MCQ preparation
ch 3
example

Date : 22/01/24
Time00

Section A

- **Write the answer of the following questions.** [Each carries 3 Marks] **[6]**

1. An electric toaster uses nichrome for its heating element. When a negligibly small current passes through it, its resistance at room temperature (27.0 °C) is found to be 75.3 Ω. When the toaster is connected to a 230 V supply, the current settles, after a few seconds, to a steady value of 2.68 A. What is the steady temperature of the nichrome element ? The temperature coefficient of resistance of nichrome averaged over the temperature range involved, is 1.70×10^{-4} °C^{-1}.

2. The resistance of the platinum wire of a platinum resistance thermometer at the ice point is 5 Ω and at steam point is 5.39 Ω. When the thermometer is inserted in a hot bath, the resistance of the platinum wire is 5.795 Ω. Calculate the temperature of the bath.

Section B

- **Write the answer of the following questions.** [Each carries 4 Marks] **[20]**

3. (a) Estimate the average drift speed of conduction electrons in a copper wire of cross-sectional area 1.0×10^{-7} m^2 carrying a current of 1.5 A. Assume that each copper atom contributes roughly one conduction electron. The density of copper is 9.0×10^3 kg/m^3 and its atomic mass is 63.5 u.

(b) Compare the drift speed obtained above with, (i) thermal speeds of copper atoms at ordinary temperatures, (ii) speed of propagation of electric field along the conductor which causes the drift motion.

4. (a) In illustration no. 1, the electron drift speed is estimated to be only a few mm s^{-1} for currents in the range of a few amperes ? How then is current established almost the instant a circuit is closed ?

(b) The electron drift arises due to the force experienced by electrons in the electric field inside the conductor. But force should cause acceleration. Why then do the electrons acquire a steady average drift speed ?

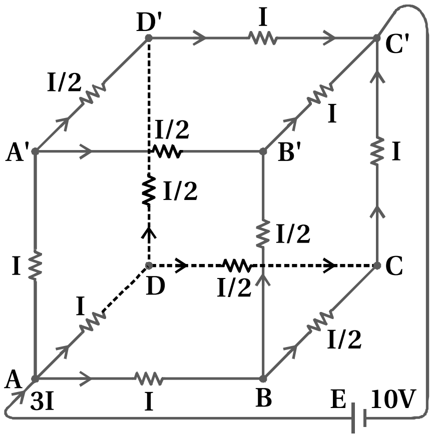
(c) If the electron drift speed is so small, and the electron's charge is small, how can we still obtain large amounts of current in a conductor ?

(d) When electrons drift in a metal from lower to higher potential, does it mean that all the 'free' electrons of the metal are moving in the same direction ?

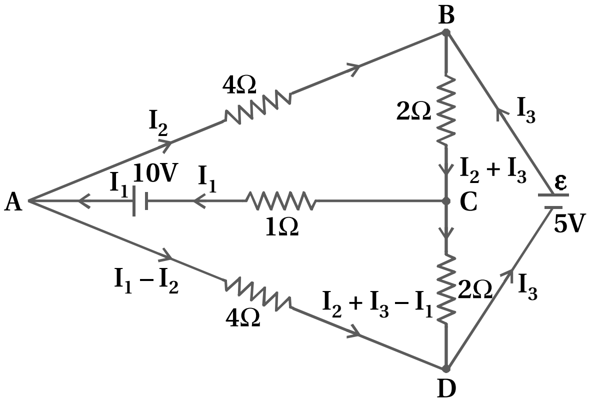
(e) Are the paths of electrons straight lines between successive collisions (with the positive ions of the metal) in the (i) absence of electric field, (ii) presence of electric field ?

5. A battery of 10 V and negligible internal resistance is connected across the diagonally opposite corners of a cubical network consisting of 12 resistors each of resistance 1 Ω (figure). Determine the equivalent resistance of the network and the current along each edge of the cube.

