

Assignment 1 : CS-E4830 Kernel Methods in Machine Learning 2020

The **deadline** for this assignment is **Thursday 06.02.2020 at 4pm**. If you have questions about the assignment, you can ask them in the 'General discussion' section on MyCourses. We will have a tutorial session regarding this assignment on 30.01.20 at 4:15 pm in TU1(1017), TUAS, Maarintie 8 (**check room**).

Please follow the **submission instructions** given in MyCourses: <https://mycourses.aalto.fi/course/view.php?id=24366§ion=4>.

Pen & Paper exercise (12 points in total)

Question 1 (2 points): Recall from Lecture 1, the form for the polynomial kernel

$$K_1(x, y) = (\langle x, y \rangle + c)^m$$

where $c \geq 0$, m is a positive integer and $x, y \in \mathbb{R}^d$.

- Prove that $K_1(x, y)$ as defined above is a valid kernel

Question 2 (4 points) Recall from lecture 2, in the context of binary classification, the Parzen window classifier assigns a test instance x based on the distance to the centroids in the following way :

$$h(x) = \begin{cases} +1 & \text{if } \|\phi(x) - c_-\|^2 > \|\phi(x) - c_+\|^2 \\ -1 & \text{otherwise.} \end{cases}$$

where c_- and c_+ represent the centroids in the feature space of the negative and positive classes respectively. Show by deriving appropriate expressions for α_i and b , that the above decision function can be written in the following form $h(x) = \text{sgn}(\sum_{i=1}^n \alpha_i k(x, x_i) + b)$ such that $k(x, x_i) = \langle \phi(x), \phi(x_i) \rangle$. Here $\text{sgn}(\cdot)$ represents the sign function, and n is the total number of training samples.

Question 3 (3 points) For $x, y \in \mathbb{R}$, check if $K_2(x, y) = \cos(x + y)$ is a valid kernel function.

Question 4 (3 points) For $x, y \in \mathcal{X} = (-1, 1)$, prove that $K_3(x, y) = \frac{1}{1-xy}$ is a valid kernel

Computer Exercise (8 points in total)

Solve the computer exercise in JupyterHub (<https://jupyter.cs.aalto.fi>). The instructions for that are given in MyCourses: <https://mycourses.aalto.fi/course/view.php?id=20602§ion=3>.