limit $\iff$ seq criterion $\iff$ $f(p+) = f(p-)$

$\forall \varepsilon > 0 \quad \exists \delta > 0$

s.t $|f(x) - L| < \varepsilon$

$0 < d(x,p) < \delta$

if $\forall \{p_n\} \to p$

then $\{f(p_n)\} \to L$

limit is unique

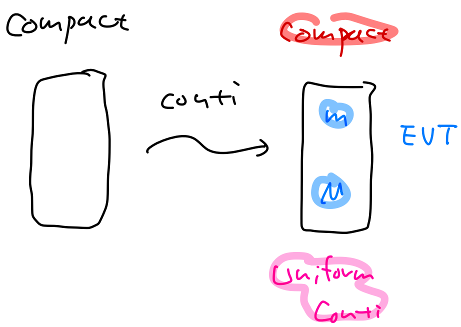
---

continuous $\iff$ topological $\iff$ $f(p) = f(p+) = f(p-)$

$\forall \varepsilon > 0 \quad \exists \delta > 0$

s.t $|f(x) - f(p)| < \varepsilon$

$d(x,p) < \delta$



IVT

$[a \; c \; b]$

conti $\rightsquigarrow$ $[m \;\; f(a) \;\; \gamma \;\; f(b) \;\; M]$

Uniform Conti

Uniform Conti

$\forall \varepsilon > 0 \quad \exists \delta > 0$

s.t $|f(x) - f(y)| < \varepsilon$

if $d(x,y) < \delta$

Lipschitz Func

$|f(x) - f(y)|$

$\leq M \cdot d(x,y)$

---

mono increasing

有 有

$f(p-) \leq f(p) \leq f(p+)$

$\shortparallel$

$\sup\limits_{x<p} f(x) \leq f(y-)$

countable

discont $\preceq \mathbb{N}$

jump $\longmapsto \mathbb{Q}$

injection

f interval / continuous ⟸ 를 위해 필요함,

then strict mono $\iff$ one-to-one

---

f then f^{-1}

interval interval

strict mono strict mono

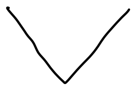
continue continue

derivatives

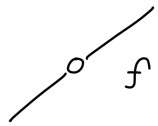
$$\lim_{x \to p} \frac{f(x) - f(p)}{x - p}$$

$f_-'(p)$ $\quad x \to p-$

$f_+'(p)$ $\quad x \to p+$



$f_-'(0) = -1$ $\qquad f_+'(0) = +1$



$f'(0+) = 1$

$f'(0-) = 1$

diffable

$\Rightarrow$ conti

chain rule

$\because$ $f(t) - f(x)$

$= (t-x)\left(f'(x) + \cancel{u(t)}\right)$

---

f on ~~(interval)~~ $[a,b]$

$p \in$ ~~interior point~~ (a,b) is local extremum

either $\begin{cases} f'(p) = 0 \\ \text{not diffable at } p \end{cases}$

Rolle's Thm

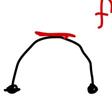
f conti $[a,b]$

diffable (a,b)

$f(a) = f(b)$

then $\exists f'(c) = 0$

$f'(c) = 0$

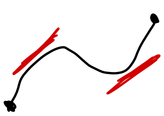


Mean Value Thm

f conti $[a,b]$

diffable (a,b)

then $\exists f'(c) = \dfrac{f(a) - f(b)}{a - b}$



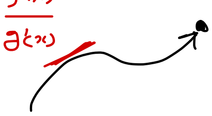
Cauchy MVT

f, g conti $[a,b]$

diffable (a,b)

then $\exists \underbrace{\dfrac{f'(c)}{g'(c)}}_{\text{순간속도의 비율}} = \underbrace{\dfrac{f(a) - f(b)}{g(a) - g(b)}}_{\text{평균속도의 비율}}$ by 철수, 영희

$\dfrac{f'(x)}{g'(x)}$



$(f(x), g(x))$

---

f on (interval)

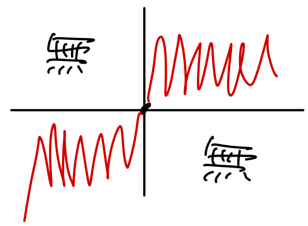
$f'(x) \geq 0 \Rightarrow$ increasing

$f'(x) > 0 \Rightarrow$ mono increasing

$\boxed{\begin{array}{l} f'(c) > 0 \ (\text{and } f' \text{ conti}) \text{ implies} \\ f(c-\delta, c) < f(c) < f(c, c+\delta) \ (\text{and increasing}) \end{array}}$

