Example 4.1

Example 4.1 A straight wire of mass 200 g and length 1.5 m carries a current of 2 A. It is suspended in mid-air by a uniform horizontal magnetic field **B** (Fig. 4.3). What is the magnitude of the magnetic field?

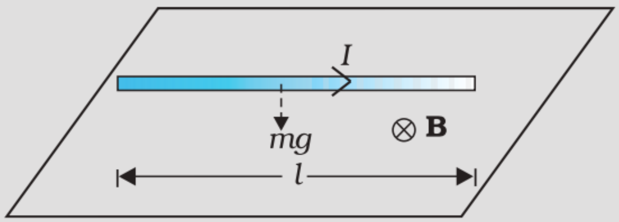


FIGURE 4.3

Solution From Eq. (4.4), we find that there is an upward force **F**, of magnitude *IlB*,. For mid-air suspension, this must be balanced by the force due to gravity:

$$m\,g = I\,lB$$

$$B = \frac{m\,g}{I\,l}$$

$$= \frac{0.2 \times 9.8}{2 \times 1.5} = 0.65 \text{ T}$$

Note that it would have been sufficient to specify m/l, the mass per unit length of the wire. The earth's magnetic field is approximately 4×10^{-5} T and we have ignored it.

Example 4.2 is useless

Example 4.3

Example 4.3 What is the radius of the path of an electron (mass 9×10^{-31} kg and charge 1.6×10^{-19} C) moving at a speed of 3×10^7 m/s in a magnetic field of 6×10^{-4} T perpendicular to it? What is its frequency? Calculate its energy in keV. (1 eV = 1.6×10^{-19} J).

Solution Using Eq. (4.5) we find

$r = mv / (qB) = 9 \times 10^{-31}\text{ kg} \times 3 \times 10^{7}\text{ m s}^{-1} / (1.6 \times 10^{-19}\text{ C} \times 6 \times 10^{-4}\text{ T})$

$= 28 \times 10^{-2}\text{ m} = 28\text{ cm}$

$\nu = v / (2\pi r) = 17 \times 10^{6}\text{ s}^{-1} = 17 \times 10^{6}\text{ Hz} = 17\text{ MHz}.$

$E = (½)mv^2 = (½)\, 9 \times 10^{-31}\text{ kg} \times 9 \times 10^{14}\text{ m}^2/\text{s}^2 = 40.5 \times 10^{-17}\text{ J}$

$\approx 4 \times 10^{-16}\text{ J} = 2.5\text{ keV}.$

Example 4.5

Example 4.5 An element $\Delta \boldsymbol{l} = \Delta x\,\hat{\mathbf{i}}$ is placed at the origin and carries a large current I = 10 A (Fig. 4.8). What is the magnetic field on the y-axis at a distance of 0.5 m. Δx = 1 cm.

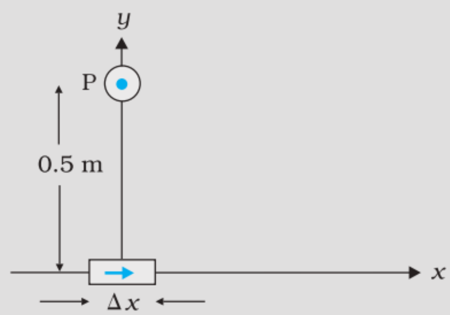


FIGURE 4.8

Solution

$$|\,d\mathbf{B}\,| = \frac{\mu_0}{4\pi}\,\frac{I\,dl\sin\theta}{r^2} \quad \text{[using Eq. (4.11)]}$$

$$dl = \Delta x = 10^{-2}\,\text{m}\,,\ I = 10\text{ A},\ \ r = 0.5\text{ m} = y,\ \ \mu_0/4\pi = 10^{-7}\,\frac{\text{T m}}{\text{A}}$$

$\theta = 90°$; $\sin\theta = 1$

$$|d\mathbf{B}| = \frac{10^{-7}\times 10\times 10^{-2}}{25\times 10^{-2}} = 4\times 10^{-8}\text{ T}$$

The direction of the field is in the +z-direction. This is so since,

$$d\boldsymbol{l}\times\mathbf{r} = \Delta x\,\hat{\mathbf{i}}\times y\,\hat{\mathbf{j}} = y\,\Delta x\left(\hat{\mathbf{i}}\times\hat{\mathbf{j}}\right) = y\,\Delta x\,\hat{\mathbf{k}}$$

