$$I(a) = \int_0^{\infty} \frac{\cos(ax)}{x^2 + b^2}\, dx$$

Feynmann Technique of Integration

$$\varphi(x,y) = \int_{a(y)}^{b(y)} f(x,y)\, dx$$

$$\varphi_y'(x,y) = \int_{a(y)}^{b(y)} \frac{\partial}{\partial y} f(x,y)\, dx + b'(y) f(b(y),y) - a'(y) f(a(y),y)$$

$$I(a) = \int_0^{\infty} \frac{\cos(ax)}{x^2+b^2}\, dx \qquad g' = \cos(ax)$$

$$g = \frac{1}{a} \sin(ax)$$

$$f = \frac{1}{x^2+b^2} \qquad dx = \frac{-2x}{(x^2+b^2)^2}$$

$$= \cancel{\frac{1}{x^2+b^2} \frac{1}{a} \sin(ax) \Big|_0^{\infty}} + \frac{1}{a} \int_0^{\infty} \frac{+2x}{(x^2+b^2)^2} \sin(ax)\, dx$$

$$\frac{1}{a} \int_0^{\infty} \frac{2x \sin(ax)}{(x^2+b^2)^2}\, dx$$

$$a\,I(a) = \int_0^\infty \frac{2x\sin(ax)}{(x^2+b^2)^2}\,dx$$

taking

$$I(a) + a\,I'(a) = \int_0^\infty \frac{2x^2\cos(ax)}{(x^2+b^2)^2}\,dx$$

$$I(a) + a\,I'(a) = 2\int_0^\infty \frac{x^2\cos(ax)}{(x^2+b^2)^2}\,dx$$

$$\frac{x^2\cos(ax)}{(x^2+b^2)^2} \quad \frac{\cos(ax)}{(x^2+b^2)} - \frac{b^2\cos(ax)}{(x^2+b^2)^2}$$

$$I(a) + a\,I'(a) = 2\left(\underbrace{\int_0^\infty \frac{\cos(ax)}{x^2+b^2}} - b^2\int_0^\infty \frac{\cos(ax)}{(x^2+b^2)^2}\,dx\right.$$

$$\cancel{I(a)} + a\,I'(a) = \cancel{2}I(a) - 2b^2\int_0^\infty \frac{\cos(ax)}{(x^2+b^2)^2}\,dx$$

$$a\,I'(a) - I(a) = -2b^2\int_0^\infty \frac{\cos(ax)}{(x^2+b^2)^2}\,dx$$

$$\cancel{I'(a)} + a\,I''(a) - \cancel{I'(a)} = +2b^2\int_0^\infty \frac{+\sin(ax)\times}{(x^2+b^2)^2}\,dx$$

$$aI''(a) = 2b^2 \int_0^\infty \frac{\sin(ax)\, x}{(x^2+b^2)^2}\, dx$$

$$\cancel{a} I''(a) = \cancel{2} b^2 \frac{\cancel{a} I(a)}{\cancel{2}}$$

$$\boxed{I''(a) = b^2 I(a)}$$

$$z^2 - b^2 = 0 \qquad (z-b)(z+b) = 0$$

$$z = b \quad z = -b \qquad y_{omo} = c_1 e^{ba} + c_2 e^{-ba}$$

$$I(0) = \int_0^\infty \frac{1}{x^2+b^2}\, dx = \frac{1}{b} \arctan\left(\frac{x}{b}\right)\Big|_0^\infty$$

$$= \frac{\pi}{2b}$$

$$y(0) = c_1 e^0 + c_2 e^0 \Rightarrow c_1 + c_2 = \frac{\pi}{2b}$$

$$y(\infty) = \lim_{a \to +\infty} \frac{1}{a} \int_0^\infty \frac{x \sin(ax)}{(x^2+b^2)}\, dx$$

$$y(\infty) = 0 \qquad c_1 e^{b\infty} + c_2 e^{-b\infty} = 0$$

$$\lim_{a \to +\infty} \qquad \to 0$$

$$\Rightarrow \quad c_1 = 0 \quad \Rightarrow \quad c_2 + c_1 = \frac{\pi}{2b}$$

$$c_2 = \frac{\pi}{2b} \quad \Rightarrow \quad \boxed{y(a) = \frac{\pi}{2b} e^{-ba}}$$

$I(1)$ $\quad y(1) = \frac{\pi}{2} e^{-1} = \boxed{\frac{\pi}{2e}}$

$b = 1$
