"A premissa é sempre verdadeira"

Hipótese { }, conclusão ⊨

A) $\{(p \to q)\} \models (p \to q) \vee \neg r$

$(p \to q)$ Hipótese — Adição

$(p \to q) \vee \neg R$ Conclusão

B) $\{\neg p \wedge (q \to r)\} \models \neg p$

Hipótese — conclusão

⊨ ~~provavel~~ Regra simplificação

C) $\{(p \to q), (q \to \neg r)\} \models (p \to \neg r)$

Silogismo hipotético

d) $\{p \to (q \to r), p\} \; q \to r$ ordem das hipóteses

$p \to B$

$B(q \to r)$ Modus Ponens:

e) $\{(q \vee r) \to \neg p, \neg(\neg p)\} \models \neg(q \vee r)$

$(q \vee r) \to \neg p$ premissa

$\neg(\neg p)$ premissa Modus

$\neg(q \vee r)$ conclusão Tollens

F) $\{(p \to q), (r \to \neg s)\} \models (p \to q) \wedge (r \to \neg s)$

$(p \to q)$ Hipó A

$(r \to \neg s)$ Hipó B

$A \wedge B$

Conjunção $\dfrac{A \quad B}{A \wedge B}$

g) $\{(p \wedge q) \vee (\neg p \wedge r),\ \neg(\neg p \wedge r) \vdash (p \wedge q)\}$

$(p \wedge q) \vee (\neg p \wedge r)$ Hip A

$\neg(\neg p \wedge r)$ Hip B

$(p \wedge q)$ Silogismo Disjuntivo

2) Justifique uma possível conclusão

a) $\{(s \vee t) \rightarrow (r \wedge q),\ (r \wedge q) \rightarrow \neg p\}$

Silogismo Hipotético $= (s \vee t) \rightarrow \neg p$

b) $\{(p \leftrightarrow q) \rightarrow \neg(r \wedge s),\ \neg\neg(r \wedge s)\}$

Modus Tollens $\neg(p \leftrightarrow q)$

c) $\{s \vee (r \wedge t),\ \sim s\}$

Silogismo disjuntivo: $(r \wedge t)$

d) $\{p \rightarrow (r \vee \sim s),\ (r \vee \sim s) \rightarrow t\}$

Silogismo Hipotético $p \rightarrow t$

e) $\{p \rightarrow r,\ \sim q \rightarrow \sim s,\ p \vee \sim q\}$

Dilema construtivo $r \vee \sim s$

f) $\{\sim p \vee \sim q,\ \sim(\sim q)\}$

Silogismo disjuntivo $= \sim p$

g) $\{P \to (\sim r \wedge q),\ \sim(\sim r \wedge q) \vee \sim s,\ \sim q \to s\}$

Dilema Destrutivo

$\sim P \vee \sim(\sim q)$

3) contrua as deduções

a) $\{(P \wedge q) \to s,\ p,\ q\} \vdash s$

1 $(P \wedge q) \to s$
2 P
3 q
4 conjunção 2, 3 = $P \wedge q$
5 - modus ponens : 1, 4 = $(P \wedge q) \to s$ $(P \wedge q)$
= s

b) $\{P \to q,\ \sim P \to r,\ \sim q\} \vdash r$

L3 + L1

L1 $P \to q$] Modus Tollens
$\sim P \to R$
L3 $\sim q$
L4 $\sim P$

L4 + L2 = $\sim P \to c$, $\sim P$ ⊢ Modus Ponens

c) $\{P \to q,\ q \to \sim\sim r,\ s \to \sim r,\ P\} \vdash \sim s$

L1 $P \to q$ ⊢ L1, L2 silogismo Hipotético
L2 $q \to r(\sim R)$ ⊢
L3 $s \to \sim R$ ⊢ L3, L6 $\sim s$
L4 P ⊢ L4, L5 Modus Ponens
L5 = $P \to \sim C$ ⊢
L6 = $\sim C$, Modus tollens

