Day 3: Generalized Linear Models

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Probability distributions

Logistic regression model

Example. Modeling the chances of getting A+ grade in the exam

Let me introduce you a student whose name is K.

K is very sincere in studies and tries everything to get A+ grade in the exams. She gives around 5 exams in a month and has been studying for last 200 months. K is disappointed today, because she got B grade in a recent exam.

K has decided to analyze his past experiences what has helped her to get A+ in the past.

K wants to model the distribution of getting grade A+ or less in the exams as a function of number of daily study hours, number of daily hours spent on twitter and number of books read in each month.

Here is the history of her grades.

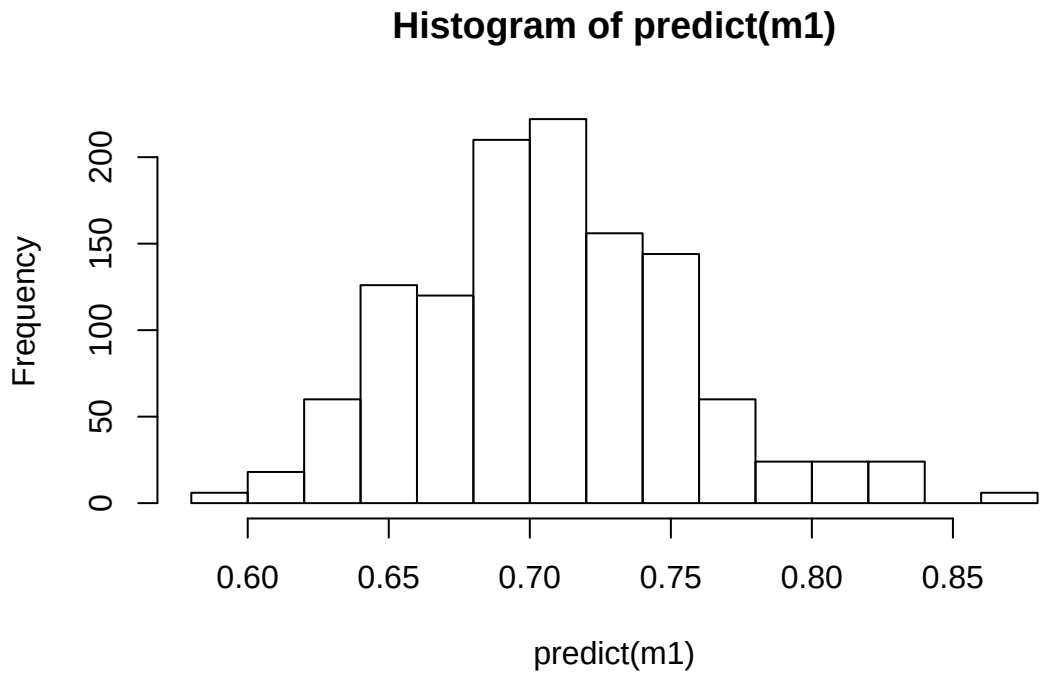
```
head(K.exams)
```

```
##   month_id year studyHours books  twitter grades
## 1        1 2005          4     0 3.255306      0
## 2        1 2005          4     0 3.255306      0
## 3        1 2005          4     0 3.255306      1
## 4        1 2005          4     0 3.255306      0
## 5        1 2005          4     0 3.255306      1
## 6        1 2005          4     0 3.255306      1
```

```
#grades=1 means K got A+ grade, grades=0 means K could not get A+

#I know true parameters, alpha=2, beta1=0.08,beta2=0.002,beta3= -0.5

#Let's see a simple linear regression can recover the parameters
m1 <- lm(grades~studyHours+books+twitter,data=K.exams)
m1
```

```
##
## Call:
## lm(formula = grades ~ studyHours + books + twitter, data = K.exams)
##
## Coefficients:
## (Intercept)   studyHours        books      twitter
##    0.944212     0.004319     0.002116    -0.088396
```

```r
# The model generates incorrect estimates

# Because 'grades' are not normally distributed

# Let's check the model predictions
hist(predict(m1))
```




```r
# Model generates invalid predictions

# Fit a logistic regression model instead
m2 <- glm(grades~studyHours+books+twitter,data=K.exams,family = binomial(link="logit"))
m2
```

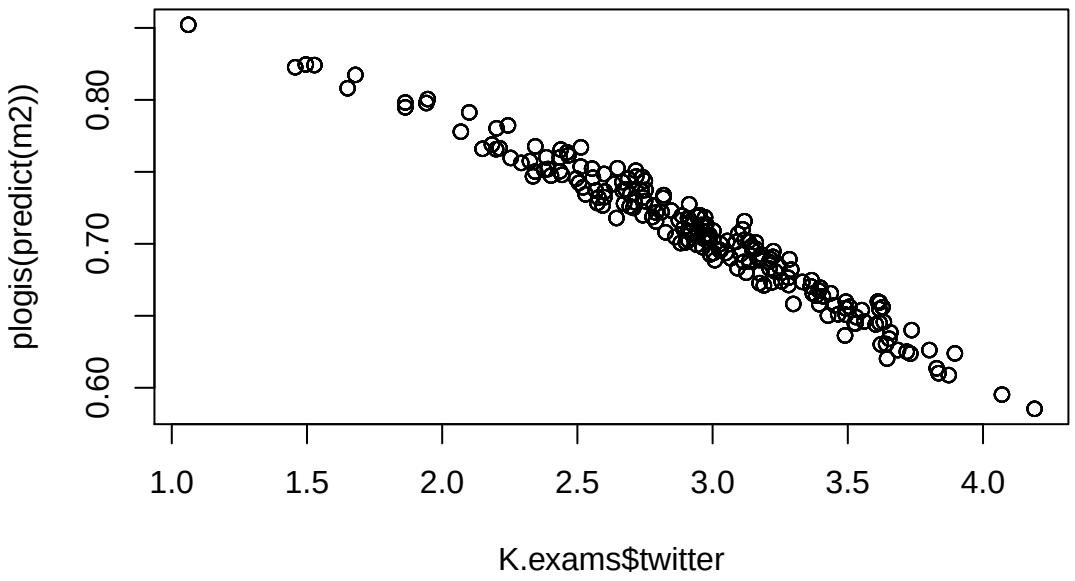
```
##
## Call:  glm(formula = grades ~ studyHours + books + twitter, family = binomial(link =
##     data = K.exams)
```

```
##
## Coefficients:
## (Intercept)   studyHours        books      twitter
##     2.08566      0.02089      0.01116     -0.44333
##
## Degrees of Freedom: 1199 Total (i.e. Null);  1196 Residual
## Null Deviance:       1450
## Residual Deviance: 1437  AIC: 1445
```

