Signal & system

Syllabus →

(1.) signal definations & its classifications.

(2.) Different operation on signal.

- (a.) Shifting
- (b.) Scaling
- (c.) Reversal
- (d.) Differentiation
- (e.) Integration
- (f.) Convolution.

(3.) Basic system operations.

- (a.) Static/dynamic
- (b.) linear/non-linear
- (c.) Causal/Non-causal
- (d.) Time invarient/time-varient
- (e.) Stable/Unstable.

(4.) Contineous time fourier series
(5.) Contineous time fourier Xform
(6.) Laplace Xform.
→ contineous time sig & sys.

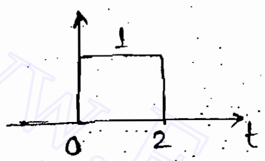
(7.) Sampling theorem
(8.) Discrete time sys.
(9.) Z-transform
→ discrete time sig & sys.

* Different operations on signal →

* Amplitude shifting
* Time shifting
* Time scaling
* Time reversal
* Amplitude Reversal
*

(1.) Time shifting →

$$x(t) \longrightarrow y(t) = x(t+k)$$

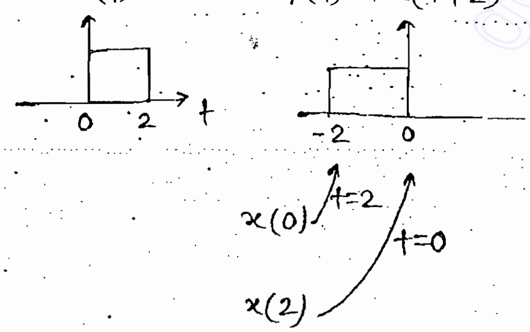


Case(1)

when $k>0$

Eg:- $k=2$

$$x(t) \longrightarrow y(t) = x(t+2)$$



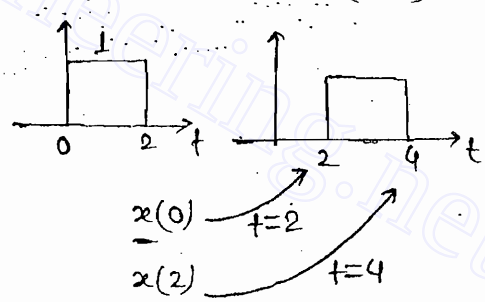
* It is a case of <u>left shifting</u>.

Case(2)

when $k<0$

Eg:- $k=-2$

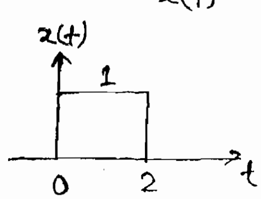
$$x(t) \longrightarrow y(t) = x(t-2)$$



* It is a case of <u>right shifting</u>

(2.) Amplitude Shifting →

$$x(t) \longrightarrow y(t) = k + x(t)$$



$x(t) = \begin{cases} 0 & , t<0 \end{cases}$

Case(1) → When $k<0$

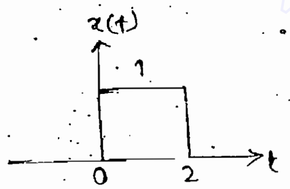
Eg:- $k=-1$

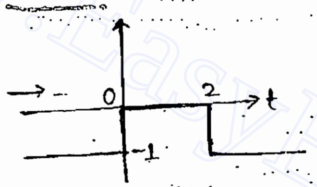
$x(t) \longrightarrow y(t) = -1 + x(t)$

$$y(t) = -1 + x(t)$$

$$= \begin{cases} -1+0 & , t<0 \\ -1+1 & ; 0 \le t \le 2 \\ -1+0 & ; t>2 \end{cases}$$

$$= \begin{cases} -1 & , t<0 \\ 0 & ; 0 \le t \le 2 \\ -1 & ; t>2 \end{cases}$$





* It is a case of downward shifting

Case(2) → When $k>0$

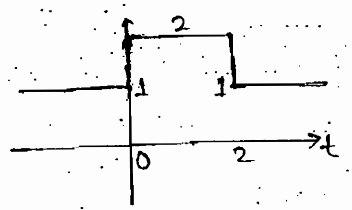
Eg:- $k=+1$

$x(t) \longrightarrow y(t) = 1 + x(t)$

$$y(t) = 1 + x(t)$$

$$= \begin{cases} 1+0 & , t<0 \\ 1+1 & ; 0 \le t \le 2 \\ 1+0 & ; t>2 \end{cases}$$

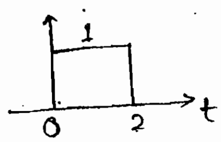
$$= \begin{cases} 1 & ; t<0 \\ 2 & ; 0 \le t \le 2 \\ 1 & ; t>2 \end{cases}$$



