Exercise 3.1 Page No: 64

1. In the matrix A

$$\begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$$

Write
(i) The order of the matrix, (ii) The number of elements,
(iii) Write the elements a_{13}, a_{21}, a_{33}, a_{24}, a_{23}.

Solution:
(i) In given matrix,
Number of rows = 3
Number of column = 4
Therefore, Order of the matrix is 3 x 4.

(ii) The number of elements in the matrix A is 3 x 4 = 12.

(iii) a_{13} = element in first row and third column = 19

a_{21} = element in second row and first column = 35

a_{33} = element in third row and third column = - 5

a_{24} = element in second row and fourth column = 12

a_{23} = element in second row and third column = 5/2

2. If a matrix has 24 elements, what are the possible orders it can have? What, if it has 13 elements?

Solution:
We know that, a matrix of order mxn having mn elements.

There are 8 possible matrices having 24 elements of orders are as follows:
1 x 24, 2 x 12, 3 x 8, 4 x 6, 24 x 1, 12 x 2, 8 x 3, 6 x 4.

Prime number 13 = 1 x 13 and 13 x 1

Again,1 x 13(Row matrix) and 13 x 1(Column matrix) are 2 possible matrices whose product is 13.

3. If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?

Solution:

We know that, a matrix of order mxn having mn elements.

There are 6 possible matrices having 18 elements of orders:
1 x 18, 2 x 9, 3 x 6, 18 x 1, 9 x 2, 6 x 3.

Again, the product of 1 and 5 or 5 and 1 is 5.

Therefore, 1 x 5 (Row matrix) and 5 x 1 (Column matrix) are 2 possible matrices.

4. Construct a 2 × 2 matrix, A = [a_{ij}], whose elements are given by:

(i) $a_{ij} = \frac{(i+j)^2}{2}$

(ii) $a_{ij} = \frac{i}{j}$

(iii) $a_{ij} = \frac{(i+2j)^2}{2}$

Solution:

(i) Construct 2x2 matrix for

$$a_{ij} = \frac{(i+j)^2}{2}$$

Elements for 2x2 matrix are: $a_{11}, a_{12}, a_{21}, a_{22}$

For a_{11}, i = 1 and j = 1

$$a_{11} = \frac{(1+1)^2}{2} = \frac{(2)^2}{2} = \frac{4}{2} = 2$$

For a_{12}, i = 1 and j = 2

$$a_{12} = \frac{(1+2)^2}{2} = \frac{(3)^2}{2} = \frac{9}{2}$$

For a_{21}, i = 2 and j = 1

$$a_{21} = \frac{(2+1)^2}{2} = \frac{(3)^2}{2} = \frac{9}{2}$$

For a_{22}, i = 2 and j = 2

$$a_{22} = \frac{(2+2)^2}{2} = \frac{(4)^2}{2} = \frac{16}{2} = 8$$

Required matrix is :

$$\begin{bmatrix} 2 & 9/2 \\ 9/2 & 8 \end{bmatrix}$$

(ii) Construct 2x2 matrix for

$$a_{ij} = \frac{i}{j}$$

Elements for 2x2 matrix are: $a_{11}, a_{12}, a_{21}, a_{22}$

For a_{11}, i = 1 and j = 1

$$a_{11}=\frac{1}{1}=1$$

For a_{12}, i = 1 and j = 2

$$a_{12}=\frac{1}{2}$$

For a_{21}, i = 2 and j = 1

$$a_{21}=\frac{2}{1}=2$$

For a_{22}, i = 2 and j = 2

$$a_{22}=\frac{2}{2}=1$$

The required matrix is

$$\begin{bmatrix} 1 & 1/2 \\ 2 & 1 \end{bmatrix}$$

(iii) Construct 2x2 matrix for

$$a_{ij}=\frac{(i+2j)^2}{2}$$

Elements for 2x2 matrix are: $a_{11}, a_{12}, a_{21}, a_{22}$

For a_{11}, i = 1 and j = 1

$$a_{11}=\frac{(1+2)^2}{2}=\frac{(3)^2}{2}=\frac{9}{2}$$

For a_{12}, i = 1 and j = 2

$$a_{12}=\frac{(1+4)^2}{2}=\frac{(5)^2}{2}=\frac{25}{2}$$

For a_{21}, i = 2 and j = 1

$$a_{21}=\frac{(2+2)^2}{2}=\frac{(4)^2}{2}=\frac{16}{2}=8$$

For a_{22}, i = 2 and j = 2

$$a_{22}=\frac{(2+4)^2}{2}=\frac{(6)^2}{2}=\frac{36}{2}=18$$

The required matrix is

$$\begin{bmatrix} 9/2 & 25/2 \\ 8 & 18 \end{bmatrix}$$

5. Construct a 3 x 4 matrix, whose elements are given by:

(i) $a_{ij}=\frac{1}{2}|-3i+j|$

(ii) $a_{ij}=2i-j$

Solution:

(i) Construct 3 x 4 matrix for

$$a_{ij}=\frac{1}{2}|-3i+j|$$

Elements for 3 x 4 matrix are: $a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24}, a_{31}, a_{32}, a_{33}, a_{34}$

For a_{11}, i = 1 and j = 1

$$a_{11}=\frac{1}{2}|-3+1|=\frac{1}{2}|-2|=\frac{1}{2}(2)=1$$

For a_{12}, i = 1 and j = 2

$$a_{12} = \frac{1}{2}|-3+2| = \frac{1}{2}|-1| = \frac{1}{2}(1) = \frac{1}{2}$$

For a_{13}, i = 1 and j = 3

$$a_{13} = \frac{1}{2}|-3+3| = \frac{1}{2}|0| = \frac{1}{2}(0) = 0$$

For a_{14}, i = 1 and j = 4

$$a_{14} = \frac{1}{2}|-3+4| = \frac{1}{2}|1| = \frac{1}{2}(1) = \frac{1}{2}$$

For a_{21}, i = 2 and j = 1

$$a_{21} = \frac{1}{2}|-6+1| = \frac{1}{2}|-5| = \frac{1}{2}(5) = \frac{5}{2}$$

For a_{22}, i = 2 and j = 2

$$a_{22} = \frac{1}{2}|-6+2| = \frac{1}{2}|-4| = \frac{1}{2}(4) = 2$$

For a_{23}, i = 2 and j = 3

$$a_{23} = \frac{1}{2}|-6+3| = \frac{1}{2}|-3| = \frac{1}{2}(3) = \frac{3}{2}$$

For a_{24}, i = 2 and j = 4

$$a_{24} = \frac{1}{2}|-6+4| = \frac{1}{2}|-2| = \frac{1}{2}(2) = 1$$

For a_{31}, i = 3 and j = 1

$$a_{31}=\frac{1}{2}|-9+1|=\frac{1}{2}|-8|=\frac{1}{2}(8)=4$$

For a_{32}, i = 3 and j = 2

$$a_{32}=\frac{1}{2}|-9+2|=\frac{1}{2}|-7|=\frac{1}{2}(7)=\frac{7}{2}$$

For a_{33}, i = 3 and j = 3

$$a_{33}=\frac{1}{2}|-9+3|=\frac{1}{2}|-6|=\frac{1}{2}(6)=3$$

For a_{34}, i = 3 and j = 4

$$a_{34}=\frac{1}{2}|-9+4|=\frac{1}{2}|-5|=\frac{1}{2}(5)=\frac{5}{2}$$

The required matrix is

$$\begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$$

(ii) Construct 3 x 4 matrix for

$$a_{ij}=2i-j$$

Elements for 3 x 4 matrix are: $a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24}, a_{31}, a_{32}, a_{33}, a_{34}$

