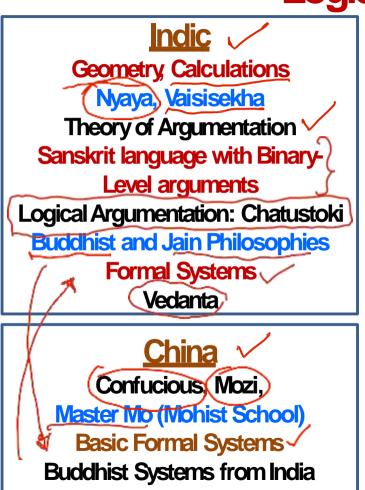
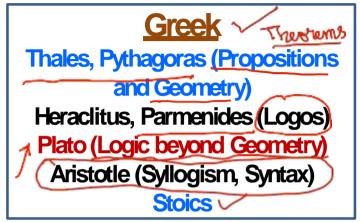
LOGICAL DEDUCTION IN A

PROPOSITIONAL LOGIC

Logic in Ancient Times





Middle East Ancient Egypt, Babylon

Arab (Avisennian Logic)

Inductive Logic

Medieval Europe

Post Aristotle

Precursor to First Order Logic

Today (**Propositional Predicate Higher Order** Logic, Numbers & **Computation Psychology** √ Philosophy Circuits

> Networks Brain/Newal Networks

PROPOSITIONAL

First Few Examples Propositional

- If I am the President then I am well-known. I am the President. So I am well-known
- If I am the President then I am well-known. I am not the President. So I am not well-known.
- If Rajat is the President then Rajat is well-known. Rajat is the President. So Rajat is well known.
- If Asha is elected VP then Rajat is chosen as G-Sec and Bharati is chosen as Treasurer. Rajat is not chosen as G-Sec. Therefore Asha is not elected VP.
- If Asha is elected VP then Rajat is chosen as G-Sec and Bharati is chosen as Treasurer. Rajat is chosen as G-Sec. Therefore Asha is elected VP.

Deduction Using Propositional Logic: Steps

Choice of Boolean Variables a, b, c, d, ...which can take values true or false

Boolean Formulae developed using well defined connectors \sim , \wedge , \vee , \rightarrow , etc, whose meaning (semantics) is given by their truth tables.

Codification of Sentences of the argument into Boolean Formulae.

Developing the <u>Deduction Process</u> as obtaining truth of a <u>Combined</u> Formula expressing the complete argument. ✓

<u>Determining the Truth</u> or Validity of the formula and thereby proving or disproving the argument and Analyzing its truth under various Interpretations.

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If I am the President then I am well-known. I am the President. So I am well-known Coding: Variables a: I am the President ~ **b**: I am well-known \checkmark Coding the sentences: F1: $a \rightarrow b$ $a \rightarrow b = 7avb$ F2: a ~ G: b 🗸 The final formula for deduction: (F1 \wedge F2) \rightarrow G, that is:

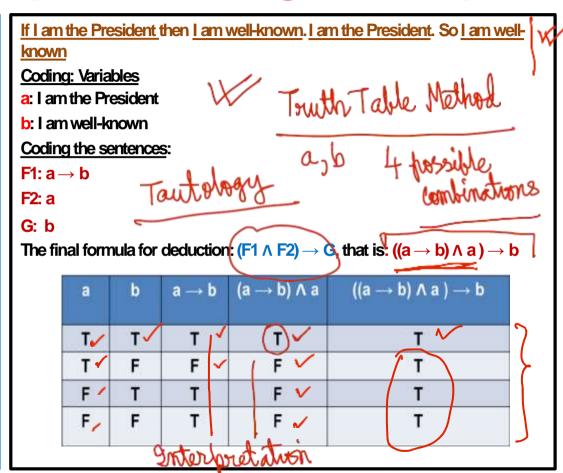
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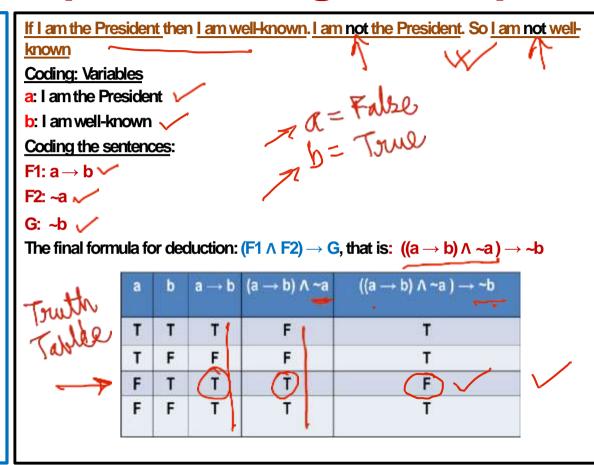
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<u>If I am the President</u> then <u>I am well-known</u>. <u>I am the President</u>. So <u>I am well-known</u>

Coding: Variables

a: I am the President

b: I am well-known

Coding the sentences:

F1: $a \rightarrow b$

F2: a

G: b

The final formula for deduction: (F1 \wedge F2) \rightarrow G, that is: ((a \rightarrow b) \wedge a) \rightarrow b

