

Combinatorial Problems w/ Recursion

Hien Luu

The only thing which can save/help you in interviews is
your thinking ability

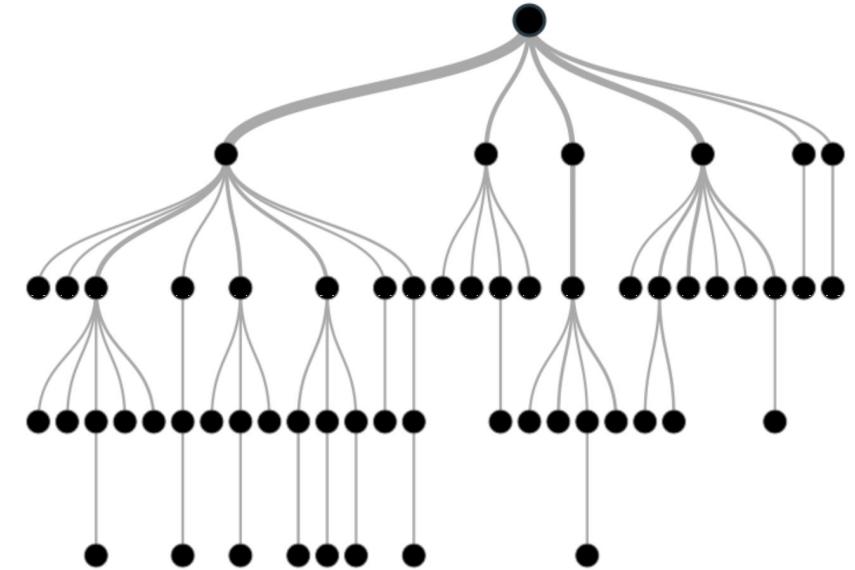


How would you explain what recursion is to a 4-year old kid?

Agenda

- Combinatorial Problems (~2hrs)
 - Exhaustive search, enumerate, generate
 - Classic problems – subsets, permutation
 - General solution Pattern
- Backtracking Problems (~1.5hrs)
 - General Solution Pattern

Popular interview questions, why?

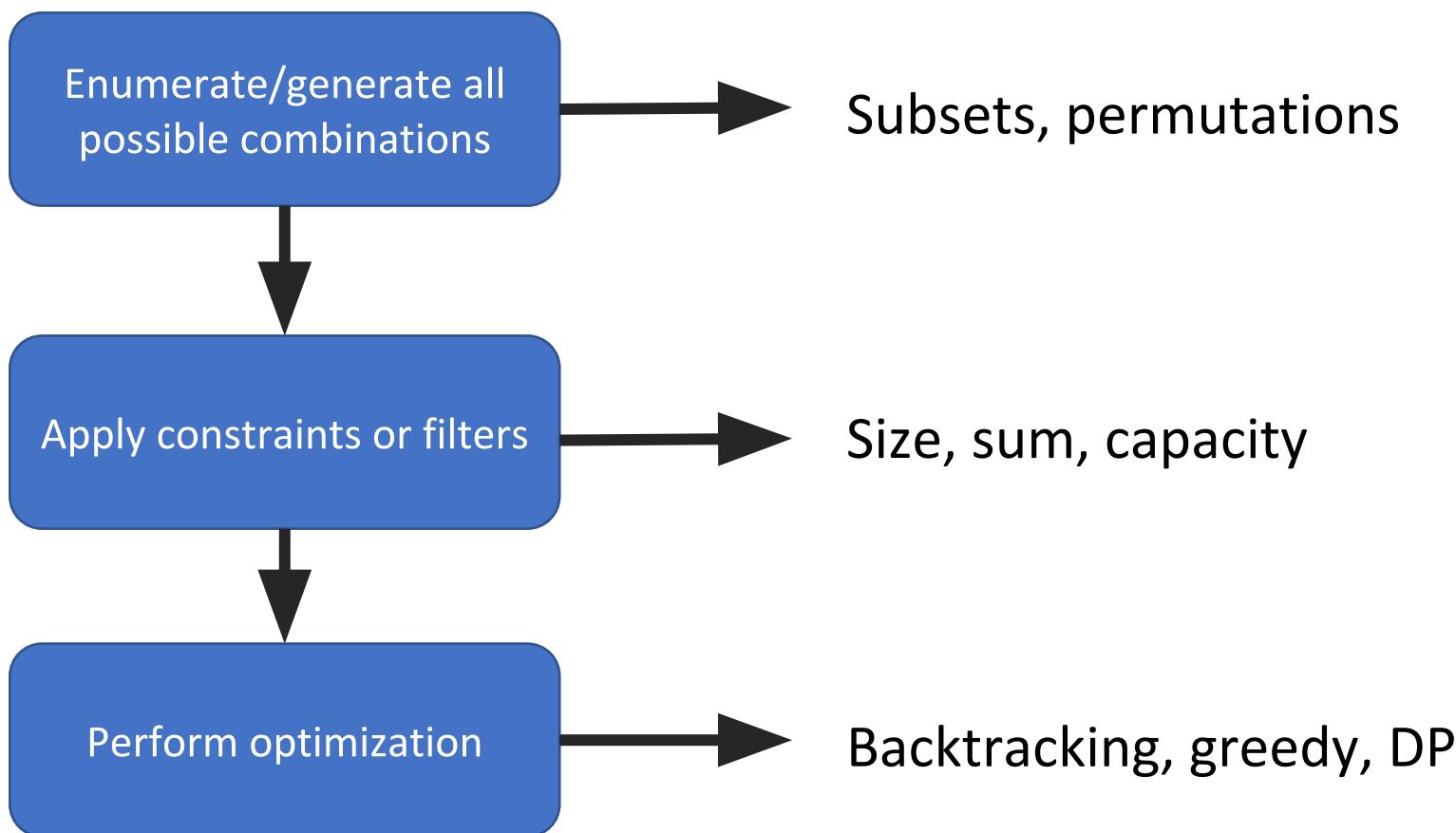


Logistics:

- break ~10:30am
- lunch at noon

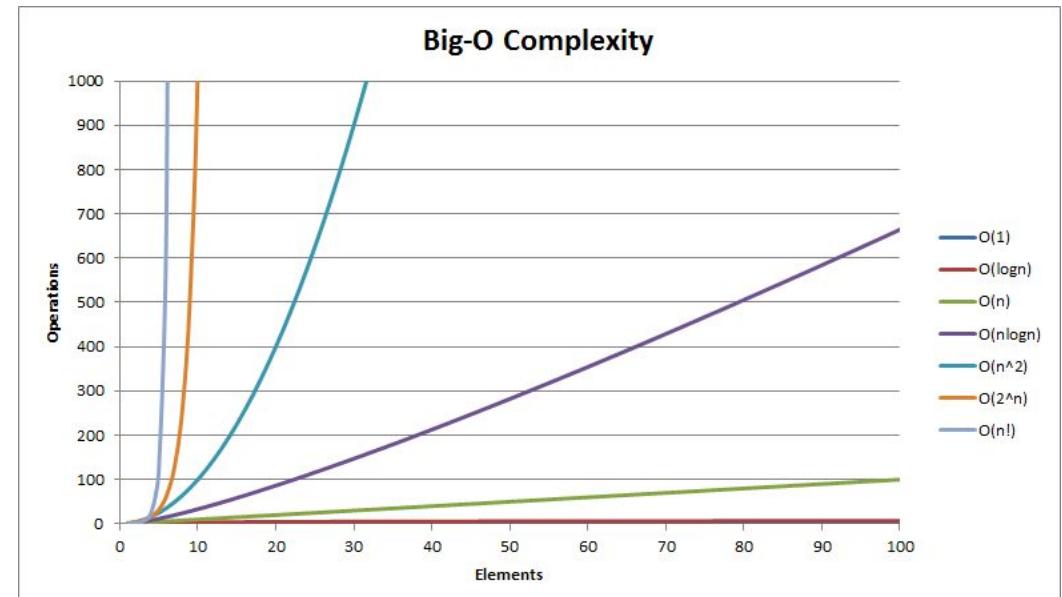
Combinatorial Problems

Layers of Complexity



Combinatorial Problems

- Common question
 - Generation, enumeration, existence
- Common pattern
 - From a set of decisions (aka subproblems) and choices
 - Lend itself to recursion
- Combinatorial explosion
 - Create challenges
 - Exponential runtime



Combinatorial Problems - Mental Model

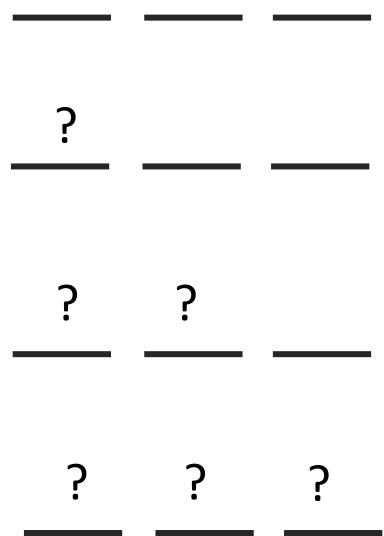
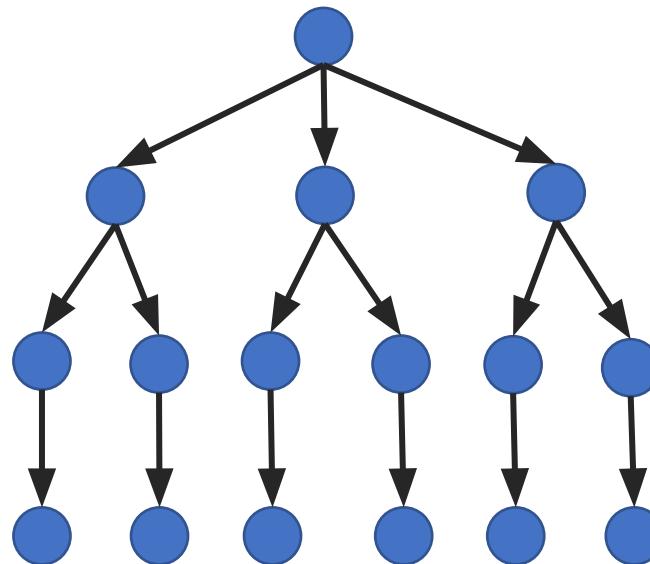
Systematic way of enumerating or generating

A series of blanks



Efficient manager
Recursion Fairy

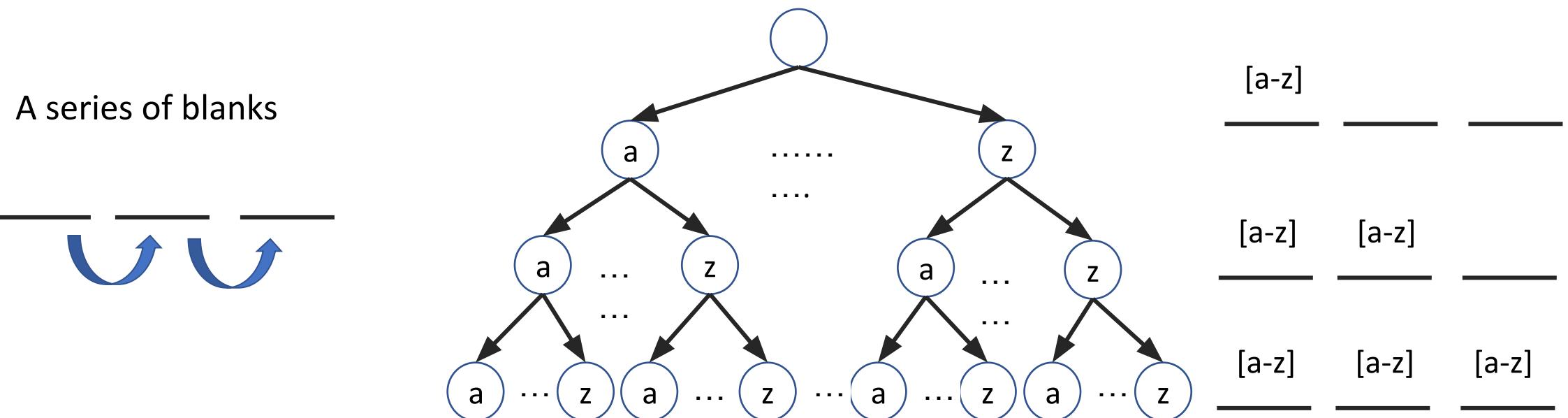
Decision Tree



Helpful visualization tool

Combinatorial Problems

- Example
 - Generate all 3-letter strings (print or collect)
 - Each blank/letter is a decision
 - Each decision has 26 possible choices



Solution Pattern – (pseudo code)

