Section 8 | Analysis of Variance and Covariance

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One-Way ANOVA

One-Way Analysis of Variance (ANOVA) is a statistical method used to compare the means of three or more groups to determine if there are statistically significant differences between them. It assesses whether the means of these groups are significantly different from each other based on the variance between group means and within-group variance.

The null hypothesis for ANOVA is that there is no significant difference between the group means, while the alternative hypothesis suggests that at least one group mean is different from the others.

Assumptions of ANOVA:

- 1. **Independence:** Observations within each group are independent.
- 2. Normality: Each group should follow a normal distribution.
- 3. Homogeneity of variance: Variances among the groups should be equal.

Steps

1. Reading the Data

```
ozone <- read.table("ozone.txt", header = TRUE)

# Converting 'wind' from character to factor
ozone[,"wind"] <- as.factor(ozone[,"wind"])

# Summary of 'max03' and 'wind' columns
summary(ozone[,c("max03", "wind")])</pre>
```

```
## max03 wind

## Min. : 42.00 East :10

## 1st Qu.: 70.75 North:31

## Median : 81.50 South:21

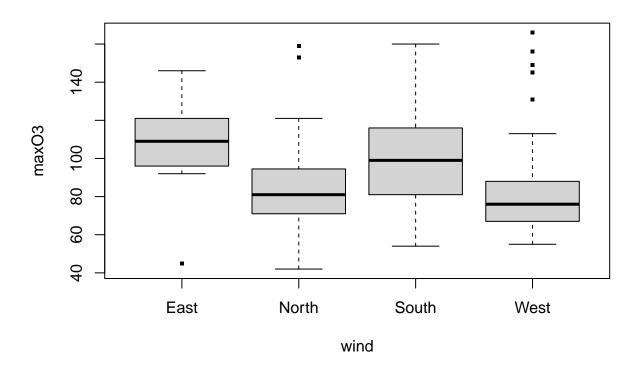
## Mean : 90.30 West :50

## 3rd Qu.:106.00

## Max. :166.00
```

2. Representing the Data

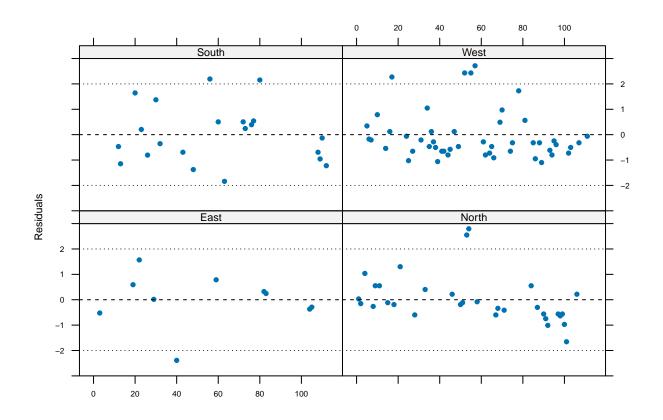
```
# Plotting a vertical boxplot of 'max03' against 'wind'
plot(max03 ~ wind, data = ozone, pch = 15, cex = 0.5)
```



3. Analyzing the Significance of the Factor

```
# Interpretation: If the p-value (Pr(>F)) is less than 0.05, # reject HO and accept the significance of 'wind'.
```

4. Conducting Residual Analysis



```
# Interpretation: Look for the majority of studentized residuals

#falling within [-2, 2]. Residuals outside this range may indicate
```

```
# potential outliers.
```