3I

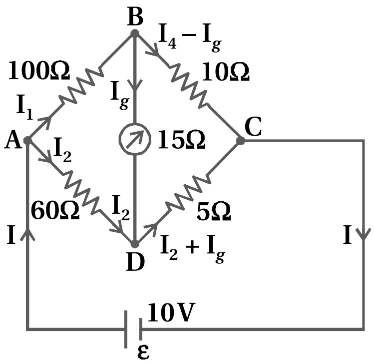


6. Determine the current in each branch of the network shown in figure.



7. The four arms of a Wheatstone bridge figure have the following resistances :

AB = 100 Ω, BC = 10 Ω, CD = 5 Ω and DA = 60 Ω



A galvanometer of 15 Ω resistance is connected across BD. Calculate the current through the galvanometer when a potential difference of 10 V is maintained across AC.

Section A

- **Write the answer of the following questions.** [Each carries 3 Marks] **[6]**

1. An electric toaster uses nichrome for its heating element. When a negligibly small current passes through it, its resistance at room temperature (27.0 °C) is found to be 75.3 Ω. When the toaster is connected to a 230 V supply, the current settles, after a few seconds, to a steady value of 2.68 A. What is the steady temperature of the nichrome element ? The temperature coefficient of resistance of nichrome averaged over the temperature range involved, is 1.70×10^{-4} °C^{-1}.

➠ Initially when extremely small current passes through nichrome filament, its temperature can be taken equal to room temperature $T_1 = 27$ °C. At this temperature its resistance is $R_1 = 75.3\ \Omega$.

➠ Now when given toaster is connected across V = 230 volt steady current passing through it (after very short time) is I = 2.68 A and its temperature increases to T_2, which is to be found out. At this temperature resistance of nichrome filament is given by formula

$R_2 = R_1\{1 + \alpha\ (T_2 - T_1)\}$

(where α = temperature coefficient of resistance)

$\therefore\ \dfrac{V}{I} = R_1\{1 + \alpha(T_2 - T_1)\}$

$(\because\ R_2 = \dfrac{V}{I}$ = resistance of hot filament)

$\therefore\ \dfrac{230}{2.68} = 75.3\left\{1 + 1.7 \times 10^{-4}(T_2 - 27)\right\}$

$\therefore\ 1.1397 = 1 + 1.7 \times 10^{-4}(T_2 - 27)$

$\therefore\ T_2 - 27 = \dfrac{0.1397}{1.7 \times 10^{-4}}$

$\therefore\ T_2 - 27 = 821.76$

$\therefore\ T_2 = 848.76$ °C

2. The resistance of the platinum wire of a platinum resistance thermometer at the ice point is 5 Ω and at steam point is 5.39 Ω. When the thermometer is inserted in a hot bath, the resistance of the platinum wire is 5.795 Ω. Calculate the temperature of the bath.

➠ Here, T_0 = ice point = 0 °C, $R_0 = 5\ \Omega$

T_1 = steam point = 100 °C, $R_1 = 5.23\ \Omega$

T_2 = ?, $R_2 = 5.795\ \Omega$

➠ $R_1 = R_0\{1 + \alpha(T_1 - T_0)\}$

$\therefore\ 5.23 = 5\{1 + \alpha(100 - 0)\}$

$\therefore\ 1.046 = 1 + \alpha(100)$

$\therefore\ 0.046 = \alpha(100)$ ... (1)

➠ $R_2 = R_0\{1 + \alpha(T_2 - T_0)\}$

$5.795 = 5\{1 + \alpha(T_2 - 0)\}$

$\therefore 1.159 = 1 + \alpha(T_2)$

$\therefore 0.159 = \alpha(T_2)$ ... (2)

➠ Taking ratio of equations (1) and (2),

$$0.2893 = \frac{100}{T_2}$$

$$\therefore T_2 = \frac{100}{0.2893} = 345.7\ °C$$

Section B

● **Write the answer of the following questions.** [Each carries 4 Marks] **[20]**

3. (a) Estimate the average drift speed of conduction electrons in a copper wire of cross-sectional area $1.0 \times 10^{-7}\ m^2$ carrying a current of 1.5 A. Assume that each copper atom contributes roughly one conduction electron. The density of copper is $9.0 \times 10^3\ kg/m^3$ and its atomic mass is 63.5 u.