For a_{11}, i = 1 and j = 1

$$a_{11}=2-1=1$$

For a_{12}, i = 1 and j = 2

$$a_{12}=2-2=0$$

For a_{13}, i = 1 and j = 3

$a_{13} = 2 - 3 = -1$

For a_{14}, i = 1 and j = 4

$a_{14} = 2 - 4 = -2$

For a_{21}, i = 2 and j = 1

$a_{21} = 4 - 3 = 3$

For a_{22}, i = 2 and j = 2

$a_{22} = 4 - 2 = 2$

For a_{23}, i = 2 and j = 3

$a_{23} = 4 - 3 = 1$

For a_{24}, i = 2 and j = 4

$a_{24} = 4 - 4 = 0$

For a_{31}, i = 3 and j = 1

$a_{31} = 6 - 1 = 5$

For a_{32}, i = 3 and j = 2

$a_{32} = 6 - 2 = 4$

For a_{33}, i = 3 and j = 3

$a_{33} = 6 - 3 = 3$

For a_{34}, i = 3 and j = 4

$a_{34} = 6 - 4 = 2$

The required matrix is

$$\begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$$

6. Find the values of x, y and z from the following equations:

(i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

(ii) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

(iii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Solution:

(i)

$$\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$$

Since both the matrices are equal, so their corresponding elements are also equal.

Find the value of unknowns by equating the corresponding elements.

4 = y
3 = z
x = 1

(ii) Since both the matrices are equal, so their corresponding elements are also equal.

Find the value of unknowns by equating the corresponding elements.

$$\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

$x+y = 6$ …(1)

$5 + z = 5 \Rightarrow z = 0$

$xy = 8$ ….(2)

From equation (1), $x = 6 – y$

Substitute the value of x in equation (2)

$(6 – y)y = 8$

$6y – y^2 = 8$

or $y^2 – 6y + 8 = 0$

$(y – 4)(y – 2) = 0$

$y = 4$ or $y = 2$

Put values of y in equation (1), $x+y = 6$, we have $y = 2$ and $y = 4$

We get $x = 4$ and $x = 2$

Therefore, $x = 2$, $y = 4$ and $z = 0$ or $x = 4$, $y = 2$ and $z = 0$.

(iii)

Since both the matrices are equal, so their corresponding elements are also equal.

Find the value of unknowns by equating the corresponding elements.

$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

$x + y + z = 9$ …(1)

$x + z = 5$ …(2)

$y + z = 7$ …(3)

equation (1) – equation (2), we get

y = 4

Equation (3): 4 + z = 7 => z = 3

Equation (2) : x + 3 = 5 => x = 2

Answer: x = 2, y = 4 and z = 3

7. Find the value of a, b, c and d from the equation:

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

Solution:

Equate the corresponding elements of the matrices:

a - b = -1 …(1)

2a + c = 5 ..(2)

2a – b = 0 …(3)

3c + d = 13 …(4)

Equation (1) - Equation (3)

-a = -1 => a = 1

Equation (1) => 1 - b = -1 => b = 2

Equation (2) => 2(1) + c = 5 = > c = 3

Equation (4) => 3(3) + d = 13 => d = 4

Therefore, a = 1, b = 2, c = 3 and d = 4

8. A = $[a_{ij}]_{m \times n}$\ is a square matrix, if

(A) m < n (B) m > n (C) m = n (D) None of these

Solution:

Option (C) is correct.

According to square matrix definition: Number of rows = number of columns (m = n)

9. Which of the given values of x and y make the following pair of matrices equal

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

(A) $x = \frac{-1}{3}$, $y = 7$ (B) Not possible to find

(C) $y = 7$, $x = \frac{-2}{3}$ (D) $x = \frac{-1}{3}$, $y = \frac{-2}{3}$

Solution:

Option (B) is correct.

Explanation:

By equating all corresponding elements, we get

3x + 7 =0 = > x = -7/3

y – 2 = 5 = > y = 7

y + 1 = 8 => y = 7

2 – 3x = 4 => x =-2/3

10. The number of all possible matrices of order 3 × 3 with each entry 0 or 1 is:

(A) 27 (B) 18 (C) 81 (D) 512

Solution:

Option (D) is correct.

The number of elements of 3x3 matrix is 9.

First element, a_{11} is 2, can be 0 or 1, similarly the number of choices for each other element is 2.

Total possible arrangements = 2^9 = 512

Exercise 3.2

Page No: 80

1. Let

$$A=\begin{bmatrix}2 & 4\\3 & 2\end{bmatrix}, B=\begin{bmatrix}1 & 3\\-2 & 5\end{bmatrix}, C=\begin{bmatrix}-2 & 5\\3 & 4\end{bmatrix}$$

Find each of the following:
(i) A + B
(ii) A – B
(iii) 3A – C
(iv) AB
(v) BA

Solution:

(i) A + B

$$\begin{bmatrix}2 & 4\\3 & 2\end{bmatrix}+\begin{bmatrix}1 & 3\\-2 & 5\end{bmatrix}=\begin{bmatrix}2+1 & 4+3\\3-2 & 2+5\end{bmatrix}=\begin{bmatrix}3 & 7\\1 & 7\end{bmatrix}$$

(ii) A – B

$$\begin{bmatrix}2 & 4\\3 & 2\end{bmatrix}-\begin{bmatrix}1 & 3\\-2 & 5\end{bmatrix}=\begin{bmatrix}2-1 & 4-3\\3+2 & 2-5\end{bmatrix}=\begin{bmatrix}1 & 1\\5 & -3\end{bmatrix}$$

(iii) 3A – C

$$3\begin{bmatrix}2 & 4\\3 & 2\end{bmatrix}-\begin{bmatrix}-2 & 5\\3 & 4\end{bmatrix}=\begin{bmatrix}6 & 12\\9 & 6\end{bmatrix}-\begin{bmatrix}-2 & 5\\3 & 4\end{bmatrix}=\begin{bmatrix}6+2 & 12-5\\9-3 & 6-4\end{bmatrix}=\begin{bmatrix}8 & 7\\6 & 2\end{bmatrix}$$

(iv) AB

$$\begin{bmatrix}2 & 4\\3 & 2\end{bmatrix}\begin{bmatrix}1 & 3\\-2 & 5\end{bmatrix}=\begin{bmatrix}2(1)+4(-2) & 2(3)+4(5)\\3(1)+2(-2) & 3(3)+2(5)\end{bmatrix}=\begin{bmatrix}-6 & 26\\-1 & 19\end{bmatrix}$$

(v) BA

$$\begin{bmatrix}1 & 3\\-2 & 5\end{bmatrix}\begin{bmatrix}2 & 4\\3 & 2\end{bmatrix}=\begin{bmatrix}1(2)+3(3) & 1(4)+3(2)\\(-2)2+5(3) & (-2)4+5(2)\end{bmatrix}=\begin{bmatrix}11 & 10\\11 & 2\end{bmatrix}$$

2. Compute the following:

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

(ii) $\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$

(iii) $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$

(iv) $\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$

Solution:

(i)

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a+a & b+b \\ -b+b & a+a \end{bmatrix} = \begin{bmatrix} 2a & 2b \\ 0 & 2a \end{bmatrix}$$

(ii)

$$\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2+2ab & b^2+c^2+2bc \\ a^2+c^2-2ac & a^2+b^2-2ab \end{bmatrix}$$

$$= \begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1+12 & 4+7 & -6+6 \\ 8+8 & 5+0 & 16+5 \\ 2+3 & 8+2 & 5+4 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$$

(iv)

$$\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 x + \sin^2 x & \sin^2 x + \cos^2 x \\ \sin^2 x + \cos^2 x & \cos^2 x + \sin^2 x \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

3. Compute the indicated products:

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ (ii) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