5. Interpreting Coefficients

```
# Summary of coefficients obtained from the linear model
summary(reg.aov1)
##
## Call:
## lm(formula = max03 ~ wind, data = ozone)
## Residuals:
##
      Min
               1Q Median
                               3Q
                                      Max
## -60.600 -16.807 -7.365 11.478 81.300
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 94.738 3.053 31.027
## wind1
                10.862
                            6.829
                                    1.590
                                            0.1147
## wind2
                -8.609
                            4.622 -1.863
                                            0.0652 .
                 7.786
                            5.205
                                   1.496
## wind3
                                           0.1376
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
## Residual standard error: 27.32 on 108 degrees of freedom
## Multiple R-squared: 0.08602,
                                   Adjusted R-squared:
## F-statistic: 3.388 on 3 and 108 DF, p-value: 0.02074
# Further constraints on the model:
# Create dummy variables for 'wind' manually
dummy_wind <- model.matrix(~ wind - 1, data = ozone) # Exclude intercept with '- 1'
# Fit the linear model using dummy variables
model <- lm(max03 ~ dummy_wind, data = cbind(ozone, dummy_wind))</pre>
# Summary of the model
summary(model)
##
## Call:
## lm(formula = max03 ~ dummy_wind, data = cbind(ozone, dummy_wind))
## Residuals:
##
               1Q Median
                               3Q
      Min
                                      Max
## -60.600 -16.807 -7.365 11.478 81.300
##
## Coefficients: (1 not defined because of singularities)
##
                      Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                        84.700
                                   3.864 21.923
                                                    <2e-16 ***
                        20.900
                                    9.464 2.208 0.0293 *
## dummy_windwindEast
```

```
## dummy windwindNorth
                        1.429
                                    6.245
                                            0.229
                                                    0.8194
## dummy_windwindSouth
                       17.824
                                    7.104
                                            2.509
                                                    0.0136 *
## dummy windwindWest
                            NA
                                       NA
                                               NA
                                                        NA
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
##
## Residual standard error: 27.32 on 108 degrees of freedom
                                   Adjusted R-squared:
## Multiple R-squared: 0.08602,
## F-statistic: 3.388 on 3 and 108 DF, p-value: 0.02074
# Constraint: Sum of alpha_i is 0
summary(lm(max03 ~ C(wind, sum), data = ozone))
##
## Call:
## lm(formula = maxO3 ~ C(wind, sum), data = ozone)
##
## Residuals:
##
      Min
               1Q Median
                               3Q
                                      Max
## -60.600 -16.807 -7.365 11.478 81.300
##
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                  94.738
                              3.053 31.027
                                              <2e-16 ***
## C(wind, sum)1
                 10.862
                              6.829
                                     1.590
                                              0.1147
                 -8.609
## C(wind, sum)2
                              4.622 -1.863
                                              0.0652 .
## C(wind, sum)3
                 7.786
                              5.205 1.496
                                              0.1376
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
## Residual standard error: 27.32 on 108 degrees of freedom
## Multiple R-squared: 0.08602,
                                   Adjusted R-squared: 0.06063
## F-statistic: 3.388 on 3 and 108 DF, p-value: 0.02074
# Using sum contrasts and re-summarizing the linear model
options(contrasts = c("contr.sum", "contr.sum"))
summary(lm(max03 ~ wind, data = ozone))
##
## Call:
## lm(formula = max03 ~ wind, data = ozone)
##
## Residuals:
##
      Min
               1Q Median
                               3Q
                                      Max
## -60.600 -16.807 -7.365 11.478 81.300
##
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                            3.053 31.027
                94.738
                                            <2e-16 ***
## wind1
                10.862
                            6.829
                                    1.590
                                            0.1147
## wind2
                -8.609
                            4.622 -1.863
                                            0.0652 .
## wind3
                 7.786
                            5.205
                                   1.496
                                            0.1376
## ---
```

```
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 27.32 on 108 degrees of freedom
## Multiple R-squared: 0.08602, Adjusted R-squared: 0.06063
## F-statistic: 3.388 on 3 and 108 DF, p-value: 0.02074
```

This explains the steps involved in the analysis of ozone data, including reading the data, representing it through a boxplot, analyzing the significance of factors using ANOVA, conducting residual analysis, and interpreting coefficients obtained from the linear model. Adjust and execute the code within R to perform the analyses on your

Multiple-Way ANOVA:

Multiple-way ANOVA, also known as factorial ANOVA, extends the concept of one-way ANOVA by allowing the simultaneous investigation of the effects of multiple categorical independent variables (factors) on a continuous dependent variable. It assesses not only the main effects of each factor but also examines the interactions between these factors.

1. Factors and Levels:

- Factors are categorical variables that divide the dataset into groups.
- Each factor can have multiple levels, representing different categories within that factor.

