CAUSAL INFERENCE IN STATISTICS

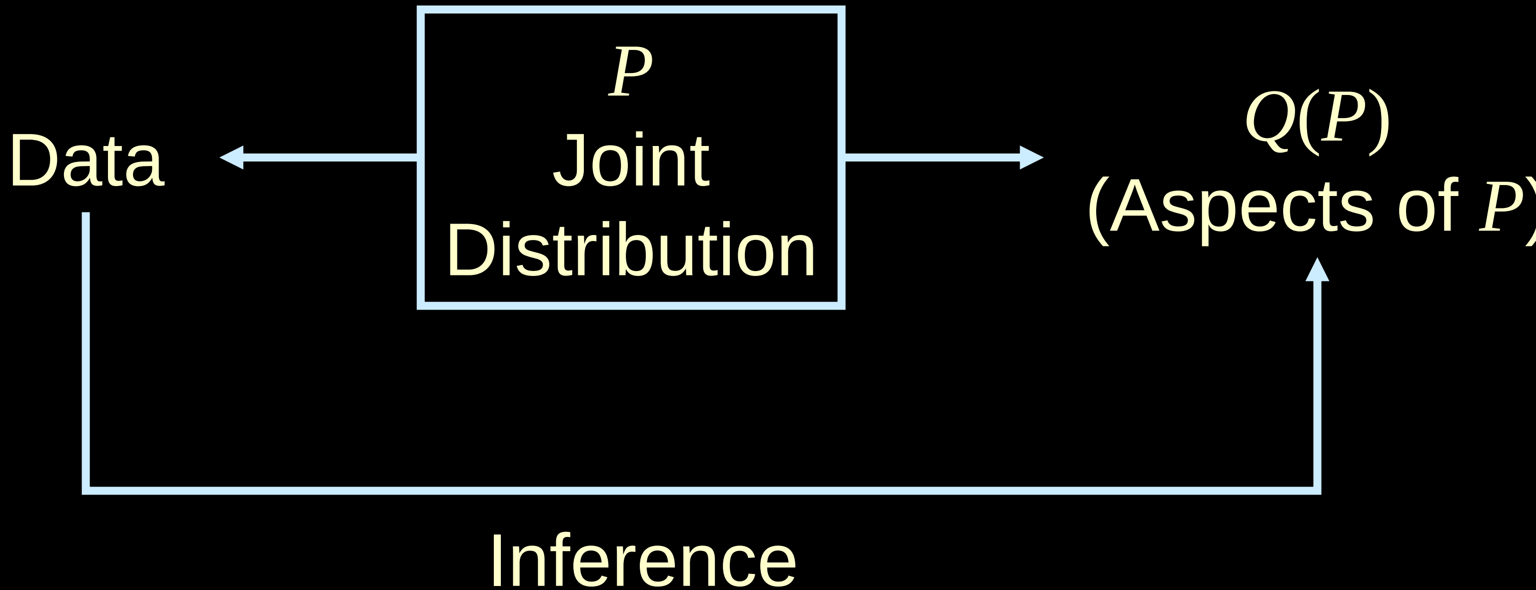
A Gentle Introduction

Judea Pearl
University of California
Los Angeles
(www.cs.ucla.edu/~judea/jsm12)

OUTLINE

- Inference: Statistical vs. Causal, distinctions, and mental barriers
- Unified conceptualization of counterfactuals, structural-equations, and graphs
- Inference to three types of claims:
  1. Causal effects and confounding
  2. Attribution (Causes of Effects)
  3. Direct and indirect effects
- Frills – external validity and transportability

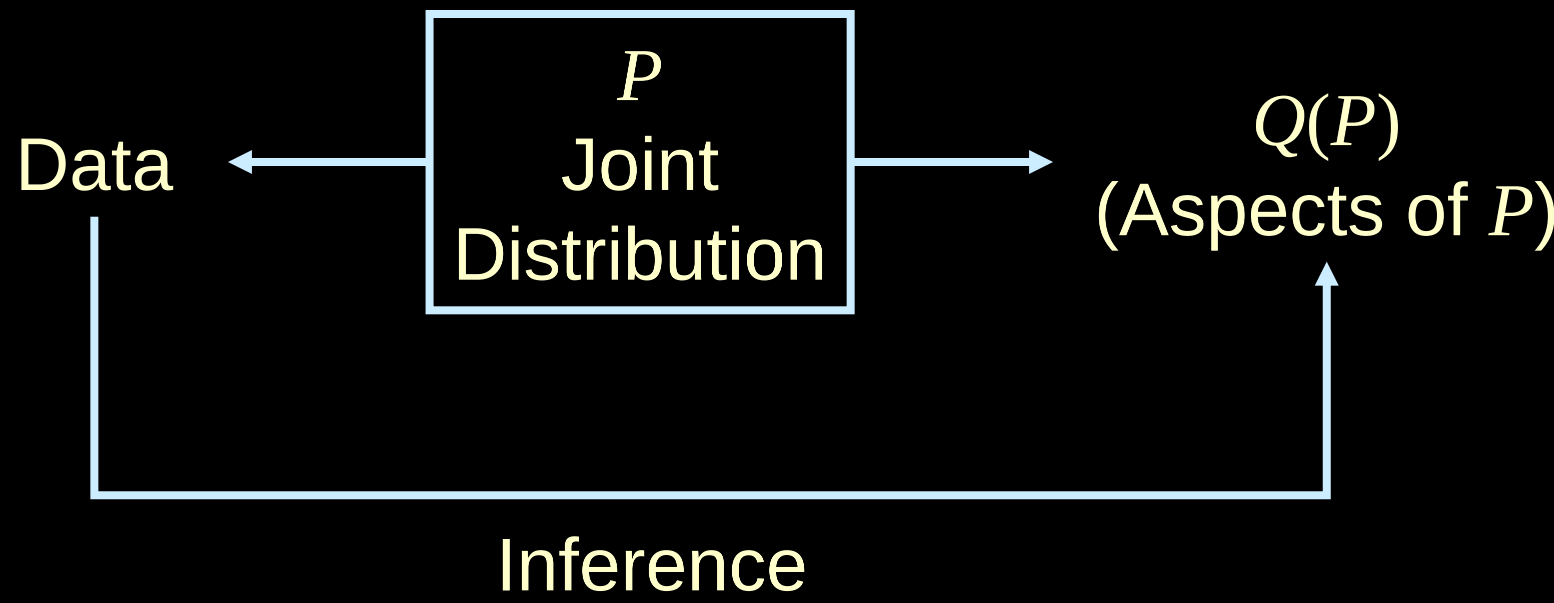
TRADITIONAL STATISTICAL INFERENCE PARADIGM



e.g., Estimate the mean of X

$$Q(P) = E(X) = \sum_{X} xP(X = x)$$

TRADITIONAL STATISTICAL INFERENCE PARADIGM

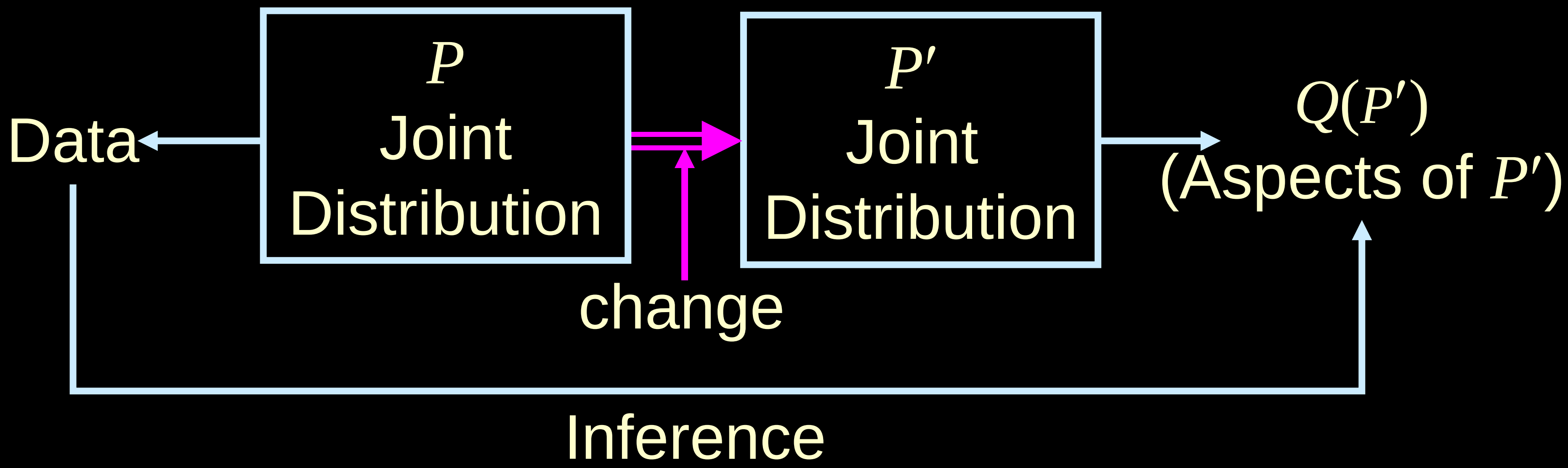


e.g., Estimate the probability that a customer who bought product A would also buy product B.

$$Q = P(B \mid A)$$

FROM STATISTICAL TO CAUSAL ANALYSIS: 1. THE DIFFERENCES

Probability and statistics deal with static relations

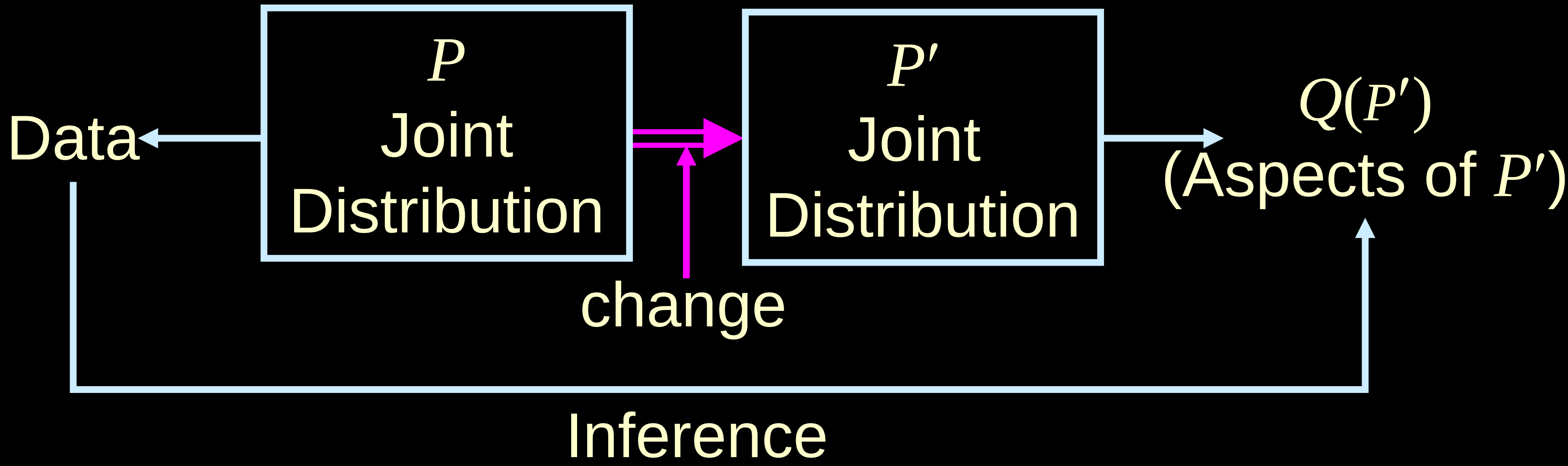


What happens when P changes?

e.g., Estimate the probability that a customer who bought A would buy B if we were to double the price.

FROM STATISTICAL TO CAUSAL ANALYSIS: 1. THE DIFFERENCES

What remains invariant when P changes say, to satisfy $P'(price=2)=1$



Note: $P'(B) \neq P\,(B \mid price = 2)$

e.g., Doubling price ≠ seeing the price doubled.

P does not tell us how it ought to change.

FROM STATISTICAL TO CAUSAL ANALYSIS: 1. THE DIFFERENCES (CONT)

1. Causal and statistical concepts do not mix.

CAUSAL	STATISTICAL
Spurious correlation	Regression
Randomization / Intervention	Association / Independence
Confounding / Effect	“Controlling for” / Conditioning
Instrumental variable	Odd and risk ratios
Ignorability / Exogeneity	Collapsibility / Granger causality
Explanatory variables	Propensity score

2.

3.

4.


The fire
shadows cast on wall
Prisoners
Roadway where puppeteers perform
PLATO'S CAVE...

FROM STATISTICAL TO CAUSAL ANALYSIS: 1. THE DIFFERENCES (CONT)

1. Causal and statistical concepts do not mix.

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2.

3.

4.

FROM STATISTICAL TO CAUSAL ANALYSIS: 2. MENTAL BARRIERS

1. Causal and statistical concepts do not mix.

CAUSAL	STATISTICAL
Spurious correlation	Regression
Randomization / Intervention	Association / Independence
Confounding / Effect	“Controlling for” / Conditioning
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Ignorability / Exogeneity	Collapsibility / Granger causality
Explanatory variables	Propensity score

2. No causes in – no causes out (Cartwright, 1989)

$$\left.\begin{array}{l}\text{statistical assumptions + data}\\ \text{causal assumptions}\end{array}\right\} \Longrightarrow \text{causal conclusions}$$

3. Causal assumptions cannot be expressed in the mathematical language of standard statistics.

4. Non-standard mathematics:
   a) Structural equation models (Wright, 1920; Simon, 1960)
   b) Counterfactuals (Neyman-Rubin (Y_x), Lewis $(x \mathrel{\Box\!\!\rightarrow} Y)$)

WHY PHYSICS IS COUNTERFACTUAL

Scientific Equations (e.g., Hooke's Law) are non-algebraic

e.g., Length (Y) equals a constant (2) times the weight (X)

$Y = 2X$	$X = 1$
$X = 1$	$Y = 2$
<u>Process information</u>	<u>The solution</u>

WHY PHYSICS IS COUNTERFACTUAL

Scientific Equations (e.g., Hooke's Law) are non-algebraic

e.g., Length (*Y*) equals a constant (2) times the weight (*X*)

Correct notation:

$$Y := 2X$$

$$X = 3 \quad \cancel{X = 1}$$

Process information

$$X = 1$$

$$Y = 2$$

The solution

Had *X* been 3, *Y* would be 6.
If we raise *X* to 3, *Y* would be 6.
Must "wipe out" *X* = 1.

