

Q  $\Rightarrow$  Given an unsorted array arr of length n. Also given a pivot element p. Rearrange the array such that all the elements less or than p are to the left of p & everything greater p are to the right of it.  
(Arrangement of left & right elements doesn't matter).

Ex  $[1, 6, 3, 8, 9, 2, 5]$   
          0 1 2 3 4 5 6

2 index  
 $p \Rightarrow 2$   
 $\rightarrow$  element  $\Rightarrow \underline{\underline{3}}$

$\rightarrow [1, 2, 3, 6, 8, 9, 5]$

arr[R] = pivot

Swap

Swap (i+1, R)

$L \rightarrow 0$   $R \rightarrow \underline{\underline{n-1}}$   
[ 1, 2, 3, 8, 9, 6, 5 ]  
↑ ↑  
i

pivot-index  $\rightarrow 2$   
pivot-element  $\rightarrow \underline{3}$

if (arr[j] <= pivot-element)  
    i++  
    Swap(arr, j, i);  
}

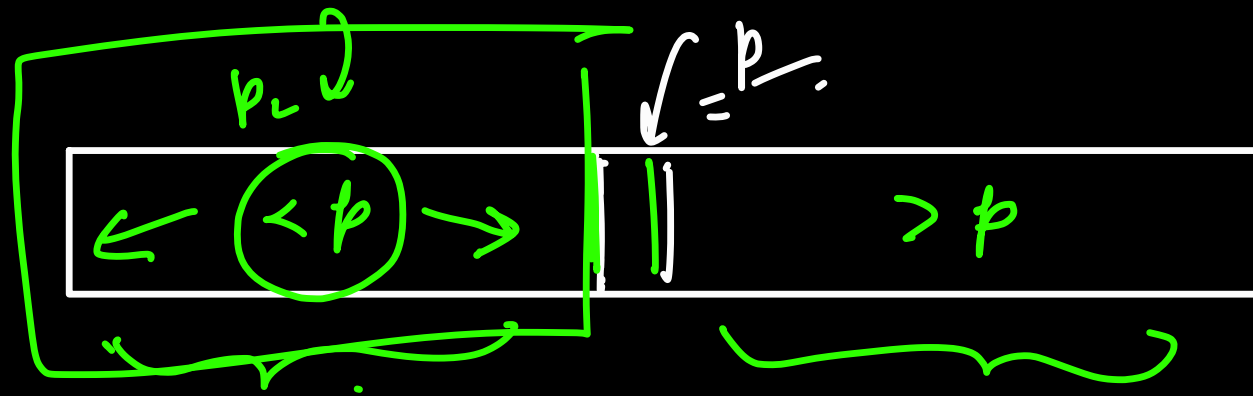
i  $\rightarrow$  everything to the left of i is less than pivot  
or equal to pivot