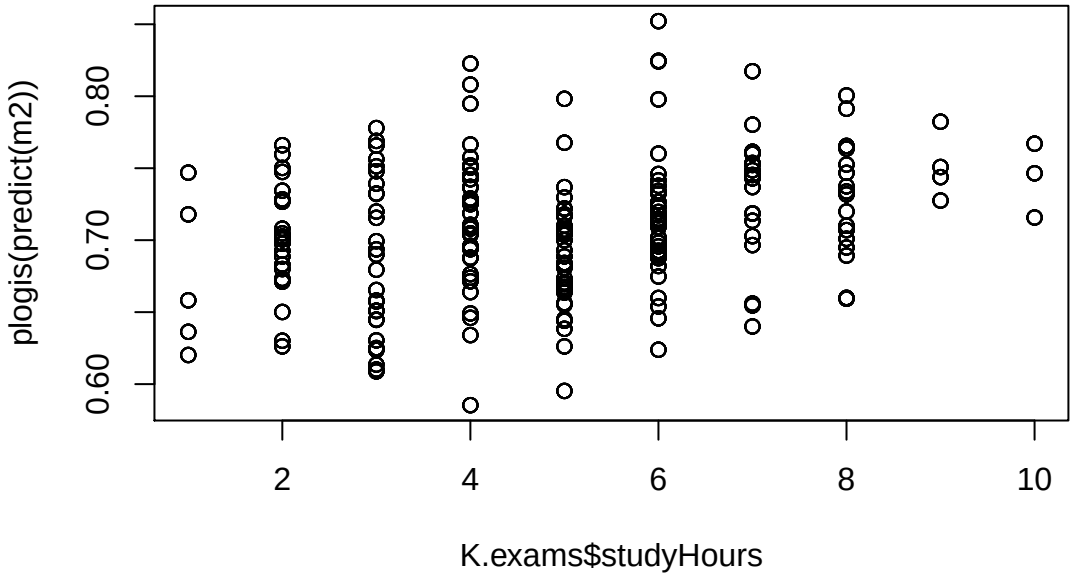
```
summary(m2)
```

```
##
## Call:
## glm(formula = grades ~ studyHours + books + twitter, family = binomial(link = "logit"
##     data = K.exams)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.9554  -1.4472   0.7814   0.8553   1.0349
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  2.08566    0.43446   4.801 1.58e-06 ***
## studyHours   0.02089    0.03172   0.659 0.510200
## books        0.01116    0.07588   0.147 0.883052
## twitter     -0.44333    0.12770  -3.472 0.000517 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 1450.5  on 1199  degrees of freedom
## Residual deviance: 1437.0  on 1196  degrees of freedom
## AIC: 1445
##
## Number of Fisher Scoring iterations: 4
```

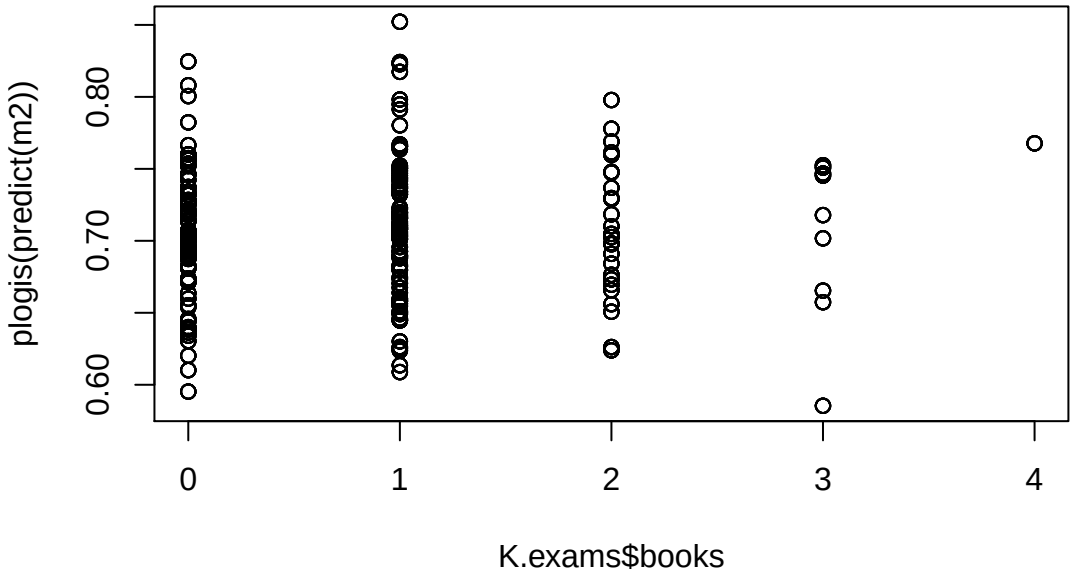
```
# The model generates correct estimates

#Predictions
plot(K.exams$twitter, plogis(predict(m2)))
```




```
plot(K.exams$studyHours, plogis(predict(m2)))
```



```r
plot(K.exams$books, plogis(predict(m2)))
```



Mixed-effect logistic regression

Let $grades_{i,j}$ indicates whether K got A+ in a exam that happened in ith month that belongs to jth year,

$logit(grades_{i,j}) = (\alpha + u_j) + \beta_1 S_i + \beta_2 B_i + \beta_3 T_i + \epsilon_{i,j}$

