k-subsets

$$\mathcal{P}_k(S) = \{ T \in \mathcal{P}(S) \mid |T| = k \}.$$

→ E.g. if $S = \{1,2,3\}$, $\mathcal{P}_2(S) = \{\{1,2\},\{1,3\},\{2,3\}\}$.

obvious way, since we already know how to compute $\mathcal{P}(S)$:

Just filter out the ones you want (size $= k$).

Say $|S| = n$. Then $|\mathcal{P}_k(S)| = \binom{n}{k} \ll |\mathcal{P}(S)|$

for k close to 0 or n.

$$\sum_{k=0}^{n} \binom{n}{k} = |\mathcal{P}(S)| = 2^n = (1+1)^n = \sum_{k=0}^{n} \binom{n}{k} 1^k 1^{n-k} \ldots$$

Algorithm outline. 2 size parameters: k, $|S|$.

Base case(s)? if $k = 0$, answer $= \{\{\}\}$.

if $k > |S|$, answer $= \{\}$.

Now assume we have a magic box that gets correct answers for $\mathcal{P}_{k'}(S')$ when $k' < k$ <u>or</u> $|S'| < |S|$.

Example from above:

$S = \{1,2,3\}$, $\mathcal{P}_2(S) = \{\{1,2\},\{1,3\},\{2,3\}\}$.

Would $\underbrace{\mathcal{P}_2(S \setminus \{3\})}_{\text{all 2-subsets w/o 3.}}$ help? Sure! $\mathcal{P}_k(S') \subseteq \mathcal{P}_k(S)$ if $S' \subseteq S$.

What's missing? All 2-subsets that have 3...

Start w/ $\mathcal{P}_{k-1}(S\setminus\{3\})$, and then add 3 to each one.

$$\mathcal{P}_2(S\setminus\{3\}) = \{\{1,2\}\}$$

$$\mathcal{P}_1(S\setminus\{3\})+3 = \{\{1\},\{2\}\}+3$$

$$= \{\{1,3\},\{2,3\}\}.$$

$\cup$ $\mathcal{P}_k(S)$. ✓

(Recall, for a <u>set of sets</u> $\mathcal{P}$ and element x, we defined $\mathcal{P}+x \triangleq \{T\cup\{x\} \mid T\in\mathcal{P}\}$.)

Regarding last project: think of it as a multidimensional <u>Cantor Set</u>