WHY PHYSICS IS COUNTERFACTUAL

Scientific Equations (e.g., Hooke's Law) are non-algebraic

e.g., Length (*Y*) equals a constant (2) times the weight (*X*)

Correct notation:
(or)

$$Y \leftarrow 2X$$

$X = 3$ $\cancel{X = 1}$

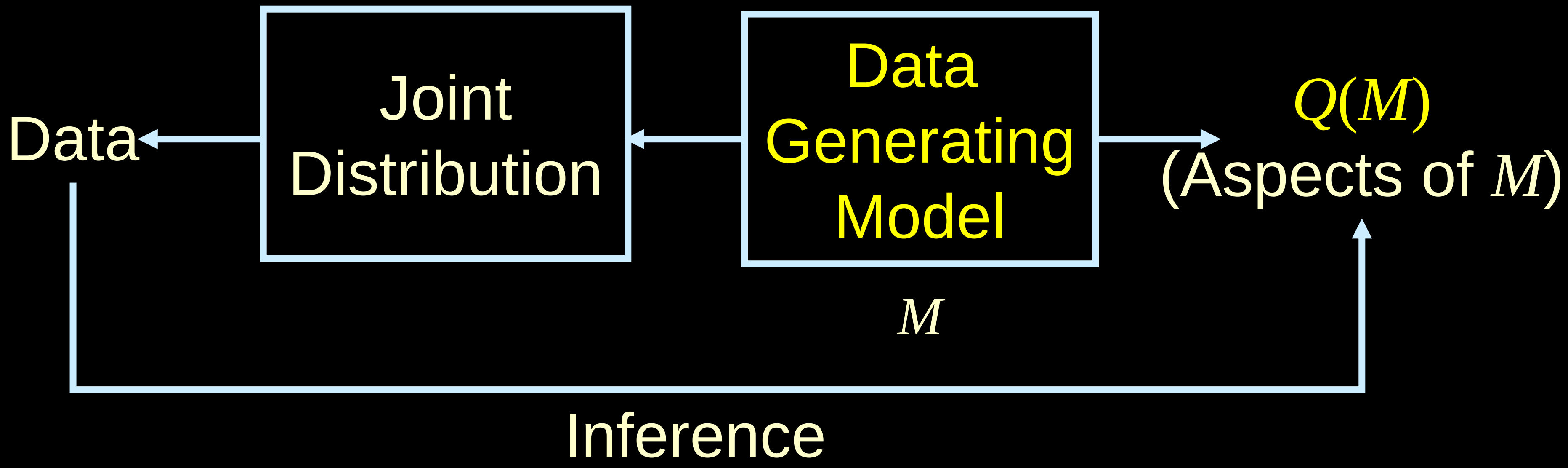
Process information

$X = 1$
$Y = 2$

The solution

Had *X* been 3, *Y* would be 6.
If we raise *X* to 3, *Y* would be 6.
Must "wipe out" $X = 1$.

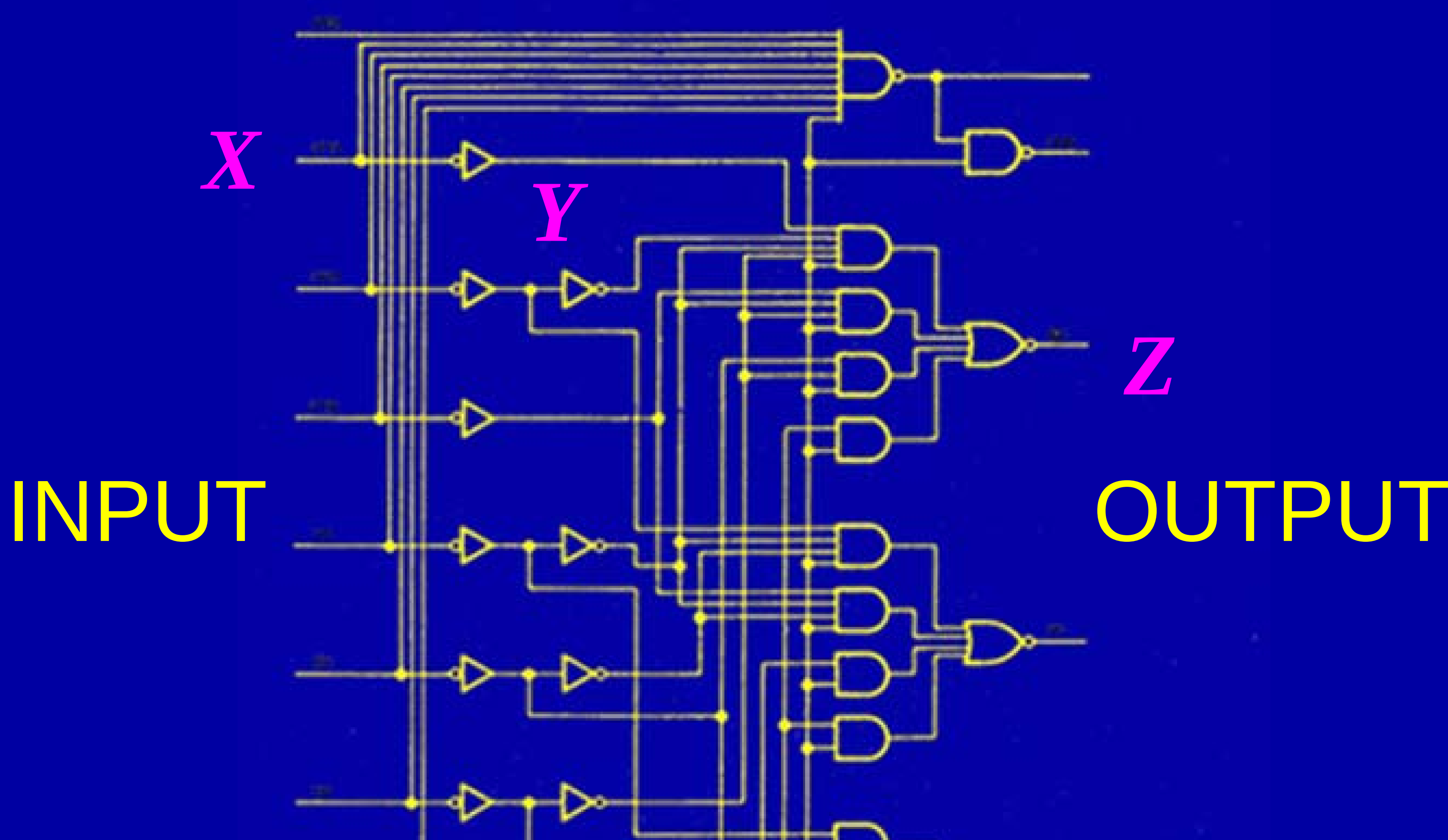
THE STRUCTURAL MODEL PARADIGM



M – Invariant strategy (mechanism, recipe, law, protocol) by which Nature assigns values to variables in the analysis.

“Think Nature, not experiment!”

FAMILIAR CAUSAL MODEL
ORACLE FOR MANIPILATION



STRUCTURAL CAUSAL MODELS

Definition: A structural causal model is a 4-tuple $\langle V, U, F, P(u)\rangle$, where

- $V = \{V_1,...,V_n\}$ are endogenous variables
- $U = \{U_1,...,U_m\}$ are background variables
- $F = \{f_1,..., f_n\}$ are functions determining V,
  $v_i = f_i(v, u)$ e.g., $y = \alpha + \beta x + u_Y$
- $P(u)$ is a distribution over U

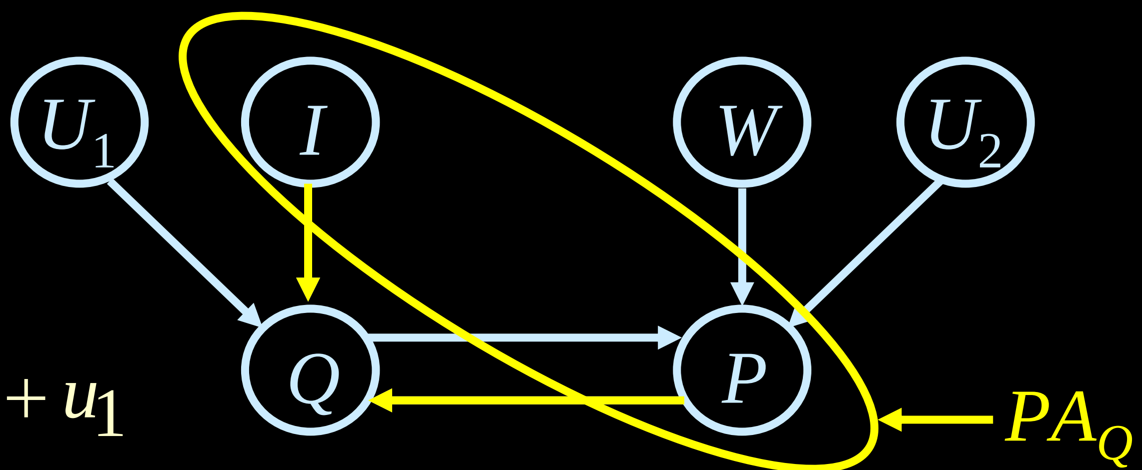
$P(u)$ and F induce a distribution $P(v)$ over observable variables

STRUCTURAL MODELS AND CAUSAL DIAGRAMS

The functions $v_i = f_i(v,u)$ define a graph

$$v_i = f_i(pa_i,u_i) \quad PA_i \subseteq V \setminus V_i \qquad U_i \subseteq U$$

Example: Price – Quantity equations in economics



$$q = b_1 p + d_1 i + u_1$$

$$p = b_2 q + d_2 w + u_2$$

SIMULATING INTERVENTIONS IN STRUCTURAL MODELS – $do(x)$

- Double the price
- Take a drug
- Raise taxes
- Make me laugh

SIMULATING INTERVENTIONS IN STRUCTURAL MODELS – *do*(*x*)

Let X be a set of variables in V.
The action ***do*(*x*)** sets X to constants x regardless of the factors which previously determined X.
***do*(*x*)** replaces all functions f_i determining X with the constant functions $X = x$, to create a mutilated model M_x

$$q = b_1 p + d_1 i + u_1$$
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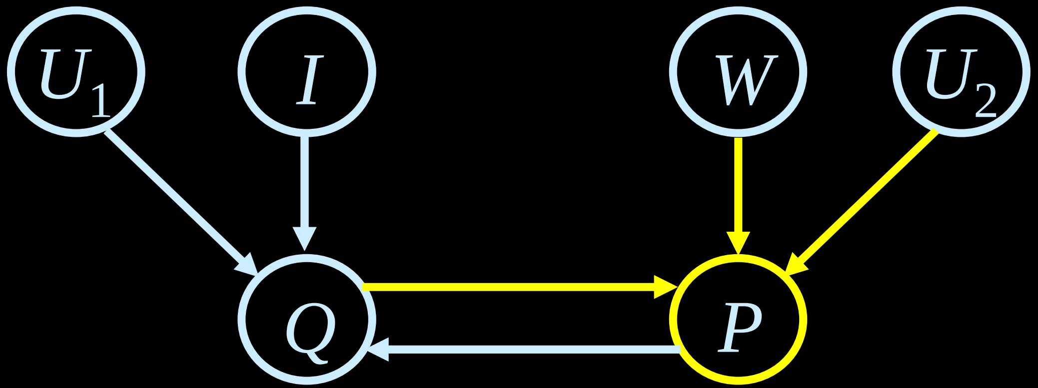
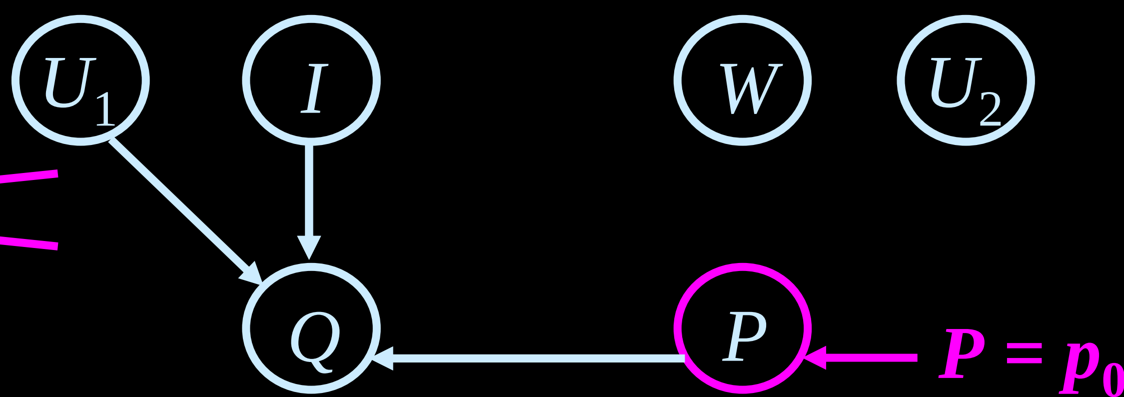


SIMULATING INTERVENTIONS IN STRUCTURAL MODELS

Let X be a set of variables in V.
The action $do(x)$ sets X to constants x regardless of the factors which previously determined X.
$do(x)$ replaces all functions f_i determining X with the constant functions $X=x$, to create a mutilated model $\boldsymbol{M_x}$

$\boxed{\boldsymbol{M_p}}$

$$q = b_1 p + d_1 i + u_1$$
$$\cancel{p = b_2 q + d_2 w + u_2}$$
$$p = p_0$$



CAUSAL MODELS AND COUNTERFACTUALS

- If I were a rich man
- Had we doubled the price

CAUSAL MODELS AND COUNTERFACTUALS

Definition:

The sentence: "Y would be y (in situation u), had X been x," denoted $\boldsymbol{Y_x(u) = y}$, means:

The solution for Y in a mutilated model M_x, (i.e., the equations for X replaced by $X = x$) with input $U=u$, is equal to y.

The Fundamental Equation of Counterfactuals:

$$\boxed{Y_x(u) = Y_{M_x}(u)}$$

CAUSAL MODELS AND COUNTERFACTUALS

Definition:

The sentence: "Y would be y (in situation u), had X been x," denoted $Y_x(u) = y$, means:

The solution for Y in a mutilated model M_x, (i.e., the equations for X replaced by $X = x$) with input $U=u$, is equal to y.

