is 0.01. The peak load is 160MW and normalised LDC is a straight line connecting points (0;1.0) and (1;0.4). It has been decided to add additional 50MW units with FOR=0.01 to meet the projected future load growth of 10% per year. Acceptable reliability level is LOLE=0.15 days/year. In what years must the units be committed in order to meet the accepted system risk level?

  Solution:

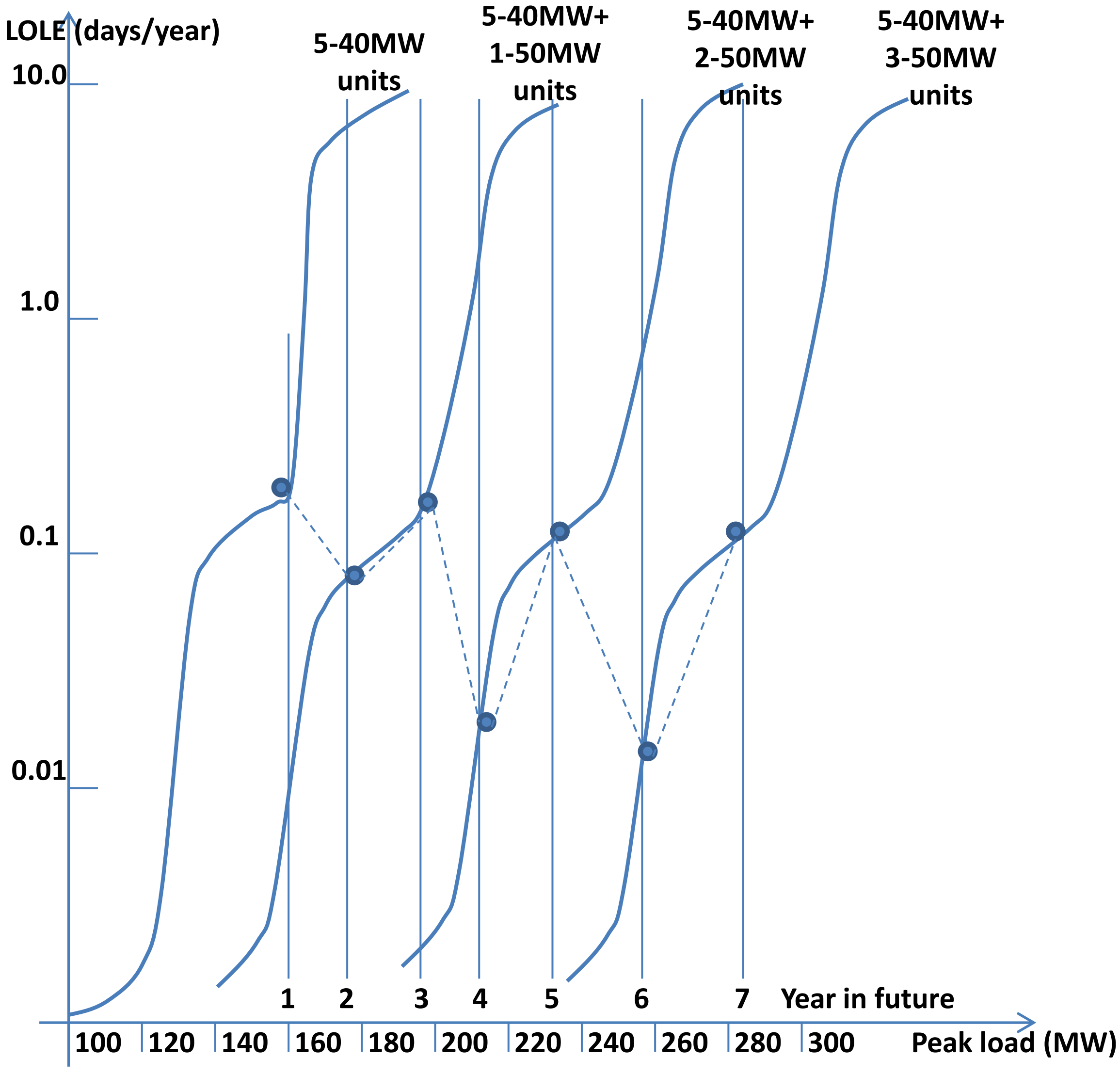
# Generation
## Capacity Expansion Planning

1. Build the capacity probability outage table for five 40MW units
2. Calculate LOLE days/year for the system of five 40MW units; peak load shall be varied
3. Calculate capacity outage table for five 40MW and one 50MW units
4. Calculate LOLE days/year for the above system; system peak is varied
5. Repeat calculations for two additional 50MW units, etc.

| | LOLE (days/year) | | | |
|---|---|---|---|---|
| System peak load (MW) | 200 MW capacity | 250 MW capacity (1 added unit) | 300 MW capacity (2 added units) | 350 MW capacity (3 added units) |
| 100 | 0.00121 | | | |
| 120 | 0.002005 | | | |
| 140 | 0.08686 | 0.001301 | | |
| 160 | 0.1506 | 0.002625 | | |
| 180 | 3.447 | 0.06858 | | |
| 200 | 6.083 | 0.1505 | 0.002996 | |
| 220 | | 2.058 | 0.03615 | |
| 240 | | 4.853 | 0.1361 | 0.00298 |
| 250 | | 6.083 | 0.18 | 0.004034 |
| 260 | | | 0.661 | 0.01175 |
| 280 | | | 3.566 | 0.1075 |
| 300 | | | 6.082 | 0.2904 |
| 320 | | | | 2.248 |
| 340 | | | | 4.88 |
| 350 | | | | 6.083 |

# Generation
## Capacity Expansion Planning

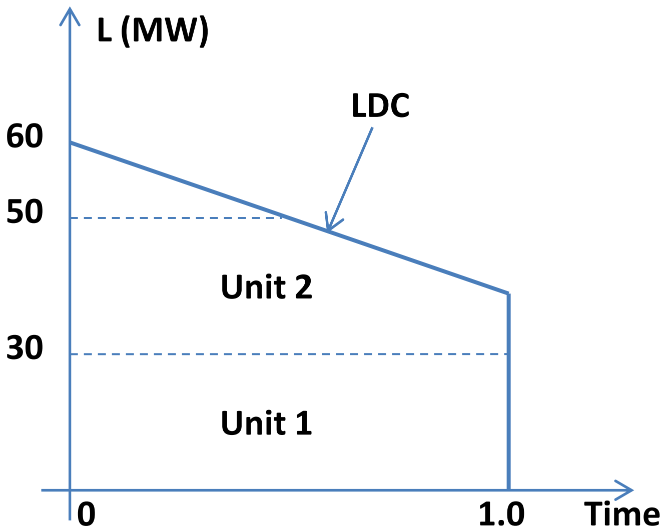
– Variation in risk with unit additions



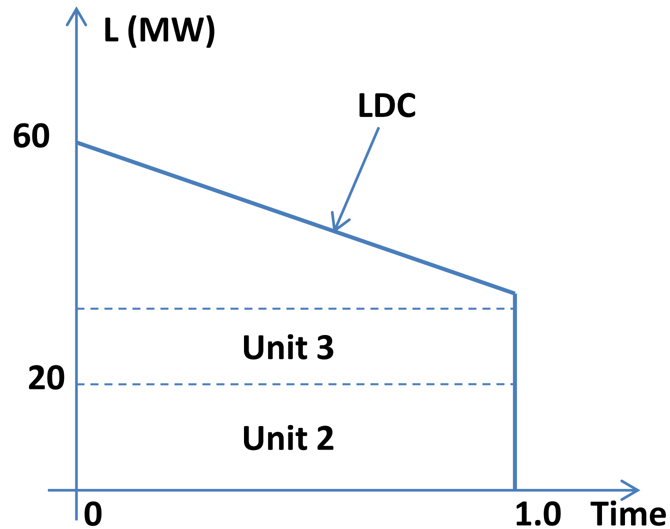
# Generation

## Expected Production of Generating Units

- Unit commitment method gives the priority list for loading of generation units (usually based on specific production costs)
- Generating units are ordered under the LDC; however their position depends on the availability of previously loaded units




Unit 1 is available




Unit 1 is NOT available

- This procedure is quite complicated particularly when the number of generating units is large
- A simpler computation procedure is given below

# Generation
## Expected Production of Generating Units

- Expected production of units can be calculated using the following algorithm:

  - Develop the priority list for generation unit loading (‘merit ordering’)
  - Assume that the entire consumption is supplied from the first unit. Find the expected non-supplied (curtailed) energy using eq. (57) and subtract it from the total energy supplied to customers. The results so obtained is expected production of the first unit.
  - Assume that the consumption is supplied from the first two units. Calculate expected non-supplied (curtailed) energy (eq. (57)) and subtract it from the total required energy. This gives expected production of the first two units. Expected production of the second unit is obtained by taking away the first unit expected production from the previous result.
  - Described algorithm is repeated for all units on the priority list.

# Generation

## Expected Production of Generating Units

- **Example 21:** LDC and reliability indicators of two units are given in Example 19. Unit 2 has lower specific production cost than unit 1. Calculate expected productions from both units.

  **Solution**

  - Unit 2 is committed first:

| Capacity in service xi (MW) | Probability pi | Duration ti (h) | Product pi x ti (h) | Curtailed energy deltaWi (MWh) | Product pi x deltaWi (MWh) |
|---|---|---|---|---|---|
| 20 | 0.95 | 66.7 | 63.3 | 667 | 633.7 |
| 0 | 0.05 | 100 | 5 | 2500 | 125 |
| | | | | | 758.7 |

  - Expected production of unit 2:
    W2 = 2500 = 758.7 = 1743.1 MWh
  - Curtailed energy LOEE was calculated in Example 19:
    LOEE = 54.3 MWh
    W1 + W2 = 2500 – 54.3 = 2445.7 MWh
    W1 = 2445.7 – 1743.1 = 704.4 MWh.

# Generation
## Operating Reserve

- Reliability techniques are applied in short-term operations planning:
  1. *Unit commitment risk:* assessment which units should be committed in any period of time in order to achieve acceptable level of system risk.
  2. *Response risk:* assessment how the committed units should be despatched (how much power each unit should generate, how much should be hold as spinning reserve, pick-up rate of spinning reserve, etc.)
- Unit commitment risk
  - At t=0 operator knows that he cannot commit any unit for T(h) – this is ‘lead time’ (if the load grows or any unit fails, another unit cannot be started)
  - Operator has to decide on committed units at t=0 and has to accept the risk of just supplying or not supplying the demand during lead time
  - The system risk level is usually specified, and load supplied and spinning reserve requirements are determined
  - Modelling is essentially capacity outage probability table where FOR is replaced with **Outage Replacement Rate (ORR)**

# Generation
## Operating Reserve

- If the repair process is neglected during lead time **T** (ie μ=0) , differential equations (2) will give:

  $$P(down) = 1 - e^{-\lambda T} \approx \lambda \cdot T = ORR \qquad (59)$$

  (note if λ in 1/yr than T in yr!)
- ORR from eq. (59) is used instead of FOR to build 'capacity outage tables'

- **Example 22:** A committed generation system consists of two 10MW units, three 20MW units and two 60MW units. Failure rates and ORR for lead times of 1, 2 and 4 hours are given in table below. If the acceptable risk level is 0.001, what is the peak load and required spinning reserve for each of the lead times?

| | | ORR for lead times of | | |
|---|---|---|---|---|
| Unit size (MW) | Failure rate lambda 1/yr | 1 hour | 2 hour | 4 hour |
| 10 | 3 | 0.000342 | 0.000685 | 0.00137 |
| 20 | 3 | 0.000342 | 0.000685 | 0.00137 |
| 60 | 4 | 0.000457 | 0.000913 | 0.001826 |

(note 0.000685 = 3 x 2/8760)

# Generation
## Operating Reserve

**Solution**

– Capacity outage probability table

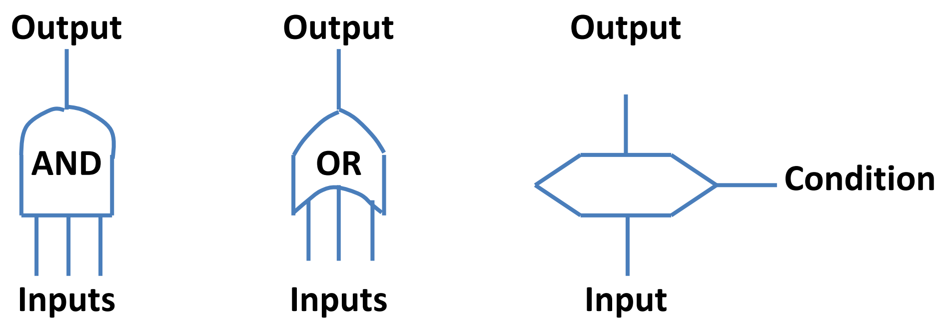
| | | Cumulative Probability | | |
|---|---|---|---|---|
| Capacity out (MW) | Capacity in (MW) | ORR=1 hour | ORR=2 hours | ORR=4 hours |
| 0 | 200 | 1 | 1 | 1 |
| 10 | 190 | 0.00262 | 0.005238 | 0.010455 |
| 20 | 180 | 0.001938 | 0.003874 | 0.00774 |
| 30 | 170 | 0.000915 | 0.001829 | 0.003665 |
| 40 | 160 | 0.000914 | 0.001826 | 0.003654 |
| 50 | 150 | 0.000914 | 0.001825 | 0.003648 |
| 60 | 140 | 0.000914 | 0.001825 | 0.003648 |
| 70 | 130 | 0.000002 | 0.000007 | 0.000028 |
| 80 | 120 | 0.000001 | 0.000005 | 0.000018 |
| 120 | 80 | | 0.000001 | 0.000003 |

– Allowed peak load and required spinning reserve for risk = 0.001

- T = 1h => Peak_Load = 170MW;  Spinning_Reserve = 30MW
- T = 2h => Peak_Load = 130MW;  Spinning_Reserve = 70MW
- T = 4h => Peak_Load = 130MW;  Spinning_Reserve = 70MW

# Complex Systems
## Fault Tree Analysis

- ***Fault tree*** is graphical representation of events (causes and consequences) that lead to a failure of the considered system (function)
- The ***events*** that lead to system failure need to be determined; their mutual ***logical connectivity*** has also to found
- Analysis is done until ***basic events*** are identified; these are usually failures of individual components
- Failure tree is constructed starting from ***top event,*** finding events that cause it and their mutual connectivity, etc (top-down approach)
- ***Logical symbols*** most frequently used are:



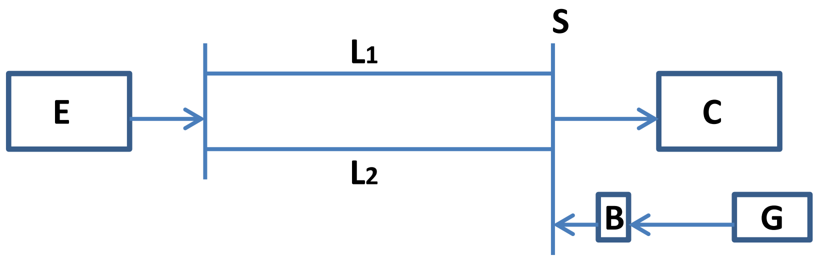

- ***Event symbols:***


- Fault tree analysis is often used for protection system reliability modelling

# Complex Systems

## Fault Tree Analysis

- **Example 23:** Construct the fault tree for the power system given below. Consumer ‘C’, connected to busbar ‘S’, is supplied from power station ‘E’ via lines $L_1$ and $L_2$. A local generator ‘G’ is connected to busbar ‘S’ via circuit breaker ‘B’.



