Velocity and acceleration

A motion of a particle can be defined if we know its position at each time

$x(t),\ y(t),\ z(t) \rightarrow \Delta x = x(t+\Delta t) - x(t)$

$\Delta y = y(t+\Delta t) - y(t)$

$\Delta z = z(t+\Delta t) - z(t)$

$\Delta \vec{r} = \vec{r}(t+\Delta t) - \vec{r}(t)$

$\vec{v} = \frac{d\vec{r}}{dt}$;

$\vec{a} = \frac{d\vec{v}}{dt}$ → acceleration is the quantity that describes the way velocity changes with time

Example: Free fall in 1D:

$x(t) = 0$

$y(t) = 0$

$z(t) = z(0) + v_0 t - \frac{1}{2} g t^2$

$v_x = 0$

$v_y = 0$

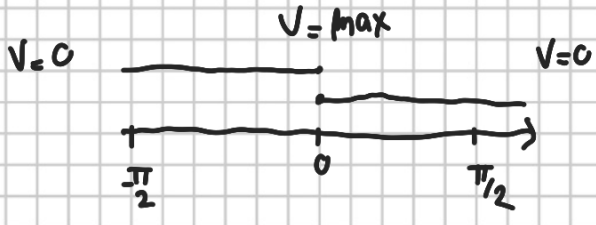
$v_z(t) = \frac{dz}{dt} = v_0 - gt$

$a_x = 0$

$a_y = 0$

$a_z = -g$

Spinning motion



→ $x(t) = \sin(\omega t)$

$v(t) = \omega \cos(\omega t)$

$a(t) = -\omega^2 \sin(\omega t)$

Note that acceleration and position have opposite sign.

Example:

A ball is thrown upwards with an initial velocity $v_0 = 9.8$ m/s. When does it reach the highest point?

$v_0 = +9{,}8$ m/s

$g = -9{,}8$ m/s^2

$v = 0 = v_0 + g\Delta t$

$-g\Delta t = v_0$

$\Delta t = -\frac{v_0}{g} = \frac{-9,8}{-9,8} = 1,0 \text{ s}$

What is the max height?

$p(t) = x_0 + v_0 t + \frac{1}{2} g t^2$

$p(1) = 0 + 9,8 \cdot 1 \frac{-9,8 \cdot 1^2}{2} = 4,9 \text{ m}$

When does it reach the ground:

$0 = 9,8 t - \frac{9,8 t^2}{2}$

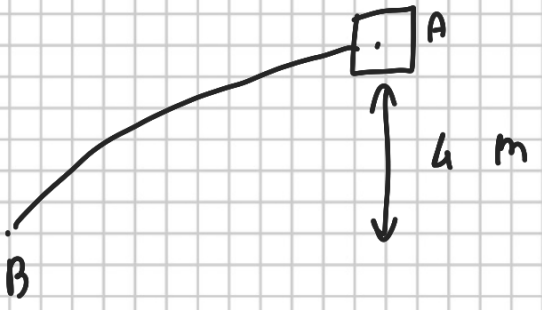
$0 = 9,8 t (1 - \frac{1}{2} t)$

$t = 0,0 \text{ v } t = 2,0 \text{ s}$

Max distance travelled: $2 \cdot h = 9,8 \text{ m}$

Exercise:

Throw keys to a friend via a window at 4 m from the ground
The person picks the keys after 2 seconds



a) v_0 of the keys

b) v when keys are picked

a) $p(2) = 4 \text{ m}$

$4 = 2v - 1g$

$v_0 = 2 + \frac{1}{2} g = 2 + 4,9 = 6,9 \text{ m/s}$

b) $v(2) = v_0 + gt = 6,9 - 9,8 \cdot 2 = 7 - 20 = -13 \text{ m/s}$