# AN AUTOMATIC TRANSMISSION MODEL FOR VEHICLE CONTROL

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#### **ABSTRACT**

This paper develops a dynamically-correct model of an automatic transmission, extending previous research results. The bond graph method is used in the modeling. The bond graph method is ideally suited for modeling "power" systems, because the method is based on generalized "power" variables. The bond graph method is capable of providing correct dynamic constraints and kinematic constraints, as well as the governing differential equations of motion. The bond graph method is used to develop 1-4 in-gear range models, as well as various upshift and downshift models, with proper dynamic and kinematic constraints.

#### 1. INTRODUCTION

An automatic transmission which transmits the power generated by an engine to the wheels is an important element in modern vehicles. In a vehicle equipped with a manual transmission, a driver determines a gear range manually by his experiences. However, an automatic transmission determines a gear range automatically considering variations such as engine speed, vehicle longitudinal speed, and throttle angle. Therefore, an automatic transmission is preferred to a manual one in view of easier driving. However, generally the fuel efficiency of an automatic transmission is approximately 90 percent that of a skilled driver using a manual transmission [1]. Also, shift feel is one of the important considerations in automatic transmissions [2], [3]. Therefore, improving both shift feel

and fuel efficiency is an important issue in automatic transmissions.

An automatic transmission is composed of planetary gears, several plate-type and band-type clutches, and elaborate hydraulic control circuits. Automatic transmission modeling has been performed previously using various methods [4]-[9]. This paper develops a dynamically-correct model of the mechanical parts of an automatic transmission using the bond graph method, extending the results in [8] and [9]. The bond graph method is applied to 1-4 ingear ranges, as well as all required various upshifts and downshifts. Note that the bond graph method is well suited for modeling a power system such as an automatic transmission, because the method is based on generalized power variables.

# 2. IN-GEAR AND UPSHIFT MODELS 2.1 Compound planetary gear model

A compound planetary gear being modeled in this paper consists of two planetary gears, as shown in Figure 1. This is one of the possibl automatic transmission configurations, and other configurations are also possible. The bond graph of the compound planetary gear in Figure 1 is shown in Figure 2. From Figure 2, the following relationship is derived [7], [8], [9]:

$$\omega_{ci} = R_{si}\omega_{si} + R_{cr}\omega_{cr} \tag{1}$$

$$\omega_{cr} = R_{ci}\omega_{ci} + R_{sr}\omega_{sr} \tag{2}$$

The clutch control schemes for achieving various gear positions are given in Table 1.

Table 1: O	peartions of	plate and	band clutches.	介:	engaging, 1:	disengaging
	F	F		11.		0,00,000,000

	Fisrt clutch	Second clutch	Third clutch	Fourth clutch	1-2 band	Reverse band
First gear	ON	OFF	OFF	OFF	ON	OFF
1-2 shift	ON	介	OFF	OFF	ON	OFF
2-1 shift	ON	U	OFF	OFF	ON	OFF
Secons gear	ON	ON	OFF	OFF	ON	OFF
2-3 shift	#	ON	Λ	OFF	<b>#</b>	OFF
3-2 shift	1	ON	<b>\</b>	OFF	⇑	OFF
Third gear	OFF	ON	ON	OFF	OFF	OFF
3-4 shift	OFF	ON	ON	Λ	OFF	OFF
4-3 shift	OFF	ON	ON	<b>#</b>	OFF	OFF
Fourth gear	OFF	ON	ON	ON	OFF	OFF
Reverse gear	ON	OFF	OFF	OFF	OFF	ON