d) $\{P \wedge q,\ p \rightarrow r,\ q \rightarrow s\} \vdash r \wedge s$

1 $P \wedge q$ → 2 ∧ Simplificação
2 $p \rightarrow r$ → 2, 4 Modus ponens = 6
3 $q \rightarrow s$
4 p
5 q → 3, 5 Modus Ponens = 7
6 r
7 s → 6, 7 Conjunção = 8
8 $r \wedge s$

e) $\{P \rightarrow (\sim q \wedge r),\ P,\ s \rightarrow q,\ s \vee t\} \vdash t$

Modus Ponens (1, 2)

1 $P \rightarrow (\sim q \wedge r)$
2 P
3 $s \rightarrow q$
4 $s \vee t$
5 $\sim q \wedge r$ → Simplificação (5)
6 $\sim q$ → Modus tollens (3, 6)
7 $\sim s$ → Silogismo Disjuntivo (7, 4)
8 t

F) $\{(P \vee q) \rightarrow (P \rightarrow (s \wedge t)),\ P \wedge r\} \vdash t \vee u$

1 $P \vee q \rightarrow (P \rightarrow (s \wedge t))$
2 $P \wedge r$ → Simplificação (2)
3 P → Modus Ponens (1, 3)
4 $(s \wedge t)$ → Simplificação (4)
5 t → Adição (5)
6 $t \vee u$

Lista 8

1. Construa as deduções

a) $\{(P \to Q), (P \wedge R)\} \vdash Q$

1 $P \to Q$
2 $P \wedge R$
3 P Simp 2
4 Q MP 1,3

b) $\{(P \wedge Q), ((P \vee R) \to S)\} \vdash (P \wedge S)$

1 $P \wedge Q$
2 $(P \vee R) \to S$
3 P Simp 1
4 $P \vee R$ ADD 3
5 S MP 2,3
6 $P \wedge S$ Conj 3,5

c) $\{(P \to (Q \to R)), (P \to Q), P\} \vdash R$

1 $P \to (Q \to R)$
2 $P \to Q$
3 P
4 Q MP 3,2
5 $Q \to R$ MP 1,3
6 R MP 4,5

d) $\{((P \vee Q) \to R), ((\neg R \vee Q) \to (P \leftrightarrow S)), (P \wedge S)\} \models (S \leftrightarrow E)$

1) $((P \wedge Q) \to R)$
2) $(\neg R \vee Q) \to \neg P$
3) $P \wedge S$
4) P simp 3
5) $P \vee Q$ ADD 4
6) R MP 1, 5
7) $R \vee Q$ ADD 6
8) $P \to (R \leftrightarrow E)$ MP 2, 7
9) $S \leftrightarrow E$ MP 4, 8

e) $\{(P \to \sim Q), (\sim P \to (R \to \sim Q)), ((R \vee \sim R) \to \sim\sim Q), \sim S\} \models \sim R$

1) $P \to \sim Q$
2) $\sim P \to (R \to \sim Q)$
3) $(\sim R \vee \sim R) \to \sim\sim Q$
4) $\sim S$
5) $\sim S \vee \sim R$ ADD 4
6) $\sim\sim Q$ MP 3, 5
7) $\sim P$ MT 1, 6
8) $R \to \sim Q$ MP 2, 7
9) $\sim R$ MT 6, 8

f) {((P ∧ Q) → r), (r → s), (t → ~u), t, (~s ∨ u)} ⊨ ~(P ∧ Q)

1	(P ∧ Q) → r	7	~s SD 5,8
2	(r → s)	8	~r MT 2,7
3	t → ~u	9	~(P ∧ Q) MT 1,8
4	t		
5	~s ∨ u		
6	~u MP 3,4		

g) {(P → Q), (Q → r), (s → t), (P ∨ s)} ⊨ (r ∨ t)