(iv) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ (v) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

(vi) $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$

Solution:

(i)

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} a(a)+b(b) & a(-b)+b(a) \\ -b(a)+a(b) & (-b)(-b)+a(a) \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1(2) & 1(3) & 1(4) \\ 2(2) & 2(3) & 2(4) \\ 3(2) & 3(3) & 3(4) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}$$

(iii)

$$\begin{bmatrix}1 & -2\\2 & 3\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\end{bmatrix} = \begin{bmatrix}1(1)+(-2)2 & 1(2)+(-2)3 & 1(3)+(-2)1\\2(1)+3(2) & 2(2)+3(3) & 2(3)+3(1)\end{bmatrix}$$

$$=\begin{bmatrix}-3 & -4 & 1\\8 & 13 & 9\end{bmatrix}$$

(iv)

$$\begin{bmatrix}2 & 3 & 4\\3 & 4 & 5\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & -3 & 5\\0 & 2 & 4\\3 & 0 & 5\end{bmatrix}$$

$$=\begin{bmatrix}2+0+12 & -6+6+0 & 10+12+20\\3+0+15 & -9+8+0 & 15+16+25\\4+0+18 & -12+10+0 & 20+20+30\end{bmatrix}$$

$$=\begin{bmatrix}14 & 0 & 42\\18 & -1 & 56\\22 & -2 & 70\end{bmatrix}$$

(v)

$$\begin{bmatrix}2 & 1\\3 & 2\\-1 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 1\\-1 & 2 & 1\end{bmatrix} = \begin{bmatrix}1 & 2 & 3\\1 & 4 & 5\\-2 & 2 & 0\end{bmatrix}$$

(vi)

$$\begin{bmatrix}3 & -1 & 3\\-1 & 0 & 2\end{bmatrix}\begin{bmatrix}2 & -3\\1 & 0\\3 & 1\end{bmatrix} = \begin{bmatrix}14 & -6\\4 & 5\end{bmatrix}$$

4. If A = $\begin{bmatrix}1 & 2 & -3\\5 & 0 & 2\\1 & -1 & 1\end{bmatrix}$, B = $\begin{bmatrix}3 & -1 & 2\\4 & 2 & 5\\2 & 0 & 3\end{bmatrix}$ and C = $\begin{bmatrix}4 & 1 & 2\\0 & 3 & 2\\1 & -2 & 3\end{bmatrix}$ then compute (A + B) and (B – C). Also, verify that A + (B – C) = (A + B) – C.

Solution:

Find A + B:

$$\begin{bmatrix}1 & 2 & -3\\5 & 0 & 2\\1 & -1 & 1\end{bmatrix}+\begin{bmatrix}3 & -1 & 2\\4 & 2 & 5\\2 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1+3 & 2-1 & -3+2\\5+4 & 0+2 & 2+5\\1+2 & -1+0 & 1+3\end{bmatrix}$$

$$=\begin{bmatrix}4 & 1 & -1\\9 & 2 & 7\\3 & -1 & 4\end{bmatrix}$$

Find B – C:

$$\begin{bmatrix}3 & -1 & 2\\4 & 2 & 5\\2 & 0 & 3\end{bmatrix}-\begin{bmatrix}4 & 1 & 2\\0 & 3 & 2\\1 & -2 & 3\end{bmatrix}$$

$$=\begin{bmatrix}3-4 & -1-1 & 2-2\\4-0 & 2-3 & 5-2\\2-1 & 0+2 & 3-3\end{bmatrix}$$

$$=\begin{bmatrix}-1 & -2 & 0\\4 & -1 & 3\\1 & 2 & 0\end{bmatrix}$$

Verify that A + (B – C) = (A + B) – C

L.H.S. = A + (B – C)

$$\begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 2-2 & -3+0 \\ 5+4 & 0-1 & 2+3 \\ 1+1 & -1+2 & 1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

R.H.S. = (A + B) – C

$$\begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4-4 & 1-1 & -1-2 \\ 9-0 & 2-3 & 7-2 \\ 3-1 & -1+2 & 4-3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

L.H.S. = R.H.S. (Verified).

5. If $A=\begin{bmatrix}\frac{2}{3} & 1 & \frac{5}{3}\\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3}\\ \frac{7}{3} & 2 & \frac{2}{3}\end{bmatrix}$ and $B=\begin{bmatrix}\frac{2}{5} & \frac{3}{5} & 1\\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5}\\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5}\end{bmatrix}$.

then compute 3 A – 5B.

Solution:

Find 3 A – 5B :

$$3\begin{bmatrix}\frac{2}{3} & 1 & \frac{5}{3}\\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3}\\ \frac{7}{3} & 2 & \frac{2}{3}\end{bmatrix} - 5\begin{bmatrix}\frac{2}{5} & \frac{3}{5} & 1\\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5}\\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5}\end{bmatrix}$$

$$=\begin{bmatrix}2 & 3 & 5\\ 1 & 2 & 4\\ 7 & 6 & 2\end{bmatrix}-\begin{bmatrix}2 & 3 & 5\\ 1 & 2 & 4\\ 7 & 6 & 2\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

6. Simplify

$$\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$$

Solution:

Simplify first matrix:

$$\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}=\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ -\sin\theta\cos\theta & \cos^2\theta\end{bmatrix}$$

Simplify second matrix:

$$\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix} = \begin{bmatrix}\sin^2\theta & -\sin\theta\cos\theta\\ \sin\theta\cos\theta & \sin^2\theta\end{bmatrix}$$

Add results obtained from both the matrices, we get,

$$\begin{bmatrix}\cos^2\theta+\sin^2\theta & \cos\theta\sin\theta-\sin\theta\cos\theta\\ -\sin\theta\cos\theta+\sin\theta\cos\theta & \cos^2\theta+\sin^2\theta\end{bmatrix}$$

We know that, $\sin^2 x + \cos^2 x = 1$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

This implies,

$$\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

7. Find X and Y, if:

(i) X + Y = $\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$ **and X – Y =** $\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$

(ii) 2X + 3Y = $\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$ **and 3X + 2Y =** $\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}$

Solution:

(i) Add both the expressions:

$$(X + Y) + (X - Y) = \begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}+\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$$

$$2X = \begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}=\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$$

Subtract both the expressions:

$$(X+Y)-(X-Y)=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}-\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$$

$$2Y \quad =\begin{bmatrix}4 & 0\\ 2 & 2\end{bmatrix}$$

$$Y=\frac{1}{2}\begin{bmatrix}4 & 0\\ 2 & 2\end{bmatrix}=\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}$$

(ii) $2X + 3Y = \begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$ ….(1) and

$3X + 2Y = \begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}$ …(2)

Multiply equation (1) by 2,

$$4X+6Y=2\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}=\begin{bmatrix}4 & 6\\ 8 & 0\end{bmatrix} \quad \ldots\ldots.(3)$$

Multiply equation (2) by 3

$$9X+6Y=3\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}=\begin{bmatrix}6 & -6\\ -3 & 15\end{bmatrix} \quad \ldots\ldots..(4)$$

Subtract equation (4) from (3)

$$-5X=\begin{bmatrix}4 & 6\\ 8 & 0\end{bmatrix}-\begin{bmatrix}6 & -6\\ -3 & 15\end{bmatrix}=\begin{bmatrix}-2 & 12\\ 11 & -15\end{bmatrix}$$

$$X=\frac{-1}{5}\begin{bmatrix}-2 & 12\\ 11 & -15\end{bmatrix}=\begin{bmatrix}\frac{2}{5} & \frac{-12}{5}\\ \frac{-11}{5} & 3\end{bmatrix}$$

