Properties of Fourier Transform

Fourier Transform: Fourier transform is the input tool that is used to decompose an image into its sine and cosine components.

Properties of Fourier Transform:

- **Linearity:**
  Addition of two functions corresponding to the addition of the two frequency spectrum is called the linearity. If we multiply a function by a constant, the Fourier transform of the resultant function is multiplied by the same constant. The Fourier transform of sum of two or more functions is the sum of the Fourier transforms of the functions.
- **Case I.**
- If h(x) -> H(f) then ah(x) -> aH(f)
- **Case II.**
  If h(x) -> H(f) and g(x) -> G(f) then h(x)+g(x) -> H(f)+G(f)
- **Scaling:**
  Scaling is the method that is used to the change the range of the independent variables or features of data. If we stretch a function by the factor in the time domain then squeeze the Fourier transform by the same factor in the frequency domain.
  If f(t) -> F(w) then f(at) -> (1/|a|)F(w/a)
- **Differentiation:**
  Differentiating function with respect to time yields to the constant multiple of the initial function.

  If f(t) -> F(w) then f'(t) -> jwF(w)
- **Convolution:**
  It includes the multiplication of two functions. The Fourier transform of a convolution of two functions is the point-wise product of their respective Fourier transforms.
- If f(t) -> F(w) and g(t) -> G(w)
  then f(t)*g(t) -> F(w)*G(w)
- **Frequency Shift:**
  Frequency is shifted according to the co-ordinates. There is a duality between the time and frequency domains and frequency shift affects the time shift.
  If f(t) -> F(w) then f(t)exp[jw't] -> F(w-w')
- **Time Shift:**
  The time variable shift also effects the frequency function. The time shifting property concludes that a linear displacement in time corresponds to a linear phase factor in the frequency domain.
  If f(t) -> F(w) then f(t-t') -> F(w)exp[-jwt']