Question 1:

Monochromatic light having a wavelength of 589 nm from the air is incident on a water surface. Find the frequency, wavelength and speed of (i) reflected and (ii) refracted light. [1.33 is the refractive index of water]

Answer:

Monochromatic light incident having wavelength,

λ = 589 nm = 589 x 10^{-9} m

Speed of light in air, **c = 3 x 10^{8} m s^{-1}**

Refractive index of water,

μ **= 1.33**

(i) The ray will be reflected back to the same medium through which it passed.

Therefore, the wavelength, speed and frequency of the reflected ray will be the same as that of the incident ray.

The frequency of light can be found from the relation:

$$v = \frac{c}{\lambda}$$

$$= \frac{3\times10^{8}}{589\times10^{-9}}$$

$$= 5.09 \times 10^{14} \text{ Hz}$$

Hence, **c = 3 x 10^{8} m s^{-1}, 5.09 × 10^{14} Hz, and 589 nm** are the speed, frequency and wavelength of the reflected light.

(b) The frequency of light which is travelling never depends upon the property of the medium. Therefore, the frequency of the refracted ray in water will be equal to the frequency of the incident or reflected light in the air.

Refracted frequency, **v = 5.09 x 10^{14} Hz**

Following is the relation between the speed of light in water and the refractive index of the water:

$$v = \frac{c}{\mu}$$

$$= v = \frac{3\times10^{8}}{1.33}$$

$$= 2.26 \times 10^{8} \text{ m s}^{-1}$$

Below is the relation for finding the wavelength of light in water:

$\lambda = \frac{v}{V}$

=

$\frac{2.26\times10^{8}}{5.09\times10^{14}}$

$= 444.007 \times 10^{-9}$ m $\mathbf{= 444.01\ nm}$

Therefore, 444.007×10^{-9} m, 444.01nm, and 5.09×10^{14} Hz are the speed, frequency, and wavelength of the refracted light.

Question 2:

What is the shape of the wavefront in each of the following cases:

(i) Light diverging from a point source.

(ii) Light emerging out of a convex lens when a point source is placed at its focus.

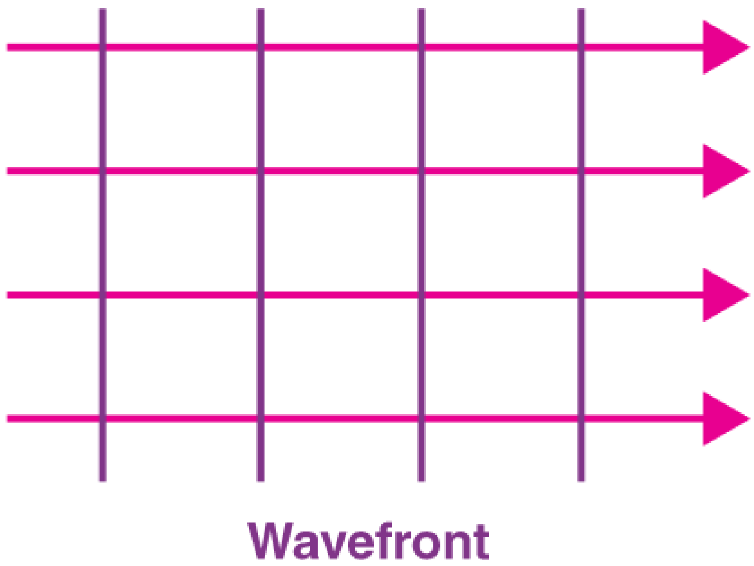
(iii) The portion of the wavefront of the light from a distant star intercepted by the Earth.

Answer:

(i) When the light diverges from a point source, the shape of the wavefront is spherical. Following is the figure of the wavefront:

Point source

(ii) The shape of the wavefront of the light emerging from a convex lens when the point source is placed at its focus is parallel odd.



(iii) When the light is coming from a distant star that is intercepted by the earth, the shape of the wavefront is plane.

Question 3:

(i) The refractive index of glass is 1.5. What is the speed of light in glass? The speed of light in a vacuum is (3.0×10^8 m s^{-1})

(ii) Is the speed of light in glass independent of the colour of light? If not, which of the two colours, red and violet, travels slower in a glass prism?