1	P → Q	6	r ∨ t DC 3,4,5
2	Q → r		
3	s → t		
4	P ∨ s		
5	P → r SH 1,2		

h) {(P → Q), (~r → (s → t)), (r ∨ (P ∨ s)), ~r} ⊨ (Q ∨ t)

1	P → Q	7	Q ∨ t DC 1,5,6
2	~r → (s → t)		
3	r ∨ (P ∨ s)		
4	~r		
5	s → t MP 2,4		
6	P ∨ s SD 3,4		

i) {(P → R), (Q → R), ~R, (P ∨ Q) ∧ (R ∨ S)} ⊨ S

1 P → R
2 Q → S
3 ~R
4 (P ∨ Q) ∧ (R ∨ S)
5 P ∨ Q Simp 4
6 ~P MT 1, 3
7 Q SD 5, 6
8 S MP 2, 7

j) {(P → Q), (Q → R), (R → S), ~S, (P ∨ t)} ⊨ t

1 P → Q
2 Q → R
3 R → S
4 ~S
5 P ∨ t
6 ~R MT 3, 4
7 ~Q MT 2, 6
8 ~P MT 1, 7
9 t SD 5, 8

K) $\{(P \to Q) \wedge (R \to S), (T \to U), (U \to V)$
$\sim Q \vee \sim V\} \vdash F \sim P \vee \sim T$

1 $(P \to Q) \wedge (R \to S)$
2 $T \to U$
3 $U \to V$
4 $\sim Q \vee \sim V$
5 $P \to Q$ simp 1
6 $T \to V$ 5, 4 2, 3
7 $\sim P \vee \sim T$ D.D 4, 5 6

L) $\{(P \wedge Q), (P \to R)\} \vdash (P \wedge R)$
1 $P \wedge Q$
2 $P \to R$
3 P simp 1
4 R MP 2, 3
5 $P \wedge R$ conj 3, 4

M) $\{(\sim P \wedge Q), (R \to P)\} \vdash (\sim P \wedge \sim R)$

1 $\sim P \wedge Q$
2 $R \to P$
3 $\sim P$ simp 1
4 $\sim R$ MT 2, 3
5 $\sim P \wedge \sim R$ conj 3, 4

n) $\{(\sim P \Rightarrow Q), \sim(R \wedge S), (P \Rightarrow (R \wedge S))\} \vdash \sim P \wedge Q$