2. Main Effects:

- The main effect of a factor refers to the influence or impact that factor has on the dependent variable, ignoring the presence of other factors.
- For example, if Factor A has three levels (low, medium, high), the main effect of Factor A measures how the dependent variable changes across these three levels, regardless of the levels of other factors.

3. Interactions:

- Interaction effects occur when the effect of one factor on the dependent variable changes based on the levels of another factor.
- For instance, if the effect of Factor A on the dependent variable differs depending on the levels of Factor B, an interaction between Factor A and Factor B exists.

4. Hypotheses in Multiple-Way ANOVA:

- The null hypothesis for main effects is that there are no differences among the levels of each factor.
- The null hypothesis for interactions is that the effects of the factors do not interact or affect each other in influencing the dependent variable.

5. Analysis:

- Multiple-way ANOVA produces an ANOVA table that includes information about main effects, interactions, degrees of freedom, sums of squares, F-values, and p-values.
- Significant p-values indicate that at least one factor or interaction has a statistically significant effect on the dependent variable.

6. Post-hoc Tests:

• If interactions are significant, post-hoc tests (e.g., Tukey HSD, Bonferroni) can be performed to determine specific differences between factor levels.

Example Interpretation:

Suppose a study investigates the effects of two factors, A and B, on the growth rate of plants. The multiple-way ANOVA results show:

• Main Effects:

- Factor A has a statistically significant main effect on plant growth (p < 0.05).
- Factor B also has a significant main effect on plant growth (p < 0.05).

• Interaction:

- There is a significant interaction between Factor A and Factor B (p < 0.05).
- This indicates that the effect of Factor A on plant growth varies depending on the levels of Factor B, and vice versa.

In conclusion, both Factor A and Factor B independently influence plant growth, and their combined effect (interaction) significantly impacts the growth rate, indicating a more complex relationship than considering each factor alone.

The interpretation involves examining both main effects and interactions to understand how each factor contributes to the outcome variable and how they miCertainly! Here's your provided R code translated into R Markdown format with explanations:

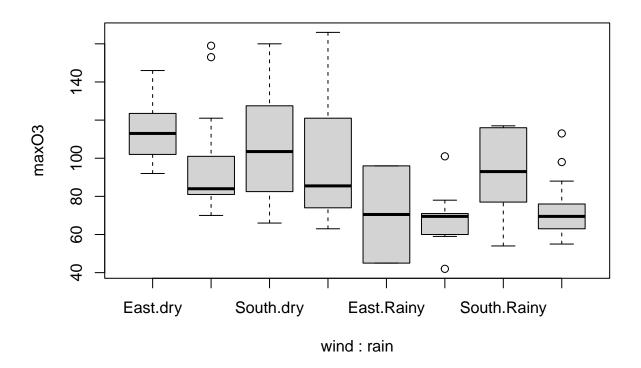
Multiple-Way ANOVA with Interaction

Step 1: Reading the Data

```
# Reading the dataset 'ozone.txt'
ozone <- read.table("ozone.txt", header = TRUE)</pre>
# Column names of the dataset
colnames (ozone)
                  "T9"
    [1] "max03"
                           "T12"
                                     "T15"
                                              "Ne9"
                                                        "Ne12"
                                                                 "Ne15"
    [8] "Wx9"
                  "Wx12"
                           "Wx15"
##
                                     "max03v" "wind"
                                                       "rain"
# Summary of columns 'maxO3', 'wind', and 'rain'
summary(ozone[, c("max03", "wind", "rain")])
##
        max03
                                              rain
                          wind
##
   Min.
           : 42.00
                      Length:112
                                          Length:112
    1st Qu.: 70.75
                      Class :character
                                          Class :character
##
   Median: 81.50
                      Mode :character
                                          Mode :character
##
   Mean
           : 90.30
    3rd Qu.:106.00
           :166.00
##
    Max.
```