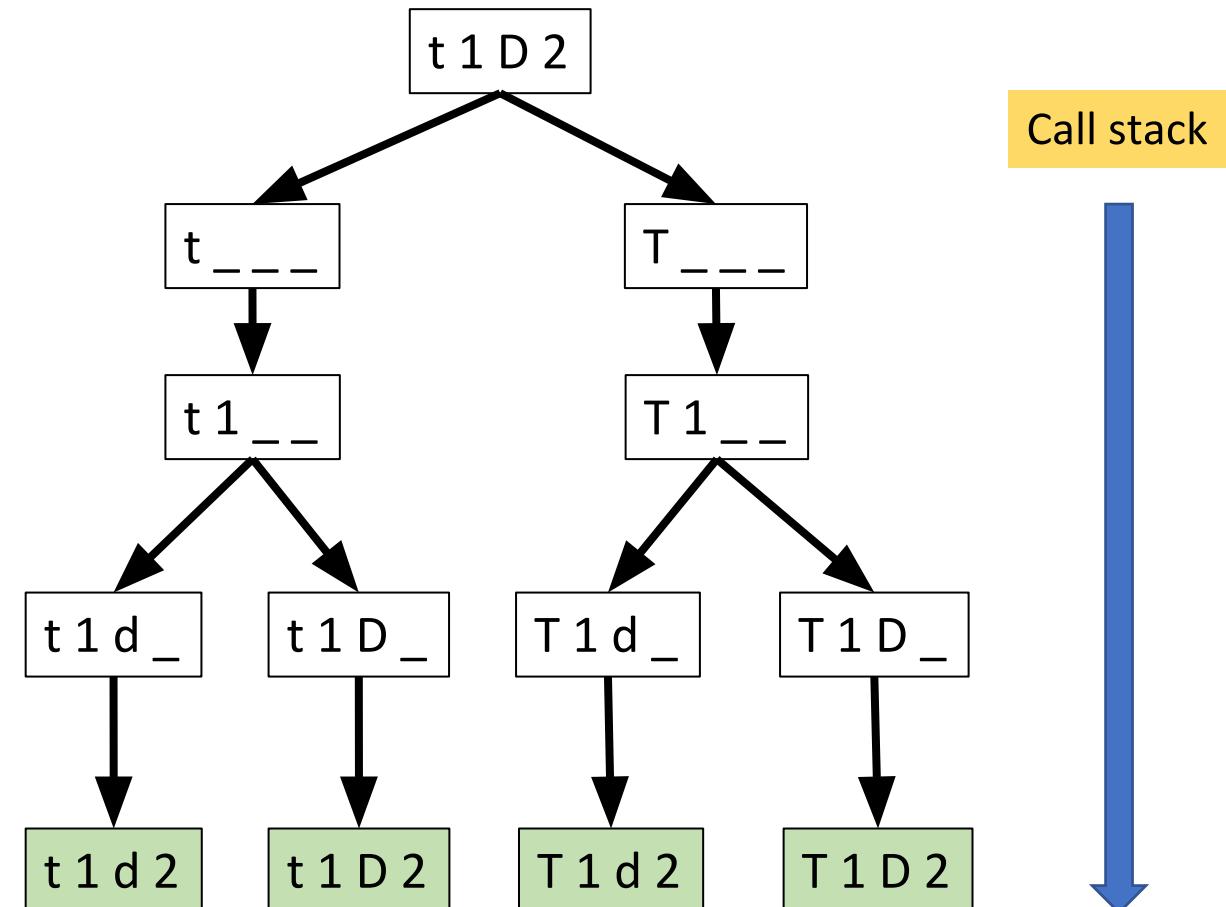
```
driver(problems) {  
    helper(problems, empty-partial-solution)  
}  
  
helper(sub problems, partial-solution) {  
    if (no more sub problems) { // base case  
        print/collect partial-solution  
    } else {  
        for (each possible choices) {  
            make a choice // add to partial-solution  
            helper(remaining-decisions, partial-solution)  
            unmake choice // if mutable data structure is used  
        }  
    }  
}
```

Letter Case Permutation

- Given a string S
 - Transform every letter to lowercase or uppercase to create another string
 - Return a list of all possible strings we could create
- Example:
 - Input: "t1D2"
 - Output: ["t1D2", "t1d2", "T1D2", "T1d2"]
 - Input: "12345"
 - Output: ["12345"]
 - Input: "abc"
 - Output: [????]

Letter Case Permutation – Decision Tree

Input: "t1D2"



Letter Case Permutation

```
// Return a list of all possible strings

List<String> driver(String input) {
    List<String> coll = new ArrayList<>();
    helper(input, 0, "", coll);
    return coll;
}

void helper(String input, int idx, String soFar, List<String> coll) {
    // base case
    // else case

}
```

Letter Case Permutation

```
void helper(String input, int idx, String soFar, List<String> coll)
{
    // base case
    // else case
}
```

Letter Case Permutation

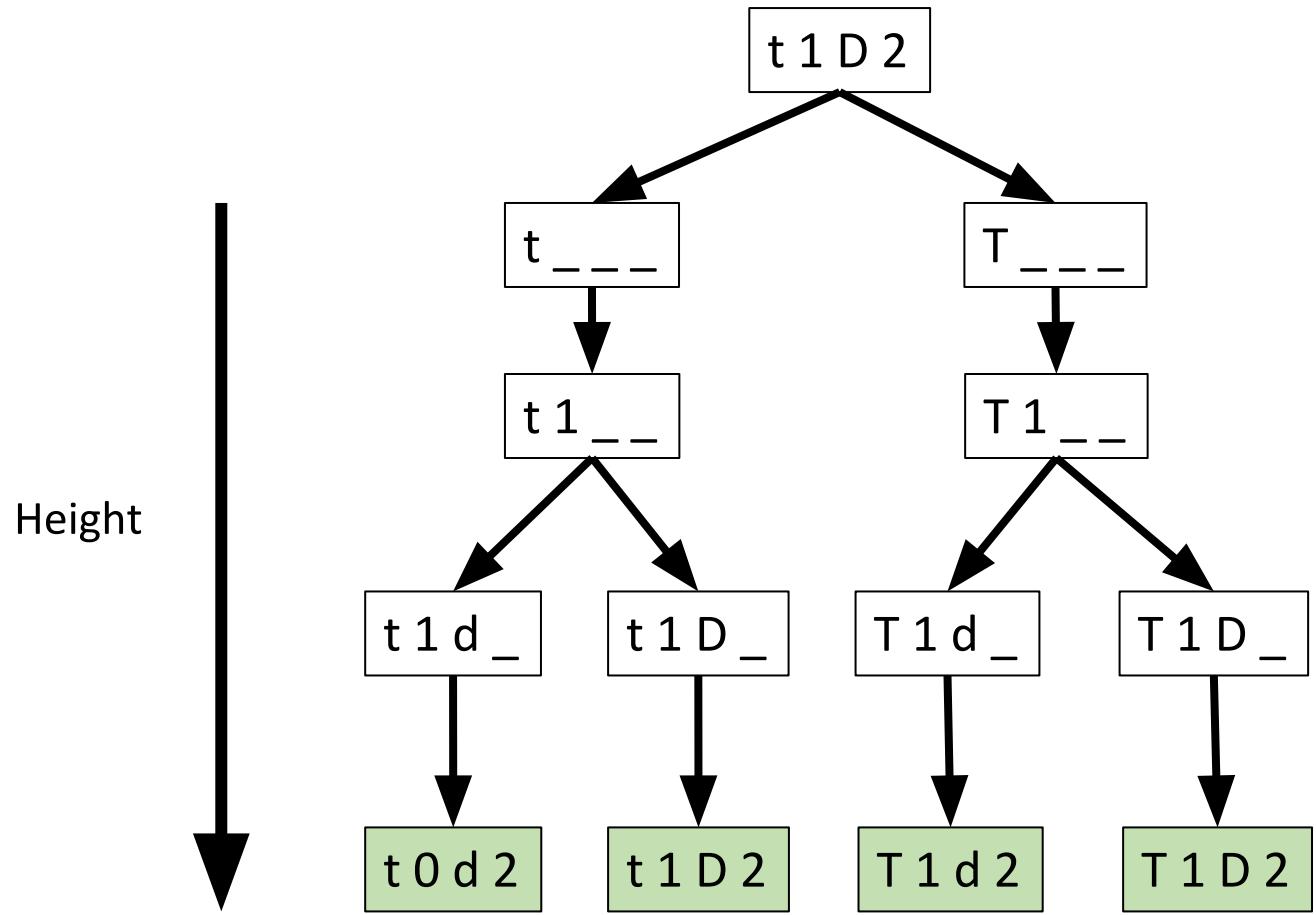
```
void helper(String input, int idx, String soFar, List<String> coll)
{
    if (idx == input.length()) {          // base case
        coll.add(soFar);
    } else {                            // else case
        char currChar = input.charAt(idx);
        if (Character.isDigit(currChar)) {
            helper(input, idx+1, soFar + currChar, coll);
        } else {
            helper(input, idx+1, soFar +
                Character.toLowerCase(currChar), coll);
            helper(input, idx+1, soFar +
                Character.toUpperCase(currChar), coll);
        }
    }
}
```

Letter Case Permutation

- Time complexity
 - # of calls * amount of work/call
- Space complexity
 - Explicit (collector) +
 - Implicit (stack)

Time complexity:
• $O(2^{n+1} * n)$

Space complexity:
• $O(2^n * n + n^2)$



Letter Case Permutation

```
// using mutable data structure
void helper(String input, int idx, List<Character> buf, List<String> coll) {
    if (idx == input.length()) {
        coll.add(new String(buf));
    } else {
        char currChar = input.charAt(idx);
        if (Character.digit(currChar)) {
            buf.append(Character.toLowerCase(currChar));
            helper(input, idx + 1, buf, coll);
            buf.remove(buf.size() - 1)
        } else {

            buf.append(Character.toLowerCase(currChar));
            helper(input, idx + 1, buf, coll);
            buf.remove(buf.size() - 1)

            buf.append(Character.toUpperCase(currChar));
            helper(input, idx + 1, buf, coll);
            buf.remove(buf.size() - 1)
        }
    }
}
```

Letter Case Permutation

```
// using mutable data structure
void helper(String input, int idx, char[] buf, List<String> coll) {
    if (idx == input.length()) {
        coll.add(new String(buf));
    } else {
        char currChar = input.charAt(idx);
        if (Character.isAlphabetic(currChar)) {
            // what should go here
        } else {
            // what should go here?
        }
    }
}
```

Letter Case Permutation

```
// using mutable data structure
void helper(String input, int idx, char[] buf, List<String> coll) {
    if (idx == input.length()) {
        coll.add(new String(buf));
    } else {
        char currChar = input.charAt(idx);
        if (Character.isDigit(currChar)) {
            buf[idx] = currChar;
            helper(input, idx + 1, buf, coll);
        } else {
            buf[idx] = Character.toLowerCase(currChar); // make a choice
            helper(input, idx + 1, buf, coll);
            buf[idx] = Character.toUpperCase(currChar); // make a choice
            helper(input, idx + 1, buf, coll);
        }
    }
}
```

Print Decimal Strings - Optional

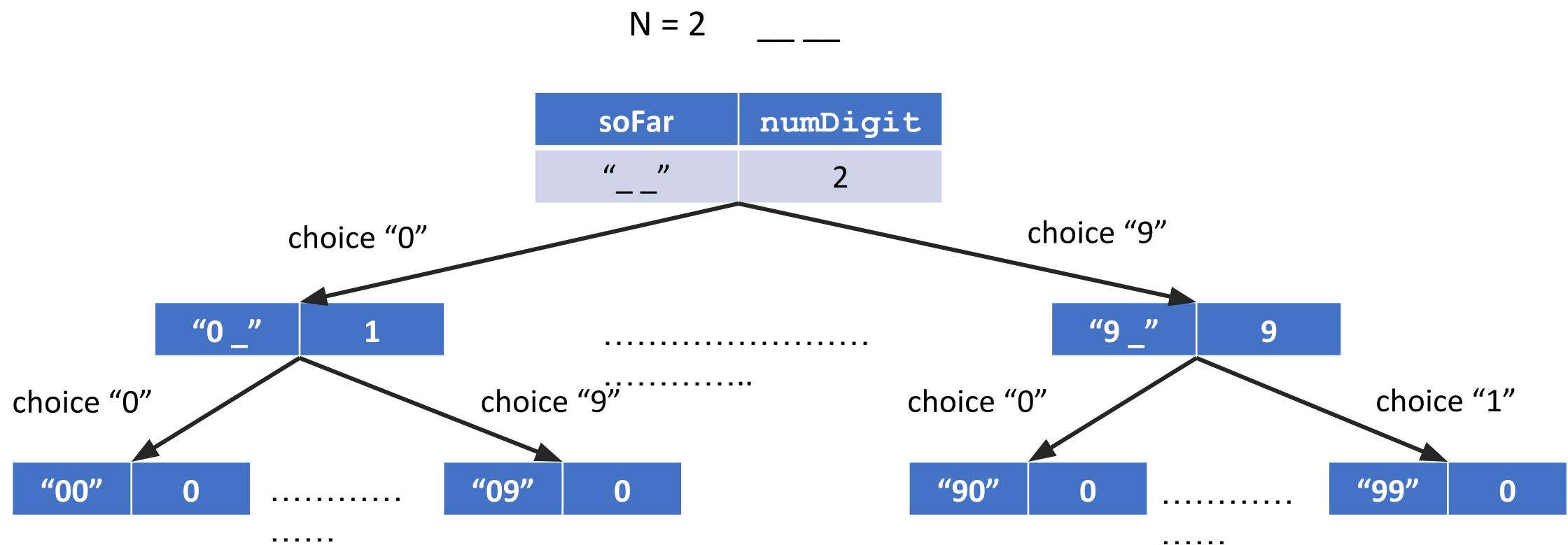
- Print all decimal numbers with n digits in ascending order
- Example:
 - N = 2
 - Output: 00, 01, 02, 03, 04, 05, 06, 07, 08, 09, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19...99
- Approach
 - What are the decisions?
 - What are the choices?
 - How to represent the partial solution?

