Неопределён интеграл

Основные интегралы

1) $\int u^n du = \frac{u^{n+1}}{n+1} + C$

2) $\int \frac{du}{u} = \ln|u| + C$

3) $\int a^u du = \frac{a^u}{\ln a} + C$, $\int e^u du = e^u + C$

4) $\int \cos u \, du = \sin u + C$

5) $\int \sin u \, du = -\cos u + C$

6) $\int \frac{du}{\cos^2 x} = \operatorname{tg} u + C$

7) $\int \frac{du}{\sin^2} = -\operatorname{ctg} u + C$

8) $\int \frac{du}{1+u^2} = \operatorname{arctg} u + C$, $\int \frac{du}{a^2+u^2} = \frac{1}{a} \operatorname{arctg} \frac{u}{a} + C$

9) $\int \frac{du}{\sqrt{1-u^2}} = \arcsin u + C$, $\int \frac{du}{\sqrt{a^2-u^2}} = \frac{1}{a} \arcsin \frac{u}{a} + C$

tall 10) $\int \frac{du}{a^2-u^2} = \frac{1}{2a} \ln\left|\frac{a+u}{a-u}\right| + C$, $\int \frac{du}{u^2-a^2} =$

long 11) $\int \frac{du}{\sqrt{u^2+a^2}} = \ln|u + \sqrt{u^2+a^2}| + C$

12) $\int \operatorname{sh} u \, du = \operatorname{ch} u + C$ $\quad \operatorname{ch}^2 u - \operatorname{sh}^2 u = 1$

13) $\int \operatorname{ch} u \, du = \operatorname{sh} u + C$

14) $\int \frac{du}{\operatorname{ch}^2 u} = \operatorname{th} u + C$

15) $\int \frac{du}{\operatorname{sh}^2 u} = -\operatorname{cth} u + C$

№ 6: 15, 16, 19

№ 6.15

$$\int\left(3x^2+2x+\frac{1}{x}\right)dx=\int 3x^2\,dx+\int 2x\,dx+\int \frac{1}{x}\,dx=$$

$$= x^3+x^2+\ln|x|+C$$

№ 6.16

$$\int\frac{2x+3}{x^4}\,dx=\int\frac{2}{x^3}\,dx+\int\frac{3}{x^4}\,dx=2\cdot\left(\frac{1}{-2\cdot x^2}\right)+3\left(\frac{1}{-3\cdot x^3}\right)+C$$

$$=-\frac{1}{x^2}-\frac{1}{x^3}+C$$

№ 6.34

$$\int\frac{dx}{5-x^2}=\frac{1}{2\sqrt{5}}\ln\left(\frac{\sqrt{5}+x}{\sqrt{5}-x}\right)+C$$

№ 6.35

$$\int\frac{dx}{\sqrt{3-x^2}}=\frac{1}{\sqrt{3}}\arcsin\frac{x}{\sqrt{3}}+C$$

Подведение под знак дифференциала

$$\int \sin^2 x \cos x dx = \int \sin^2 x \, d(\sin x) = \frac{\sin^3 x}{3} + C$$

$$\cos x dx = d(\sin x)$$

$$\frac{d(\sin x)}{dx} = \cos x$$

1) $$\int \sin^2 x \cos x dx = \left| \begin{array}{l} \sin x = t \\ d(\sin x) = dt \\ \cos x dx = dt \end{array} \right| = t^2 dt = \frac{t^3}{3} + C$$

2) $$\int \frac{dx}{5x+4} = \left| \begin{array}{l} d(5x+4) = 5dx \\ dx = \frac{1}{5} d(5x+4) \end{array} \right. = \frac{1}{5} \int \frac{d(5x+4)}{5x+4} = \frac{\ln|5x+4|}{5} + C$$

3) $$\int \frac{\sqrt{\operatorname{arctg} x}}{1+x^2} dx = \int \sqrt{\operatorname{arctg} x} \cdot \frac{1}{x^2+1} dx =$$

$$= \sqrt{\operatorname{arctg} x} \cdot d(\operatorname{arctg} x) = \frac{2 \operatorname{arctg}^{\frac{3}{2}} x}{3} + C$$

№6 44, 45, 48, 52, 60, 61

№6.44

$$\int \sqrt{x+3} \, dx = \left| \begin{array}{l} d(\sqrt{x+3}) = \frac{1}{2\sqrt{x+3}} dx \\ dx = 2\sqrt{x+3} \, d(\sqrt{x+3}) \end{array} \right| =$$

$$= \int \sqrt{x+3} \; 2 \cdot \sqrt{x+3} \cdot d(\sqrt{x+3}) =$$

$$= 2t^2 dt = \frac{2}{3} t^3 + C = \frac{2}{3}(x+3)\sqrt{x+3} + C$$

№6.45

$$\int (3-4\sin x)^{\frac{1}{3}} \cos x dx = \left| \begin{array}{l} d(3-4\sin x) = -4\cos x dx \\ \cos x dx = \frac{d(3-4\sin x)}{-4} \end{array} \right| =$$

$$= \frac{u^{\frac{1}{3}} du}{-4} = -\frac{1}{4} \cdot \frac{3}{4} u^{\frac{4}{3}} = -\frac{3}{16} (3-4\sin x)^{\frac{4}{3}}$$

№6.48

$$\int \frac{dx}{x\ln^2 x} = \left| d\ln x = \frac{1}{x}\,dx \right| = \int \ln^{-2}x \cdot d\ln x =$$

$$= \frac{-1}{\ln x} + C$$

№6.52

$$\int ctg\,x\,dx = \int \frac{\cos x}{\sin x}\,dx = \int \frac{d(\sin x)}{\sin x} =$$

$$= \ln|\sin x| + C$$

№6.60

$$\int x\cdot 5^{-x^2}\,dx = \int \frac{x\,dx}{5^{x^2}} = \left| \begin{matrix} d(x^2) = 2x\,dx \\ x\,dx = \frac{d(x^2)}{2} \end{matrix} \right| =$$

$$= \int \frac{5^{-x^2}}{2}\,d(x^2) = \frac{1}{2}\,\frac{5^{-x^2}}{\ln 5} + C = \frac{5^{-x^2}}{2\ln 5}$$

№6.61

$$\int \frac{dx}{1-4x^2} = \left| \begin{matrix} d(2x) = -8x\,dx \\ dx = \frac{d(1-4x^2)}{-8x} \end{matrix} \right| =$$

$$\int \frac{x-1}{(x+2)^2}\,dx = \int \frac{x+2-3}{(x+2)^2}\,dx = \int\left(\frac{1}{x+2} - \frac{3}{(x+2)^2}\right)dx =$$

$$= \left| d(x+2) = dx \right| = \int \frac{d(x+2)}{x+2} - \int \frac{3\,d(x+2)}{(x+2)^2} =$$

$$= \ln|x+2| + \frac{3}{x+2} + C$$