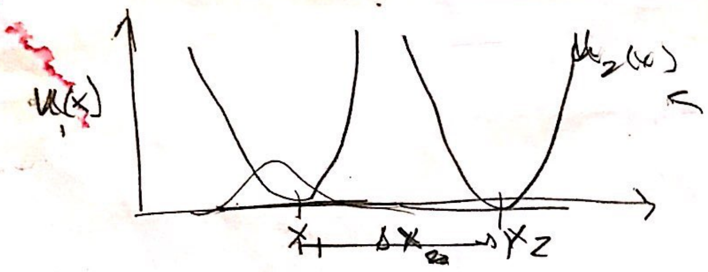


① 4/22/21

← in units of kT. $u_2(x) = \beta U_2(x)$

assume k is a constant!

$$u_1(x) = \frac{1}{2}k(x-x_1)^2$$
$$u_2(x) = \frac{1}{2}k(x-x_2)^2$$

$$\Delta u = u_2 - u_1(x) = \frac{k}{2}\left[\cancel{x^2} - 2x\cdot x_2 + x_2^2 - \cancel{x^2} + 2x\cdot x_1 - x_1^2\right]$$

$$\Delta u = \frac{k}{2}\left(-2x(x_2-x_1) + x_2^2 - x_1^2\right)$$

$$= -k\left[x(\Delta x) + \frac{x_2^2 - x_1^2}{2}\right]$$

so for equal $k_1 = k_2 = k$, Δu is linearly relates to x

$$x = -\left(\frac{\Delta u}{k} + \frac{x_2^2 - x_1^2}{2}\right)\frac{1}{\Delta x}$$

so $P_1(x)\,dx \propto \dfrac{e^{-\frac{k}{2}(x-x_1)^2}\,dx}{\int \Sigma\,dx}$

$$P(\Delta u)\,d\Delta u = e^{-\frac{k}{2}\left\{-\left(\frac{\Delta u}{k} + \frac{x_2^2 - x_1^2}{2}\right)\frac{1}{\Delta x} + x_1\right\}^2}\left(\frac{dx}{d\Delta u}\right)d\Delta u$$

② 4/22/21

$$\frac{dv}{d\Delta u} = -\frac{1}{k\Delta x}$$

$$\int_{+\infty}^{-\infty} P(\Delta u)\, d\Delta u = \int_{+\infty}^{-\infty} e^{-\frac{k}{2}\left\{\left(\frac{\Delta u}{k} + \frac{x_2^2 - x_1^2}{2}\right)^2 \frac{1}{\Delta x^2} + 2x_1\left(\frac{\Delta u}{k} + \frac{x_2^2 - x_1^2}{2}\right)\frac{1}{\Delta x} + x_1^2\right\}} \cdot \left(-\frac{1}{k\Delta x}\right)$$

flip

$$\int_{-\infty}^{+\infty} = \left[e^{-\frac{k}{2}\left\{\frac{\Delta u^2}{k^2\Delta x^2} + 2\left(\frac{x_2^2-x_1^2}{2}\right)\frac{\Delta u}{k\Delta x^2} + \left(\frac{x_2^2-x_1^2}{2}\right)^2\frac{1}{\Delta x^2} + \frac{\Delta u}{k}\cdot 2x_1\frac{1}{\Delta x} + 2x_1\left(\frac{x_2^2-x_2^2}{2}\right)\frac{1}{\Delta x} + x_1^2\right\}}\right] \cdot \left(\frac{+1}{k\Delta x}\right) d\Delta u$$

$$= \left[e^{-\frac{k}{2}\left\{\left(\frac{\Delta u}{k\Delta x}\right)^2\left[\frac{1}{\cancel{k^2\Delta x^2}}\right] + 2\left(\frac{\Delta u}{k\Delta x}\right)\left[\left(\frac{x_2^2-x_1^2}{2}\right)\frac{1}{\cancel{k}\Delta x^{\cancel{2}}} + \frac{x_1}{\cancel{k\Delta x}}\right] + \left[\left(\frac{x_2^2-x_1^2}{2}\right)\frac{1}{\Delta x} + x_1\right]^2 - \left[\left(\frac{x^2-x_1^2}{2}\right)\frac{1}{\Delta x} + x_1\right]^2 + \left(\frac{x_2^2-x_1^2}{2}\right)^2\frac{1}{\Delta x^2} + 2x_1\left(\frac{x_2^2-x_2^2}{2}\right)\frac{1}{\Delta x} + x_1^2\right\}}\right]\frac{1}{k\Delta x}\, d\Delta u$$

complete the square!