Print Decimal Strings – Decision Tree

N = 2

— —

Print Decimal Strings



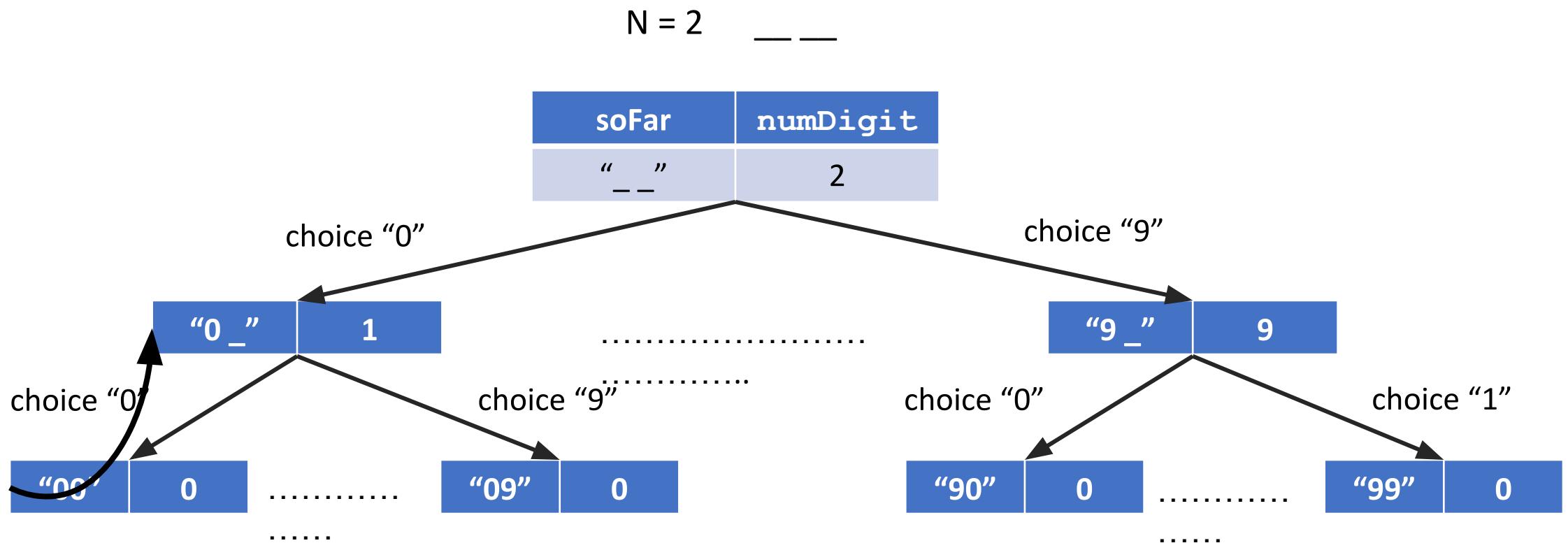
All Decimal Strings

```
driver(int numDigit) {  
    helper(numDigit, new ArrayList<Integer>());  
}  
  
helper(int numDigit, List<Integer> soFar) {  
    // base case  
    // else case  
}
```

All Decimal Strings

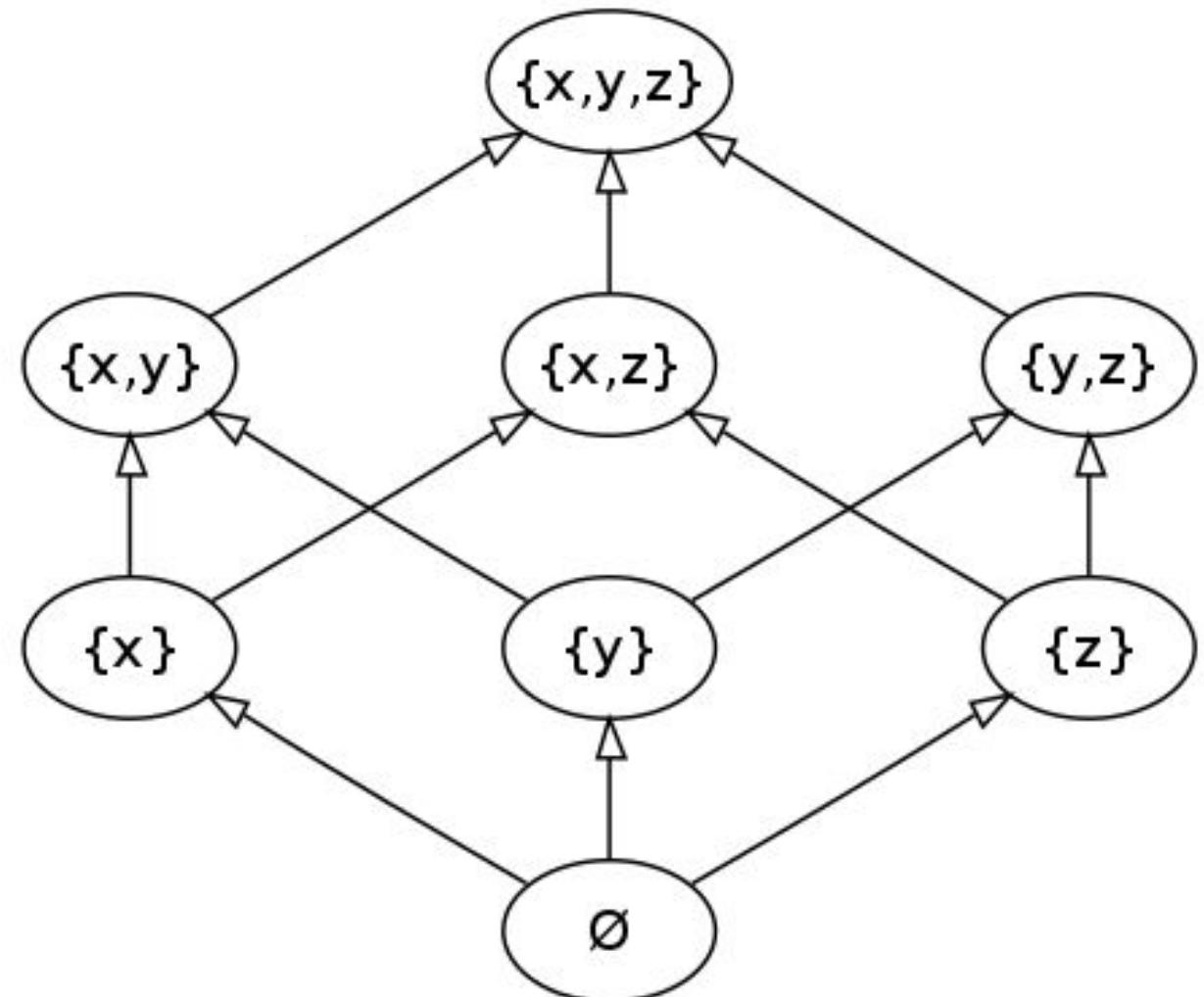
```
driver(int numDigit) {  
    helper(numDigit, new ArrayList<Integer>());  
}  
  
helper(int numDigit, List<Integer> soFar) {  
    if (numDigit == 0) {  
        System.out.println(soFar);  
    } else {  
        for (int choice = 0; choice <= 9; choice++) {  
            soFar.add(choice);  
            helper(numDigit-1, soFar);  
            soFar.remove(soFar.size()-1); // very important step  
        }  
    }  
}
```

Print Decimal Strings



Generate Powerset

Classical Combinatorial
Structure

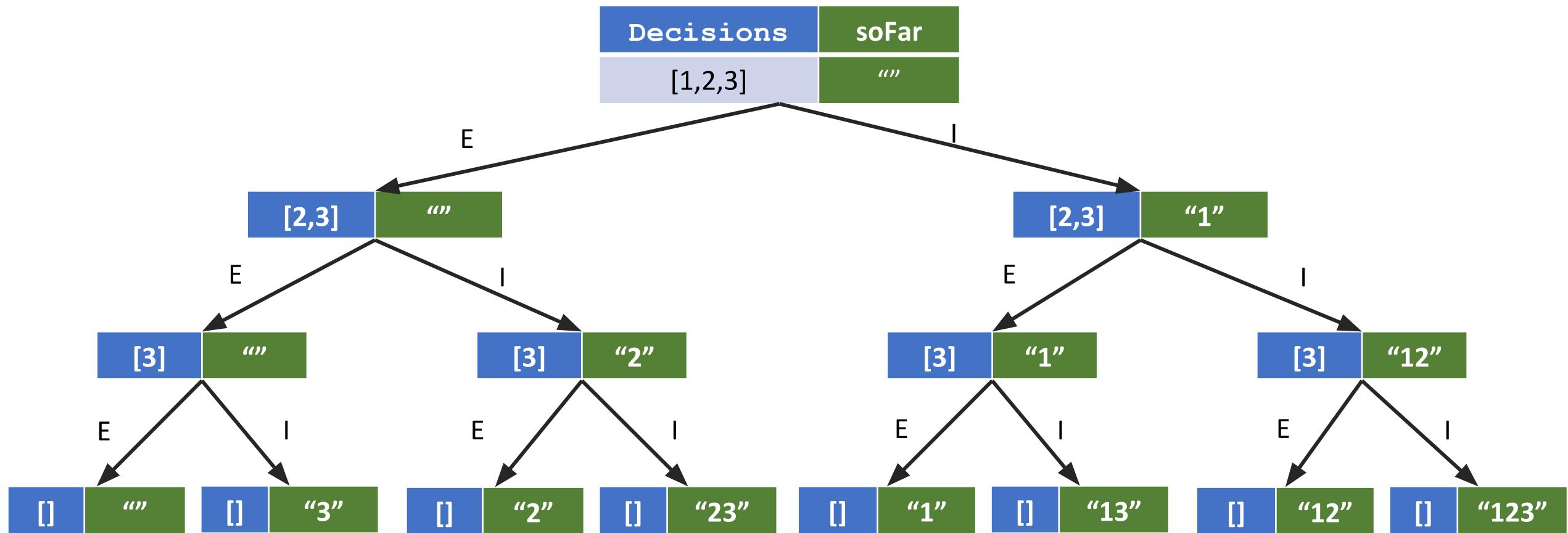


All Subsets aka Powerset

- Given a set of distinct integers, return all subsets
 - A subset can be empty or all the integers
 - An integer can be a part of or not a part of a subset
- Example:
 - Input: [1,2,3]
 - Output: {},{1},{2},{3},{1,2},{1,3},{2,3},{1,2,3}
- Approach
 - What are the decisions?
 - What are the choices?
 - How to represent the partial solution?

All subsets – printSubsets([1,2,3])

All subsets – printSubsets([1,2,3])



$$O(n) = O(n-1) + O(n-1) \implies 2O(n-1)$$

All subsets – printSubset([1,2,3])

```
List<String> driver(int[] numbers) {  
    // decision, partial solution, collector  
    List<String> coll = new ArrayList<>();  
    helper(???);  
}  
  
// let's write code together  
void helper(???) {  
}
```

All subsets – printSubset([1,2,3])

```
// using mutable data structure
helper(int[] numbers, int idx, StringBuffer soFar,
       List<String> coll) {
    if (idx == numbers.length) {
        coll.add(soFar.toString()); // a copy of soFar
    } else {
        helper(numbers, idx+1, soFar); // exclude

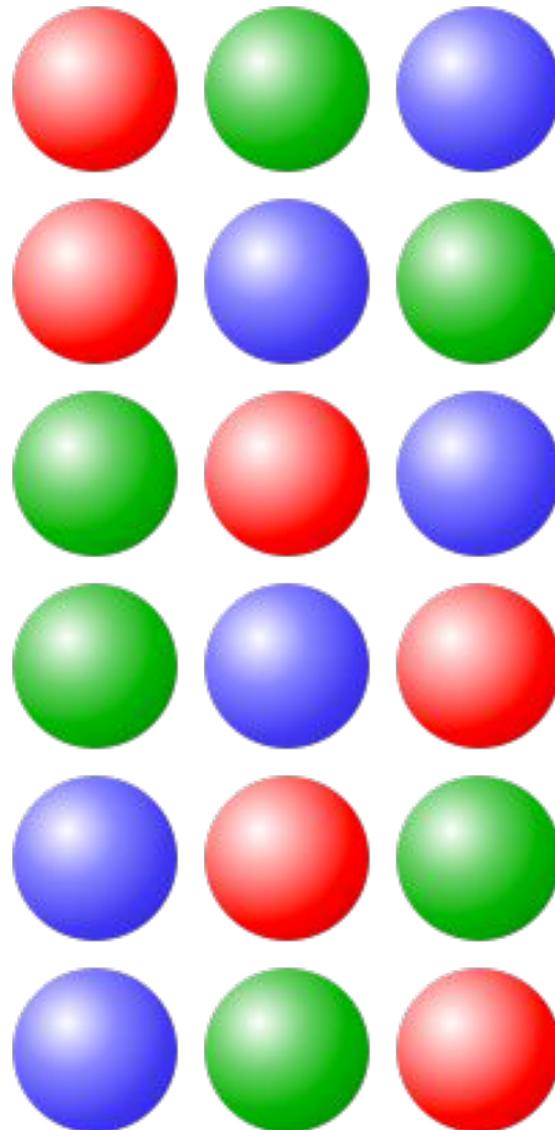
        soFar.append(numbers[idx]); // include
        helper(numbers, idx+1, soFar);
        soFar.deleteCharAt(soFar.length()-1);
    }
}
```

All subsets – printSubset([1,2,3])

- Time complexity
 - $O(2^n)$
- Space complexity when returning all subsets
 - Mutable: $O(2^n * n + n)$
- Space complexity when only printing subsets
 - ??