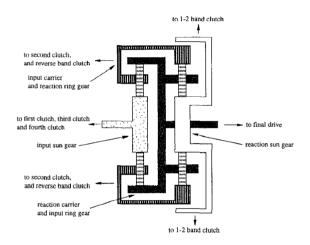


Figure 1: A compound planetary gear arrangement

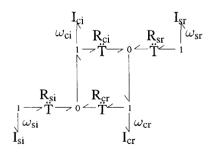


Figure 2: Bond graph for a compound planetary gear

# 2.2 First gear, one-to-two shift, and second gear

In [8] and [9], the first gear range, 1-2 shift, and second gear range models of an automatic transmission were developed together with simulation and experiment results. Since only the first gear, 1-2 shift, and second gear models are considered in [8] and [9], the kinematic constraint that the speed of the r.s.g. is zero and the corresponding dynamic requirements were considered. From the third gear, however, the r.s.g. free-wheels. Therefore, the reaction torque on the r.s.g. by the pressure acting on the 1-2 band clutch and the torque transmitted to the r.s.g. by the kinematics and dynamics of the compound planetary gear set must be considered, in order to precisely know if the transmission is in the first or second gear range. That is, in the first gear. 1-2 shift, and second gear, an additional dynamic constraint on the r.s.g. must be considered. The dynamic constraints on the r.s.g. for the first gear, 1-2 shift, and second gear, are as follows (the bond graphs of first gear, 1-2 shift, and second gear are found in [8] and [9]):

$$RT_{12B} = \frac{R_{sr}}{R_{ci}} \left( \frac{T_t - (I_t + I_{si})\dot{\omega}_t}{R_{si}} - I_{ci}\dot{\omega}_{ci} \right) + \frac{R_{sr}}{R_{ci}} \left( 1 - \frac{1}{R_{si}}T_{c2} \right)$$
(3)  

$$RT_{12B} = \frac{R_{sr}}{R_{ci}} (T_{c2} - \frac{I_{si}}{R_{si}}\dot{\omega}_{si} - I_{ci}\dot{\omega}_{ci})$$
(4)  

$$RT_{12B} = \frac{R_{sr}}{R_{ci}} \left( T_t - -(I_t + I_{ci})\dot{\omega}_t - \frac{I_{si}}{R_{si}}\dot{\omega}_{si} \right)$$
(5)

For the speed of the r.s.g. to be zero, in the first gear, 1-2 shift, and second gear, the torque on the r.s.g.

by the 1-2 band pressure must be larger than the reaction torque transmitted to the r.s.g. Therefore, an additional dynamic constraint is:

$$T_{12B} > RT_{12B}$$
 (6)

#### 2.3 Second-to-thrid shift model

2.3.1 Torque phase model. Until the 1-2 band pressure becomes sufficiently small, the speed of the r.s.g. remains zero. Hence, the 2-3 shift torque phase is bond graphed as in Figure 3. In Figure 3, it can be seen that if the 1-2 band pressure is not decreased sufficiently before the third clutch pressure builds up, the turbine torque can be transmitted through either the i.s.g. or the i.c.g.

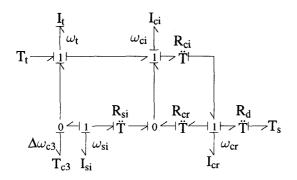


Figure 3: Bond graph for torque phase of 2-3 shift

From the bond graph model, the state equations are:

$$I_{t2}\dot{\omega}_t = T_t + \left(1 - \frac{R_2}{R_1}\right)T_{c3} - R_2R_3T_s \tag{7}$$

the kinematic constraints are:

$$\omega_t = \omega_{ci}, \ \dot{\omega}_t = \dot{\omega}_{ci} \tag{8}$$

$$\omega_{si} = \frac{R_2}{R_1} \omega_{ci}, \ \frac{R_2}{R_1} \dot{\omega}_t = \dot{\omega}_{ci} \tag{9}$$

$$\omega_{cr} = R_2 \omega_{ci}, \ \dot{\omega}_{cr} = R_2 \dot{\omega}_{ci} \tag{10}$$

$$\omega_{sr} = 0 \text{ (held by 1 - 2 band clutch)} \tag{11}$$

and the dynamic constraints are:

$$T_{c2} > RT_{c2up} = T_t + T_{c3} - I_t \dot{\omega}_t$$

$$T_{c2} > RT_{c2down} = R_2 I_{cr} \dot{\omega}_{cr} + \frac{R_2}{R_1} I_{si} \dot{\omega}_{si}$$