$u_j \sim Normal(0, \sigma_u^2)$

```r
K.exams
```

```
##       month_id year studyHours books  twitter grades
## 1            1 2005          4     0 3.255306      0
## 2            1 2005          4     0 3.255306      0
## 3            1 2005          4     0 3.255306      1
## 4            1 2005          4     0 3.255306      0
## 5            1 2005          4     0 3.255306      1
## 6            1 2005          4     0 3.255306      1
## 7            2 2005         10     0 2.740096      1
## 8            2 2005         10     0 2.740096      1
## 9            2 2005         10     0 2.740096      1
## 10           2 2005         10     0 2.740096      0
```

```
##  11          2 2005          10      0 2.740096       1
##  12          2 2005          10      0 2.740096       1
##  13          3 2005           3      2 2.183897       1
##  14          3 2005           3      2 2.183897       0
##  15          3 2005           3      2 2.183897       1
##  16          3 2005           3      2 2.183897       1
##  17          3 2005           3      2 2.183897       1
##  18          3 2005           3      2 2.183897       1
##  19          4 2005           6      0 3.175874       0
##  20          4 2005           6      0 3.175874       1
##  21          4 2005           6      0 3.175874       1
##  22          4 2005           6      0 3.175874       1
##  23          4 2005           6      0 3.175874       0
##  24          4 2005           6      0 3.175874       1
##  25          5 2005           7      1 2.201050       1
##  26          5 2005           7      1 2.201050       0
##  27          5 2005           7      1 2.201050       1
##  28          5 2005           7      1 2.201050       0
##  29          5 2005           7      1 2.201050       0
##  30          5 2005           7      1 2.201050       0
##  31          6 2005           3      0 3.835551       1
##  32          6 2005           3      0 3.835551       1
##  33          6 2005           3      0 3.835551       1
##  34          6 2005           3      0 3.835551       1
##  35          6 2005           3      0 3.835551       1
##  36          6 2005           3      0 3.835551       1
##  37          7 2005           4      0 2.706841       1
##  38          7 2005           4      0 2.706841       0
##  39          7 2005           4      0 2.706841       1
##  40          7 2005           4      0 2.706841       1
##  41          7 2005           4      0 2.706841       1
##  42          7 2005           4      0 2.706841       0
##  43          8 2005           4      2 2.980095       1
##  44          8 2005           4      2 2.980095       1
##  45          8 2005           4      2 2.980095       1
##  46          8 2005           4      2 2.980095       1
##  47          8 2005           4      2 2.980095       0
##  48          8 2005           4      2 2.980095       1
##  49          9 2005           5      4 2.344215       1
##  50          9 2005           5      4 2.344215       1
##  51          9 2005           5      4 2.344215       1
##  52          9 2005           5      4 2.344215       0
##  53          9 2005           5      4 2.344215       1
##  54          9 2005           5      4 2.344215       1
##  55         10 2005           3      1 3.873360       1
##  56         10 2005           3      1 3.873360       1
```

```
## 57          10 2005          3     1 3.873360      1
## 58          10 2005          3     1 3.873360      1
## 59          10 2005          3     1 3.873360      1
## 60          10 2005          3     1 3.873360      0
## 61          11 2005          5     1 3.617550      1
## 62          11 2005          5     1 3.617550      0
## 63          11 2005          5     1 3.617550      1
## 64          11 2005          5     1 3.617550      1
## 65          11 2005          5     1 3.617550      1
## 66          11 2005          5     1 3.617550      1
## 67          12 2005          4     1 1.456345      1
## 68          12 2005          4     1 1.456345      0
## 69          12 2005          4     1 1.456345      1
## 70          12 2005          4     1 1.456345      0
## 71          12 2005          4     1 1.456345      1
## 72          12 2005          4     1 1.456345      0
## 73          13 2006          5     1 3.331862      1
## 74          13 2006          5     1 3.331862      1
## 75          13 2006          5     1 3.331862      1
## 76          13 2006          5     1 3.331862      1
## 77          13 2006          5     1 3.331862      0
## 78          13 2006          5     1 3.331862      1
## 79          14 2006          3     2 3.492623      1
## 80          14 2006          3     2 3.492623      0
## 81          14 2006          3     2 3.492623      1
## 82          14 2006          3     2 3.492623      0
## 83          14 2006          3     2 3.492623      0
## 84          14 2006          3     2 3.492623      0
## 85          15 2006          6     0 3.551657      0
## 86          15 2006          6     0 3.551657      1
## 87          15 2006          6     0 3.551657      1
## 88          15 2006          6     0 3.551657      1
## 89          15 2006          6     0 3.551657      0
## 90          15 2006          6     0 3.551657      1
## 91          16 2006          2     1 3.426616      1
## 92          16 2006          2     1 3.426616      1
## 93          16 2006          2     1 3.426616      0
## 94          16 2006          2     1 3.426616      0
## 95          16 2006          2     1 3.426616      1
## 96          16 2006          2     1 3.426616      0
## 97          17 2006          7     1 2.736608      0
## 98          17 2006          7     1 2.736608      1
## 99          17 2006          7     1 2.736608      1
## 100         17 2006          7     1 2.736608      1
## 101         17 2006          7     1 2.736608      1
## 102         17 2006          7     1 2.736608      0
```

```
## 103        18 2006          3     3 3.451710      1
## 104        18 2006          3     3 3.451710      1
## 105        18 2006          3     3 3.451710      0
## 106        18 2006          3     3 3.451710      1
## 107        18 2006          3     3 3.451710      1
## 108        18 2006          3     3 3.451710      1
## 109        19 2006          7     0 2.976139      0
## 110        19 2006          7     0 2.976139      0
## 111        19 2006          7     0 2.976139      0
## 112        19 2006          7     0 2.976139      1
## 113        19 2006          7     0 2.976139      1
## 114        19 2006          7     0 2.976139      0
## 115        20 2006          2     1 2.991009      1
## 116        20 2006          2     1 2.991009      0
## 117        20 2006          2     1 2.991009      1
## 118        20 2006          2     1 2.991009      1
## 119        20 2006          2     1 2.991009      0
## 120        20 2006          2     1 2.991009      1
## 121        21 2006          3     0 2.292431      1
## 122        21 2006          3     0 2.292431      0
## 123        21 2006          3     0 2.292431      1
## 124        21 2006          3     0 2.292431      1
## 125        21 2006          3     0 2.292431      1
## 126        21 2006          3     0 2.292431      1
## 127        22 2006          5     2 2.667878      1
## 128        22 2006          5     2 2.667878      1
## 129        22 2006          5     2 2.667878      0
## 130        22 2006          5     2 2.667878      1
## 131        22 2006          5     2 2.667878      0
## 132        22 2006          5     2 2.667878      0
## 133        23 2006          2     2 2.253453      1
## 134        23 2006          2     2 2.253453      1
## 135        23 2006          2     2 2.253453      1
## 136        23 2006          2     2 2.253453      1
## 137        23 2006          2     2 2.253453      1
## 138        23 2006          2     2 2.253453      1
## 139        24 2006          4     1 3.135685      1
## 140        24 2006          4     1 3.135685      1
## 141        24 2006          4     1 3.135685      1
## 142        24 2006          4     1 3.135685      1
## 143        24 2006          4     1 3.135685      1
## 144        24 2006          4     1 3.135685      0
## 145        25 2007          8     0 3.094756      1
## 146        25 2007          8     0 3.094756      1
## 147        25 2007          8     0 3.094756      0
## 148        25 2007          8     0 3.094756      0
```

```
## 149        25 2007          8     0 3.094756      1
## 150        25 2007          8     0 3.094756      1
## 151        26 2007          5     0 2.983304      1
## 152        26 2007          5     0 2.983304      1
## 153        26 2007          5     0 2.983304      0
## 154        26 2007          5     0 2.983304      1
## 155        26 2007          5     0 2.983304      1
## 156        26 2007          5     0 2.983304      0
## 157        27 2007          2     2 2.963188      1
## 158        27 2007          2     2 2.963188      1
## 159        27 2007          2     2 2.963188      1
## 160        27 2007          2     2 2.963188      0
## 161        27 2007          2     2 2.963188      0
## 162        27 2007          2     2 2.963188      1
## 163        28 2007          2     2 3.685463      1
## 164        28 2007          2     2 3.685463      0
## 165        28 2007          2     2 3.685463      1
## 166        28 2007          2     2 3.685463      1
## 167        28 2007          2     2 3.685463      1
## 168        28 2007          2     2 3.685463      1
## 169        29 2007          2     2 2.402211      1
## 170        29 2007          2     2 2.402211      1
## 171        29 2007          2     2 2.402211      0
## 172        29 2007          2     2 2.402211      1
## 173        29 2007          2     2 2.402211      1
## 174        29 2007          2     2 2.402211      1
## 175        30 2007          8     1 2.462087      1
## 176        30 2007          8     1 2.462087      1
## 177        30 2007          8     1 2.462087      1
## 178        30 2007          8     1 2.462087      1
## 179        30 2007          8     1 2.462087      1
## 180        30 2007          8     1 2.462087      0
## 181        31 2007          6     1 2.973480      1
## 182        31 2007          6     1 2.973480      1
## 183        31 2007          6     1 2.973480      1
## 184        31 2007          6     1 2.973480      1
## 185        31 2007          6     1 2.973480      1
## 186        31 2007          6     1 2.973480      1
## 187        32 2007          5     0 3.491676      1
## 188        32 2007          5     0 3.491676      1
## 189        32 2007          5     0 3.491676      0
## 190        32 2007          5     0 3.491676      1
## 191        32 2007          5     0 3.491676      0
## 192        32 2007          5     0 3.491676      1
## 193        33 2007          6     1 2.634795      1
## 194        33 2007          6     1 2.634795      1
```