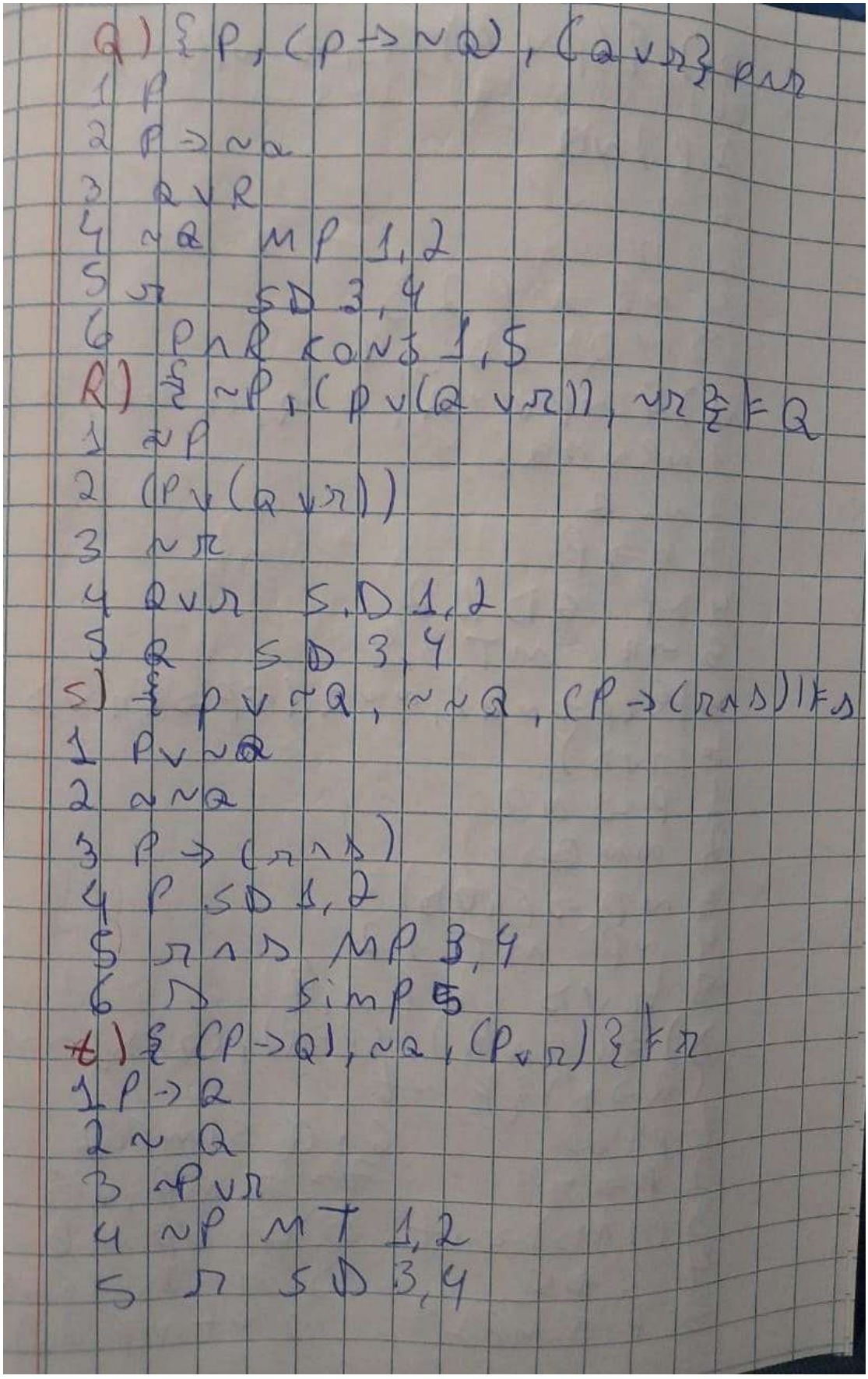
1 $\sim P \Rightarrow Q$
2 $\sim (R \wedge S)$
3 $P \Rightarrow (R \wedge S)$
4 $\sim P$ MT 2, 3
5 Q MP 1, 4
6 $\sim P \wedge Q$ CONJ 4, 5

o) $\{(P \vee Q), \sim R, (Q \Rightarrow R)\} \vdash P$

1 $P \vee Q$
2 $\sim R$
3 $Q \Rightarrow R$
4 $\sim Q$ MT 2, 3
5 P S.D 1, 4

p) $\{(P \wedge Q), (R \vee S), (P \Rightarrow \sim S)\}$

1. $P \wedge Q$
2 $R \vee S$
3 $P \Rightarrow \sim S$
4 P SIMP 1
5 $\sim S$ MP 3, 4
6 R SD 2, 5


Q) {P, (P → ~Q), (Q ∨ R)} ⊢ P ∨ R
1 P
2 P → ~Q
3 Q ∨ R
4 ~Q MP 1,2
5 R SD 3,4
6 P ∧ R Conj 1,5
R) {~P, (P ∨ (Q ∨ R)), ~R} ⊢ Q
1 ~P
2 (P ∨ (Q ∨ R))
3 ~R
4 Q ∨ R S.D 1,2
5 Q S D 3,4
S) {P ∨ ~Q, ~~Q, (P → (R ∧ S))} ⊢ S
1 P ∨ ~Q
2 Q ∨ ~Q
3 P → (R ∧ S)
4 P S D 1,2
5 R ∧ S MP 3,4
6 S simp 5
T) {(P → Q), ~Q, (P ∨ R)} ⊢ R
1 P → Q
2 ~ Q
3 ~P ∨ R
4 ~P M T 1,2
5 R S D 3,4

