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Solomahy CDF

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1a) $a_i = (a_{ix}, a_{iy})$
$b_i = (b_{ix}, b_{iy})$ $\Rightarrow$ $\begin{bmatrix} b_{1x} & b_{2x} & b_{3x} \\ b_{1y} & b_{2y} & b_{3y} \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \alpha & \beta & t_x \\ \gamma & \rho & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{1x} & a_{2x} \\ a_{1y} & a_{2y} \\ 1 & 1 \end{bmatrix}$

$$\begin{bmatrix} \alpha & \beta & t_x \\ \gamma & \rho & t_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} b_{1x} & b_{2x} & b_{3x} \\ b_{1y} & b_{2y} & b_{3y} \\ 1 & 1 & 1 \end{bmatrix}.$$

rank $A = n$.

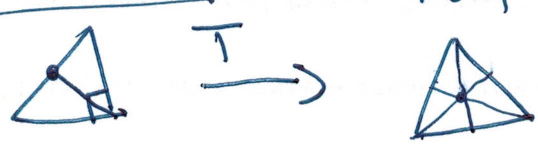
b) 2D homography determined by 4 point mappings
2D similarity requires 3 point mappings.

c) $C_x = \dfrac{a_{1x} + a_{2x} + a_{3x}}{3}$ $\quad C_y = \dfrac{a_{1y} + a_{2y} + a_{3y}}{3}$

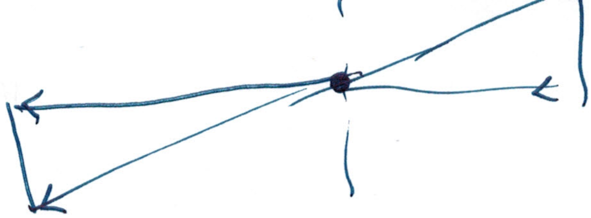
$$A c = A \begin{bmatrix} c_x \\ c_y \\ 1 \end{bmatrix} = A \begin{bmatrix} \frac{a_1}{3} + \frac{a_2}{3} + \frac{a_3}{3} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{Aa_1 + Aa_2 + Aa_3}{3} \\ 1 \end{bmatrix}$$

Centroid is affine invariant.

Circumcenter is not, as angles can change

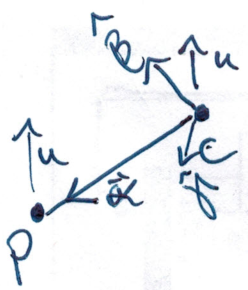


2 a)



Light passes through singa point, opposite of viewing plane, therefore flipping the image.

b)



$$\hat{\alpha} = \frac{\cancel{P-C}\,(\vec{C}-\vec{P})}{\|\vec{C}-\vec{P}\|} \qquad 1$$

$$\vec{B} = \hat{a} \times \hat{u} \qquad 2$$

$$\hat{B} = \frac{\vec{b}}{\|\vec{b}\|}$$

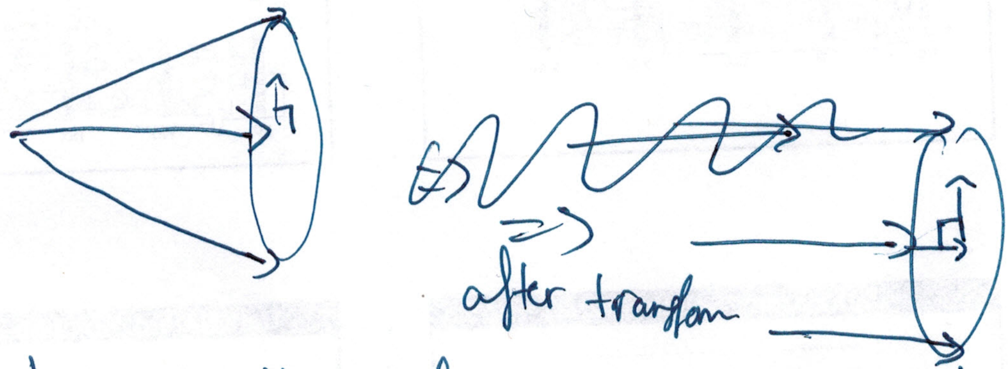
$$\hat{\gamma} = \hat{a} \times \hat{B} \qquad 3$$

$$\begin{bmatrix} \alpha_x & \alpha_y & \alpha_z & 0 \\ \beta_x & \beta_y & \beta_z & 0 \\ \gamma_x & \gamma_y & \gamma_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -c_x \\ & 1 & 0 & -c_y \\ & & 1 & -c_z \\ 0 & & & 1 \end{bmatrix}$$

World to Camera

c) Perspectie Transform

↳ Vectors orthogonal to gaze will remain orthogonal and parallel after perspectie transform.



d) Yes, If not orthogonal, then the lines wile angle after tranform and extend infinitely to convergence point.