- Joint probabilities of counterfactuals:

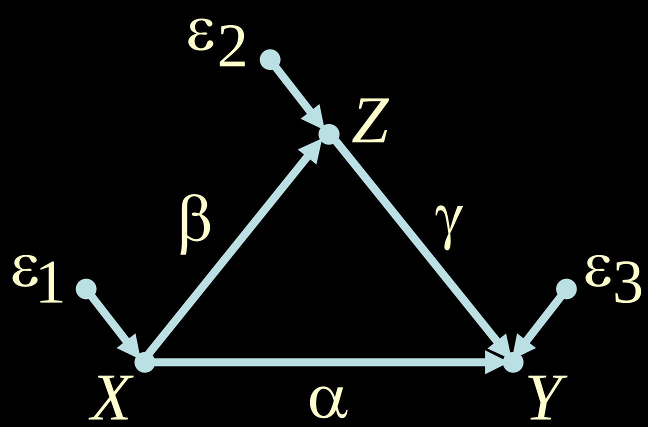
$$P(Y_x = y, Z_w = z) = \sum_{u: Y_x(u)=y, Z_w(u)=z} P(u)$$

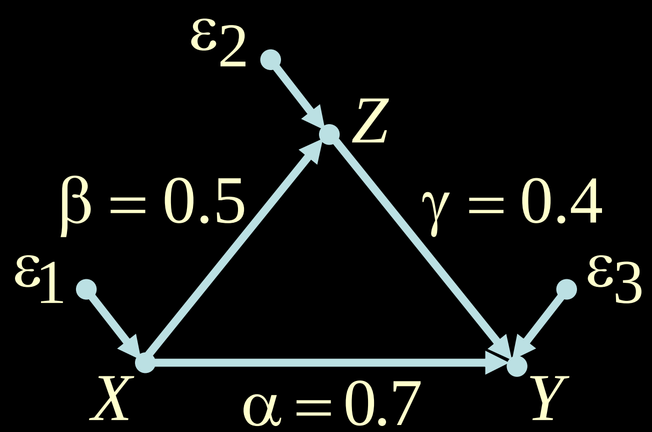
In particular:

$$P(y \mid do(x)) = P(Y_x = y) = \sum_{u: Y_x(u)=y} P(u)$$

$$PN(Y_{x'} = y' \mid x, y) = \sum_{u: Y_{x'}(u)=y'} P(u \mid x, y)$$

READING COUNTERFACTUALS FROM SEM





X = Treatment $\qquad x = \varepsilon_1$

Z = Study Time $\qquad z = \beta x + \varepsilon_2$

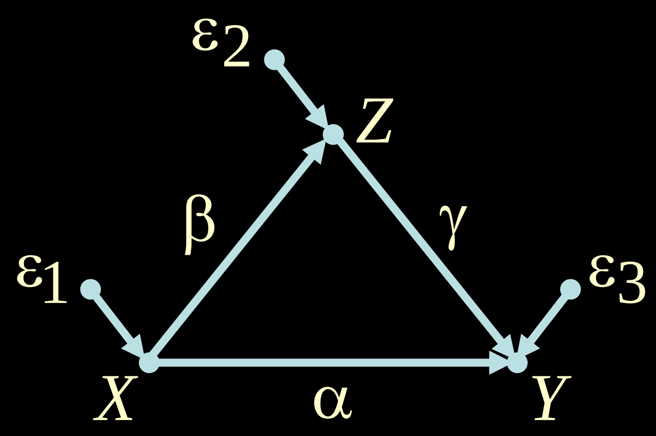
Y = Score $\qquad y = \alpha x + \gamma z + \varepsilon_3$

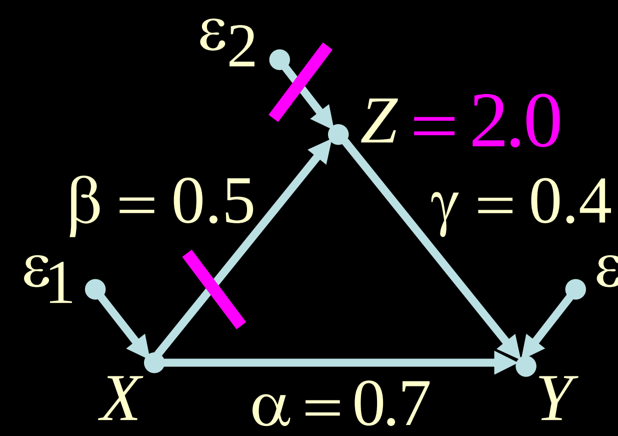
Data shows: $\alpha = 0.7$, $\beta = 0.5$, $\gamma = 0.4$

A student named Joe, measured $X = 0.5$, $Z = 1.0$, $Y = 1.9$

Q_1: What would Joe's score be had he doubled his study time?

READING COUNTERFACTUALS FROM SEM





X = Treatment

Z = Study Time

Y = Score

$x = \varepsilon_1$

$\cancel{z = \beta x + \varepsilon_2}$ $\quad z = 2.0$

$y = \alpha x + \gamma z + \varepsilon_3$

Data shows: $\alpha = 0.7$, $\beta = 0.5$, $\gamma = 0.4$

A student named Joe, measured $X = 0.5$, $Z = 1.0$, $Y = 1.9$

Q_1: What would Joe's score be had he doubled his study time?

Answer: $Y_{Z=2} = 0.7\ 0.5 + 0.4\ 2.0 + \varepsilon_3 = 2.30$

REGRESSION VS. STRUCTURAL EQUATIONS (THE CONFUSION OF THE CENTURY)

Regression (claimless, nonfalsifiable):

$$Y = ax + \varepsilon_Y$$

Structural (empirical, falsifiable):

$$Y = bx + u_Y$$

Claim: (regardless of distributions):

$$E(Y \mid do(x)) = E(Y \mid do(x), do(z)) = bx$$

The mothers of all questions:

Q. When would b equal a?

A. When all back-door paths are blocked, $(u_Y \perp\!\!\!\perp X)$

Q. When is b estimable by regression methods?

A. Graphical criteria available

THE FIVE NECESSARY STEPS OF CAUSAL ANALYSIS

Define: Express the target quantity Q as property of the model M.

Assume: Express causal assumptions in structural or graphical form.

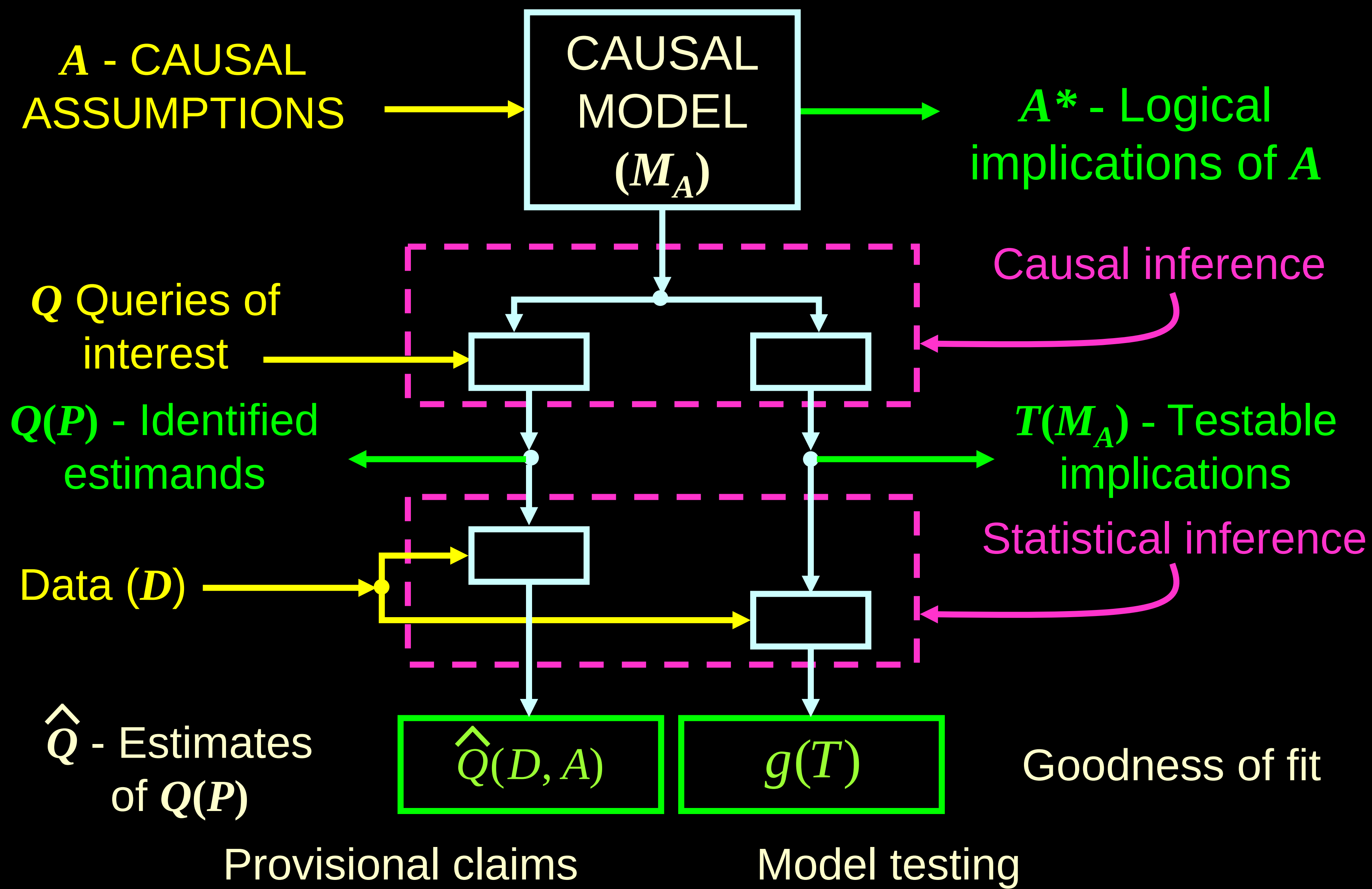
Identify: Determine if Q is identifiable.

Estimate: Estimate Q if it is identifiable; approximate it, if it is not.

Test: If M has testable implications

Repeat if necessary

THE LOGIC OF CAUSAL ANALYSIS



THE FIVE NECESSARY STEPS FOR EFFECT ESTIMATION

Define: Express the target quantity Q as a property of the model M.

$$\boxed{P(Y_x = y) \quad \text{or} \quad P(y \mid do(x))}$$

Assume: Express causal assumptions in structural or graphical form.

Identify: Determine if Q is identifiable.

Estimate: Estimate Q if it is identifiable; approximate it, if it is not.

Test: If M has testable implications

THE FIVE NECESSARY STEPS FOR AVERAGE TREATMENT EFFECT

Define: Express the target quantity Q as a property of the model M.

$$\text{ATE} \equiv E(Y \mid do(x_1)) - E(Y \mid do(x_0))$$

Assume: Express causal assumptions in structural or graphical form.

Identify: Determine if Q is identifiable.

Estimate: Estimate Q if it is identifiable; approximate it, if it is not.

Test: If M has testable implications

THE FIVE NECESSARY STEPS FOR DYNAMIC POLICY ANALYSIS

Define:	Express the target quantity Q as a property of the model M.
	$P(y \mid do(x = g(z))$
Assume:	Express causal assumptions in structural or graphical form.
Identify:	Determine if Q is identifiable.
Estimate:	Estimate Q if it is identifiable; approximate it, if it is not.
Test:	If M has testable implications

THE FIVE NECESSARY STEPS FOR TIME VARYING POLICY ANALYSIS

Define: Express the target quantity Q as a property of the model M.

$$P(y \mid do(X = x, Z = z, W = w))$$

Assume: Express causal assumptions in structural or graphical form.

Identify: Determine if Q is identifiable.

Estimate: Estimate Q if it is identifiable; approximate it, if it is not.

Test: If M has testable implications

THE FIVE NECESSARY STEPS FOR TREATMENT ON TREATED

Define: Express the target quantity Q a property of the model M.

$$\text{ETT} = P(Y_x = y \mid X = x')$$

Assume: Express causal assumptions in structural or graphical form.

Identify: Determine if Q is identifiable.

Estimate: Estimate Q if it is identifiable; approximate it, if it is not.

Test: If M has testable implications

THE FIVE NECESSARY STEPS FOR INDIRECT EFFECTS

Define: Express the target quantity Q a property of the model M.

$$\text{IE} = E[Y_{x,Z(x')}] - E[Y_x]$$

Assume: Express causal assumptions in structural or graphical form.

Identify: Determine if Q is identifiable.

Estimate: Estimate Q if it is identifiable; approximate it, if it is not.

Test: If M has testable implications

THE FIVE NECESSARY STEPS FROM DEFINITION TO ASSUMPTIONS

Define: Express the target quantity Q as a property of the model M.

Assume: Express causal assumptions in structural or graphical form.

Identify: Determine if Q is identifiable.

Estimate: Estimate Q if it is identifiable; approximate it, if it is not.

Test: If M has testable implications

FORMULATING ASSUMPTIONS
THREE LANGUAGES

1. English: Smoking (X), Cancer (Y), Tar (Z), Genotypes (U)

2. Counterfactuals:

$$Z_x(u) = Z_{yx}(u),$$

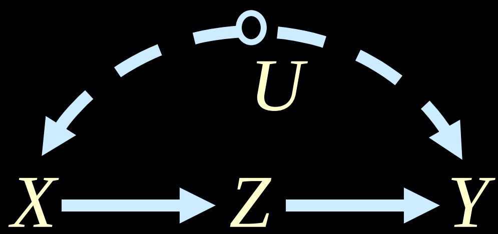
$$X_y(u) = X_{zy}(u) = X_z(u) = X(u),$$

$$Y_z(u) = Y_{zx}(u),$$

$$Z_x \perp\!\!\!\perp \{Y_z, X\}$$

Not too friendly:
Consistent?, complete?, redundant?, plausible?, testable?

3. Structural:



$$x = f_1(u, \varepsilon_1) \qquad y = f_3(z, u, \varepsilon_3)$$

$$z = f_2(x, \varepsilon_2) \qquad \varepsilon_1 \perp\!\!\!\perp \varepsilon_2 \perp\!\!\!\perp \varepsilon_3$$

FROM Q AND ASSUMPTIONS TO IDENTIFICATION

Define: Express the target quantity Q as a function $Q(M)$ that can be computed from any model M.

Assume: Express causal assumptions in structural or graphical form.

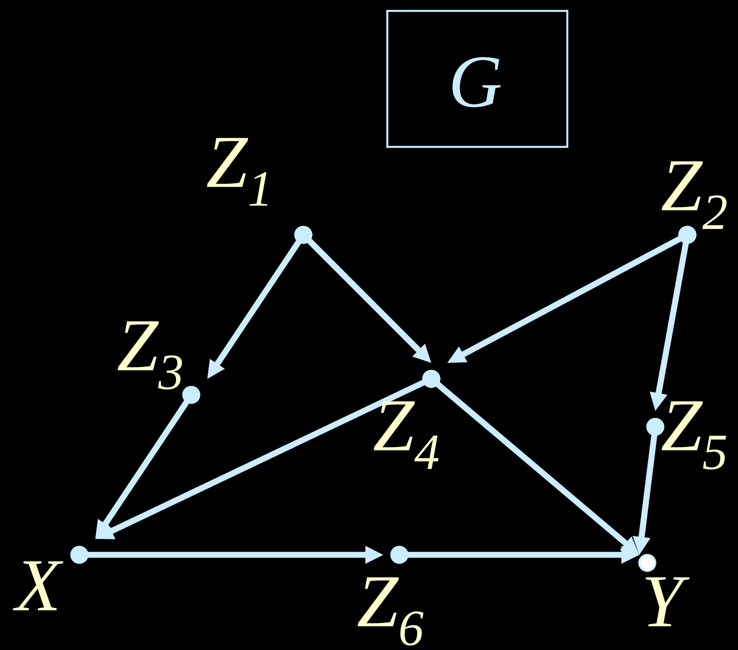
Identify: Determine if Q is identifiable. SOLVED!!!

Estimate: Estimate Q if it is identifiable; approximate it, if it is not.