(b) Compare the drift speed obtained above with, (i) thermal speeds of copper atoms at ordinary temperatures, (ii) speed of propagation of electric field along the conductor which causes the drift motion.

➠ (a) If valency of given element is p then its one atom will have p no. of free electrons. Hence, its total N no. of atoms, in total volume V will have pN no. of free electrons. Hence, free electron number density of this element would be :

➠ $I = 1.5$ A, $A = 1 \times 10^{-7}$, $e = 1.6 \times 10^{-19}$

Density $= 9.0 \times 10^3\ kg/m^3$

Atomic mass = 63.54 u

$$\text{No. of moles} = \frac{n}{N_A} = \frac{M}{M_0}$$

$$\therefore n = \frac{M \cdot N_A}{M_0}$$

$$\therefore n = \frac{\rho \cdot V \cdot N_A}{M_0} \qquad (\because M = \rho V)$$

$$\therefore n = \frac{9 \times 10^3 \times 1 \times 6.02 \times 10^{23}}{63.5 \times 10^{-3}}$$

(For 'n' volume $V = 1 m^3$)

$\therefore n = 8.5 \times 10^{28}\ m^{-3}$

$I = nAv_d e$

$$v_d = \frac{I}{nAe} = \frac{1.5}{8.5 \times 10^{28} \times 1 \times 10^{-7} \times 1.6 \times 10^{-19}}$$

$= 1.1 \times 10^{-3}$ m/s

$= 1.1$ mm/s

(b) (i) At ordinary temperature (room temperature), T = 27 °C

= (27 + 273) K = 300 K, if rms value of vibrational speed of copper atom (known as thermal speed because vibrations of atoms are due to absorption of thermal energy from surrounding) is v_{rms} then we have the formula,

$$\frac{1}{2}m < v^2 > = \frac{3}{2} k_B T$$

(Where k_B = Boltzman constant

$= 1.38 \times 10^{-23}$ Jmol^{-1}K^{-1}

T = absolute temperature)

[Here, m = Atomic mass]

$$\therefore \; < v^2 > = \frac{3k_B T}{m}$$

$$\therefore \; \sqrt{< v^2 >} = \sqrt{\frac{3k_B T}{m}}$$

$$\therefore \; v_{rms} = \sqrt{\frac{3k_B T}{m}} \quad \left(\because \sqrt{< v^2 >} = v_{rms}\right)$$

$$\therefore \; v_{rms} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300}{63.5 \times 1.66 \times 10^{-27}}}$$

$\therefore \; v_{rms} = 343.2$ m/s

Now,

$$\frac{v_d}{v_{rms}} = \frac{1.098 \times 10^{-3}}{343.2} = 3.2 \times 10^{-6}$$

$\therefore \; v_d = 3.2 \times 10^{-6} \, v_{rms}$

(ii) When a metallic wire is connected across a battery, electric field is found to be established inside the wire with a speed almost equal to velocity of light in vacuum, $c = 3 \times 10^8$ m/s.

Now,

$$\frac{v_d}{c} = \frac{1.1 \times 10^{-3}}{3 \times 10^8} = 3.66 \times 10^{-12}$$

$\therefore \; v_d = 3.66 \times 10^{-12} \, c$

4. (a) In illustration no. 1, the electron drift speed is estimated to be only a few mm s^{-1} for currents in the range of a few amperes ? How then is current established almost the instant a circuit is closed ?

(b) The electron drift arises due to the force experienced by electrons in the electric field inside the conductor. But force should cause acceleration. Why then do the electrons acquire a steady average drift speed ?