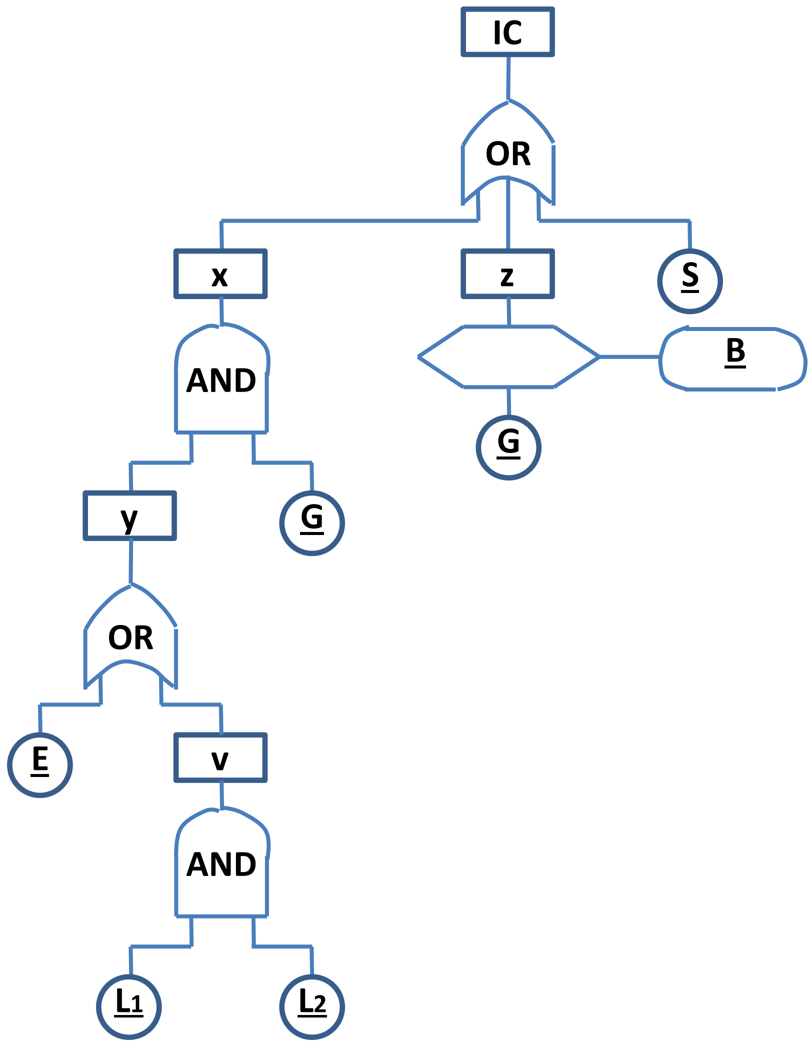

Solution

Symbols used in the figure below denote:

- IC = supply interruption of customer ‘C’
- x = simultaneous failure of generator ‘G’ and the system supplying busbar ‘S’
- z = failure of generator ‘G’ and failure of breaker ‘B’
- v = simultaneous failure of both lines
- $\underline{S}$, $\underline{E}$, $\underline{G}$, $\underline{B}$, $\underline{L_1}$, $\underline{L_2}$ = failure of, respectively, busbar ‘S’, power station ‘E’, generator ‘G’, breaker ‘B’ and lines ‘$L_1$’ and ‘$L_2$’

# Complex Systems
## Fault Tree Analysis

- Fault tree:



# Complex Systems
## Fault Tree Analysis

- System unavailability is calculated from the fault tree; general expressions for *mutually dependent* events need to be used
- Probability that at least one of the events $D_i$ i=1,2,...,m is going to occur (logical **OR**):

$$P\{D\} = \sum_{i=1}^{m} P\{D_i\} - \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} P\{D_i \cdot D_j\} + \sum_{i=1}^{m-2} \sum_{j=i+1}^{m-1} \sum_{k=j+1}^{m} P\{D_i \cdot D_j \cdot D_k\} - \ldots (-1)^{m-1} P\{\prod_{i=1}^{m} D_i\} \quad (60)$$

- Probability that all events will occur (logical **AND**):

$$P\{\prod_{i=1}^{m} D_i\} = P\{D_1\} \cdot P\{D_2 \mid D_1\} \cdot P\{D_3 \mid D_1 \cdot D_2\} \ldots P\{D_m \mid \prod_{i=1}^{m-1} D_i\} \qquad (61)$$

- Previous **Example 23**:

  P{IC} = P{x} + P{z} + P{$\underline{S}$} - P{x|z}P{z} - P{x}P{$\underline{S}$} - P{z}P{$\underline{S}$} + P{$\underline{S}$}P{x|z}P{z} (*x & y are dependent*)

  P{x} = P{y}P{$\underline{G}$}

  P{z} = P{$\underline{B}$|$\underline{G}$}P{$\underline{G}$}

  P{x|z} = P{xz}/P{z} = P{y} (*see diagram*)

  P{y} = P{$\underline{E}$} + P{v} – P{$\underline{E}$}P{v}

  P{v} = P{$\underline{L}_1$}P{$\underline{L}_2$}

- Minimum cuts can also be created from the fault tree

# Complex Systems
## State Enumeration

- Power system states are enumerated and each system state is analysed in turn
- A system state is determined by the status of all system components; sum of all state probabilities is equal unity
- State enumeration is often used for reliability analysis of composite transmission and generation systems (‘DigSilent’ software)
- Algorithm:
  1. Take the next system state, say ‘k’, into consideration. Its probability is:

     $$P_k = P\{\prod_{i=1}^{n} e_i\} = \prod_{i=1}^{n} P_i \qquad (62)$$

     $e_i$ = status of component ‘i’; ‘n’ = number of components; $P_i$ = availability if component ‘i’ is in service, unavailability if it is out of service
  2. Analyse state ‘k’ using appropriate power system model(s). If the system failed to deliver its function, recalculate the failure probability $P_f$ :

     $$P_f = P_{fs} + P_k \qquad (63)$$

     ‘s’ = index of the previous ‘failed’ state

# Complex Systems
## State Enumeration

3. Calculate expected value of the reliability quantity ‘V’:

$$E\{V\} = E\{V\}_s + P_k \cdot V_k \qquad (64)$$

4. Calculate total probability of all enumerated states ‘$P_0$’:

$$P_0 = P_{0s} + P_k \qquad (65)$$

   If probability (1-$P_0$) is grater than pre-specified threshold value, return to step No. 1. Otherwise stop.

- An alternative stopping criterion is specification of the outage order for simultaneous outages which will be studied; this is typically 3$^{rd}$ or 4$^{th}$ order
- Generating units are less reliable than transmission components; higher order of unit outages is often studied

# Complex Systems
## State Enumeration

- **Example 24**: Consider the simple power system given in Example 23, where all components are mutually independent. Calculate system state probabilities of the intact network, single component outages, double outages and triple outages with both generators out of service. Assume generator unavailability of 15%, line unavailability of 2% and busbar/breaker unavailability of 0.5%.

  Solution: Homework!

# Complex Systems
## Monte Carlo Simulation

- Widely used method in all areas of reliability analysis, particularly for complex systems
- The name comes from Monte Carlo casinos
- ‘Similar’ to state enumeration method, however system states are generated randomly
- MC simulation is often combined with state enumeration
- Essential Algorithm:
    1. Generate a system state by random generation of system components
    2. Analyse the system state using ‘appropriate’ power system model(s) to calculate reliability indicators.
    3. Test whether the convergence criterion has been satisfied. If yes go to step No. 4, otherwise return to step No. 1.
    4. Final processing of simulation data obtained in the above loop.

# Complex Systems
## Monte Carlo Simulation

- Essential features:

Assume 'Q' is a system unavailability and '$x_i$' is zero-one indicator variable ($x_i = 0$ if system in up-state; $x_i = 1$ if system in down-state). Then:

-Expected value of unavailability $$\overline{Q} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

-Variance of 'x' $$V(x) = \frac{1}{N}\sum_{i=1}^{N}(x_i - \overline{Q})^2 = \overline{Q} - \overline{Q}^2 \quad (\sum_{i=1}^{N} x_i^2 = \sum_{i=1}^{N} x_i)$$

-Variance of unavailability $$V(\overline{Q}) = V(x)/N = (\overline{Q} - \overline{Q}^2)/N$$

-Coefficient of variation $$\alpha = \sqrt{V(\overline{Q})}/\overline{Q} = \sqrt{(1-\overline{Q})/(N\cdot\overline{Q})}$$

-Number of samples (iterations) $$N = (1-\overline{Q})/(\alpha^2 \cdot \overline{Q})$$

-Standard deviation of estimate $$\sigma = \sqrt{V(\overline{Q})} = \sqrt{V(x)}/\sqrt{N} \tag{66}$$

**1.** For a desired accuracy level α, required number of samples 'N' depends on system unavailability but does not depend on system size (large systems are OK)

**2.** The number of samples 'N' is inversely proportional to the system unavailability.

**3.** Coefficient of variation can be used as the convergence criterion (variation coefficient of EENS has the lowest convergence rate).

**4.** Standard deviation can be reduced by decreasing the *sample variance* & *increasing 'N'*.

# Complex Systems
## Monte Carlo Simulation

- Random number generation
  - Random numbers are generated to determine status of a component (up/down) or value of any other probabilistic variable
  - Pseudorandom generators are used for this purpose; requirements are:
    - The random number is uniformly distributed in range [0,1]
    - There should be minimal correlation between random numbers (independence)
    - The repeat period should be very long.
  - The multiplicative congruential generator makes use of the recursive formula:

$$x_{i+1} = a \cdot x_i (\bmod m) = a \cdot x_i - m[a \cdot x_i / m]$$

$$U_i = x_i / m \qquad (67)$$

[$ax_i/m$] = largest positive integer in $ax_i/m$; for example, 32(mod(30))=2;

$U_i$ = random number in interval [0;1]

  - The mixed congruential generator:

$$x_{i+1} = (a \cdot x_i + c)(\bmod m) = (a \cdot x_i + c) - m[(a \cdot x_i + c)/m]$$

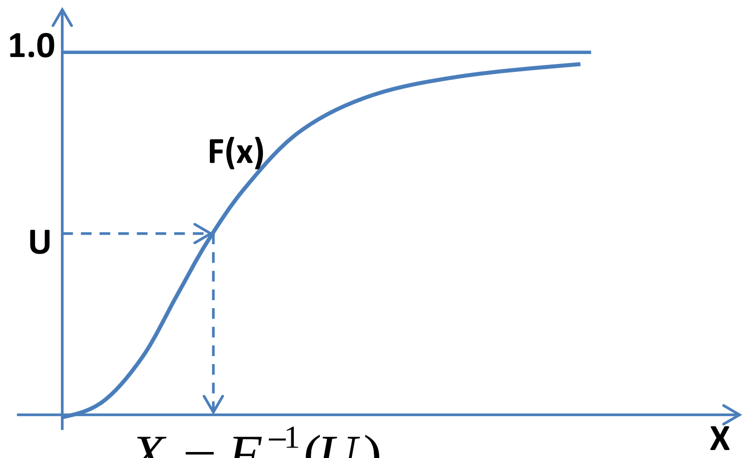
$$U_i = x_i / m$$

$$m = 2^{31}\ ,\quad a = 314159269\ ,\ \ c = 453806245$$

$$m = 2^{35}\ ,\quad a = 5^{15}\ ,\ \ c = 1 \qquad (68)$$

# Complex Systems
## Monte Carlo Simulation

- Generation of random variables
  - Essential idea
    1. Generate a uniformly distributed random number U between [0,1]
    2. Calculate random variable X from the cumulative pdf F(X)




$$X = F^{-1}(U)$$

  - Tabulating procedure (look-up table)
    - Interval [0,1] is divided into 'k' subintervals which have the same length 1/k (k=500)
    - Discrete values of the cumulative probability pdf are midpoints 0.5/k, 1.5/k, 2.5/k...
    - Generated random number U is in the i-th interval, random variable X is inverse of the cumulative pdf at (i-0.5)/k
  - Exponentially distributed random variable

$$f(x) = \lambda \cdot e^{-\lambda x};\ F(x) = 1 - e^{-\lambda x};$$

$$X = F^{-1}(U) = -\frac{1}{\lambda}\ln(1-U);\ X' = -\frac{1}{\lambda}\ln(U) \qquad (69)$$

(variable (1-U) distributes uniformly in the interval [0,1])

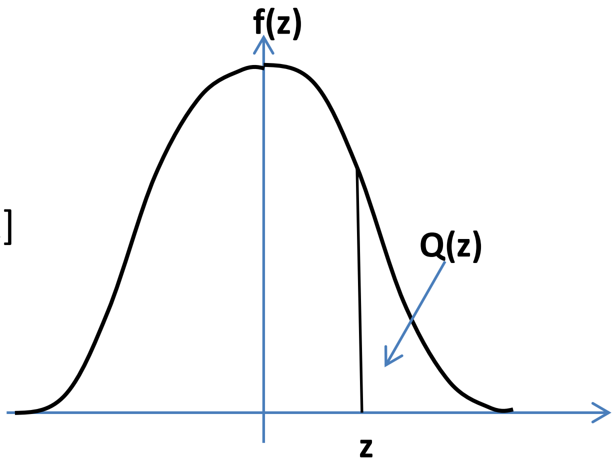
# Complex Systems
## Monte Carlo Simulation