Generate Permutations

- Rearranging the members of a set into a sequence
- Order matters



Generating Permutations

- Given a string of characters, print all the possible permutations
- Definition: the arrangement of characters in which the order is important
- Example:
 - Input: “HAT”

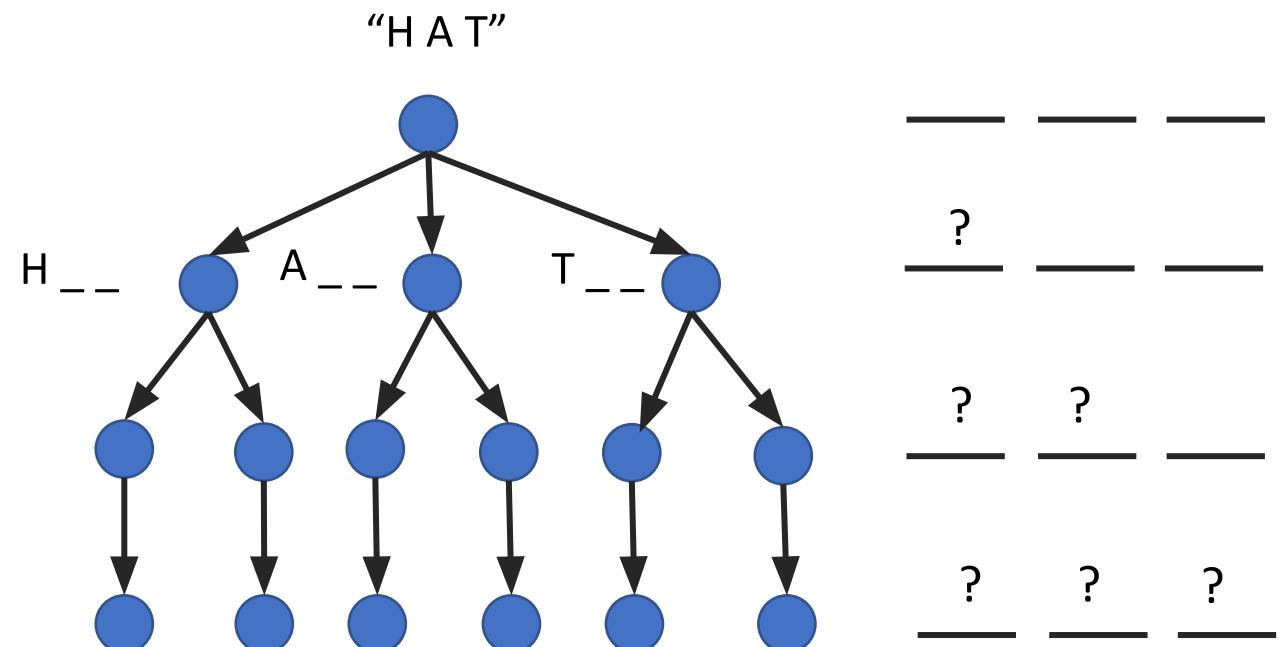
HAT
HTA
AHT
ATH
THA
TAH

H A T
— — —

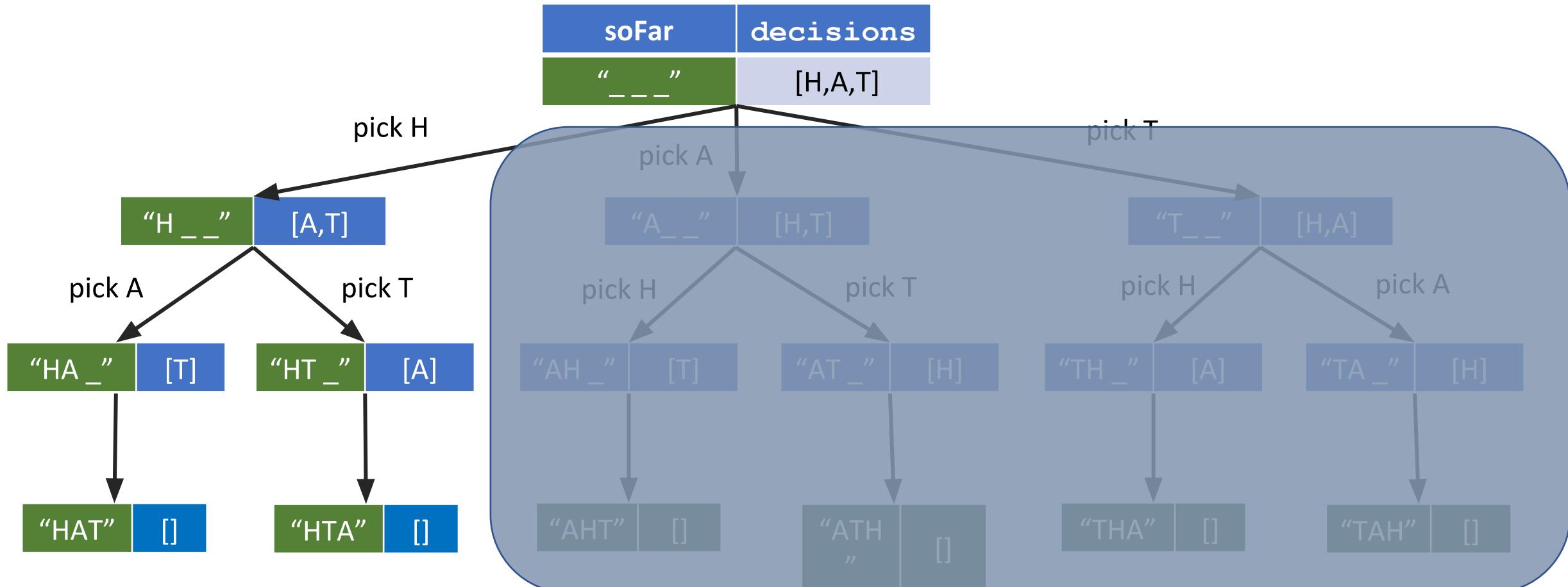
Generating Permutations

- Rearranging the members of a set
 - Print them
- Example:
 - Input: “HAT”

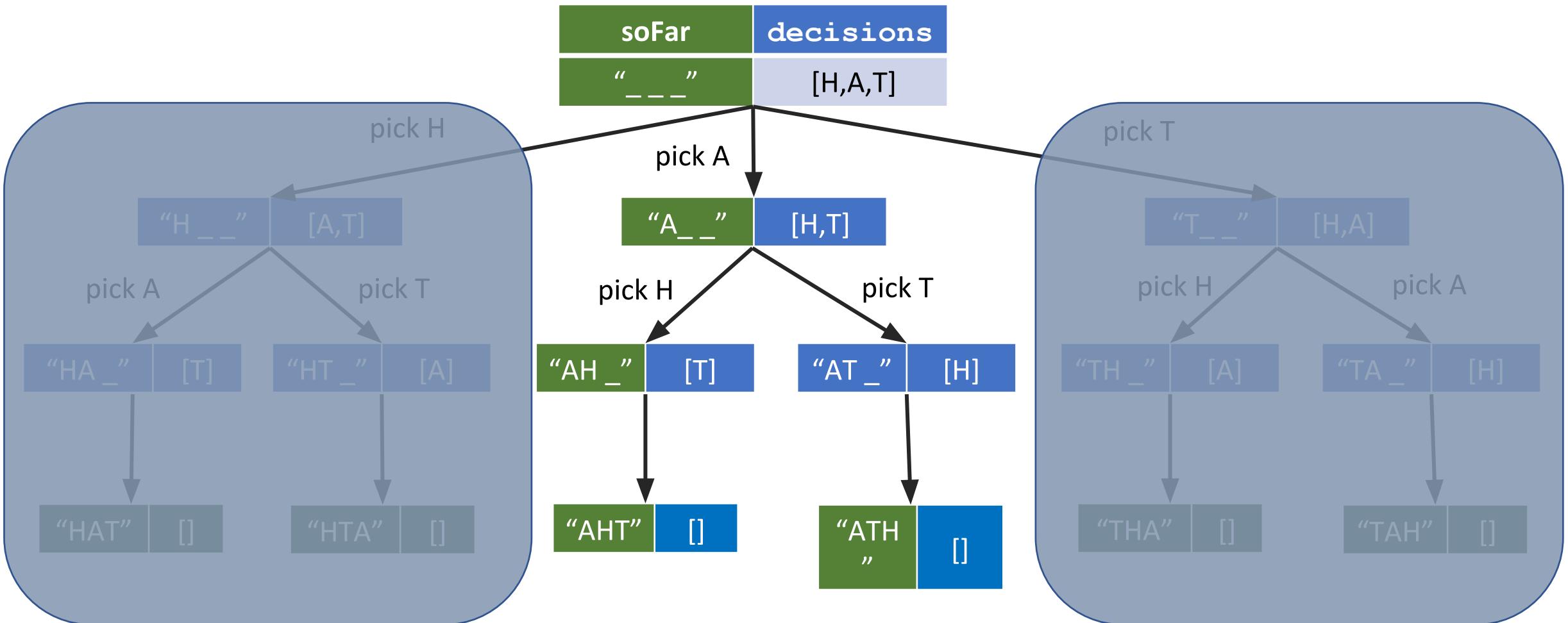
HAT
HTA
AHT
ATH
THA
TAH



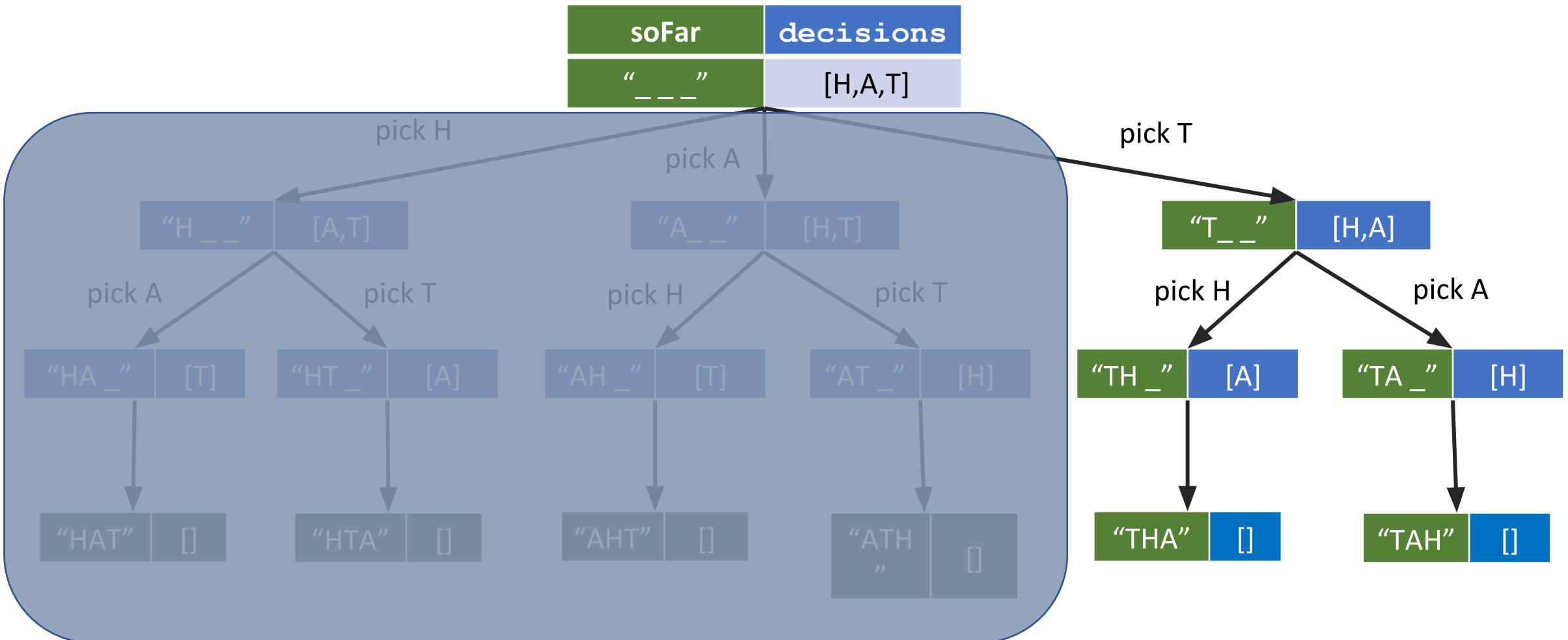
Generating Permutations



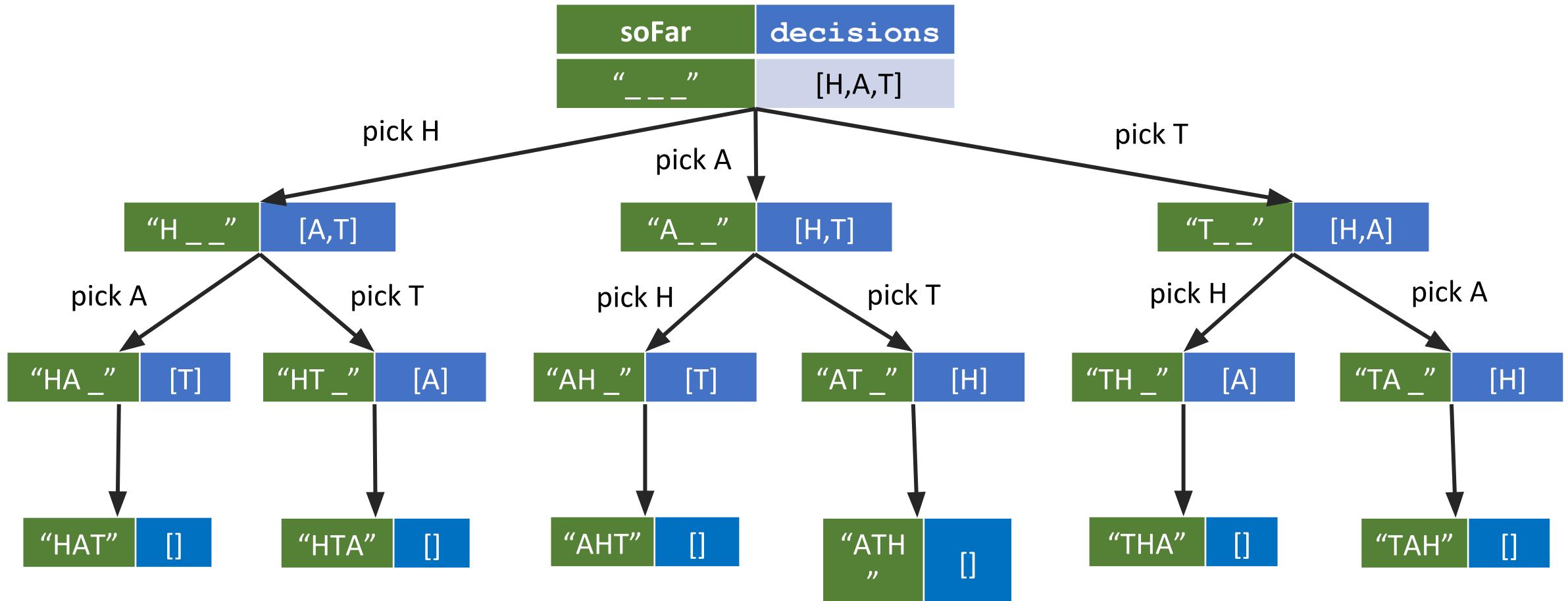
Generating Permutations



Generating Permutations

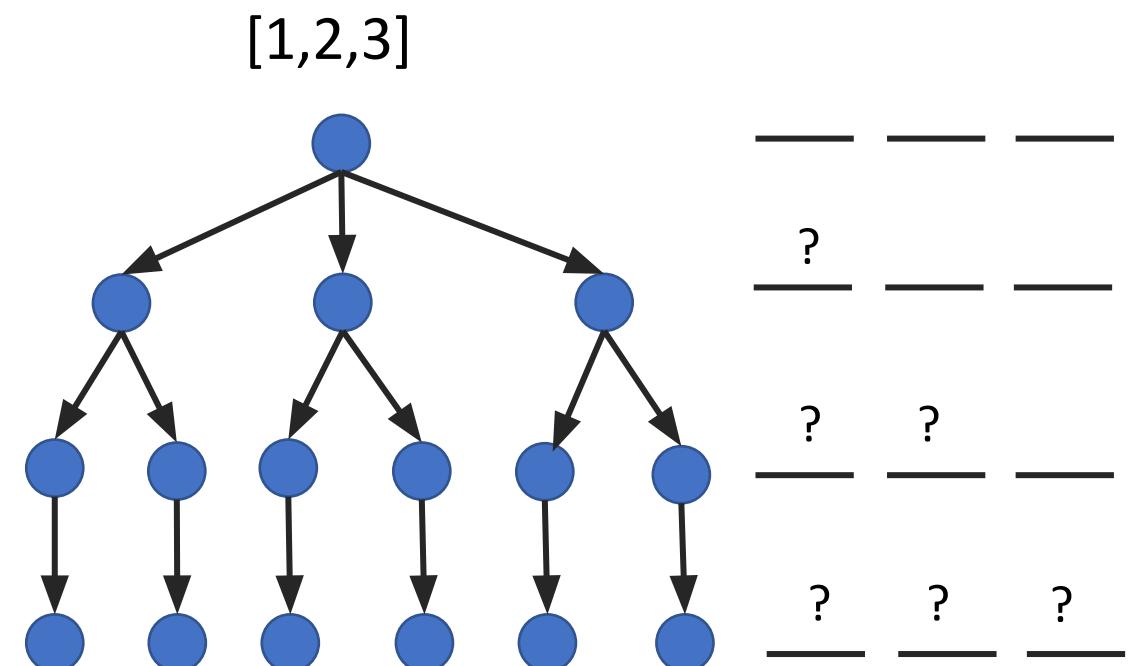


Generating Permutations



Generating Permutations

- Rearranging the members of a set
 - Print them
- Approach:
 - What are the decisions?
 - What are the choices?
 - How to represent the subproblems?
 - Remove the chosen choice from the decision list
 - Move the choice to the front and use an index to tell subordinate where to start from