(c) If the electron drift speed is so small, and the electron's charge is small, how can we still obtain large amounts of current in a conductor ?

(d) When electrons drift in a metal from lower to higher potential, does it mean that all the 'free' electrons of the metal are moving in the same direction ?

(e) Are the paths of electrons straight lines between successive collisions (with the positive ions of the metal) in the (i) absence of electric field, (ii) presence of electric field ?

➠ (a) When a battery is connected across a conducting wire, electric field is found to be established in that wire almost with a speed of light. This causes instantaneous local drift of every free electron and that too with a tremendous acceleration because its value given by $a = \frac{eE}{m}$ is found to be very large (because m being mass of electron is extremely small equal to 9.1×10^{-31} kg)

➠ Hence all the free electrons start drifting simultaneously which produces instantaneous current in the wire. This current achieves its steady value within a very short time. (That is why bulb gets on almost at the same moment when we switch it on).

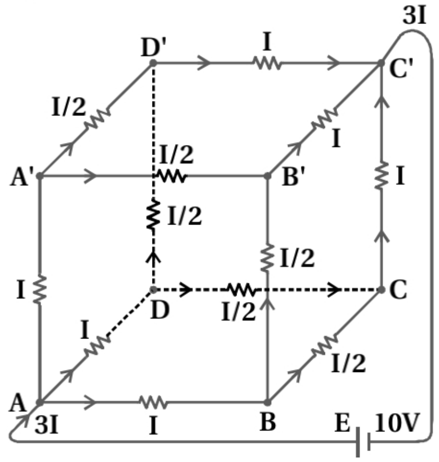
(b) In a current carrying conductor, each free electron is accelerated instantaneously by acceleration $a = \frac{eE}{m}$ which increases the drift speed of free electron until it collides with randomly oscillating positive ion in the metal. After collision, free electron comes to rest. Again it gets accelerated, again its drift speed increases, again it undergoes another collision, again it comes to rest and so on. Here frequency of such collision is very large and so time lapsed between two successive collisions (known as relaxation time) is very small. Hence, on average we can consider all the free electrons drifting continuously.

(c) Because free electron number density is very large in the metal approximately 10^{29} per $1\ m^3$, it contributes majority in producing large current in the metallic wires.

(d) No, drift velocities of all the free electrons are not in the same direction because free electrons have to move in the direction of resultant velocity which is a superposition of drift velocity and thermal velocity.

(e) In the absence of external electric field, paths of free electrons between two successive collisions are straight but in the presence of external electric field, these paths are curved (or non linear) in general.

5. A battery of 10 V and negligible internal resistance is connected across the diagonally opposite corners of a cubical network consisting of 12 resistors each of resistance 1 Ω (figure). Determine the equivalent resistance of the network and the current along each edge of the cube.

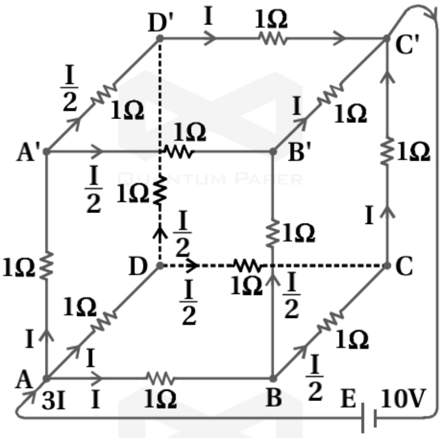


➠ Figure shows cubical network consisting of 12 resistors each of resistance R = 1 Ω.

➠ A battery of ε = 10 V is connected between the vertices A and C' are opposite, one of it is diagonal with negligible internal resistance and a current of 3A is flowing through the circuit.

➠ Based on symmetry the currents flowing through each arm and their direction are assumed as shown in the figure.