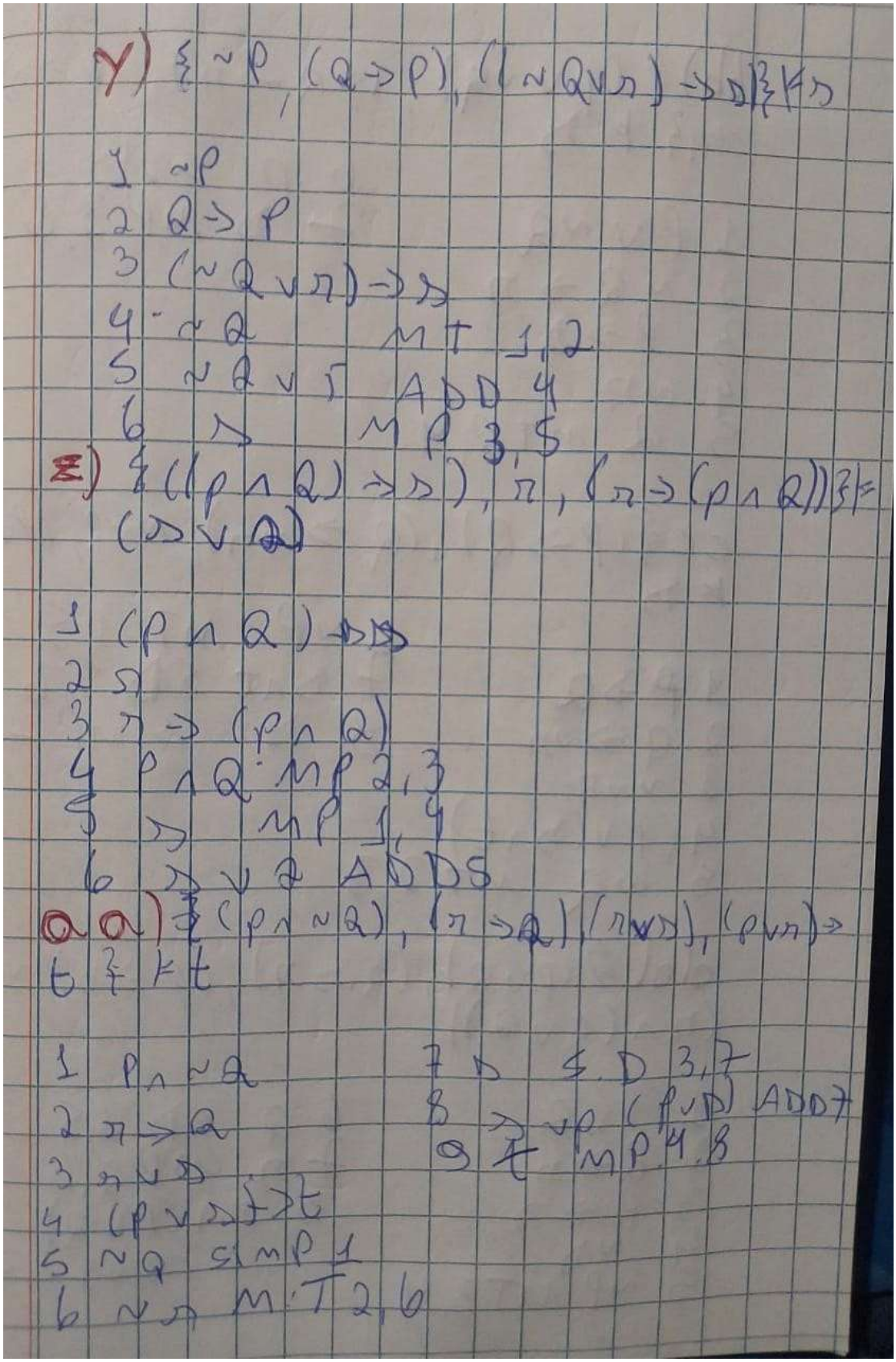
e) $\{(p \vee \sim q), (r \to \sim p, r\}$

