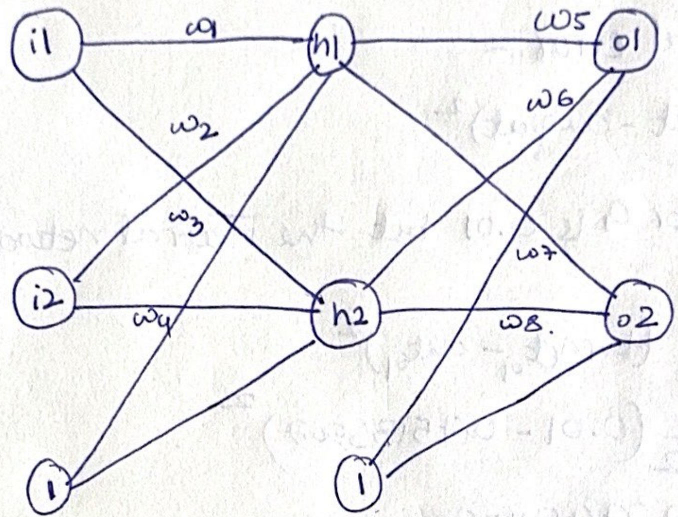
Back Propagation using Gradient Descent



Step 1:- The forward pass:-

→ The total net input for h_1

$$net_{h_1} = w_1 * i_1 + w_2 * i_2 + b_1 * 1$$

$$net_{h_1} = (0.15 \times 0.05) + (0.2 \times 0.1) + (0.35 \times 1)$$

$$= 0.3775$$

→ Using logistic function

$$out_{h_1} = \frac{1}{1+e^{-net_{h_1}}} = \frac{1}{1+e^{-0.3775}} = 0.593269992$$

→ For h_2:-

$$out_{h_2} = 0.596884378$$

→ Output for o_1:-

$$net_{o_1} = w_5 * out_{h_1} + w_6 * out_{h_2} + b_2 * 1$$

$$net_{o_1} = (0.4 \times 0.593269992) + (0.45 \times 0.596884378) + (0.6 \times 1)$$

$$= 1.105905967$$

$$out_{o_1} = \frac{1}{1+e^{-net_{o_1}}} = 0.75136507$$

For O_2 :-

$out_{o_2} = 0.772928465$.

→ Calculating the total error :-

$$E_{total} = \sum \frac{1}{2} (target - output)^2$$

→ The target output for o_1 is 0.01 but the neural network output 0.75136507,

$$\therefore \text{Error } E_{o1} = \frac{1}{2} (target_{o1} - out_{o1})^2$$

$$= \frac{1}{2} (0.01 - 0.75136507)^2$$

$$= 0.274811083.$$

$$\text{Error } E_{o_2} = 0.023560026.$$

$$E_{Total} = E_{o1} + E_{o2}$$

$$= 0.298371109.$$

Step 2: The Backward Pass

$$\frac{\partial E_{Total}}{\partial w_5} = \frac{\partial E_{total}}{\partial out_{o1}} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial w_5}.$$

$$\frac{\partial E_{total}}{\partial out_{o1}} = 2 * \frac{1}{2} (target_{o1} - out_{o1})^{2-1} * -1 + 0$$

$$\frac{\partial E_{total}}{\partial out_{o1}} = -(target_{o1} - out_{o1})$$

$$= -(0.01 - 0.75136507)$$

$$= 0.74136507.$$

$$\frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial out_{o1}} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial w_5}$$

$$= 0.7413607 \times 0.186815602 \times 0.59326992$$

$$= 0.082167041$$

→ To decrease the error, we then subtract this value from the current weight.

$$w_5^+ = w_5 - \eta * \frac{\partial E_{total}}{\partial w_5}$$

$= 0.4 - 0.5 \times 0.082167041$

$= 0.35891648$

→ For other weights

$w_6^+ = 0.408666186$

$w_7^+ = 0.511301270$

$w_8^+ = 0.561370121$

step 3: Hidden layer.

$$\frac{\partial E_{o1}}{\partial net_{o1}} = \frac{\partial E_{o1}}{\partial out_{o1}} \times \frac{\partial out_{o1}}{\partial net_{o1}} = 0.74136507 \times 0.186815602$$

$= 0.138498562$

And $\frac{\partial net_{o1}}{\partial out_H} = w_5$

$net_{o1} = w_5 \times out_{h1} + w_6 \times out_{h2} + b_2 * 1$

$$\frac{\partial net_{o1}}{\partial out_{h1}} = w_5 = 0.40$$

$$\Rightarrow \frac{\partial E_{o1}}{\partial out_{h1}} = \frac{\partial E_{o1}}{\partial net_{o1}} \times \frac{\partial net_{o1}}{\partial out_{h1}} = 0.138498562 \times 0.40$$

$= 0.055399425$

$$\rightarrow \frac{\partial E_{o2}}{\partial out_{h1}} = 0.019049119$$

$$\therefore \cancel{\frac{\partial E_{total}}{\partial out_{h1}} = \frac{\partial E_{o1}}{\partial out_{h1}} + \frac{\partial E_{o2}}{}}$$

For $\frac{\partial E_{o2}}{\partial out_{h1}} = -0.019049119$

$$\therefore \frac{\partial E_{total}}{\partial out_{h1}} = \frac{\partial E_{o1}}{\partial out_{h1}} + \frac{\partial E_{o2}}{\partial out_{h1}}$$

$$= 0.055399425 + (-0.019049119)$$

$$= 0.036350306$$

$\Rightarrow$ $out_{h1} = \frac{1}{1+e^{-net_{h1}}}$

$$\frac{\partial out_{h1}}{\partial net_{h1}} = out_{h1}(1 - out_{h1}) = 0.59326999(1 - 0.59326999)$$

$$= 0.241300709$$

$\rightarrow$ $net_{h1} = w_1 * i_1 + w_3 * i_2 + b_1 * 1.$

$$\frac{\partial net_{h1}}{\partial w_1} = i_1 = 0.05.$$

$$\therefore \frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out_{h1}} \times \frac{\partial out_{h1}}{\partial net_{h1}} \times \frac{\partial net_{h1}}{\partial w_1}.$$

$$\frac{\partial E_{total}}{\partial w} = 0.000438568$$

$$\therefore w_1^+ = w_1 - \eta \frac{\partial E_{total}}{\partial w} = 0.149780716.$$

$w_2^+ = 0.19956143$

$w_3^+ = 0.24975114$

$w_4^+ = 0.29950229.$