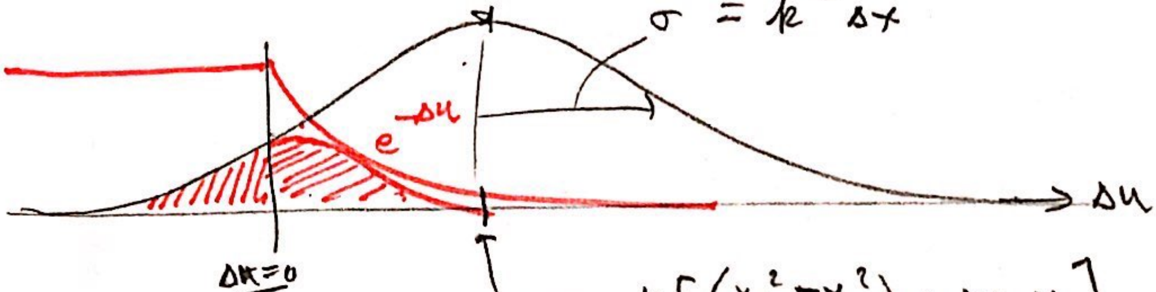
$$= \left[e^{-\frac{k}{2}\left\{\text{BIG CONSTANT}\right\}}\left(\frac{1}{k\Delta x}\right)\right] e^{-\frac{k}{2}\left\{\left(\frac{\Delta u}{k\Delta x}\right) - \left[\left(\frac{x_2^2-x_1^2}{2}\right)\frac{1}{\Delta x} + x_1\right]\right\}^2}$$

$$= \left[\quad .. \quad\right] e^{-\frac{k}{2}\left(\frac{1}{k^2\Delta x^2}\right)\left\{\Delta u - k\Delta x\left[\quad .. \quad\right]\right\}^2}$$

so it's a Gaussian of $\cancel{2}\sigma^2 = \cancel{2}k\Delta x^2$

centered at $\cancel{k}\,\cancel{\Delta x}\left[\left(\frac{x_2^2-x_1^2}{2}\right)\frac{k}{\cancel{\Delta x}} + k\Delta x \cdot x_1\right] = k\left[\left(\frac{x_2^2-x_1^2}{2}\right) + \Delta x\cdot x_1\right]$

③ 4/22/2021

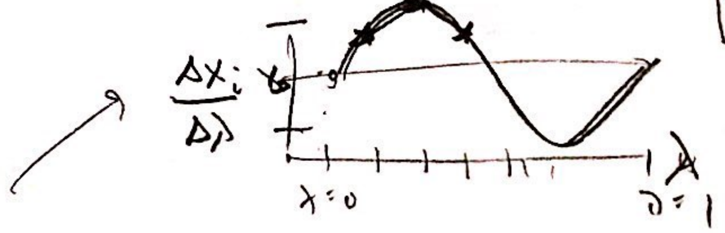


$$\Delta u_0 = k\left[\left(\frac{x_2^2 - x_1^2}{2}\right) + \Delta x \cdot x_1\right]$$

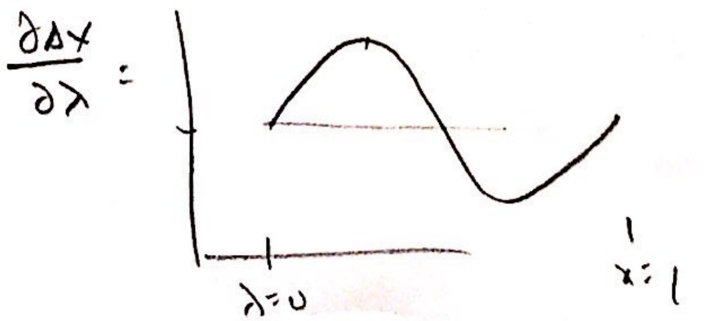
$$\langle P_{acc\,1\to2}\rangle = \int \min(1, \exp(-\Delta u)) \cdot \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(\Delta u - \Delta u_0)^2}{2\sigma^2}\right)$$

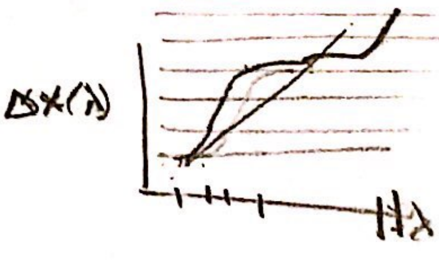
~ something involving erf(.) ...
Mathematica?

$$\boxed{x_0(\lambda) = x_0(1 + 0.8\sin(2\pi\lambda))}$$



$$\int_0^\lambda \left(\frac{\partial \Delta x}{\partial \lambda}\right) \partial\lambda = \Delta x(\lambda)$$





$$\frac{x_2^2 - x_1^2}{2} + (x_2 - x_1)x_1$$

$$\frac{(x_2 - x_1)(x_2 + x_1)}{2} + (x_2 - x_1)x_1$$

$$\frac{\Delta x}{\cancel{2}}\left(\frac{x_1 + x_2}{2} + x_1\right)$$

$$x_2 = \cancel{\Delta x}\, x_1 + \Delta x$$

$$\Rightarrow \Delta x \left(\frac{x_1 + (x_1 + \Delta x)}{2} + x_1\right)$$

$$\Delta x\left(\frac{\Delta x}{2} + \frac{2x_1}{2} + x_1\right)$$

$$= \frac{\Delta x^2}{2} + 2\Delta x\, x_1$$