Various Models

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1 Static Model

Economy is made up of infinite workers who provide one unit of labor inelastically. Each worker is endowed with two productivity profiles, one for urban labor and one for rural labor. Every worker is normalized to provide one efficiency unit of labor in the rural sector. Workers get an urban productivity from a Pareto distribution:

$$z \sim 1 - z^{-\theta} \tag{1}$$

Where theta is the shape parameter.

The rural production function:

$$Y_r = A_r N_r^{\phi} \tag{2}$$

Where N_r is the number of workers in the rural area, A_r is the rural technology, and $0 < \phi < 1$ to get decreasing returns to labor in the rural sector.

The urban production function:

$$Y_u = A_u N_u \tag{3}$$

Where A_u is the urban technology and N_u is in efficiency units of labor.

The Rural and Urban wages per efficiency unit are:

$$w_r = \frac{\partial Y_r}{\partial N_r} = A_r \phi N_r^{\phi-1} \tag{4}$$

$$w_u = \frac{\partial Y_u}{\partial N_u} = A_u \tag{5}$$

Additionally, there is a cost T of migrating to work in the urban sector. Thus we can find z^* which is the threshold of indifference on the choice of sector. Workers with urban productivity above z^* would work in the urban sector and workers with urban productivity below z^* would work in the rural sector.

$$A_r \phi N_r^{\phi-1} = z^* A_u - T$$

$$z^* = \frac{A_r \phi N_r^{\phi-1} + T}{A_u}$$

$$\text{Where } N_r = \int_0^{z^*} f(z)dz$$

2 2 Period Model

2.1 Endowments

Once again there are infinite workers in the economy who each provide one unit of labor inelastically. Every person is normalized to provide one efficiency unit of labor in the rural sector. The urban productivity is characterized as:

$$z = \mu + \sigma$$
$$\mu \sim 1 - \mu^{-\theta}$$
$$\sigma \sim 1 - \sigma^{-\beta}$$

Where μ is an observed part of a person's skills, and σ is an unobserved shock on their urban productivity which becomes observed after one period of living in an urban setting. β and θ are shape parameters for their respective Pareto distributions.

The production functions are the same as in the static model:

$$Y_r = A_r N_r^{\phi}$$
$$\text{Where } N_r = \int_0^{z^*} f(z)dz$$
$$Y_u = A_u N_u$$
$$\text{Where } N_u = \int_{z^*}^{\infty} (\mu + \sigma) f(z)dz$$

Where A_r and A_u are the rural and urban technologies, N_r and N_u are the efficiency units in each sector of the economy, and $0 < \phi \leq 1$.

Wages per efficiency unit are also defined in the same way as the static model:

$$w_r = \frac{\partial Y_r}{\partial N_r} = A_r \phi N_r^{\phi - 1} \tag{6}$$
$$w_u = \frac{\partial Y_u}{\partial N_u} = A_u \tag{7}$$

2.2 Period 1

In the first period every person decides whether to remain in the rural sector or migrate to the urban sector. Migrating to the urban sector incurs a fixed cost T.

Workers maximize utility according to:

$$v(\mu, \sigma, N_r^1) = max(v(\mu, E(\sigma), N_r^1 | rural), v(\mu, E(\sigma) | migrate))$$
$$\text{Where:}$$
$$v(\mu, E(\sigma), N_r^1 | rural) = A_r \phi {N_r^1}^{\phi - 1} + \beta E(v(\mu, E(\sigma), N_r^2))$$
$$v(\mu, E(\sigma) | migrate) = A_u(\mu + E(\sigma)) - T + \beta E(v(\mu, E(\sigma), N_r^2))$$

We define the threshold values of μ^* and consequently z^* as:

$$\mu^* + E(\sigma) = z^* = \frac{A_r \phi N_r^{\phi-1} + T}{A_u}$$

The total production in the economy is:

$$Y_r = A_r N_r^{\phi}$$
$$\text{Where: } N_r = \int_0^{z^*} f(z)dz$$
$$Y_u = A_u N_u$$
$$\text{Where: } N_u = \int_{z^*}^{\infty} zf(z)dz$$

2.3 Period 2

In the second period agents who have worked in the urban sector observe their urban productivities in full (i.e. know what their σ is) and decide whether or not to remain in the urban sector. They make this choice according to:

$$v(\mu, \sigma, N_r^2) = max(v(N_r^2|rural), v(\mu, \sigma|urban))$$
$$\text{Where:}$$
$$v(N_r^2|rural) = A_r \phi N_r^{2^{\phi-1}}$$
$$v(\mu, \sigma|urban) = (\mu + \sigma)A_u$$

Those who worked in the rural sector in the first period still have not observed their σ and make their choice according to:

$$v(\mu, \sigma, N_r^2) = max(v(N_r^2|rural), v(\mu, \sigma|urban))$$
$$\text{Where:}$$
$$v(N_r^2|rural) = A_r \phi N_r^{2^{\phi-1}}$$
$$v(\mu, \sigma|urban) = (\mu + E(\sigma))A_u - T$$

In this case there doesn't exist a threshold value for z. Instead we have two indifference equations. One for those who migrated in the first period, and one for those who didn't.

Indifference equation for urban workers in first period:

$$A_r \phi N_r^{2^{\phi-1}} = (\mu + \sigma)A_u$$

Indifference equation rural workers in first period:

$$A_r \phi N_r^{2^{\phi-1}} = (\mu + E(\sigma))A_u - T$$

The total production in the economy is:

$$Y_r = A_r N_r^{\phi}$$
$$Y_u = A_u N_u$$