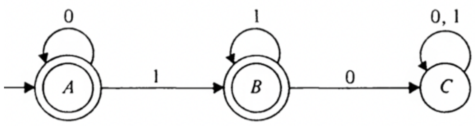
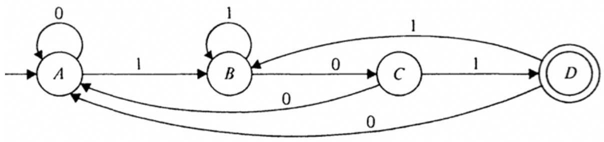
EXERCISES ON THE FIRST 5 CLASSES:
REGULAR LANGUAGES

Exercise 1: Give a DFA accepting the following languages over the alphabet $\{0,1\}$:

a) The set of all strings ending in 00;
b) The set of all strings with three consecutive 0's;
c) The set of all strings such that every block of five consecutive symbols contains at least two 0's;
d) The set of all strings beginning with a 1 which, interpreted as the binary representation of an integer, is congruent to zero modulo 5.

Exercise 2: Describe in words the languages accepted by the following DFAs:





Then, write down two regular expressions equivalent to the automata above.

Exercise 3: Give a NFA accepting the languages of all binary sequences that contain two 0's that are separated by a string whose length is $4i$, for some $i > 0$.

Exercise 4: Construct the DFA's equivalent to the following NFA's:

a) $(\{p,q,r,s\}\ ,\ \{0,1\}\ ,\ \delta_{\mathrm{a}}\ ,\ p\ ,\ \{s\})$

b) $(\{p,q,r,s\}\ ,\ \{0,1\}\ ,\ \delta_{\mathrm{b}}\ ,\ p\ ,\ \{q,s\})$

where δ_{a} and δ_{b} are defined as follows:

	0	1
p	p, q	p
q	r	r
r	s	—
s	s	s

	0	1
p	q, s	q
q	r	q, r
r	s	p
s	—	p

Exercise 5: Given two alphabets Σ and Γ, let us define a *(regular) substitution* to be a function

$$f : \Sigma \rightarrow REG(\Gamma)$$

i.e. a function that associates a regular language on Γ to every character of Σ. Substitutions are extended to strings as follows:

$$f(\varepsilon) = \varepsilon \qquad\qquad f(wa) = f(w)\,f(a)$$

and, consequently, are extended to languages by having $f(L) = \{\,f(w) \mid w \in L\}$.

Prove that regular languages are closed under regular substitutions, i.e., if L is regular and f is a regular substitution, then also $f(L)$ is regular.

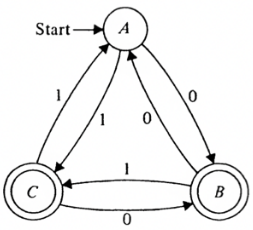
Exercise 6: Write regular expressions for each of the following languages over the alphabet $\{0,1\}$:

1. The set of strings with at most one pair of consecutive 0s and at most one pair of consecutive 1s
2. The set of strings in which every pair of adjacent 0s appears before any pair of adjacent ls
3. The set of all strings not containing 101 as a substring.

Exercise 7: Construct a (deterministic or nondeterministic) automata equivalent to the following regular expressions:

a) 10 + (0 + 11)0*1
b) 01[((10)* + 111)* +0]*1

Exercise 8: Construct a regular expression equivalent to the following DFA:



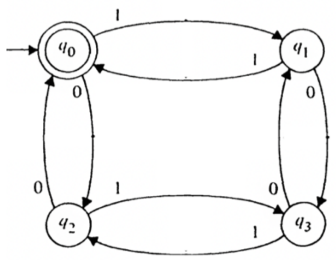
Then, build the NFA associated to the regular expression found, by using the construction used for passing from REs to NFA.

Exercise 9: Construct a left-linear and a right-linear grammar for the languages:

a) (0 + 1)*00(0 + 1)*
b) 0*(1(0 + 1))*
c) (((01 + 10)*11)*00)*

For all right linear grammars devised, write down the associated equivalent NFA.

Exercise 10: Give a regular grammar equivalent to the following DFA:



Exercise 11: Prove that $\{0^n 1^m 0^{n+m} \mid n,m > 0\}$ is not regular.

Exercise 12: Are $\{0^{2n} \mid n > 0\}$ and $\{0^{2^n} \mid n > 0\}$ regular? Justify your answer, by either providing a non-regularity proof or by providing a grammar/automaton/regular expression that generate these languages.