Answer:

(i) Refractive Index of glass,

μ **= 1.5**
Speed of light, c = 3×10^8 ms^{-1}

The relation for the speed of light in a glass is:

$v = \frac{c}{\mu}$

=

$\frac{3\times10^8}{1.5}$

=

$2 \times 10^8 m/s$

Hence, the speed of light in glass is 2 × 10^8 m s^{-1}

(ii) The speed of light is dependent on the colour of the light. For a white light, the refractive index of the violet component is greater than the refractive index of the red component. So, the speed of violet light is less than the speed of red light in the glass. This will reduce the speed of violet light in a glass prism when compared with red light.

Question 4:

In Young's double-slit experiment, the slits are separated by 0.28 mm, and the screen is placed 1.4 m away. 1.2cm is the distance between the central bright fringe and the fourth bright fringe. Determine the wavelength of light used in the experiment.

Answer:

Distance between the slits and the screen, **D = 1.4 m**

and the distance between the slits, d = 0.28 mm = 0.28 x 10^{-3} m

Distance between the central fringe and the fourth (n = 4) fringe,

u = 1.2cm **= 1.2 × 10^{-2} m**

For constructive interference, the following is the relation for the distance between the two fringes:

$$u = n\,\lambda\,\frac{D}{d}$$

Where, **n = order of fringes**

= 4

λ **= Wavelength of light used**

Rearranging the formula, we get

$\lambda = \frac{ud}{nD}$

=

$\frac{1.2\times10^{-2}\times0.28\times10^{-3}}{4\times1.4}$

= 6 × 10^{-7} m = 600 nm

Therefore, 600 nm is the wavelength of the light.

Question 5:

In Young's double-slit experiment using the monochromatic light of wavelength λ, the intensity of light at a point on the screen where path difference is λ is K units. What is the intensity of light at a point where the path difference is

$\frac{\lambda}{3}$ **?**

Answer:

Let I_1 and I_2 be the intensity of the two light waves. Their resultant intensities can be obtained as:

$$I' = I_1 + I_2 + 2\sqrt{I_1\ I_2}\ cos\phi$$

Where,

ϕ = Phase difference between the two waves

For monochromatic light waves:

$I_1 = I_2$

Therefore

$$I' = I_1 + I_2 + 2\sqrt{I_1\ I_2}\ cos\phi$$

=

$$2I_1 + 2I_1\ cos\phi$$

Phase difference =

$\frac{2\pi}{\lambda} \times Path\ difference$

Since path difference = λ,

Phase difference,

$\phi = 2\pi$

and I' = K [Given]
Therefore

$I_1 = \frac{K}{4}$ . . . . **(i)**

When path difference= $\frac{\lambda}{3}$

Phase difference,

$\phi = \frac{2\pi}{3}$

Hence, the resultant intensity:

$$I'_g = I_1 + I_1 + 2\sqrt{I_1\,I_1}\,cos\frac{2\pi}{3}$$

$$= 2I_1 + 2I_1(-\tfrac{1}{2})$$

Using equation (i), we can write:

$I_g = I_1 = \frac{K}{4}$

Hence, the intensity of light at a point where the path difference is

$\frac{\lambda}{3}$ is $\frac{K}{4}$ units.

Question 6: A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes in Young's double-slit experiment.

(a) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 650 nm.

(b) What is the least distance from the central maximum where the bright fringes due to both wavelengths coincide?

Answer:

Wavelength of the light beam,

λ_1 **= 650 nm**

Wavelength of another light beam,

λ_2 **= 520 nm**

Distance of the slits from the screen = D

Distance between the two slits = d

(i) Distance of the n[th] bright fringe on the screen from the central maximum is given by the relation,

x = $n\,\lambda_1\left(\frac{D}{d}\right)$

For the third bright fringe, **n = 3**

Therefore, x =

$$3 \times 650\frac{D}{d} = 1950\frac{D}{d}nm$$

(b) Let, the n^{th} bright fringe due to wavelength λ_2 and $(n-1)^{th}$ bright fringe due to wavelength

λ_2 coincide on the screen. The value of n can be obtained by equating the conditions for bright fringes:

$$n\lambda_2 = (n-1)\lambda_1$$

520n = 650n – 650

650 = 130n

Therefore, n = 5

Hence, the least distance from the central maximum can be obtained by the relation:

x =

$$n\,\lambda_2\,\frac{D}{d}$$
=
$$5 \times 520\frac{D}{d} = 2600\frac{D}{d}$$
nm

Note: The value of d and D are not given in the question.

Question 7:

In a double-slit experiment, 0.2° is found to be the angular width of a fringe on a screen placed 1 m away. The wavelength of light used is 600 nm. What will be the angular width of the fringe if the entire experimental apparatus is immersed in water? Take the refractive index of water to be $\frac{4}{3}$.