Generating Permutations

```
void driver (String input) {  
    helper(input, "");  
}  
  
void helper(String remaining, String soFar) {
```

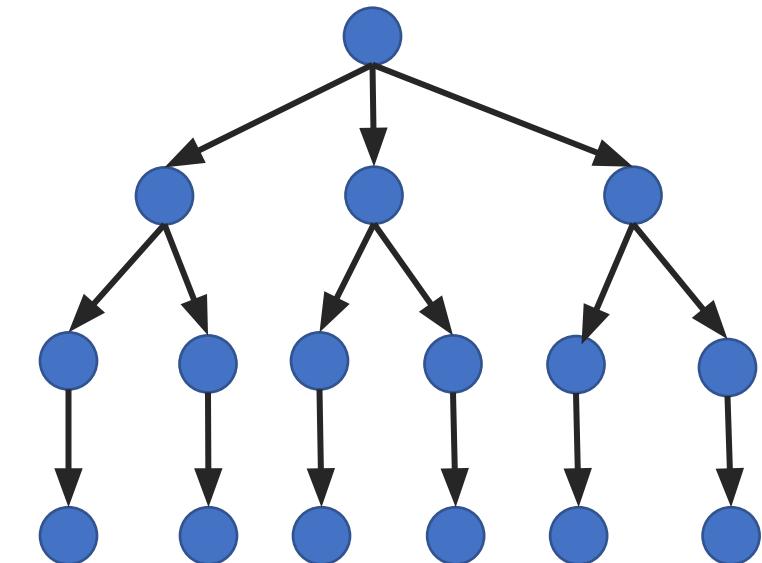
Generating Permutations

```
void driver (String input) {  
    helper(input, "");  
}  
  
void helper(String remaining, String soFar) {  
    if (remaining.isEmpty()) {  
        System.out.println(soFar);  
    } else {  
        for (int i = 0; i < remaining.length(); i++) { // choices  
            String prefix = remaining.substring(0,i);  
            String suffix = remaining.substring(i+1);  
            helper(prefix+suffix, soFar + remaining.charAt(i));  
        }  
    }  
}
```

Generating Permutations

Runtime & Space Complexity Analysis of Permutation

```
void helper(String remaining, String soFar) {  
    if (remaining.isEmpty()) {  
        System.out.println(soFar);  
    } else {  
        for (int i = 0; i < remaining.length(); i++) {  
            String prefix = remaining.substring(0,i);  
            String suffix = remaining.substring(i+1);  
            helper(prefix+suffix, soFar + remaining.charAt(i));  
        }  
    }  
}
```



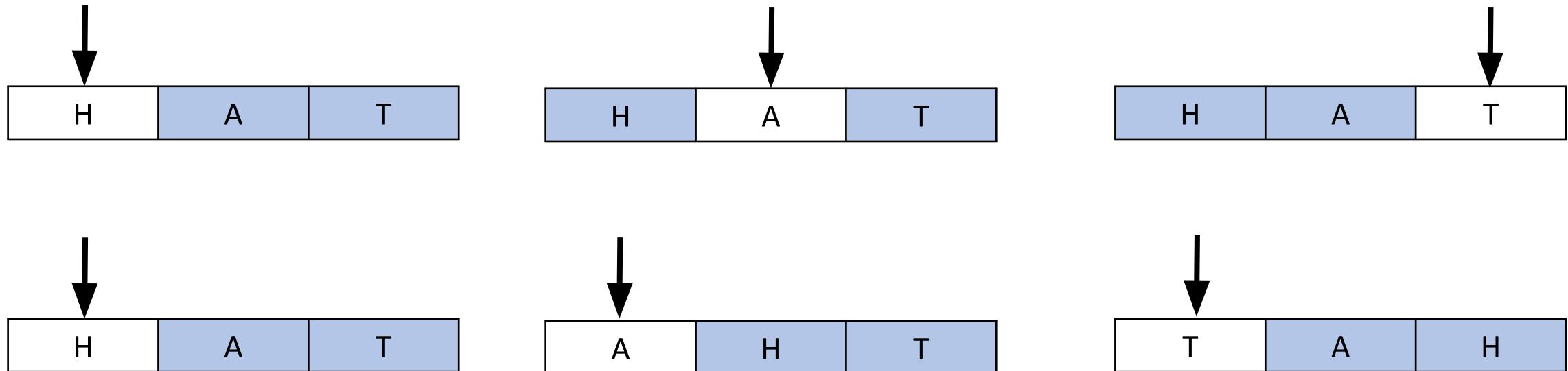
- Time complexity
 - # of calls * amount of work/call
- Space complexity
 - Explicit (collector) + Implicit (stack)

Generating Permutations

Using Mutable Data Structure for Subproblems

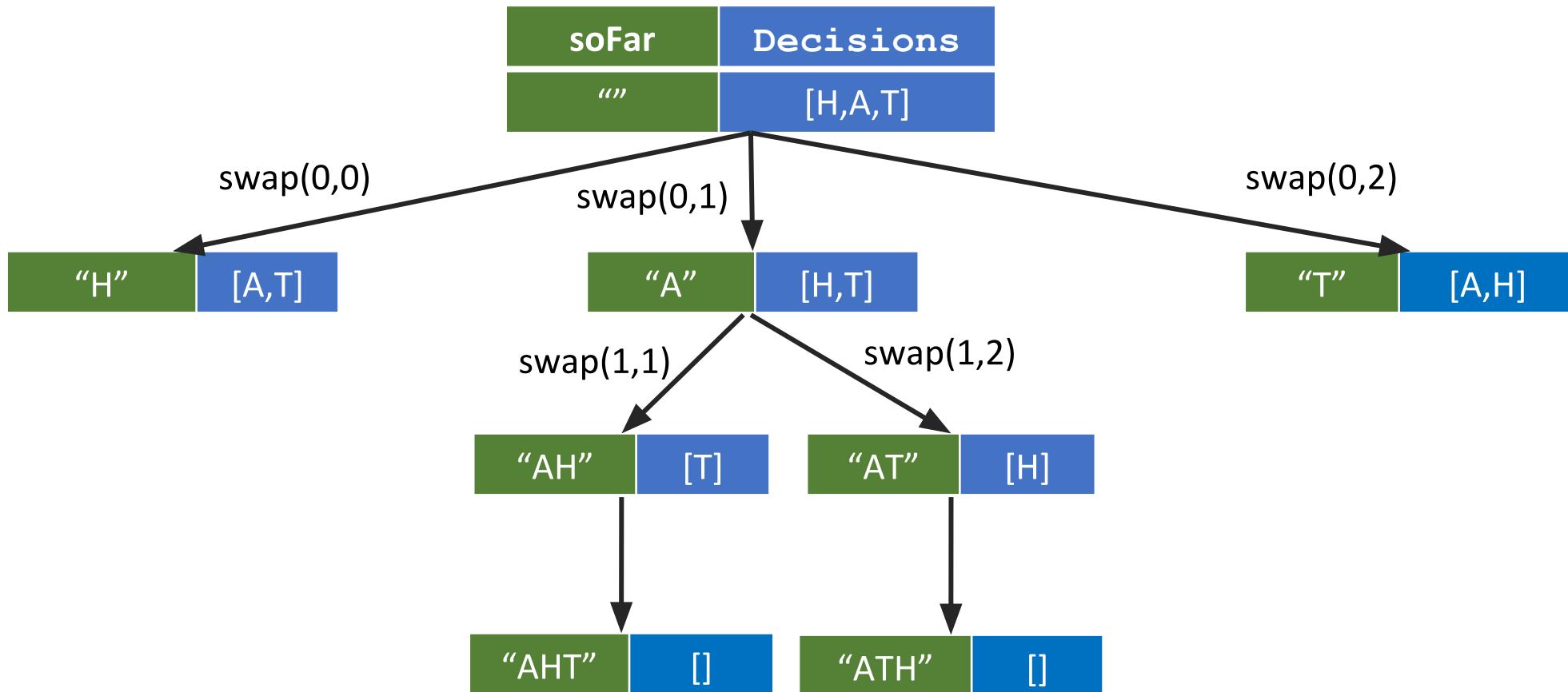
?
— — —

Input: [H,A,T], idx=0

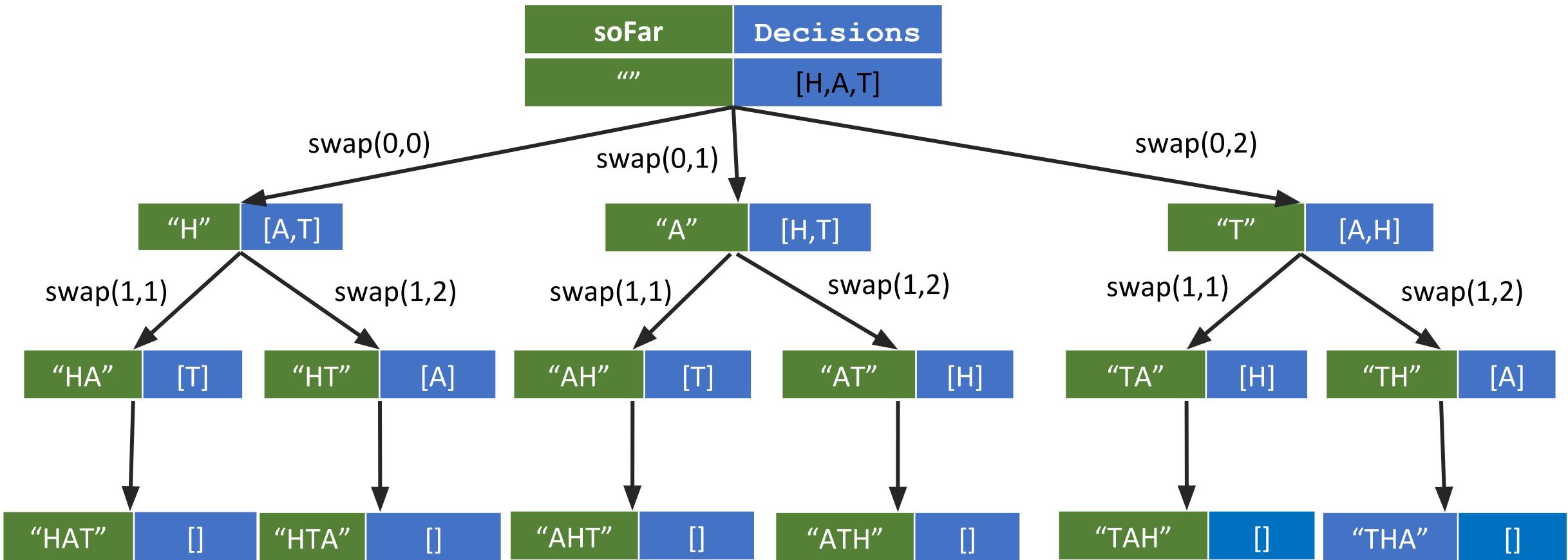


How to pass down the remaining work to subordinates?

