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OPERATORS

- * Arithmetic : +, -, *, /, %
- * Relational : >, <, >=, <=, ==, !=
- * Logical : &, |, !
- * Bitwise: &, |, ^, <<, >>, ~
- * Others: sizeof, =, -----

a	b	$a \& b$	$a b$	$a \oplus b$	$\sim a$
0	0	0	0	0	1
0	1	0	1	1	1
1	0	0	1	1	0
1	1	1	1	0	0

$$\text{int } a = 5, b = 7$$

$$c = a \& b$$

$$\begin{array}{r} a = 0101 \\ b = 0111 \\ \hline c = \underline{0101} = 5. \end{array}$$

$$c = a | b \quad a = 5, b = 7.$$

$$a = 0101$$

$$b = 0111$$

$$c = \underline{0111} = 7.$$

$$a = 5, b = 7 \quad \wedge \quad (\text{if same } a/b = 0 \text{ else, } a/b = 1)$$

$$c = a \oplus b$$

$$a = 0101 \rightarrow 5$$

$$b = \underline{0111} \rightarrow 7$$

$$a \oplus b = \underline{0110} \rightarrow 2$$

$$c = \sim a \rightarrow a = 5.$$

$$\downarrow \quad \text{int 4 bytes = 32 bits}$$

$$a = 0\ 00\ldots\ 00101$$

$$(\sim a) = \underline{1}\ 11\ldots\ 11010$$

↓ ↓
-ve.

TO do in executor.

$a \& a = a$	$a \& 0 = 0$	$a \& 1 = \text{even } a \% 2$
$0 \& a = a$	$a 0 = a$	$a 1 \rightarrow \text{even}(a+1)$
$a \wedge a = 0$	$a \wedge 0 = a$	$\downarrow \rightarrow \text{odd}(a)$

$$a \wedge 1 \rightarrow \text{even}(a+1) \quad \downarrow \rightarrow \text{odd}(a-1)$$

$$a \& 1 = a \% 2.$$

$$a = 5 = 0101$$

$$a \& 1 = \frac{0001}{0\ 001} = 1$$

} odd ①

$$a \& 1 = 1 \text{ (odd)}$$

$$a \& 1 = 0 \text{ (even)}$$

$$a = 6 = 0110$$

$$a \& 1 = \frac{0001}{0\ 000} = 0 \rightarrow \text{even}$$

$$a = 5 = 0101$$

$$a \& 1 = \frac{0001}{0\ 101} = 5$$

} odd

$$a = 6 = 0110$$

$$a \& 1 = \frac{0101}{0} = 0$$

} even

$$a \& 1 = 0 \rightarrow 6$$

$$\frac{0001}{0111} \rightarrow 7$$

$a \& 1 = a \text{ (odd)}$

$a \& 1 = 0 + 1 \text{ (even)}$

$$a = 5 = 101 \rightarrow 5$$

$$a \& 1 = \frac{001}{100} \rightarrow 1 \quad (4)$$

} odd

$$a = 6 = 110 \rightarrow 6$$

$$\frac{001}{101} \rightarrow 7$$

} even

$a \& 1 = a + 1 \text{ for (even)}$

$a - 1 \text{ for (odd)}$

Bitwise operators support the associative and
the commutative properties.

* $a \& b = b \& a$

* $a \& (b \& c) = (a \& b) \& c$

* $a \& b = b \& a$.

left shift and Right Shift.

left shift: \ll

\ll

$$7 : \begin{array}{l} 00000111 \\ 00001110 \end{array} = 7$$

$$= 7 \times 2^1$$

$7 \ll 1 :$ $\begin{array}{l} \swarrow \\ \text{old discarded} \end{array}$ $00011100 = 7 \times 2^2$

$7 \ll 2 :$ $00111000 = 7 \times 2^3$

$7 \ll 3 :$ $00111000 = 7 \times 2^3$

discarded/circled back will depend on the language or the compiler. In our session we treat it as discarded.