Answer:

Distance of the screen from the slits, **D = 1 m**

Wavelength of light used,

λ_1 **= 600 nm**

Angular width of the fringe in air

θ_1 **= 0.2°**

Angular width of the fringe in water= θ_2

Refractive index of water,

$$\mu = \frac{4}{3}$$

$$\mu = \frac{\theta_1}{\theta_2}$$ is the relation between the refractive index and the angular width

$$\theta_2 = \frac{3}{4}\theta_1$$
$$= \frac{3}{4} \times 0.2 = 0.15$$

Therefore, 0.15° is the reduction in the angular width of the fringe in water.

Question 8: What is the [Brewster] angle for air-to-glass transition? (Refractive index of glass = 1.5.)

Answer:

Refractive index of glass,

μ =1.5

Consider Brewster angle = θ

Following is the relation between the Brewster angle and the refractive index:

$$tan\theta = \mu$$
$$\theta = tan^{-1}(1.5)$$
$$= 56.31°$$

Therefore, the Brewster angle for air-to-glass transition is 56.31°

Question 9: Light of wavelength 5000 Armstrong falls on a plane reflecting surface. What are the wavelength and frequency of the reflected light? For what angle of incidence is the reflected ray normal to the incident ray?

Answer:

Wavelength of incident light,

[λ]
= 5000 Armstrong **= 5000 x 10^{-10} m**
Speed of light, **c =3 x 10^{8} m**

Following is the relation for the frequency of incident light:

v =

$$\frac{c}{\lambda} = \frac{3 \times 10^{8}}{5000 \times 10^{-10}} = 6 \times 10^{14}$$

The wavelength and frequency of incident light are equal to the reflected ray. Therefore, 5000 Armstrong and

6×10^{14} Hz is the wavelength and frequency of the reflected light. When the reflected ray is normal to the incident ray, the sum of the angle of incidence,

$\angle i$ and angle of reflection,

$\angle r$, is 90°.
From the laws of reflection, we know that the angle of incidence is always equal to the angle of reflection

$\angle i + \angle r = 90°$
i.e.

$\angle i + \angle i$

= 90°

Hence,

$$\angle i = \frac{90}{2}$$

= 45°

Therefore, 45° is the angle of incidence.

Question 10:

Estimate the distance for which ray optics is a good approximation for an aperture of 4 mm and wavelength of 400 nm.

Answer:

Fresnel's distance (Z_F) is the distance which is used in ray optics for a good approximation. Following is the relation,

$$Z_F = \frac{a^2}{\lambda}$$

Where,

Aperture width, a = 4 mm **= 4 × 10^{-3} m**

Wavelength of light,

λ = 400 nm **= 400 × 10^{-9} m**

$Z_F = \frac{(4\times10^{-3})^2}{400\times10^{-9}}$ **= 40m**

Therefore, 40 m is the distance for which the ray optics is a good approximation.

Question 11: The 6563 Å Hα line emitted by hydrogen in a star is found to be red-shifted by 15 Å. Estimate the speed with which the star is receding from the Earth.

Answer:

λ = 6563 Å

Δλ = 15 Å

Since the star is receding, the velocity (v) is negative.

Δλ = – vλ/c

v = – cΔλ/λ

$= -(3 \times 10^8) \times (15 \text{ Å}/ 6563 \text{ Å})$

$= -6.86 \times 10^5$ m/s

Question 12: Explain how Corpuscular theory predicts the speed of light in a medium, say, water, to be greater than the speed of light in a vacuum. Is the prediction confirmed by experimental determination of the speed of light in water? If not, which alternative picture of light is consistent with the experiment?

Answer:

According to Newton's Corpuscular theory, the velocity of light in the denser medium (water) is greater than the velocity of light in the rarer medium (vacuum). This was experimentally wrong.

At the angle of incidence (i) of the light of velocity v, the angle of refraction is r.

Due to the change in the medium, the change in the velocity of light in water is v.

