Based on the mathematical theory of Score-Based Generative Models (Diffusion Models), the answer is **YES**.

The two equations describe the **same marginal probability distributions** $p_t(\mathbf{x})$ for all time t.

Here is the concise, detailed English proof note using the Fokker-Planck Equation.

Proof of Equivalent Marginal Distributions

1. Context Identification

- **Top Equation (SDE):** This is the **Reverse-time SDE**. It describes a stochastic process with random noise injection ($d\bar{w}_t$).

$$dx = [\mathbf{f}(\mathbf{x}, t) - g^2(t)\nabla \log p_t(\mathbf{x})]dt + g(t)d\bar{w}_t$$

- **Bottom Equation (ODE):** This is the **Probability Flow ODE**. It describes a deterministic process (no noise term).

$$dx = [\mathbf{f}(\mathbf{x}, t) - \frac{1}{2}g^2(t)\nabla \log p_t(\mathbf{x})]dt$$

2. The Derivation (Via Fokker-Planck Equation)

We start with the general **Forward SDE**:

$$dx = \mathbf{f}(\mathbf{x}, t)dt + g(t)d\mathbf{w}$$

The evolution of the probability density $p_t(\mathbf{x})$ is governed by the **Fokker-Planck Equation (Kolmogorov Forward Equation)**:

$$\frac{\partial p_t(\mathbf{x})}{\partial t} = -\nabla \cdot (\mathbf{f}(\mathbf{x}, t)p_t(\mathbf{x})) + \frac{1}{2}g^2(t)\nabla^2 p_t(\mathbf{x})$$

We can rewrite the diffusion term (second term) using the identity $\nabla p_t = p_t \nabla \log p_t$:

$$\begin{aligned}\frac{1}{2}g^2(t)\nabla^2 p_t(\mathbf{x}) &= \frac{1}{2}g^2(t)\nabla \cdot (\nabla p_t(\mathbf{x})) \\ &= \frac{1}{2}g^2(t)\nabla \cdot (p_t(\mathbf{x})\nabla \log p_t(\mathbf{x}))\end{aligned}$$

Now, substitute this back into the Fokker-Planck equation:

$$\frac{\partial p_t(\mathbf{x})}{\partial t} = -\nabla \cdot (\mathbf{f}(\mathbf{x}, t) p_t(\mathbf{x})) + \nabla \cdot \left(\frac{1}{2} g^2(t) p_t(\mathbf{x}) \nabla \log p_t(\mathbf{x}) \right)$$

3. Converting to ODE Form (Continuity Equation)

We combine the terms inside the divergence operator $\nabla \cdot$:

$$\frac{\partial p_t(\mathbf{x})}{\partial t} = -\nabla \cdot \left[\left(\mathbf{f}(\mathbf{x}, t) - \frac{1}{2} g^2(t) \nabla \log p_t(\mathbf{x}) \right) p_t(\mathbf{x}) \right]$$

This equation is exactly the **Continuity Equation** (also known as the Liouville Equation) for a probability density moving according to a deterministic velocity field $\mathbf{v}_t(\mathbf{x})$.

The velocity field is:

$$\mathbf{v}_t(\mathbf{x}) = \mathbf{f}(\mathbf{x}, t) - \frac{1}{2} g^2(t) \nabla \log p_t(\mathbf{x})$$

4. Conclusion

This velocity field defines the **Probability Flow ODE**:

$$d\mathbf{x} = \left[\mathbf{f}(\mathbf{x}, t) - \frac{1}{2} g^2(t) \nabla \log p_t(\mathbf{x}) \right] dt$$

Therefore:

1. The **ODE** (bottom equation) is derived directly from the Fokker-Planck equation of the forward process.
2. The **Reverse SDE** (top equation) is constructed to reverse the forward process.
3. **Result:** Both equations result in the **exact same marginal probability density function** $p_t(\mathbf{x})$ at every time step t, despite one being stochastic and the other deterministic.

Summary Note

- **SDE vs. ODE:** The SDE relies on random noise ($g(t)d\bar{w}$) to shape the distribution. The ODE replaces the diffusion effect of that noise with an extra drift term ($\frac{1}{2} g^2 \nabla \log p$).
- **Why the coefficient change?**
  - Top (SDE): Coefficient is $-1 \cdot g^2$.
  - Bottom (ODE): Coefficient is $-\frac{1}{2} \cdot g^2$.
  - The difference of $\frac{1}{2} g^2$ exactly compensates for the lack of stochastic diffusion in the ODE.

Would you like me to explain more about how this is applied in sampling (generation) or the difference in computational cost between the two?