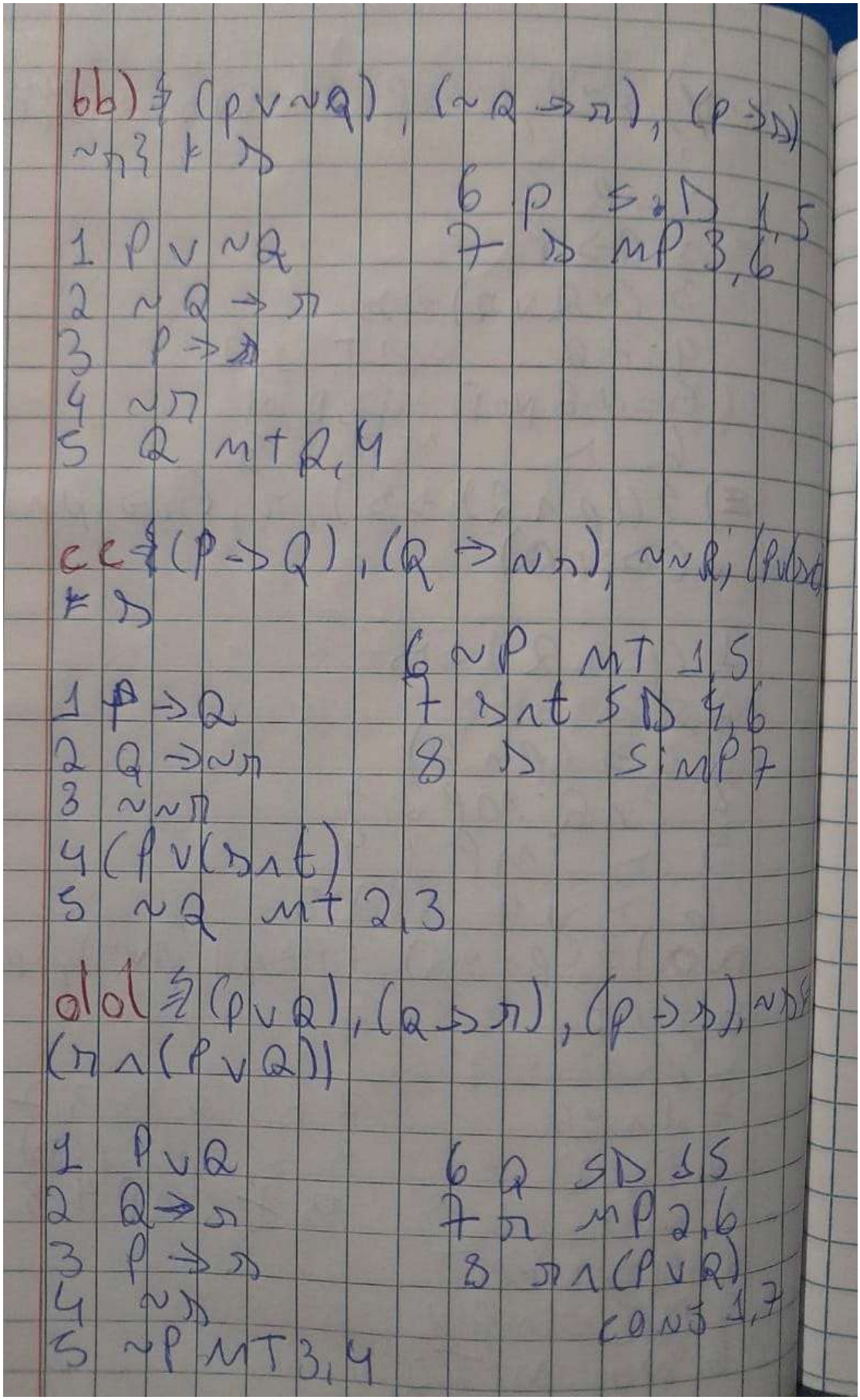
1 $p \vee \sim q$
2 $r \to \sim p$
3 r
4 $\sim p$ MP 2,3
5 $\sim q$ SD 1,4