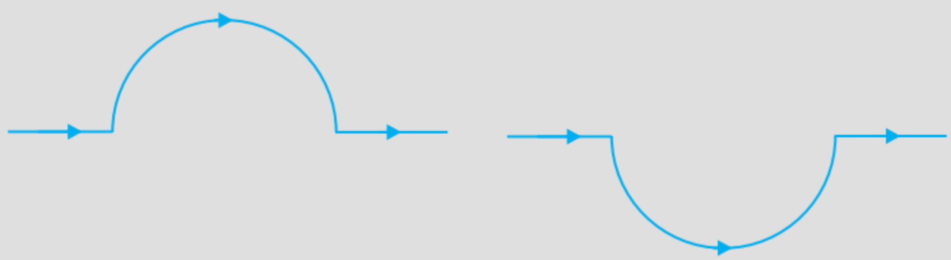
We remind you of the following cyclic property of cross-products,

$$\hat{\mathbf{i}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}};\ \hat{\mathbf{j}}\times\hat{\mathbf{k}} = \hat{\mathbf{i}};\ \hat{\mathbf{k}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}$$

Note that the field is small in magnitude.

Example 4.6

Example 4.6 A straight wire carrying a current of 12 A is bent into a semi-circular arc of radius 2.0 cm as shown in Fig. 4.11(a). Consider the magnetic field **B** at the centre of the arc. (a) What is the magnetic field due to the straight segments? (b) In what way the contribution to **B** from the semicircle differs from that of a circular loop and in what way does it resemble? (c) Would your answer be different if the wire were bent into a semi-circular arc of the same radius but in the opposite way as shown in Fig. 4.11(b)?



(a) (b)

FIGURE 4.11

Solution

(a) $d\mathbf{l}$ and $\mathbf{r}$ for each element of the straight segments are parallel. Therefore, $d\boldsymbol{l} \times \mathbf{r} = 0$. Straight segments do not contribute to $|\mathbf{B}|$.

(b) For all segments of the semicircular arc, $d\boldsymbol{l} \times \mathbf{r}$ are all parallel to each other (into the plane of the paper). All such contributions add up in magnitude. Hence direction of **B** for a semicircular arc is given by the right-hand rule and magnitude is half that of a circular loop. Thus **B** is 1.9×10^{-4} T normal to the plane of the paper going into it.

(c) Same magnitude of **B** but opposite in direction to that in (b).

Example 4.7

Example 4.7 Consider a tightly wound 100 turn coil of radius 10 cm, carrying a current of 1 A. What is the magnitude of the magnetic field at the centre of the coil?

Solution Since the coil is tightly wound, we may take each circular element to have the same radius R = 10 cm = 0.1 m. The number of turns N = 100. The magnitude of the magnetic field is,

$$B = \frac{\mu_0 NI}{2R} = \frac{4\pi \times 10^{-7} \times 10^2 \times 1}{2 \times 10^{-1}} = 2\pi \times 10^{-4} = 6.28 \times 10^{-4}\ \text{T}$$

Example 4.8 Figure 4.13 shows a long straight wire of a circular cross-section (radius a) carrying steady current I. The current I is uniformly distributed across this cross-section. Calculate the magnetic field in the region $r < a$ and $r > a$.

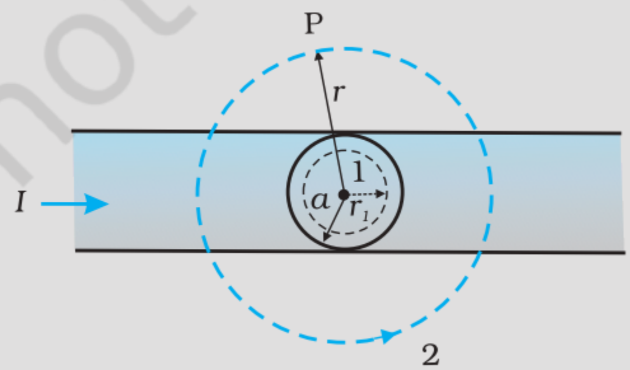


FIGURE 4.13

Solution (a) Consider the case $r > a$. The Amperian loop, labelled 2, is a circle concentric with the cross-section. For this loop,