```
## 195        33 2007          6     1 2.634795      1
## 196        33 2007          6     1 2.634795      0
## 197        33 2007          6     1 2.634795      1
## 198        33 2007          6     1 2.634795      1
## 199        34 2007          4     0 3.024169      1
## 200        34 2007          4     0 3.024169      1
## 201        34 2007          4     0 3.024169      1
## 202        34 2007          4     0 3.024169      1
## 203        34 2007          4     0 3.024169      1
## 204        34 2007          4     0 3.024169      0
## 205        35 2007          2     1 2.343508      1
## 206        35 2007          2     1 2.343508      1
## 207        35 2007          2     1 2.343508      0
## 208        35 2007          2     1 2.343508      0
## 209        35 2007          2     1 2.343508      1
## 210        35 2007          2     1 2.343508      0
## 211        36 2007          6     2 3.222877      1
## 212        36 2007          6     2 3.222877      0
## 213        36 2007          6     2 3.222877      1
## 214        36 2007          6     2 3.222877      1
## 215        36 2007          6     2 3.222877      1
## 216        36 2007          6     2 3.222877      1
## 217        37 2008          5     0 4.070054      1
## 218        37 2008          5     0 4.070054      1
## 219        37 2008          5     0 4.070054      0
## 220        37 2008          5     0 4.070054      0
## 221        37 2008          5     0 4.070054      0
## 222        37 2008          5     0 4.070054      1
## 223        38 2008          6     3 3.132805      1
## 224        38 2008          6     3 3.132805      1
## 225        38 2008          6     3 3.132805      1
## 226        38 2008          6     3 3.132805      0
## 227        38 2008          6     3 3.132805      1
## 228        38 2008          6     3 3.132805      1
## 229        39 2008          4     0 2.391273      0
## 230        39 2008          4     0 2.391273      1
## 231        39 2008          4     0 2.391273      1
## 232        39 2008          4     0 2.391273      1
## 233        39 2008          4     0 2.391273      1
## 234        39 2008          4     0 2.391273      1
## 235        40 2008          4     1 2.910382      0
## 236        40 2008          4     1 2.910382      1
## 237        40 2008          4     1 2.910382      1
## 238        40 2008          4     1 2.910382      1
## 239        40 2008          4     1 2.910382      0
## 240        40 2008          4     1 2.910382      1
```

```
## 241        41 2008          8     1 2.819067      1
## 242        41 2008          8     1 2.819067      1
## 243        41 2008          8     1 2.819067      0
## 244        41 2008          8     1 2.819067      0
## 245        41 2008          8     1 2.819067      1
## 246        41 2008          8     1 2.819067      1
## 247        42 2008          8     0 3.284072      0
## 248        42 2008          8     0 3.284072      0
## 249        42 2008          8     0 3.284072      0
## 250        42 2008          8     0 3.284072      1
## 251        42 2008          8     0 3.284072      1
## 252        42 2008          8     0 3.284072      1
## 253        43 2008          7     2 3.628767      1
## 254        43 2008          7     2 3.628767      0
## 255        43 2008          7     2 3.628767      0
## 256        43 2008          7     2 3.628767      0
## 257        43 2008          7     2 3.628767      1
## 258        43 2008          7     2 3.628767      1
## 259        44 2008          3     1 2.599938      1
## 260        44 2008          3     1 2.599938      1
## 261        44 2008          3     1 2.599938      1
## 262        44 2008          3     1 2.599938      1
## 263        44 2008          3     1 2.599938      1
## 264        44 2008          3     1 2.599938      0
## 265        45 2008          8     1 2.438376      1
## 266        45 2008          8     1 2.438376      1
## 267        45 2008          8     1 2.438376      1
## 268        45 2008          8     1 2.438376      0
## 269        45 2008          8     1 2.438376      1
## 270        45 2008          8     1 2.438376      1
## 271        46 2008          3     1 3.066708      1
## 272        46 2008          3     1 3.066708      1
## 273        46 2008          3     1 3.066708      1
## 274        46 2008          3     1 3.066708      0
## 275        46 2008          3     1 3.066708      0
## 276        46 2008          3     1 3.066708      1
## 277        47 2008          2     1 3.622049      0
## 278        47 2008          2     1 3.622049      1
## 279        47 2008          2     1 3.622049      1
## 280        47 2008          2     1 3.622049      0
## 281        47 2008          2     1 3.622049      1
## 282        47 2008          2     1 3.622049      1
## 283        48 2008          6     0 2.557316      1
## 284        48 2008          6     0 2.557316      0
## 285        48 2008          6     0 2.557316      1
## 286        48 2008          6     0 2.557316      0
```

```
## 287        48 2008          6     0 2.557316      1
## 288        48 2008          6     0 2.557316      1
## 289        49 2009          7     1 3.118421      0
## 290        49 2009          7     1 3.118421      0
## 291        49 2009          7     1 3.118421      1
## 292        49 2009          7     1 3.118421      1
## 293        49 2009          7     1 3.118421      0
## 294        49 2009          7     1 3.118421      0
## 295        50 2009          9     0 2.913429      0
## 296        50 2009          9     0 2.913429      0
## 297        50 2009          9     0 2.913429      1
## 298        50 2009          9     0 2.913429      1
## 299        50 2009          9     0 2.913429      1
## 300        50 2009          9     0 2.913429      1
## 301        51 2009          8     0 2.817432      1
## 302        51 2009          8     0 2.817432      1
## 303        51 2009          8     0 2.817432      1
## 304        51 2009          8     0 2.817432      0
## 305        51 2009          8     0 2.817432      1
## 306        51 2009          8     0 2.817432      1
## 307        52 2009          2     0 3.189734      0
## 308        52 2009          2     0 3.189734      0
## 309        52 2009          2     0 3.189734      0
## 310        52 2009          2     0 3.189734      1
## 311        52 2009          2     0 3.189734      1
## 312        52 2009          2     0 3.189734      1
## 313        53 2009          6     0 1.495756      1
## 314        53 2009          6     0 1.495756      1
## 315        53 2009          6     0 1.495756      1
## 316        53 2009          6     0 1.495756      1
## 317        53 2009          6     0 1.495756      1
## 318        53 2009          6     0 1.495756      1
## 319        54 2009          3     2 2.443186      1
## 320        54 2009          3     2 2.443186      0
## 321        54 2009          3     2 2.443186      1
## 322        54 2009          3     2 2.443186      1
## 323        54 2009          3     2 2.443186      1
## 324        54 2009          3     2 2.443186      1
## 325        55 2009          4     0 2.568361      0
## 326        55 2009          4     0 2.568361      0
## 327        55 2009          4     0 2.568361      0
## 328        55 2009          4     0 2.568361      1
## 329        55 2009          4     0 2.568361      1
## 330        55 2009          4     0 2.568361      1
## 331        56 2009          5     0 3.232275      1
## 332        56 2009          5     0 3.232275      0
```

```
## 333        56 2009          5     0 3.232275      1
## 334        56 2009          5     0 3.232275      0
## 335        56 2009          5     0 3.232275      1
## 336        56 2009          5     0 3.232275      1
## 337        57 2009          4     1 1.863524      1
## 338        57 2009          4     1 1.863524      1
## 339        57 2009          4     1 1.863524      1
## 340        57 2009          4     1 1.863524      1
## 341        57 2009          4     1 1.863524      1
## 342        57 2009          4     1 1.863524      1
## 343        58 2009          4     0 2.777525      1
## 344        58 2009          4     0 2.777525      0
## 345        58 2009          4     0 2.777525      1
## 346        58 2009          4     0 2.777525      0
## 347        58 2009          4     0 2.777525      0
## 348        58 2009          4     0 2.777525      1
## 349        59 2009          6     1 2.958924      1
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## 799       134 2016          7     1 1.679056      0
## 800       134 2016          7     1 1.679056      1
## 801       134 2016          7     1 1.679056      1
## 802       134 2016          7     1 1.679056      1
## 803       134 2016          7     1 1.679056      1
## 804       134 2016          7     1 1.679056      1
## 805       135 2016          4     1 3.382669      0
## 806       135 2016          4     1 3.382669      1
## 807       135 2016          4     1 3.382669      1
## 808       135 2016          4     1 3.382669      0
## 809       135 2016          4     1 3.382669      1
## 810       135 2016          4     1 3.382669      1
## 811       136 2016          2     2 3.218813      1
## 812       136 2016          2     2 3.218813      0
## 813       136 2016          2     2 3.218813      0
## 814       136 2016          2     2 3.218813      1
## 815       136 2016          2     2 3.218813      1
## 816       136 2016          2     2 3.218813      1
## 817       137 2016          7     0 3.736091      0
## 818       137 2016          7     0 3.736091      0
## 819       137 2016          7     0 3.736091      1
## 820       137 2016          7     0 3.736091      1
## 821       137 2016          7     0 3.736091      0
## 822       137 2016          7     0 3.736091      1
## 823       138 2016          5     1 3.506386      0
## 824       138 2016          5     1 3.506386      1
## 825       138 2016          5     1 3.506386      0
## 826       138 2016          5     1 3.506386      1
## 827       138 2016          5     1 3.506386      1
## 828       138 2016          5     1 3.506386      1
## 829       139 2016          8     3 2.717433      1
## 830       139 2016          8     3 2.717433      1
## 831       139 2016          8     3 2.717433      1
## 832       139 2016          8     3 2.717433      1
## 833       139 2016          8     3 2.717433      1
## 834       139 2016          8     3 2.717433      1
## 835       140 2016          7     0 3.159712      1
## 836       140 2016          7     0 3.159712      0
## 837       140 2016          7     0 3.159712      1
## 838       140 2016          7     0 3.159712      1
```