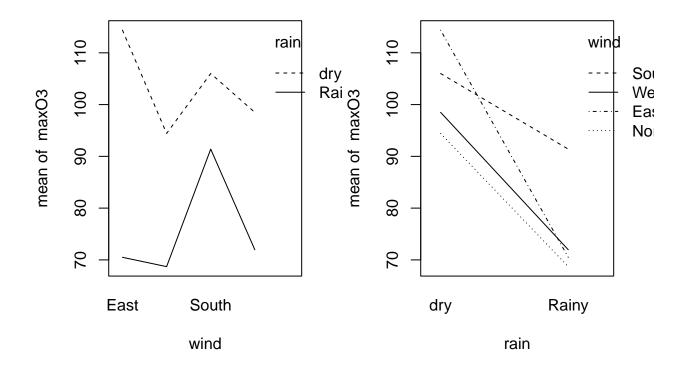
Step 2: Representing the Data

```
# Creating boxplots and interaction plots for 'max03' against 'wind' and 'rain'
boxplot(max03 ~ wind * rain, data = ozone)
```



```
par(mfrow = c(1, 2)) # Setting up a side-by-side plotting layout

# Interaction plots for 'max03' with respect to 'wind' and 'rain'
with(ozone, interaction.plot(wind, rain, max03))
with(ozone, interaction.plot(rain, wind, max03))
```



Step 3: Choosing the Model

```
# Fitting the multiple-way ANOVA model with interaction
mod.int \leftarrow lm(max03 \sim wind * rain, data = ozone)
anova(mod.int)
## Analysis of Variance Table
##
## Response: max03
##
              Df Sum Sq Mean Sq F value
                                             Pr(>F)
## wind
                    7586 2528.7 4.1454
                                            0.00809 **
                   16159 16159.4 26.4910 1.257e-06 ***
## wind:rain
               3
                    1006
                           335.5 0.5500
                                            0.64929
## Residuals 104
                  63440
                           610.0
## ---
                   0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
## Signif. codes:
# Check for interaction: If p-value is > 0.05, there's no #significant interaction
# Fitting a model without interaction to check main effects
mod.without.int \leftarrow lm(max03 \sim wind + rain, data = ozone)
anova(mod.without.int)
```

Analysis of Variance Table

```
##
## Response: max03
             Df Sum Sq Mean Sq F value
##
                 7586 2528.7 4.1984 0.007514 **
## wind
## rain
              1 16159 16159.4 26.8295 1.052e-06 ***
## Residuals 107 64446
                        602.3
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
# If both factors (wind and rain) are significant, we keep the model
```

Step 4: Interpreting the Coefficients

```
# Converting 'wind' and 'rain' to factors for model specification
ozone$wind <- as.factor(ozone$wind)</pre>
ozone$rain <- as.factor(ozone$rain)</pre>
# Specifying contrasts and fitting the model
model <- lm(max03 ~ C(wind, sum) + C(rain, sum), data = ozone)</pre>
summary(model)
##
## Call:
## lm(formula = max03 ~ C(wind, sum) + C(rain, sum), data = ozone)
##
## Residuals:
##
      Min
                1Q Median
                                3Q
                                       Max
## -42.618 -15.664 -3.712 8.295 67.990
##
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
                 90.135
                              2.883 31.260 < 2e-16 ***
## (Intercept)
## C(wind, sum)1
                 7.786
                               6.164 1.263
                                                0.209
## C(wind, sum)2
                 -8.547
                               4.152 -2.059
                                                0.042 *
## C(wind, sum)3
                   5.685
                               4.694
                                      1.211
                                                0.228
                                      5.180 1.05e-06 ***
## C(rain, sum)1
                 12.798
                               2.471
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
## Residual standard error: 24.54 on 107 degrees of freedom
## Multiple R-squared: 0.2692, Adjusted R-squared: 0.2419
## F-statistic: 9.856 on 4 and 107 DF, p-value: 7.931e-07
options(contrasts = c("contr.sum", "contr.sum"))
# Writing the models with and without interaction
summary(lm(max03 ~ wind + rain + wind:rain, data = ozone)) # With interaction
##
## Call:
## lm(formula = max03 ~ wind + rain + wind:rain, data = ozone)
```

```
##
## Residuals:
                1Q Median
##
       Min
                                       Max
  -40.000 -15.971
                   -3.462
                             7.635
                                    67.500
##
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 89.4831
                            3.2603
                                    27.446
                                            < 2e-16 ***
## wind1
                 2.9544
                            7.6345
                                     0.387
                                             0.6996
## wind2
                -7.9189
                            4.6782
                                    -1.693
                                             0.0935
## wind3
                 9.2169
                            5.5358
                                     1.665
                                             0.0989
                            3.2603
## rain1
                13.8428
                                     4.246 4.75e-05 ***
## wind1:rain1
                 8.0947
                            7.6345
                                     1.060
                                             0.2915
                                             0.8347
## wind2:rain1
                -0.9785
                            4.6782
                                    -0.209
## wind3:rain1
                            5.5358
                                    -1.182
                                             0.2399
               -6.5428
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
## Residual standard error: 24.7 on 104 degrees of freedom
## Multiple R-squared: 0.2807, Adjusted R-squared:
## F-statistic: 5.797 on 7 and 104 DF, p-value: 1.092e-05
summary(lm(max03 ~ wind + rain, data = ozone)) # Without interaction
##
## Call:
  lm(formula = max03 ~ wind + rain, data = ozone)
##
## Residuals:
##
       Min
                1Q
                   Median
                                3Q
                                       Max
  -42.618 -15.664
                    -3.712
                             8.295
                                    67.990
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
                             2.883
                                    31.260
## (Intercept)
                 90.135
                                            < 2e-16 ***
## wind1
                  7.786
                             6.164
                                     1.263
                                              0.209
## wind2
                 -8.547
                             4.152
                                    -2.059
                                               0.042 *
                                     1.211
## wind3
                  5.685
                             4.694
                                               0.228
## rain1
                 12.798
                             2.471
                                     5.180 1.05e-06 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 24.54 on 107 degrees of freedom
## Multiple R-squared: 0.2692, Adjusted R-squared: 0.2419
## F-statistic: 9.856 on 4 and 107 DF, p-value: 7.931e-07
```

