1\. 1. $\binom{9}{3\,3\,3}$

2\. $9!$

3\. $\binom{9}{3\,3\,3}\cdot\frac{1}{3!} = \frac{9!}{3!\,3!\,3!}\cdot\frac{1}{3!} = \frac{7!}{3}\cdot\frac{1}{3!}$

4\. $\binom{9}{2\,2\,5}\cdot\frac{1}{2!} = \frac{9!}{2!\,2!\,5!}\cdot\frac{1}{2!}$

5\. 1

5\. 1. Each and every element of S can go to any element of S':

$$\underbrace{k \cdot k \cdot k \dots}_{n \text{ times}} = k^n$$

2\. If $n = k$, 1, else if $k > n$:

$$\# = \sum_{i=0}^{k-(n-1)} \prod_{j=0}^{n-1} (k - i)$$

$k = 3$

$n = 2$

$\{1, 2, 3, 4\}$

5\. 1. k^n

2\. $\binom{k}{n}$

4\. $\frac{k!}{(k-n)!}$

3\. $\prod_{i=1}^{n} \frac{k!}{(k-i)!}$

6\. 1. No

2\. $p(A) =$

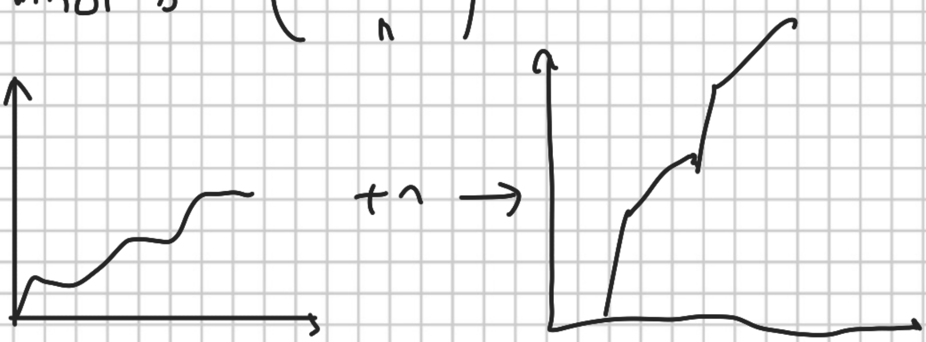
3\. $\frac{1}{3}$

4\. Yes

5\. 5. It is possible to encode it as a functions we can count:

\# non-decreasing functions $\{1 \dots n\} \to \{1 \dots k\}$ $\longleftrightarrow$ \# strictly decreasing f. $\{1 \dots n\} \to \{1 \dots n+k-1\}$

which is $\binom{n+k-1}{n}$



POOL:

- 50 D. E. EX.
- 30 G. EX.
- 10 S. EX.

BOB:

- 20 "
- 10 "
- 5 "

$A = \{$BOB Knows 4/4 Ex. taken at random from pool$\}$

1. $p(A) = \dfrac{\text{\# of sets of 4 out of 10 G. exercises}}{\text{\# of sets of 4 G. exercises}} = \dfrac{\binom{10}{4}}{\binom{30}{4}} = \dfrac{\frac{10!}{\cancel{4!}\,6!}}{\frac{30!}{\cancel{4!}\,26!}} = \dfrac{10 \cdot 9 \cdot 8 \cdot 7}{30 \cdot 29 \cdot 28 \cdot 27} =$

$= 0{,}0077$

2. $\binom{50}{5}\binom{40}{4}\binom{10}{1}$

3. $p(\{\text{BOB solves all exercise}\}) = \dfrac{\binom{20}{5}\binom{10}{4}\binom{5}{1}}{\binom{50}{5}\binom{40}{4}\binom{10}{1}}$

4. $p(\{3|2|1\}) = \cancel{\dfrac{\binom{5}{3}\binom{17}{2}\binom{4}{2}\binom{8}{2}\binom{10}{1}}{\binom{50}{5}\binom{40}{4}\binom{10}{1}}} \dfrac{\left(\binom{20}{3} - \binom{20}{4}\right)\left(\binom{10}{2} - \binom{10}{3}\right)\binom{5}{1}}{\binom{50}{5}\binom{40}{4}\binom{10}{1}}$

- 40 CARDS
- $A = \{1, 2, 3$ of a given seed$\}$

$p(A) = \dfrac{\text{\# of ways the specific 3 cards appear in a deck out of 10 out 40}}{\text{\# of ways 10 cards can be taken out of 40}} = \dfrac{\binom{37}{7}}{\binom{40}{10}} = \dfrac{\frac{\cancel{37!}}{\cancel{7!}\,\cancel{30!}}}{\frac{\cancel{40!}\,40 \cdot 39 \cdot 38}{\cancel{10!}\,\cancel{30!}}} = \dfrac{10 \cdot 9 \cdot 8}{40 \cdot 39 \cdot 38} = 0{,}012$

$10 \cdot 9 \cdot 1$

- $B = \{1, 2, 3$ of two different seeds$\}$

$p(B) = \dfrac{\binom{4}{2}\binom{34}{4}}{\binom{40}{10}} = \dfrac{\frac{\cancel{4!}}{2! \cdot 2!} \cdot \frac{\cancel{34!}}{\cancel{30!}\,\cancel{4!}}}{\frac{\cancel{40!}\,40 \cdot 39 \cdot 38 \cdot 37 \cdot 36 \cdot 35}{10!\,\cancel{30!}}} = 0{,}00033$

$p(C) = \binom{4}{1} \cdot p(A) = \frac{4!}{1!\,3!} \cdot p(A) = 0{,}049$ ← not adjusted for over counting

$\binom{4}{1} \cdot p(A) - \binom{4}{2} p(B) + \binom{4}{3} \dfrac{\binom{31}{1}}{\binom{40}{10}} =$ √

$= \sum_{k=1}^{4} (-1)^{k-1} \sum_{1 \leq i_1 \leq i_2 < i_k < i_n} |A_{i_1} \cap A_{i_2} A_{i_3} \dots|$

6. Count the number of non-surjection functions, that is, those that miss at least one value. Note that the sets of these functions are not disjoint:

$$\text{surj. f: } K^n - \underbrace{|A_1 \cup A_2 \ldots \cup A_K|} \rightarrow K^n - \left(\sum_{j=1}^{K} (-1)^{j-1} \sum_{1 \le i_1 \le i_j \le K} |A_{i_1} \cap A_{i_2} \cap A_{i_3} \cap A_{i_4} \ldots A_{i_j}|\right)$$

$$A_j = \{\text{functions to } \{1 \ldots K\} \setminus \{j\}\}$$

$$K^n - \sum_{j=1}^{K} (-1)^{j-1} \binom{K}{j} (K-j)^n$$

A nicer way to write it:

$j' = K - j$

$$= \sum_{j'=0}^{K-1} (-1)^{K-j'} \binom{K}{j'} j'^n$$

And now, the total is:

$$\sum_{j'=0}^{K} (-1)^{K-j'} \binom{K}{j'} j'^n$$