Generating Permutations



Generating Permutations



Show the swapping and Unswapping

Generating Permutations

```
void driver(int[] input) {  
    helper(input, 0, "");  
}  
  
void helper(int[] input, int idx, String soFar) {  
    if (idx == input.length) {  
        System.out.println(soFar);  
    } else {  
        ??  
    }  
}
```

Generating Permutations

```
void driver(int[] input) {  
    helper(input, 0, "");  
}  
  
void helper(int[] input, int idx, String soFar) {  
    if (idx == input.length) {  
        System.out.println(soFar);  
    } else {  
        for (int i = idx; i < input.size(); i++) {  
            swap(input, idx, i);  
            helper(input, idx+1, soFar + input[i]);  
            swap(input, idx, i); //restore state  
        }  
    }  
}
```

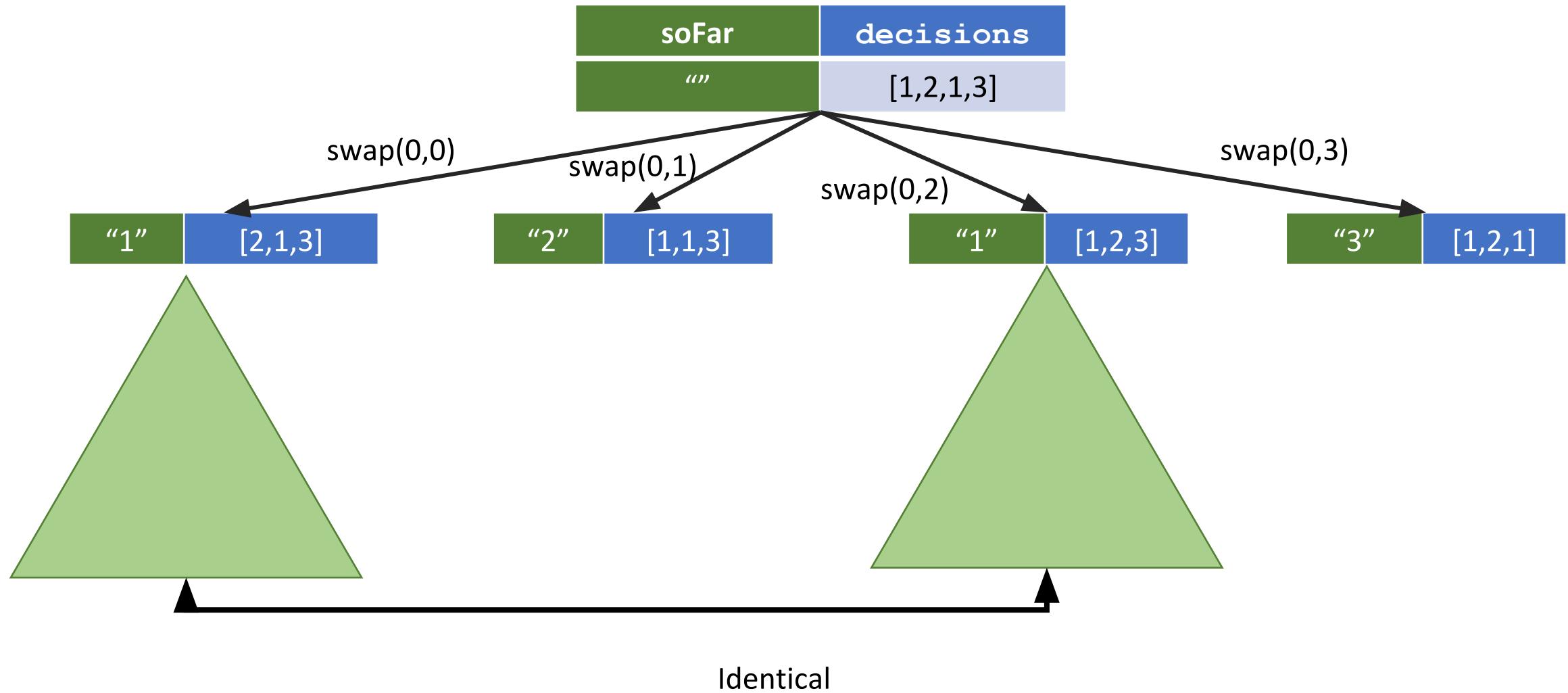
Generating Permutations

```
void helper(int[] input, int idx, StringBuffer soFar) {  
    if (idx == input.length) {  
        System.out.println(soFar.toString());  
    } else {  
        for (int i = idx; i < input.size(); i++) {  
            swap(input, idx, i);  
            soFar.append(input[i]);  
  
            helper(input, idx+1, soFar);  
            soFar.deleteCharAt(soFar.length()-1);  
  
            swap(input, idx, i); //restore state  
        }  
    }  
}
```

Generating Permutations w/ Duplicates

- What if input has duplicates
 - If don't handle them properly then there will be duplicates in the output
- Example:
 - Input: [1,2,3,1,2]
- Approach
 - ??

Generating Permutations w/ Duplicates



Generating Permutations w/ Duplicates

- What if input has duplicates
 - If don't handle them properly then there will be duplicates in the output
- Example:
 - Input: [1,2,3,1,2]
- Approach
 - Keep track of the choices
 - If seen it before, then skip it
 - Don't make the same choice twice

Generating Permutations w/ Duplicates

```
void helper(int[] input, int idx, String soFar) {  
    // base case  
    // other case  
}
```

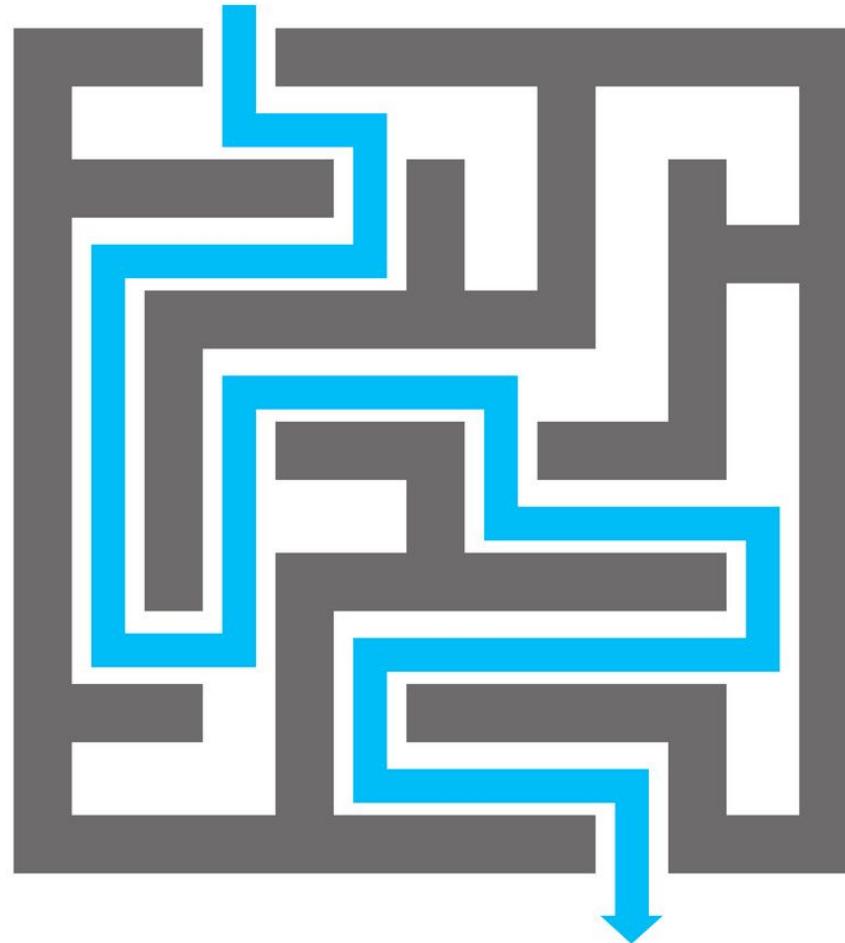
Generating Permutations w/ Duplicates

```
void helper(int[] input, int idx, StringBuffer soFar) {  
    if (idx == input.length) {  
        System.out.println(soFar.toString());  
    } else {  
        Set<Integer> cache = new HashSet<Integer>();  
        for (int i = idx; i < input.length; i++) {  
            if (cache.contains(input[i])) continue;  
            cache.add(input[i]);  
  
            swap(input, idx, i);  
            soFar.append(input[i]);  
            helper(input, idx+1, soFar);  
            soFar.deleteCharAt(soFar.length()-1);  
            swap(input, idx, i); //restore state  
        }  
    }  
}
```

Exhaustive Search w/ Constraints

Backtracking

- Find one or all possible solutions that meet a certain set of constraints
- Incrementally build candidates to solutions
- Abandon a candidate solution as soon as it can't satisfy the given constraints



Backtracking

- Find solution(s) by trying all possible paths with constraints
 - Determine all possible solutions
 - Determine feasibility (one solution)
 - Determine the number of possible solutions
 - Determine the best (maximum, minimum) solution
- Applications
 - Parsing languages
 - Games: n-queens, Sudoku, Minesweeper

Backtracking - general approach

- Incremental approach
 - Perform exhaustive search w/o constraints
 - Apply the constraints
 - Add the backtracking steps at appropriate places
- Runtime & space complexity
 - Still exponential in worst-case scenario

Backtracking – all possible solutions

```
helper(sub problems, partial-solution) {  
    if (no more sub problems) { // base case  
        print/collect partial-solution  
    } else {  
        for (each possible choices) {  
            make a choice // add to partial-solution  
            helper(remaining-decisions, partial-solution)  
            unmake choice // if mutable data structure is used  
        }  
    }  
}
```

Backtracking – feasibility

```
helper(sub problems, partial-solution) {  
    if (violate constraint) { // backtracking case  
        return;  
    }  
  
    if (no more sub problems) { // base case  
        print/collect partial-solution  
    } else {  
        for (each possible choices) {  
            make a choice // add to partial-solution  
            helper(remaining-decisions, partial-solution)  
            unmake choice // if mutable data structure is used  
        }  
    }  
}
```

Backtrack Problem – Subset Size

Medium

1236

62

Add to List

Share

Given two integers n and k , return all possible combinations of k numbers out of $1 \dots n$.

Example:

Input: $n = 4$, $k = 2$

Output:

[

[2,4],

[3,4],

[2,3],

[1,2],

[1,3],

[1,4],

]

Backtrack Problem – Subset Size

```
// w/o enforcing the constraint
helper(int[] numbers, int idx, StringBuffer soFar,
       List<String> coll, int k) {
    if (idx == numbers.length) {
        coll.add(soFar.toString()); // a copy of soFar
    } else {
        helper(numbers, idx+1, soFar); // exclude
        soFar.append(numbers[idx]); // include
        helper(numbers, idx+1, soFar);
        soFar.deleteCharAt(soFar.length()-1);
    }
}
```

Backtrack Problem – Subset Size

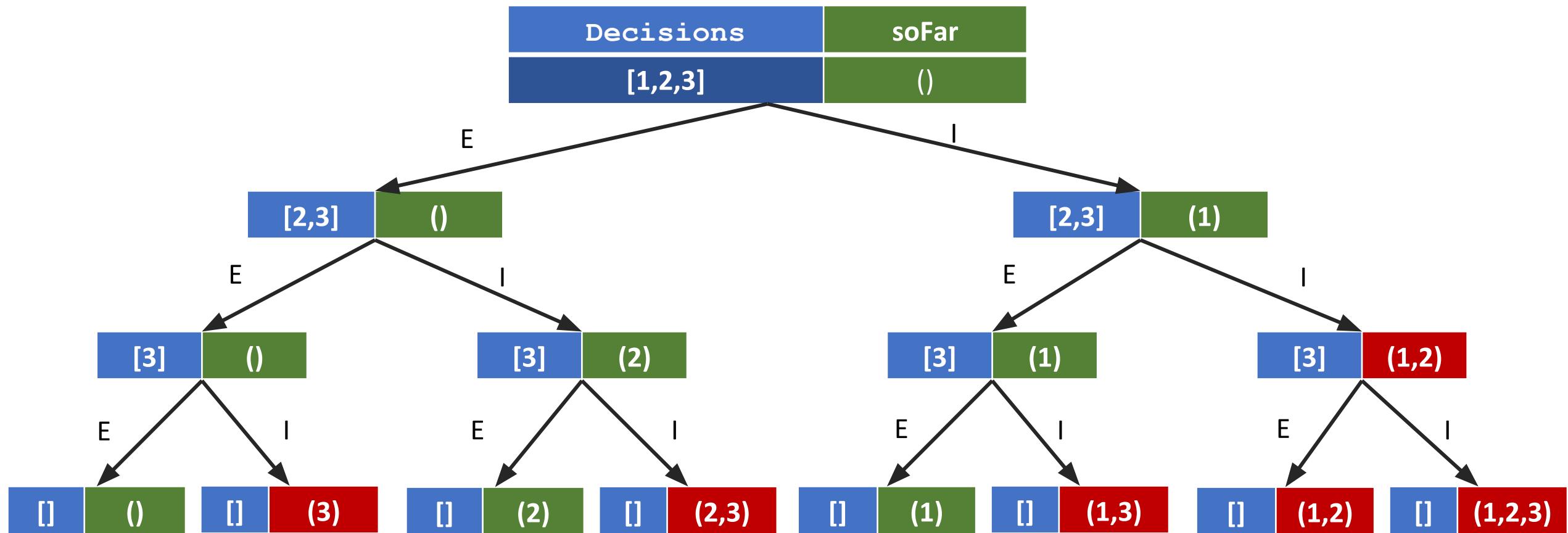
```
// with enforcing the constraint
helper(int[] numbers, int idx, StringBuffer soFar,
       List<String> coll, int k) {
    if (soFar.length() == k) { // backtracking case
        coll.add(soFar.toString());
    } if (idx == numbers.length) { // base case
        return;
    } else {
        helper(numbers, idx+1, soFar); // exclude

        soFar.append(numbers[idx]);      // include
        helper(numbers, idx+1, soFar);
        soFar.deleteCharAt(soFar.length()-1);
    }
}
```

Backtrack Problem – Subset Sum

- Print all subsets of a set of distinct integers – [1,2,3]
 - With sum equals to $K = 2$
- What question(s) should we ask about the integers?
 - All positive integers
 - All positive integers with 0
 - Positive, 0 and negative
- Approach
 - When to compute the subset sum?
 - At the leaf level
 - On the fly

All subsets – Subset Sum([1,2,3], 2)



violate the constraint

Backtrack Problem – Subset Sum

```
// printing subsets w/o constraint
void helper(int[] input, int idx, List<Integer> path, int target)
{
    if (idx == input.length) {
        System.out.println(path);
    } else {
        helper(input, idx+1, path, target); // exclude

        path.add(input[idx]);
        helper(input, idx+1, path, target); // include
        path.remove(path.size()-1);
    }
}
```

Backtrack Problem – Subset Sum w/ pruning

```
void helper(int[] input, int idx, List<Integer> path,
           int sumSoFar, int target) {

    if (sumSoFar == target) System.out.println(path); return;
    if (idx == input.length || sumSoFar > target) {
        return;
    } else {
        helper(input, idx+1, path, sumSoFar, target); // exclude

        // include
        path.add(input[idx]);
        helper(input, idx+1, path, sumSoFar + input[idx] ,target);
        path.remove(path.size()-1);
    }
}
```

Backtrack Problem – Subset Sum

- Print all subsets of a set of distinct integers
 - With sum equals to K
- What question(s) should we ask about the integers?
 - All positive integers
 - All positive integers with 0
 - Positive, 0 and negative

Backtrack Problem – Dice Roll Sum



- Print all combinations of dice values that add up to given sum
 - Each dice has 6 values: {1,2,3,4,5,6}
- For example:
 - Dice=2, Sum=7
 - Output: {1,6}, {2,5}, {3,4}, {4,3},{5,2}, {6,1}
 - Dice=3, Sum=7
 - Output: {1,1,5}, {1,2,4}, {1,3,3}, {1,4,2},{1,5,1},
 - {2,1,4}, {2,2,3}, {2,3,2}, {2,4,1}
 - {3,1,3}, {3,2,2}, {3,3,1}
 - {4,1,2}, {4,2,1}, {5,1,1}

Backtrack Problem – Dice Roll Sum



- Print all combinations of dice values that add up to given sum
 - Each dice has 6 values: {1,2,3,4,5,6}
 - `diceSum(numDice, target sum)`
 - `diceSum(3, 7)`
- Approach
 - What are the decisions to iterate over?
 - What are the choices at each decision?
 - Base case?
 - What does the recursion tree look like?

