LOGICAL DEDUCTION IN A

INFERENCING BY RESOLUTION REFUTATION

Predicate Logic

Wherever Mary goes, so does the lamb. Mary goes to school. So the lamb goes to school.

No contractors are dependable. Some engineers are contractors. Therefore some engineers are not dependable.

All dancers are graceful. Ayesha is a student. Ayesha is a dancer. Therefore some student is graceful.

Every passenger is either in first class or second class. Each passenger is in second class if and only if he or she is not wealthy. Some passengers are wealthy. Not all passengers are wealthy. Therefore some passengers are in second class.

New Additions in Proposition (First Order Logic)

Variables Constants Prodicate Symbols and

Variables, Constants, Predicate Symbols and

New Connectors: 3 (there exists), \(\forall\)

Wherever Mary goes, so does the Lamb. Mary goes to School. So the Lamb goes to School.

Predicate: goes(x,y) to represent x goes to y

New Connectors: **∃** (there exists), **∀**(for all)

F1: $\forall x (goes(Mary, x) \rightarrow goes(Lamb, x))$

F2: goes(Mary, School) ground instance

G: goes(Lamb, School) ~

To prove: (F1 \wedge F2) \rightarrow G) is always true

OLLAN ~

Inferencing in Predicate Logic

Domain: D

Constant Symbols: M, N, O, P,

Variable Symbols: x,y,z,....

Function Symbols: F(x), G(x,y),

H(x,y,z)

<u>Predicate Symbols</u>: p(x), q(x,y),

r(x,y,z),

Connectors: $^{\sim}$, $^{\wedge}$, $^{\vee}$, \rightarrow , $\stackrel{\triangle}{=}$, $\stackrel{\vee}{+}$

Terms:

Well-formed Formula:

Free and Bound Variables:

Interpretation, Valid, Non-Valid,

Satisfiable, Unsatisfiable

What is an Interpretation? Assign a domain set D, map constants, functions, predicates suitably. The formula will now

have a truth value

Example:

F1: $\forall x(g(M, x) \rightarrow g(L, x))$

F2: g(M, S)

G: g(L, S)

Interpretation 1: D = {Akash, Baby, Home, Play, Ratan, Swim},

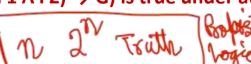
etc.,

Interpretation 2: D = Set of Integers, etc., INFINITE ATIONS
How many interpretations can there be?

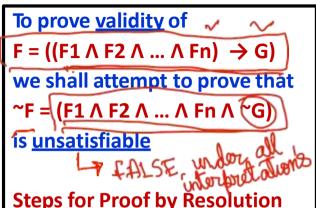
To prove <u>Validity</u>, means $(F1 \land F2) \rightarrow G)$ is true under all interpretations

To prove \$atisfiability means (F1 \land F2) \rightarrow G) is true under at

least one interpretation



Resolution Refutation for Propositional Logic



Steps for Proof by Resolution
Refutation: () () () ()

- 1. Convert of Clausal Form / Conjunctive Normal Form (CNF, Product of Sums).
- Generate new clauses using the resolution rule.
- 3. At the end, either Faise will be derived if the formula ^F is unsatisfiable implying

Fis valid.

If Asha is elected VP then Rajat is chosen as G-Sec and Bharati is chosen as Treasurer. Rajat is not chosen as G-Sec. Therefore Asha is not elected VP.

F1: (a → (b ∧ c)) = (~a ∨ b) ∧ (~a ∨ c)

Clauses of Clause Form: ~F
= (C1 \(\text{C2} \\ \text{C3} \\ \text{C4}\)

where: C1: (\(\text{~a V b} \)

C2: (\(\text{~a V c} \)

C3: \(\text{~b} \)

C4: a

To prove that \(\text{~F is False} \)

(Proof by showing that ((C1 π C2) \rightarrow C3) is a valid formula). \checkmark

To prove unsatisfiability use the Resolution Rule repeatedly to reach a situation where we have two contradictory clauses of the form C1 = a and $C2 = \sim a$ from which False can be derived.