We know that

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

Substituting the value of X

$$2\begin{bmatrix} \frac{2}{5} & \frac{-12}{5} \\ \frac{-11}{5} & 3 \end{bmatrix} + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

By further calculation

$$\begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix} + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

It can be written as

$$3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix}$$

So we get

$$3Y = \begin{bmatrix} 2-\frac{4}{5} & 3+\frac{24}{5} \\ 4+\frac{22}{5} & 0-6 \end{bmatrix} = \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix}$$

Here

$$Y = \frac{1}{3}\begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

8. Find X, if Y = $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and 2X + Y = $\begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

Solution:

$$2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - Y$$

$$2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$

9. Find x and y if

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

Solution:

Solving left hand side expression, we get

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix}$$

Now,

$$\begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

Find x and y:

Equate corresponding elements of the matrices:

2 + y = 5 => y = 3

2x + 2 = 8 => x = 3

10. Solve the equation for x, y , z and t and if

$$2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

Solution:

$$2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

$$\begin{bmatrix} 2x+3 & 2z-3 \\ 2y+0 & 2t+6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

Find x, y , z and t by equating corresponding entries:

2x + 3 = 9 => x = 3

2y = 12 => y = 6

2z – 3 = 15 => z = 9

2t + 6 = 18 => t = 6

11. If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$ **find the values of x and y.**

Solution:

$$x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

Find x and y by equating corresponding entries

2x – y = 10 …(i)

3x + y = 5 …(ii)

Add both the equation,

5x = 15 => x = 3

Put value of x in equation (ii),

9 + y = 5 => y =-4

12. Given $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$**, find the values of x, y, z and w.**

Solution:

$$3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$$

$$\begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{bmatrix}$$

Find x, y, z and w by equating corresponding entries

3x = x + 4 => **x = 2**

3z = -1 + z + w ……(1)

3y = 6 + x + y ……(2)

3w = 2w + 3 => **w = 3**

Put value of w in equation (1)

3z = -1 + z + 3 => 2z = 2 => **z = 1**

Put value of x in equation (2)

3y = 6 + 2 + y => **y = 4**.

13. If $F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$**, show that F(x) F(y) = F(x + y).**

Solution:

Change x to y

$$F(y) = \begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$F(x)\,F(y) = \begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}\cos x\cos y-\sin x\sin y+0 & -\cos x\sin y-\sin x\cos y+0 & 0-0+0\\ \sin x\cos y+\cos x\sin y+0 & -\sin x\sin y+\cos x\cos y+0 & 0+0+0\\ 0+0+0 & 0+0+0 & 0+0+1\end{bmatrix}$$

$$= \begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$F(x + y) = \begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$$

F(x) F(y) = F (x + y)
Hence proved.

14. Show that

(i) $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

Solution:

(i) L.H.S.: $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

Multiply both the matrices

$$\begin{bmatrix} 5(2)+(-1)3 & 5(1)+(-1)4 \\ 6(2)+7(3) & 6(1)+7(4) \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$$

R.H.S.: $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$

Multiply both the matrices

$$= \begin{bmatrix} 2(5)+1(6) & 2(-1)+1(7) \\ 3(5)+4(6) & 3(-1)+4(7) \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$$

L.H.S. ≠ R.H.S.

(ii) L.H.S.: $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

Multiply both the matrices

$$= \begin{bmatrix} 1(-1)+2(0)+3(2) & 1(1)+2(-1)+3(3) & 1(0)+2(1)+3(4) \\ 0(-1)+1(0)+0(2) & 0(1)+1(-1)+0(3) & 0(0)+1(1)+0(4) \\ 1(-1)+1(0)+0(2) & 1(1)+1(-1)+0(3) & 1(0)+1(1)+0(4) \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

R.H.S.:

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1(1)+1(0)+0(1) & (-1)2+1(1)+0(1) & (-1)3+1(0)+0(0) \\ 0(1)+(-1)0+1(1) & (0)2+1(-1)+1(1) & (0)3+0(-1)+1(0) \\ 2(1)+3(0)+4(1) & 2(2)+3(1)+4(1) & 2(3)+3(0)+4(0) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$$

L.H.S. ≠ R.H.S.

15. Find $A^2 - 5A + 6I$, if A is

$$\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

Solution:

Find A^2 = A X A

$$\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

Find 5 A

$$5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix}$$

Find 6I

$$6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now,

$$A^2 - 5A + 6I = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$$

16. If $A=\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$**, prove that $A^3 - 6A^2 + 7A + 2I = 0$**

Solution:

$$A=\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$$

$$A^2 = A \times A = \begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1+0+4 & 0+0+0 & 2+0+6\\ 0+0+2 & 0+4+0 & 0+2+3\\ 2+0+6 & 0+0+0 & 4+0+9\end{bmatrix}$$

$$=\begin{bmatrix}5 & 0 & 8\\ 2 & 4 & 5\\ 8 & 0 & 13\end{bmatrix}$$

$$6A^2 = 6\begin{bmatrix}5 & 0 & 8\\ 2 & 4 & 5\\ 8 & 0 & 13\end{bmatrix} = \begin{bmatrix}30 & 0 & 48\\ 12 & 24 & 30\\ 48 & 0 & 78\end{bmatrix}$$

$$A^3 = A^2 \times A$$

$$=\begin{bmatrix}5 & 0 & 8\\ 2 & 4 & 5\\ 8 & 0 & 13\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}21 & 0 & 34\\ 12 & 8 & 23\\ 34 & 0 & 55\end{bmatrix}$$

Now,

$$A^3 - 6A^2 + 7A + 2I = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Which is a zero matrix.

Therefore, $A^3 - 6A^2 + 7A + 2I = 0$ (Proved)

17. If A = $\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,find k so that $A^2 = kA - 2I$.

Solution:

$A^2 = kA - 2I$

$$\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k-0 \\ 4k-0 & -2k-2 \end{bmatrix}$$

Equate all the corresponding values to find the value of k,

1 = 3k – 2 => k = 1

-2 = -2k => k = 1

4 = 4k => k = 1

-4 = -2k – 2 => k = 1

The value of k is 1.

18. If A = $\begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that

$$I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Solution:

$$I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

$$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

Now,

R.H.S. =

$$(I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha + \sin\alpha\tan\frac{\alpha}{2} & -\sin\alpha + \cos\alpha\tan\frac{\alpha}{2} \\ -\cos\alpha\tan\frac{\alpha}{2} + \sin\alpha & \sin\alpha\tan\frac{\alpha}{2} + \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\cos\alpha\cos\frac{\alpha}{2} + \sin\alpha\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \dfrac{-\sin\alpha\cos\frac{\alpha}{2} + \cos\alpha\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \\ \dfrac{-\cos\alpha\sin\frac{\alpha}{2} + \sin\alpha\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \dfrac{\sin\alpha\sin\frac{\alpha}{2} + \cos\alpha\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\cos\left(\alpha - \dfrac{\alpha}{2}\right)}{\cos\dfrac{\alpha}{2}} & \dfrac{-\sin\left(\alpha - \dfrac{\alpha}{2}\right)}{\cos\dfrac{\alpha}{2}} \\ \dfrac{\sin\left(\alpha - \dfrac{\alpha}{2}\right)}{\cos\dfrac{\alpha}{2}} & \dfrac{\cos\left(\alpha - \dfrac{\alpha}{2}\right)}{\cos\dfrac{\alpha}{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\cos\dfrac{\alpha}{2}}{\cos\dfrac{\alpha}{2}} & \dfrac{-\sin\dfrac{\alpha}{2}}{\cos\dfrac{\alpha}{2}} \\ \dfrac{\sin\dfrac{\alpha}{2}}{\cos\dfrac{\alpha}{2}} & \dfrac{\cos\dfrac{\alpha}{2}}{\cos\dfrac{\alpha}{2}} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\dfrac{\alpha}{2} \\ \tan\dfrac{\alpha}{2} & 1 \end{bmatrix}$$

L.H.S.
Hence proved.