– Normally distributed random variable
  - Numerical procedure described earlier
  - Algorithm
    - Generate a uniformly distributed number Uε[0,1]
    - Calculate normally distributed variable X from:

$$X = \frac{z \quad if\ 0.5 < U \le 1.0}{\dfrac{0 \quad if \qquad U = 0.5}{-z\ if \quad 0 \le U < 0.5}}\ ;$$

$$Q = \frac{1-U \quad if\ 0.5 < U \le 1.0}{U \quad if\ 0 \le U \le 0.5} \qquad (70)$$

('z' and 'Q' are given in equations (15))




– Weibull distributed random variable

$$f(x) = \frac{\beta}{\alpha^{\beta}} x^{\beta-1} \exp[-(x/\alpha)^{\beta}];\ \ F(x) = 1 - \exp[-(x/\alpha)^{\beta}];$$

$$X = \alpha \cdot [-\ln(1-U)]^{1/\beta};\ X = \alpha \cdot [-\ln(U)]^{1/\beta} \qquad (71)$$

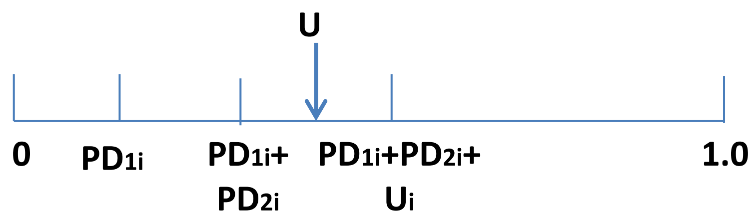
(variable (1-U) is uniformly distributed between [0,1])

# Complex Systems
## Monte Carlo Simulation

- Status of system components
  - Component 'i' unavailability is $U_i$, availability $A_i$ (two-state model)


If uniformly distributed random number $U \leq U_i$, component is in down state, otherwise ($U_i < U \leq 1.0$) component is in up state

  - Component 'i' has two derated stated whose probabilities are $PD_1$ and $PD_2$




The above example shows that component 'i' is in derated state 2.

- Variance reduction techniques

-Reducing sample variance V(x) can reduce standard deviation of estimate σ (eq. (66)) which is equivalent as increasing the number of iterations

-Different techniques are applied:

- Control variates
- Importance sampling (events which make greater contribution to the results have higher P)
- Stratified sampling (more samples are drawn from subintervals which make grater contribution)
- Antithetic Variates

# Complex Systems
## Monte Carlo Simulation

- Independent (non-sequential) Monte Carlo simulation
  - One system state is independent from another; system state is defined by states of its components
  - Assume $S_i$ is state of component 'i', $Pf_i$ is probability of failure and $U_i$ is randomly generated number distributed uniformly between [0,1]. State $S_i=0$ (success state) if $U_i \geq Pf_i$, otherwise $S_i=0$ (failure state)
  - System state with 'm' components is $S=(S_1, S_2, ..., S_m)$ , its probability is P(S) and the set of system states is 'G'
  - Estimated expectation of the reliability index function F(S) is

$$\overline{F} = \sum_{S \in G} F(S)P(S) = \sum_{S \in G} F(S)\frac{n(S)}{N} \qquad (72)$$

    (n(S)=number of occurrences of state 'S', N=number of samples)
  - Stopping criterion when expected value of reliability function is sought:

# Complex Systems
## Monte Carlo Simulation

1. Confidence intervals are:

$$P\left\{\overline{F} - t_{\alpha/2}^{N-1} \cdot \frac{s_F}{\sqrt{N}} \le F \le \overline{F} + t_{\alpha/2}^{N-1} \cdot \frac{s_F}{\sqrt{N}}\right\} = 1 - \alpha \qquad (73)$$

$t_{\alpha/2}^{N-1}$ = value of student 't' distribution for (N-1) degrees of freedom and probability α/2; often replaced with *normal distribution value β*

$s_F$ = standard deviation estimate of reliability function F

α = risk level

2. If $\varepsilon_F$ is pre-specified tolerance, required number of iterations is:

$$N \ge (t_{\alpha/2}^{N-1} \cdot s_F / \varepsilon_F)^2 \qquad (74)$$

– Stopping criterion when probability $\overline{P}$ is calculated:

$$N \ge \beta^2 \cdot \overline{P} \cdot (1 - \overline{P}) / (\varepsilon_P)^2 \qquad (75)$$

β = value of normal distribution for probability α/2; $\varepsilon_P$ = tolerance

– Independent Monte Carlo simulation can not give frequencies and durations

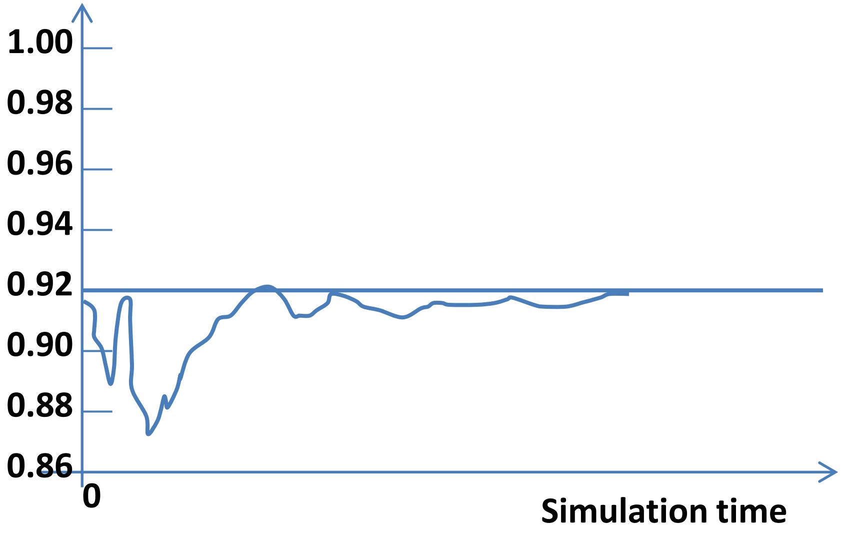
# Complex Systems
## Monte Carlo Simulation

**Example 25:** A system consists of two parallel components, whose failure rates are 0.001 f/hr and 0.0024 f/hr. Repair rates are 0.003 r/hr and 0.005 r/hr. Calculate system availability using analytical expression and Monte Carlo simulation.

Solution:

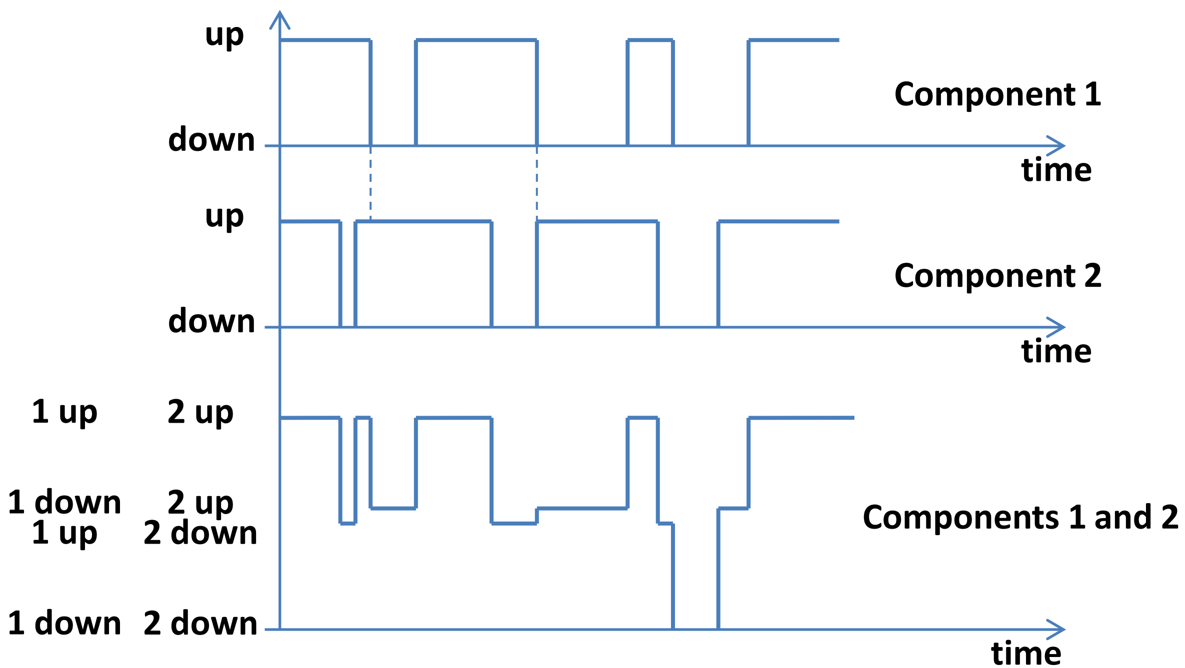
-System unavailability is 0.081081 and availability 0.918919

-Monte Carlo simulation results are given in the figure below



# Complex Systems
## Monte Carlo Simulation

- Sequential Monte Carlo simulation (state duration sampling)
  - In systems where subsequent states are highly dependent on previous states (hydro, wind, storage, etc.)
  - Sampling the probability distribution of the component state duration (ie up and down times)
  - Sequential simulation of one year of system operation is repeated many times (until convergence is obtained)
  - Essential idea is given in the figure below



# Complex Systems
## Monte Carlo Simulation

- It is assumed that two-state models and exponential pdfs are used
- Global Algorithm:
  1. Specify the initial state of each component which is usually up state.
  2. Sample the duration of each component residing in its present state. If $U_i$ is uniformly distributed random number and pdf is exponential, the state duration $T_i$ is (eq. (69))

     $$T_i = -\ln(U_i)/\lambda_i$$

     ($\lambda_i$ = failure rate if in up state; $\lambda_i$ = repair rate if in down state)
  3. Repeat Step 2 in the given time span (year) for all components. Chronological component state transition process has the form shown in the previous figure.
  4. Calculate the chronological system state transition process by combining chronological component transition processes of all components (figure!)
  5. Conduct system analysis for each different system state to obtain the reliability index F(S) and to calculate expectation E(F)
- Advantages of the approach:
  1. It can be used to calculate frequencies and durations
  2. Probability distributions of reliability indices can be calculated

# Complex Systems
## Monte Carlo Simulation

- Convergence criteria:
  - There are no explicit expressions as in the case of non-sequential simulation
  - Total simulation time 'T' shall be 'sufficiently long' (say 1000 years or more)
  - Do the sequential MC simulation for 'T' and then 'T+deltaT'. If the expected value of the reliability index is "almost the same" in both cases, stop. Otherwise increase T and continue simulations.
  - Another criterion is to keep total simulation time 'T' constant and to change initial conditions of system components. If both results are 'the same', stop.
- Assume a system has experienced 3 failures in 8 years, the durations being 0.019 yr, 0.204 yr and 0.346 yr. The average failure duration is 0.189 yr (= (0.019+0.204+0.346)/3 ), failure frequency is 0.375 1/yr ( 3/8 ) and failure probability 0.071 (= 0.189 x 0.375).

# Complex Systems
## Monte Carlo Simulation

– **Example 26:** Sampled durations of two components are given in the table (x=up state; y=down state). Find system states. Which are the failure states if the system is: a) Parallel connection; b) Series connection?

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Component 1** | **x** | **y** | **x** | **y** | **x** | **y** | **x** | **y** | **x** | **y** |
| **T (h)** | **13600** | **34** | **6240** | **19** | **5410** | **118** | **23860** | **23** | **800** | **27** |
| **Component 2** | **x** | **y** | **x** | **y** | **x** | **y** | **x** | **y** | **x** | **y** |
| **T (h)** | **6600** | **99** | **7150** | **13** | **5290** | **48** | **4420** | **104** | **1778** | **111** |

– **Solution:** Homework!