$L=0$   
 $[23, 9, 18, 32, 50, 61]$   
 $R=n-1$   
 $\uparrow$   
 $i$

pivot-index = 1  
pivot-element = 32

$O(n)$  time

$O(1)$  space



Quick Sort

2 3

1	3	4	6	8	9	10	13
---	---	---	---	---	---	----	----

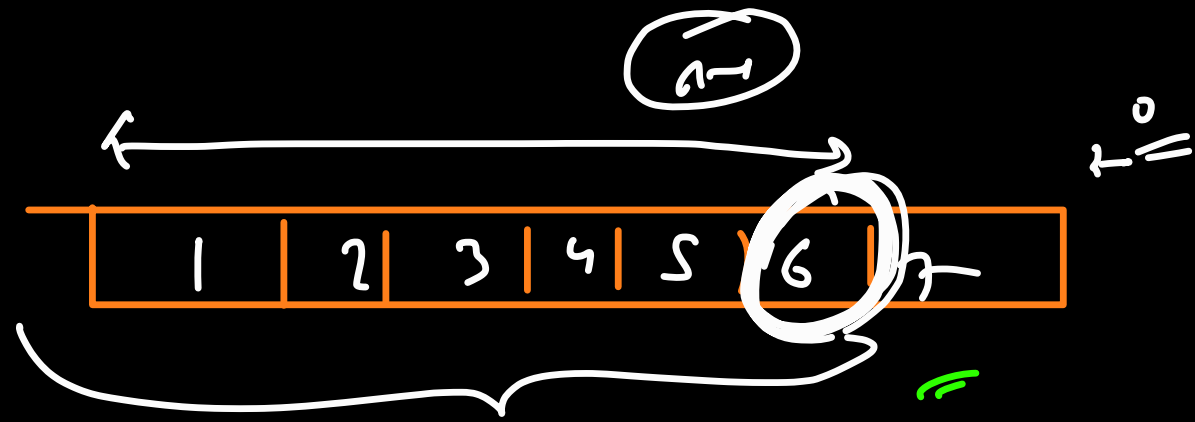
//

$f(arr, l, r) = \text{partition}(l, r, \text{pivotindex})$   
 $f(arr, l, \text{pivotindex} - 1)$   
 $f(arr, \text{pivotindex} + 1, r)$

↙  
Quicksort  
↘

↗ The index at which  
pivot resides  
often partitions

$O(n^2)$

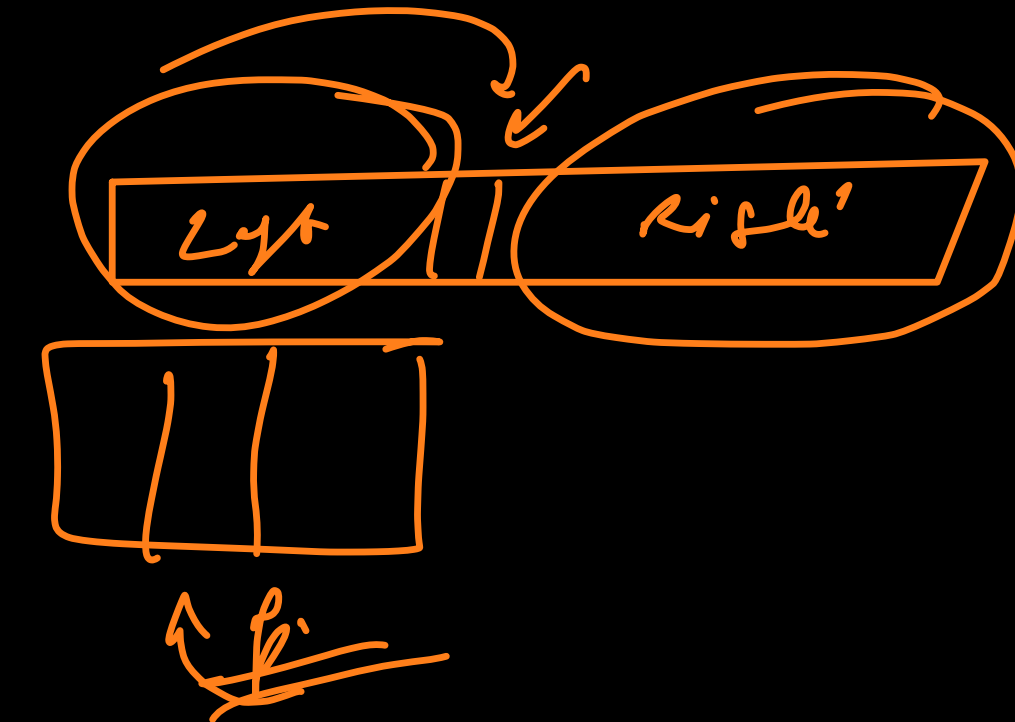
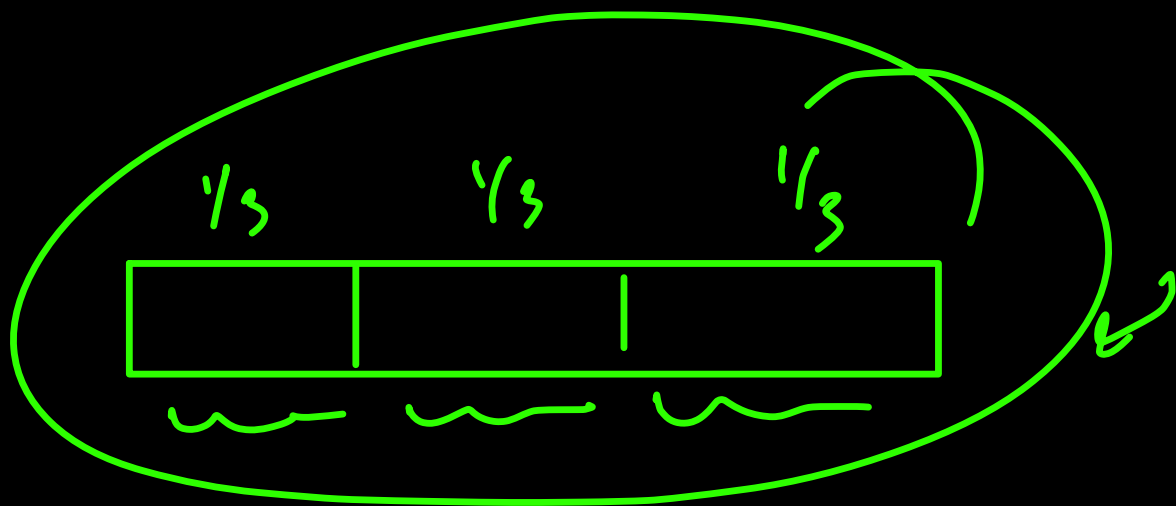