If the propositional formula is satisfiable then we will not reach a contradiction and eventually no new clauses will be derivable.

For propositional logic the procedure terminates.

Resolution Rule is Sound and Complete

Applying Resolution Refutation

then a new clause C3 = b V c can be derived.

(Proof by showing that ((C1 \land C2) \rightarrow C3) is a valid formula).

To prove unsatisfiability use the Resolution Rule repeatedly to reach a situation where we have two contradictory clauses of the form C1 = a and C2 = ~a from which False can be derived.

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Resolution Rule is Sound and Complete

If Asha is elected VP then Rajat is chosen as G-Sec and Bharati is chosen as Treasurer. Rajat is not chosen as G-Sec. Therefore Asha is not elected VP. 6 (F1KF2) ->G F1: $(a \rightarrow (b \land c)) = (\sim a \lor b) \land (\sim a \lor c)$ F2: ~b ~ FINF2 NTG ~**G**: a Clauses of Clause Form: ~F **New Clauses Derived** = (C1 \wedge C2 \wedge C3 \wedge C4) **C5:** ~a (Using C1 and C3) where: C1: (~a) V b) ^ C6: False (using C4 and C5) C2: (~a V c) ^ C3: ~b < C4: a 🗸 To prove that ~F is False

Example

Let C1 = a V b and C2 = ~a V c then a new clause C3 = b V c can be derived.

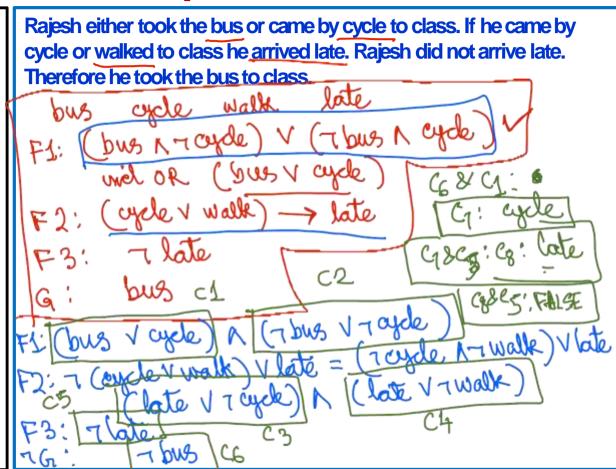
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Resolution Rule is Sound and Complete



Resolution Refutation for Predicate Logic

Given a formula F which we wish to check for validity, we first check if there are any free variables. We then quantify all free variables universally.

Create F' = ~F and check for unsatisfiability of F'

STEPS:

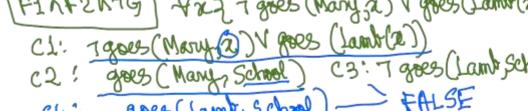
Conversion to Clausal (CNF) Form:

- Handling of Variables and Quantifiers, Ground Instances
- **Applying the Resolution Rule:**
- Concept of Unification
- Principle of Most General Unifier (mgu)
- Repeated application of Resolution Rule using mgu
- F1: \(\forall x(\text{goes(Mary, x)} \)\(\forall \text{goes(Lamb, x)}\)\(\forall \text{goes(Mary, School)}\)
- G: goes(Lamb, School)

To prove: (F1 \wedge F2) \rightarrow G) is valid

CONVERSION TO CLAUSAL FORM IN PREDICATE LOGIC

- Remove implications and other Boolean symbols converting to equivalent forms using , V, Λ
- Move negates (~) inwards as close as possible
- 3. Standardize (Rename) variables to make them unambiguous
- Remove Existential Quantifiers by an appropriate new function /constant symbol taking into account the variables dependent on the quantifier (Skolemization)
- 5. Drop Universal Quantifiers
- **6.** Distribute V over Λ and convert to CNF