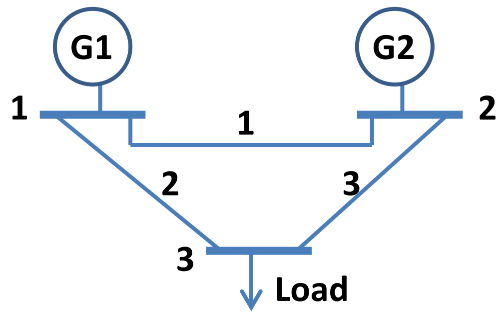
# Composite Generation and Transmission System

- National Grid transmission system:
- England and Wales: 400kV & 275kV
- Scotland: 275kV & 132kV
- Submarine cable presented

# Composite Generation and Transmission Systems

## Reliability – Adequacy Indices

- Composite system adequacy assessment ("steady-state reliability")




- '***System Indices***' (whole system) and '***Node Indices***' (individual buses)
- '***Annualised Indices***' (at peak load & expressed on a one-year basis) and '***Annual Indices***' (multi-step annual load duration curve)
- *Basic* Adequacy Indices (System Indices or Node Indices)

  1. ***Probability of Load Curtailments – PLC:***

     $$PLC = \sum_{i \in S} P_i \qquad (76.1)$$

     ($P_i$=probability of state 'i'; S=set of all states with load curtailment)

  2. Expected Frequency of Load Curtailments – EFLC (occ./yr):

     Hard task, replaced with Expected Number of Load Curtailments – ENLC:

     $$ENLC = \sum_{i \in S} F_i ; \qquad F_i = P_i \cdot \sum_{k \in N} \lambda_k \qquad (76.2)$$

     (N = set of all departure rates corresponding to state 'i')

# Composite Generation and Transmission Systems
## Reliability – Adequacy Indices

3. Expected Duration of Load Curtailments – EDLC (hr/yr):

$$EDLC = PLC \cdot 8760 \qquad (76.3)$$

4. Average Duration of Load Curtailments (hr/disturbance):

$$ADLC = EDLC / EFLC \approx EDLC / ENLC \qquad (76.4)$$

5. Expected Load Curtailments – ELC (MW/yr):

$$ELC = \sum_{i \in S} C_i \cdot F_i \qquad (76.5)$$

($C_i$ = ***total*** load curtailed in state 'i')

6. ***Expected Demand Not Supplied – EDNS (MW):***

$$EDNS = \sum_{i \in S} C_i \cdot P_i \qquad (76.6)$$ ***Different from ELC!***

7. ***Expected Energy Not Supplied – EENS (MWh/yr)***

$$EENS = \sum_{i \in S} C_i \cdot F_i \cdot D_i = \sum_{i \in S} 8760 C_i \cdot P_i \qquad (76.7)$$

($D_i$ = duration of state 'i')

# Composite Generation and Transmission Systems
## Reliability – Adequacy Indices

- Additional *System* Adequacy Indices (systems of different sizes)

8. Bulk Power Interruption Index – BPII (MW/MW-yr):

$$BPII = \sum_{i \in S} C_i \cdot F_i / L \qquad (76.8)$$

(L = annual system peak load in MW)

9. ***Bulk Power/Energy Curtailment Index*** – BPECI (MWh/MW-yr):

$$BPECI = EENS / L \qquad (76.9)$$

10. Bulk Power-Supply Average MW Curtailment Index – BPACI (MW/disturbance):

$$BPACI = ELC / EFLC \qquad (76.10)$$

***11. Modified Bulk Power Curtailment Index*** – MBPCI (MW/MW)

$$MBPCI = EDNS / L \qquad (76.11)$$

12. Severity Index – SI (system min/yr):

$$SI = BPECI \cdot 60 \qquad (76.12)$$

# Composite Generation and Transmission Systems

## Reliability – Adequacy Indices

- **Example 27:** Generation and line data for the simple 3-node system are given in tables below. Peak load is 110MW, losses of 5 MW are added to the peak load and line capacities in MW are obtained by multiplying the MVA capacities by 0.95 (power factor). Analyse all first and second order outages and calculate adequacy indices ELC, ENLC, EENS and EDLC.

| Power Station | No. of Units | Capacity (MW) | Unavailability | Failure rate (f/yr) | Repair rate (r/yr) |
|---|---|---|---|---|---|
| 1 | 4 | 20 | 0.01 | 1 | 99 |
| 2 | 2 | 30 | 0.05 | 3 | 57 |
| Total | 6 | 140 | | | |

| Line | Node 1 | Node 2 | Failure rate (f/yr) | Repair time (h) | Rating (MVA) |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 4 | 8 | 80 |
| 2 | 1 | 3 | 5 | 8 | 100 |
| 3 | 2 | 3 | 3 | 10 | 90 |

### Solution

- State probability = product of component (un)availabilities
- State frequency = state probability x (sum of all failure and repair rates from that state)
- Available capacity = consider both generation and line (multiply by 0.95) capacities
- State duration = 8760 x state probability / state frequency
- ELC = state frequency x curtailed load; ENLC = state frequency
- EENS = 8760 x state probability x curtailed load; EDLC = 8760 x state probability

# Composite Generation and Transmission Systems
## Reliability – Adequacy Indices

– Adequacy indices

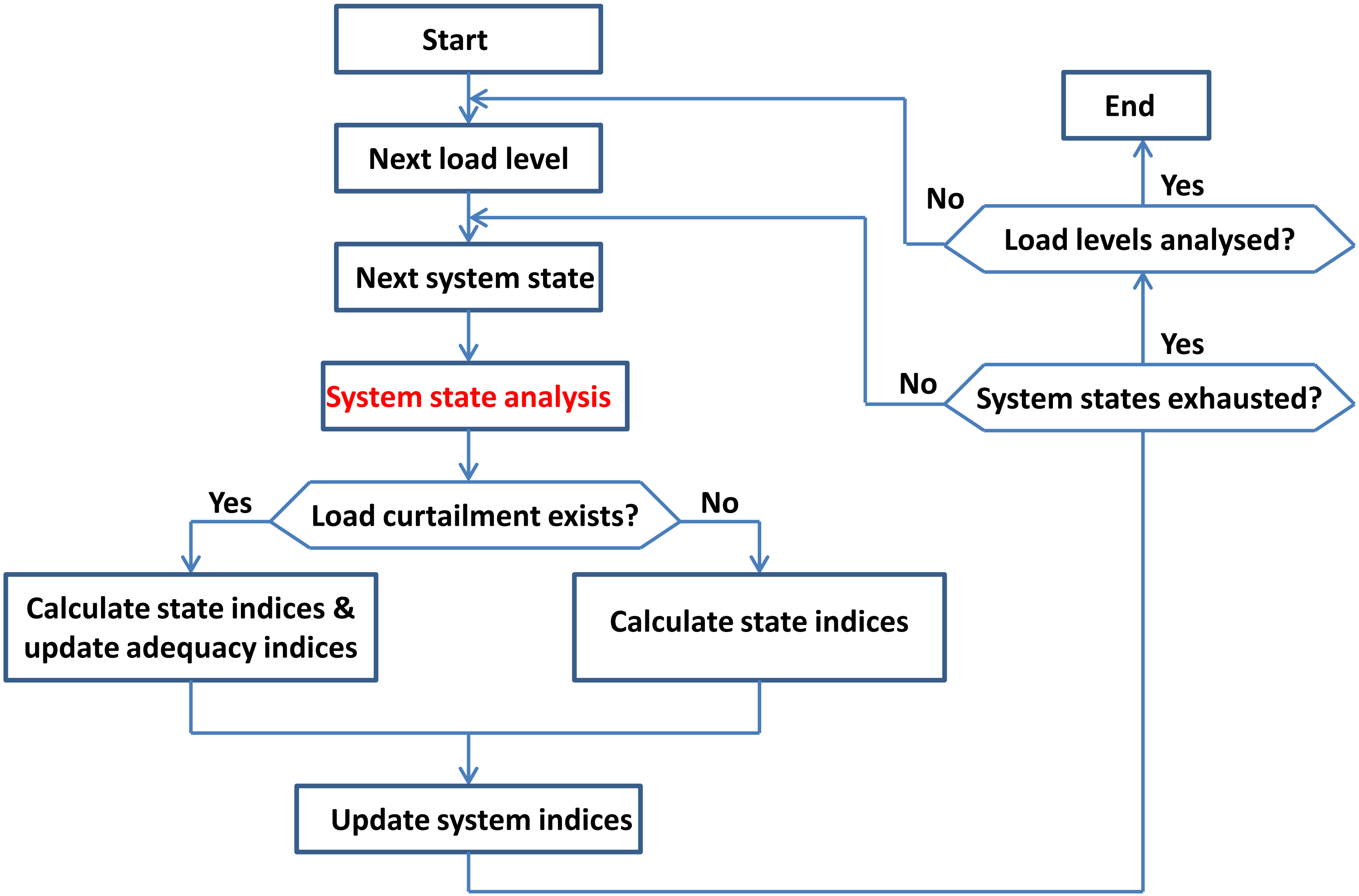
| State | Elements out | State Probability P | State Frequency F (occ/yr) | G&T Capacity Available (MW) | Load Curtailed 0/1 | State Duration (h) | Curtailed Load (MW) | ELC (MW) | ENLC (occ/yr) | EENS (MWh/yr) | EDLC (h) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | 0.85692158 | 18.85227476 | 140 | 0 | 398.18 | 0 | 0 | 0 | 0 | 0 |
| 2 | G1 | 0.03462309 | 4.1547708 | 120 | 0 | 73 | 0 | 0 | 0 | 0 | 0 |
| 3 | G1, G1 | 0.00052449 | 0.11436062 | 100 | 1 | 40.18 | 15 | 1.71541 | 0.1143606 | 68.917986 | 4.594532 |
| 4 | G1, G2 | 0.00364454 | 0.63414996 | 90 | 1 | 50.34 | 25 | 15.8537 | 0.63415 | 798.15426 | 31.92617 |
| 5 | G1, L1 | 0.00012648 | 0.15329376 | 120 | 0 | 7.23 | 0 | 0 | 0 | 0 | 0 |
| 6 | G1, L2 | 0.0001581 | 0.1914591 | 86 | 1 | 7.23 | 29 | 5.55231 | 0.1914591 | 40.163724 | 1.384956 |
| 7 | G1, L3 | 0.00011857 | 0.11774001 | 95 | 1 | 8.82 | 20 | 2.3548 | 0.11774 | 20.773464 | 1.038673 |
| 8 | G2 | 0.09020227 | 6.85537252 | 110 | 1 | 115.26 | 5 | 34.2769 | 6.8553725 | 3950.8594 | 790.1719 |
| 9 | G2, G2 | 0.00237374 | 0.3085862 | 80 | 1 | 67.38 | 35 | 10.8005 | 0.3085862 | 727.78868 | 20.79396 |
| 10 | G2, L1 | 0.00032951 | 0.38783327 | 110 | 1 | 7.44 | 5 | 1.93917 | 0.3878333 | 14.432538 | 2.886508 |
| 11 | G2, L2 | 0.00041188 | 0.48438029 | 86 | 1 | 7.45 | 29 | 14.047 | 0.4843803 | 104.634 | 3.608069 |
| 12 | G2, L3 | 0.00030891 | 0.29315559 | 95 | 1 | 9.23 | 20 | 5.86311 | 0.2931556 | 54.121032 | 2.706052 |
| 13 | L1 | 0.0031303 | 3.4840239 | 140 | 0 | 7.87 | 0 | 0 | 0 | 0 | 0 |
| 14 | L1, L2 | 0.0000143 | 0.0315029 | 60 | 1 | 3.84 | 55 | 1.73266 | 0.0315029 | 6.88974 | 0.125268 |
| 15 | L1, L3 | 0.00001072 | 0.02128992 | 95 | 1 | 4.41 | 35 | 0.74515 | 0.0212899 | 3.286752 | 0.093907 |
| 16 | L2 | 0.00391288 | 4.35112256 | 86 | 1 | 7.8777 | 29 | 126.183 | 4.3511226 | 994.02804 | 34.27683 |
| 17 | L2, L3 | 0.0000134 | 0.026599 | 0 | 1 | 4.41 | 110 | 2.92589 | 0.026599 | 12.91224 | 0.117384 |
| 18 | L3 | 0.00293466 | 2.6265207 | 95 | 1 | 9.79 | 20 | 52.5304 | 2.6265207 | 514.15243 | 25.70762 |
| Total | | 0.99975942 | | | | | | 276.519 | 16.444 | 7311.114 | 919.432 |

# Composite Generation and Transmission Systems
## Global Algorithm

- Software tools
- Global algorithm can be applied to both ‘State Enumeration’ and ‘Monte Carlo Simulation’ procedures
- State Enumeration method:
  - Multi-step load duration curve (LDC) is usually applied
  - Analysed system states are up to the pre-specified outage order
  - Calculated adequacy indices are those already presented
  - Frequency and duration method cannot be applied
  - Stopping criterion is outage order which can be different for generation units and transmission lines (and their combination)
- Monte Carlo Simulation method:
  - Either Independent or Sequential simulation
  - Load levels can be either sampled, or enumerated (multi-step LDC)
  - Frequency and duration method can be applied in Sequential MC Simulation
  - Well defined stopping criterion in Independent MC Simulation, not so good in Sequential MC Simulation

# Composite Transmission and Generation Systems

## Global Algorithm



# Composite Generation and Transmission Systems
## Analysis of System States

- Optimisation models based on AC or DC loadflow models
- DC loadflow model:

$$\begin{vmatrix} PG_2 - PD_2 \\ PG_3 - PD_3 \\ \dots \\ PG_N - PD_N \end{vmatrix} = \begin{vmatrix} B_{22} & B_{23} & \dots & B_{2N} \\ B_{32} & B_{33} & \dots & B_{3N} \\ \dots & \dots & \dots & \dots \\ B_{N2} & B_{N3} & \dots & B_{NN} \end{vmatrix} x \begin{vmatrix} \theta_2 \\ \theta_3 \\ \dots \\ \theta_N \end{vmatrix}$$

$$B_{ii} = \sum 1/x_{ij} \; i = 2,3,\dots,N \; ; B_{ij} = 1/x_{ij} \quad i \neq j$$

$$P_{ij} = (\theta_i - \theta_j)/x_{ij} \; i \neq j \qquad (77)$$

PG = (N-1) vector of active power generations;

PD = (N-1) vector of active power demands;

B = (N-1)x(N-1) susceptance matrix;

θ = (N-1) vector of node angles;

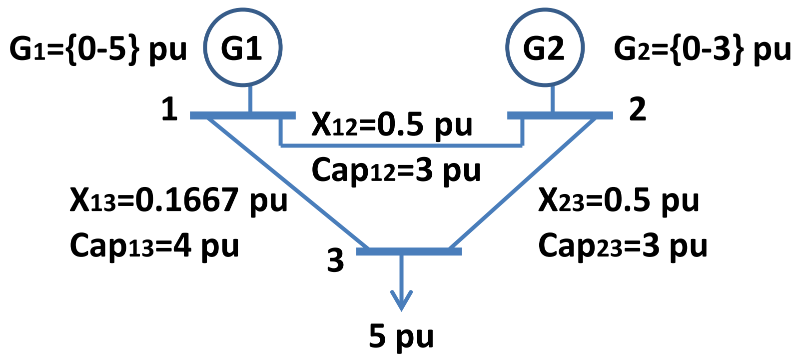
$x_{ij}$ = reactance of branch i-j (positive);

N = number of nodes (node No. 1 is slack)

# Composite Transmission and Generation Systems

## Analysis of System States

- **Example 28-1:** A 3-node network is shown in the figure below. Generator 2 is cheaper than generator 1 and it operates at full output. Calculate branch power flows for the intact network and all single outages of lines and generators. Are there line overloads? Assume node No. 1 is the slack node.




- Solution: Homework!