$$\boxed{N \ll i = N \times 2^i} \rightarrow \text{(derived)}$$

Right Shift: \gg

\gg

$$20 : \rightarrow 0\underset{\curvearrowright}{0}\underset{\curvearrowright}{0}\underset{\curvearrowright}{1}\underset{\curvearrowright}{0}\underset{\curvearrowright}{1}\underset{\curvearrowright}{0}\underset{\curvearrowright}{0} \rightarrow 20$$

$$20 \gg 1 \rightarrow 0\underset{\curvearrowright}{0}\underset{\curvearrowright}{0}\underset{\curvearrowright}{0}\underset{\curvearrowright}{0}\underset{\curvearrowright}{1}\underset{\curvearrowright}{0}\underset{\curvearrowright}{1}\underset{\curvearrowright}{0} \rightarrow 10 = \frac{20}{2^1}$$

$$20 \gg 2 \rightarrow 0\underset{\curvearrowright}{0}\underset{\curvearrowright}{0}\underset{\curvearrowright}{0}\underset{\curvearrowright}{0}\underset{\curvearrowright}{0}\underset{\curvearrowright}{1} \rightarrow 5 = \frac{20}{2^2}$$

By deriving we get :-

$$\boxed{N \gg i = \frac{N}{2^i}}$$

problem:-

int powerof2(int N)

{

y

Brute force method :- (Method 1)

```
int powerof2(int n)
{
    int res = 1
    for (int i = 0; i < n; i++)
    {
        res *= 2;
    }
    return res;
}
```

Using Bitwise operation :- (Method 2)

$$N * 2^i = N \ll i$$

$$a * 2^i = a \ll i$$

$$1 * 2^N \Rightarrow a = 1$$

$$i = N$$

$$\boxed{\text{Power}(N) = 1 \ll N}$$

int powerof2(int n)

}

return (1 << n)

}

eg: int powerof2(int 3)

↑

return (1 << 3)

Y

0 0 0

1 0 0 0

$2^3 \ 2^2 \ 2^1 \ 2^0$

0 0 0 1

$1 \ll 3 = 1 0 0 0$ = 8

range of integer : $[-2^{31}, 2^{31} - 1]$
 if we wanted to perform i will be 30
 then max value of i will be in the range of int
 so that 2^i will be in the range of int.

(1u << N)
 ↳ typecasting the int to unsigned
 will ~~not~~ increase the range of i from
 30 to 31 as unsigned range is
 $= [2^{32} - 1]$
 max of 31 can be allowed.

(1u1l << N)

$$2^{63} - 1 = 62$$

(1ull << N)

$$2^{64} - 1 = 63$$

Problem :-

N i

5 2

10 3

8 2

7 1

check the i^{th} bit &

return true if set

010 → true

1010 → true

1000 → false

0111 → true

bool checkbit (int N, int i)

↑

return ((N >> i) & 1) != 0

(or)

return ((N >> i) % 2) != 0

3

shift the
 i^{th} bit of
 the number
 to the end
 and check
 the number is
 odd or even

$\overset{2}{\textcircled{0}} \textcircled{0} 1$ $(5, 2)$
 we should move 1 to end
 so right by 2 times (i times)

0 0 0 1

$$\begin{array}{r}
 0 0 0 1 \\
 \times 0 0 0 1 \\
 \hline
 \textcircled{1} \neq 0 \quad \text{hence set}
 \end{array}$$

$\overset{N}{\textcircled{0}} \textcircled{0} 0 0 0$ $(8, 2)$
 $\overset{i}{\curvearrowright}$

$$(8, 2) \rightarrow 1 0 0 0$$

$$\begin{array}{r}
 0 0 1 0 \\
 8 \quad 0 0 0 1 \\
 \hline
 0 0 0 0 = 0 \text{ (not set)}
 \end{array}$$

Alternate approach:-

$(5, 2) = 0 \textcircled{1} 0 1$ instead of moving
 $\textcircled{0} 0 0$ it bit to end
 move the 1 from

$N \rightarrow 0 1 0 1 \rightarrow N$ the end 1^{st} position

$$\begin{array}{r}
 1 \ll \overset{i}{\textcircled{1}} \rightarrow 0 1 0 0 \\
 (N \& (1 \ll i)) \rightarrow \frac{0 1 0 0}{(1 \ll i)}
 \end{array}$$

$\neq 0$ hence set

return $N \& (1 \ll i) != 0$

If we observe $1 \ll i == (N \& (1 \ll i))$

so we can also check as

return $(N \& (1 \ll i)) == (1 \ll i)$

$$1 \leq N \leq 10^9, 0 \leq i, \leq 30$$

problem:- checking how many bits are set in a given number.

ilp olp

N
5 \rightarrow 0101 \rightarrow 2 bits are set

16 \rightarrow 10000 \rightarrow 1 bit is set

12 \rightarrow 01100 \rightarrow 2 bits are set.