а	b	a → b	(a → b) ∧ a	$((a \to b) \land a) \to b$
Т	Т	Т	Т	Т
Т	F	F	F	T
F	Т	Т	F	T
F	F	Т	F	T

If <u>Rajat is the President</u> then <u>Rajat is well-known</u>. <u>Rajat is the President</u>. So <u>Rajat is well-known</u>

Coding: Variables

a: Rajat is the President \

b: Rajat is well-known 🍾

Coding the sentences:

F1: $a \rightarrow b \checkmark$

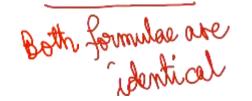
F2: a 🗸

G: b \

The final formula for deduction:

(F1 \wedge F2) \rightarrow G,

that is: $((a \rightarrow b) \land a) \rightarrow b$



If Asha is elected VP then Rajat is chosen as G-Sec and Bharati is chosen as Treasurer. Rajat is not chosen as G-Sec. Therefore Asha is not elected VP. a: Asha is elected VP b: Rajat is chosen G-Sec C. : Bharati is chosen Treasmon If Asha is elected VP then Rajat is chosen as G-Sec and Bharati is chosen as Treasurer. Rajat is chosen as G-Sec. Therefore Asha is elected VP. FINF2)is true under all interpretations or not.

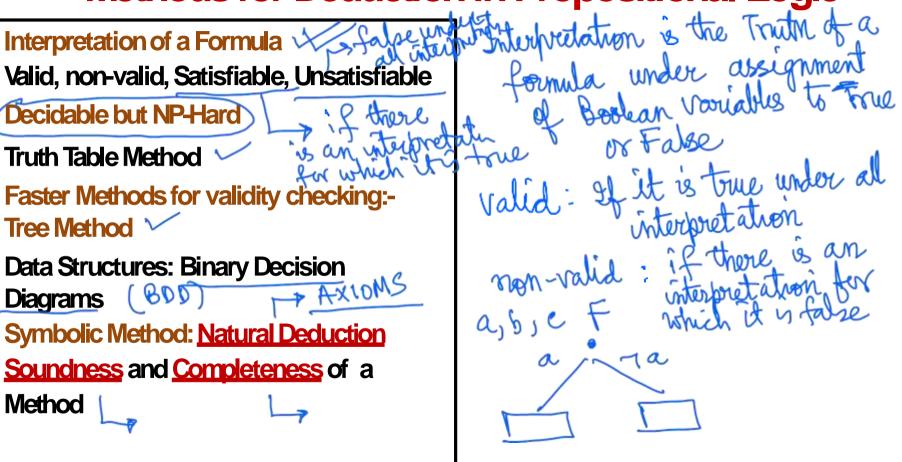
More Examples

If Asha is elected VP then Rajat is chosen as G-Sec or Bharati is chosen as Treasurer. Rajat is not chosen as G-Sec. Therefore if Asha is elected as VP then Bharati is chosen as Treasurer

a: Asha is elected VP 6. Rajat is chosen G-Sec C: Bharati is Chosen Treasurer [(F1 NF2) -> G]

If Asha is elected VP then either Rajat is chosen as G-Sec or Bharati is chosen as Treasurer but not both. Rajat is not chosen as G-Sec. Therefore if Asha is elected as VP then Bharati is chosen as Treasurer F1: [a -> ((b MTC) V (TbNC)) 6: (a>c) (F1KF2)-7G

Methods for Deduction in Propositional Logic



Methods for Deduction in Propositional Logic

Interpretation of a Formula

Valid, non-valid, Satisfiable, Unsatisfiable

Decidable but NP-Hard

Truth Table Method

Faster Methods for validity checking:-Tree Method

Data Structures: Binary Decision

Diagrams

Symbolic Method: Natural Deduction

Soundness and Completeness of a

Method

NATURAL DEDUCTION: Modus Ponens: (a \rightarrow b), a :- therefore b Modus Tollens: $(a \rightarrow b)$, ~b :- therefore ~a \checkmark Hypothetical Syllogism: (a \rightarrow b), (b \rightarrow c):therefore (a \rightarrow c) \checkmark Disjunctive Syllogism: (a V b), ~a:- therefore b Constructive Dilemma: $(a \rightarrow b) \land (c \rightarrow d)$, $(a \lor b) \land (c \rightarrow d)$ c):- therefore (b V d) Destructive Dilemma: (a \rightarrow b) \land (c \rightarrow d), (~b V ~d):-therefore (~a V ~c) Simplification: a \wedge b:- therefore a \checkmark Conjunction: a, b:- therefore a ∧ b Addition: a :- therefore a V b Natural Deduction is Sound and Complete

Insufficiency of Propositional Logic

Wherever Mary goes, so does the lamb. Mary goes to school. So the lamb goes to school.

No contractors are dependable. Some engineers are contractors. Therefore some engineers are not dependable.

All dancers are graceful. Ayesha is a student. Ayesha is a dancer. Therefore some student is graceful.

Every passenger is either in first class or second class. Each passenger is in second class if and only if he or she is not wealthy. Some passengers are wealthy. Not all passengers are wealthy. Therefore some passengers are in second class.

Thank you