Test: If M has testable implications

THE PROBLEM OF CONFOUNDING

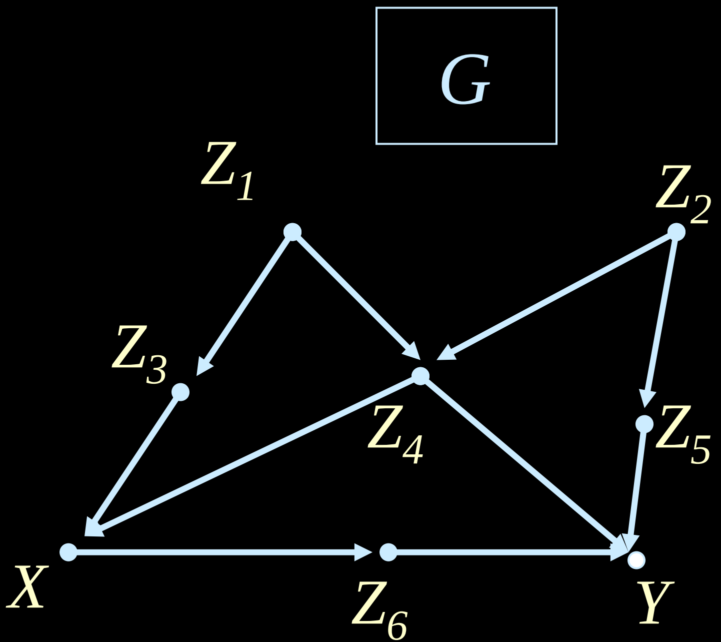
Find the effect of X on Y, $P(y|do(x))$,
given measurements on auxiliary variables $Z_1, \ldots, Z_k$

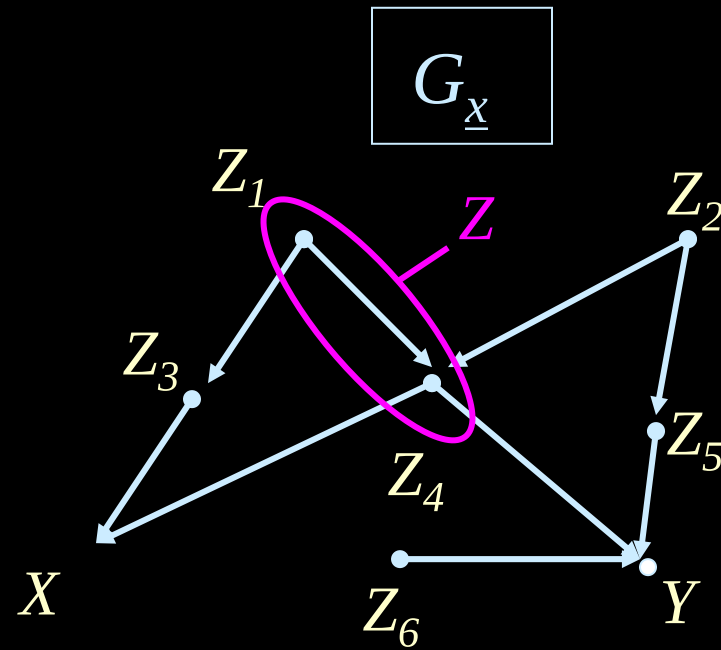


Can $P(y|do(x))$ be estimated if only a subset, Z, can be measured?

ELIMINATING CONFOUNDING BIAS THE BACK-DOOR CRITERION

$P(y \mid do(x))$ is estimable if there is a set Z of variables that d-separates X from Y in $G_{\underline{X}}$.



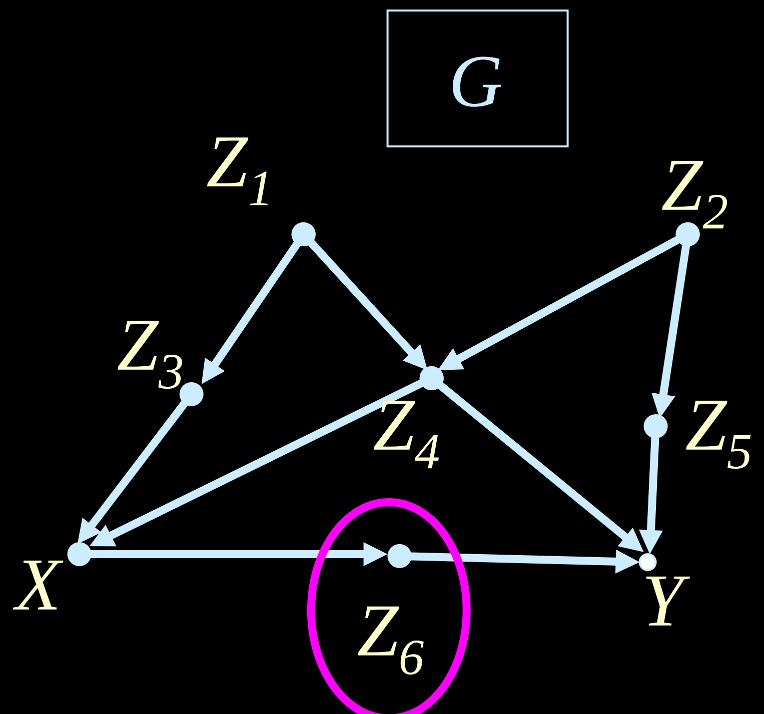


Moreover, $P(y \mid do(x)) = \sum_z P(y \mid x,z)\, P(z)$ ("adjusting" for Z)

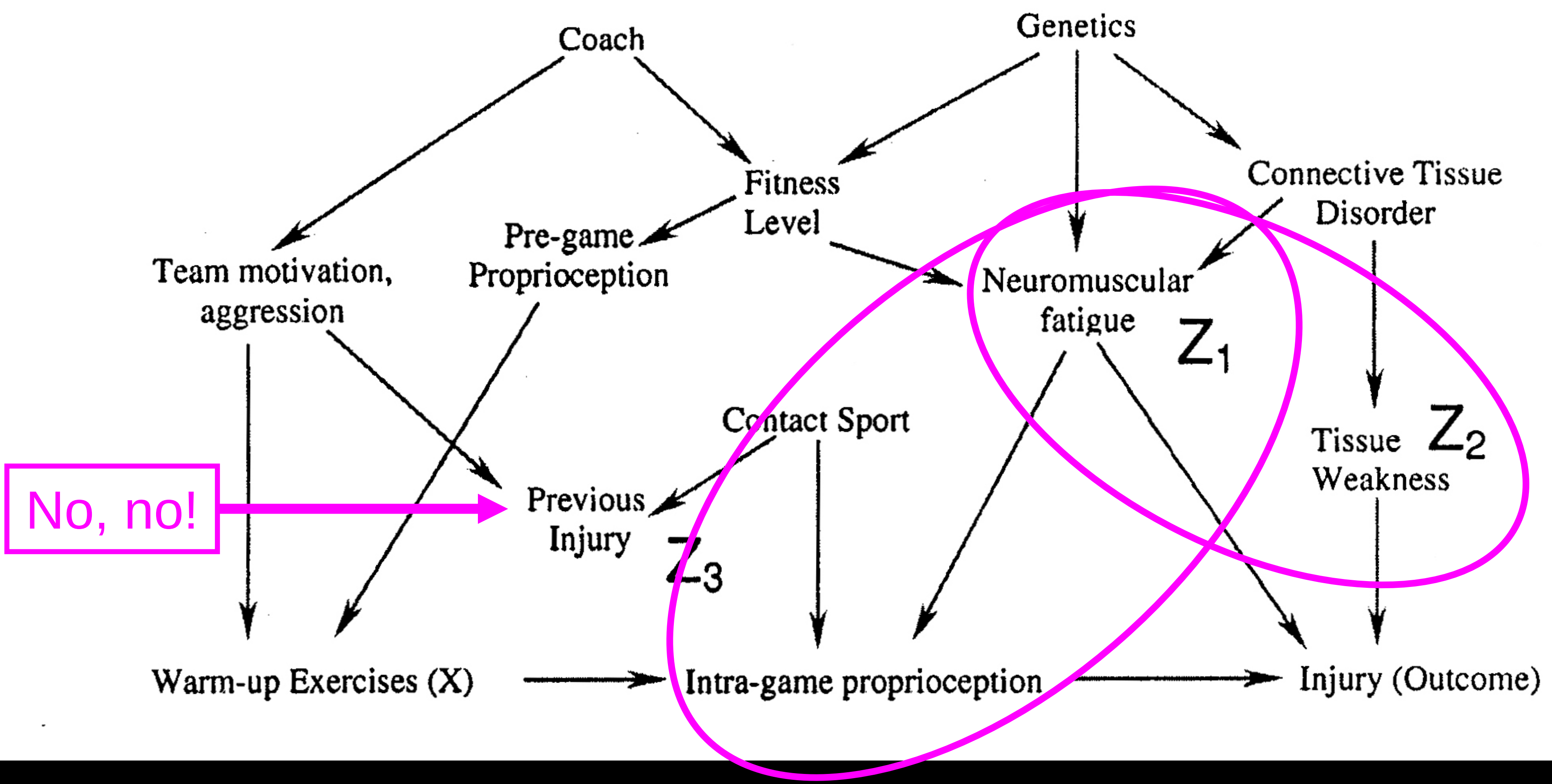
EFFECT OF INTERVENTION BEYOND ADJUSTMENT

Theorem (Tian-Pearl 2002)

We can identify $P(y|do(x))$ if there is no child Z of X connected to X by a confounding path.



EFFECT OF WARM-UP ON INJURY (After Shrier & Platt, 2008)



COUNTERFACTUALS AT WORK

ETT – EFFECT OF TREATMENT ON THE TREATED

1. Regret:
   I took a pill to fall asleep.
   Perhaps I should not have?

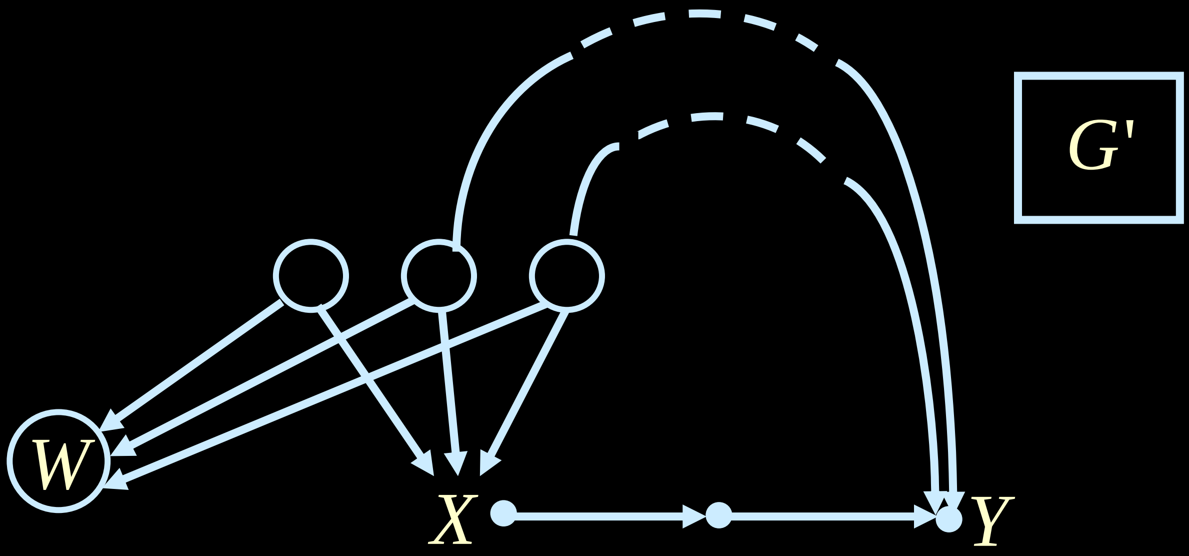
2. Program evaluation:
   What would terminating a program do to those enrolled?

$$P(Y_x = y \mid x')$$

IDENTIFICATION OF COUNTERFACTUALS

$$\text{ETT} = P(Y_X = y \mid x')$$

ETT is identifiable in G iff $P(\mathrm{y} \mid do(x), w)$ is identifiable in G'

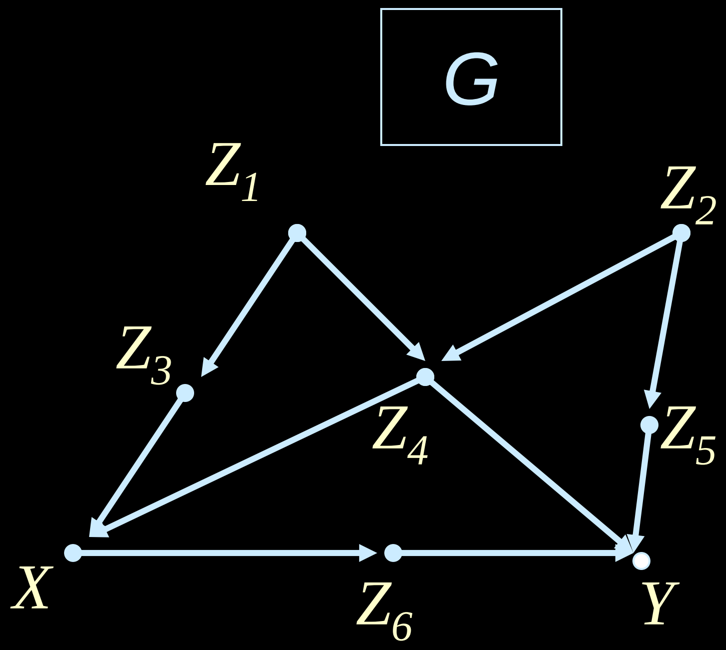


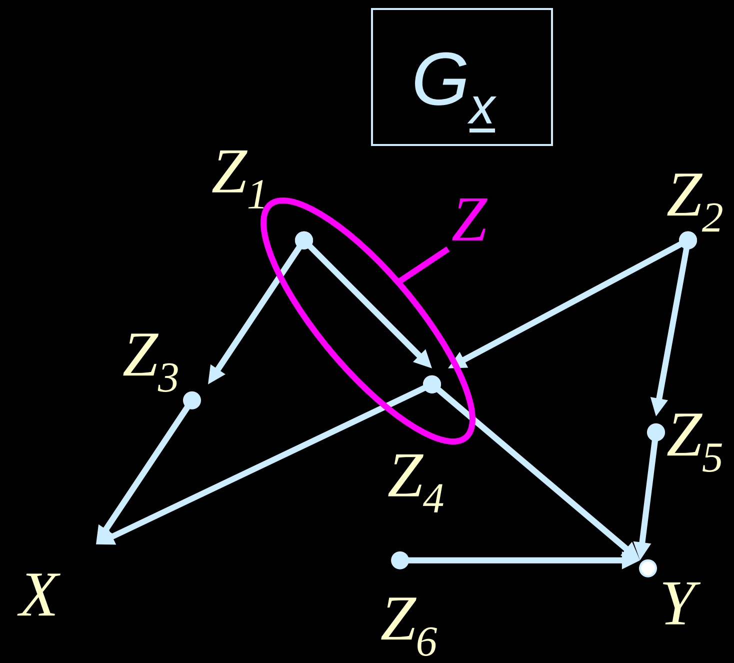
Moreover, $\text{ETT} = P(Y_X = y \mid x') = P(y \mid do(x), w) \Big|_{G'} \Big|_{w=x'}$

Complete graphical criterion (Shpitser-Pearl, 2009)

ETT - THE BACK-DOOR CRITERION

ETT is identifiable in G if there is a set Z of variables that *d*-separates X from Y *in* $G_{\underline{X}}$.





Moreover, ETT $= \sum_z P(y \mid x, z)\, P(z \mid x')$

- "Standardized morbidity"

FROM IDENTIFICATION TO ESTIMATION

Define: Express the target quantity Q as a function $Q(M)$ that can be computed from any model M.

e.g., $Q = P(y \mid do(x))$

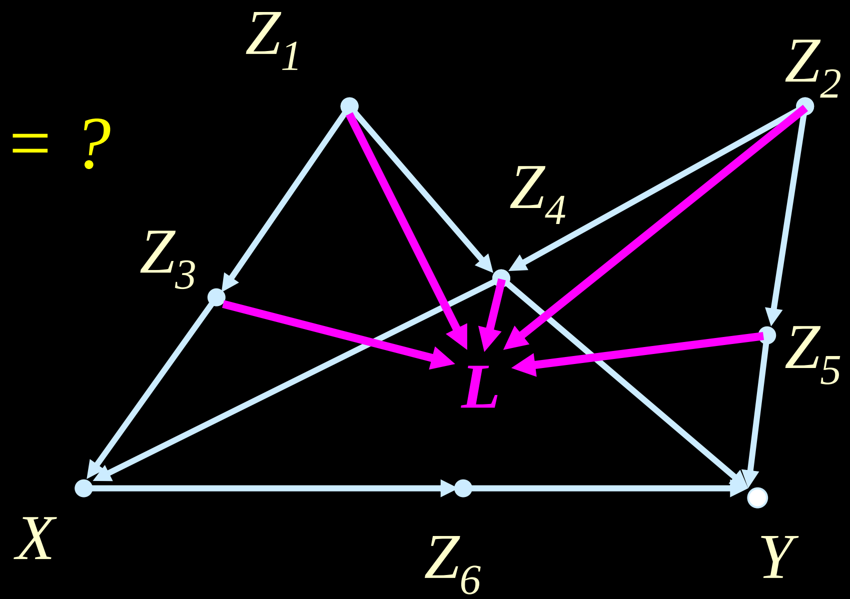
Assume: Formulate causal assumptions using ordinary scientific language and represent their structural part in graphical form.

