Energy types

- **Kinetic** → associated to the motion of a particle
  ↳ $K = \frac{1}{2}mv^2$ $[K] = ML^2T^{-2} =$ Joules
  A mass with $v = 0$ m/s has no Kinetic energy.

- **Potential**
  For example, in gravity:

In the system there is a total of K and U that get transformed into one another. $E = K + U$

Law of conservation of energy

All the forces can be derived from U:

$F(x) = -\frac{dV}{dx}$ → derivative of the potential energy

↓ force acting on a particle

consequently,

$V(x) = -\int F(x)\,dx$

↓

The $-$ sign means that force has always the direction that lowers the potential energy. It minimizes it.

So, for example,

$K = \frac{1}{2}mv^2$

$E = \frac{1}{2}mv^2 + V(x)$ → potential energy

Let's proof energy is constant:

E const. means $\frac{dE}{dt} = 0$

$\frac{dK}{dt} = \frac{d}{dt}\frac{1}{2}mv^2 = \frac{1}{2}m\,2v\frac{dv}{dt} = mva$

$$\frac{dV}{dt} = \frac{dV}{dx}\frac{dx}{dt} = \frac{dV}{dx}\cdot v$$

Potential derivative

$$\frac{dE}{dt} = mva + \frac{dV}{dx}\cdot v = v\left(ma + \frac{dV}{dx}\right)$$

So, if E is a constant, the derivative is 0:

$$ma + \frac{dV}{dx} = 0$$

$ma - F(x) \Rightarrow F(x) = ma \rightarrow$ for Newton's law to be constant, energy must be constant

An example:

We have a particle in 2D, x and y, and mass m.
We have $V = \frac{1}{2}K(x^2+y^2)$, the potential energy.

Derive the equations of motion:

$F(x) = -\frac{dV}{dx}$:

$$\left.\begin{aligned} F_x &= -\frac{dV}{dx} = -Kx = ma_x \rightarrow a_x = -\frac{K}{m}x \\ F_y &= -\frac{dV}{dy} = -Ky = ma_y \rightarrow a_y = -\frac{K}{m}y \end{aligned}\right\} \vec{a} = -\frac{K}{m}\vec{r} \Rightarrow$$: this is circular motion

Stable equilibrium

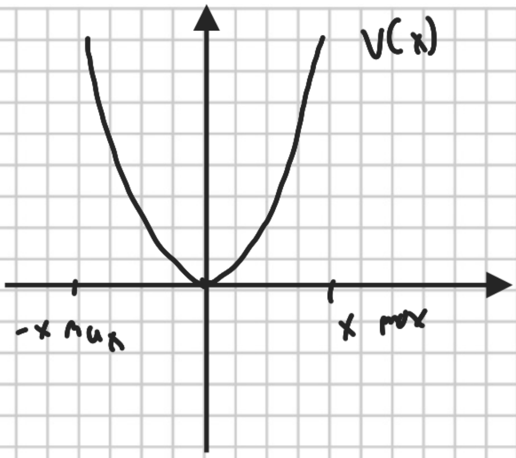
A system will return in its state of equilibrium even after having the state modified. It's different from other kinds of equilibrium.

Hook's law for springs: $F - Kx$ ↳ displacement

We can try to get the potential from Hooke:

$$F(x) = -\frac{dV}{dx} \rightarrow V(x) = -\int F(x)\,dx = -\int -Kx\,dx = K\int x\,dx = \frac{1}{2}Kx^2 + c$$

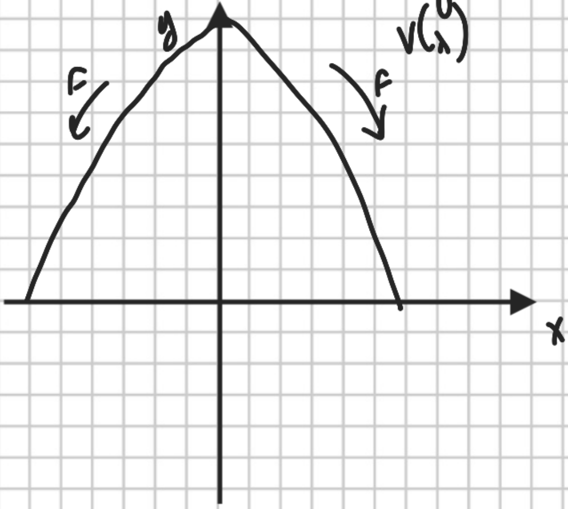
We can see that this is a symmetric distribution:



$x=0$ is the natural length

The force will be always pushing to $x=0$ and it will return there $\rightarrow$ the stable equilibrium point.