19. A trust fund has Rs. 30,000 that must be invested in two different types of bonds.The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs.30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of:

(a) Rs. 1800 (b) Rs. 2000

Solution:

Let x be the investment in first bond, then the investment in the second bond will be Rs. (30,000 – x).

Interest paid by first bond is 5% per year and interest paid by second bond is 7% per year.

Matrix of investment [x 30000-x]

Matrix of annual interest per year is

$$\begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix}$$

To obtain an annual total interest of Rs. 1800, we have

$$[x \quad 30000-x]\begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 1800$$

$$\left[\frac{5x}{100} + \frac{7(30000-x)}{100}\right] = 1800$$

$$\frac{210000-2x}{100} = 1800$$

210000- 2x = 180000

x = 15000

The investment in first bond is Rs. 15,000

And investment in second bond is Rs. (30000 – 15000) = Rs. 15,000

To obtain an annual total interest of Rs. 2000, we have

$$[x \quad 30000-x]\begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 2000$$

$$\frac{210000-2x}{100} = 2000$$

or x = Rs. 5000

The investment in first bond is Rs. 5,000

And investment in second bond is Rs. (30000 – 5000) = Rs. 25,000

20. The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs. 80, Rs. 60 and Rs. 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra. Assume X, Y, Z, W and P are matrices of order 2 × n, 3 × k, 2 × p, n × 3 and p × k, respectively. Choose the correct answer in Exercises 21 and 22.

Solution:
Let the selling prices of each book as a 3 x 1 matrix

$$\begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}$$

Total amount received by selling all books

$$12\begin{bmatrix} 10 & 8 & 10 \end{bmatrix}\begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix} = \begin{bmatrix} 120 & 96 & 120 \end{bmatrix}\begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}$$

= 9600 + 5760 + 4800
= 20160

Total amount received by selling all the books is Rs. 20160.

21. The restriction on n, k and p so that PY + WY will be defined are:

(A) k = 3, p = n **(B) k is arbitrary, p = 2**

(C) p is arbitrary, k = 3 **(D) k = 2, p = 3**

Solution:
Option (A) is correct.

PY + WY = P(order of matrix, p x k) x Y(order of matrix, 3 x k) + W(order of matrix, n x k) Y(order of matrix, 3 x k)
Here k = 3 and p = n

22. If n = p, then the order of the matrix 7X – 5Z is:
(A) p × 2 (B) 2 × n (C) n × 3 (D) p × n
Solution: Option (B) is correct.

The order of matrices X and Z are equal, since n = p
The order of 7X – 5Z is same as the order of X and Z.
The order of 7X – 5Z is either 2 x n or 2 x p. (Given n = p)

Exercise 3.3 Page No: 88

1. Find the transpose of each of the following matrices:

(i) $\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

Solution:

(i) Let A = $\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$, Then A' = $\begin{bmatrix} 5 & \frac{1}{2} & -1 \end{bmatrix}$

(ii) Let A = $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$, Then A' = $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

(iii) Let A = $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$, Then A' = $\begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$

2. If A = $\begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ **and B =** $\begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$ **then verify that:**

(i) $(A+B)' = A' + B'$ **(ii)** $(A - B)' = A' - B'$

Solution:

(i) $A + B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}$

Now, $(A+B)' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$

Again,

$$A' + B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$$

Proved.

(ii) $A - B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix}$

$$(A - B)' = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$$

$$A' - B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ -2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$$

Hence proved.

3. If A’ = $\begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ **and B =** $\begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ **then verify that:**

(i) $(A+B)' = A' + B'$

(ii) $(A-B)' = A' - B'$

Solution:

$$A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

As we know that (A')' = A, we have

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 5 & 4 & 4 \end{bmatrix}$$

LHS:

$$(A + B)' = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$$

RHS:

$$A' + B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$$

LHS = RHS

(ii)

$$A - B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix}$$

LHS

$(A-B)' = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

RHS:

$A' - B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

LHS = RHS

4. If A' = $\begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$ then find (A + 2B)'.

Solution:

$(A')' = A = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$

Find A + 2 B

$= \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + 2\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$

And

$(A+2B)' = \begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$

5. For the matrices A and B, verify that (AB)' = B'A', where:

(i) **A =** $\begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$, **B =** $\begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

(ii) **A =** $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, **B =** $\begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$

Solution: LHS

$$AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} \begin{bmatrix} -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$$

$$(AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

RHS:

$$B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

LHS = RHS

(ii)

$$AB = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}$$

LHS

$$(AB)' = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

RHS:

$$B'A' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

LHS = RHS

6.
(i) If A = then verify that A'A = I.

(ii) If A = then verify that A'A = I.

Solution:

(i) L.H.S. = AA' = $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}'$

$$= \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \cos\alpha\sin\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \cos\alpha\sin\alpha & \sin^2\alpha + \cos^2\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I = \text{R.H.S.}$$

(ii)

L.H.S. = A'A = $\begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}'\begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix} = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix}\begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$

$$= \begin{bmatrix} \sin^2\alpha + \cos^2\alpha & \sin\alpha\cos\alpha - \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha - \sin\alpha\cos\alpha & \cos^2\alpha + \sin^2\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I = \text{R.H.S.}$$

7. (i) Show that the matrix A = $\begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a symmetric matrix.

(ii) Show that the matrix A = $\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is a skew symmetric matrix.

Solution:

According to the symmetric matrix definition: A' = A

$$A' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} = A$$

A is a symmetric matrix.

(ii) According to the skew symmetric matrix definition: A' = -A

$$A' = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= -A$$

A is a skew symmetric matrix.

8. For a matrix A = $\begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that:

(i) (A + A') is a symmetric matrix.

(ii) (A – A') is a skew symmetric matrix.

Solution:
(i)

$$A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$$

$$A + A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$$

$$(A+A')' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = A + A'$$

(A + A') is a symmetric matrix.

(ii)

$$A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$$

$$A - A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$(A - A')' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = -(A - A')$$

(A – A') is a skew symmetric matrix.

9. Find 1/2 (A + A') and 1/2(A – A') when A is

$$\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}.$$

Solution:

$$A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} \text{ then } A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

Now, A + A' is

$$\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} + \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So, 1/2 (A + A') is

$$\frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Again ,

$$A - A' = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} - \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{bmatrix}$$

$$1/2\,(A - A') = 1/2 \begin{bmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{bmatrix} = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

10. Express the following matrices as the sum of a symmetric and skew symmetric matrix:

(i) $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ **(iv)** $\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$

Solution:

(i)

Let $A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ then, $A' = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$

Symmetric matrix = 1/2 (A + A')

$$= \frac{1}{2}\left(\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$$

And Skew symmetric matrix = 1/2(A – A')

$$= \frac{1}{2}\left(\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

Again,

Symmetric matrix + Skew symmetric matrix =

$$\begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$$

Which is A.

Given matrix is sum of Symmetric matrix and Skew symmetric matrix .

(ii) Let $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ then, $A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

Symmetric matrix = 1/2 (A + A')

$$= \frac{1}{2}\left(\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

And Skew symmetric matrix = 1/2(A – A')

$$= \frac{1}{2}\left(\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Symmetric matrix + Skew symmetric matrix =

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Which is A.

Given matrix is sum of Symmetric matrix and Skew symmetric matrix .

