Errors

A measurement is the experimental result of a series of operations. The key element to assess is the reliability.

A true value exists.
The definition of error is

$$\text{error} = |\text{measured} - \text{true value}|$$

Error Origin:

1. Instrumental limitations
2. Accidental causes

They can be classified in:

1. Systematic errors → can be suppressed
2. Statistical → cannot

Example of error measurement:

Depth of a well using the time it takes for a stone to reach the bottom.

We know that $h = \frac{1}{2} g t^2$

$g \to 10 \ m/s^2$

Uncertainties:

1. Clock calibration → systematic error
2. Sync between clock start/stop and stone → accidental
3. Air friction → systematic
4. Finite sound propagation time → systematic
5. Variation in air density with time → accidental

Result of a measurement

This is a distribution of results. Multiple results can build more accurate results. Mean formula:

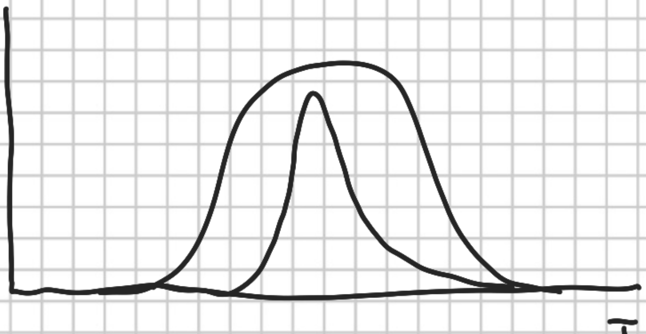
$$\langle T \rangle = \frac{1}{N}\sum_{i=1}^{N} T_i$$

The error for distributions is standard deviation:

$$\varepsilon_T = \sqrt{\frac{1}{N-1}\sum_{i=0}^{n}(T_i - \langle T \rangle)}$$

→ standard deviation estimates the error you make when reporting $\langle T \rangle$ as a result.

Distributions

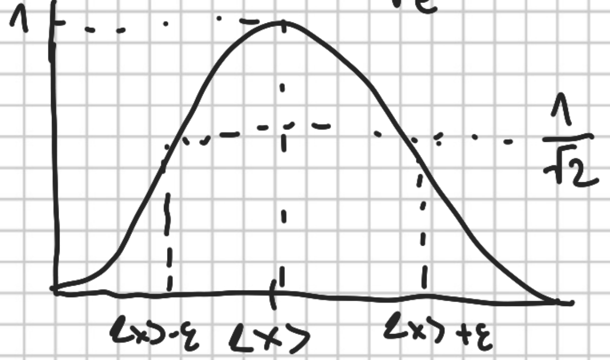


The Gaussian distribution explains many natural phenomena. Its equation is

$$f(x) = A e^{-\frac{(x-\langle x \rangle)^2}{2\varepsilon^2}}$$

When $x - \langle x \rangle = \varepsilon \rightarrow f(x) = \frac{A}{\sqrt{e}}$

We have this graph for $\frac{f(x)}{A}$



The answer of the distribution is equal to the Area:

$$\text{Area } (\langle x \rangle - \varepsilon, \langle x \rangle + \varepsilon)$$

it is always the 65%

The integral of a probability distribution is the probability.

- Random walkers example:
  1000 times → sum 500 times either 1 or -1
  The results will form a Gaussian distribution.

Central limit theorem of statistics → SEARCH

Error propagation

With two measurements: l_1, l_2, and their error ε_1, ε_2

$L_{TOT} = l_1 \pm l_2$

$\varepsilon_{TOT} = \varepsilon_1 + \varepsilon_2$

With product:

For example:

$v = \frac{L}{\Delta t}$; $\varepsilon_v = \left|\frac{\partial v}{\partial L}\right| \varepsilon_L + \left|\frac{\partial v}{\partial t}\right| \varepsilon_{\Delta t}$

$\varepsilon_v = \frac{\varepsilon_L}{\Delta t} + \frac{L}{\Delta t^2} \varepsilon_{\Delta t}$

$\varepsilon_L = \varepsilon_{l_1} + \varepsilon_{l_2}$ → $\frac{\varepsilon_v}{v} = \frac{\varepsilon_L}{L} + \frac{\varepsilon_{\Delta t}}{\Delta t}$

In general, if we have a quantity that's a function of other quantities:

$a = f(x, y, z \ldots)$

$\varepsilon_a^2 = \left(\frac{df}{dx}\right)^2 \varepsilon_x^2 + \left(\frac{df}{dy}\right)^2 \varepsilon_y^2 \ldots$

How to report a result:

9.82 ± 0.51 → the error should have as many digits as the measurement