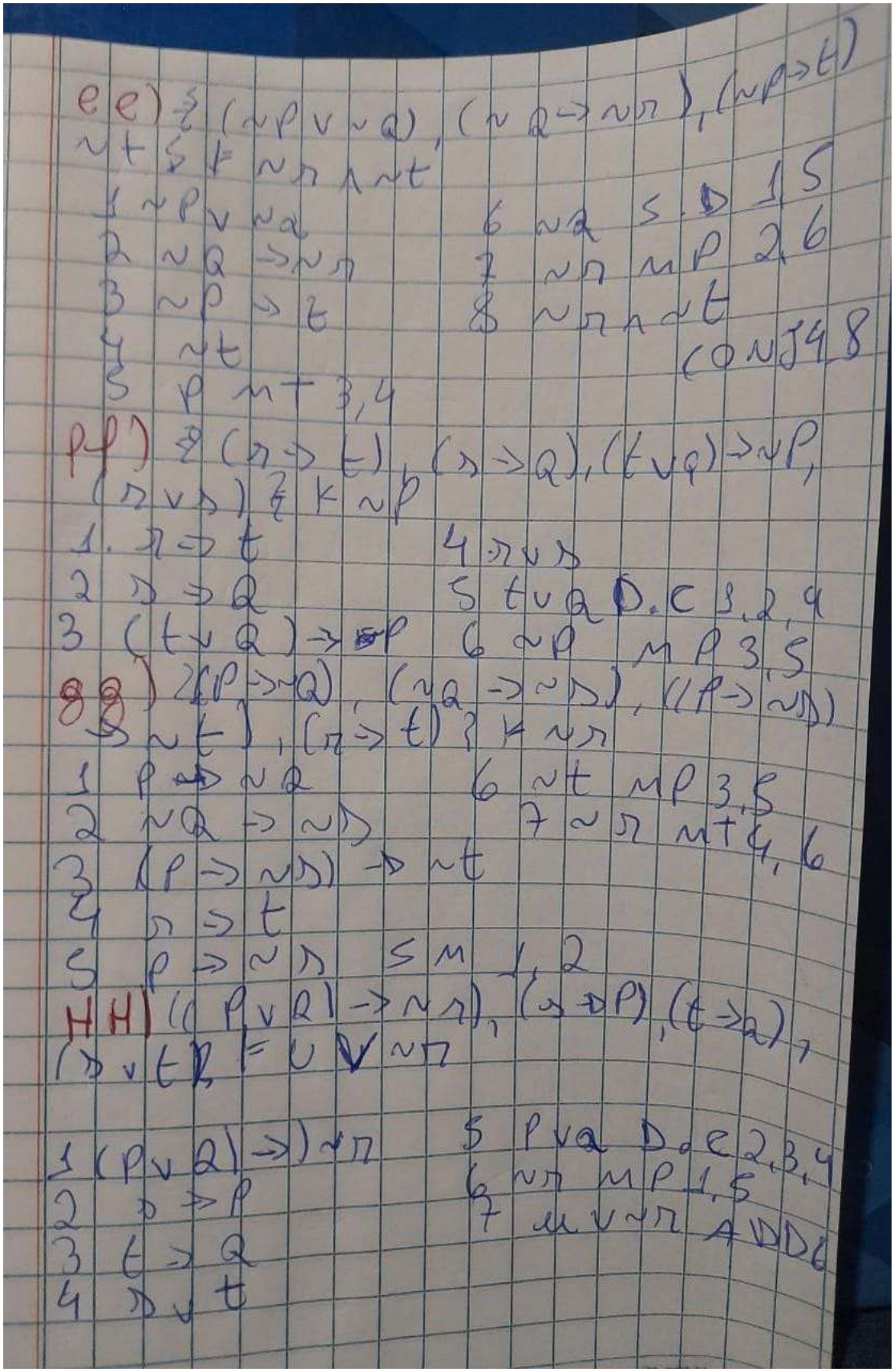
v) $\{\sim p \vee \sim q, \sim\sim q, (r \to p)\} \vdash \sim r$
1 $\sim p \vee \sim q$
2 $\sim\sim q$
3 $r \to p$
4 $\sim p$ SD 1,2
5 $\sim r$ MT 3,4