(iii)

Let $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ then, $A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

Symmetric matrix = 1/2 (A + A')

$$= \frac{1}{2}\left(\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix}$$

And Skew symmetric matrix = 1/2(A – A')

$$= \frac{1}{2}\left(\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}$$

Symmetric matrix + Skew symmetric matrix =

$$\begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

Which is A.

Given matrix is sum of Symmetric matrix and Skew symmetric matrix .

(iv) Let $A = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$ then, $A' = \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$

Symmetric matrix = 1/2 (A + A')

$$\frac{1}{2}\left(\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$$

And Skew symmetric matrix = 1/2(A – A')

$$\frac{1}{2}\left(\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}\right) = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$$

Symmetric matrix + Skew symmetric matrix =

$$\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$$

Which is A.
Given matrix is sum of Symmetric matrix and Skew symmetric matrix .

Choose the correct answer in Exercises 11 and 12.
11. If A and B are symmetric matrices of same order, AB – BA is a:
(A) Skew-symmetric matrix
(B) Symmetric matrix
(C) Zero matrix
(S) Identity matrix
Solution:
Option (A) is correct.

A and B are symmetric matrices so we get
$A' = A$ and $B' = B$ .... (1)
We know that
$(AB - BA)' = (AB)' - (BA)'$ $[(A - B)' = A' - B']$
So we get
$= B'A' - A'B'$ $[(AB)' = B'A']$
$= BA - AB$ [Using equation (1)]
$= - (AB - BA)$

12.
If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, and $A + A' = I$, then the value of α is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) π (D) $\frac{3\pi}{2}$

Solution:

Option (B) is correct.

$$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By equating corresponding terms,

$$2\cos\alpha = 1$$

$$\cos\alpha = \cos\frac{\pi}{3}$$

$$\alpha = \frac{\pi}{3}$$

Exercise 3.4 Page No: 97

Using elementary transformations, find the inverse of each of the matrices, if it exists in Exercises 1 to 17.

1.

$$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

As we know, A = I A

$$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$[R_2 \to R_2 - 2R_1]$$

$$\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$$

$$\left[R_2 \to \frac{1}{5} R_2\right.$$

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A$$

$$[R_1 \to R_1 + R_2]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A$$

Therefore, $A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$

2.

$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

As A = AI = $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A \quad [R_1 \to R_1 - R_2]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A \quad [R_2 \to R_2 - R_1]$$

$$A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

3. $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

As we know, A = AI

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A \quad [R_2 \to R_2 - 2R_1]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} A \quad [R_1 \to R_1 - 3R_2]$$

$$A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

4.

$$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

As we know, A = AI

$$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Again,

$$\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A \quad [R_2 \to R_2 - 2R_1]$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix} A \quad [R_1 \leftrightarrow R_2]$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 5 & -2 \end{bmatrix} A \quad [R_2 \leftrightarrow R_2 - 2R_1]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix} A \quad [R_1 \to R_1 - R_2]$$

Therefore, the inverse of given matrix is:

$$A^{-1} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$$

5.

$$\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$

As we know, A = AI

$$\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Again,

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} A \quad [R_2 \to R_2 - 3R_1]$$

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} A \quad [R_1 \to R_1 - R_2]$$

$$A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$$

6.

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

As we know, A = AI

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Again,

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A \quad [R_1 \leftrightarrow R_2]$$

$$\begin{bmatrix} 1 & 3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix} A \quad [R_2 \to R_2 - 2R_1]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ 1 & -2 \end{bmatrix} A \quad [R_1 \to R_1 + 3R_2]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A \quad [R_2 \to (-1)R_2]$$

$$A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

7. $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

As we know, A = IA

$$\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\begin{bmatrix} 6 & 2 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} A \quad [R_1 \to 2R_1]$$

$$\begin{bmatrix} 1 & 0 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix} A \quad [R_1 \to R_1 - R_2]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -10 & 6 \end{bmatrix} A \quad [R_2 \to R_2 - 5R_1]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} A \quad \left[R_2 \to \frac{1}{2}R_2\right]$$

$$A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$$

8.

$$\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

As we know, A = IA

$$\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A \quad [R_1 \to R_1 - R_2]$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix} A \quad [R_2 \to R_2 - 3R_1]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix} A \quad [R_1 \to R_1 - R_2]$$

$$A^{-1} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$$

9.

$$\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$

As we know, A = IA

$$\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A \quad [R_1 \to R_1 - R_2]$$

$$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} A \quad [R_2 \to R_2 - 2R_1]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix} A \quad [R_1 \to R_1 - 3R_2]$$

$$A^{-1} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$$

10.

$$\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$

As we know, A = IA

$$\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\begin{bmatrix} -1 & 1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A \quad [R_1 \to R_1 + R_2]$$

$$\begin{bmatrix} 1 & -1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix} A \quad [R_1 \to (-1) R_1]$$

$$\begin{bmatrix} 1 & -1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -4 & -3 \end{bmatrix} A \quad [R_2 \to R_2 + 4R_1]$$

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 2 & \frac{3}{2} \end{bmatrix} A \quad \left[R_2 \to \frac{-1}{2} R_2\right]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix} A \quad [R_1 \to R_1 + R_2]$$

$$A^{-1} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$$

11.

$$\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$

As we know, A = IA

$$\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\begin{bmatrix} 1 & -2 \\ 2 & -6 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} A \quad [R_1 \leftrightarrow R_2]$$

$$\begin{bmatrix} 1 & -2 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} A \quad [R_2 \to R_2 - 2R_1]$$

$$\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-1}{2} & 1 \end{bmatrix} A \quad \left[R_2 \to \frac{-1}{2} R_2\right]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ \frac{-1}{2} & 1 \end{bmatrix} A \quad [R_1 \to R_1 + 2R_2]$$

$$A^{-1} = \begin{bmatrix} -1 & 3 \\ -1/2 & 1 \end{bmatrix}$$

12.

$$\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

As we know, A = IA

$$\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ 0 & 1 \end{bmatrix} A \quad \left[R_1 \to \frac{1}{6}R_1\right]$$

$$\begin{bmatrix} 1 & \frac{-1}{2} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ \frac{1}{3} & 1 \end{bmatrix} A \quad [R_2 \to R_2 + 2R_1]$$

All entries in second row of left side are zero, so A^{-1} does not exist.

13.

$$\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

We know that, A = IA

$$\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying: $R_1 \to R_1 + R_2$

$$\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A$$

Applying: $R_2 \to R_2 + R_1$

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} A$$

Applying: $R_1 \to R_1 + R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} A$$

$$A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

14.

$\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$

We know that, A = IA

$$\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying: $\left[R_1 \to \frac{1}{2} R_1\right]$

$$\begin{bmatrix} 1 & \frac{1}{2} \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying: $R_2 \to R_2 - 4R_1$

$$\begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -2 & 1 \end{bmatrix} A$$

All entries in second row of left side are zero, so inverse of the matrix does not exist.

15.

$\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

We know that, A = IA

$$\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying: $R_1 \to R_1 - R_3$

and $R_1 \to (-1)R_1$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying: $R_2 \to R_2 - 2R_1$
and $R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 5 \\ 0 & -5 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & -2 \\ 3 & 0 & -2 \end{bmatrix} A$$

Applying: $R_2 \leftrightarrow R_3$

and $R_2 \to \left(\frac{-1}{5}\right) R_2$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ \frac{-3}{5} & 0 & \frac{2}{5} \\ 2 & 1 & -2 \end{bmatrix} A$$

Applying: $R_1 \to R_1 - R_2$

and $R_3 \to \frac{1}{5} R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-2}{5} & 0 & \frac{3}{5} \\ \frac{-3}{5} & 0 & \frac{2}{5} \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{bmatrix}$$

Applying: $[R_2 \to R_2 + R_3]$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} A$$

$$A^{-1} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$$

16.