$L = 2\pi r$

I_e = Current enclosed by the loop = I

The result is the familiar expression for a long straight wire

$B(2\pi r) = \mu_0 I$

$$B = \frac{\mu_0 I}{2\pi r} \qquad [4.19(a)]$$

$$B \propto \frac{1}{r} \quad (r > a)$$

Now the current enclosed I_e is not I, but is less than this value. Since the current distribution is uniform, the current enclosed is,

$$I_e = I\left(\frac{\pi r^2}{\pi a^2}\right) = \frac{Ir^2}{a^2}$$

Using Ampere's law, $B(2\pi r) = \mu_0 \frac{Ir^2}{a^2}$

$$B = \left(\frac{\mu_0 I}{2\pi a^2}\right) r \qquad [4.19(b)]$$

$B \propto r \quad (r < a)$

$B \propto r \qquad (r < a)$

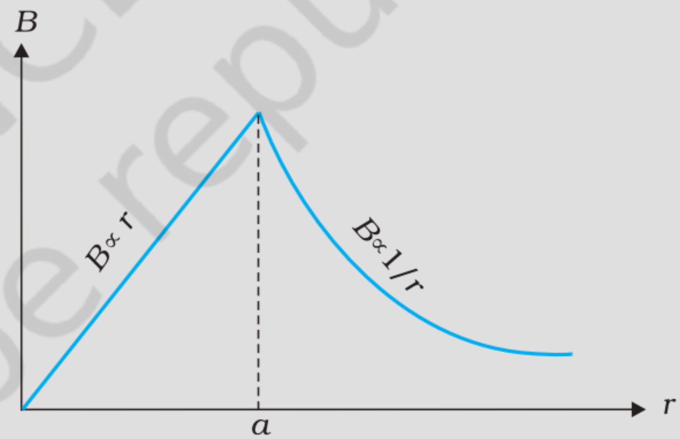


FIGURE 4.14

Figure (4.14) shows a plot of the magnitude of **B** with distance r from the centre of the wire. The direction of the field is tangential to the respective circular loop (1 or 2) and given by the right-hand rule described earlier in this section.

This example possesses the required symmetry so that Ampere's law can be applied readily.

Example 4.9

Example 4.9 A solenoid of length 0.5 m has a radius of 1 cm and is made up of 500 turns. It carries a current of 5 A. What is the magnitude of the magnetic field inside the solenoid?

Solution The number of turns per unit length is,

$$n = \frac{500}{0.5} = 1000 \text{ turns/m}$$

The length $l = 0.5$ m and radius $r = 0.01$ m. Thus, $l/a = 50$ i.e., $l >> a$. Hence, we can use the *long* solenoid formula, namely, Eq. (4.20)

$$B = \mu_0 n I$$
$$= 4\pi \times 10^{-7} \times 10^3 \times 5$$
$$= 6.28 \times 10^{-3} \text{ T}$$

Example 4.10 is useless

Example 4.11 A 100 turn closely wound circular coil of radius 10 cm carries a current of 3.2 A. (a) What is the field at the centre of the coil? (b) What is the magnetic moment of this coil?

The coil is placed in a vertical plane and is free to rotate about a horizontal axis which coincides with its diameter. A uniform magnetic field of 2T in the horizontal direction exists such that initially the axis of the coil is in the direction of the field. The coil rotates through an angle of 90° under the influence of the magnetic field. (c) What are the magnitudes of the torques on the coil in the initial and final position? (d) What is the angular speed acquired by the coil when it has rotated by 90°? The moment of inertia of the coil is 0.1 kg m^2.

Solution

(a) From Eq. (4.16)

$$B = \frac{\mu_0 N I}{2R}$$

Here, N = 100; I = 3.2 A, and R = 0.1 m. Hence,

$$B = \frac{4\pi \times 10^{-7} \times 3.2}{2 \times 10^{-1}} = \frac{4 \times 10^{-5} \times 10}{2 \times 10^{-1}} \qquad \text{(using } \pi \times 3.2 = 10\text{)}$$

$$= 2 \times 10^{-3}\ \text{T}$$

The direction is given by the right-hand thumb rule.

(b) The magnetic moment is given by Eq. (4.30),

$m = N I A = N I \pi r^2 = 100 \times 3.2 \times 3.14 \times 10^{-2} = 10\ \text{A m}^2$

The direction is once again given by the right-hand thumb rule.

(c) $\tau = |\mathbf{m} \times \mathbf{B}|$ [from Eq. (4.29)]

$= m\,B \sin \theta$

Initially, $\theta = 0$. Thus, initial torque $\tau_i = 0$. Finally, $\theta = \pi/2$ (or 90º). Thus, final torque $\tau_f = m\,B = 10 \times 2 = 20$ N m.