# Composite Generation and Transmission Studies
## Analysis of System States

- Minimum load curtailment (MLC) model
  - A Linear Programming optimisation model if the DC loadflow is applied
  - Load curtailments are modelled as additional (fictitious) nodal generation at load nodes (ie nodes with consumption)
  - Curtailed loads are prioritised using weighting factors; most important loads that should not be curtailed have highest priority factors
  - Objective function: minimisation of the sum of (weighted) curtailed loads at all consumer nodes
  - Constraints:
    - DC loadflow equations
    - Generation limits
    - Branch flow limits
    - Limits on load curtailments

# Composite Generation and Transmission Studies
## Analysis of System States

- Mathematical model:

$$\min z = \sum_{i \in NC} w_i \cdot C_i$$

$$PG + C - PD = B \cdot \theta$$

$$\sum_{i=1}^{N} PG_i + \sum_{i=1}^{N} C_i = \sum_{i=1}^{N} PD_i$$

$$PG^{\min} \leq PG \leq PG^{\max}$$

$$0 \leq C \leq PD$$

$$|(\theta_i - \theta_j)/x_{ij}| \leq Cap_{ij} \quad i \neq j \qquad (78)$$

NC = set of consumer nodes;

$w_i$ = weighting factor;

$C_i$ = curtailed load at node 'i' (elements of vector 'C');

PG,PD = (N-1) vectors of generation and demand;

$PG_{min}$, $PG_{max}$= (N-1) vectors of generation limits;

$Cap_{ij}$ = thermal limit (capacity) of branch i-j;

Variables are vectors **PG**, **C** and **θ**

# Composite Generation and Transmission Studies
## Analysis of System States

- **Example 28-2:** Specify MLC model for the 3-node intact network given in the example 28-1. What is the load curtailment in each studied contingency case ?

  Solution

$$\min z = w_3 \cdot C_3$$

$$\begin{vmatrix} PG_2 \\ 0 \end{vmatrix} + \begin{vmatrix} 0 \\ C_3 \end{vmatrix} - \begin{vmatrix} 0 \\ 5 \end{vmatrix} = \begin{vmatrix} 4 & -2 \\ -2 & 8 \end{vmatrix} \cdot \begin{vmatrix} \theta_2 \\ \theta_3 \end{vmatrix}$$

$$PG_1 + PG_2 + C_3 = 5$$

$$0 \le PG_1 \le 5$$

$$0 \le PG_2 \le 3$$

$$0 \le C_3 \le 5$$

$$|(0-\theta_2)/0.5| \le 3$$

$$|(0-\theta_3)/0.16667| \le 4$$

$$|(\theta_2-\theta_3)/0.5| \le 3$$

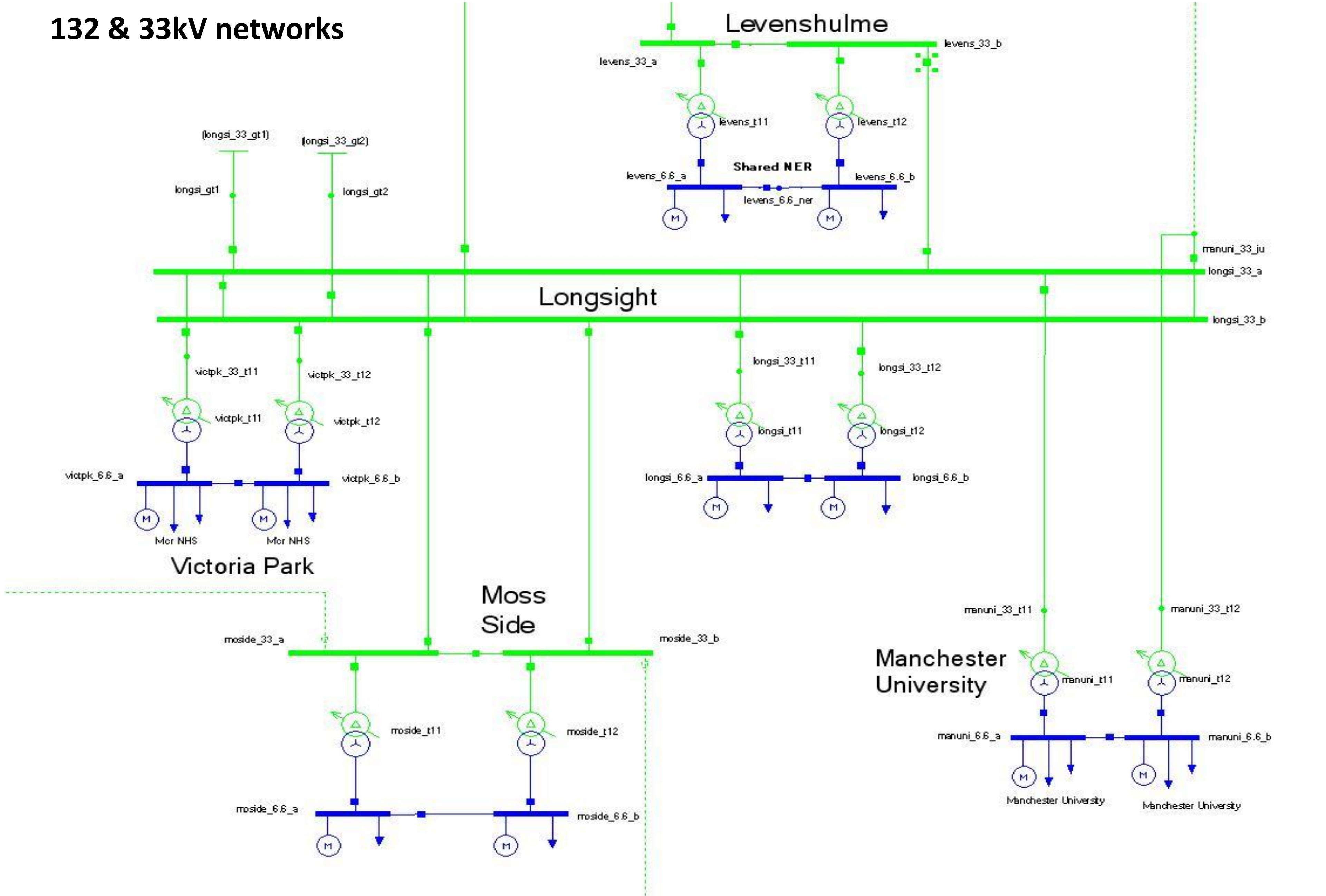
Load curtailments at node No. 3 in each contingency case are found by bringing the line flows into specified limits. Homework!

# Composite Generation and Transmission Studies
## Analysis of System States

- Additional features
  - Nodal loads: completely dependent, independent, partially correlated
  - Availability of primary resources (eg. wind, hydro)
  - Line common mode outages: a model already shown
  - Multi-state model for base generating units (derated states)
  - Peak generating units: a model already shown
  - Transformer substations with/without spare transformer
  - Normal and adverse weather conditions
  - Etc...
- System operation features shall be modelled

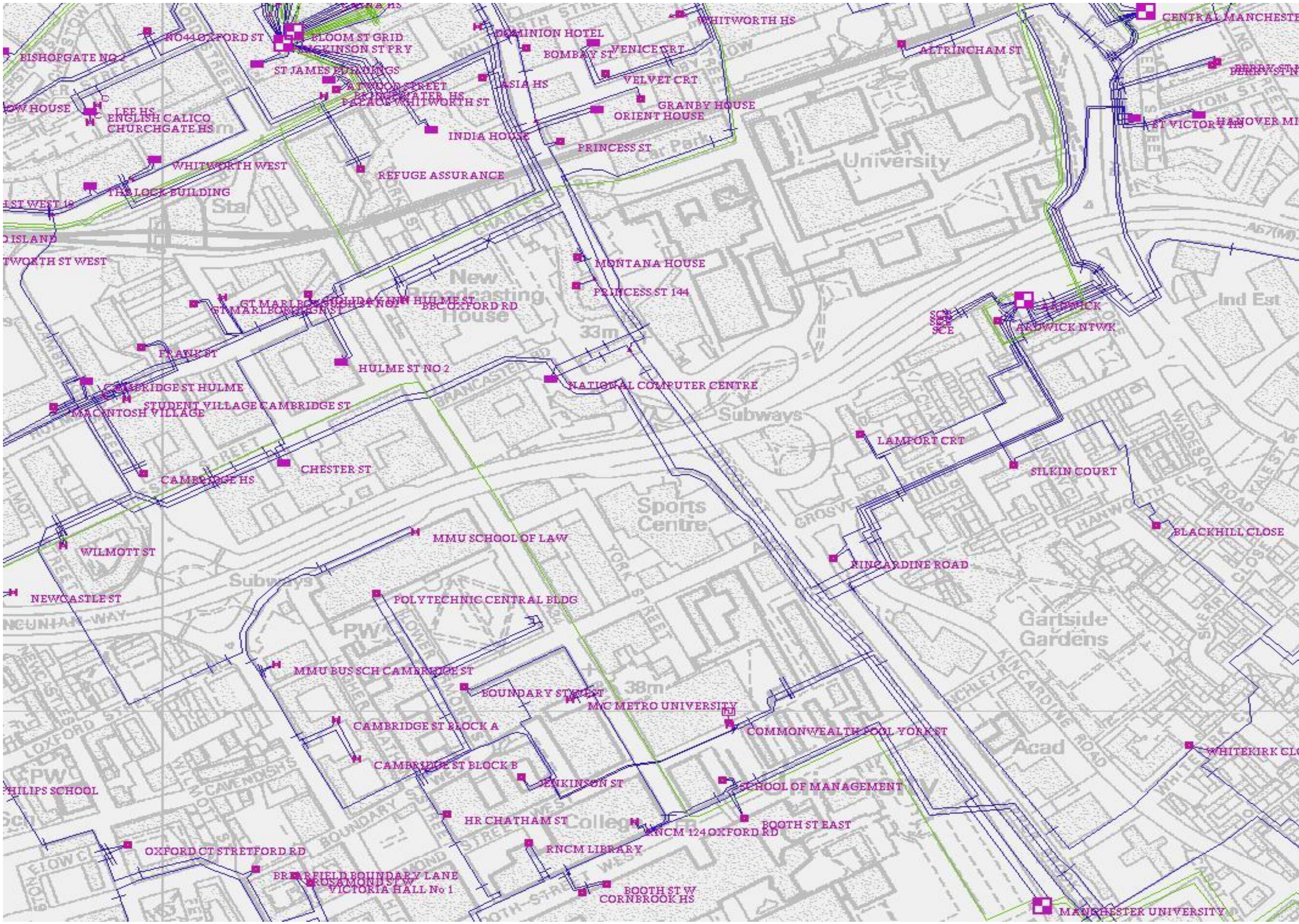
# Distribution Networks – Types of Networks (IPSA)

**132 & 33kV networks**



# Distribution Networks – Types of Networks (DINIS)

## 11 & 6.6kV networks



# Distribution Networks – Types of Networks (GIS)

**LV networks**

# Distribution Networks
## Evaluation Techniques

- UK distribution networks:
  - 132 kV and 33 kV networks are meshed
  - 11 and 6.6 kV networks are either radial (majority) or meshed (minority)
  - LV networks are either radial (majority) or meshed (minority)
- Greatest contribution to unreliability comes from 11(6.6) kV networks and than LV networks (radial operation)
- A typical 11(6.6) kV radial feeder has one or more normally open points which can supply customers under outage conditions
- *Monte Carlo Simulation* technique can be applied to both radial and meshed networks
- *Analytical* methods can be applied to radial networks; they are based on simplified (un)availability calculation of components connected in series:

$$\lambda_S = \sum_{i=1}^{n} \lambda_i$$

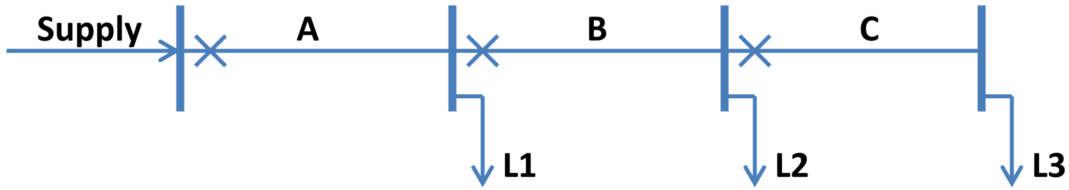
$$r_S = \{\sum_{i=1}^{n} \lambda_i \cdot r_i\} / \lambda_S$$

(n=number of components; $\lambda_i$=failure rate; $r_i$=repair time)

$$U_S = \lambda_S \cdot r_S = \sum_{i=1}^{n} \lambda_i \cdot r_i \qquad (32)$$

# Distribution Networks
## Evaluation Techniques

- **Example 29:** A radial three-load point feeder consists of three sections which are protected by a circuit breaker. Section (line/cable) availability data are given in the table below. Calculate unavailability (hours/year) of each load point.




| Section | Failure rate (f/yr) | Repair time (hours) | (Repair rate) (r/yr) |
|---|---|---|---|
| A | 0.2 | 6 | 1460 |
| B | 0.1 | 5 | 1752 |
| C | 0.15 | 8 | 1095 |

Solution

Load point 'L3':

$$\lambda_{L3} = 0.2 + 0.1 + 0.15 = 0.45$$

$$U_{L3} = 0.2 \cdot 6 + 0.1 \cdot 5 + 0.15 \cdot 8 = 2.9$$

$$r_{L3} = 2.9 / 0.45 = 6.44$$

All load points:

| Load point | Failure rate (f/yr) | Repair time (hours) | Unavailability (hours/yr) |
|---|---|---|---|
| L1 | 0.2 | 6 | 1.2 |
| L2 | 0.3 | 5.7 | 1.7 |
| L3 | 0.45 | 6.4 | 2.9 |

-Note 1: A fault triggers *nearest circuit breaker* to trip!

-Note 2: Real-life feeders do NOT have circuit breakers at each section!

