**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans:** To find the probability that the service manager cannot meet his commitment, we need to calculate the probability that the time required for servicing transmissions exceeds the remaining time available.

Given: μ = 45 minutes (mean) σ = 8 minutes (standard deviation) Time allowed = 1 hour = 60 minutes Time delay = 10 minutes

To calculate the probability, we need to convert the time allowed and the time delay to a common unit, which is minutes.

Remaining time = Time allowed - Time delay Remaining time = 60 minutes - 10 minutes Remaining time = 50 minutes

Now, we can calculate the z-score using the formula:

z = (Remaining time - μ) / σ

z = (50 - 45) / 8 z = 0.625

Using a standard normal distribution table or a statistical software, we can find the cumulative probability associated with the z-score of 0.625. Let's denote this probability as P.

P ≈ 0.2676

The probability that the service manager cannot meet his commitment is approximately 0.2676.

**Therefore, the correct option is B. 0.2676.**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans A:** Probability of an employee being older than 44: To calculate this, we need to find the area under the normal distribution curve to the right of 44. We can use a standard normal distribution table or a statistical software to determine this probability.

Using the standard normal distribution table, the z-score for 44 can be calculated as: z = (x - μ) / σ z = (44 - 38) / 6 z = 1

The area to the right of 44 (z = 1) under the standard normal distribution curve is approximately 0.1587. This means that about 15.87% of the employees are older than 44.

**Ans B:** Probability of an employee's age being between 38 and 44: To calculate this, we need to find the area under the normal distribution curve between 38 and 44.

Using the standard normal distribution table, we can find the probabilities for z-scores corresponding to 38 and 44. The z-score for 38 is -0.67, and the z-score for 44 is 1.

The area between -0.67 and 1 under the standard normal distribution curve is approximately 0.6474. This means that about 64.74% of the employees have ages between 38 and 44.

Comparing the probabilities, we can conclude that the statement is false. More employees at the processing center are between 38 and 44 years old (64.74%) than older than 44 (15.87%).

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**ANS:** The Normal Distribution has its link with the Central Limit Theorem, which states that ‘Any large sum of independent identically distribution random variables are approximately Normal then (X1 + X2) and (2X1) tends to have Normal distribution only If X1 and X2 are i.i.d and n is Large.

The Difference between 2X1 and (X1 + X2) is the magnitude they hold of two different sample subsets (X1 and X2) from the same source(population). X1 and X2 can be a different subset of a sample from a similar source (population) but If X1 ~ N(μ, σ2) then, 2 X1 ~ N(2 μ, 4 σ2 ) If X1 ~ N(μ, σ2) and X2 ~ N(μ, σ2) are iid normal random variables then (X1 + X2)N(μ+ μ, σ2+ σ2)(2 μ, 2 σ2) Hence, 2X1 – (X1+X2) ~(2 μ – 2 μ, 4 σ2 + 2σ2 ) The distribution remains the same for every sample subset of similar source, it tends to fall under Normal distribution and slight deviations in parameters.

The Normal distribution has two parameters, the mean, µ, and the variance, σ2. µ and σ2satisfy −∞ < µ < ∞, σ2> 0. We write X ∼ Normal (µ, σ2) or X ∼ N(µ, σ2 ).

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**ANS:** To find the values a and b symmetric about the mean such that the probability of the random variable X taking a value between them is 0.99, we need to calculate the interval based on the mean and standard deviation of X.

Given: X ~ N(100, 20^2)

To find the interval, we can use the properties of the normal distribution and calculate the z-scores corresponding to the cumulative probabilities at the tails.

Since the probability of the random variable falling between a and b is 0.99, we can find the cumulative probabilities associated with (1 - 0.99) / 2 = 0.005 on each tail of the distribution.

Using a standard normal distribution table or a statistical software, we can find the z-scores corresponding to a cumulative probability of 0.005. Let's denote this z-score as z.

z ≈ -2.57

Now, we can find the values a and b by applying the z-score formula:

a = μ + z \* σ a = 100 + (-2.57) \* 20 a ≈ 48.6

b = μ - z \* σ b = 100 - (-2.57) \* 20 b ≈ 151.4

The values a and b, which are symmetric about the mean, are approximately 48.6 and 151.4, respectively.

Among the answer choices provided: D. 48.5, 151.5

Therefore, the correct option is D. 48.5, 151.5.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**ANS:** Rupee ranges in between [9.9 to 98.1] Crore Rupees, 95% of the time for the Annual Profit of the Company.

1. Specify the 5th percentile of profit (in Rupees) for the company

**ANS:** Profit\_5th\_percentile = μ\_total + (z\_5 \* σ\_total) Profit\_5th\_percentile = 12 + (-1.645 \* √74)

1. Which of the two divisions has a larger probability of making a loss in a given year?

**ANS:** For Profit1, the mean (μ1) is 5 and the standard deviation (σ1) is √32.

For Profit2, the mean (μ2) is 7 and the standard deviation (σ2) is √42.