$$+ R_2 R_d T_s + \frac{R_2}{R_1} T_{c3}$$

$$T_{12B} > RT_{12B} = \frac{R_{sr}}{R_1} \left( T_t + \frac{R_{cr}}{R_{si}} T_{c3} - (I_t + I_{ci}) \dot{\omega}_t - \frac{I_s i}{R_1} \dot{\omega}_s i \right)$$

$$(14)$$

2.3.2 Speed phase model. As the pressure of the 1-2 band clutch is decreased, the reaction torque is also decreased. This implies that the dynamic constraint cannot be satisfied. At this instant, the r.s.g. starts to rotate, and speed phase starts. The bond graph of the speed phase is shown in Figure 4. This model is an intermediate process in which the speed of the turbine is equal to that of the i.s.g., and the r.s.g. free-wheels.

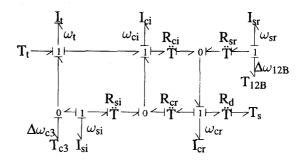


Figure 4: Bond graph for speed phase of 2-3 shift

From Figure 4, the state equations are:

$$I_{t23}\dot{\omega}_t = T_t + \left(1 - \frac{1}{R_{ci}}\right)T_{c3} - \frac{R_{ci}}{R_{sr}}T_{12B} + I_{s23}\dot{\omega}_{cr}$$
(15)

$$I_{cr23}\dot{\omega}_{cr} = \frac{T_{12B}}{R_{sr}} + \left(1 - \frac{1}{R_{si}}\right)T_{c3} - R_dT_s + I_{s23}\dot{\omega}_t$$
(16)

where

$$I_{t23} = I_t + I_{ci} + \frac{I_{si}}{R_{si}^2} + \left(\frac{R_{ci}}{R_{sr}}\right)^2 I_{sr}$$
 (17)

$$I_{cr23} = I_{cr} + \left(\frac{R_{cr}}{R_{si}}\right)^2 I_{si} + \frac{I_{sr}}{R_{sr}^2}$$
 (18)

$$I_{s23} = \left(\frac{R_{cr}}{R_{si}^2}\right) I_{si} + \left(\frac{R_{ci}}{R_{sr}^2}\right) I_{sr}$$
 (19)

the kinematic constraints are:

$$\omega_t = \omega_{ci}, \ \dot{\omega}_t = \dot{\omega}_{ci} \ (20)$$

$$\omega_{ci} = R_{si}\omega_{si} + R_{cr}\omega_{cr}, \ \dot{\omega}_{ci} = R_{si}\dot{\omega}_{si} + R_{cr}\dot{\omega}_{cr} \ (21)$$

$$\omega_{cr} = R_{sr}\omega_{sr} + R_{ci}\omega_{ci}, \ \dot{\omega}_{cr} = R_{sr}\dot{\omega}_{sr} + R_{ci}\dot{\omega}_{ci} \ (22)$$

and the dynamic constraints are:

$$T_{c2} > RT_{c2up} = T_t + T_c 3 - I_t \dot{\omega}_t$$
 (23)  
 $T_{c2} > RT_{c2down} = I_{ci} \dot{\omega}_{ci} + \frac{R_2}{R_1} I_{si} \dot{\omega}_{si} + R_2 I_{cr} \dot{\omega}_{cr} + R_2 R_d T_s - \frac{R_2}{R_1} T_{c3}$  (24)

# 2.4 Third gear model

In the 2-3 shift, if the r.s.g. free-wheels, and the i.s.g. locks up with the turbine, the shift from second to third gear is ended. Fig. 5 is the bond graph of the third gear range of an automatic transmission. In Figure 5, the turbine is connected to both the i.s.g. and the i.c.g. In addition, in the third gear, all the gears of the compound planetary gear rotate in the same speed to that of the turbine. Because of this, the third gear range is called direct drive mode [10].