Randomized  
Quick sort

$O(n \log n)$

$n-1 + n-2 + n-3 + \dots$

$O(n^2)$



$$T(n) \overset{\substack{\rightarrow \text{last element} \\ \text{as pivot}}}{=} T(n-1) + T(0) + O(n)$$

↓  
no. of ops

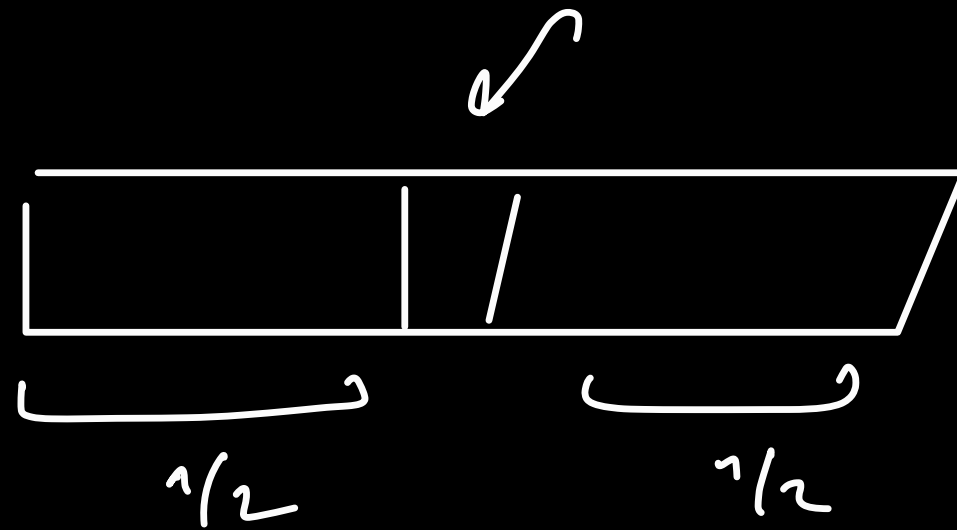
to do quick  
sort on  
a n length  
array →

$$\begin{aligned}
 T(n) &= \underbrace{T(n-1)} + O(n) \\
 &= \underbrace{T(n-2) + O(n)} + O(n) \\
 &= T(n-3) + O(n) + O(n) + O(n)
 \end{aligned}$$

⋮

$$\Rightarrow T(1) \longrightarrow \underline{O(n^2)}$$





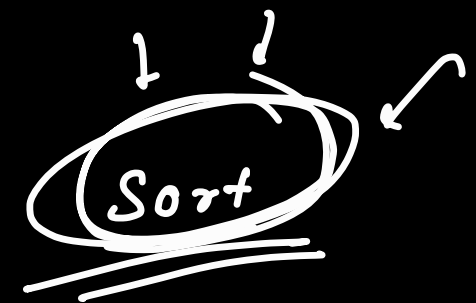
$$T(n) = T(n/2) + T(n/2) + O(n)$$

$$T(n) = 2T(n/2) + O(n)$$

Recursion

$$\underline{\underline{\Omega(n \log n)}}$$

	Merge	Quick
Time $\rightarrow$	$\Omega(n \log n)$	$\Omega(n \log n)$
	$\Theta(n \log n)$	$\Theta(n \log n)$
	$O(n \log n)$	$O(n^2)$



<u>Space</u>	$O(n)$	$O(\log n)$
In place $\rightarrow$	No	<u>Yes</u>
Stability	<u>Yes</u>	<u>No</u>