Using Snells law,

c sin i = v sin r ——(1)

The relation between the velocities and the refractive index is

v/c = μ ———(2)

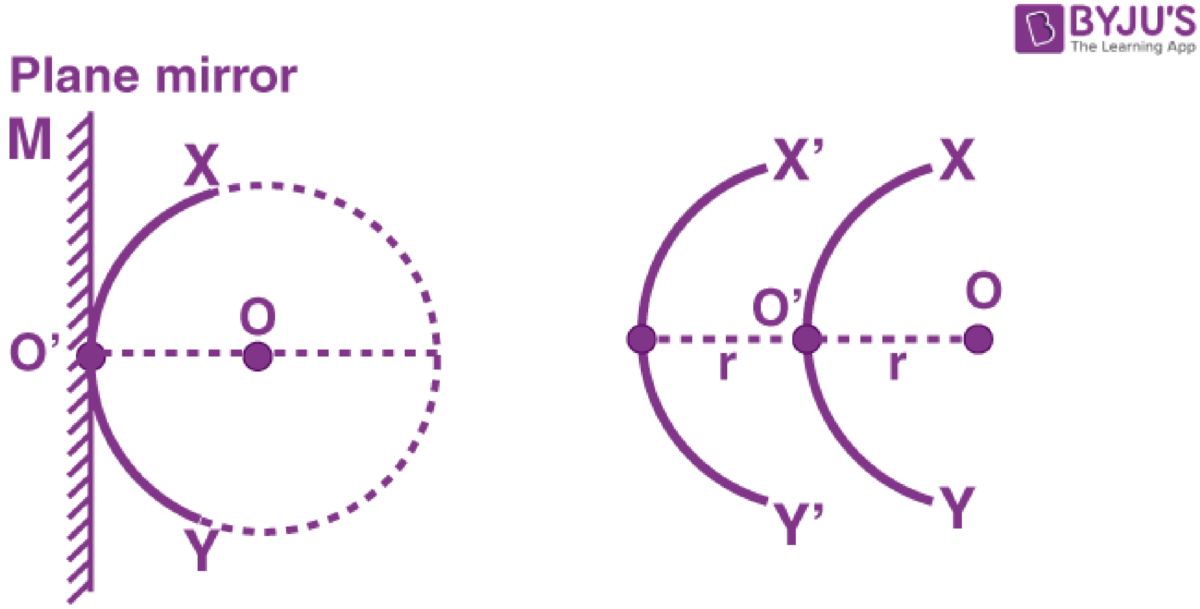
v/c = sin i/sin r = μ ——-(3)

But μ> 1, so v > c is not possible.

Huygens' wave theory is consistent with the experiment.

Question 13: You have learnt in the text how Huygens' principle leads to the laws of reflection and refraction. Use the same principle to deduce directly that a point object placed in front of a plane mirror produces a virtual image whose distance from the mirror is equal to the object's distance from the mirror.

Answer:



Consider an object O placed in front of the plane mirror MO' at a distance r. The object is taken as the centre point O, and a circle is drawn such that it just touches the plane mirror at point O'.

According to Huygens' Principle, XY is the wavefront of the incident light. If the mirror was not present, then a similar wavefront X'Y' (as XY) would form behind O' at distance r. X'Y' can be considered as a virtual reflected ray for the plane mirror. Therefore, a point object placed in front of the plane mirror produces a virtual image whose distance from the mirror is equal to the object distance (r).

Question 14: Let us list some of the factors which could possibly influence the speed of wave propagation:
(i) Nature of the source
(ii) The direction of propagation
(iii) The motion of the source and/or observer
(iv) Wavelength
(v) The intensity of the wave. On which of these factors, if any, does (a) the speed of light in a vacuum (b) the speed of light in a medium (say, glass or water) depend?

Answer:

(a) The speed of light in the vacuum does not depend on any of the factors listed.

(b) The speed of light in the medium depends on the wavelength of the light in that medium.

Question 15: For sound waves, the Doppler formula for frequency shift differs slightly between the two situations: (i) source at rest; observer moving, and (ii) source moving; observer at rest. The exact Doppler formulas for the case of light waves in a vacuum are, however, strictly identical for these situations. Explain why this should be so. Would you expect the formulas to be strictly identical for the two situations in the case of light travelling in a medium?

Answer:

The Doppler formula differs slightly between the two situations because sound waves can travel only through a medium. The motion of the observer relative to the medium is different in both cases. Hence, the doppler formula is different.

Light waves can propagate in a vacuum. In a vacuum, the speed of light does not depend on the motion of the observer and the source.

When light travels in the medium, the doppler formula for the two cases will be different.

Question 16: In a double-slit experiment using the light of wavelength 600 nm, the angular width of a fringe formed on a distant screen is 0.1°. What is the spacing between the two slits?

