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Assignment 9

H9.1 MAX SAT Optimization problem :

Given a CNF formula ϕ with variables $\{x_1, x_2, \ldots, x_n\}$

For $i = 1$ to n :

if $E[\#\text{satisfied clauses} \mid X_{i-1}, x_i = \text{true}]$

$\geq E[\#\text{satisfied clauses} \mid X_{i-1}, x_i = \text{false}]$

then

$x_i = \text{true}$

else $x_i = \text{false}$

Here X_i is the truth-assignment of previous variables $\{x_1, \ldots, x_i\}$ by the above algorithm.

As we can see, it is a deterministic greedy algorithm maximizing the # satisfied clause.

H9.2 Given : A set S with $|S| = 2^k = n$

We find a family of sets s.t. OPTIMAL solution for MIN SET COVER has two sets and algorithm picks $\log_2 n = k$ sets.

Let $S_0, S_1, \dots, S_k$ be pairwise disjoint sets of sizes $\{1, 1, 2, 4, 8, \dots, 2^{k-1}\}$. And there are two more disjoint sets T_0 and T_1 such that each exactly half of elements of S.

Our set family is $\{T_0, T_1, S_0, S_1, \dots, S_k\}$.

Clearly OPTIMAL solution has size $2 = \{T_0, T_1\}$.

But our greedy algorithm can pick up sets $\{S_0, S_1, \dots, S_k\}$, in the order $S_k, S_{k-1}, \dots S_1, S_0$.

$\therefore$ size of solution by greedy algo. $= k+1$

$$= \log_2 n + 1$$

The above size can of course be reduced to $\log_2 n$ by merging S_0 and S_1, i.e. $S_1 = S_0 \cup S_1$.

$\therefore$ Approximation ratio $\alpha = \dfrac{\log_2 n}{2}$.

H9.3

(a) 3SAT parameterised by # variables $n = |X|$ in $\phi(x)$.

To show : It is in FPT.

Sol : A TM m can check all possible truth assigments.

$\#$ assignments $= 2^k$ $(k = n, \text{parameter})$

time taken to check if a assignment satisfies ϕ
$= O(n)$

$\therefore$ Time taken to decide 3SAT by m
$= 2^k \cdot n$

$\therefore$ 3SAT is in FPT when parameterized by # variables.

(b) To show : 3SAT is in XP when parameterized by size of smallest backdoor $Z \subseteq X$ for a satisfiable formula $\phi = \phi(x)$

Sol : Since, any truth assignment of a backdoor set reduces ϕ to T or F, means truth assignment of other variables doesn't affect the result.

Algorithm :

Given a formula ϕ, a bound k

1. For every set $Z \subseteq X$ s.t. $|Z| = k$.
2. Assign 'true' to X/Z variables.
3. Check for all truth assignments of Z:
   If any assignment of Z along with X/Z reduces to truth, then accept.
4. Reject.

We can check the time complexity of the above algorithm :

$$= \underbrace{{}^{n}C_{k}}_{\text{\# backdoors}} \cdot \underbrace{2^{k}}_{\text{\# of truth assignments for each backdoor}} \cdot \underbrace{n}_{\text{Time taken to check if } \phi \text{ reduces to true or false.}}$$

Here, $f(k(n)) = {}^{n}C_{k} \cdot 2^{k}$

$p(|n|) = |n|$

$\therefore$ The above problem is in XP.