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

We know that, A = IA

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying: $R_2 \to R_2 + 3R_1$

and $R_3 \to R_3 - 2R_1$

and $R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -1 & 4 \\ 0 & 9 & -11 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 0 & 1 \\ 3 & 1 & 0 \end{bmatrix} A$$

Applying: $R_2 \to (-1)R_2$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -4 \\ 0 & 9 & -11 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix} A$$

Applying: $R_1 \to R_1 - 3R_2$

and $R_3 \to R_3 - 9R_2$

and $R_3 \to \frac{1}{25} R_3$

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ 2 & 0 & -1 \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

Applying: $R_1 \to R_1 - 10R_3$

and $R_2 \to R_2 + 4R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

$$A^{-1} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

17.

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

We know that, A = IA

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying: $R_2 \to R_2 - 2R_1$
and $R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying: $R_2 \to R_2 - 2R_1$
and $R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & -2 & -5 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 0 & 1 \\ 5 & -2 & 0 \end{bmatrix} A$$

Applying: $R_1 \to R_1 - R_2$
and $R_3 \to R_3 + 2R_2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & -1 \\ 0 & 0 & 1 \\ 5 & -2 & 2 \end{bmatrix} A$$

Applying: $R_1 \to R_1 + R_3$

and $R_2 \to R_2 - 3R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

18.
Matrices A and B will be inverse of each other only if

(A) AB = BA
(B) AB = BA = 0
(C) AB = 0, BA = I
(D) AB = BA = I

Solution:

Option (D) is correct.

Consider A as a square matrix of order m, and if another square matrix of same order exists of order m,
We know that
AB = BA = I where B is the inverse of A.
Here A is the inverse of B.
Hence, the matrices A and B will be inverse of each other only if AB = BA = I.

Miscellaneous Exercise Page No: 100

1. Let A = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ show that $(aI + bA)^n = a^n I + na^{n-1}\, bA$ where I is the identity matrix of order 2 and n ∈ N.

Solution:
Use Mathematical Induction:

Step 1: Result is true for n = 1

$$(aI + bA)^n = a^n I + na^{n-1} bA$$

Step 2: Assume that result is true for n = k
So,
$$(aI + bA)^k = a^k I + ka^{k-1} bA$$

Step 3: Prove that, result is true for n = k + 1
That is,
$$(aI + bA)^{k+1} = a^{k+1} I + (k+1) a^k bA$$
L.H.S.:
$$(aI + bA)^{k+1} = (aI + bA)^k (aI + bA)$$

$$= (a^k I + ka^{k-1} bA)(aI + bA)$$

$$= a^{k+1} I \times I + ka^k bAI + a^k bAI + ka^{k-1} b^2 A.A$$
Here, A.A = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ = 0
This implies
$$= a^{k+1} I + (k+1) a^k bA$$

= R.H.S.

Thus, result is true.

Therefore, p(n) is true.

2. If A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, prove that A^n = $\begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$, n ∈ N

Solution:
Let us say, $p(n) = A^n$

Use Mathematical Induction:

Step 1: Result is true for n = 1

$$p(1) = A = \begin{bmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Step 2: Assume that result is true for n = k
So,

$$p(1) = A = \begin{bmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$p(k) = A^k = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$$

Step 3: Prove that, result is true for n = k + 1
That is,

$$p(k+1) = A^{k+1} = \begin{bmatrix} 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \end{bmatrix}$$

L.H.S.:

$A^{k+1} = A^k A$

$$\begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 3^{k} & 3^{k} & 3^{k} \\ 3^{k} & 3^{k} & 3^{k} \\ 3^{k} & 3^{k} & 3^{k} \end{bmatrix}$$

= R.H.S.

Thus, result is true.

Therefore, By Mathematical Induction p(n) is true for all natural numbers.

3. If A = $\begin{bmatrix} \mathbf{3} & \mathbf{-4} \\ \mathbf{1} & \mathbf{-1} \end{bmatrix}$ then prove that A^n = $\begin{bmatrix} \mathbf{1+2n} & \mathbf{-4n} \\ \mathbf{n} & \mathbf{1-2n} \end{bmatrix}$ where n is any positive integer.

Solution:

Use Mathematical Induction:

Step 1: Result is true for n = 1.

$A^1 = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

Step 2: Assume that result is true for n = k
So,

$$A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$$

Step 3: Prove that, result is true for n = k + 1
That is,

$$A^{k+1} = \begin{bmatrix} 1+2(k+1) & -4(k+1) \\ (k+1) & 1-2(k+1) \end{bmatrix}$$

L.H.S.:

$A^{k+1} = A^k A$

$$=\begin{bmatrix} 1+2(k+1) & -4(k+1) \\ (k+1) & 1-2(k+1) \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

Using result from step 2.

$$=\begin{bmatrix} 3+6k-4k & -4-8k+4k \\ 3k+1-2k & -4k-1+2k \end{bmatrix}$$

$$=\begin{bmatrix} 3+2k & -4-4k \\ 1+k & -1-2k \end{bmatrix}$$

$$=\begin{bmatrix} 1+2(k+1) & -4(k+1) \\ (k+1) & 1-2(k+1) \end{bmatrix}$$

= R.H.S.

Thus, result is true.

Therefore, By Mathematical Induction result is true for all positive integers.

4. If A and B are symmetric matrices, prove that AB – BA is a skew symmetric matrix.

Solution:
Step 1: If A and B are symmetric matrices, then A' = A and B' = B ... (i)

Step 2: (AB – BA)' = (AB)' – (BA)'

(AB – BA)' = B'A' – A'B' [Using Reversal law]

(AB – BA)' = BA – AB [Using eq. (i)]

(AB – BA)' = – (AB – BA)

Therefore, (AB – BA) is a skew symmetric.

5. Show that the matrix B'AB is symmetric or skew symmetric according as A is symmetric or skew symmetric.

Solution:

We know that, (AB)' = B' A'

(B'AB)' = [B'(AB)]' = (AB)' (B')'

This implies, (B'AB)' = B'A'B ..say equation (1)

If A is a symmetric matrix, then A' = A

Using eq. (i) (B'AB)' = B'AB

Therefore, B'AB is a symmetric matrix.

Again,

If A is a skew symmetric matrix then A' = – A

Using equation (i), (B'AB)' = B'(– A)B = – B'AB

So, B'AB is a skew symmetric matrix.

6. Find the values of x, y, z if the matrix A = $\begin{bmatrix} \mathbf{0} & \mathbf{2y} & \mathbf{z} \\ \mathbf{x} & \mathbf{y} & -\mathbf{z} \\ \mathbf{x} & -\mathbf{y} & \mathbf{z} \end{bmatrix}$ satisfies the equation A'A = I.

Solution:

Given matrix is A = $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$

Transpose of A = A' = $\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$

Now, A'A = I (Given)

$$\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0+x^2+x^2 & 0+xy-xy & 0-xz+xz \\ 0+xy-xy & 4y^2+y^2+y^2 & 2yz-yz-yz \\ 0-zx+zx & 2yz-yz-yz & z^2+z^2+z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This implies:

$$\begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

To find the values of unknowns, equate corresponding matrix entries:
We have,

$2x^2 = 1 \Rightarrow x = \pm\frac{1}{\sqrt{2}}$

$6y^2 = 1 \Rightarrow y = \pm\frac{1}{\sqrt{6}}$ and

$3z^2 = 1 \Rightarrow z = \pm\frac{1}{\sqrt{3}}$

7. For what value of x.

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

Solution:

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

$$\begin{bmatrix} 1+4+1 & 2+0+0 & 0+2+2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

$$[6 \quad 2 \quad 4]\begin{bmatrix}0\\2\\x\end{bmatrix}=0$$

$$[0+4+4x] \quad =0$$

Therefore, 4 + 4x = 0 => x = -1.