Answer:

The wavelength of the light, $\lambda = 600$ nm

The angular width of the fringe formed, $\theta = 0.1^0 = 0.1\ \pi/180$

Spacing between the slits, $d = \lambda/\theta = (600 \times 10^{-9} \times 180)/(0.1 \times 3.14)$

$= 108000 \times 10^{-9}/0.314$

$d = 3.44 \times 10^{-4}$ m

Question 17: Answer the following questions:
(a) In a single slit diffraction experiment, the width of the slit is made double the original width. How does this affect the size and intensity of the central diffraction band?
(b) In what way is diffraction from each slit related to the interference pattern in a double-slit experiment?
(c) When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the shadow of the obstacle. Explain why?
(d) Two students are separated by a 7 m partition wall in a room 10 m high. If both light and sound waves can bend around obstacles, how is it that the students are unable to see each other even though they can converse easily?
(e) Ray optics is based on the assumption that light travels in a straight line. Diffraction effects (observed when light propagates through small apertures/slits or around small obstacles) disprove this assumption. Yet the ray optics assumption is so commonly used in understanding location and several other properties of images in optical instruments. What is the justification?

Answer:

(a) In a single slit diffraction experiment, if the width of the slit is made double the original width, the size of the central diffraction band reduces to half, and the intensity of the band increases four times.

(b) If the width of each slit is comparable to the wavelength of light used, the interference pattern thus obtained in the double-slit experiment is modified by diffraction from each of the two slits.

(c) When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the shadow of the obstacle. The bright spot is formed due to the constructive interference of the light waves that get diffracted from the edges of the circular obstacle.

(d) The obstacle bends the waves by a large angle if the wavelength of the wave is comparable with the size of the obstacle. The wavelength of the light wave is much smaller than the size of the wall. Therefore, the diffraction angle is also very small. As a result, the students will not be able to see each other. On the other hand, the size of the wall and the wavelength of the sound wave is comparable. Hence, the diffraction angle is large. Therefore, students can hear each other.

(e) The size of the aperture in the optical instruments is much larger than the wavelength of light. Therefore, the diffraction effect of light is negligible in these instruments. Thus, the assumption of light travelling in a straight line can be used in these instruments.

Question 18: Two towers on top of two hills are 40 km apart. The line joining them passes 50 m above a hill halfway between the towers. What is the longest wavelength of radio waves which can be sent between the towers without appreciable diffraction effects?

Answer:

Distance between the towers = 40 km

Height of the line joining the hills, d = 50 m

The radial spread of the radio waves must not exceed 50 m

Aperture a = d = 50 m

The hill is located halfway between the towers. Therefore, Fresnel's distance is Z_p = 20 km

Fresnel's distance can be given by the equation, $Z_p = a^2/\lambda$

$\lambda = a^2/Z_p$

$= (50)^2/(20 \times 10^3)$

$= 250/20 = 12.5 \times 10^{-3}$ m = 12.5 cm

Question 19: A parallel beam of light of wavelength 500 nm falls on a narrow slit, and the resulting diffraction pattern is observed on a screen 1 m away. It is observed that the first minimum is at a distance of 2.5 mm from the centre of the screen. Find the width of the slit.

Answer:

The wavelength of the beam of light, λ = 500 nm

Distance between the slit and the screen, D= 1 m

Distance of the first minimum from the centre of the screen, $x = 2.5$ mm $= 2.5 \times 10^{-3}$ m

First minima, n = 1

Consider the equation, $n\lambda = xd/D$

$\Rightarrow d = n\lambda D/x = (1 \times 500 \times 10^{-9} \times 1)/(2.5 \times 10^{-3})$

$= 200 \times 10^{-6}$ m

Question 20: Answer the following questions:
(a) When a low-flying aircraft passes overhead, we sometimes notice a slight shaking of the picture on our TV

screen. Suggest a possible explanation.
(b) As you have learnt in the text, the principle of linear superposition of wave displacement is basic to understanding intensity distributions in diffraction and interference patterns. What is the justification for this principle?

Answer:

(a) The weak radar signals from the aircraft interfere with the TV signal received by the antenna.

(b) This is because superposition follows from the linear character of a differential equation that governs wave motion.

Question 21: In deriving the single slit diffraction pattern, it was stated that the intensity is zero at angles of nλ/a. Justify this by suitably dividing the slit to bring out the cancellation.

Answer:

Let “a” be the width of a single slit. The single slit is further divided into n smaller slits of width a’.

a’ = a/n

Each of the smaller slits should produce zero intensity for the single slit to produce zero intensity.

For this to happen, the angle of diffraction, $\theta = \lambda/a'$

$\Rightarrow \theta = \lambda/(a/n)$

or, $\theta = n\lambda/a$

Therefore, in deriving the single slit diffraction pattern, it was stated that the intensity is zero at angles of nλ/a.