Assignment 02

Bayesian models & data analysis

2025-01-28

0.1 Part 1: Probability

In an experiment, two coins a and b are tossed simultaneously. Suppose the probability of getting heads on the coin a is 0.6 and the probability of getting heads on the coin b is 0.4.

1. What is the probability of getting heads on both the coins a and b?

(Hint: The outcomes of the coin a and b are independent events, i.e., the outcome of coin a does not affect the outcome of coin b. Suppose H_a represents the event that coin a gives heads and H_b represents the event coin b gives heads. You already know $P(H_a)$ and $P(H_b)$. You can calculate the probability of joint occurrence events H_a and H_b: the probability of getting heads on both the coins as follows: $P(H_aH_b) = P(H_a) \times P(H_b)$.)

2. What is the sample space of the experiment? (What are all the possible outcomes that can occur when the coins a and b are tossed?)
3. Calculate the probability of occurrence of each outcome in the sample space.

(Hint: Since the events are independent, you can compute the joint probability for each outcome using the information given in the problem.) 4. Consider two events A and B in the sample space of the experiment.

A: heads appear on either coin a or coin b or both
B: heads appear only on coin a

- What is the probability of occurrence of event A?
- What is the probability of occurrence of event B?
- What is joint probability of occurrence of the events A and B?

(The event of joint occurrence of A and B will be $A \cap B$).

0.2 Part 2: Discrete random variables

5. In a visual word recognition experiment, a participant has to recognize whether the word shown on screen is a meaningful word (e.g., "book") or a non-word (e.g., "bktr"). The participant is asked to answer "yes" if the word is meaningful, and "no" if it is a non-word. Suppose a participant is shown 50 words, what is the probability that the participant will correctly recognize 45 out of 50 words? You are given that the probability of correctly recognizing a word is 0.9.

(Hint: Use the binomial distribution, $f(k, n, p) = \frac{n!}{k!(n-k)!}p^k(1-p)^{n-k}$. In this question, you are given $k = 45$, $n = 50$, $p = 0.9$.)

6. Suppose in a small city, 10 road accidents happen on average in a single day. The probability of k number of road accidents in a day is given by the probability mass function $f(k, \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$ where $\lambda = 10$. What is the probability of zero road accidents in a day?