# Distribution Networks
## Interruption Indices

- Calculated failure rate, outage duration and unavailability are long-term averages
- Customer orientated indices:
  - System Average Interruption Frequency Index – SAIFI

$$SAIFI = \frac{total\,number\,of\,\,customer\,\mathrm{int}\,erruptions}{total\,number\,of\,\,customers\,served} = \frac{\sum \lambda_i \cdot N_i}{\sum N_i} \quad (79.1)$$

    ($\lambda_i$ = failure rate of load point 'i'; $N_i$ = number of customers at load point 'i')

    Distribution companies call this index 'Customer Interruptions' – **CIs**

  - System Average Interruption Duration Index – SAIDI

$$SAIDI = \frac{sum\,of\,\,customer\,\mathrm{int}\,erruption\,durations}{total\,number\,of\,\,customers\,served} = \frac{\sum U_i \cdot N_i}{\sum N_i} \quad (79.2)$$

    ($U_i$ = annual outage time in h/yr; $N_i$ = number of customers at load point 'i')

    Distribution companies call this index 'Customer Minutes Lost' - **CMLs**

# Distribution Networks
## Interruption Indices

– Average Service Availability (Unavailability) Index – ASAI (ASUI)

$$ASAI = \frac{customer\ hours\ of\ available\ service}{customer\ hours\ demanded} = \frac{\sum 8760 \cdot N_i - \sum U_i \cdot N_i}{\sum 8760 \cdot N_i} \qquad (79.3)$$

$$ASUI = \frac{customer\ hours\ of\ unavailable\ service}{customer\ hours\ demanded} = 1 - ASAI \qquad (79.4)$$

- Load- and energy-orientated indices:
  – Energy Not Supplied – ENS

$$ENS = total\ energy\ not\ supplied\ by\ the\ system = \sum LA_i \cdot U_i \qquad (79.5)$$

($LA_i$ = average load at load point ‘i’; $U_i$ = annual unavailability (h) at load point ‘i’

  – Average Energy Not Supplied – AENS

$$AENS = \frac{total\ energy\ not\ supplied}{total\ number\ of\ customers} = \frac{\sum LA_i \cdot U_i}{\sum N_i} \qquad (79.6)$$

# Distribution Networks
## Interruption Indices

- **Example 30:** Consider the three-node network from previous example where load point indices are given in the second table. Calculate the customer- and load-orientated indices (additional data are presented below).

| Load point | Number of customers | Average load demand (kW) |
|---|---|---|
| L1 | 200 | 1000 |
| L2 | 150 | 700 |
| L3 | 100 | 400 |
| Total | 450 | 2100 |

Solution

SAIFI = (0.2x200+0.3x150+0.45x100)/(200+150+100) = 0.289 interr/customer yr

SAIDI = (1.2x200+1.7x150+2.9x100)/(450) = 1.74 hours/customer yr

ASAI = [450x8760-(1.2x200+1.7x150+2.9x100)]/(450x8760) = 0.999801

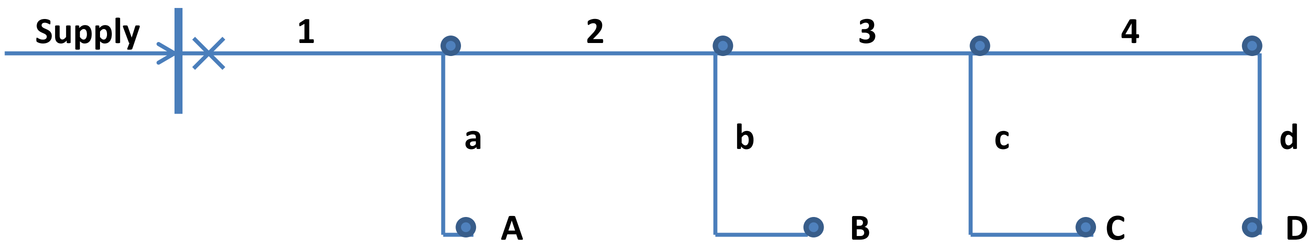
ASUI = 1-0.999801 = 0.000199

ENS = 1000x1.2+700x1.7+400x2.9 = 3550 kWh/yr

AENS = 3550/450 = 7.89 kWh/customer yr

# Distribution Networks
## Radial Systems

- A typical (11kV) radial network is shown in figure below; there is only one circuit breaker, laterals (spurs) are not protected and there is no NOP




- If a section or lateral is faulty, the incoming circuit breaker will trip; this means that all load points A, B, C & D will have *identical* reliability indices
- Assume reliability parameters given in table below. Calculate load point indices

| Section/ Lateral | Length (km) | Failure rate (f/yr) | Repair time (h) |
|---|---|---|---|
| 1 | 2 | 0.2 | 4 |
| 2 | 1 | 0.1 | 4 |
| 3 | 3 | 0.3 | 4 |
| 4 | 2 | 0.2 | 4 |
| | | | |
| a | 1 | 0.2 | 2 |
| b | 3 | 0.6 | 2 |
| c | 2 | 0.4 | 2 |
| d | 1 | 0.2 | 2 |

# Distribution Networks
## Radial Systems

| Component failure | Load point A | | | Load point B | | | Load point C | | | Load point D | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Rate (f/yr) | Repair time (h) | U (h/yr) | Rate (f/yr) | Repair time (h) | U (h/yr) | Rate (f/yr) | Repair time (h) | U (h/yr) | Rate (f/yr) | Repair time (h) | U (h/yr) |
| 1 | 0.2 | 4 | 0.8 | 0.2 | 4 | 0.8 | 0.2 | 4 | 0.8 | 0.2 | 4 | 0.8 |
| 2 | 0.1 | 4 | 0.4 | 0.1 | 4 | 0.4 | 0.1 | 4 | 0.4 | 0.1 | 4 | 0.4 |
| 3 | 0.3 | 4 | 1.2 | 0.3 | 4 | 1.2 | 0.3 | 4 | 1.2 | 0.3 | 4 | 1.2 |
| 4 | 0.2 | 4 | 0.8 | 0.2 | 4 | 0.8 | 0.2 | 4 | 0.8 | 0.2 | 4 | 0.8 |
| | | | | | | | | | | | | |
| a | 0.2 | 2 | 0.4 | 0.2 | 2 | 0.4 | 0.2 | 2 | 0.4 | 0.2 | 2 | 0.4 |
| b | 0.6 | 2 | 1.2 | 0.6 | 2 | 1.2 | 0.6 | 2 | 1.2 | 0.6 | 2 | 1.2 |
| c | 0.4 | 2 | 0.8 | 0.4 | 2 | 0.8 | 0.4 | 2 | 0.8 | 0.4 | 2 | 0.8 |
| d | 0.2 | 2 | 0.4 | 0.2 | 2 | 0.4 | 0.2 | 2 | 0.4 | 0.2 | 2 | 0.4 |
| Total | 2.2 | 2.73 | 6 | 2.2 | 2.73 | 6 | 2.2 | 2.73 | 6 | 2.2 | 2.73 | 6 |

(Total failure rate = sum of all rates; Total unavailability = sum of all unavailabilities; Total repair time = Total unavailability/Total failure rate)

- Assume additional load point data given in the table below

  SAIFI=2.2 interruptions/customer yr

  SAIDI=6.0 hours/customer yr

  ASUI=0.000685;  ASAI=0.999315

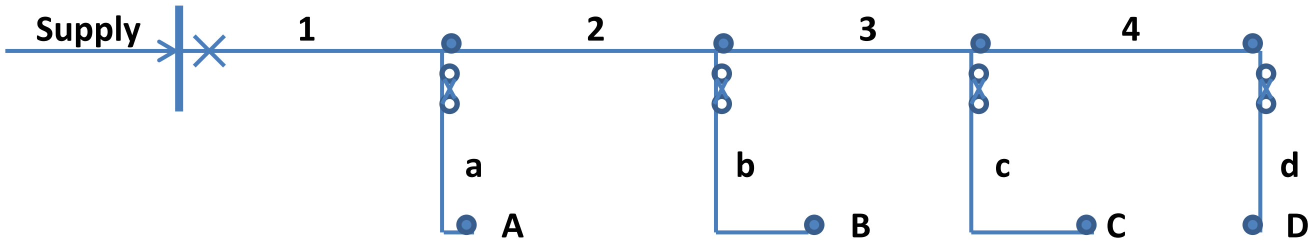
  ENS=84.0 MWh/yr

  AENS=28.0 kWh/customer yr

| Load point | Number of customers | Average load (kW) |
|---|---|---|
| A | 1000 | 5000 |
| B | 800 | 4000 |
| C | 700 | 3000 |
| D | 500 | 2000 |

# Distribution Networks
## Radial Systems

- Effect of lateral (spur) protection
- Fuses are installed at tee-points in each lateral; a fault in a lateral is cleared by the fuse and other load points remain connected




| Component failure | Load point A | | | Load point B | | | Load point C | | | Load point D | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Rate (f/yr) | Repair time (h) | U (h/yr) | Rate (f/yr) | Repair time (h) | U (h/yr) | Rate (f/yr) | Repair time (h) | U (h/yr) | Rate (f/yr) | Repair time (h) | U (h/yr) |
| 1 | 0.2 | 4 | 0.8 | 0.2 | 4 | 0.8 | 0.2 | 4 | 0.8 | 0.2 | 4 | 0.8 |
| 2 | 0.1 | 4 | 0.4 | 0.1 | 4 | 0.4 | 0.1 | 4 | 0.4 | 0.1 | 4 | 0.4 |
| 3 | 0.3 | 4 | 1.2 | 0.3 | 4 | 1.2 | 0.3 | 4 | 1.2 | 0.3 | 4 | 1.2 |
| 4 | 0.2 | 4 | 0.8 | 0.2 | 4 | 0.8 | 0.2 | 4 | 0.8 | 0.2 | 4 | 0.8 |
| a | 0.2 | 2 | 0.4 | | | | | | | | | |
| b | | | | 0.6 | 2 | 1.2 | | | | | | |
| c | | | | | | | 0.4 | 2 | 0.8 | | | |
| d | | | | | | | | | | 0.2 | 2 | 0.4 |
| Total | 1 | 3.6 | 3.6 | 1.4 | 3.14 | 4.4 | 1.2 | 3.33 | 4 | 1 | 3.6 | 3.6 |

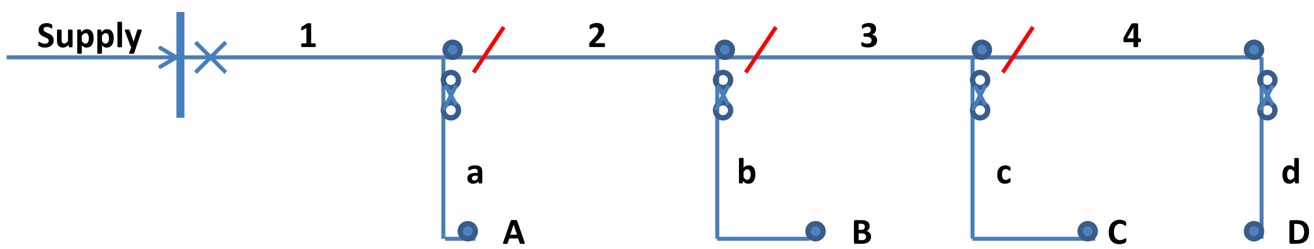
(Total failure rate = sum of all rates; Total unavailability = sum of all unavailabilities; Total repair time = Total unavailability/Total failure rate)

SAIFI=1.15 interr/customer yr; SAIDI=3.91 hours/customer yr; ASUI=0.000446

ASAI=0.999554; ENS=54.8MWh; AENS=18.3 kWh/customer yr

# Distribution Networks
## Radial Systems

- Effect of isolators (switches)
- Isolators are installed along feeder main and they are not fault-breaking devices -> the main circuit breaker operates in case of a fault
- When a fault has been detected, an isolator can be opened and the breaker reclosed to supply other load points
- Assume switching time is 0.5 hours (repair time is 4 hours)



# Distribution Networks
## Radial Systems

| Component failure | Load point A | | | Load point B | | | Load point C | | | Load point D | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Rate (f/yr) | Repair time (h) | U (h/yr) | Rate (f/yr) | Repair time (h) | U (h/yr) | Rate (f/yr) | Repair time (h) | U (h/yr) | Rate (f/yr) | Repair time (h) | U (h/yr) |
| 1 | 0.2 | 4 | 0.8 | 0.2 | 4 | 0.8 | 0.2 | 4 | 0.8 | 0.2 | 4 | 0.8 |
| 2 | 0.1 | 0.5 | 0.05 | 0.1 | 4 | 0.4 | 0.1 | 4 | 0.4 | 0.1 | 4 | 0.4 |
| 3 | 0.3 | 0.5 | 0.15 | 0.3 | 0.5 | 0.15 | 0.3 | 4 | 1.2 | 0.3 | 4 | 1.2 |
| 4 | 0.2 | 0.5 | 0.1 | 0.2 | 0.5 | 0.1 | 0.2 | 0.5 | 0.1 | 0.2 | 4 | 0.8 |
| | | | | | | | | | | | | |
| a | 0.2 | 2 | 0.4 | | | | | | | | | |
| b | | | | 0.6 | 2 | 1.2 | | | | | | |
| c | | | | | | | 0.4 | 2 | 0.8 | | | |
| d | | | | | | | | | | 0.2 | 2 | 0.4 |
| Total | 1 | 1.5 | 1.5 | 1.4 | 1.89 | 2.65 | 1.2 | 2.75 | 3.3 | 1 | 3.6 | 3.6 |

(Total failure rate = sum of all rates; Total unavailability = sum of all unavailabilities; Total repair time = Total unavailability/Total failure rate)