If we had something like, the point $x=0$ is of unstable equilibrium.



An example of stable equilibrium:

$V(x) = 4\varepsilon\left[\left(\frac{\gamma}{x}\right)^{12} - \left(\frac{\gamma}{x}\right)^{6}\right] \rightarrow$ potential energy of a system of 2 atoms in a molecule. They have interactions because of electrons.

ε and γ are parameters.

Let's find the average distance: We must minimize the $V(x)$! To get s.e.

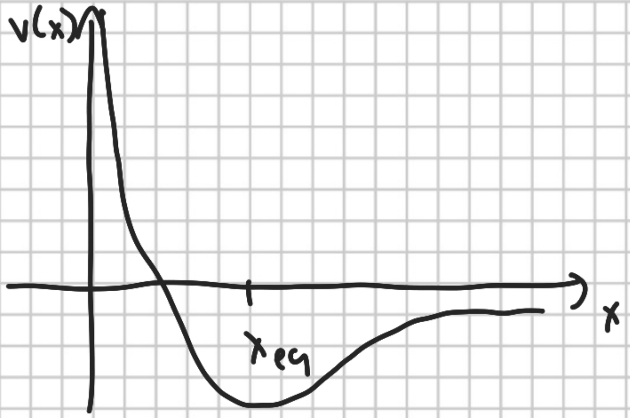
$$\frac{dV(x)}{dx} = 0$$

DO IT AT HOME

$$\frac{dV(x)}{dx} = \frac{2\gamma^{12}}{x^{13}} = \frac{\gamma^{6}}{x^{7}}$$

This goes to $x = 2^{1/6} \cdot \gamma$

Note that $\gamma = 0{,}263$ nm, So $x_{eq} \sim$ nm.



This the graph of $V(x)$.

Potential energy of gravity

im i this a particle of mass m, going from i to f.

The complete formula for gravitation is:

$$\vec{F}_g = -\frac{GMm}{r^2}\hat{r}$$

Newton's Gravity constant

Let's calculate the potential enrgy at point f:

$$U_f = -\int_{r_i}^{r_f} F(r)\,dr + U_i = GMm\int_{r_i}^{r_f}\frac{dr}{r^2} + U_i = -GMm\left(\frac{1}{r_f} - \frac{1}{r_i}\right) + U_i$$

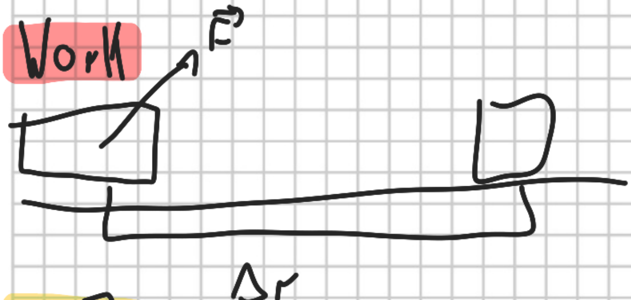
initial potential energy

- coming from the integral

What is U_i? We can say it's infinitely far, so the mass does not feel the earth force, so $U_i = 0$, $r_i = \infty$, so we can write:

$$U_f = -\frac{GMm}{r}$$ ← potential energy for gravity

Work



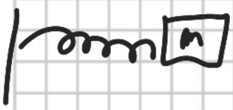
W = scalar quantity = $\vec{F}\cdot\Delta\vec{r}$

external force

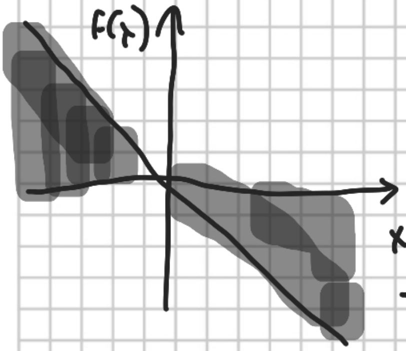
displacement

$[W] = Nm = \text{Joules}$

Let's try to look at a spring:



$F = -kx$



$$W = \int \vec{F} \cdot d\vec{r} \qquad (dW = \vec{F} \cdot d\vec{r})$$

→ $W=0$ → because it's a spring!