Backtrack Problem – Dice Roll Sum

```
void driver(int numDice, int targetSum) {  
    helper(??);  
}  
  
void helper(??) {  
}
```

Backtrack Problem – Dice Roll Sum

```
void helper(int numDice, int targetSum, List<Integer> path) {  
  
    if (numDice == 0) {  
        if (computeSum(path) == targetSum) {  
            System.out.println(path);  
        }  
    } else {  
        for (int value = 1; value <= 6; value++) {  
            path.add(value);  
            helper(numDice-1, targetSum, path);  
            path.remove(path.size()-1);  
        }  
    }  
}  
// what are the opportunities for pruning?
```

Backtrack Problem – Dice Roll Sum

- Analyzing **helper**
 - Exhaustive search even for paths that not lead to success
 - Current sum is already too high
- What are the opportunities for pruning
 - We must roll every remaining dice
 - How can we take advantage of that info.?
 - $\text{sumSoFar} + 1 * \text{numDice} > \text{targetSum}$
 - $\text{sumSoFar} + 6 * \text{numdice} < \text{targetSum}$

Backtrack Problem – Dice Roll Sum

```
void helper(int numDice, int targetSum, int sumSoFar
           List<Integer> path) {
    if (numDice == 0) {
        if (computeSum(path) == targetSum)
            System.out.println(path);
    } else if (sumSoFar + 1*numDice > targetSum ||
               sumSoFar + 6*numDice < targetSum) {
        return;
    } else {
        for (int value = 1; value <= 6; value++) {
            path.add(value);
            helper(numDice-1, targetSum, sumSoFar + value, path);
            path.remove(path.size()-1);
        }
    }
}
```

Backtrack Problem – Generate Parentheses

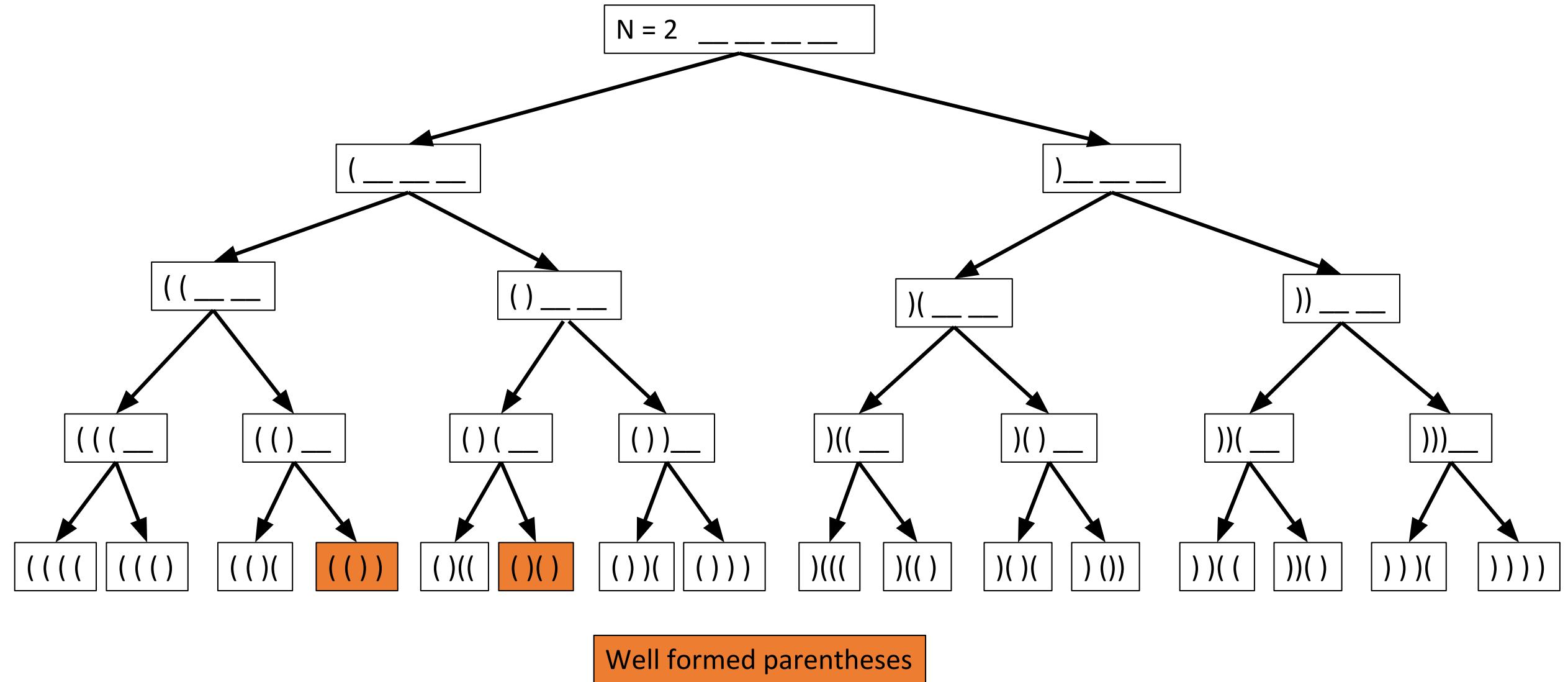
- Given n pairs of parentheses
 - Generate all combinations of well-formed parentheses
- For example
 - N=1 - [“()”]
 - N=2 - [“(())”, “() ()”]
 - N = 3 - [“(() ())”, “((()) ”, “(() () () ”, “() (() () ”, “() (() () ”]

How comfortable do you feel you can solve this problem at this point?

Backtrack Problem – Generate Parentheses

- Given n pairs of parentheses
 - Print all combinations of well-formed parentheses
- For example
 - N=1 - [“()”]
 - N=2 - [“(())”, “() ()”]
 - N = 3 - [“(() ())”, “((()) ”, “(() () () ”, “() (() () ”, “() (() () ”]
- Approach
 - What are the decisions?
 - What are the choices?
 - What are the constraints? When to apply them?

Backtrack Problem – Generate Parentheses



Backtrack Problem – Generate Parentheses

```
// without constraints - print
void driver(int n) {
    helper(2*n, "");
}

// what pattern can we leverage? subproblems? partial solution?
void helper(int remaining, String soFar) {
    ???
}
```

Backtrack Problem – Generate Parentheses

```
// without constraints - print
void driver(int n) {
    helper(2*n, "");
}

// what pattern can we leverage? subproblems? partial solution?
void helper(int remaining, String soFar) {
    if (remaining == 0) {
        if (isWellFormed(soFar)) print(soFar);
    } else {
        helper(remaining-1, soFar + "(");
        helper(remaining-1, soFar + ")");
    }
}
```

Backtrack Problem – Generate Parentheses

```
//"()()"
boolean isWellFormed(String input) {
    ??
}
```

Backtrack Problem – Generate Parentheses

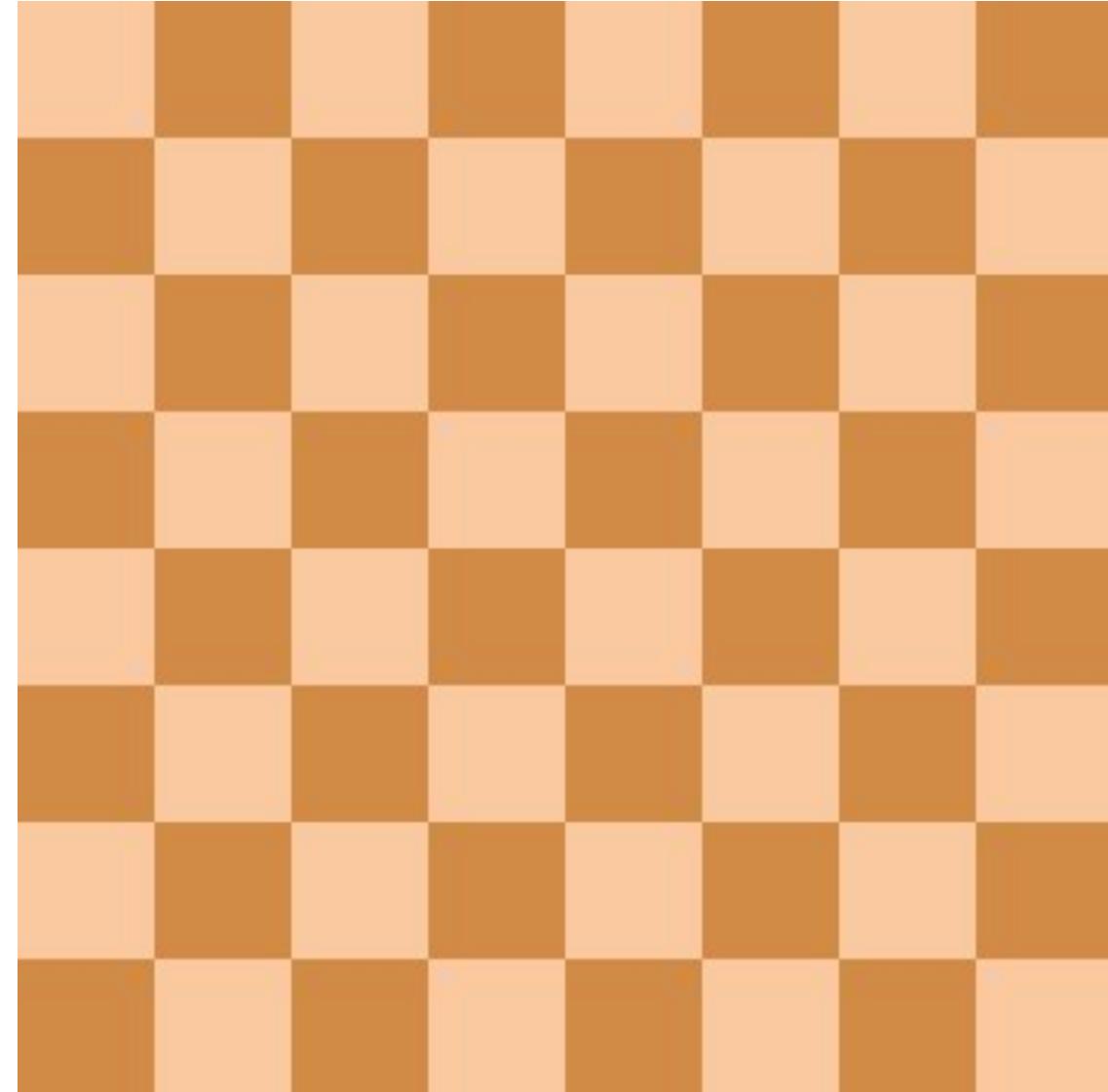
- Given n pairs of parentheses
 - Generate all combinations of well-formed parentheses
- For example
 - N=1 - [“()”]
 - N=2 - [“(())”, “() ()”]
 - N=3 - [“(() ())”, “((()) ”, “(() () () ”, “() (() () ”, “() (() () ”]
- Formalize the constraints
 - The number of remaining “(“ must be \leq to the remaining “)” at any point in time

Backtrack Problem – Generate Parentheses

```
void driver(int n) {  
    helper(n, n, new StringBuffer());  
}  
  
void helper(int lParanCnt, int rParanCnt, StringBuffer soFar) {  
    if (lParanCnt > rParanCnt) return;  
    if (lParanCnt < 0 || rParanCnt < 0) return;  
  
    if (lParanCnt == 0 && rParanCnt == 0) {  
        System.out.print(soFar.toString());  
    } else {  
        soFar.append("(");  
        helper(lParanCnt - 1, rParanCnt, soFar);  
        soFar.removeCharAt(soFar.length()-1);  
  
        soFar.append(")");  
        helper(lParanCnt, rParanCnt - 1, soFar);  
        soFar.removeCharAt(soFar.length()-1);  
    }  
}
```

