Delivery Routing Optimization

CVRP Solution & Analysis

Problem Definition

The Challenge

Design optimal delivery routes for 2 vehicles serving 20 locations from a central depot, minimizing total distance while respecting capacity constraints.

Key Constraints

Each location visited exactly once
Vehicle capacity: 100 units
All routes start/end at depot
Minimize total travel distance

1 Depot
20 Locations
2 Vehicles

→ Find optimal routes

Problem Understanding: The Inputs

Distance Matrix (21×21)

Distances between all nodes (0-20)

Symmetric matrix

JSON format

Demands (21 nodes)

5-15 units per location

Depot demand = 0

Total: 190 units

Vehicle Capacity

Each vehicle: 100 units max
2 identical vehicles

All data provided in cvrp_problem_data.json

Solution Approach: Method Overview

Optimization Approach

Mixed Integer Programming (MIP) using Gurobi optimizer

Model Components

Binary routing variables: x_ijk = 1 if vehicle k travels from i to j

Binary assignment variables: y_ik = 1 if vehicle k services location i

Objective: Minimize total distance traveled

Constraints: Visit once, capacity limits, depot start/end, flow conservation

Solution Approach: Key Technical Decisions

Three-Index Arc Formulation

→ Explicit vehicle routes • Clear capacity tracking • Natural for 2-vehicle problem

Dynamic Subtour Elimination

→ Lazy constraints added as needed • Avoids exponential size • Callback for cycles

Why Gurobi?

→ Industry-standard MIP solver • Efficient branch-and-cut • Built-in callback support

Results: Optimal Routes Found

215

✓ Proven Optimal (0% gap)

Capacity Validation

✓ Both vehicles within 100-unit limit
✓ High utilization (92% & 98%)
✓ Efficient load distribution

Vehicle 0

0 → 5 → 14 → 6 → 10 → 9 → 19 → 13 → 4 → 3 → 1 → 0

Load: 92/100 units (92%) | Distance: 106 units

Vehicle 1

0 → 2 → 20 → 15 → 16 → 18 → 7 → 8 → 17 → 12 → 11 → 0

Load: 98/100 units (98%) | Distance: 109 units

Results: Solution Performance

144 sec
Solve Time
Optimization completed in under 3 minutes

101,642
Nodes Explored
Branch-and-bound tree size

5,020
Lazy Constraints
Subtour cuts added dynamically

0.0%
Optimality Gap
Proven optimal solution

Insights & Lessons Learned

Key Insights

Lazy constraint approach significantly reduced initial model size while maintaining optimality

Near-perfect capacity utilization (92% & 98%) demonstrates efficient route assignment

Three-index formulation makes vehicle-specific routing explicit and intuitive

Lessons Learned

Learned Gurobi and advanced features like callbacks for dynamic constraint generation

Began transition from optimal control background to MILP optimization techniques

Subtour elimination is critical - initial attempts without lazy constraints produced invalid routes

Post-processing validation is essential for verifying capacity and connectivity

Possible Extensions & Future Work

More Vehicles

Scale to 3+ vehicles

Handle heterogeneous fleet (different capacities)
Multiple depots

Time Windows

Add delivery time constraints

Service time at each location
Route duration limits

Dynamic Elements

Real-time traffic data integration

Dynamic demand updates
Vehicle breakdowns/re-routing

Alternative Approaches: Could explore heuristics (Clarke-Wright, genetic algorithms) for larger instances or metaheuristics for approximate solutions with time constraints

Thank You

Questions?