8. If A = $\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}$, show that $A^2 - 5A + 7I = 0$.

Solution:

$A^2 = A\,A = \begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix} = \begin{bmatrix}8 & 5\\-5 & 3\end{bmatrix}$

$5A = 5\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix} = \begin{bmatrix}15 & 5\\-5 & 10\end{bmatrix}$

$7I = 7\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}7 & 0\\0 & 7\end{bmatrix}$

Now, $\mathbf{A^2 - 5A + 7I = 0}$

L.H.S.

$A^2 - 5A + 7I = \begin{bmatrix}8 & 5\\-5 & 3\end{bmatrix} - \begin{bmatrix}15 & 5\\-5 & 10\end{bmatrix} + \begin{bmatrix}7 & 0\\0 & 7\end{bmatrix}$

$$=\begin{bmatrix}8-15+7 & 5-5+0\\-5+5+0 & 3-10+7\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

R.H.S.

Hence Proved.

9.

Find x, if $\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$

Solution:

$$\begin{bmatrix} x-0-2 & 0-10-0 & 2x-5-3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

$$\left[(x-2)x-10(4)+(2x-8)1\right] = 0$$

$$[x^2 - 2x - 40x + 2x - 8] = 0$$

$$[x^2 - 48] = [0]$$

or $x^2 - 48 = 0 \Rightarrow x = \pm 4\sqrt{3}$

10. A manufacturer produces three products x, y, z, which he sells in two markets. Annual sales are indicated below:

Market	**Products**	**Products**	**Products**
I	**10,000**	**2,000**	**18,000**
II	**6,000**	**20,000**	**8,000**

(a) If unit sales prices of x, y, and z are Rs. 2.50, Rs. 1.50 and Rs. 1.00 respectively, find the total revenue in each market with the help of matrix algebra.

(b) If the unit costs of the above three commodities are Rs. 2.00, Rs. 1.00 and 50 paise respectively. Find the gross profit.

Solution:

(a)
If unit sales prices of x, y, and z are Rs. 2.50, Rs. 1.50 and Rs. 1.00 respectively.
Total revenue in market I and II can be shown with the help of matrix as: Basically Revenue Matrix

$$\begin{bmatrix} 10{,}000 & 2{,}000 & 18{,}000 \\ 6{,}000 & 20{,}000 & 8{,}000 \end{bmatrix}\begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}$$

Solving above matrix, we have,

$$= \begin{bmatrix} 25{,}000 + 3{,}000 + 18{,}000 \\ 15{,}000 + 30{,}000 + 8{,}000 \end{bmatrix}$$

$$= \begin{bmatrix} 46{,}000 \\ 53{,}000 \end{bmatrix}$$

Therefore, the total revenue in Market I = Rs. 46,000 and in Market II = Rs. 53,000.

(b) If the unit costs of the above three commodities are Rs. 2.00, Rs. 1.00 and 50 paise respectively.
Total cost prices of all the products in market I and II can be shown with the help of matrix as: Basically Cost Matrix

$$\begin{bmatrix} 10{,}000 & 2{,}000 & 18{,}000 \\ 6{,}000 & 20{,}000 & 8{,}000 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$$

Solving above matrix, we have,

$$\begin{bmatrix} 20{,}000 + 2{,}000 + 9{,}000 \\ 12{,}000 + 20{,}000 + 4{,}000 \end{bmatrix}$$

$$= \begin{bmatrix} 31{,}000 \\ 36{,}000 \end{bmatrix}$$

From (a) and (b),

The profit collected in two markets is given in matrix form as

Profit matrix = Revenue matrix – Cost matrix

$$\begin{bmatrix} 46{,}000 \\ 53{,}000 \end{bmatrix} - \begin{bmatrix} 31{,}000 \\ 36{,}000 \end{bmatrix} = \begin{bmatrix} 15{,}000 \\ 17{,}000 \end{bmatrix}$$

Therefore, the gross profit in market I and market II = Rs. 15000 + Rs. 17000 = Rs. 32,000.

11. Find the matrix X so that X

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

Solution:

Let X = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

We have,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

Equate all the corresponding elements:

a + 4b = -7 ….(1)

2a + 5b = -8 …(2)

3a + 6b = - 9 ..(3)

c + 4d = 2 …(4)

2c +5d = 4 …(5)

3c + 6d =6 ….(6)

Solving (1) and (2), we have a = 1 and b = -2
Solving (4) and (5), we have c = 2 and d = 0

So X = $\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

12. If A and B are square matrices of the same order such that AB = BA, then prove by induction that $AB^n = B^nA$. Further, prove that $(AB)^n = A^nB^n$ for all n ∈ N.

Solution:

Use Mathematical Induction, to prove $AB^n = B^nA$

Step 1: Result is true for n = 1

AB = BA

Step 2: Assume that result is true for n = k
So,
$AB^k = B^kA$

Step 3: Prove that, result is true for n = k + 1
That is,

$AB^{k+1} = B^{k+1} A$

L.H.S.:

$AB^{k+1} = AB^k B$

Using result of Step 2, we have

$= B^kA B$

$= B^{k+1} A$

= R.H.S.

Thus, by Mathematical Induction the result is true.

Again, prove that $(AB)^n = A^nB^n$

Use Mathematical Induction:

Step 3: Result is true for n = 1

$(AB)' = AB$

Step 4: Assume that result is true for n = k
So, $(AB)^k = A^kB^k$

Step 3: Prove that, result is true for n = k + 1
That is, $(AB)^{k+1} = A^{k+1}B^{k+1}$

L.H.S.: $(AB)^{k+1} = (AB)^k\ (AB)$

$= A^kB^k\ (AB)$ (using step 2 result)

$= A^k\ (B^k A)\ B$

$= A^k\ (A B^k)\ B$

$= (A^k\ A)\ (B^k B)$

$= A^{k+1}B^{k+1}$

= R.H.S.
Thus, result is true for n = k+1.
Therefore, by Mathematical Induction we have $(AB)^n = A^nB^n$ for all $n \in N$.

13. If A = $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then:

(A) $1+\alpha^2+\beta\gamma=0$ (B) $1-\alpha^2+\beta\gamma=0$

(C) $1-\alpha^2-\beta\gamma=0$ (D) $1+\alpha^2-\beta\gamma=0$

Solution:
Option (C) is correct.

$A^2 = I$

$$\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \alpha^2+\beta\gamma & 0 \\ 0 & \beta\gamma+\alpha^2 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

or $\alpha^2+\beta\gamma=1$

or $1-\alpha^2-\beta\gamma=0$

14. If the matrix A is both symmetric and skew symmetric, then:

(A) A is a diagonal matrix

(B) A is a zero matrix

(C) A is a square matrix

(D) None of these

Solution:
Option (B) is correct.

Consider A is a symmetric and skew symmetric matrix $A' = A$ and $A' = -A$

$A = -A$

$A + A = 0$

$2A = 0 \Rightarrow A = 0$

Hence, A is a zero matrix.

15. If A is a square matrix such that $A^2 = A$, then $(I + A)^3 – 7A$ is equal to:

(A) A **(B) I – A**

(C) I **(D) 3A**

Solution:

Option (C) is correct.

Explanation:
$(I + A)^3 - 7A = I^3 + A^3 + 3I^2A + 3IA^2 - 7A$

$= I + A^3 + 3A + 3A^2 - 7A$

$= I + A^2 A + 3A + 3A - 7A \quad [A^2 = A]$

$= I + A A - A$

$= I + A - A$

$= I$