Instructions

- *Note: This final set of Homework Problems contains three additional exercises addressing some of topics from the second part of the course, i.e. the area of the 2nd midterm exam. The problems will be graded as usual, but no associated tutorial session is currently planned; this can however be arranged if there is demand. The exam itself covers the topics from NP-completeness to parameterised algorithms (Lectures 8–17 and Tutorials 4–9). The exam will most likely be organised online via MyCourses at the originally announced time, viz. Tue 7 April, 1–4 p.m., with exact details still to be confirmed.*
- You should solve Homework Problems H10.1–H10.3 on your own, and submit your solutions via the MyCourses interface by the deadline of Tue 31 Mar, 23:59, These problems will be individually graded on a scale of 0–2 points per problem.
- In preparing your solutions to the Homework Problems:
  1. Justify your solutions, be precise, and provide sufficient detail so that it is easy to follow your reasoning.
  2. Submit your solutions as an easily readable, single pdf file, which is either typeset or written in full sentences and clean handwriting.
  3. **[Code of Conduct]** You can discuss the problems with your colleagues and the course's teaching staff, but you must write the presentations of your solutions *independently* and *individually*, without any notes from such discussions.

Homework Problems

H10.1 Show that the following DIRECTED CYCLE COVER problem is **NP**-complete:

> INSTANCE: A directed graph $G = (V, E)$ and an integer K.
> QUESTION: Is there a subset of at most K arcs (directed edges) $F \subseteq E$, such that any directed cycle in the graph G contains at least one arc from the set F?[1]

(*Hint:* Reduction from VERTEX COVER; represent vertices by arcs. Remember to validate both directions of the reducibility condition.)

(2 points)

[1] In other words, given a directed graph G, what is the smallest number of arcs that need to be removed to make it acyclic?

H10.2 Show that the following decision problem is in class Π_2^p:

INTEGER EXPRESSION EQUIVALENCE (IE-EQ): Do two *integer expressions* E_1 and E_2, built out of binary numbers by operations + ("sum") and | ("or") describe the same set? E.g.

$$(001 \mid 011) + (010) = (011 \mid 101),$$

or in decimal notation: $(1 \mid 3) + (2) = (3 \mid 5)$. (*Hint:* Consider first the complementary problem of INTEGER EXPRESSION INEQUIVALENCE (IE-NEQ). It might also be helpful to think about the simpler problem of INTEGER EXPRESSION MEMBERSHIP (IE-M): Given a binary number x and an integer expression E, does x belong to the set described by E?)

Additional fact: The problem IE-EQ is in fact Π_2^p-complete, as established by L. Stockmeyer in 1973. (You do not need to prove this result.) (2 points)

H10.3 Consider the optimisation problems MIN SET COVER and MIN DOMINATING SET from Tutorial 4. (Problems C4.3 and H4.3, respectively.) Show that if, for some constant $\alpha > 1$, there is an α-approximation algorithm for MIN DOMINATING SET, then there is also an α-approximation algorithm for MIN SET COVER. (2 points)