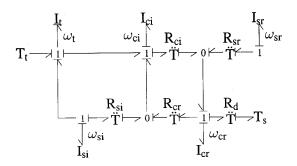


Figure 5: Bond graph for third gear

From the bond graph, the state equations are:

$$I_{t3} = T_t - R_3 R_d T_s (25)$$

where

$$I_{t3} = I_t + I_{si} + I_{sr} + I_{ci} + I_{sr}$$
 (26)

$$R_3 = 1.0$$
 (27)

the kinematic constraints are:

$$\omega_t = \omega_{ci} = \omega_{si} = \omega_{cr} = \omega_{sr} \tag{28}$$

$$\dot{\omega}_t = \dot{\omega}_{ci} = \dot{\omega}_{si} = \dot{\omega}_{cr} = \dot{\omega}_{sr} \tag{29}$$

and the dynamic constraints are:

$$T_{c2} > RT_{c2up} = T_t + R_{si}R_4R_dT_s$$

$$-\left(I_t + I_{si} + \frac{R_{ci}R_{si}^2}{R_{ci}R_{si}^2}I_{sr}\right)\dot{\omega}_t$$

$$+ \frac{R_{si}}{R_{cr}}\left(I_sr + \frac{R_{cr}R_r}{R_{sr}}I_sr\right)\dot{\omega}_{cr} \qquad (30)$$

$$T_{c2} > RT_{c2down} = R_{si}\left(I_ci - \frac{R_{ci}R_r}{R_{sr}}I_sr\right)\dot{\omega}_t$$

$$+ R_{cr}(I_{ci} + R_4^2I_{cr} + R_r^2I_{sr})\dot{\omega}_{cr}$$

$$+ R_4R_dT_s \qquad (31)$$

$$T_{c3} > RT_{c3up} = \frac{R_{si}}{R_{cr}}(-I_{s23}\dot{\omega}_t)$$

$$+I_{cr23}\dot{\omega}_{cr} + R_dT_s)$$

$$T_{c3} > RT_{c3down} = -T_t + \left(\frac{R_r}{R_{sr}}I_{sr}\right)\dot{\omega}_{cr}$$

$$+\left(I_t + I_{ci} + \frac{R_{ci}R_r}{R_sr}I_{sr}\right)\dot{\omega}_t + R_4R_dT_s$$
 (33)

where

$$R_r = \frac{1 - R_{ci}R_{cr}}{R_{cr}R_{sr}} \tag{34}$$

$$R_4 = \frac{1}{R_{cr}} \tag{35}$$

### 2.5 Third-to-fourth shift model

**2.5.1 Torque phase.** With the fourth clutch pressure on-coming, the 3-4 shift is initiated. The bond graph of the torque phase of the 3-4 shift is shown in Figure 6. The state equations as well as all constraints can be written from this bond graph using the same procedures.

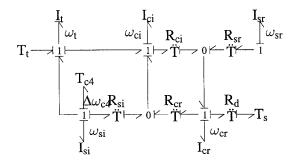


Figure 6: Bond graph for torque phase of 3-4 shift

2.5.2 Speed phase. As the pressure of the fourth clutch increases, the fourth clutch capacity increases. Since the role of the fourth clutch is to hold the i.s.g. stationary, if the fourth clutch torque is larger than the torque transmitted to the i.s.g., the third clutch sprag stops transmitting the power from the turbine to the i.s.g. Physically, this occurs when the reaction torque on the third clutch sprag goes to zero. The reaction torque is

$$RT_{sp3} = R_{si} \left( (I_t + I_{ci})\dot{\omega}_t - \frac{R_{ci}}{R_{sr}} I_{sr}\dot{\omega}_{sr} - T_t \right) - T_{c4}$$
(36)

At this time, the speed phase of 3-4 shift starts. During the speed phase, the i.s.g. speed keeps decreasing and eventually goes to zero. The 3-4 speed phase is bond-graphed in Figure 7. Again, all equations can be easily written from the bond graph.

Figure 7: Bond graph for speed phase of 3-4 shift

# 2.6 Fourth gear model

When the i.s.g. speed goes to zero in the 3-4 shift, the transmission is in the fourth gear. The bond graph model is depicted in Fig. 8. In the fourth gear, the r.s.g. free-wheels times faster than the turbine, and the r.c.g. rotates times faster than the turbine. Hence, the fourth gear range is called over-drive mode [10]. Note that the fourth gear range is distinguished from the second gear range whether the r.s.g. free-wheels and the i.s.g. is held stationary or the r.s.g. is held stationary and the i.s.g. free-wheels.