Basic approach:-

```
int count = 0;  
for (int i = 0; i <= 30; i++)
```

{
 if (Checkbit(N, i))

 {
 count += 1;
 }

}

return count;

here the check happens for all 30 bits.

which is not needed so to optimize it

Approach 2

```
while (N != 0)
```

{
 if ((N & 1) != 0)

 {
 count += 1;

}

N >> 1;

y

0...0, 000101 \rightarrow 1st
↓
31st

010 2ndite

001 \rightarrow 3rd.

000 \rightarrow stop

=

approach : 3

<u>N</u>	<u>(N-1)</u>	<u>$N \& (N-1)$</u>
5 : 0101	4 : 0100	0100 (4)
12 : 1100	11 : 1011	1000 (8)
16 : 10000	15 : 1111	00000 (0)
7 : 0111	6 : 0110	0110 (6)

int count = 0;

while ($N \neq 0$)

{

$N = N \& (N-1)$;

}

return count;

problem :-

Generate a Number with only i^{th} bit & j^{th}

bit :-

<u>i</u>	<u>j</u>	
5	2	5th bit 2nd bit → 000100 → 36
3	1	→ 001010 → 10
4	3	→ 011000 → 24
3	3	→ 001000 → 8.

$$N = 2^i + 2^j \quad (\text{if } i \neq j)$$

but if $i=j$ then it gives wrong o/p

so 1 operation needs to be performed.

$$\begin{array}{r}
 2^5 | 2^2 \\
 \hline
 100000 \\
 000100 \\
 \hline
 100100 \rightarrow 36
 \end{array}$$

$$2^3 \mid 2^3 = \begin{array}{r} 001000 \\ 001000 \\ \hline 001000 \end{array} \rightarrow 8$$

int
func numberwithij (int i, int j)

{

return $(1 \ll i) \mid (1 \ll i) \rightarrow 0 \leq i, i \leq 30$

if $0 \leq i, i \leq 60$

y

if $(0 \leq i, i \leq 60)$

then $(1 \ll i) \mid (1 \ll i)$

problem:-

Generate a number which will have the
x 1's followed by y 0's.

i/p	x	y
3	2	
2	2	
4	3	

\leftarrow
 $\begin{array}{r} 011100 \\ 11100 \\ \hline 1100 \end{array} \rightarrow 28$

\leftarrow
 $\begin{array}{r} 1100 \\ 3210 \\ \hline 10 \end{array} \rightarrow 12$

\leftarrow
 $\begin{array}{r} 0111000 \\ 6543210 \\ \hline 0 \end{array} \rightarrow 120$

$$3+2-1 = 4$$

travel from $i=y$ to $i=x+y-1$.

$x=3, y=2 \Rightarrow 11100 \rightarrow$ starts from 2nd place

and goes till 4th place ($3+2-1$)

for(int i=y ; $i <= x+y-1$; i++)

{
 $\text{res} += \boxed{1 \ll i}; (\text{res} += 2^i)$.

y

alternative approach

x	y
3	2

$$\begin{array}{r} 76843210 \\ \rightarrow 00011100 \end{array}$$

$$2^x = 8 : 2^3 \rightarrow 1000$$

$$2^y - 1 = 7 : (8-1) \rightarrow 0111$$

00000011 → shift by y bits
00011100

$$(2^x - 1) \ll y \rightarrow ((1 \ll x) - 1) \ll y$$

$$a \ll i = a * 2^i$$

$$(2^x - 1)(2^y)$$

$$= 2^{x+y} - 2^y$$

$$= 2^x \cdot 2^y - 2^y$$

$$= \boxed{(1 \ll x)(1 \ll y) - (1 \ll y)}$$

$$= \boxed{(1 \ll (x+y)) - (1 \ll y)}$$