Conservative and non-conservative forces

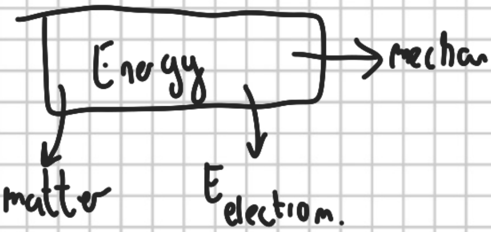
→ Consider an object and surrounding system
→ The object and the rest of the system interact via a force
→ Force acts on object 1 and does work W_1
→ Things are inverted: the force does work W_2
↓
We try to put things back into place
→ If the force is conservative, $W_1 = -W_2$
↓
An example of non-conservative force is friction: you cannot re-extract the heat caused by friction.

A non-conservative force has no potential.
The work of a c. force moving on a closed circuit is 0.

Non conservative forces = dissipative forces
↓
Mechanical energy is not conserved

Isolated and non-isolated system

In an isolated system → $\Delta E = 0$ (no contact with the external world)
nonisolated →



so, $\Delta E = \sum T$
↓
all the transferred energies

A ball on a plane is an example of i.s.
In an isolated system, we have

$$\Delta E_{mec} = \Delta K + \Delta U = 0$$

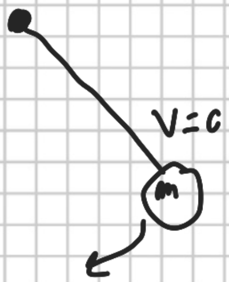
In a non-isolated,

$\Delta E_{mec} = \Delta E_{int}$ — linked to the temperature of the system

dissipated energy

Example:

The pendulum:



At $v=0$, $U \to max$ $K=0$

At $30°$, $U \to \frac{1}{2}$ $K=\frac{1}{2}$

At midpoint, $U \to 0$ $K = max$

If it is an i.s., the pendulum goes on forever.

Note:

For most exercises, U won't be

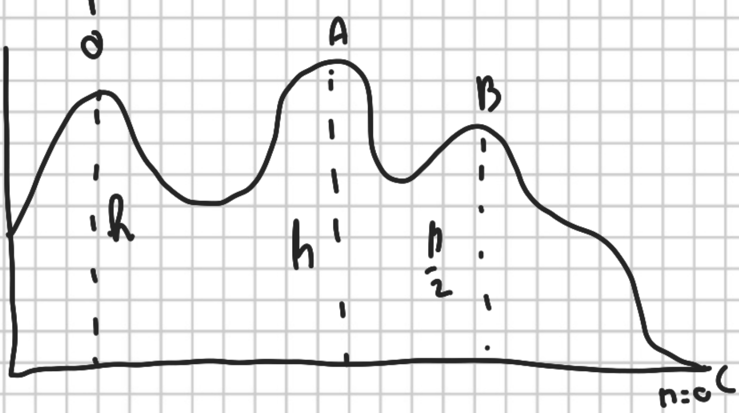
$$\Delta U = -GM_T m\left(\frac{1}{r_f} - \frac{1}{r_i}\right) = GM_T m\left(\frac{r_f - r_i}{r_f r_i}\right)$$

but, since we are close to the Earth surface, $r_f r_i \sim R^2$

$$\Delta U = \frac{GM_T m \Delta y}{R^2} = mg\Delta y$$

$$g = \frac{GM_t}{R_T^2}$$

Example:



A block of mass m starts sliding down at point O.

v? in A, B, C

There is v_0; this an i.s.

$V_A = V_0 \longrightarrow K_A = \frac{1}{2} m v^2$

$U_A = mgh$

$U_C = 0$

$K_C = \frac{1}{2} m v_0^2 + mgh$

$U_B = mg\frac{h}{2}$

$K_B = \frac{1}{2} m v_0^2 + mg\frac{h}{2} \longrightarrow \frac{1}{2}\cancel{m} v_0^2 + \cancel{m}gh = \frac{1}{2}\cancel{m} v_B^2 + \cancel{m} g \frac{h}{2}$

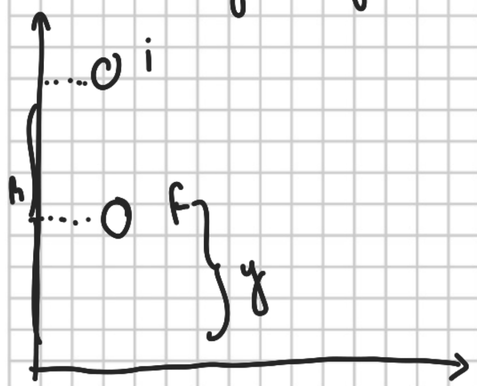
$v_0^2 + 2gh = v_B^2 + gh$

$v_B^2 = v_0^2 - gh$

$V_B = \sqrt{v_0^2 + gh}$

Another example:

Free-falling object:



Determine the velocity at height y.

a) $v_i = 0$

$K_f + U_f = K_i + U_i$

$\frac{1}{2}\cancel{m} v_f^2 + \cancel{m} g y = \quad + \cancel{m} g h$

$v_f^2 + 2gy = \quad + 2gh$

$v_f = \sqrt{2g(h-y)}$

b) $v_i \neq 0$

$K_f + U_f = K_i + U_i$

$\frac{1}{2}\cancel{m} v_f^2 + \cancel{m} g y = \frac{1}{2} \cancel{m} v_i^2 + \cancel{m} g h$

$v_f^2 + 2gy = v_i^2 + 2gh$

$v_f = \sqrt{2g(h-y) + v_i^2}$

$$\begin{cases} v_f = v_0 + gt \\ x_f = \frac{1}{2} g t^2 + v_0 t + x_i \end{cases}$$

$0 = \frac{1}{2} g t^2 + v_0 t + x_i - x_f$

Another example:

- Object with mass m sliding
- $v_0 = 0$
- loop of radius r

a) $V_A = ?$

b) $N_A = ?$

a) $E_i = mgh$

$E_A = 2mgr + \frac{1}{2} m v_A^2$

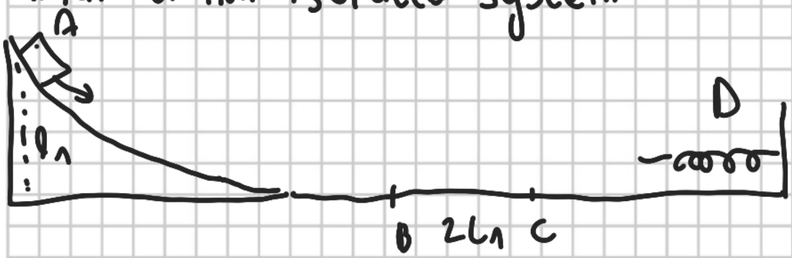
$mgh = 2mgr + \frac{1}{2} m v_a^2$

$v_a^2 = 2g(h - 2r)$

b) $N + mg = \frac{v_a^2}{r} \cdot m$

$N = m\left(\frac{v_a^2}{r} - g\right) = \frac{mg}{r}(2h - 5R)$

With a non-isolated system:



$V_A = 0$

$V_D = 0$

part with friction

What is the $\mu_d = ?$

$$\underbrace{K_A + U_A}_{E_{ma}} - \Delta E_{int} = \underbrace{K_D + U_D}_{E_{md}} + \underbrace{U_{SD}}_{\text{Spring}}$$

$$0 + mgl_1 - \underbrace{\Delta E_{int}}_{2mgL_1\mu_d} = 0 + 0 + \frac{1}{2}Kx^2$$

$$mgL_1 - 2mgL_1\mu_d = \frac{1}{2}Kx^2$$

$$2mgL_1\mu_d = mgL_1 - \frac{1}{2}Kx^2$$

$$\mu_d = \frac{1}{2} - \frac{Kx^2}{4mgL_1}$$

Power

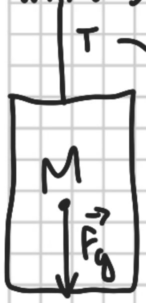
$\langle P \rangle = \frac{W}{\Delta t}$ – the average power of a Force

$P = \frac{dW}{dt} = F \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$ – instant velocity

$[P] = ML^2T^{-3}$ (joules per second, or watt)

Example:

What's the power delivered by the engine of an elevator?



T → the engine needs to produce the required tension

- $a = 0$ a)
- $a > 0$ b)

a) $T - F_g - F_d = 0$

dissipation

$\vec{T} = (\vec{F}_g + Mg)$

$\vec{P} = \vec{T} \cdot \vec{v} = T \cdot v = (F_g + Mg)v$

b) $T - F_g - F_d > 0 = Ma$

$T = M(a+g) + F_d$

$(T_b > T_a)$

$\frac{P_b}{P_A} = 1 + \frac{Ma}{F+Mg} > 1$

not constant!