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## 839      140 2016          7     0 3.159712      0
## 840      140 2016          7     0 3.159712      1
## 841      141 2016          5     0 2.949493      0
## 842      141 2016          5     0 2.949493      1
## 843      141 2016          5     0 2.949493      1
## 844      141 2016          5     0 2.949493      0
## 845      141 2016          5     0 2.949493      1
## 846      141 2016          5     0 2.949493      1
## 847      142 2016          2     0 2.881669      0
## 848      142 2016          2     0 2.881669      1
## 849      142 2016          2     0 2.881669      1
## 850      142 2016          2     0 2.881669      1
## 851      142 2016          2     0 2.881669      0
## 852      142 2016          2     0 2.881669      1
## 853      143 2016          6     0 2.385297      1
## 854      143 2016          6     0 2.385297      1
## 855      143 2016          6     0 2.385297      1
## 856      143 2016          6     0 2.385297      0
## 857      143 2016          6     0 2.385297      1
## 858      143 2016          6     0 2.385297      1
## 859      144 2016          8     3 2.648295      1
## 860      144 2016          8     3 2.648295      0
## 861      144 2016          8     3 2.648295      1
## 862      144 2016          8     3 2.648295      0
## 863      144 2016          8     3 2.648295      1
## 864      144 2016          8     3 2.648295      1
## 865      145 2017          8     1 2.100192      1
## 866      145 2017          8     1 2.100192      1
## 867      145 2017          8     1 2.100192      1
## 868      145 2017          8     1 2.100192      0
## 869      145 2017          8     1 2.100192      0
## 870      145 2017          8     1 2.100192      1
## 871      146 2017          5     1 2.875236      1
## 872      146 2017          5     1 2.875236      1
## 873      146 2017          5     1 2.875236      1
## 874      146 2017          5     1 2.875236      0
## 875      146 2017          5     1 2.875236      1
## 876      146 2017          5     1 2.875236      1
## 877      147 2017          3     0 2.577860      0
## 878      147 2017          3     0 2.577860      1
## 879      147 2017          3     0 2.577860      1
## 880      147 2017          3     0 2.577860      0
## 881      147 2017          3     0 2.577860      0
## 882      147 2017          3     0 2.577860      1
## 883      148 2017          6     0 2.782600      0
## 884      148 2017          6     0 2.782600      1
```

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## 885       148 2017          6     0 2.782600      1
## 886       148 2017          6     0 2.782600      0
## 887       148 2017          6     0 2.782600      1
## 888       148 2017          6     0 2.782600      1
## 889       149 2017          3     0 3.002640      1
## 890       149 2017          3     0 3.002640      1
## 891       149 2017          3     0 3.002640      1
## 892       149 2017          3     0 3.002640      1
## 893       149 2017          3     0 3.002640      1
## 894       149 2017          3     0 3.002640      0
## 895       150 2017          3     1 2.741895      1
## 896       150 2017          3     1 2.741895      1
## 897       150 2017          3     1 2.741895      0
## 898       150 2017          3     1 2.741895      1
## 899       150 2017          3     1 2.741895      0
## 900       150 2017          3     1 2.741895      1
## 901       151 2017          5     0 2.989072      1
## 902       151 2017          5     0 2.989072      1
## 903       151 2017          5     0 2.989072      0
## 904       151 2017          5     0 2.989072      1
## 905       151 2017          5     0 2.989072      1
## 906       151 2017          5     0 2.989072      1
## 907       152 2017          5     0 3.407852      0
## 908       152 2017          5     0 3.407852      1
## 909       152 2017          5     0 3.407852      0
## 910       152 2017          5     0 3.407852      1
## 911       152 2017          5     0 3.407852      1
## 912       152 2017          5     0 3.407852      1
## 913       153 2017          3     1 3.718179      0
## 914       153 2017          3     1 3.718179      0
## 915       153 2017          3     1 3.718179      1
## 916       153 2017          3     1 3.718179      1
## 917       153 2017          3     1 3.718179      0
## 918       153 2017          3     1 3.718179      1
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## 921       154 2017          5     1 2.988084      1
## 922       154 2017          5     1 2.988084      0
## 923       154 2017          5     1 2.988084      1
## 924       154 2017          5     1 2.988084      1
## 925       155 2017          3     1 2.520534      1
## 926       155 2017          3     1 2.520534      1
## 927       155 2017          3     1 2.520534      0
## 928       155 2017          3     1 2.520534      1
## 929       155 2017          3     1 2.520534      1
## 930       155 2017          3     1 2.520534      1
```