3I



- Next, take a closed loop, say ABCC'EA and apply Kirchoff's second rule :

$$-IR - \frac{IR}{2} - IR + \varepsilon = 0$$

$$\therefore \frac{5IR}{2} = \varepsilon \qquad \dots (1)$$

- If equivalent resistance of given network is R_{eq} and current of 3I is passed through it then,

$$3IR_{eq} = \varepsilon \qquad \dots (2)$$

Comparing equations (1) and (2),

$$3IR_{eq} = \frac{5IR}{2}$$

$$\therefore R_{eq} = \frac{5}{6}R$$

Now, if equivalent resistance R = 1 Ω, then

$$R_{eq} = \frac{5}{6} \times 1 = \frac{5}{6}\,\Omega$$

- Current passing through the battery,

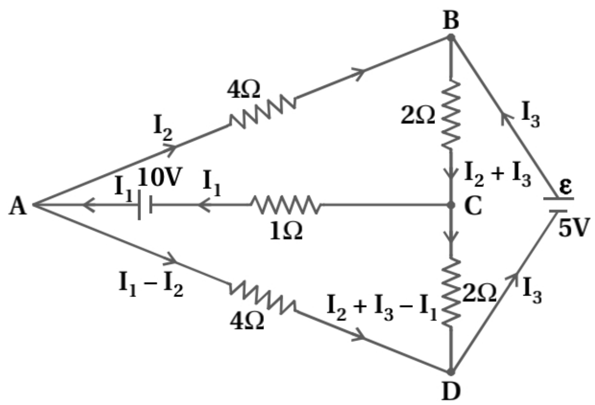
$$3I = \frac{\varepsilon}{R_{eq}} = \frac{10}{5/6} = \frac{60}{5}$$

$\therefore 3I = 12$ A

$\therefore I = 4$ A

$\therefore$ Current passing through AA', AD, AB, D'C', B'C' and CC' vertices are 4 A and A'B', A'D', DD', DC, BB' and BC then current passing through vertices is $\frac{I}{2} = \frac{4}{2} = 2$A

6. Determine the current in each branch of the network shown in figure.



➠ Applying KVL ($\Sigma IR = \Sigma\varepsilon$) in the closed loop ADCA, we get

$-4(I_1 - I_2) + 2(I_2 + I_3 - I_1) - I_1 = -10$

$\therefore\ 4(I_1 - I_2) - 2(I_2 + I_3 - I_1) + I_1 = 10$

$\therefore\ 7I_1 - 6I_2 - 2I_3 = 10$ ... (1)

➠ Similarly for closed loop ABCDA,

$-4I_2 - 2(I_2 + I_3) - I_1 = -10$

$\therefore\ 4I_2 + 2(I_2 + I_3) + I_1 = 10$

$\therefore\ I_1 + 6I_2 + 2I_3 = 10$ ... (2)

➠ For closed loop BCDεB,

$-2(I_2 + I_3) - 2(I_2 + I_3 - I_1) = -5$

$\therefore\ 2(I_2 + I_3) + 2(I_2 + I_3 - I_1) = 5$

$\therefore\ 4I_2 + 4I_3 - 2I_1 = 5$

$\therefore\ 2I_1 - 4I_2 - 4I_3 = -5$

$\therefore\ I_1 - 2I_2 - 2I_3 = -2.5$ ... (3)

➠ Adding equations (1) and (2),

$8I_1 = 20$

$\therefore\ I_1 = \frac{5}{2}\,A$ ... (4)

➠ Adding equations (2) and (3),

$2I_1 + 4I_2 = 7.5$

$\therefore\ 2\left(\frac{5}{2}\right) + 4I_2 = 7.5$

$\therefore\ 5 + 4I_2 = 7.5$

$\therefore\ I_2 = \frac{2.5}{4} = \frac{25}{40} = \frac{5}{8}\,A$ ... (5)

