AMIT YADAV

Assigment 6

H6.1 To prove: $PH \subseteq PSPACE$

Sol: We know that $PH = \bigcup_{i=1} \Sigma_i P$

Using induction on i

Base Case: $\Sigma_1 P = \exists P = NP \subseteq PSPACE$

$\Pi_1 P = \forall P = \text{Co-NP} \subseteq PSPACE$ (Because Co-NP $\subseteq$ co-PSPACE $=$ PSPACE)

Induction step:

Hypothesis: Let $\Sigma_i P$ and $\Pi_i P \subseteq PSPACE$. — (1)

Then $\Sigma_{i+1} P = \exists(\Pi_i P)$.

Since $\Pi_i P$ can be solved in PSPACE,

Claim: A DTM can check all possible values of guesses in a duck-tailing manner, using only PSPACE.

$L \in \Sigma_{i+1} P = \exists(\Pi_i P) \Rightarrow$ ~~$x \in L$~~ ~~iff~~ $\exists R \in \Pi_i P$

s.t. $x \in L$ iff $\exists y$ s.t $(x,y) \in R$.

Now, checking if $(x,y) \in R$ is in PSPACE because $\Pi_i P \subseteq PSPACE$ (I.S. — (1)).

A DTM M checks all values of 'y' in duck-tailing fashion.
Space taken = length of (largest) maximum branch for a guess.
= polynomial space

(using induction hypo. $\Pi_i^P \subseteq PSPACE$. So check for any 'y' takes PSPACE.

$\therefore \quad \Sigma_{i+1}^P \subseteq PSPACE.$

Similarly for $\Pi_{i+1}^P = \forall(\Sigma_i^P)$. Since $\Sigma_i^P \subseteq PSPACE$, checking for all guesses in duck-tailing manner.
And reject string if it is rejected for any guess.

$\therefore \quad \Pi_{i+1}^P \subseteq PSPACE.$

$\therefore \quad \forall\, i \geq 1, \quad \Sigma_i^P \subseteq PSPACE.$

$\therefore \quad \bigcup_{i \geq 1} \Sigma_i^P \subseteq PSPACE$

$\Rightarrow \quad PH \subseteq PSPACE.$

H 6.2 Covering radius.

Given: $H \in \{0,1\}^{m \times n}$ and ~~R~~ r.

Sol: Let R be a relation such that

$(H\#r, x, c) \in R$ iff $Hc = 0 \pmod 2$ and
$x \in \{0,1\}^n$
$c \in \{0,1\}^n$ $d(x, c) \leq r$.

Now Covering radius $\in \Pi_2^P$ if $R \in P$ and x and c are of polyn. length.

Proof: $|x| = |c| = n$ since, $x, c \in \{0,1\}^n$

Time for checking in R:

Checking if $Hc = 0$ takes $O(mn)$ time.

" " $d(x,c) \leq r$ takes $O(n)$ time.

$\therefore$ R runs in polynomial time and certificates are also polyn. length.

Also

$H\#r \in C.R.$ ~~iff~~ iff $\forall x \in \{0,1\}^n$, $\exists c \in \{0,1\}^n$ s.t. $(H\#r, x, c) \in R$.

That is every $x \in \{0,1\}^n$ should be in covering radius of distance, from C.