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1). $T(n) = 3T(n-1) + 12n$

Given: $T(0) = 5$

Find $T(1) = ?$

put $n = 1$ —

$T(1) = 3T(0) + 12 \times 1$

$T(1) = 3 * 5 + 12$

$\boxed{T(1) = 27}$

$T(2) = 3T(1) + 12 \times 2$

$T(2) = 3 \times 27 + 24$

$\boxed{T(2) = 105}$

2). Subsitution method:-

a. $T(n) = T(n-1) + c$

$T(n-1) = T(n-2) + c$ —— ①

$T(n) = T(n-2) + 2c$ —— ②

$T(n) = T(n-3) + 3c$ —— ③

$\downarrow$ k times

$T(n) = T(n-k) + kc$

$k = n-1$

$T(n) = T(n-n+1) + (n-1)c$

$T(n) = T(1) + cn - c$

$T(n) = 1 + cn - c$

$\boxed{T(n) \rightarrow O(n)}$

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b. $T(n) = 2\,T(n/2) + n$ ——— ①

$T(n) = 4\,T\left(\frac{n}{2^2}\right) + 2n$ ——— ②

$T(n) = 16\,T\left(\frac{n}{2^3}\right) + 3n$ ——— ③

| k Times

$\cancel{T(n)} = 16\,T\left(\frac{n}{2^k}\right) + kn$

$\frac{n}{2^k} = 1$

$n = 2^k$

$\log_2 n = k \cancel{\log_2 2}$

$\boxed{k = \log_2 n}$

$T(n) = 16\,T\left(\frac{n}{2^{\log_2 n}}\right) + \log_2 n \cdot n$

$= 16\,T\left(\frac{\cancel{n}}{\cancel{n \log_2 2}}\right) + n \cdot \log_2 n$

$T(n) = 16\,T(1) + n \cdot \log_2 n$

$T(n) = 16 + n\log_2 n$

$\boxed{O(n\log_2 n)}$

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c. $T(n) = 2T(n/2) + c$ —— ①

$T(n) = 4T\left(\frac{n}{2^2}\right) + 2c$ —— ②

$T(n) = 16T\left(\frac{n}{2^3}\right) + 3c$ —— ③

$T(n) = 16T\left(\frac{n}{2^k}\right) + kc$ —— k Times

$\frac{n}{2^k} = 1$

$n = 2^k$

$\log_2 n = k \log_2 2$

$\boxed{k = \log_2 n}$

$T(n) = 16T\left(\frac{n}{2^{\log_2 n}}\right) + \log_2 n \cdot c$

$T(n) = 16T(1) + \log_2 n \cdot c$

$T(n) = 16 + c \cdot \log_2 n$

$\boxed{O(\log_2 n)}$

d. $T(n) = T(n/2) + c$

$T(n) = T\left(\frac{n}{2^2}\right) + 2c$

$T(n) = T\left(\frac{n}{2^3}\right) + 3c$

$$T(n) = T\left(\frac{n}{2^k}\right) + k \cdot c$$

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

$$\log_2 n = k \log_2 2$$

$$\boxed{\log_2 n = k}$$

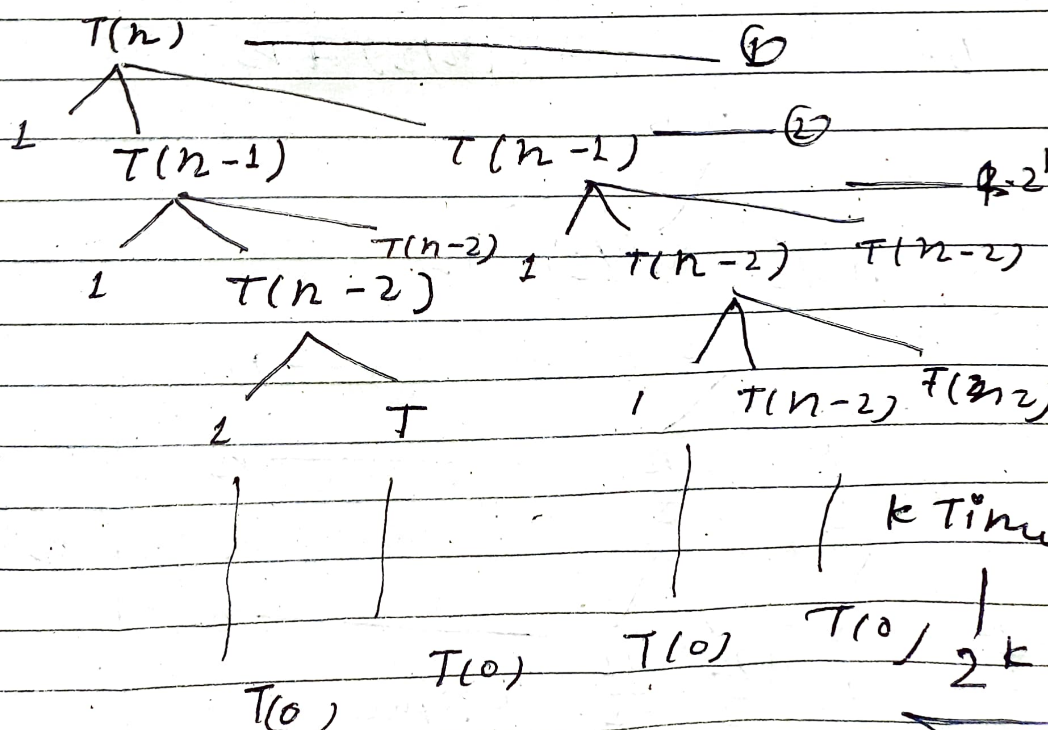
$$T(n) = T\left(\frac{\cancel{n}}{\cancel{2^{\log_2 n}}}\right) + \log_2 n \cdot c$$