Identify: Determine if Q is identifiable.

Estimate: Estimate Q if it is identifiable; approximate it, if it is not.

PROPENSITY SCORE ESTIMATOR (Rosenbaum & Rubin, 1983)

$P(y \mid do(x)) = ?$



$$L(z_1, z_2, z_3, z_4, z_5) \equiv P(X = 1 \mid z_1, z_2, z_3, z_4, z_5)$$

Theorem: $\sum_z P(y \mid z, x) P(z) = \sum_l P(y \mid L = l, x) P(L = l)$

Adjustment for L replaces Adjustment for Z

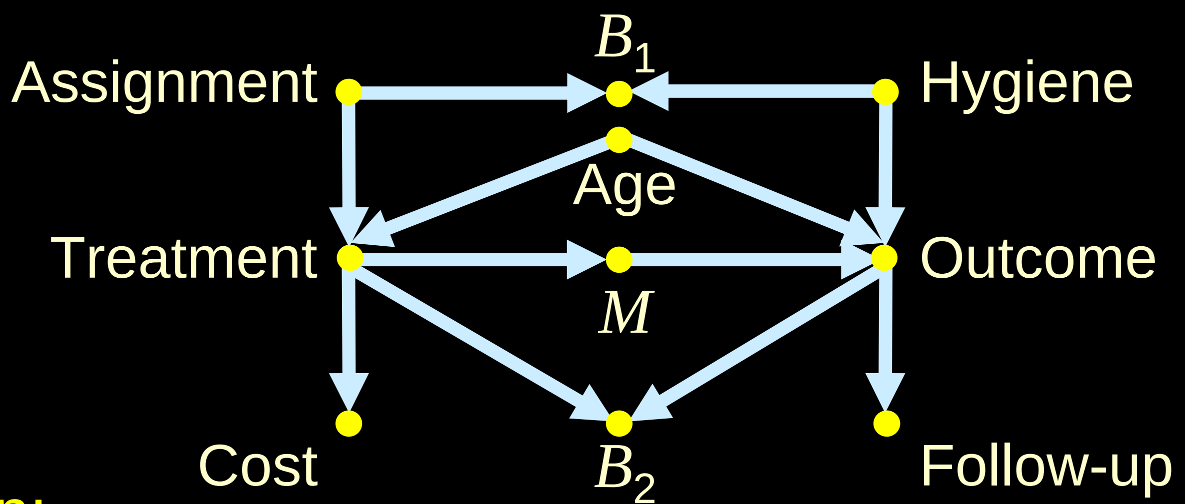
WHAT PROPENSITY SCORE (PS) PRACTITIONERS NEED TO KNOW

$$L(z) = P(X = 1 \mid Z = z)$$

$$\sum_z P(y \mid z, x) P(z) = \sum_l P(y \mid l, x) P(l)$$

1. The assymptotic bias of PS is EQUAL to that of ordinary adjustment (for same Z).
2. Including an additional covariate in the analysis CAN SPOIL the bias-reduction potential of others.
3. Choosing sufficient set for PS, requires knowledge about the model.

WHICH COVARIATES MAY / SHOULD BE ADJUSTED FOR?



Question:

Which of these eight covariates may be included in the propensity score function (for matching) and which should be excluded.

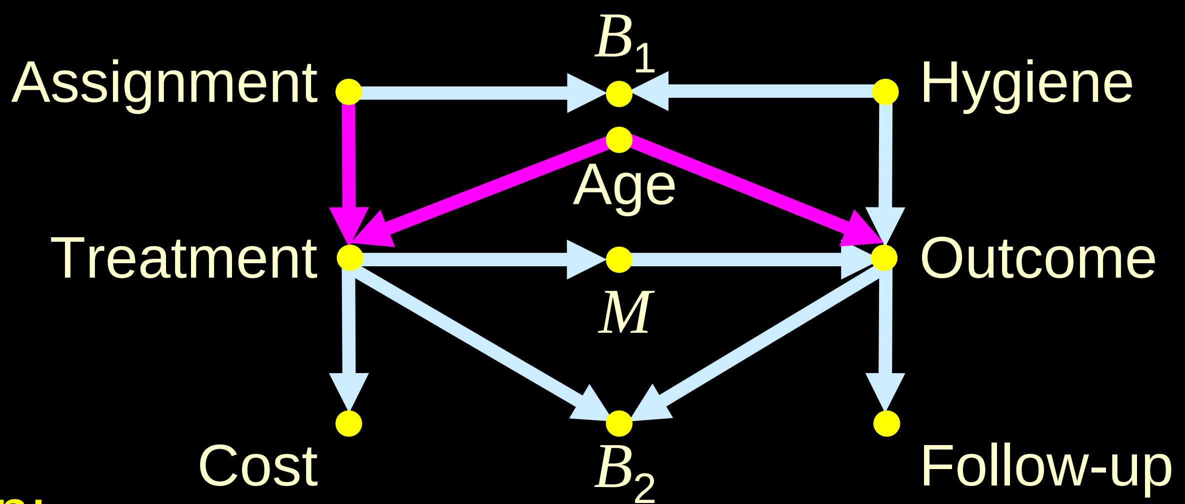
Answer:

Must include: Age

Must exclude: B_1, M, B_2, Follow-up, Assignment without Age

May include: Cost, Hygiene, {Assignment + Age}, {Hygiene + Age + B_1} , more . . .

WHICH COVARIATES MAY / SHOULD BE ADJUSTED FOR?



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WHAT PROPENSITY SCORE (PS) PRACTITIONERS NEED TO KNOW

$$L(z) = P(X = 1 \mid Z = z)$$

$$\sum_{z} P(y \mid z, x) P(z) = \sum_{1} P(y \mid l, x) P(l)$$

1. The assymptotic bias of PS is EQUAL to that of ordinary adjustment (for same Z).
2. Including an additional covariate in the analysis CAN SPOIL the bias-reduction potential of others.
3. Choosing sufficient set for PS, requires knowledge about the model.
4. That any empirical test of the bias-reduction potential of PS, can only be generalized to cases where the causal relationships among covariates, observed and unobserved is the same.

THE STRUCTURAL-COUNTERFACTUAL SYMBIOSIS

1. Express assumptions in structural or graphical language.

2. Express queries in counterfactual language.

3. Translate (1) into (2) for algebraic analysis,
   Or (2) into (1) for graphical analysis.

4. Use either graphical or algebraic machinery
   to answer the query in (2).

GRAPHICAL – COUNTERFACTUALS TRANSLATION

Every causal graph expresses counterfactuals assumptions, e.g., $X \rightarrow Y \rightarrow Z$

1. Missing arrows $\quad Y \leftarrow Z \qquad\qquad Y_{x,z}(u) = Y_x(u)$

2. Missing arcs $\quad Y \qquad Z \qquad\qquad Y_x \perp\!\!\!\perp Z_y$

consistent, and readable from the graph.

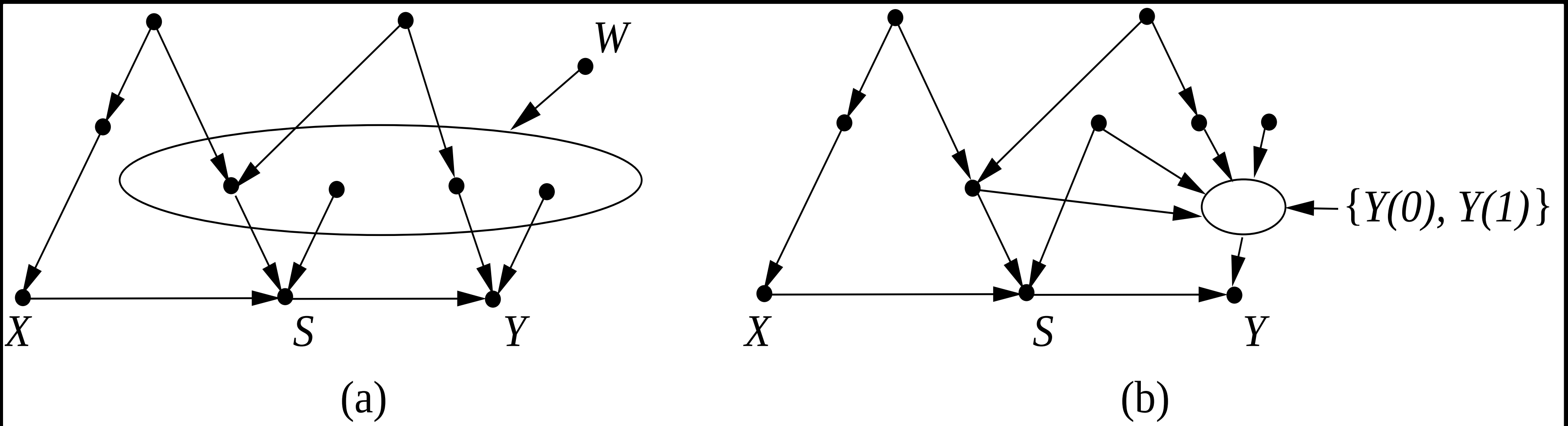
Every theorem in SCM is a theorem in Potential-Outcome Model, and conversely.

DEMYSTIFYING CONDITIONAL IGNORABILITY

$$\{Y(0), Y(1)\} \perp\!\!\!\perp X \mid Z$$ (Ignorability)

$$(X \perp\!\!\!\perp Y \mid Z)_{G_{\underline{X}}}$$ (Back-door)

Where in the graph are $\{Y(0), Y(1)\}$?
W plays the role of $\{Y(0), Y(1)\}$ in the graph



DETERMINING THE CAUSES OF EFFECTS (The Attribution Problem)

- Your Honor! My client (Mr. A) died BECAUSE he used that drug.

-

DETERMINING THE CAUSES OF EFFECTS (The Attribution Problem)

- Your Honor! My client (Mr. A) died BECAUSE he used that drug.

- Court to decide if it is MORE PROBABLE THAN NOT that A would be alive BUT FOR the drug!

$PN = P(? \mid A$ is dead, took the drug$) \geq 0.50$

THE ATTRIBUTION PROBLEM

Definition:

1. What is the meaning of $PN(x,y)$:
   "Probability that event y would not have occurred if it were not for event x, given that x and y did in fact occur."

Answer:

$$PN(x, y) = P(Y_{x'} = y' \mid x, y)$$

Computable from M

THE ATTRIBUTION PROBLEM

Definition:

1. What is the meaning of $PN(x,y)$:
   "Probability that event y would not have occurred if it were not for event x, given that x and y did in fact occur."

Identification:

2. Under what condition can $PN(x,y)$ be learned from statistical data, i.e., observational, experimental and combined.

PARTIAL IDENTIFICATION
(Tian and Pearl, 2000)

- Bounds given combined nonexperimental and experimental data

$$\max\left\{\begin{matrix} 0 \\ \dfrac{P(y)-P(y_{x'})}{P(x,y)} \end{matrix}\right\} \leq PN \leq \min\left\{\begin{matrix} 1 \\ \dfrac{P(y'_{x'})}{P(x,y)} \end{matrix}\right\}$$

- Identifiability under monotonicity (Combined data)

$$PN = \frac{P(y|x)-P(y|x')}{P(y|x)} + \frac{P(y|x')-P(y_{x'})}{P(x,y)}$$

corrected Excess-Risk-Ratio

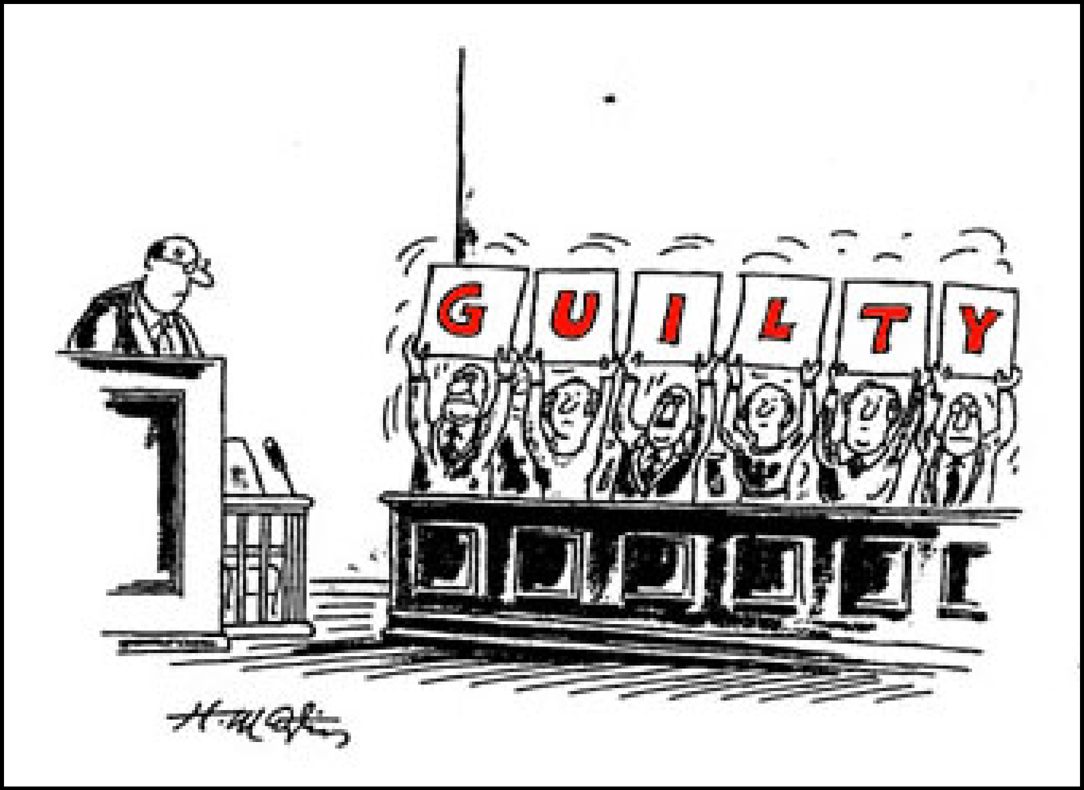
CAN FREQUENCY DATA DECIDE LEGAL RESPONSIBILITY?