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## 931       156 2017          5     2 2.750234      1
## 932       156 2017          5     2 2.750234      1
## 933       156 2017          5     2 2.750234      1
## 934       156 2017          5     2 2.750234      0
## 935       156 2017          5     2 2.750234      1
## 936       156 2017          5     2 2.750234      1
## 937       157 2018          5     0 3.657715      0
## 938       157 2018          5     0 3.657715      1
## 939       157 2018          5     0 3.657715      1
## 940       157 2018          5     0 3.657715      0
## 941       157 2018          5     0 3.657715      1
## 942       157 2018          5     0 3.657715      0
## 943       158 2018          1     0 2.644481      0
## 944       158 2018          1     0 2.644481      0
## 945       158 2018          1     0 2.644481      1
## 946       158 2018          1     0 2.644481      0
## 947       158 2018          1     0 2.644481      1
## 948       158 2018          1     0 2.644481      0
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## 950       159 2018          4     1 2.495928      1
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## 952       159 2018          4     1 2.495928      0
## 953       159 2018          4     1 2.495928      1
## 954       159 2018          4     1 2.495928      1
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## 956       160 2018          5     1 2.811650      1
## 957       160 2018          5     1 2.811650      1
## 958       160 2018          5     1 2.811650      1
## 959       160 2018          5     1 2.811650      1
## 960       160 2018          5     1 2.811650      0
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## 962       161 2018          8     0 3.159778      0
## 963       161 2018          8     0 3.159778      1
## 964       161 2018          8     0 3.159778      1
## 965       161 2018          8     0 3.159778      1
## 966       161 2018          8     0 3.159778      1
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## 968       162 2018          4     1 3.560940      1
## 969       162 2018          4     1 3.560940      1
## 970       162 2018          4     1 3.560940      1
## 971       162 2018          4     1 3.560940      1
## 972       162 2018          4     1 3.560940      1
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## 974       163 2018          5     0 3.210862      1
## 975       163 2018          5     0 3.210862      0
## 976       163 2018          5     0 3.210862      1
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## 978       163 2018          5     0 3.210862      1
## 979       164 2018          5     1 2.943828      0
## 980       164 2018          5     1 2.943828      0
## 981       164 2018          5     1 2.943828      1
## 982       164 2018          5     1 2.943828      1
## 983       164 2018          5     1 2.943828      1
## 984       164 2018          5     1 2.943828      1
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## 986       165 2018          5     2 3.398308      1
## 987       165 2018          5     2 3.398308      1
## 988       165 2018          5     2 3.398308      1
## 989       165 2018          5     2 3.398308      0
## 990       165 2018          5     2 3.398308      1
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## 992       166 2018          6     0 3.054539      1
## 993       166 2018          6     0 3.054539      0
## 994       166 2018          6     0 3.054539      1
## 995       166 2018          6     0 3.054539      0
## 996       166 2018          6     0 3.054539      1
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## 1005      168 2018          6     1 3.086478      0
## 1006      168 2018          6     1 3.086478      0
## 1007      168 2018          6     1 3.086478      1
## 1008      168 2018          6     1 3.086478      1
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## 1010      169 2019          6     0 2.925454      1
## 1011      169 2019          6     0 2.925454      1
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## 1013      169 2019          6     0 2.925454      0
## 1014      169 2019          6     0 2.925454      1
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## 1016      170 2019          1     0 3.489742      0
## 1017      170 2019          1     0 3.489742      1
## 1018      170 2019          1     0 3.489742      0
## 1019      170 2019          1     0 3.489742      1
## 1020      170 2019          1     0 3.489742      1
## 1021      171 2019          5     2 3.246917      1
## 1022      171 2019          5     2 3.246917      1
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## 1023      171 2019          5     2 3.246917      1
## 1024      171 2019          5     2 3.246917      1
## 1025      171 2019          5     2 3.246917      1
## 1026      171 2019          5     2 3.246917      0
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## 1028      172 2019          9     3 2.716166      0
## 1029      172 2019          9     3 2.716166      1
## 1030      172 2019          9     3 2.716166      0
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## 1032      172 2019          9     3 2.716166      1
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## 1034      173 2019          4     0 1.650161      1
## 1035      173 2019          4     0 1.650161      1
## 1036      173 2019          4     0 1.650161      1
## 1037      173 2019          4     0 1.650161      1
## 1038      173 2019          4     0 1.650161      1
## 1039      174 2019          2     1 3.091597      0
## 1040      174 2019          2     1 3.091597      1
## 1041      174 2019          2     1 3.091597      1
## 1042      174 2019          2     1 3.091597      1
## 1043      174 2019          2     1 3.091597      0
## 1044      174 2019          2     1 3.091597      1
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## 1046      175 2019          7     2 2.972007      0
## 1047      175 2019          7     2 2.972007      0
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## 1049      175 2019          7     2 2.972007      1
## 1050      175 2019          7     2 2.972007      1
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## 1052      176 2019          7     1 2.598704      1
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## 1054      176 2019          7     1 2.598704      1
## 1055      176 2019          7     1 2.598704      1
## 1056      176 2019          7     1 2.598704      1
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## 1058      177 2019          3     1 3.828493      0
## 1059      177 2019          3     1 3.828493      1
## 1060      177 2019          3     1 3.828493      1
## 1061      177 2019          3     1 3.828493      0
## 1062      177 2019          3     1 3.828493      1
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## 1064      178 2019         10     1 3.117980      1
## 1065      178 2019         10     1 3.117980      1
## 1066      178 2019         10     1 3.117980      1
## 1067      178 2019         10     1 3.117980      1
## 1068      178 2019         10     1 3.117980      1
```

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## 1069      179 2019          6     1 2.886421      0
## 1070      179 2019          6     1 2.886421      1
## 1071      179 2019          6     1 2.886421      0
## 1072      179 2019          6     1 2.886421      1
## 1073      179 2019          6     1 2.886421      0
## 1074      179 2019          6     1 2.886421      1
## 1075      180 2019          3     1 3.464294      1
## 1076      180 2019          3     1 3.464294      0
## 1077      180 2019          3     1 3.464294      1
## 1078      180 2019          3     1 3.464294      1
## 1079      180 2019          3     1 3.464294      1
## 1080      180 2019          3     1 3.464294      1
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## 1083      181 2020          8     1 3.611246      1
## 1084      181 2020          8     1 3.611246      1
## 1085      181 2020          8     1 3.611246      1
## 1086      181 2020          8     1 3.611246      1
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## 1089      182 2020          5     0 3.603279      1
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## 1092      182 2020          5     0 3.603279      1
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## 1095      183 2020          6     1 3.147590      0
## 1096      183 2020          6     1 3.147590      1
## 1097      183 2020          6     1 3.147590      1
## 1098      183 2020          6     1 3.147590      0
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## 1101      184 2020          6     2 3.144376      1
## 1102      184 2020          6     2 3.144376      0
## 1103      184 2020          6     2 3.144376      0
## 1104      184 2020          6     2 3.144376      1
## 1105      185 2020          6     0 2.912839      0
## 1106      185 2020          6     0 2.912839      1
## 1107      185 2020          6     0 2.912839      0
## 1108      185 2020          6     0 2.912839      1
## 1109      185 2020          6     0 2.912839      1
## 1110      185 2020          6     0 2.912839      1
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## 1113      186 2020          7     1 2.944383      1
## 1114      186 2020          7     1 2.944383      1
```