SAIFI = 1.15 interruptions/customer yr

SAIDI = 2.58 hours/customer yr

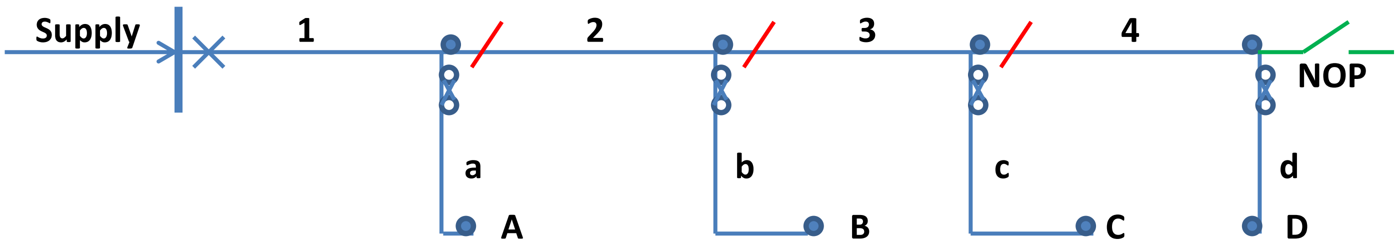
ASUI = 0.000294; ASAI = 0.999706

ENS = 35.2 MWh/yr

AENS = 11.7 kWh/customer yr

# Distribution Networks
## Radial Systems

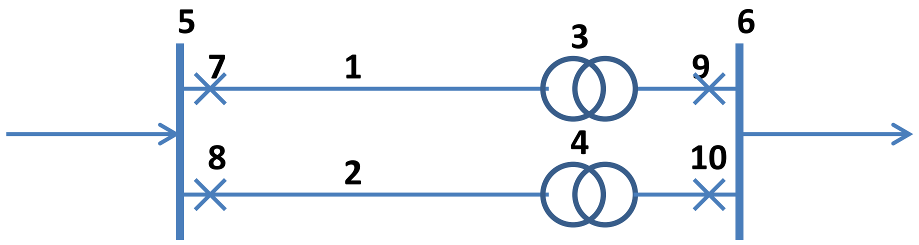
- Effect of transferring load
- Radial distribution feeders are usually connected to adjacent radial circuit through normally open point(s) – NOP(s)
- A part of feeder load can be supplied from the adjacent feeder through NOP
- The greatest effect is for the load points furthest from the supply point




| Component failure | Load point A | | | Load point B | | | Load point C | | | Load point D | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Rate (f/yr) | Repair time (h) | U (h/yr) | Rate (f/yr) | Repair time (h) | U (h/yr) | Rate (f/yr) | Repair time (h) | U (h/yr) | Rate (f/yr) | Repair time (h) | U (h/yr) |
| 1 | 0.2 | 4 | 0.8 | 0.2 | 0.5 | 0.1 | 0.2 | 0.5 | 0.1 | 0.2 | 0.5 | 0.1 |
| 2 | 0.1 | 0.5 | 0.05 | 0.1 | 4 | 0.4 | 0.1 | 0.5 | 0.05 | 0.1 | 0.5 | 0.05 |
| 3 | 0.3 | 0.5 | 0.15 | 0.3 | 0.5 | 0.15 | 0.3 | 4 | 1.2 | 0.3 | 0.5 | 0.15 |
| 4 | 0.2 | 0.5 | 0.1 | 0.2 | 0.5 | 0.1 | 0.2 | 0.5 | 0.1 | 0.2 | 4 | 0.8 |
| | | | | | | | | | | | | |
| a | 0.2 | 2 | 0.4 | | | | | | | | | |
| b | | | | 0.6 | 2 | 1.2 | | | | | | |
| c | | | | | | | 0.4 | 2 | 0.8 | | | |
| d | | | | | | | | | | 0.2 | 2 | 0.4 |
| Total | 1 | 1.5 | 1.5 | 1.4 | 1.39 | 1.95 | 1.2 | 1.88 | 2.25 | 1 | 1.5 | 1.5 |

# Distribution Networks
## Ring Networks

- Ring networks are very common at 33kV
- Full analysis can be done using Monte Carlo simulation
- A simplified solution will be presented
- Consider a dual transformer feeder
  - It will be assumed that two busbars and circuit breakers are 100% reliable




- Approximate equations for two components in parallel:

$$\lambda_{PP} = \frac{\lambda_1 \cdot \lambda_2 (r_1 + r_2)}{1 + \lambda_1 \cdot r_1 + \lambda_2 \cdot r_2} \approx \lambda_1 \cdot \lambda_2 (r_1 + r_2)$$

$$r_{PP} = \frac{r_1 \cdot r_2}{r_1 + r_2}$$

$$U_{PP} = \lambda_P \cdot r_P = \lambda_1 \cdot \lambda_2 \cdot r_1 \cdot r_2 \qquad (35)$$

# Distribution Networks
## Ring Networks

- Approximate expressions for three components in parallel

$$\lambda_{PP} \approx \lambda_1 \cdot \lambda_2 \cdot \lambda_3 (r_1 \cdot r_2 + r_1 \cdot r_3 + r_2 \cdot r_3)$$

$$r_{PP} = \frac{r_1 \cdot r_2 \cdot r_3}{r_1 \cdot r_2 + r_1 \cdot r_3 + r_2 \cdot r_3}$$

$$U_{PP} = \lambda_{PP} \cdot r_{PP} = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot r_1 \cdot r_2 \cdot r_3 \qquad (36)$$

- Assume component reliability data given below. Calculate reliability indices:

| Component | Failure rate (f/yr) | Repair time (h) |
|---|---|---|
| 1 | 0.5 | 10 |
| 2 | 0.5 | 10 |
| 3 | 0.01 | 100 |
| 4 | 0.01 | 100 |

  – Series connection of line 1 (or 2) and transformer 3 (or 4):

$$\lambda_{13} = \lambda_{24} = 0.5 + 0.01 = 0.51 \ f/yr; \ U_{13} = U_{24} = 0.5 \cdot 10 + 0.01 \cdot 100 = 6 \ h/yr$$

$$r_{13} = r_{24} = 6/0.51 = 11.76 \ hours$$

# Distribution Networks
## Ring Networks

- Parallel connection of two transformer-feeders:

$$\lambda_{PP} = 0.51 \cdot 0.51(11.76 + 11.76)/8760 = 6.984 \cdot 10^{-4} \;\; f/yr$$

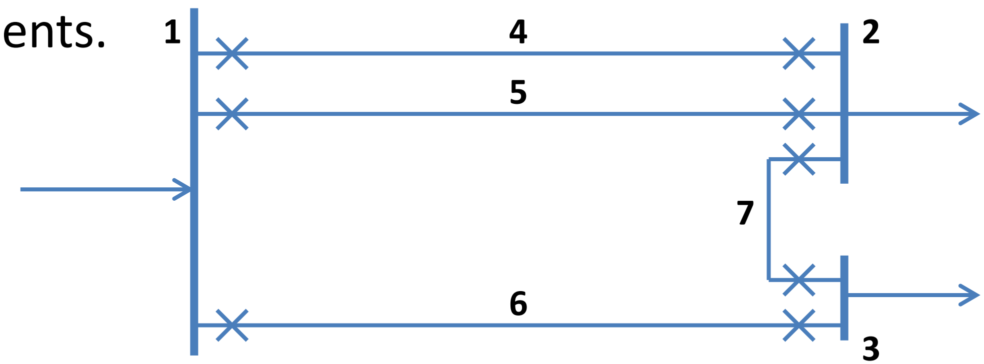
$$r_{PP} = \frac{11.76 \cdot 11.76}{11.76 + 11.76} = 5.88\, hours$$

$$U_{PP} = \lambda_{PP} \cdot r_{PP} = 4.107 \cdot 10^{-3} \;\; h/yr$$

- The technique is not suitable for further development:
  - Weather effects (normal and adverse weather conditions)
  - Scheduled maintenance
  - Transient failures
  - Different modes of failure, etc.

# Distribution Networks
## Total Loss of Continuity

- Similar approach to the previous one (series and parallel connection of components)
- Average load disconnected = peak load x load factor
- Average energy not supplied = average load disconnected x annual outage time
- Consider meshed-ring system with two load points. Failure rates of lines 4-7 are 0.02 f/yr, of busbars 1-3 are 0.01 f/yr, repair times of lines 4-7 are 10 hours and of busbars 1-3 are 5 hours. Load point 2: peak load 20MW, load factor 0.75, 2000 customers. Load point 3: peak load 10MW, load factor 0.75, 1000 customers. Calculate reliability indices using approximate expressions for series and parallel connection of components.




  – Outage event: parallel connection (second- and third-order)
  – Parallel connections: expression for lambda - ***both repair times must be divided by 8760 h***!
  – Totals: "series connection" of outage events (***any*** outage event causes supply interruption)

# Distribution Networks
## Total Loss of Continuity

| Outage event | Failure rate (f/yr) | Repair time (hours) | Unavailability U (hours/yr) | Average load disconnected (MW) | Average energy not supplied (MWh/yr) |
|---|---|---|---|---|---|
| **Load point 2** | | | | | |
| 1 | 0.01 | 5 | 0.05 | 15 | 0.75 |
| 2 | 0.01 | 5 | 0.05 | 15 | 0.75 |
| 4+5+6 | 3.13E-11 | 3.33 | 1.04E-10 | 15 | 1.56E-09 |
| 4+5+3 | 1.04E-11 | 2.5 | 2.60E-11 | 15 | 3.90E-10 |
| 4+5+7 | 3.13E-11 | 3.33 | 1.04E-10 | 15 | 1.56E-09 |
| Total | 2.00E-02 | 5 | 1.00E-01 | 15 | 1.5 |
| | | | | | |
| **Load point 3** | | | | | |
| 1 | 0.01 | | 0.05 | 7.5 | 0.375 |
| 3 | 0.01 | | 0.05 | 7.5 | 0.375 |
| 6+2 | 3.42E-07 | | 1.14E-06 | 7.5 | 8.55E-06 |
| 6+7 | 9.13E-07 | | 4.57E-06 | 7.5 | 3.43E-05 |
| 4+5+6 | 3.13E-11 | | 1.04E-10 | 7.5 | 7.80E-10 |
| Total | 2.00E-02 | | 1.00E-01 | 7.5 | 0.75 |
| SAIFI=0.02 interruptions/customer yr;  SAIDI=0.10 hours/customer yr | | | | | |
| ASAI=0.999989;  ASUI=1.142E-05 | | | | | |
| ENS=2.25 MWh/yr;  AENS=0.75 kWh/customer yr | | | | | |

- Small contribution from second-order, negligible from third-order events
- Main contribution from busbar faults (first-order event)

# Distribution Networks
## Partial Loss of Continuity

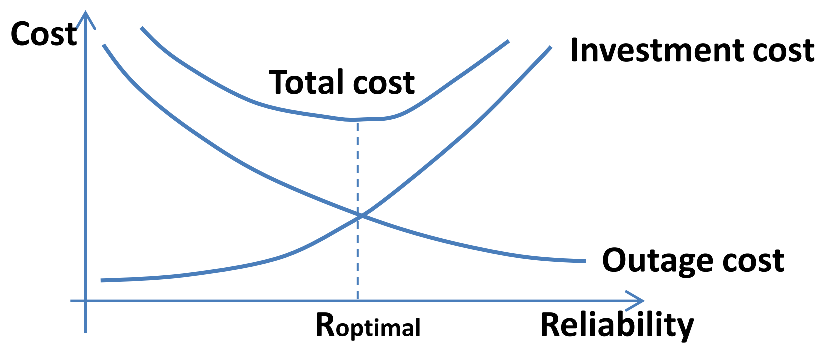
- A partial loss of continuity occurs when a part of the load cannot be supplied
- First-order and second-order outages are studied
- Selection of second-order outages:
  - All outages if the number is small
  - Manually determine from experience which outages are ‘critical’
  - The third-order minimal cut sets can be used to identify second-order PLOC events. All second-order combinations from each third-order minimal cut set for the considered load point.
- Load flow model (preferable ‘AC’) needs to be applied to find out whether network constraints have been violated
- Circuit overloads and voltages outside permissible limits are usually considered
- Load curtailment (shedding) can be done in several ways:
  - Load is shed at the receiving end of the overloaded line
  - Load is reduced proportionally at all load points that can affect the overload (‘sensitivity analysis’)
  - Optimal load shedding -> minimum load curtailment model (AC or DC)

# Distribution Networks
## Further Considerations

- Protection failures (eg stuck circuit breaker), busbar failures
- Transient and temporary failures (eg lightning strike)
  - Permanent outages: damage of components
  - Temporary outages: no damage, restored by manual switching or fuse replacement
  - Transient outages: no damage, restored by automation (automatic switching)
- Inclusion of scheduled maintenance
- Inclusion of weather effects
- More complex configurations (eg ring networks)
- More complex analysis...

# Reliability Cost Assessment
## Fundamentals

- Total cost is a sum of investment, operational and outage costs
- Typical 'cost v reliability' curve is shown below



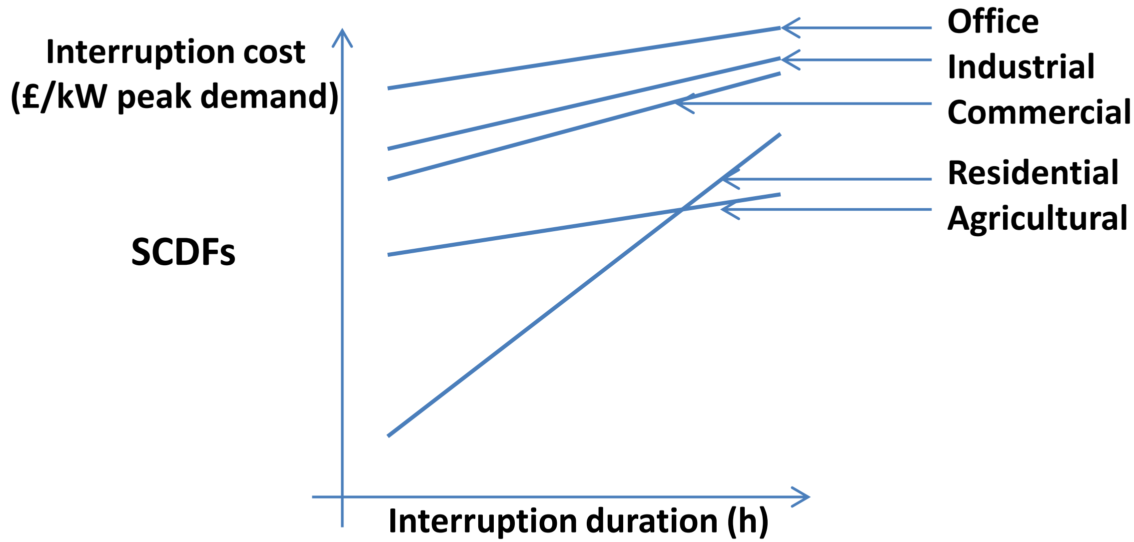

- Outage costs can be seen by utility or customer/society
- Utility outage costs:
  - Loss of revenue from customers not served
  - Loss of incentive reward – regulatory framework in the UK
  - Increased expenditure due to maintenance and repair...
- Customer outage costs:
  - Industry costs due to lost manufacture, damaged equipment, etc.
  - Residential customer costs, such as spoiled deep frozen food, alternative heating, etc.
  - Costs difficult to quantify, such as loss of convenience, looting, rioting, etc.