Figure 8: Bond graph for fourth gear

#### 2.7 Reverse gear model

In the reverse gear, the turbine torque is supplied to the i.s.g. by the first clutch, and the i.c.g. is held to ground by the reverse band, and the r.s.g. free-wheels. This word model is bond graphed in Figure 9.

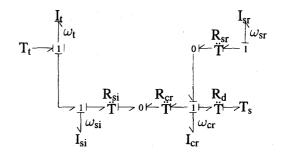


Figure 9: Bond graph for reverse gear

## 3. DONWSHIFT MODELS

#### 3.1 Two-to-one shift model

The shifting down from second to first gear begins with the second clutch pressure off-going. The bond graph model of the 2-1 shift is identical to that of the 1-2 shift in [8] and [9]. The equations of motion as well as the kinematics and dynamic constraints are also identical.

#### 3.2 Third-to-second shift model

3.2.1 Torque phase. With the first clutch pressure on-coming, the third clutch pressure off-going, and the 1-2 band clutch pressure on-coming, the torque phase of the 3-2 shift is initiated. In the torque phase, the dynamic constraints on the third gear range are satisfied with the 1-2 band clutch pressure being not zero. The torque phase of the 3-2 shift is bond graphed in Figure 10.

$$T_{t} \xrightarrow{I_{t}} T_{t} \xrightarrow{\omega_{ci}} R_{ci} \xrightarrow{R_{sr}} T_{\omega_{sr}} \xrightarrow{\omega_{sr}} T_{12B}$$

$$R_{si} \xrightarrow{R_{cr}} R_{d} \xrightarrow{R_{d}} T_{12B}$$

$$R_{si} \xrightarrow{R_{cr}} T_{\omega_{sr}} T_{s}$$

$$I_{si} \xrightarrow{I_{cr}} T_{cr}$$

Figure 10: Bond graph for torque phase of 3-2 shift

**3.2.2 Speed phase.** The speed phase model of the 3-2 shift is identical to the 2-3 shift model. In Figure 10, if the third clutch pressure decreases and the turbine is disconnected to the i.s.g., the shift model becomes as Figure 4.

## 3.3 Fourth-to-third shift model

In the 4-3 shift, the i.s.g. rotates with being reduced. As mentioned in the 3-4 shift, the third clutch transmits power through the one-way sprag. Therefore, Figure 7 is the bond graph when the fourth clutch pressure is not zero and the speed of the turbine is equal to that of the i.s.g. If the fourth clutch pressure is not zero, and the speed of the i.s.g. is slower than that of the turbine, the bond graph for the 4-3 shift is in Figure 11. In Figure 11, it can be seen that both the r.s.g. and the r.c.g. are free-wheeling.

$$T_{t} \xrightarrow{I_{t}} U_{t} \qquad U_{ci} \xrightarrow{I_{ci}} R_{ci} \qquad R_{sr} \xrightarrow{I_{sr}} U_{s}$$

$$T_{t} \xrightarrow{I_{t}} T \xrightarrow{I_{t}} 0 \xrightarrow{I_{t}} T \xrightarrow{I_{t}} T$$

$$R_{si} \qquad R_{cr} \qquad R_{d} \qquad T_{s}$$

$$I_{si} \qquad I_{cr} \qquad U_{cr}$$

Figure 11: Bond graph for the i.s.g. free-wheeling

#### 4. SIMULATION RESULTS

The developed model is implemented as a module in a powertrain simulation tool called, "AUTOTOOL". For detailed simulation reseults, see [11].

### 5. CONCLUSION

In this paper, a mathematical model of an automatic transmission was developed using the bond graph method. The dynamic and kinematic constraints, as well as the governing differential equations of motions for the 1-4 in-gear ranges, upshifts, and downshifts were developed. The addition of proper dynamic and kinematic constraints allows the correct determination of in-gear ranges and shift conditions. The shift command influences only the pressures of each clutch, and the proper range of the automatic transmission is determined by the dynamic and kinematic constraints internally in a simulation. This allows dynamically-correct simulations of automatic transmissions. The developed model can be used in the design of a hydraulic pressure controller,

intelligent cruise controller, and vehicle platooning controller.

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