This provides a step-by-step analysis for multiple-way ANOVA with interaction. It reads the data, represents it through boxplots and interaction plots, chooses the appropriate model based on interaction significance, and interprets the coefficients considering the inclusion or exclusion of interaction terms in the model. Adjust and execute the code within R to perform the analyses on your dataset ght interact in influencing it.

Analysis of Covariance:

1. ANOVA vs. ANCOVA:

- ANOVA compares group means based on categorical independent variables (factors).
- ANCOVA extends ANOVA by including one or more continuous variables (covariates) to adjust for their influence on the dependent variable.

2. Assumptions:

- ANCOVA assumes that there is a linear relationship between the covariate(s) and the dependent variable within each group.
- Homogeneity of regression slopes: The relationship between the covariate(s) and the dependent variable should be consistent across groups.

3. Purpose:

- ANCOVA aims to increase the precision and accuracy of the comparison between group means by removing variance associated with the covariate(s).
- It helps to reduce error variance, increases statistical power, and can lead to a better understanding of the relationship between the independent variable (factor) and the dependent variable after controlling for covariate(s).

4. Model Equation:

- The general form of the ANCOVA model is: $Y_{ij} = \beta_0 + \beta_1 X_{ij} + \beta_2 G_j + \epsilon_{ij}$
 - $-Y_{ij}$ = the dependent variable for the i-th subject in the j-th group.
 - $-X_{ij}$ = the covariate for the i-th subject in the j-th group.
 - $-G_{j}$ = the j-th group indicator variable.
 - $-\beta_0$ = the intercept.
 - $-\beta_1$ = the slope coefficient for the covariate.
 - $-\beta_2$ = the group mean difference coefficient.
 - $-\epsilon_{ij}$ = the error term.

5. Hypotheses:

- Null hypothesis (H0): The group means are equal after adjusting for the covariate(s).
- Alternative hypothesis (H1): At least one group mean is different after adjusting for the covariate(s).

6. Analysis:

- ANCOVA generates an ANOVA table that includes information on the effect of the covariate(s), the effect of the group factor, and their interaction.
- F-tests are used to determine the significance of the covariate, group factor, and interaction terms.