$f'(0) = 1$

$$\begin{cases} f(x) = x + 2x^2 \sin\left(\frac{1}{x}\right) \\ f'(x) = 1 + 4x\sin\left(\frac{1}{x}\right) \\ \qquad - 2\cos\left(\frac{1}{x}\right) \quad \text{oscilate ,} \end{cases}$$



not increasing

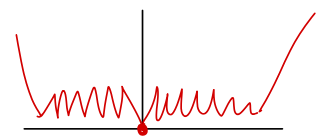
$\boxed{\begin{array}{l} f \text{ is conti on } (a, b) \\ f \ \begin{array}{l} \text{decreasing at } c- \\ \text{increasing at } c+ \end{array} \ \begin{array}{c} \Longrightarrow \\ \not\Longleftarrow \end{array} \ c \text{ is local min} \end{array}}$

$f'(0) = 0$

$f(x) = x^4\left(2 + \sin\left(\frac{1}{x}\right)\right)$

$f'(x) = 4x^3\left(2 + \sin\left(\frac{1}{x}\right)\right) - x^2 \cdot \cos\left(\frac{1}{x}\right)$

oscilate ,



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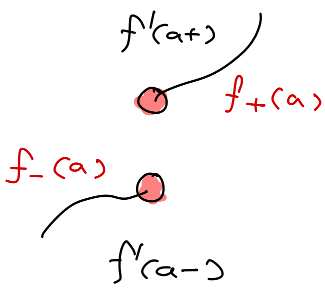
f conti $[a, b)$

diffable (a, b)

if $f'(a+)$ exists

then $f_+(a) = f'(a+)$



$\because$ MVT

$$\frac{f(a+h) - f(a)}{h} = f'(c)$$

$h \to 0+$ then $c \to a+$

$(LHS) \to f_+(a)$

Darboux

f diffable on I

$f'(a) < \lambda < f'(b)$

then $\exists\, f'(c) = \lambda$

$\because$ $g(x) = f(x) - \lambda x$

$g(a) \searrow \quad \nearrow g(b)$

$\times$

local min

$\boxed{\begin{array}{l} f'(a)\text{가 존재} \\ \Rightarrow f'\text{는 } a\text{에서} \begin{cases} \text{연속} \\ \text{oscilate} \\ \cancel{\text{jump}} \text{ disconti} \\ \cancel{\text{removable}} \text{ disconti} \end{cases} \end{array}}$

---

f diffable on I

f' do not jump

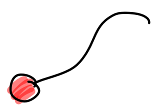
$\forall x \ f'(x) \neq 0 \Rightarrow f$ strict mono $\Rightarrow$ f one-to-one, f^{-1} conti $\Rightarrow f^{-1}$ diffable .

① $f'(c) > 0 \not\Longrightarrow (c-\delta, c+\delta)$ 에서 증가

f'가 c에서 연속이면 성립

② $\lim_{x \to a+} f'(x)$가 존재한다면,

$$f'_+(a) = \lim_{x \to a+} f'(x)$$



③ f'가 불연속이더라도 IVP가 성립한다.

$f'(a) < \lambda < f'(b)$ 이면

$\exists\ f'(c) = \lambda$

④ $f'(a)$ well defined $\Longrightarrow$ continuous (O)

oscillate (O)

jump disconti (X)

⑤ $f'(x) \neq 0$ in I then f strict mono

hence f^{-1} diffable.

로피탈의 정리

① f, g real valued, diffable on (a, b)

② $g'(x) \neq 0$ in (a, b)

③ $\lim_{x \to a+} \frac{f'(x)}{g'(x)} = L \in [-\infty, \infty]$

④ $\begin{cases} \lim_{x \to a+} f(x) = 0 \text{ and } \lim_{x \to a+} g(x) = 0 \\ \lim_{x \to a+} g(x) = \pm\infty \end{cases}$

then $\lim_{x \to a+} \frac{f(x)}{g(x)} = L$

$\because$ define $f(a) = g(a) = 0$

more fixed

GMVT $\frac{f(x_n) - f(a)}{g(x_n) - g(a)} = \frac{f'(c_n)}{g'(c_n)}$

$x_n \to a+$ then $\lim_{x \to a+} \frac{f(x) - f(a)}{g(x) - g(a)} = \lim_{x \to a+} \frac{f'(x)}{g'(x)}$

$\because$

$\lim_{x \to \infty} \frac{f'(x)}{g'(x)} = L$,

more fixed

GMVT $\frac{f(x) - f(a+)}{g(x) - g(a+)} = \frac{f'(\xi)}{g'(\xi)}$

then $(\cdots)$ $\frac{f(x)}{g(x)} = \frac{f'(\xi)}{g'(\xi)}\left(1 - \frac{g(a+)}{g(x)}\right) + \frac{f(a+)}{g(x)}$