Exercise Paper 2

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1 Power series

Exercise .1. Given the following series, find:

- the center of the serie x_0;
- the convergence radius R;
- the convergence set E.

$$\sum_{n=1}^{\infty} log(\frac{1}{n^2+1}+1), \quad \sum_{n=1}^{\infty} \frac{9^n-8^n}{(3n^2+1)5^n}(4x+1)^n, \quad \sum_{n=1}^{\infty}(-1)^n\frac{9^n-5^n}{(4n+7)8^n}(x-\frac{1}{2})$$
$$\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}-1}{3log(n+1)}(x-5)^n, \quad \sum_{n=1}^{\infty} \frac{n3^n+1}{2^{2n+1}-9}(7x-e)^n, \quad \sum_{n=1}^{\infty} \frac{n!(2^n+5)}{(n+3)!}(x+2)^n.$$
$$\sum_{n=1}^{\infty} \frac{(3n)^n-2n!}{2^{n+1}n^{n-1}+5}(x+\frac{1}{2})^n, \quad \sum_{n=1}^{\infty} \frac{17^n}{14^n(n+4)}(x+6)^n, \quad \sum_{n=1}^{\infty} \frac{7^n}{17^n(n+8)}(x-12)^n$$

2 Numerical series

Exercise .2. Given the following numerical series, analyse their convergence:

$$\sum_{n=1}^{\infty} \sqrt[7]{\frac{n^6+n^3}{n^15+1}}, \quad \sum_{n=1}^{\infty} \sqrt[5]{\frac{n^4+n^3}{n^9+1}}, \quad \sum_{n=1}^{\infty} n^3(cos(\frac{1}{n})-1)^2$$
$$\sum_{n=1}^{\infty} \frac{2^n}{3^n}(n^2+sin(e^n)), \quad \sum_{n=1}^{\infty} sin(\frac{n^2+5}{n^4+3n^3-6n+2}), \quad \sum_{n=1}^{\infty} \sqrt{\frac{n^2+7n+13}{(n+4)^6}}$$

3 ODE first order

Exercise .3. Given the following ODEs (or Cauchy's problems), find the solution: (Note that if you have two different starting values you have to solve two different problems)

$$y' = y^2-2y-3,\ y(0)=0; \quad xy' = -y+4x^2,\ y(0)=0,\ y(0)=3; \quad y' = y+x;$$
$$y' = xln(x^2+1)(1+y^2),\ y(0)=2; \quad y' = \frac{x^3+y^3}{xy^2}; \quad y' = \frac{1}{1+x^2},\ y(0)=4.$$

4 ODE second order

Exercise .4. Given the following ODEs or Cauchy's problems find a set of solutions or the exact one:

$$y'' + 3y = t - 3,\ y(0) = y'(0) = 0; \quad y'' + 2y' + y = sin(2t),\ y(0) = \frac{1}{2}, y'(0) = 1;$$

$$y'' - 2y' - 3y = e^{4t},\ y(0) = y'(0) = 0y'' + 2y' + 2y = sin(t) + cos(t);$$

$$y'' + 4y = t - cos(3t); \quad y'' + 4y = t + cos(t),\ y(0) = 0,\ y(0) = 1;$$

$$y'' - 2y' + y = 6te^{7t},\ y(0) = y'(0) = 0.$$