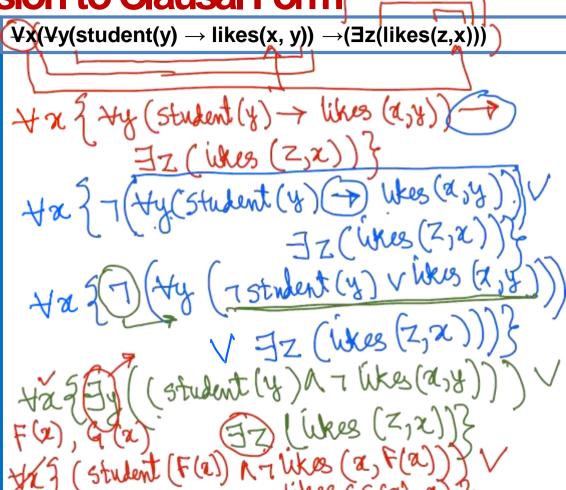


Conversion to Clausal Form

1. Remove implications and other Boolean symbols converting to

equivalent forms using ~, V, A

- 2. Move negates (~) inwards as close as possible
- 3. Standardize (Rename) variables to make them unambiguous
- 4. Remove Existential Quantifiers
 by an appropriate new function
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- 5. Drop Universal Quantifiers
- 6. Distribute V over Λ and convert to CNF



Substitution, Unification, Resolution

Consider clauses:

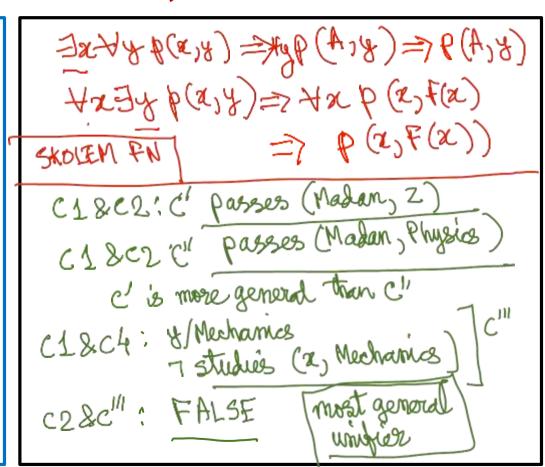
- C1: ~studies(x,y) V passes(x,y) ~
- C2: studies(Madan,z)
- C3: ~passes(Chetan, Physics) -
- C4: ~passes(w, Mechanics) /

What new clauses can we derive by the resolution principle?

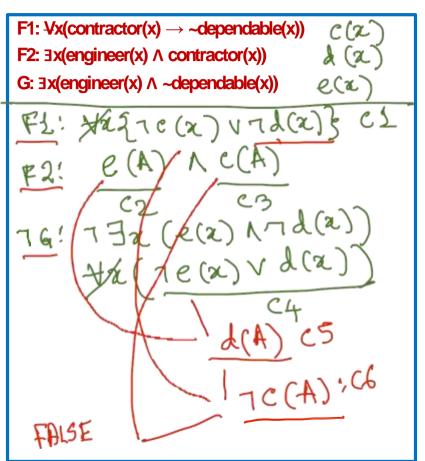
Ground Clause and a more general clause

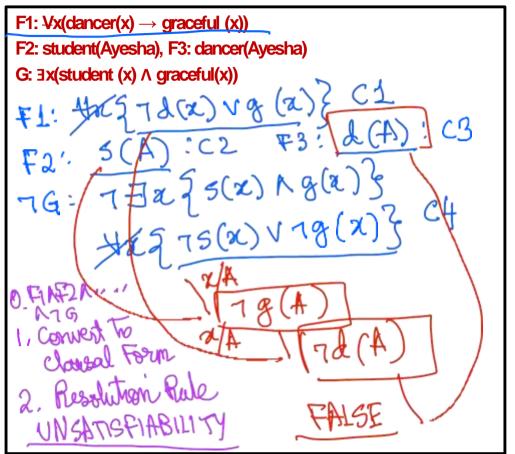
Concept of substitution / unification and the Most General Unifier (mgu)

Resolution Rule for Predicate
Calculus: Repeated Application of
Resolution using mgu



Examples





Thank you