	Experimental		Nonexperimental	
	$do(x)$	$do(x')$	x	x'
Deaths (y)	16	14	2	28
Survivals (y')	984	986	998	972
	1,000	1,000	1,000	1,000

- Nonexperimental data: drug usage predicts longer life
- Experimental data: drug has negligible effect on survival
- Plaintiff: Mr. A is special.
  1. He actually died
  2. He used the drug by choice
- Court to decide (given both data):
  Is it more probable than not that A would be alive but for the drug?

$$PN \triangleq P(Y_{x'} = y' \mid x, y) > 0.50$$

SOLUTION TO THE ATTRIBUTION PROBLEM



- WITH PROBABILITY ONE $1 \le P(y'_{x'} \mid x,y) \le 1$
- Combined data tell more that each study alone

EFFECT DECOMPOSITION
(direct vs. indirect effects)

1. Why decompose effects?

2. What is the definition of direct and indirect effects?

3. What are the policy implications of direct and indirect effects?

4. When can direct and indirect effect be estimated consistently from experimental and nonexperimental data?

WHY DECOMPOSE EFFECTS?

1. To understand how Nature works

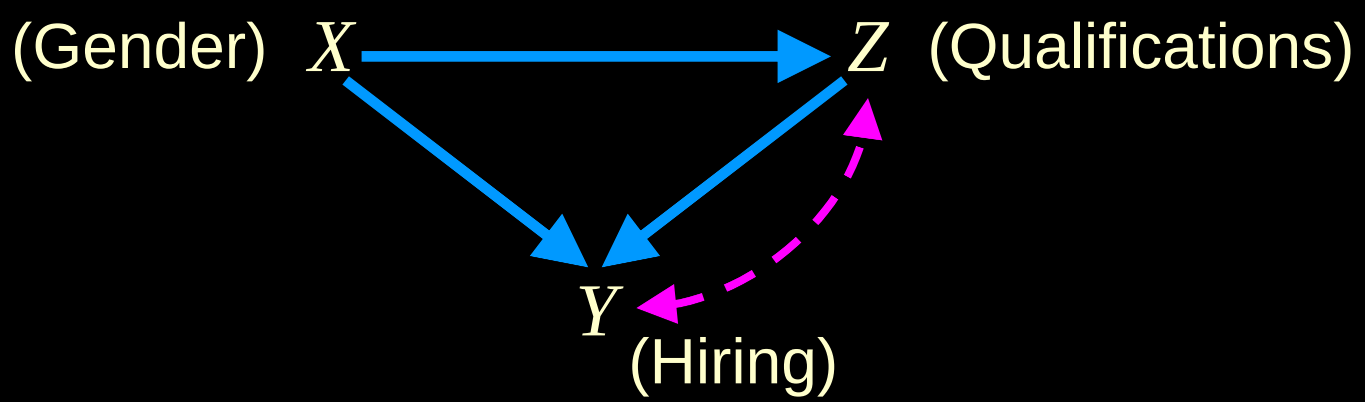
2. To comply with legal requirements

3. To predict the effects of new type of interventions:

   Signal routing, rather than variable fixing

LEGAL IMPLICATIONS OF DIRECT EFFECT

Can data prove an employer guilty of hiring discrimination?



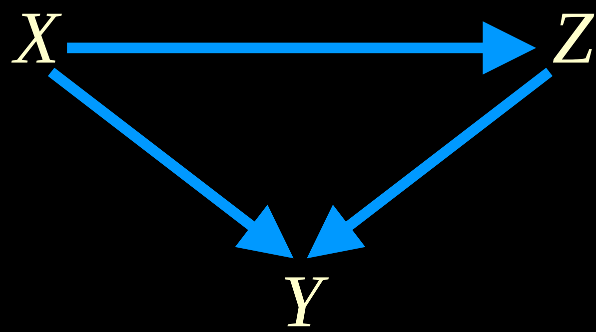
What is the direct effect of X on Y ?

$$E(Y \mid do(x_1), do(z)) - E(Y \mid do(x_0), do(z))$$

(averaged over z) Adjust for *Z*? No! No!

NATURAL INTERPRETATION OF AVERAGE DIRECT EFFECTS

Robins and Greenland (1992) – “Pure”



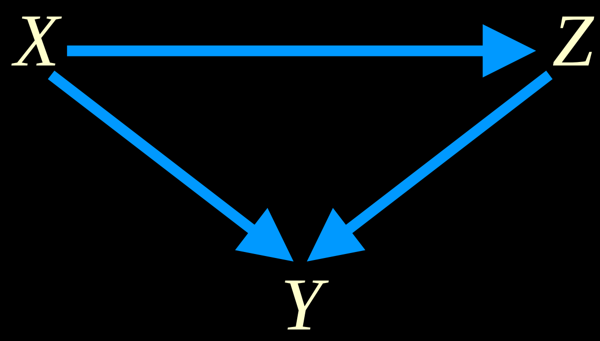
$$z = f(x, u)$$
$$y = g(x, z, u)$$

Natural Direct Effect of *X* on *Y*: $DE(x_0, x_1; Y)$
The expected change in *Y*, when we change *X* from x_0 to x_1 and, for each *u*, we keep *Z* constant at whatever value it attained before the change.

$$E[Y_{x_1 Z_{x_0}} - Y_{x_0}]$$

In linear models, *DE* = Controlled Direct Effect $= \beta(x_1 - x_0)$

DEFINITION OF INDIRECT EFFECTS



$$z = f(x, u)$$
$$y = g(x, z, u)$$

Indirect Effect of *X* on *Y*: $IE(x_0, x_1; Y)$
The expected change in *Y* when we keep *X* constant, say at x_0, and let *Z* change to whatever value it would have attained had *X* changed to x_1.

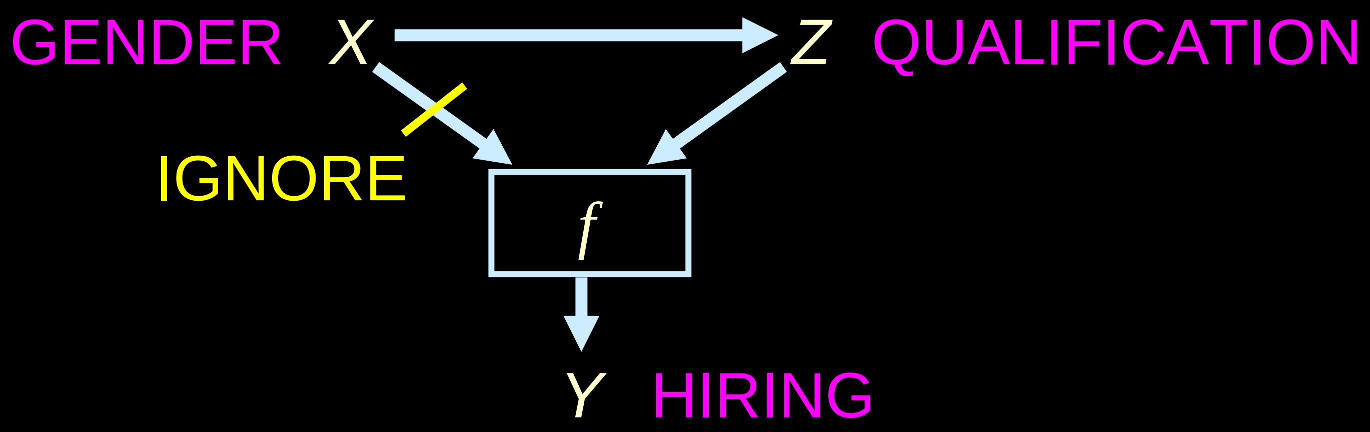
$$E[Y_{x_0 Z_{x_1}} - Y_{x_0}]$$

In linear models, *IE = TE - DE*

POLICY IMPLICATIONS OF INDIRECT EFFECTS

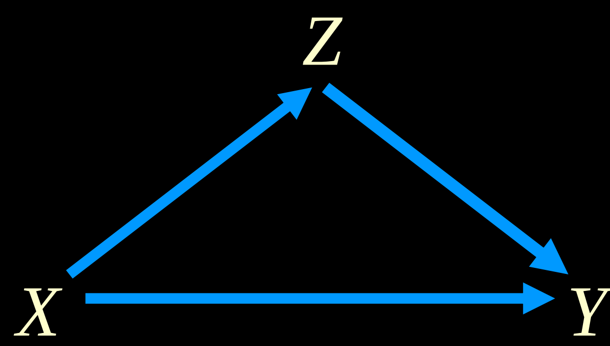
What is the indirect effect of X on Y?

The effect of Gender on Hiring if sex discrimination is eliminated.



Blocking a link – a new type of intervention

MEDIATION FORMULAS IN UNCONFOUNDED MODELS



$$z = f(x, u_1)$$

$$y = g(x, z, u_2)$$

$$x \perp\!\!\!\perp u_1 \perp\!\!\!\perp u_2$$

$$DE = \sum_z [E(Y \mid x_1, z) - E(Y \mid x_0, z)] P(z \mid x_0)$$

$$IE = \sum_z [E(Y \mid x_0, z)[P(z \mid x_1) - P(z \mid x_0)]$$

$$TE = E(Y \mid x_1) - E(Y \mid x_0)$$

$IE =$ Fraction of effect explained by mediation

$TE - DE =$ Fraction of effect owed to mediation

SUMMARY OF MEDIATION RESULTS

1. Formal semantics of path-specific effects, based on disabling mechanisms, instead of value fixing.

2. Path-analytic techniques extended to nonlinear and nonparametric models.

3. Meaningful (graphical) conditions for estimating direct and indirect effects from experimental and nonexperimental data.

EXTERNAL VALIDITY
From Threats to Licenses

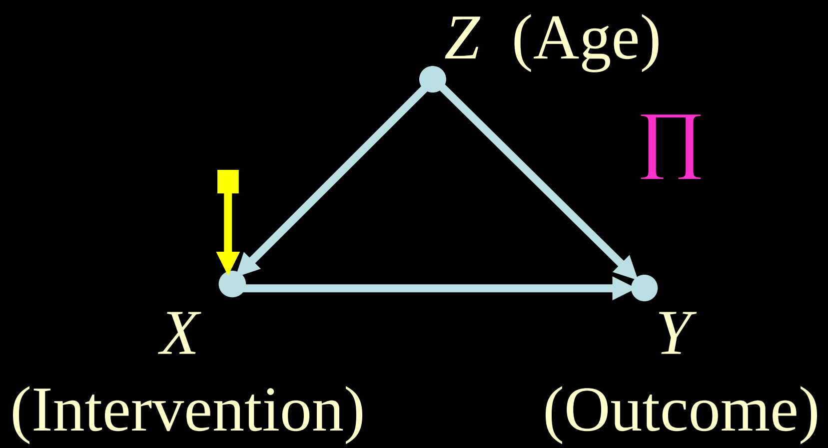
- “`External validity’ asks the question of generalizability: To what population, settings, treatment variables, and measurement variables can this effect be generalized?”
  (Shadish, Cook and Campbell 2002)
- “An experiment is said to have `external validity’ if the distribution of outcomes realized by a treatment group is the same as the distribution of outcome that would be realized in an actual program.”
  (Manski, 2007)
- "A threat to external validity is an explanation of how you might be wrong in making a generalization."
  (Wikipedia 2011, after Trochin)
- “A license of validity is a set of theoretical assumptions that neutralizes all conceivable threats.”
  (Anon, 2011)

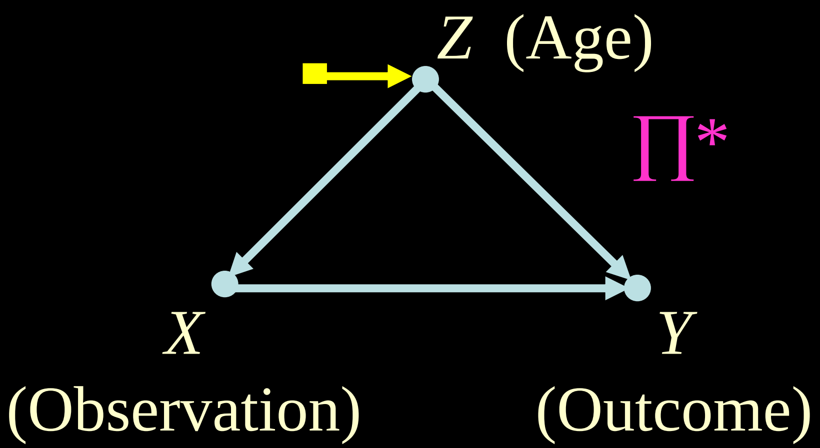
TRANSPORTABILITY ACROSS DOMAINS

1. A Theory of causal transportability
   When can causal relations learned from experiments be transferred to a different environment in which no experiment can be conducted?
2. A Theory of statistical transportability
   When can statistical information learned in one domain be transferred to a different domain in which
   a. only a subset of variables can be observed? Or,
   b. only a few samples are available?
3. Applications to Meta Analysis
   Combining results from many diverse studies

MOTIVATION

WHAT CAN EXPERIMENTS IN LA TELL ABOUT NYC?





Experimental study in LA

Measured: $P(x, y, z)$

$P(y \mid do(x), z)$

Observational study in NYC

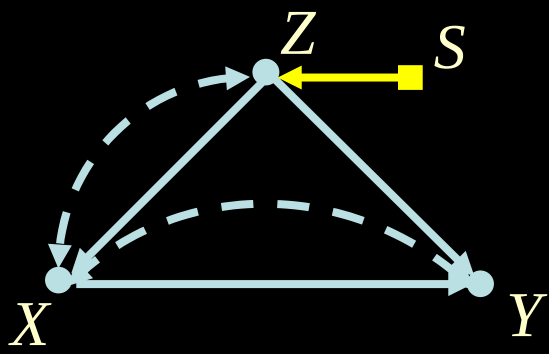
Measured: $P^*(x, y, z)$

$P^*(z) \neq P(z)$

Needed: $P^*(y \mid do(x)) = \; ? \; = \sum_z P(y \mid do(x), z) P^*(z)$

Transport Formula (calibration): $F(P, P_{do}, P^*)$

TRANSPORT FORMULAS DEPEND ON THE STORY

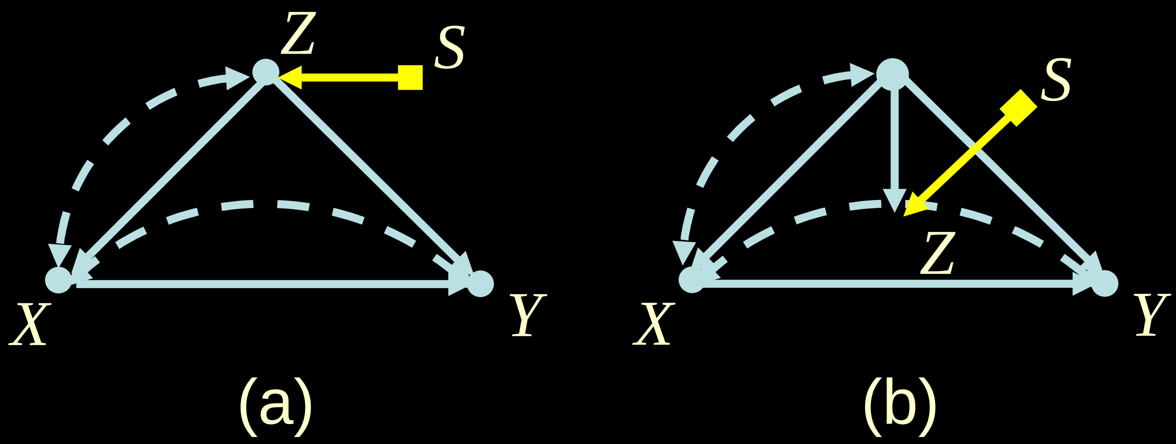


(a)