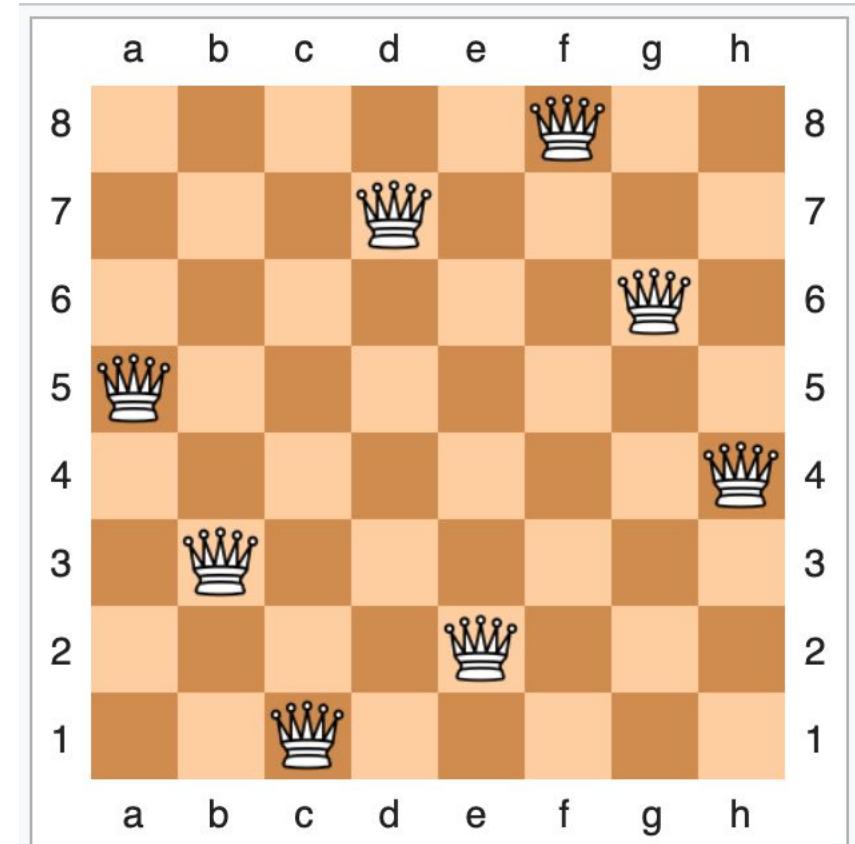
Backtrack - Venerable N-Queens Problem

- Given an NxN board, place N queens such that they can't attack each other
 - A queen can attack another queen if they are on same row or column or diagonal line
- Return all possible boards



Backtrack Problem - N Queens

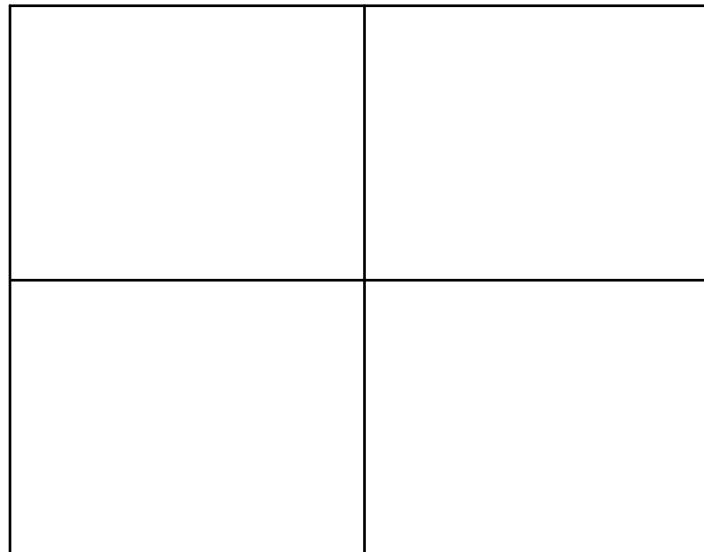
- Published in 1848 by chess composer Max Bezzel
- First solution was published 1850
- For an 8x8 board
 - 4,426,165,368 possible arrangements
 - Only 92 are valid



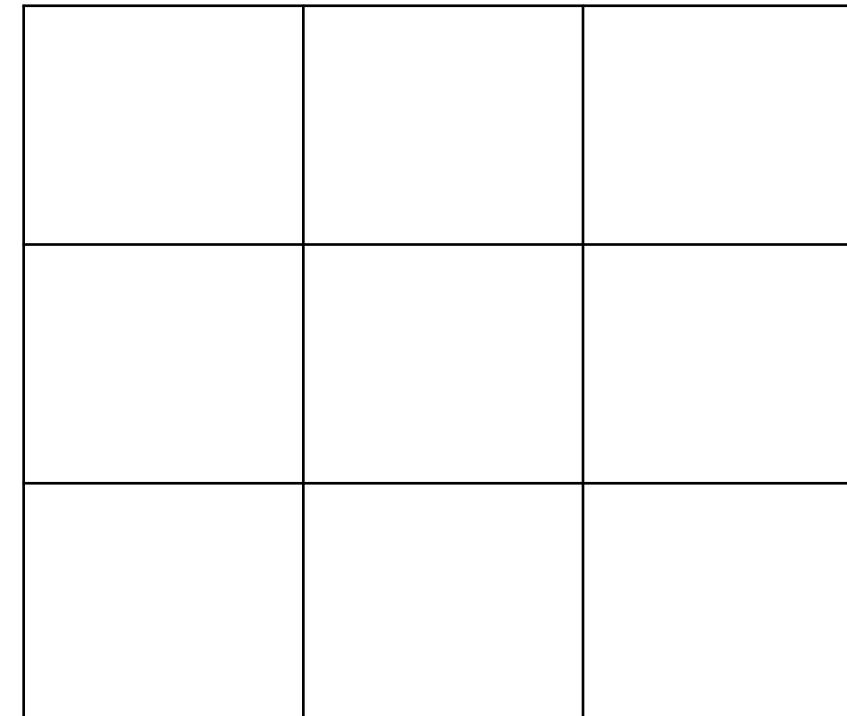
Backtrack Problem - N Queens

A queen can attack another queen if they are on same row or column or diagonal line

2 x 2

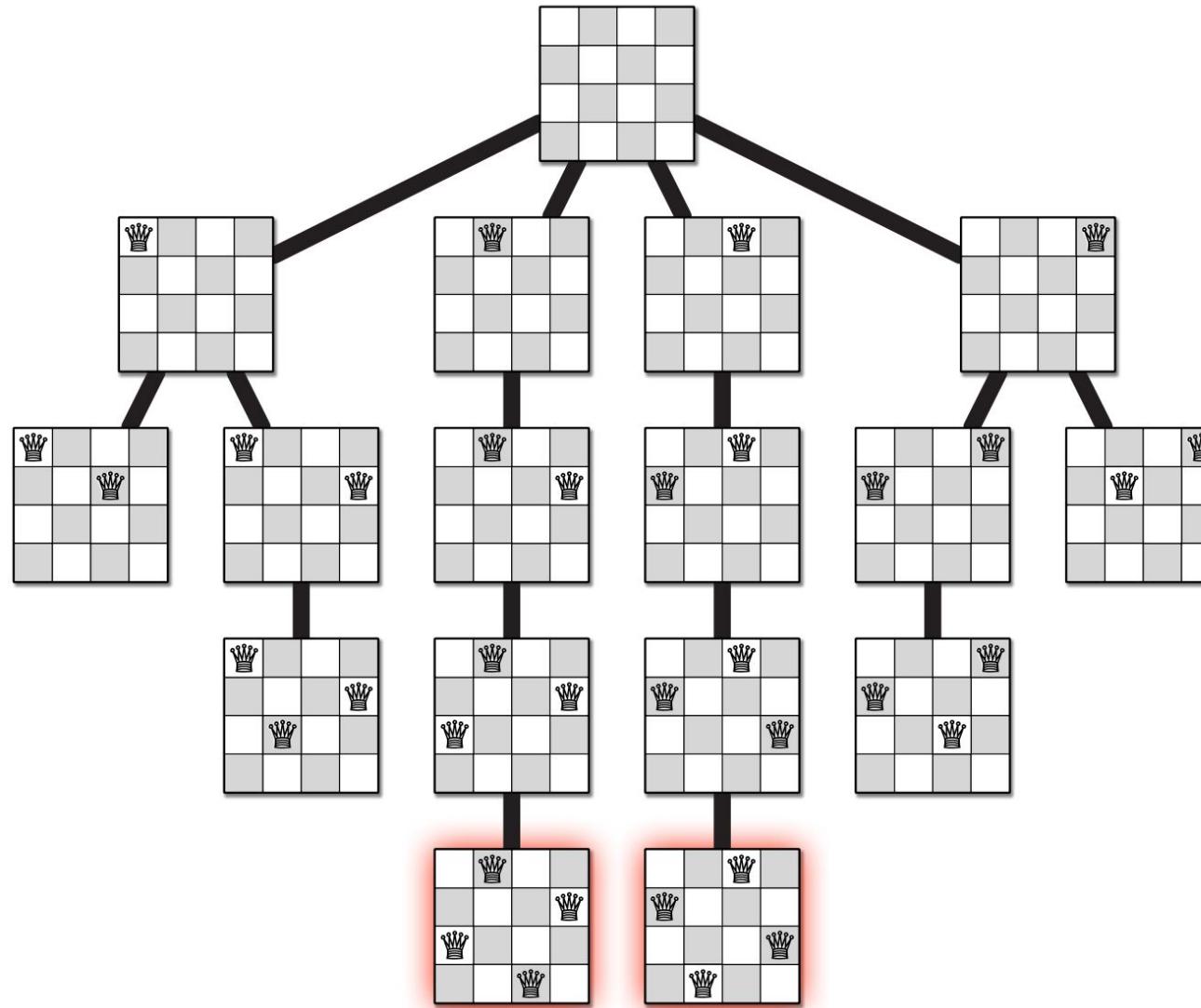


3 x 3



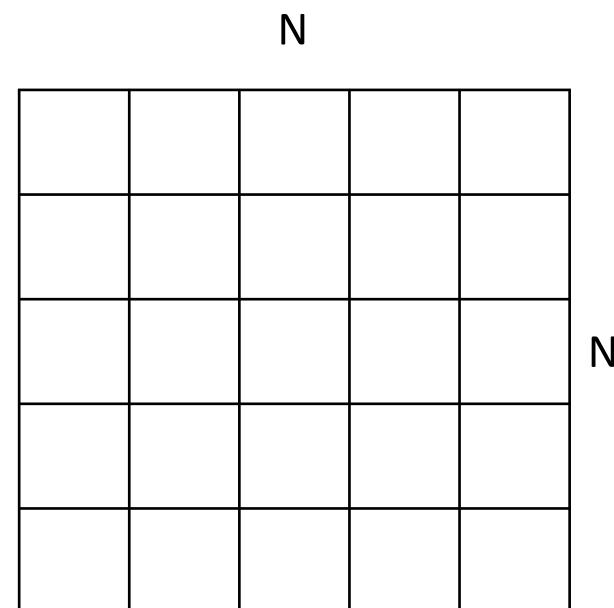
Backtrack Problem - N Queens

4 Queens



Backtrack Problem - N Queens

- Start with naïve approach - without the constraints
 - Place each queen in all possible cells
 - How many possible choices for 1st queen?
 - How many possible choices for 2nd queen?
 - How many possible choices for 3rd queen?
- $N^2 \times N^2 - 1 \times N^2 - 2$, etc...



Backtrack Problem - N Queens

- Start with a slightly better approach - with row constraint
 - At the minimum - each queen must be on a separate **row**
 - Place a queen in all possible columns in each row
 - Check if each combination meets the constraints?
 - How many possible choices for 1st queen, 2nd queen, 3rd queen?
 - $N \times N \times N \Rightarrow N^N$



--	--	--	--	--	--	--

Each cell represents a row
Each cell value represents the column

Backtrack Problem - N Queens

- A better approach with row & column constraint
 - Place one queen per row and explore all possible different columns
 - Check if each combination meets the constraints?
 - How many possible choices for 1st queen, 2nd queen, 3rd queen?
 - $N, N-1, N-2 \Rightarrow N!$

1-D array



Index -> row number

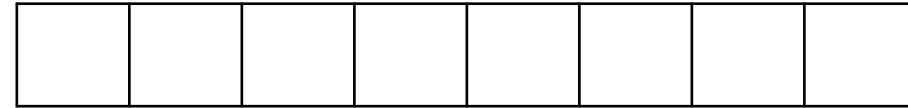
Value -> column number

Backtrack Problem - N Queens

- Translate to code
 - Which template from earlier problems we can leverage?
 - How many decisions?
 - How many choices at each decision?
 - Need a way to keep track which queens have placed on the board?
 - The board can be represented by 1-d array
 - since each queen must not be on the same row
 - Index represents the row number
 - The value in each cell represents the column
 - How to represent the subproblems?
 - How to represent the partial solution?

Backtrack Problem - N Queens

Decision tree/general strategy



Backtrack Problem - N Queens

```
List<List<Integer>> driver(int boardSize) {  
    List<List<Integer>> boards = new ArrayList<>();  
    List<Integer> board = new ArrayList<>();  
    helper(??)  
    return boards;  
}  
  
void helper(??) {  
    // what are the decision?  
    // what are the choices at each decision?  
}
```

Backtrack Problem - N Queens

```
// write code here
void helper(??) {
    // what are the decision?
    // what are the choices at each decision?
}
```

Backtrack Problem - N Queens

```
void helper(int boardSize, int row, List<Integer> board,
           List<List<Integer>> coll) {

    if (row == boardSize) {
        coll.add(new ArrayList<>(board));
    } else {
        // explore possible columns for a particular row
        for (int col = 0; col < boardSize; col++) {
            if (isSafeToPlaceQueenAt(board, row, col)) {
                board.append(col);
                helper(boardSize, row+1, board, coll);
                board.remove(board.size()-1);
            }
        }
    }
}
```

Backtrack Problem - N Queens

```
boolean isSafeToPlaceQueenAt(List<Integer> board, int row, int col) {  
    // check for column conflict  
    // check for diagonal conflict  
}
```

	0	1	2	3	4	5
0					Q	
1						
2						
3			Q			
4						
5						

Q1: (0,4), Q2: (3,1)

$\text{ABS}(0-3) == \text{ABS}(4-1)$

Backtrack Problem - N Queens

```
boolean isSafeToPlaceQueenAt(List<Integer> board, int row2,
                             int col2) {
    for (int col1 : board) {
        if (col1 == col2) return false;
    }
    // diagonal
    for (int row1 = 0; row1 < board.size(); row1++) {
        int xDist = Math.abs(row1 - row2);
        int yDist = Math.abs(board.get(row1) - col2);

        if (xDist == yDist) return false;
    }
    return true;
}
```

Summary

- Identify the problem type
 - Permutation
 - Combination
- Visualize by drawing recursion tree
- Identify the decision points and choices
 - How to represent the subproblems
 - How to represent the partial solution (mutable vs immutable data structure)
- Apply the solution template
 - Base case and recursive case
 - Apply the constraints if needed
 - Abstract out the utility/helper function

The only thing which can save/help you in interviews is
your thinking ability



Thank You

“Tell me and I will forget. Show me and I will remember. Involve me and I will understand.”

Quick survey at the end of the class