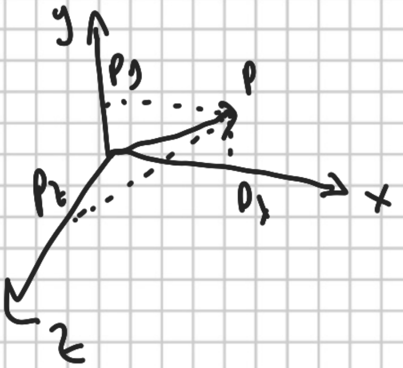
Cartesian plane



- The origin is arbitrary
- The cartesian system → system 3 axes, perpendicular
- Time is independent from position and velocity
- Reference system (x, y, z, t) → specifies a point p
- $\hat{x} \cdot \hat{y} = \hat{x} \cdot \hat{z} = \hat{y} \cdot \hat{z} = 0$
- $\hat{x} \times \hat{z} = \hat{y}$
- $\hat{x} \times \hat{y} = \hat{z}$
- $\hat{y} \times \hat{z} = \hat{x}$

If the direction is c.clockwise, the direction is upwards and vice versa

$\vec{p} = (p_x, p_y, p_z)$

$|\vec{p}| = \sqrt{p_x^2 + p_y^2 + p_z^2}$

$a \cdot \vec{p} = (a p_x, a p_y, a p_z)$

$\vec{r} + \vec{p} = (r_x + p_x, r_y + p_y, r_z + p_z)$

$\vec{r} \cdot \vec{p} = \cos\alpha |\vec{r}||\vec{p}|$ → if they're perpendicular, it's 0

$\vec{r} \cdot \vec{p} = \sin\alpha |\vec{r}||\vec{p}|$ → if they're parallel, it's 0

Unitarian vector: $\hat{p} = \frac{\vec{p}}{|\vec{p}|}$

Euclidean Notation:

$\vec{r} \cdot \vec{p} = r_i \cdot p_i = \sum r_i p_i$

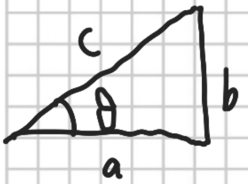
$\vec{r} \cdot \vec{p} = (r_x p_x \hat{x}\hat{x} + r_y p_y \hat{y}\hat{y} + r_z p_z \hat{z}\hat{z})$

$\vec{r} \cdot \vec{p} = r_x p_x + r_y p_y + r_z p_z$

Vectorial product:

$\vec{p} \times \vec{r} = (p_x \hat{x} + p_y \hat{y} + p_z \hat{z}) \times (r_x \hat{x}, r_y \hat{y}, r_z \hat{z})$

Trigonometry



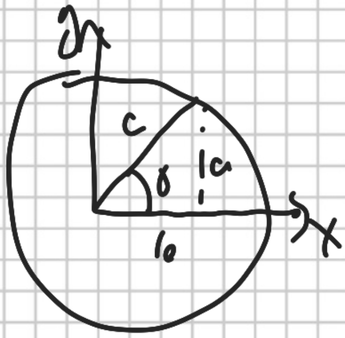
$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$a^2 + b^2 = c^2$

$\sin^2\theta + \cos^2\theta = 1$

The difference between sin and cos is $\frac{\pi}{2}$



$b = \cos\theta\, c$

$a = \sin\theta\, c$

$p = (b, a)$

(c, θ)

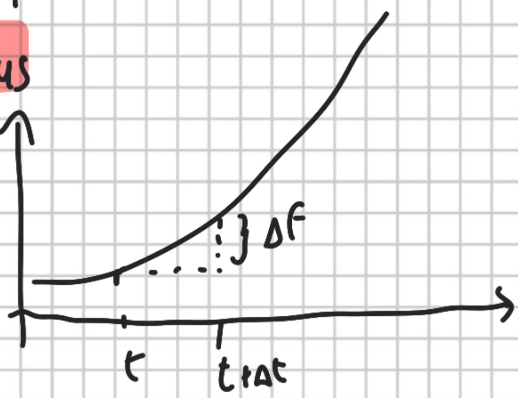
$c = \sqrt{x^2 + y^2}$

$\theta = \operatorname{atan}\frac{y}{x}$

Elements of differential calculus

Δf Function variation

$\Delta f = f(t + \Delta t) - f(t)$



Limit:

$$\lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t} = \frac{d f(t)}{dt}$$

Example:

$f(t) = t^2$

$f(t + \Delta t) = (t + \Delta t)^2 = t^2 + 2\Delta t t + \Delta t^2$

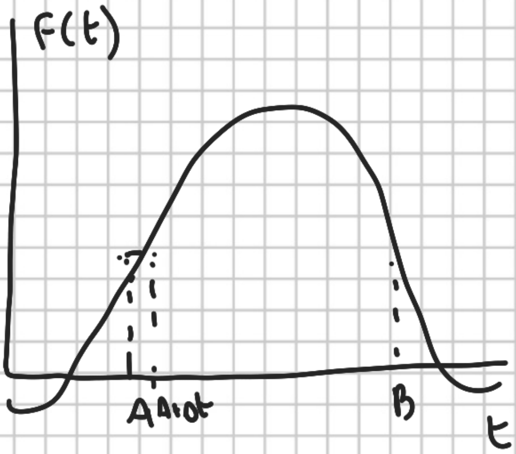
$f(t + \Delta t) - f(t) = \Delta t^2 + 2\Delta t t$

$$g(t) = \frac{f(t + \Delta t) - f(t)}{\Delta t} = \frac{\Delta t^2 + 2\Delta t t}{\Delta t} = \Delta t + 2t$$

$$\lim_{\Delta t \to 0} g(t) = 2t$$

Elements of integral calculus

An integral can be thought as a sequential sum.



The integral is equivalent to the area under the graph.

It is the sum of small areas:

$$\Delta A: f(a)\cdot \Delta t \to A = \sum_{i=0}^{n} f(t)\,\Delta t \to \text{area sum}$$

$\downarrow$

correct result only when $\Delta t \to 0$ and $n \to \infty$

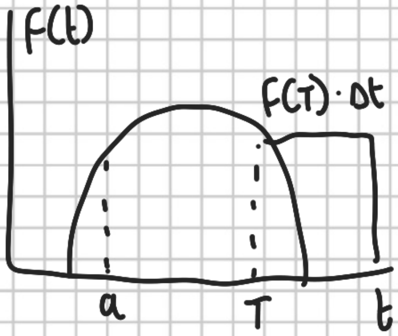
$$A = \int_a^b f(t)\,dt = \lim_{\Delta t \to 0} \sum_i f(t)\,\Delta t$$

$$F(T) = \int_a^T f(t)\,dt$$

The fundamental theorem of calculus:

$$F(t) = \int f(t)\,dt; \qquad f(t) = \frac{dF(t)}{dt}$$

Demonstration:



$$F(\Delta T + t) = \int_a^{T+\Delta t} f(t)\,dt$$

$$F(T+\Delta t) - F(T) = f(T)\cdot \Delta t$$

$$\frac{F(T+\Delta t) - F(T)}{\Delta t} = f(t) \to \frac{dF}{dt} = \lim_{\Delta t \to 0} \frac{F(T+\Delta t) - F(T)}{\Delta t} = f(T)$$

Example of integral:

$$f(t) = t^n$$

$$F(t) = \int f(t)\,dt \; ; \quad f(t) = \frac{dF(t)}{dt} = t^n$$

$$F(t) = \frac{t^{n+1}}{n+1} + C$$