a) $\boldsymbol{Z}$ represents age

$$P^*(y \mid do(x)) = \sum_z P(y \mid do(x), z) P^*(z)$$

TRANSPORT FORMULAS DEPEND ON THE STORY



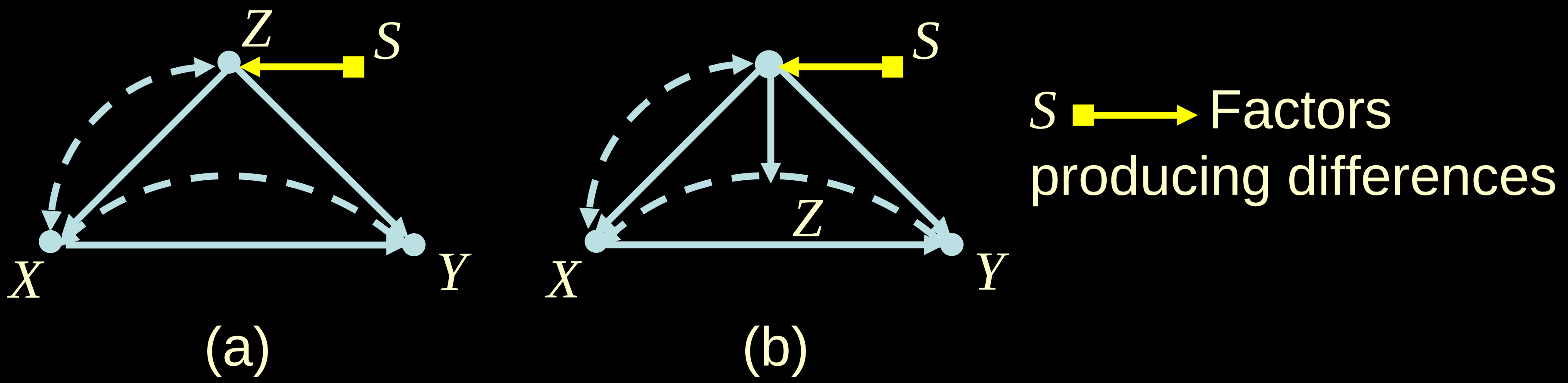
a) ***Z*** represents age

$$P^*(y \mid do(x)) = \sum_z P(y \mid do(x), z) P^*(z)$$

b) ***Z*** represents language skill

$$P^*(y \mid do(x)) = P(y \mid do(x))$$

TRANSPORT FORMULAS DEPEND ON THE STORY



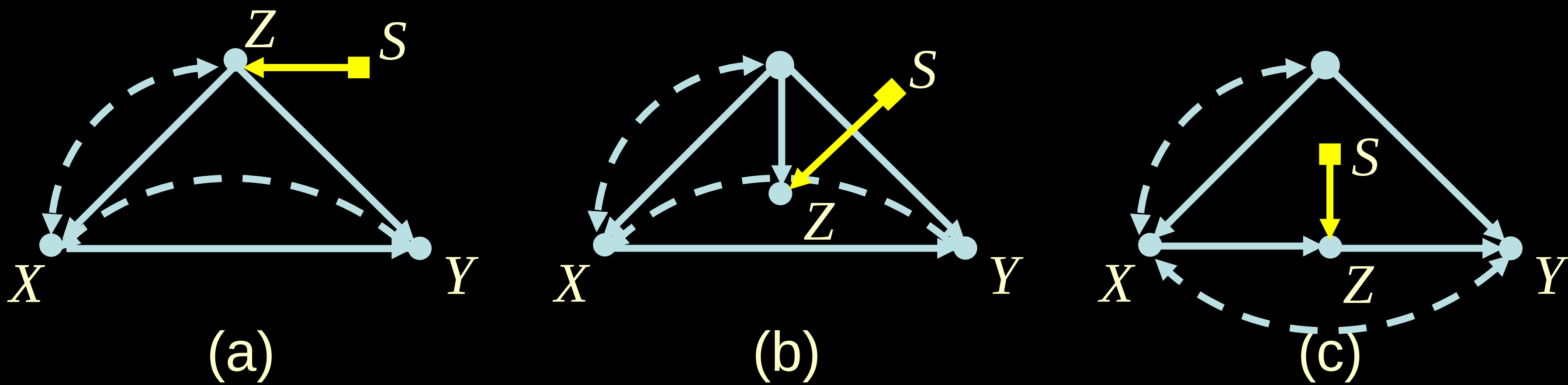
a) $\boldsymbol{Z}$ represents age

$$P^*(y \mid do(x)) = \sum_z P(y \mid do(x), z) P^*(z)$$

b) $\boldsymbol{Z}$ represents language skill

$$P^*(y \mid do(x)) = \ ?$$

TRANSPORT FORMULAS DEPEND ON THE STORY



a) ***Z*** represents age

$$P^*(y \mid do(x)) = \sum_z P(y \mid do(x), z) P^*(z)$$

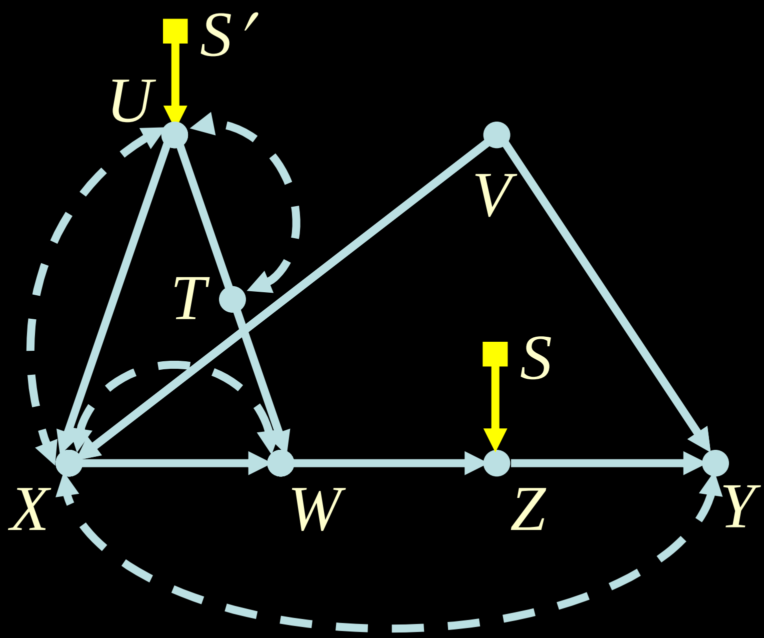
b) ***Z*** represents language skill

$$P^*(y \mid do(x)) = P(y \mid do(x))$$

c) ***Z*** represents a bio-marker

$$P^*(y \mid do(x)) = \sum_z P(y \mid do(x), z) P^*(z \mid x)$$

GOAL: ALGORITHM TO DETERMINE IF AN EFFECT IS TRANSPORTABLE
Back to Transportability



INPUT: Annotated Causal Graph

S → Factors creating differences

OUTPUT:

1. Transportable or not?
2. Measurements to be taken in the experimental study
3. Measurements to be taken in the target population
4. A transport formula

$$P^*(y \mid do(x)) =$$
$$f[P(y,v,z,w,t,u \mid do(x));\ P^*(y,v,z,w,t,u)]$$

TRANSPORTABILITY REDUCED TO CALCULUS

Theorem 1

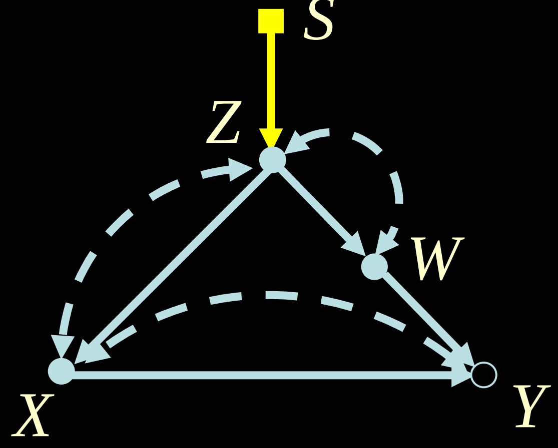
A causal relation ***R*** is transportable from Π to Π^* if and only if ***R*(Π*)** is reducible, using the rules of ***do***-calculus, to an expression in which ***S*** appears only as a conditioning variable in ***do***-free terms.

$$R(\Pi^*) = P^*(y \mid do(x)) = P(y \mid do(x), s)$$

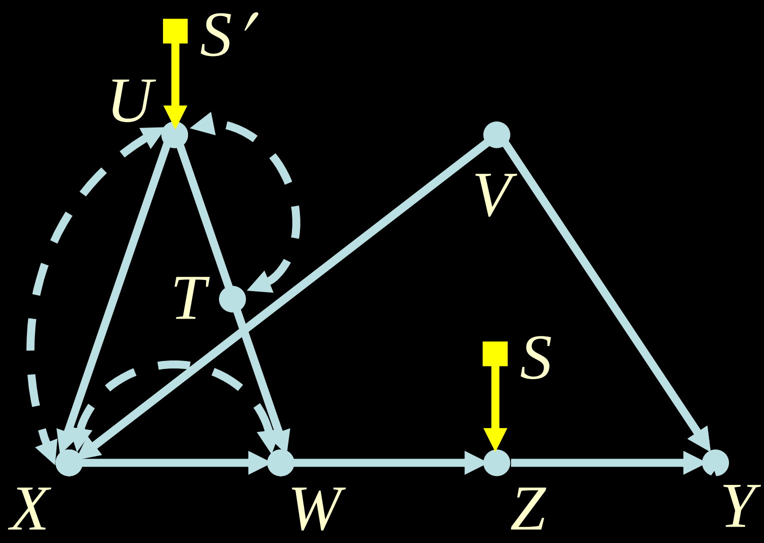
$$= \sum_w P(y \mid do(x), s, w) P(w \mid do(x), s)$$

$$= \sum_w P(y \mid do(x), w) P(w \mid s)$$

$$= \sum_w P(y \mid do(x), w) P^*(w)$$



RESULT: ALGORITHM TO DETERMINE IF AN EFFECT IS TRANSPORTABLE



INPUT: Annotated Causal Graph

S → Factors creating differences

OUTPUT:

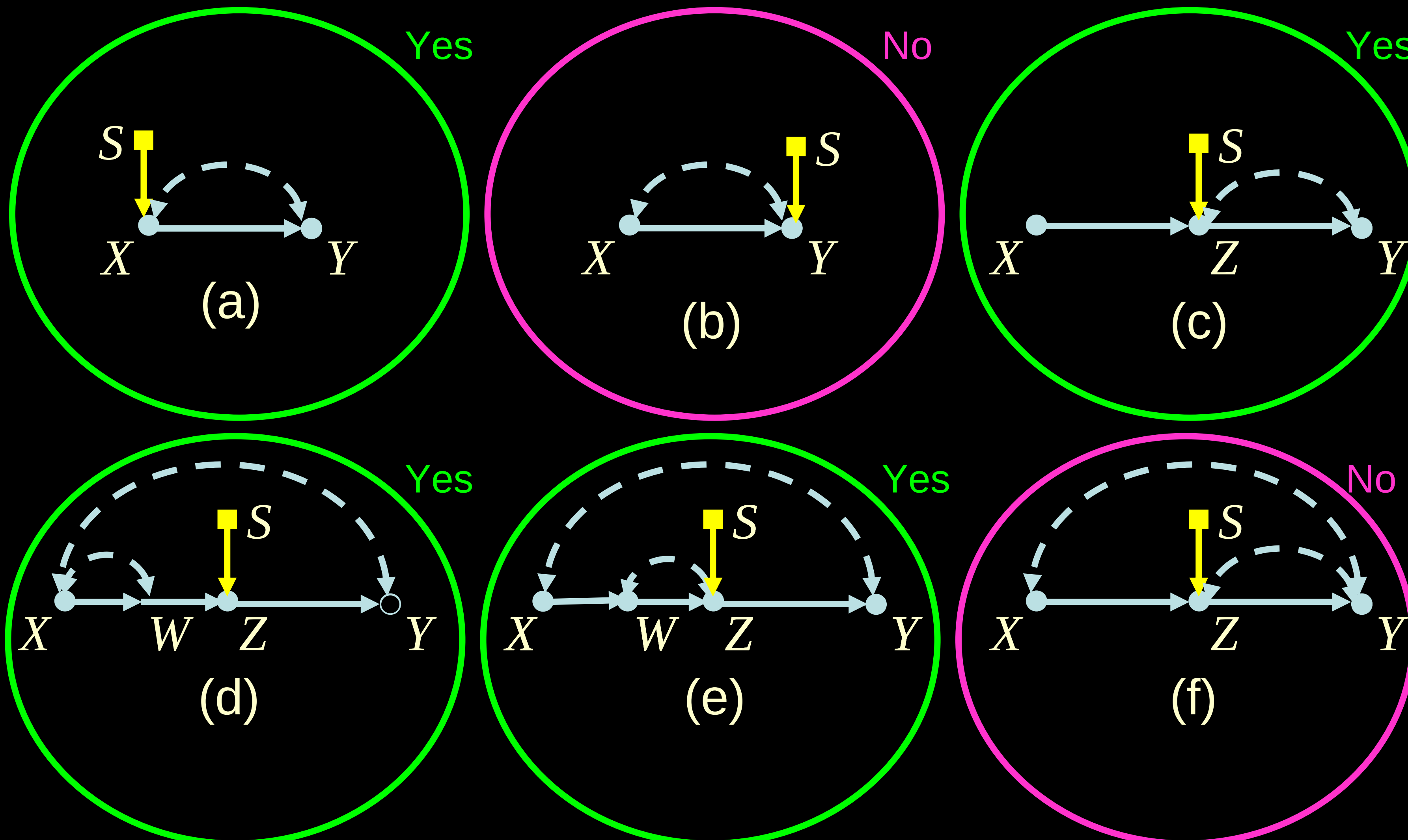
1. Transportable or not?
2. Measurements to be taken in the experimental study
3. Measurements to be taken in the target population
4. A transport formula

$$P^*(y \mid do(x)) =$$

$$\sum_z P(y \mid do(x), z) \sum_w P^*(z \mid w) \sum_t P(w \mid do(x), t)\, P^*(t)$$

WHICH MODEL LICENSES THE TRANSPORT OF THE CAUSAL EFFECT $X \rightarrow Y$

S External factors creating disparities



STATISTICAL TRANSPORTABILITY

Why should we transport statistical information?

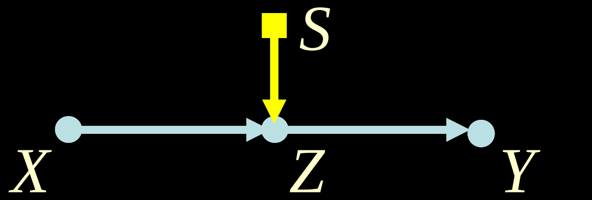
i.e., Why not re-learn things from scratch ?

1. Measurements are costly.
   Limit measurements to a subset V^* of variables called "scope".
2. Samples are scarce.
   Pooling samples from diverse populations will improve precision, if differences can be filtered out.

STATISTICAL TRANSPORTABILITY

Definition: (Statistical Transportability)

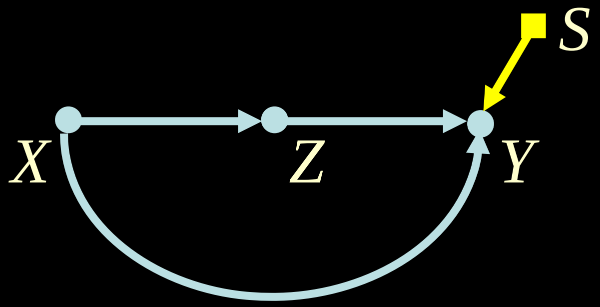
A statistical relation ***R*(*P*)** is said to be *transportable* from Π to Π***** over *V* * if ***R*(*P**)** is identified from ***P*, *P**(*V* *),** where ***V**** is a subset of variables.



