



Reinforcement Learning

Q-LEARNING ALGORITHM

Autonomous Cars

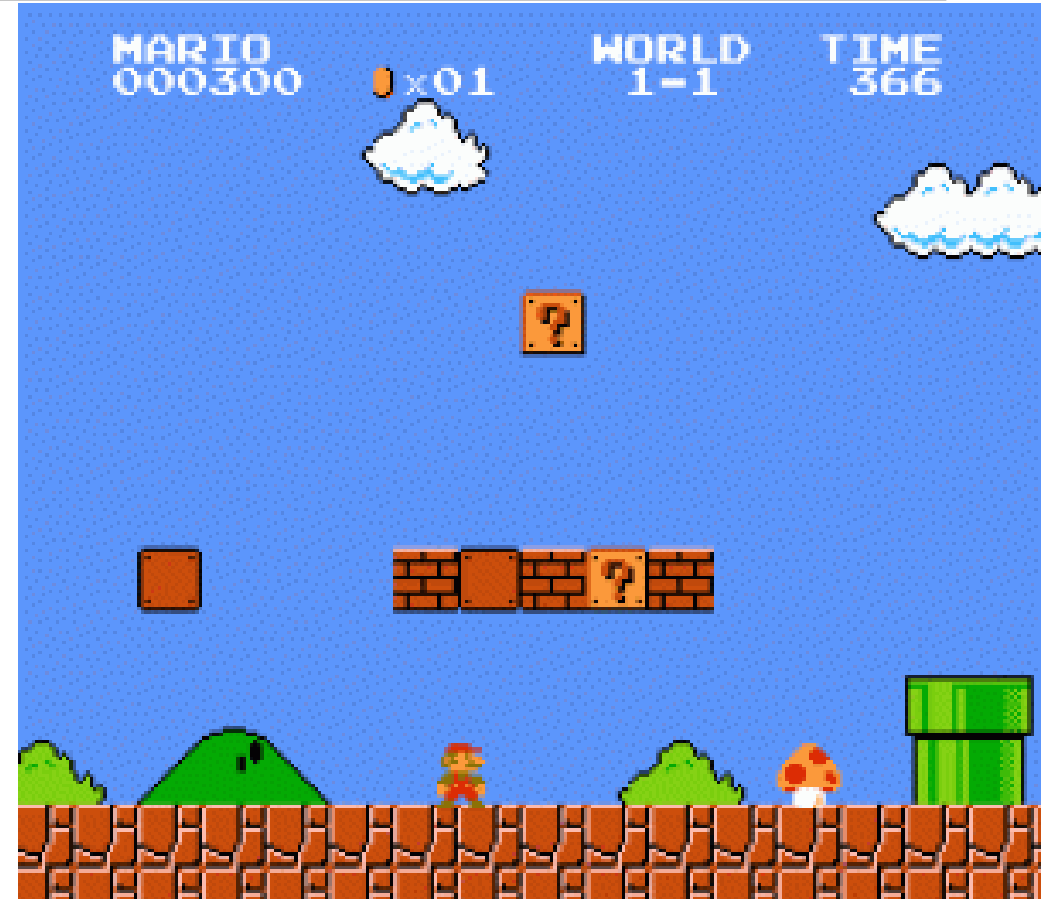
State space:

?



Autonomous Cars

- Simpler environment
- Fewer variables on the state space
- Controlled by Open AI Gym.



Markov Decision Process

- Mathematical framework for modeling decision making.
- Describes the best action for each state in the MDP, known as the optimal policy.
- Is a 4-tuple (S, A, P_a, R_a) .

$$\pi(s) := \operatorname{argmax}_a \left\{ \sum_{s'} P(s' | s, a) (R(s' | s, a) + \gamma V(s')) \right\}$$

$$V(s) := \sum_{s'} P_{\pi(s)}(s, s') (R_{\pi(s)}(s, s') + \gamma V(s'))$$

MDP has the following workflow:

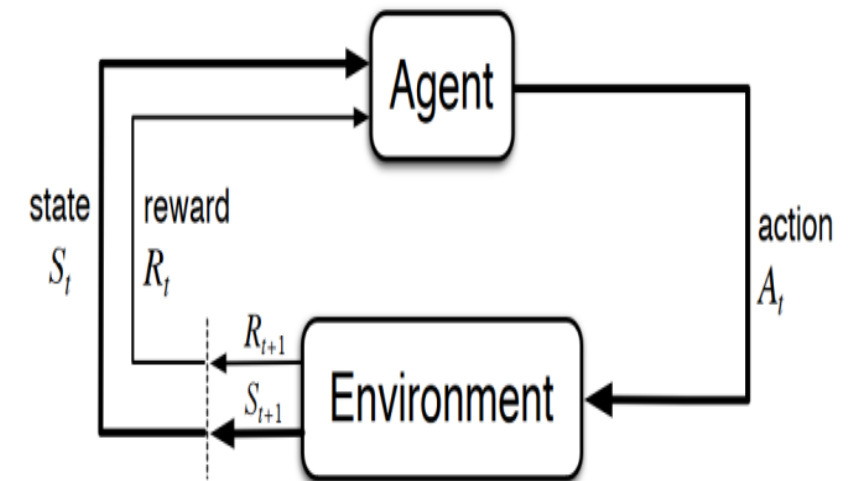


Abbildung 1: Markov Decision Processes Workflow. Retrieved from <http://deeplizard.com/learn/video/my207WNoeyA>

Q learning Algorithm

Algorithm 1: *Q*-learning: Learn function $Q : \mathcal{X} \times \mathcal{A} \rightarrow \mathbb{R}$

Require:

Sates $\mathcal{X} = \{1, \dots, n_x\}$

Actions $\mathcal{A} = \{1, \dots, n_a\}$, $A : \mathcal{X} \Rightarrow \mathcal{A}$

Reward function $R : \mathcal{X} \times \mathcal{A} \rightarrow \mathbb{R}$

Black-box (probabilistic) transition function $T : \mathcal{X} \times \mathcal{A} \rightarrow \mathcal{X}$

Learning rate $\alpha \in [0, 1]$, typically $\alpha = 0.1$

Discounting factor $\gamma \in [0, 1]$

procedure QLEARNING($\mathcal{X}, A, R, T, \alpha, \gamma$)

Initialize $Q : \mathcal{X} \times \mathcal{A} \rightarrow \mathbb{R}$ arbitrarily

while Q is not converged **do**

Start in state $s \in \mathcal{X}$

while s is not terminal **do**

$a \leftarrow \pi(s)$

$r \leftarrow R(s, a)$ ▷ Receive the reward

$s' \leftarrow T(s, a)$ ▷ Receive the new state

$Q(s', a) \leftarrow (1 - \alpha) \cdot Q(s, a) + \alpha \cdot (r + \gamma \cdot \max_{a'} Q(s', a'))$

$s \leftarrow s'$

$Q^* \leftarrow Q$

$\pi^* = \max_{a \in \mathcal{A}} Q^*(s, a)$

return π^*, Q^*

Determines best policy to achieve goal state.

Key items:

- States, actions
- Q table
- Policy Function
- Reward Function
- Transition Function

Desktop Test

The Bellman Equation:

$$Q(s_t, a_t) \leftarrow (1 - \alpha) \cdot \underbrace{Q(s_t, a_t)}_{\text{old value}} + \underbrace{\alpha}_{\text{learning rate}} \cdot \overbrace{\left(\underbrace{r_t}_{\text{reward}} + \underbrace{\gamma}_{\text{discount factor}} \cdot \underbrace{\max_a Q(s_{t+1}, a)}_{\text{estimate of optimal future value}} \right)}^{\text{learned value}}$$

Desktop Test

For this test, assume that every action is enumerated from 0 to 5 and leads to the state of the same number, i.e. action 5 leads to state 5. Then let the Q table be initialized to 0:

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Let R be a matrix with the following values, arbitrarily picked for this example:

$$R = \begin{bmatrix} -1 & -1 & -1 & -1 & 0 & -1 \\ -1 & -1 & -1 & 0 & -1 & 100 \\ -1 & -1 & -1 & 100 & -1 & -1 \\ -1 & 0 & 0 & -1 & 0 & -1 \\ 0 & -1 & -1 & 0 & -1 & 100 \\ -1 & 0 & -1 & -1 & 0 & 100 \end{bmatrix}$$

Desktop Test

Q(1,5)

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



Q(5,2)

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.1 & 0 & 0 & 0 & 0 \end{bmatrix}$$



Q(2,3)

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.1 & 0 & 0 & 0 & 0 \end{bmatrix}$$