1\. 1. $25 \cdot 24 = 600$ or $\binom{25}{2} \cdot 2$

2\. $A = \{$a person is selected either as president or secretary$\}$

$$p(\bar{A}) = \frac{24 \cdot 23}{25 \cdot 24} \quad \text{or} \quad \frac{\binom{24}{1}}{\binom{25}{2}} = \frac{24}{\frac{25!}{23!\,2!}} = \frac{2}{25}$$

$\leftarrow$ 1 is forced: there are 24 remaining choices

$$p(A) = 1 - p(\bar{A}) = 1 - \frac{\cancel{24} \cdot 23}{25 \cdot \cancel{24}} = \frac{2}{25} = 0{,}08$$

We could even

$$\underset{\uparrow p}{\frac{1}{25}} \cdot \frac{24}{24} + \frac{24}{25} \cdot \underset{\uparrow s}{\frac{1}{24}} = \frac{2}{25}$$

2\. RICE. $4! = 24$

PASTA: $\frac{5!}{2!} = 5 \cdot 4 \cdot 3 = 60$

POTATOES: $\frac{8!}{2! \cdot 2!} = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 3 \cdot 2 = 10'080$

3\. It is $\binom{n}{k}$ for a given k.

4\. 1. $\binom{13}{10} = \frac{13!}{10!\,3!} = \frac{13 \cdot \cancel{12} \cdot 11}{\cancel{6}} = 286$

2\. $\binom{11}{8} = \frac{11!}{8!\,3!} = \frac{11 \cdot 10^{5} \cdot 9^{3}}{\cancel{6}} = 165$

3\. $\binom{11}{9} + \binom{11}{9} = 2 \cdot \frac{11!}{9! \cdot 2!} = \cancel{2} \cdot \frac{11 \cdot 10}{\cancel{2}} = 110$

5\. 1. ABCDEFG $\rightarrow 7! = 7!$ possible words

CDEFG $\rightarrow 5!$ other words

$$\frac{6 \cdot 2 \cdot 5!}{7!} = \frac{2}{7}$$

2\. A B C D E F G $\quad \frac{7!}{\cancel{7}!}$

$$\frac{\cancel{7} \cdot 2 \cdot 5!}{7!} = \frac{2}{6} = \frac{1}{3}$$

6\. Total possible arrangements: $\binom{52}{5} = \frac{52!}{5!\,47!}$

1\. Number of pokers: $13 \cdot 48$

So $p(A) = \frac{13 \cdot 48}{\frac{52!}{5!\,47!}} = \frac{13 \cdot 48 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48} = 0{,}00024$

2\. Number of colours: $\binom{13}{5} \cdot 4$

So $p(b) = \frac{\binom{13}{5} \cdot 4}{\frac{52!}{5!\,47!}} = \frac{\frac{13!}{5!\,8!} \cdot 4}{\frac{52!}{5!\,47!}} = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 4}{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48} =$

$= 0{,}00198$

3. # of full: $13 \cdot \binom{4}{3} \cdot 12 \cdot \binom{4}{2}$

So $p(c) = \dfrac{13 \cdot \binom{4}{3} \cdot 12 \cdot \binom{4}{2}}{\frac{52!}{5!\,47!}} = 0{,}00288$

4. # of doubles: $\binom{13}{2} \cdot \binom{4}{2} \cdot \binom{4}{2} \cdot 48$

$$p(d) = \dfrac{13 \cdot 4 \cdot 3 \cdot 12 \cdot 4 \cdot 3 \cdot 48 - \overbrace{(13 \cdot 4 \cdot 12 \cdot 4 \cdot 3)}^{\text{fulls}}}{\frac{52!}{5!\,47!}} = 0{,}412$$

5. # of tris $= 13 \cdot \binom{4}{3} \cdot 49 \cdot 48$

$$p(e) = 13 \cdot 4 \cdot 49 \cdot 48 - (13 \cdot 4 \cdot 12 \cdot 4 \cdot 3 + 13 \cdot 48) = 0{,}0439$$

7. 1. $\binom{200}{100}! \cdot \binom{100}{50} \cdot \binom{50}{50} = \dfrac{200!}{100! \cdot 100!} \cdot \dfrac{100!}{50! \cdot 50!} \cdot 1$

Double the number.

2. $\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{3}{4} + \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{3}{4} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} =$

$= \frac{3}{64} + \frac{3}{64} + \frac{\cancel{1}8}{\cancel{8}64} = \frac{14}{64} = \frac{7}{32}$

A	B	W
1	1	2/3
2	2	1/3
3	3	1/2

8. 2. $P_\beta(\{\omega\}) = \frac{1}{Z_\beta}$

3. $\lim\limits_{\beta \to \infty}$

3. With order $1 \cdot \frac{3}{51} \cdot \frac{2}{50} \cdot \frac{48}{49} \cdot \frac{3}{48} \cdot \binom{5}{3}$

↑ Whatever order

$|\Omega| = n$

$H : \Omega \to \mathbb{R}$

$\beta \geq 0$

$$p(\{\omega\}) = \frac{e^{-\beta H(\omega)}}{Z_\beta} \leftarrow \text{normalize: } \sum_{\omega \in \Omega} \mathbb{P}(\{\omega\}) = 1$$

1. Find Z_β s.t. that it normalizes:

$$\sum_{\omega \in \Omega} \frac{e^{-\beta H(\omega)}}{Z_\beta} = 1$$

$$\frac{1}{Z_\beta} \sum_{\omega \in \Omega} \frac{e^{-\beta H(\omega)}}{Z_\beta} = 1 \Rightarrow Z_\beta = \sum_{\omega \in \Omega} \frac{e^{-\beta H(\omega)}}{Z_\beta}$$

2. If $\beta = 0$, $e^{-\beta H(\omega)} = 1$, so it is the same for everyone. And Z_β, consequently, is $|\Omega|$.

3. $E_m = \{\omega \in \Omega : H(\omega) = m\}$

$$m = \min_{\omega \in \Omega} H(\omega)$$

$$\lim_{\beta \to \infty} \mathbb{P}(\omega) = 0 \quad \text{if } \omega \notin E_m$$
$$> \quad \omega \in E_m$$

add and subtract m

$$\lim_{\beta \to \infty} \frac{e^{-\beta(H(\omega) - m)}}{Z_\beta} \cdot e^{-\beta m}$$

$$Z_\beta = \sum_{\omega \in \Omega} e^{-\beta(H(\omega) - m)} \cdot e^{-\beta m}$$

$$Z_\beta = e^{-\beta m} \cdot \left(\underbrace{\sum_{\omega \in E_m} 1}_{|E_m|} + \underbrace{\sum_{\omega \notin E_m} e^{-\beta(\underbrace{H(\omega) - m}_{>0})}}_{\leq |\Omega| \cdot e^{-\beta\delta}} \right)$$

$\delta \to$ the next value H takes along the minimum (H is discrete)

$\to 0$ since $\beta \to \infty$

into the limit

$$\frac{Z_\beta}{e^{-\beta m}} = |E_m| + o(1)$$

so,

in the set of minima, >0; otherwise, 0

$$\lim_{\beta \to \infty} \frac{e^{-\beta(H(\omega)-m)}}{Z_\beta} \cdot e^{-\beta m} = \frac{1}{|E_m|}$$