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## 1115      186 2020          7     1 2.944383      1
## 1116      186 2020          7     1 2.944383      0
## 1117      187 2020          7     0 2.434803      0
## 1118      187 2020          7     0 2.434803      0
## 1119      187 2020          7     0 2.434803      1
## 1120      187 2020          7     0 2.434803      1
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## 1122      187 2020          7     0 2.434803      1
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## 1124      188 2020          4     1 2.920022      1
## 1125      188 2020          4     1 2.920022      0
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## 1127      188 2020          4     1 2.920022      1
## 1128      188 2020          4     1 2.920022      1
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## 1132      189 2020          1     1 2.335769      0
## 1133      189 2020          1     1 2.335769      1
## 1134      189 2020          1     1 2.335769      0
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## 1136      190 2020          8     1 3.617684      1
## 1137      190 2020          8     1 3.617684      1
## 1138      190 2020          8     1 3.617684      1
## 1139      190 2020          8     1 3.617684      1
## 1140      190 2020          8     1 3.617684      1
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## 1143      191 2020          5     3 2.909173      0
## 1144      191 2020          5     3 2.909173      0
## 1145      191 2020          5     3 2.909173      0
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## 1149      192 2020          7     1 2.666103      0
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## 1152      192 2020          7     1 2.666103      1
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## 1154      193 2020          7     1 2.554312      1
## 1155      193 2020          7     1 2.554312      1
## 1156      193 2020          7     1 2.554312      1
## 1157      193 2020          7     1 2.554312      1
## 1158      193 2020          7     1 2.554312      0
## 1159      194 2020          4     2 2.711466      1
## 1160      194 2020          4     2 2.711466      1
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## 1161      194 2020          4     2 2.711466      0
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## 1164      194 2020          4     2 2.711466      1
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## 1167      195 2020          6     1 2.847638      0
## 1168      195 2020          6     1 2.847638      1
## 1169      195 2020          6     1 2.847638      1
## 1170      195 2020          6     1 2.847638      1
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## 1173      196 2020          4     0 2.212312      1
## 1174      196 2020          4     0 2.212312      1
## 1175      196 2020          4     0 2.212312      1
## 1176      196 2020          4     0 2.212312      0
## 1177      197 2020          3     1 3.176436      0
## 1178      197 2020          3     1 3.176436      1
## 1179      197 2020          3     1 3.176436      1
## 1180      197 2020          3     1 3.176436      1
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## 1182      197 2020          3     1 3.176436      0
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## 1187      198 2020          6     1 2.911608      1
## 1188      198 2020          6     1 2.911608      1
## 1189      199 2020          6     2 3.895949      0
## 1190      199 2020          6     2 3.895949      1
## 1191      199 2020          6     2 3.895949      1
## 1192      199 2020          6     2 3.895949      0
## 1193      199 2020          6     2 3.895949      0
## 1194      199 2020          6     2 3.895949      0
## 1195      200 2020          8     2 3.111133      1
## 1196      200 2020          8     2 3.111133      0
## 1197      200 2020          8     2 3.111133      1
## 1198      200 2020          8     2 3.111133      1
## 1199      200 2020          8     2 3.111133      1
## 1200      200 2020          8     2 3.111133      1
```