$$T(n) = T(1) + \log_2 n \cdot c$$

$$\boxed{O(\log_2 n)}$$

3. Recursive tree -

a. $T(n) = 2T(n-1) + 1$



$$1+2+2^2+2^3+ \cdots 2^k = 2^{k+1}-1$$

∵ GP series –

$$a + ar + ar^2 + \cdots ar^k = a\left(\frac{r^{k+1}}{r-1}\right)$$

$a = 1, \; r = 2$

$$= 1\left(\frac{2^{k+1}}{2-1}\right)$$

$$= 2^{k+1}$$

assum –

$n - k = 0$

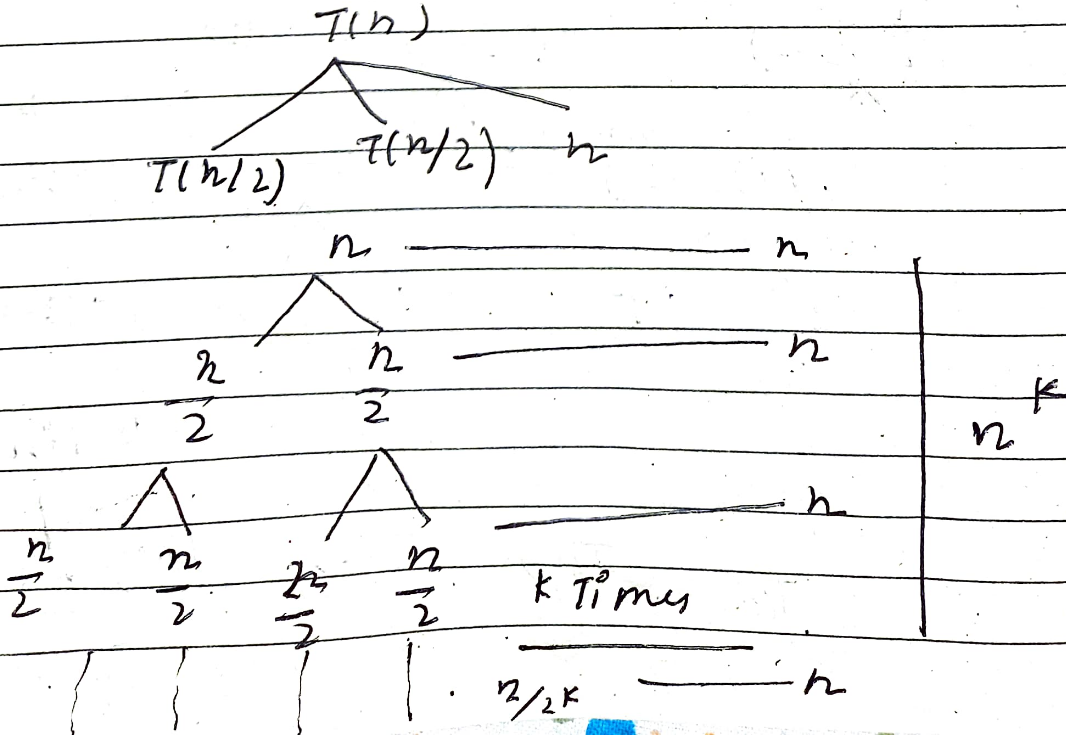
$n = k$

$$1+2+2^2+ \cdots 2^k = 2^{k+1}-1$$

$$= 2^{n+1}-1$$

$$\boxed{T(n) = O(2^n)}$$

b. $T(n) = 2T(n/2) + n$



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assum - $\frac{n}{2^k} = 1$

$n = 2^k$

$\boxed{k = \log n}$

$\boxed{T(n) = n \log n}$