3 a) $\vec{\nabla} f = \left\langle \frac{-2x\left(R-\sqrt{x^2+y^2}\right)}{\sqrt{x^2+y^2}}, \frac{-2y\left(R-\sqrt{x^2+y^2}\right)}{\sqrt{x^2+y^2}}, 2z \right\rangle$ 4

$$\hat{n} = \frac{\vec{\nabla} f}{\|\vec{\nabla} f\|}$$

b) $\underbrace{\hat{n}}_{= \vec{\nabla} f(P_0)} \cdot \left(\langle x, y, z \rangle - \underbrace{P_0}_{\text{point of tangent}} \right) = 0$

c) $x = R\cos\lambda,\ y = R\sin\lambda,\ z = r$

$$f(x,y,z) = \left(R - \underbrace{\sqrt{x^2+y^2}}_{=R}\right)^2 + \cancel{r^2} - \cancel{r^2} = 0$$

$$= 0 = 0 \checkmark$$

d) $\frac{\partial g(\lambda)}{\partial \lambda} = \langle -R\sin\lambda, R\cos\lambda, 0 \rangle$

e) $\vec{\nabla} f \cdot \frac{\partial g(\lambda)}{\partial \lambda} = 0$ if lies on plane $\longrightarrow$

Cont e)

~~$-2R\cos\lambda$~~ $\dfrac{\partial \vec{g}(\lambda)}{\partial \lambda} = \langle -y, x, 0 \rangle.$

$\Rightarrow \vec{\nabla} f \cdot \dfrac{\partial \vec{g}(\lambda)}{\partial \lambda} = \dfrac{-2x(-y)(R-\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}}.$

$- \dfrac{2yx(R-\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}} + 0$

$= 0 \Rightarrow \dfrac{\partial \vec{g}(\lambda)}{\partial \lambda}$ lives in plane.

4 a) $B_1(t) = (1-t)^3 P_1 + 3(1-t)^2 t P_2 + 3(1-t)t^2 P_3 + t^3 P_4$

$B_2(t) = (1-t)^3 P_4 + 3(1-t)^2 t P_5 + 3(1-t)t^2 P_6 + t^3 P_7$

Share point: $B_1(1) = B_2(0)$

$$B_1'(t) = -3(1-t)^2 P_1 + 3(1-t)^2 P_2 + 6(1-t)tP_3 + 3t^2 P_4 - 6(1-t)tP_2 - 3t^2 P_3$$

$$B_2'(t) = -3(1-t)^2 P_4 + 3(1-t)^2 P_5 + 6(1-t)tP_6 + 3t^2 P_7 - 6(1-t)tP_5 - 3t^2 P_6$$

$$\boxed{B_1'(1) = -3P_3 + 3P_4 \qquad B_2'(0) = -3P_4 + 3P_5}$$

b) $B_1''(t) = +6(1-t)P_1 - 6(1-t)P_2 - 6tP_3 + 6tP_4 + 6tP_2 + 6(1-t)P_3 - 6(1-t)P_2 - 6tP_3$

$$\boxed{\begin{aligned} B_1''(1) &= 6P_2 - 12P_3 + 6P_4 \\ B_2''(0) &= 6P_4 - 12P_5 + 6P_6 \end{aligned}}$$

c) $-3P_3 + 3P_4 = -3P_4 + 3P_5 \Rightarrow \boxed{2P_4 - P_3 = P_5}$

$P_2 - 2P_3 = -2P_5 + P_6 \Rightarrow \boxed{P_2 - 2P_3 + 2P_5 = P_6}$

$\cancel{P_2 - 2P_3 + 2P_4 = P_6}$

d) Affine, C_0 continuity, local change control and smoothness, ease of computat

Invariance.

Affine → ~~Allows operation reordering.~~

↳ Apply operations to control point

↳ Not curve

C_0 → guaranteed

Local Control → Due to form of Bezier Curv

it is more smooth than simple polynomial fitting.

Ease of Compute → Affine, only recalculate control points, Natural weighted sum.

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