```
# Random intercept adjustment
library(lme4)
```

```
## Loading required package: Matrix
```

```
m3 <- glmer(grades~studyHours+books+twitter+(1|year),data=K.exams,family = binomial(link
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(m3)
```

```
## Generalized linear mixed model fit by maximum likelihood (Laplace
##   Approximation) [glmerMod]
##  Family: binomial  ( logit )
## Formula: grades ~ studyHours + books + twitter + (1 | year)
##    Data: K.exams
##
##      AIC      BIC   logLik deviance df.resid
##   1447.0   1472.4   -718.5   1437.0     1195
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -2.4010 -1.3601  0.5976  0.6645  0.8416
##
## Random effects:
##  Groups Name        Variance Std.Dev.
##  year   (Intercept) 1e-14    1e-07
## Number of obs: 1200, groups:  year, 16
##
## Fixed effects:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  2.08566    0.43445   4.801 1.58e-06 ***
## studyHours   0.02089    0.03172   0.659 0.510199
## books        0.01116    0.07588   0.147 0.883052
## twitter     -0.44333    0.12770  -3.472 0.000517 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##             (Intr) stdyHr books
## studyHours -0.439
## books      -0.154  0.016
## twitter    -0.912  0.097 -0.001
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
# Random intercept and slope adjustment
m3 <- glmer(grades~studyHours+books+twitter+(1+twitter|year),data=K.exams,family = binom
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(m3)
```

```
## Generalized linear mixed model fit by maximum likelihood (Laplace
##   Approximation) [glmerMod]
##  Family: binomial  ( logit )
## Formula: grades ~ studyHours + books + twitter + (1 + twitter | year)
##    Data: K.exams
##
##      AIC      BIC   logLik deviance df.resid
##   1448.3   1483.9   -717.1   1434.3     1193
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -2.2595 -1.3145  0.5964  0.6545  0.9574
##
## Random effects:
##  Groups Name        Variance Std.Dev. Corr
##  year   (Intercept) 1.1367   1.0662
##         twitter     0.1417   0.3764   -1.00
## Number of obs: 1200, groups:  year, 16
##
## Fixed effects:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  2.11492    0.52344   4.040 5.34e-05 ***
## studyHours   0.02403    0.03268   0.735  0.46207
## books        0.03491    0.07819   0.446  0.65524
## twitter     -0.46283    0.16234  -2.851  0.00436 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##             (Intr) stdyHr books
## studyHours -0.383
## books      -0.130  0.038
## twitter    -0.936  0.085 -0.008
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

Poisson regression model

```
covid <- read.table("COVID19data.csv",sep=",",header=T)
m4 <- glm(cases ~ month,data=subset(covid,cases>0),family=poisson())
summary(m4)
```

```
##
## Call:
## glm(formula = cases ~ month, family = poisson(), data = subset(covid,
##     cases > 0))
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
##  -91.86   -53.52   -36.19   -22.57  1184.15
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) 5.3343213  0.0004806   11099   <2e-16 ***
## month       0.2512666  0.0000515    4879   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 304741254  on 41990  degrees of freedom
## Residual deviance: 277589568  on 41989  degrees of freedom
## AIC: 277857472
##
## Number of Fisher Scoring iterations: 7
```

```
m5 <- glmer(cases ~ month+(month|country),data=subset(covid,cases>0),family=poisson())
summary(m5)
```

```
## Generalized linear mixed model fit by maximum likelihood (Laplace
##   Approximation) [glmerMod]
##  Family: poisson  ( log )
## Formula: cases ~ month + (month | country)
##    Data: subset(covid, cases > 0)
##
##       AIC       BIC    logLik  deviance  df.resid
##  30632692  30632735 -15316341  30632682     41986
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -221.79   -8.48   -2.12    4.22  765.29
##
## Random effects:
##  Groups  Name        Variance Std.Dev. Corr
##  country (Intercept) 6.02420  2.4544
##          month       0.07798  0.2792   -0.49
## Number of obs: 41991, groups:  country, 214
##
```

```
## Fixed effects:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  2.85431    0.12310   23.19   <2e-16 ***
## month        0.19072    0.01793   10.63   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##       (Intr)
## month -0.374
```

Mixed-effect poisson regression

Mixed-effect lognormal model