w) $\{(p \to \sim q), \sim\sim q, (\sim p \to (r \vee s))\} \vdash (r \vee s)$
1 $p \to \sim q$
2 $q \sim q$
3 $\sim p \to (r \vee s)$
4 $\sim p$ MT 1,2
5 $r \vee s$ MP 3,4

x) $\{(p \wedge q), (p \to r), (s \wedge s) \to \sim t, (q \to s)\} \vdash \sim t$
1 $p \wedge q$
2 $p \to r$
3 $(r \wedge s) \to \sim t$
4 $q \to s$
5 p SimP 1
6 q SimP 1
7 r MP 2,5
8 s MP 4,6
9 $r \wedge s$ Conj 7,8
10 $\sim t$ MP 3,9




bb) ⊨ (p ∨ ¬q), (¬q → r), (p → s), ¬r ⊢ s
1 p ∨ ¬q
2 ¬q → r
3 p → s
4 ¬r
5 q MT 2, 4
6 p ∨ s 1, 5
7 s MP 3, 6
cc) ⊨ (p → q), (q → ¬r), ¬¬s, (p ∨ s) ⊢ s
1 p → q
2 q → ¬r
3 ¬¬r
4 (p ∨ (s ∧ t))
5 ¬q MT 2, 3
6 ¬p MT 1, 5
7 s ∧ t 5 DS 4, 6
8 s ∧ E MP 7
dd) ⊨ (p ∨ q), (q → r), (p → s), ¬s ⊢ (r ∧ (p ∨ q))
1 p ∨ q
2 q → r
3 p → s
4 ¬s
5 ¬p MT 3, 4
6 q DS 1, 5
7 r MP 2, 6
8 r ∧ (p ∨ q) CONJ 1, 7


ee) {(¬P ∨ ¬Q), (¬Q → ¬R), (¬P → t)}
¬t ⊢ ¬R ∧ ¬t
1 ¬P ∨ ¬Q
2 ¬Q → ¬R
3 ¬P → t
4 ¬t
5 P MT 3,4
6 ¬Q S.D 1,5
7 ¬R MP 2,6
8 ¬R ∧ ¬t
CONJ 4,8
ff) {(R → t), (S → Q), (t ∨ Q) → ¬P,
(R ∨ S)} ⊢ ¬P
1. R → t
2 S → Q
3 (t ∨ Q) → ¬P
4 R ∨ S
5 t ∨ Q D.C 3,2,4
6 ¬P MP 3,5
gg) {(P → Q), (¬Q → ¬S), ((P → ¬S)
¬t), (R → t)} ⊢ ¬R
1 P → ¬Q
2 ¬Q → ¬S
3 (P → ¬S) → ¬t
4 S → t
5 P → ¬S SM 1,2
6 ¬t MP 3,5
7 ¬R MT 4,6
HH) {(P ∨ Q) → ¬R), (S → P), (t → Q),
(S ∨ t)} ⊢ U ∨ ¬R
1 (P ∨ Q) → ¬R
2 S → P
3 t → Q
4 S ∨ t
5 P ∨ Q D.C 2,3,4
6 ¬R MP 1,5
7 U ∨ ¬R ADD 6