7. Interpretation:

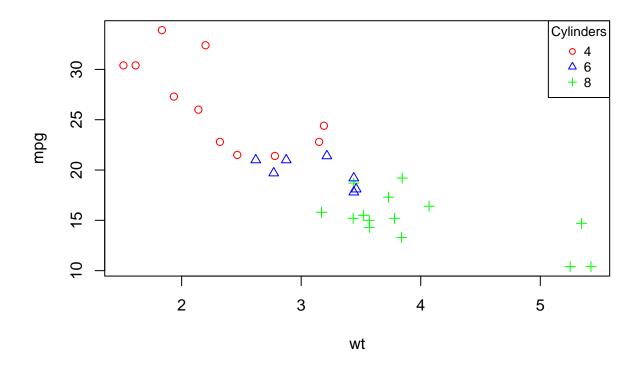
• Significant interaction between group and covariate indicates that the effect of the group factor on the dependent variable varies based on the levels of the covariate.

ANCOVA is a valuable tool in experimental design and observational studies where controlling for covariates is crucial for accurate comparison among groups. It allows researchers to better understand the relationship between the independent variable and the dependent variable while considering the influence of covariate(s).

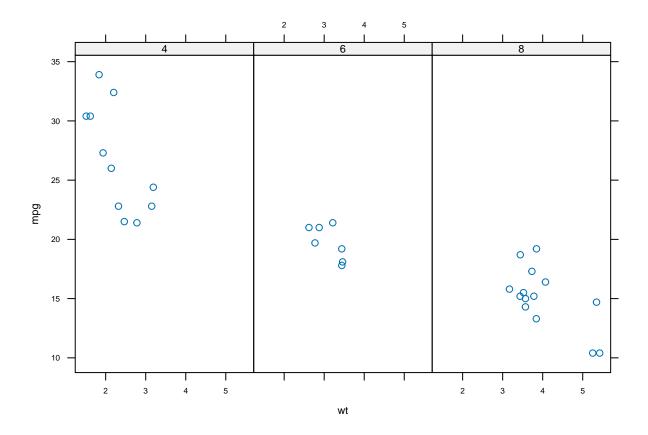
Analysis of Covariance

Step 1: Read The Data

```
# Load the mtcars dataset and view its structure
data(mtcars)
str(mtcars)
                   32 obs. of 11 variables:
## 'data.frame':
## $ mpg : num 21 21 22.8 21.4 18.7 18.1 14.3 24.4 22.8 19.2 ...
## $ cyl : num 6 6 4 6 8 6 8 4 4 6 ...
## $ disp: num 160 160 108 258 360 ...
## $ hp : num 110 110 93 110 175 105 245 62 95 123 ...
## $ drat: num 3.9 3.9 3.85 3.08 3.15 2.76 3.21 3.69 3.92 3.92 ...
## $ wt : num 2.62 2.88 2.32 3.21 3.44 ...
## $ qsec: num 16.5 17 18.6 19.4 17 ...
## $ vs : num 0 0 1 1 0 1 0 1 1 1 ...
## $ am : num 1 1 1 0 0 0 0 0 0 ...
## $ gear: num 4 4 4 3 3 3 3 4 4 4 ...
## $ carb: num 4 4 1 1 2 1 4 2 2 4 ...
# Select columns 'mpg', 'cyl', and 'wt' from mtcars
dat <- mtcars[, c("mpg", "cyl", "wt")]</pre>
summary(dat)
##
                                         wt
                        cyl
        mpg
## Min. :10.40 Min. :4.000 Min. :1.513
## 1st Qu.:15.43 1st Qu.:4.000 1st Qu.:2.581
## Median :19.20 Median :6.000 Median :3.325
## Mean :20.09 Mean :6.188
                                   Mean :3.217
## 3rd Qu.:22.80 3rd Qu.:8.000
                                   3rd Qu.:3.610
## Max. :33.90 Max. :8.000
                                   Max. :5.424
Step 2: Representing the Data
cyl_colors <- c("red", "blue", "green")</pre>
# Convert 'cyl' column to a factor with appropriate levels
dat$cyl <- factor(dat$cyl, levels = c(4, 6, 8))</pre>
# Check the levels of 'cyl' column after conversion
levels(dat$cyl)
## [1] "4" "6" "8"
# Creating a plot: mpg against wt with different colors and shapes #for each 'cyl'
plot(mpg ~ wt, pch = as.numeric(cyl), col = cyl_colors[as.numeric(dat$cyl)], data = dat,
    xlim = range(dat$wt), ylim = range(dat$mpg))
# Adding a legend with colors
legend("topright", legend = levels(dat$cyl), pch = 1:nlevels(dat$cyl), col = cyl_colors, title = "Cylin"
```



```
# Creating an xyplot to visualize relationships between mpg and wt for each 'cyl'
library(lattice)
xyplot(mpg ~ wt | cyl, data = dat)
```