➠ Substituting values of I_1 and I_2 in equation (3),

$2.5 - 2\left(\frac{5}{8}\right) - 2I_3 = -2.5$

$\therefore\ 2.5 - \frac{5}{4} + 2.5 = 2I_3$

$\therefore\ 10 - 5 + 10 = 8I_3$

$\therefore\ 15 = 8I_3$

$\therefore\ I_3 = \frac{15}{8}\,A$ ... (6)

Current passing through the each branch :

In branch AC : $I_1 = \frac{5}{2}\,A$

In branch AB : $I_2 = \frac{5}{8}\,A$

In branch BED : $I_3 = \frac{15}{8}$ A

In branch AD : $I_1 - I_2 = \frac{5}{2} - \frac{5}{8} = \frac{15}{8}$ A

In branch BC : $I_2 + I_3 = \frac{5}{8} + \frac{15}{8} = \frac{5}{2}$ A

In branch CD : $I_2 + I_3 - I_1 = \frac{5}{8} + \frac{15}{8} - \frac{5}{2} = 0$

➠ Verify Kirchoff's loop rule by taking potential difference of loop BADεB,

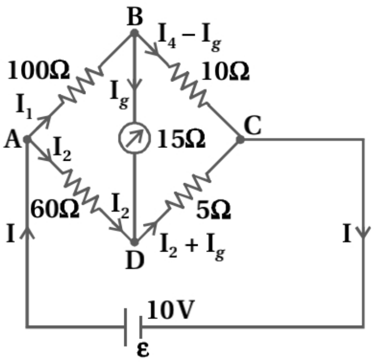
$$4I_2 - 4(I_1 - I_2) + \varepsilon = -4 \times \frac{5}{8} - 4\left(\frac{5}{2} - \frac{5}{8}\right) + 5$$

$$= -\frac{5}{2} - 4\left(\frac{15}{8}\right) + 5$$

$$= -\frac{5}{2} - \frac{5}{2} + 5$$

$$= 0$$

7. The four arms of a Wheatstone bridge figure have the following resistances :
AB = 100 Ω, BC = 10 Ω, CD = 5 Ω and DA = 60 Ω



A galvanometer of 15 Ω resistance is connected across BD. Calculate the current through the galvanometer when a potential difference of 10 V is maintained across AC.

➠ Applying loop rule for closed loop BADB, we get,

$100I_1 - 60I_2 + 15I_g = 0$

$\therefore 20I_1 - 12I_2 + 3I_g = 0$ ... (1)

(Where I_g = current passing through galvanometer)

➠ Applying loop rule for closed loop BCDB, we get,

$-10(I_1 - I_g) + 5(I_2 + I_g) + 15I_g = 0$

$\therefore -10I_1 + 10I_g + 5I_2 + 5I_g + 15I_g = 0$

$\therefore -10I_1 + 5I_2 + 30I_g = 0$

Dividing by – 5, we get,

$2I_1 - I_2 - 6I_g = 0$ ... (2)

➠ Applying KVL for closed loop ADCεA, we get

$-60I_2 - 5(I_2 + I_g) = -10$

$\therefore -65I_2 - 5I_g = -10$

Dividing by – 5, we get

$$13I_2 + I_g = 2 \qquad \dots (3)$$

➠ Multiplying equation (2) by 10, we get

$$20I_1 - 10I_2 - 60I_g = 0 \qquad \dots (4)$$

➠ Subtracting equation (4) from equation (1),

$$-2I_2 + 63I_g = 0$$

$$\therefore I_2 = \frac{63}{2} I_g \qquad \dots (5)$$

➠ From equation (3) and (5),

$$13\left(\frac{63}{2} I_g\right) + I_g = 2$$

$$\therefore 819I_g + 2I_g = 4$$

$$\therefore 821I_g = 4$$

$$\therefore I_g = \frac{4}{821} = 0.00487 \text{ A} = 4.87 \times 10^{-3} \text{ A}$$

$$= 4.87 \text{ mA}$$