# Reliability Cost Assessment
## Customer Damage Functions

- Basic customer survey methods:
  1. Contingent valuation method
  2. Direct costing method
  3. Indirect costing method
- Contingent valuation method:
  - Customer’s willingness to pay (**WTP**) to avoid an interruption
  - Customer’s willingness to accept (**WTA**) compensation for having an interruption
  - WTA values are significantly higher than WTP values
  - Recent study in the UK (2013): one non-supplied kWh is around £17
- Direct costing method:
  - Evaluation of direct costs associated with particular outage scenario
  - Industry, commercial/retail markets
- Indirect costing method:
  - Economic principle of substitution – valuation of a replacement good is used to measure worth of the original good
  - Residential sector
  - Hypothetical insurance policy to compensate for possible interruption effects,...

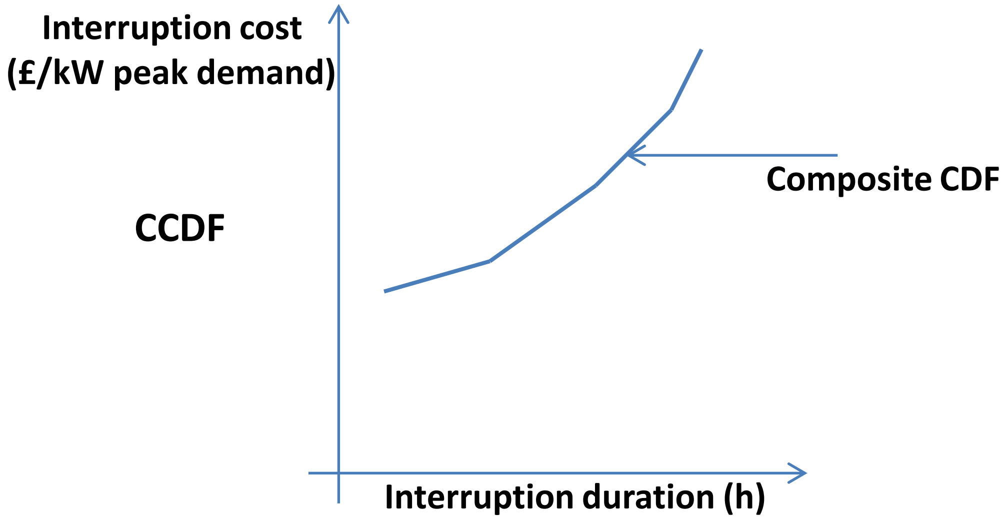
# Reliability Cost Assessment
## Customer Damage Functions

- Sector Customer Damage Function (SCDF)
  - Unit interruption cost is a function of interruption duration for a sector
  - There are 7 main sectors: larger users, industrial, commercial, agriculture, residential, governmental and offices
  - Reported customer interruption costs (£) are normalised using the annual peak (£/kW peak)
  - Weighted average unit interruption cost for each section is calculated:
    - Short interruption duration => weight is % of the annual peak demand
    - Long interruption duration => weight is % of the annual energy consumption



# Reliability Cost Assessment
## Customer Damage Functions

- Composite Customer Damage Function (CCDF)
  - Unit interruption cost is a function of interruption duration for a customer mix at a bus, in an area, or in a whole system
  - Customer mix in terms of energy consumptions or peak demand percentages must be known
  - Interruption shorter than 0.5 hours => weighting factor is percentage of annual peak load
  - Interruption longer than 0.5 hours => weighting factor is percentage of annual energy consumption



# Reliability Cost Assessment
## Generation Systems

- Reliability indicators have been calculated
- We are interested in the monetary value associated with unserved energy
- Expected Energy Not Served (EENS) in MWh/yr:

$$EENS = \sum_{i=1}^{N} C_i \cdot F_i \cdot D_i \qquad (80.1)$$

N=number of loss load events; $C_i$=load curtailment of event ‘i’ in MW; $F_i$=frequency of event ‘i’ in occ./yr; $D_i$=duration of event ‘i’ in hours

- Expected Interruption Cost (EIC) in £/yr:

$$EIC = \sum_{i=1}^{N} C_i \cdot F_i \cdot W(D_i) \qquad (80.2)$$

$W(D_i)$=customer damage function for duration $D_i$ of loss load event ‘i’

- In sequential Monte Carlo simulation EIC is:

$$EIC = \frac{1}{M} \sum_{i=1}^{N} W(D_i) \cdot E_i / D_i \qquad (80.3)$$

M=number of simulated years; N=total number of loss load events; $E_i$=energy not supplied of interruption ‘i’ in MWh; $D_i$=duration of interruption ‘i’ in hours

# Reliability Cost Assessment
## Composite Systems

- Minimum load curtailment model (eq. (78))
  - Objective function can be modified: weighting factors '$w_i$' are replaced with customer damage functions $W_i(D_k)$ at node 'i'

$$\min z = \sum_{i=1}^{NC} W_i(D_k) \cdot C_i \qquad (81.1)$$

NC=number of load nodes; $D_k$=duration of system state 'k' in h; $C_i$=load curtailment at node 'i'

  - System state duration $D_k$ needs to be determined ($p_k$=$F_k$ x $D_k$)
- Annual bus and system indices:

$$EENS_i = \sum_{k=1}^{N} C_{ik} \cdot F_k \cdot D_k; \qquad EENS = \sum_{i=1}^{NC} EENS_i \qquad (81.2)$$

$$EIC_i = \sum_{k=1}^{N} C_{ik} \cdot F_k \cdot W_i(D_k); \quad EIC = \sum_{i=1}^{NC} EIC_i \qquad (81.3)$$

N=number of system states sampled; $F_k$=frequency of system state 'k' (occ./yr); $EENS_i$=expected energy not served at bus 'i'; $EIC_i$=expected interruption cost at bus 'i'

# Reliability Cost Assessment
## Distribution Networks

- Calculation of reliability indices has been done
- Previous equations (81) can be applied

# Reliability of Substations
## Fundamentals

- Substations:
  - Transformer substations
  - Substations without transformation
  - Step-up transformation in power stations
- Reliability analysis is done in following cases:
  - Comparison of alternative solutions for a substation
  - Calculation of risk/damage cost of an important customer supplied from the substation
- Large number of components, fortunately many of them are highly reliable (eg switches, measuring devices, etc.)
- Methods for reliability analysis:
  - Minimal cut sets
  - Failure tree analysis
  - Failure state analysis – systematic enumeration of substation states
  - ***Selective enumeration***

# Reliability of Substations
## Selective Enumeration

- Definitions:
  - Critical event ‘E’: event which interrupts substation function
  - Functional block: set of components where repair/maintenance of one component leaves out of service all components
  - ‘B’ – disconnection (repair and/or maintenance) of functional block B (B=BR+BM)
  - ‘BR’ – repair/replacement of functional block B
  - ‘BM’ – maintenance of functional block B
  - ***Active failure***: failure which triggers adjacent circuit breakers
  - ‘BA’ – active failure of functional block B
  - $r_i$, $r''_i$, $s_i$ – repair time, maintenance time and switching time in case of active failure
- Critical event ‘E’:

$$E = \sum_j B_j + \sum_h BA_h + \sum_{i,k} B_i \cdot B_k + \sum_{p,q} BA_p \cdot B_q$$

$$B_i \cdot B_k = (BR_i + BM_i) \cdot (BR_k + BM_k) = \ldots$$

$$BA_p \cdot B_q = BA_p \cdot (BR_q + BM_q) = \ldots \qquad (82)$$

*j, h, i, k, p, q* – indices of functional blocks

- Definition of critical event ‘E’ should be done by several engineers!

# Reliability of Substations
## Selective Enumeration

- Unreliability 'U' and interruption frequency 'f' of a substation is:

$$U = P\{E\} = \sum_i P\{D_i\}\;; \qquad f = f\{E\} = \sum_i f\{D_i\} \qquad (83)$$

  $D_i$ = any event on right hand side of expression (82)

- Probabilities and frequencies of individual events:

$$P\{BA_h\} = f_h^{'} \cdot s_h\,,\; f\{BA_h\} = f_h^{'};\;\; P\{BR_j\} = U_j\,, f\{BR_j\} = f_j;\;\; P\{BM_j\} = U_j^{"}\,, f\{BM_j\} = f_j^{"}$$

$$P\{BR_j \cdot BR_k\} = P\{BR_j\} \cdot P\{BR_k\}\,,\;\; f\{BR_j \cdot BR_k\} = f\{BR_j\} \cdot P\{BR_k\} + f\{BR_k\} \cdot P\{BR_j\}$$

$$P\{BR_i \cdot BM_k\} = P\{BR_i\} \cdot P\{BM_k\} \cdot \frac{r_k^{"}}{r_i + r_k^{"}}\,,\;\; f\{BR_i \cdot BM_k\} = f\{BR_i\} \cdot P\{BM_k\}$$

$$P\{BA_i \cdot BR_k\} = P\{BA_i\} \cdot P\{BR_k\}\,,\;\; f\{BA_j \cdot BR_k\} = f\{BA_j\} \cdot P\{BR_k\} + f\{BR_k\} \cdot P\{BA_j\}$$

$$P\{BA_i \cdot BM_k\} = P\{BA_i\} \cdot P\{BM_k\} \cdot \frac{r_k^{"}}{s_i + r_k^{"}}\,,\;\; f\{BA_i \cdot BM_k\} = f\{BA_i\} \cdot P\{BM_k\} \qquad (84)$$

$U_j$ , $f_j$ – series connection of components in a block (standard outage formulae)

$f'_h$ – (frequency of breaker active failures) + probability of breaker failure x
(frequency of active failures of components protected by breaker)

$f''_j$ – maximum frequency of all component frequencies (simultaneous maintenance)

$U''_j$ – maintenance unavailability based on all component maintenance times

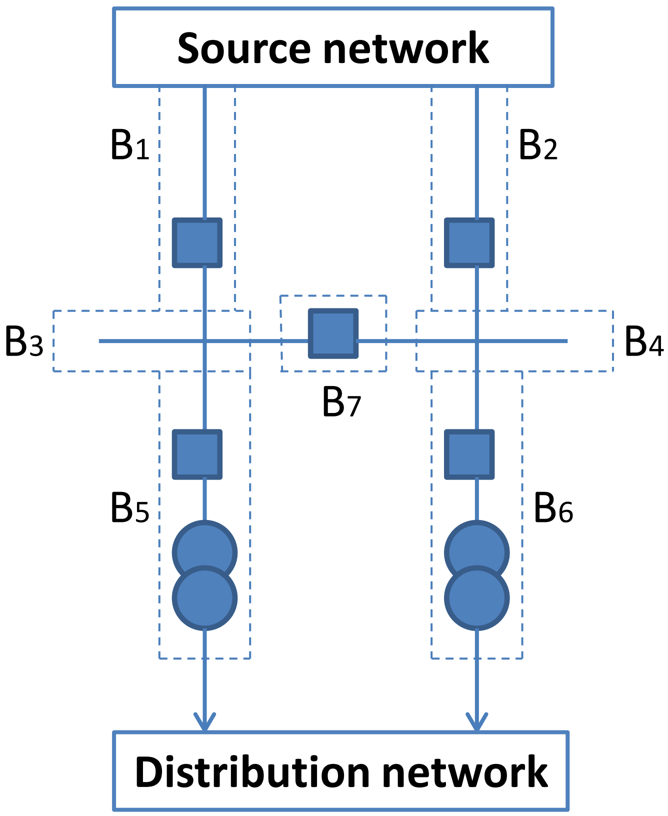
# Reliability of Substations
## Selective Enumeration

- A transformer substation is shown in the figure below. Maintenance of circuit breakers is only done. Switches are not presented – busbar switches are usually included in busbar blocks $B_3$ and $B_4$. Analysed substation function is supply of the distribution network. Define the critical event 'E' which will enable calculation of reliability indices.

Solution:

$$E = B_1 \cdot B_2 + B_1 \cdot BR_4 + B_2 \cdot BR_3 + BR_3 \cdot BR_4$$
$$+ BR_3 \cdot B_6 + BR_4 \cdot B_5 + B_5 \cdot B_6 + BA_1 \cdot B_6$$
$$+ BA_2 \cdot B_5 + BA_5 \cdot B_2 + BA_6 \cdot B_1 + BA_7$$

Watch out active failures!



# Reliability Centered Maintenance
## Basic Tasks

- Maintenance: field assessment, overhaul, refurbishment & replacement
- Taking a component out of service for maintenance increases operation risk of the whole system
- ‘Outage planning’ department determines the ‘best’ network configurations (by stages) during maintenance period
- Fundamental principle of reliability centered maintenance (RCM) is that impact on system reliability is determined
- Traditional building blocks, such as physical condition of components, their failure history and aging status, safety concerns in maintenance, workforce limitations and environment impacts are retained in the RCM
- *Preventive* maintenance is applied to major equipment in order to prevent major failures; it requires a higher annual budget
- Corrective maintenance is applied to less important devices and to failed major plants; it results in higher failure risk and reliability cost

# Reliability Centered Maintenance
## Basic Tasks

- Comparison between Maintenance Alternatives

  Compare different maintenance scheme for a single component or a maintenance sequence associated with multiple components

  1. Select the alternatives based on equipment physical condition and constraints in implementing maintenance.
  2. Build the risk evaluation model – different models for generation, transmission, distribution and substations; any component to be maintained is modelled as *deterministic* planned outage.
  3. Evaluate the risks of all the selected alternatives. The 'best' index is expected energy not supplied (EENS); if it is difficult to calculate it, use the 'pure' probability index.
  4. If necessary conduct an economic analysis; The total cost includes the maintenance cost and the increased system reliability cost due to maintenance.
  5. Determine the best maintenance alternative using either the lowest system risk or the lowest total cost.

# Reliability Centered Maintenance

## Basic Tasks

- Lowest-Risk Maintenance Scheduling

  Determination of timing and duration of maintenance activities due to time-dependent factors (loads, generation, reservoirs, line ratings).

  1. Build the risk evaluation model for the system in which the maintenance activity is carried out; the model must be able to simulate time-dependent factors.
  2. Evaluate the system risk with the maintenance outage shifted over all possible time intervals; Generally, Monte Carlo simulation is more suitable than the enumeration technique.
  3. Determine the lowest-risk maintenance schedule by comparing the results

- Ranking the Importance of Components

  The value of a component depends on the damage caused by its absence from the system.

  1. Build a risk assessment model and evaluate the system risk for the base case when all components are in service but can randomly fail.
  2. Evaluate the risk when a component is out of service and all other components can randomly fail; repeat this step for all components considered for ranking.
  3. Calculate the differences in the system risk between the base case and the cases with each component out of service; create the ranking list.