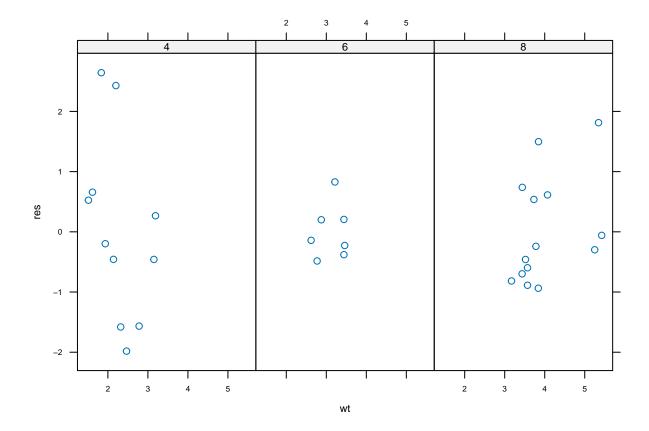
Step 3: Choosing the Model

```
# Fitting models to choose the appropriate one
global \leftarrow lm(mpg \sim -1 + cyl + cyl:wt, data = dat)
slopeU \leftarrow lm(mpg \sim -1 + cyl + wt, data = dat)
interceptU <- lm(mpg ~ cyl:wt, data = dat)</pre>
# Conducting nested model tests using ANOVA
anova(slopeU, global)
## Analysis of Variance Table
##
## Model 1: mpg ~ -1 + cyl + wt
## Model 2: mpg ~ -1 + cyl + cyl:wt
              RSS Df Sum of Sq
##
     Res.Df
                                       F Pr(>F)
## 1
         28 183.06
## 2
         26 155.89 2
                           27.17 2.2658 0.1239
anova(interceptU, global)
## Analysis of Variance Table
## Model 1: mpg ~ cyl:wt
```

```
## Model 2: mpg ~ -1 + cyl + cyl:wt
## Res.Df
             RSS Df Sum of Sq F Pr(>F)
## 1
       28 220.37
## 2
        26 155.89 2 64.476 5.3769 0.01111 *
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
# Interpreting nested model tests to select the appropriate model
# Conducting additional tests to check null slope or intercept #models
simple <- lm(mpg ~ wt, data = dat)</pre>
anova(simple, slopeU)
## Analysis of Variance Table
##
## Model 1: mpg ~ wt
## Model 2: mpg ~ -1 + cyl + wt
## Res.Df RSS Df Sum of Sq F Pr(>F)
## 1
       30 278.32
       28 183.06 2 95.263 7.2856 0.002835 **
## 2
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
anova1 <- lm(mpg ~ cyl, data = dat)</pre>
anova(anova1, slopeU)
## Analysis of Variance Table
## Model 1: mpg ~ cyl
## Model 2: mpg ~ -1 + cyl + wt
            RSS Df Sum of Sq F Pr(>F)
## Res.Df
## 1
       29 301.26
        28 183.06 1 118.2 18.08 0.000213 ***
## 2
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
```

Step 4: Conducting the Residual Analysis

```
# Visualizing residuals against wt for each 'cyl'
xyplot(rstudent(slopeU) ~ wt | cyl, ylab = "res", data = dat)
```



This part walks through the analysis of covariance steps. It starts with reading the dataset, representing the data through plots, choosing the model via nested model tests, and concludes with conducting residual analysis by visualizing the residuals against wt for each 'cyl'. Execute the code in an R environment to perform these analyses on your dataset. Adjustments can be made according to the specific dataset and variables being studied.