Exercise Paper 1

Francesco Palandra, Stefano Ramagnano

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1 Riemann's Integrals (rational functions)

Exercise .1. Compute the following integrals using rational functions method

$$\frac{4}{3}\int_0^1 \frac{1}{x^2-x-2}dx, \quad \int \frac{1}{x^2-5x+6}dx, \quad \int \frac{2x^3+x^2+4}{(x^2+4)^2}dx$$

$$\int \frac{x^3+5x^2+4x+4}{x^2+1}dx, \quad \int_0^1 \frac{x-2}{9x^2+1}dx, \quad \int \frac{x^2-x}{2x+1}dx$$

$$\int_1^3 \frac{x^2}{x-4}dx, \quad \int \frac{x^3+1}{5x^2+6}dx, \quad \int \frac{2x^4+4}{x^3+1}dx$$

$$\int \frac{x+1}{x-3}dx, \quad \int \frac{x^2}{5x^2+4x+1}dx.$$

2 Notable limits, Taylor's Expansion

Exercise .2. Compute the following limits using notable limits

$$\lim_{x\to 0} \frac{1-cos(3x)}{x^2}, \quad \lim_{x\to\infty}(1+\frac{1}{2x})^{3x}.$$

Exercise .3. Compute the following limits using notable limits

$$\lim_{x\to 0} \frac{ln(1-4x)}{x}, \quad \lim_{x\to 0} \frac{sin(x)}{ln(1+x)}, \quad \lim_{x\to 0} \frac{1-cos(x)}{ln(1+x^2)}$$

$$\lim_{x\to 0} \frac{cos^3(x)-1}{sin(x^2)}, \quad \lim_{x\to 0} \frac{sin(3x)}{\sqrt{1-cos(3x)}}, \quad \lim_{x\to +\infty} xsin(\frac{\pi}{x}).$$

Exercise .4. Compute the following integrals using Taylor's expansion

$$\lim_{x\to 0} \frac{log(1+\frac{x}{2})}{sin(3x)}, \quad \lim_{x\to 0} \frac{cos^2(x)+x^2-1}{x^4}, \quad \lim_{x\to 0} \frac{e^{3x}-sin(3x)-1}{log(1-2x)}.$$

Exercise .5. Compute the following limits using Taylor's expansion

$$\lim_{x\to 0} \frac{tan(x)-x}{x^5} - \frac{1}{3x^2}, \quad \lim_{x\to -\infty} \frac{e^x}{(1+\frac{1}{x})x^2}, \quad \lim_{x\to 0} \frac{sin(x)-xe^x+x^2cos(x)}{(e^x-1)^3}$$

$$\lim_{x\to 0^+} \frac{x^2log(1+x)+tan(x)}{sin(x)+\sqrt{x}}, \quad \lim_{x\to 0} \frac{sin^2(x)-sin(x^2)}{x^2(cos^2(x)-cos(x^2))}.$$

3 Riemann's Integrals (substitution method)

Exercise .6. Compute the following integrals

$$\int \frac{2-x}{\sqrt{x}(\sqrt{x}+1)}dx, \quad \int \frac{\ln^2(x)+1}{x\ln(x)}dx, \quad \int \frac{\sqrt{x}}{\sqrt{x}-x^{\frac{1}{3}}}dx$$

$$\int \sqrt{7-x^2}dx, \quad \int_0^{\frac{\pi}{2}} \frac{\cos(x)}{1+\sin^2(x)}dx, \quad \int_{-1}^0 \frac{2x+1}{x+2}dx$$

$$\int \frac{\ln(x)}{x(\ln^4(x)+1)}dx, \quad \int_1^2 \frac{dx}{x\log(2x)}, \quad \int \frac{e^{3x}}{e^{2x}-3e^x-10}dx$$

4 Riemann's Integrals (improper integrals)

Exercise .7. Analyse the following integrals

$$\int_0^{+\infty} e^{-3x}dx, \quad \int_1^2 \frac{dx}{x\ln(x)}, \quad \int_0^1 (x+1)e^x\ln(x)dx$$

$$\int_0^{+\infty} \frac{4x}{4x^8+1}dx, \quad \int_0^{+\infty} \frac{e^{\sqrt{x}}}{\sqrt{x}}dx, \quad \int_2^{+\infty} \frac{\sqrt{x+2}-2}{x^2-3x+2}dx$$

$$\int_0^\infty \frac{|\sin(x)|}{x^2+x}dx, \quad \int_1^\infty \frac{\cos(x)}{\sqrt{x}}dx, \quad \int_0^\infty e^{-x^2}dx$$

5 Riemann's Integrals (do your best!)

Exercise .8. Compute the following integrals

$$\int \frac{x^4+3x^3+x^2-7x+5}{x^2}dx, \quad \int xe^x dx.$$

Exercise .9. Compute the following integrals

$$\int \frac{1}{4x+3}dx, \quad \int \frac{1}{4x^2+1}dx$$

Exercise .10. Compute the following definite integrals

$$\int_1^5 \frac{1}{x^{\frac{1}{x}}}dx, \quad \int_0^5 \frac{1}{e^x}dx, \quad \int_1^4 \frac{1+\sqrt{x}}{x^2}dx.$$

Exercise .11. Compute the following integrals

$$\int \frac{x+3}{x^2+4x+4}dx, \quad \int \frac{2x+4}{x^2-x+10}dx.$$