(d) From Newton's second law,

$$\mathcal{I} \frac{\mathrm{d}\omega}{\mathrm{d}t} = mB \sin\theta$$

where $\mathcal{I}$ is the moment of inertia of the coil. From chain rule,

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{\mathrm{d}\omega}{\mathrm{d}\theta}\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{\mathrm{d}\omega}{\mathrm{d}\theta}\omega$$

Using this,

$$\mathcal{I}\ \omega \mathrm{d}\omega = m\,B \sin\theta\, \mathrm{d}\theta$$

Integrating from $\theta = 0$ to $\theta = \pi/2$,

$$\mathcal{I} \int_0^{\omega_f} \omega\, \mathrm{d}\omega = mB \int_0^{\pi/2} \sin\theta\, \mathrm{d}\theta$$

$$\mathcal{I} \frac{\omega_f^2}{2} = -m\,B \cos\theta \Big|_0^{\pi/2} = m\,B$$

$$\omega_f = \left(\frac{2\,mB}{\mathcal{I}}\right)^{1/2} = \left(\frac{2 \times 20}{10^{-1}}\right)^{1/2} = 20\text{s}^{-1}$$

Example 4.12

Example 4.12

(a) A current-carrying circular loop lies on a smooth horizontal plane. Can a uniform magnetic field be set up in such a manner that the loop turns around itself (i.e., turns about the vertical axis).

(b) A current-carrying circular loop is located in a uniform external magnetic field. If the loop is free to turn, what is its orientation of stable equilibrium? Show that in this orientation, the flux of the total field (external field + field produced by the loop) is maximum.

(c) A loop of irregular shape carrying current is located in an external magnetic field. If the wire is flexible, why does it change to a circular shape?

Solution

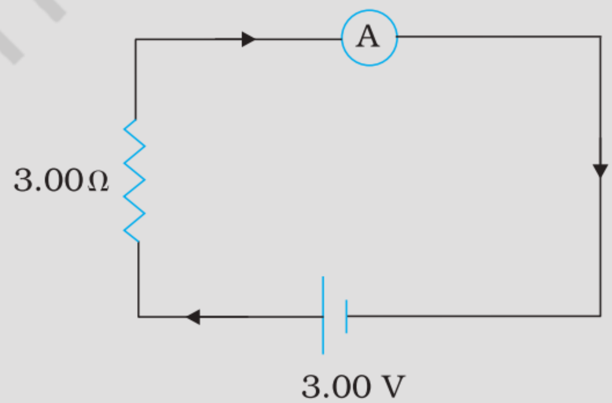
(a) No, because that would require $\boldsymbol{\tau}$ to be in the vertical direction. But $\boldsymbol{\tau} = I\,\mathbf{A} \times \mathbf{B}$, and since $\mathbf{A}$ of the horizontal loop is in the vertical direction, τ would be in the plane of the loop for any $\mathbf{B}$.

(b) Orientation of stable equilibrium is one where the area vector $\mathbf{A}$ of the loop is in the direction of external magnetic field. In this orientation, the magnetic field produced by the loop is in the same direction as external field, both normal to the plane of the loop, thus giving rise to maximum flux of the total field.

(c) It assumes circular shape with its plane normal to the field to maximise flux, since for a given perimeter, a circle encloses greater area than any other shape.

Example 4.13

Example 4.13 In the circuit (Fig. 4.23) the current is to be measured. What is the value of the current if the ammeter shown (a) is a galvanometer with a resistance R_G = 60.00 Ω; (b) is a galvanometer described in (a) but converted to an ammeter by a shunt resistance r_s = 0.02 Ω; (c) is an ideal ammeter with zero resistance?



Solution

(a) Total resistance in the circuit is,

$R_G + 3 = 63\,\Omega$. Hence, I = 3/63 = 0.048 A.

(b) Resistance of the galvanometer converted to an ammeter is,

$$\frac{R_G\, r_s}{R_G + r_s} = \frac{60\quad \Omega \times 0.02\Omega}{(60 + 0.02)\Omega} \simeq 0.02\Omega$$

Total resistance in the circuit is,

$0.02\,\Omega + 3\,\Omega = 3.02\,\Omega$. Hence, I = 3/3.02 = 0.99 A.

(c) For the ideal ammeter with zero resistance,

I = 3/3 = 1.00 A