R*=*P** (*y* | *x*)** is transportable over ***V* = **{*X*,*Z*},** i.e., ***R*** is estimable without re-measuring ***Y***

$$R = \sum_z P^*(z \mid x) P(z \mid y)$$

Transfer Learning



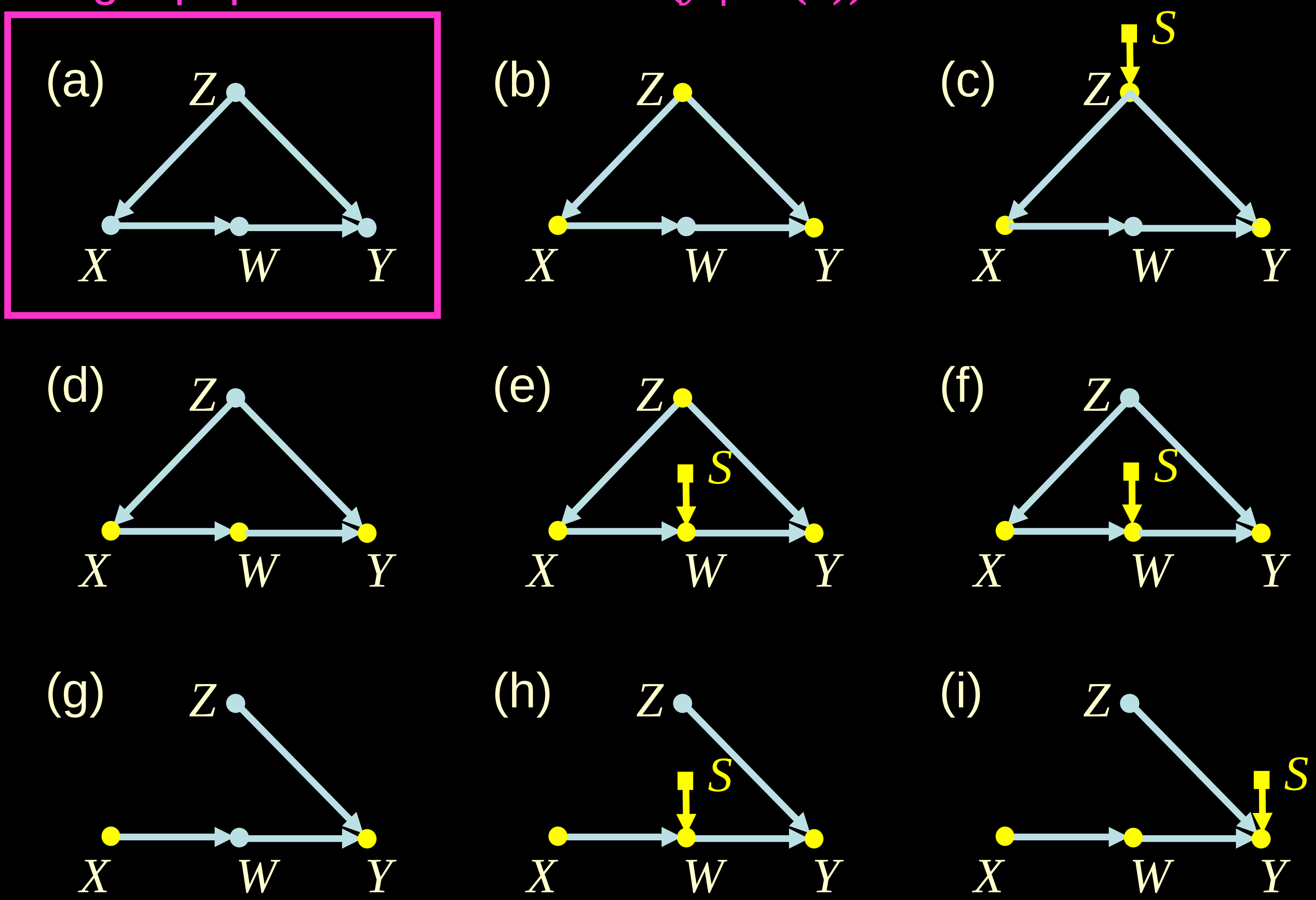
If few samples (N_2) are available from Π* and many samples (N_1) from Π, then estimating ***R*** = ***P**(*y* | *x*)** by

$$R = \sum_z P^*(y \mid x, z) P(z \mid x)$$

achieves a much higher precision

META-ANALYSIS OR MULTI-SOURCE LEARNING

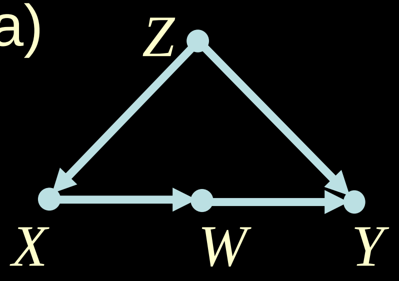
Target population Π^* $R = P^*(y \mid do(x))$



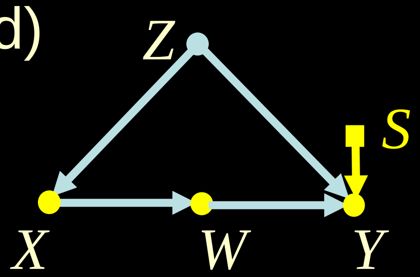
CAN WE GET A BIAS-FREE ESTIMATE OF THE TARGET QUANTITY?

Target population Π^* $\boldsymbol{R = P^*(y \mid do(x))}$

(a)



(d)



Is R identifiable from (d) and (h) ?

$$R = \sum_w P^*(y \mid do(x), w) P^*(w \mid do(x))$$

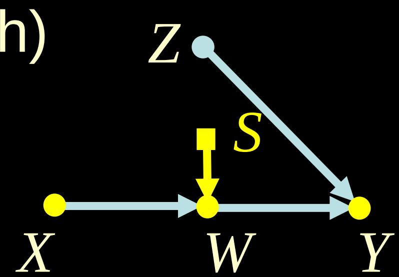
$$\sum_w P_{(h)}(y \mid do(x), w) P_{(d)}(w \mid do(x))$$

$$\sum_w P_{(h)}(y \mid do(x), w) P_{(d)}(w \mid x)$$

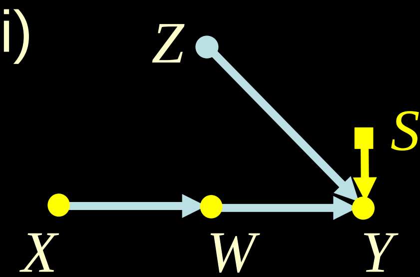
$R(\Pi^*)$ is identifiable from studies (d) and (h).

$R(\Pi^*)$ is not identifiable from studies (d) and (i).

(h)



(i)



FROM META-ANALYSIS TO META-SYNTHESIS

The problem

How to combine results of several experimental and observational studies, each conducted on a different population and under a different set of conditions, so as to construct an aggregate measure of effect size that is "better" than any one study in isolation.

Definition (Meta-Estimability)

A relation R is said to be "meta estimable" from a set of populations $\{\Pi_1, \Pi_2, ..., \Pi_K\}$ to a target population Π^* iff it is identifiable from the information set $I = \{I(\Pi_1), I(\Pi_2), ..., I(\Pi_K), I(P^*)\}$.

FROM META-ANALYSIS TO META-SYNTHESIS (Cont.)

Theorem

$\{\Pi_1, \Pi_2, \ldots, \Pi_K\}$ – a set of studies.

$\{D_1, D_2, \ldots, D_k\}$ – selection diagrams (relative to Π^*).

A relation $R(\Pi^*)$ is "meta estimable" if it can be decomposed into terms of the form:

$$Q_k = P(V_k \mid do(W_k), Z_k)$$

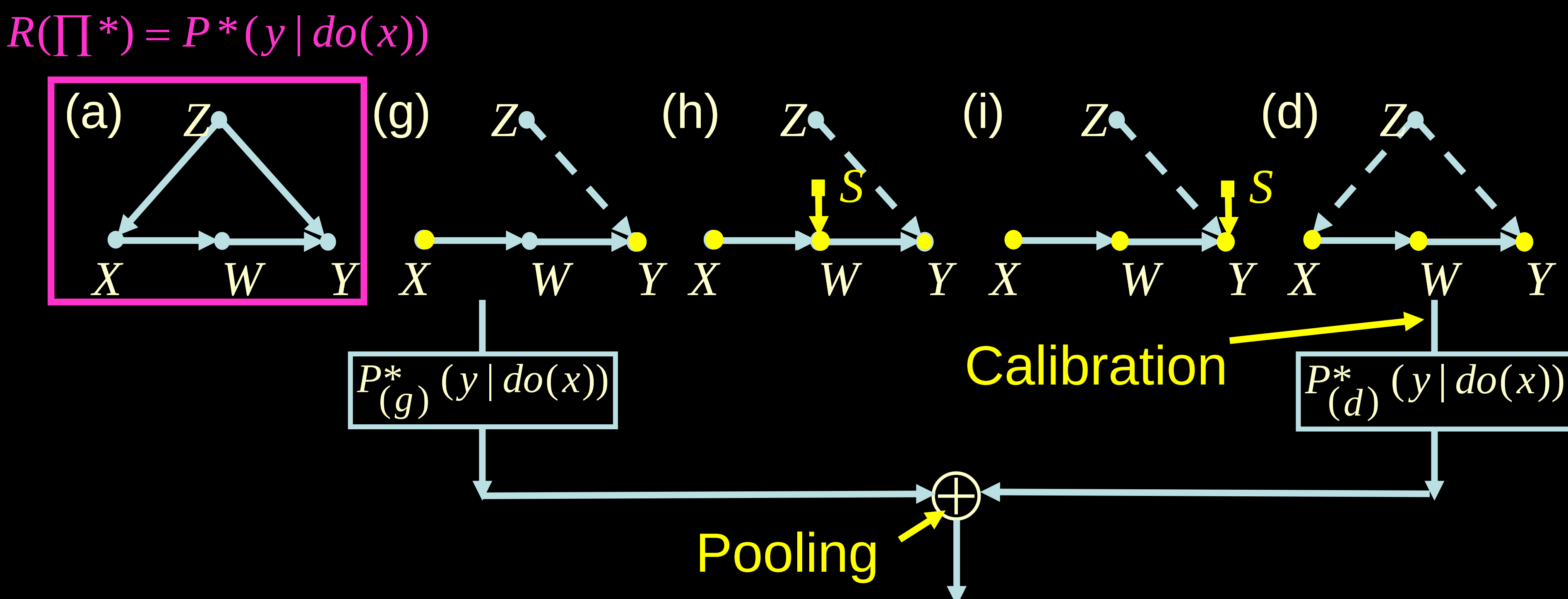
such that each Q_k is transportable from D_k.

Open-problem: Systematic decomposition

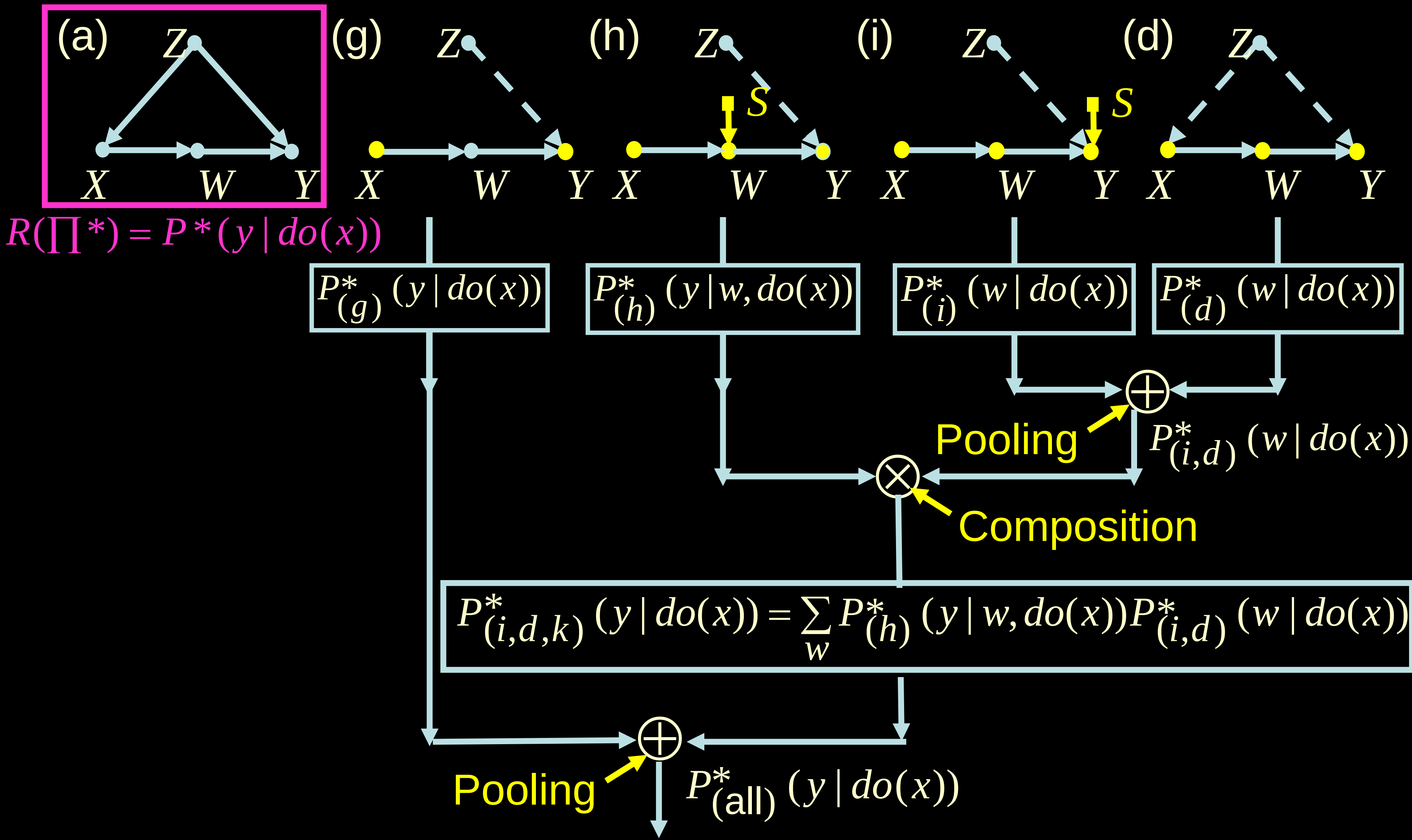
BIAS VS. PRECISION IN META-SYNTHESIS

Principle 1: Calibrate estimands before pooling (to minimize bias)

Principle 2: Decompose to sub-relations before calibrating (to improve precision)



BIAS VS. PRECISION IN META-SYNTHESIS (Cont.)



CONCLUSIONS

I TOLD YOU CAUSALITY IS SIMPLE

- Principled methodology for causal and counterfactual inference (complete)
- Unification of the graphical, potential-outcome and structural equation approaches
- Friendly and formal solutions to century-old problems and confusions.

CONCLUSIONS

He is wise who bases causal inference on an explicit causal structure that is defensible on scientific grounds.

(Aristotle 384-322 B.C.)

From Charlie Poole

QUESTIONS???

Now is the time!