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Assignment 2

H2.1 Given : $\exists M_1$ s.t. $L(M_1) = L_1$

$\exists M_2$ s.t. $L(M_2) = L_2$

where M_1 and M_2 are non-deterministic TM

and $L_1, L_2 \in NP$

① Unions

Take a Non-det. TM M such that on any input 'x',

① Run M_1 on x

if M_1 accepts x, then accept

② Run M_2 on x

if M_2 accepts x, then accept

③ Else reject

We can see that 'M' will take time in order of max of time of M_1 and M_2.

So, $L(M) \in NP$

$\therefore$ NP is closed under unions

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(ii) Intersection

Take a Non-det. TM m such that on input 'x'

(1) Run m_1 on x,
 if m_1 rejects x, then reject

(2) Run m_2 on x,
 if m_2 rejects x, then reject

(3) Else accept.

Time complexity for m will also be maximum of time complexity of m_1 and m_2.

$\therefore$ $L(m) \in NP$

$\therefore$ NP is closed under intersection.

HW-2

(2) To show: $NP \subseteq PSPACE \subseteq EXP$

Part 1 $PSPACE \subseteq EXP$

Our space is restricted to some polynomial space. And since, a TM has finite states, we can find total number of possible config. of TM.

$$\# \text{conf.} = |K| \times |\Sigma|^{n(s)} \times n(s)$$

where $|K|$ is # states

$|\Sigma|$ is size of alphabets

$n(s)$ is the space available.

Now, all the conf. can be visited in above given time i.e. $O(|K| * |\Sigma|^{n(s)} * n(s))$.

So, this TM shouldn't take more than this exponential time.

So, PSPACE $\subseteq$ EXP.

(The same holds for multi-tape TM because ~~they~~ the time is still exponential)

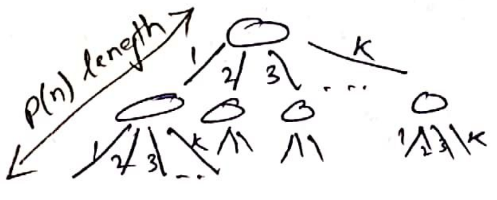
Part 2

NP $\subseteq$ PSPACE

Given a Non-det. TM M, deciding a string in non-deterministic poly. time.

Let's say, maximum 'k' choices of next config. are possible at any givn. config.

i.e.



So we can construct a Det. TM M', which can run one sequence of config. at a time, in a BFS kind of order.

i.e. run one size 1st step : 1 and check if string accepted or not

⋮

kth step : k " " " " " "

run two size steps : 1 1 " " " " " "

1 2 " " " " "

⋮

1 k

2 1

⋮

2 k

⋮

k k " " " " "

This way, a running all seq. of config. will take exponential time. But it will take polynomial space.

space $= O($~~max.~~ length of biggest sequence$)$

$= O(p(n))$

$\therefore$ all NP problems can be run in a DTM using PSPACE.

H2.3

Halting problem : TM M_H and input 'h'

Construct a TM 'M' such that on input 'x'

① If $|x| \leq 7$, then accept it.

② Else

(2.1) Run M_H on h

If it halts, then accept x

Else reject x

Now $L(M) = \begin{cases} \{x \mid |x| \leq 7\} & \text{if } M_H \text{ doesn't halt on } h \\ \text{Everything} & \text{otherwise} \end{cases}$

Now, if we have a TM M' which takes input a TM and tells if it accepts all strings of length ≤ 7, and only those, then using M', we can decide whether M_H halts on h or not.

If M' accepts all |string| ≤ 7 and only those, then M_H doesn't halt

Else $\longrightarrow$ M_H halts on h.

Since M_H is not possible (i.e. HP is undecidable)

$\therefore$ M is also not possible.

$\therefore$ The above problem is undecidable.