* It is a case of upward shifting

(3.) Time Scaling →

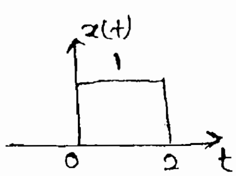
$x(t) \longrightarrow y(t) = x(at)$

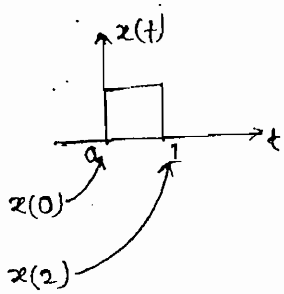


Case(1) → When $a>1$

Ep:- $a=2$

$x(t) = y(t) = x(2t)$



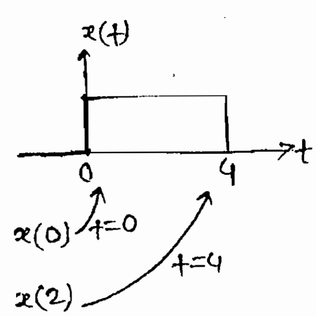


Time Compression

Case(2) → When $a<1$

Eg:- $a=0.5$

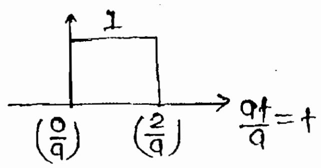
$x(t) = y(t) = x(0.5t)$



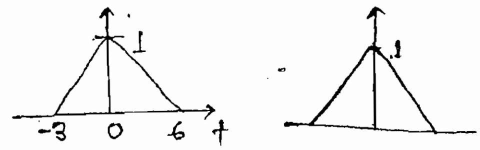
Time expansion

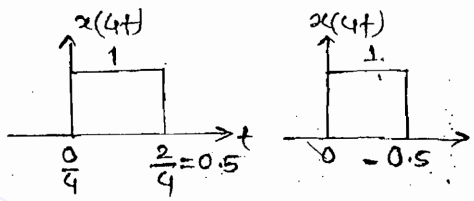
Rule General →

$x(at) \longrightarrow t$



Ex:- $x(t) \longrightarrow x(-3t)$

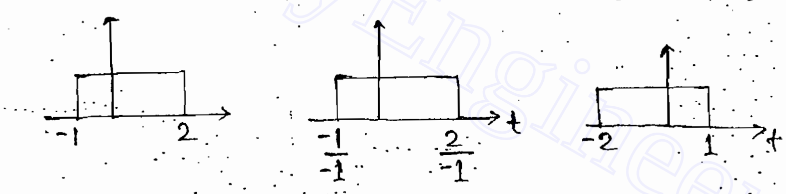




(4.) Time-reversal →

$$x(t) = y(t) = x(-t)$$

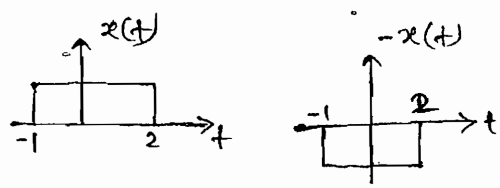
* Time reversal is a special case of time scaling in which signal folding will take palce around y-axis. $x(-t) = a(=-1)$



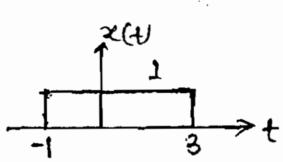
(5.) Amplitude Reversal →

$$x(t) \longrightarrow y(t) = -x(t)$$

* In this case, signal folding will take place about x-axis



Q.→



Draw signal $y(t)$ if $y(t) = 2x(2t+1)$

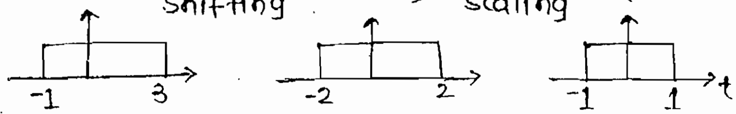
Sol^n→ 1st method→

$$x(t) \xrightarrow[\text{scaling}]{\text{time}} x(2t) \xrightarrow[\text{shifting}]{\text{time}} x[2(t+0.5)]$$

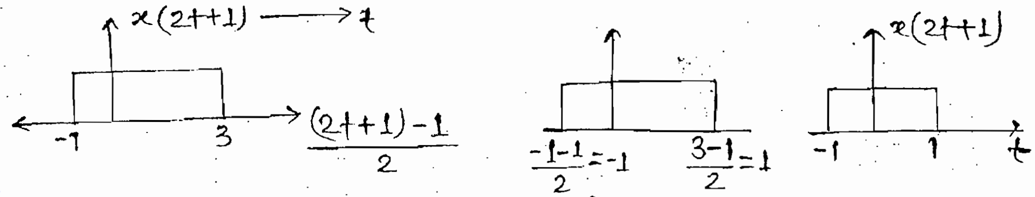
↑ 1 ↑ ↑ y(t)

2nd method →

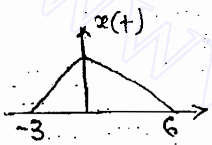
$x(t) \xrightarrow[\text{shifting}]{\text{Time}} x(t+1) \xrightarrow[\text{scaling}]{\text{time}} x(2t+1)$



3rd method → (Trick)



Q. →

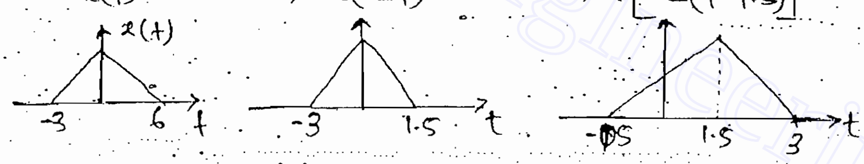


draw sig $y(t)$ if $y(t) = x(-2t+3)$

Soln → 1st method →

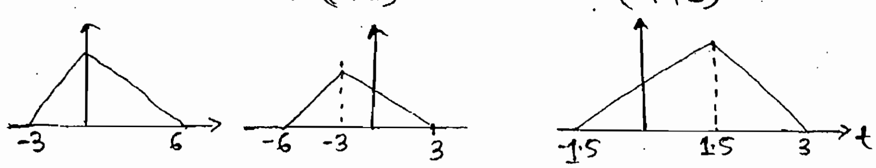
$y(t) = x[-2(t-1.5)]$

$x(t) \longrightarrow x(-2t) \longrightarrow x[-2(t-1.5)]$

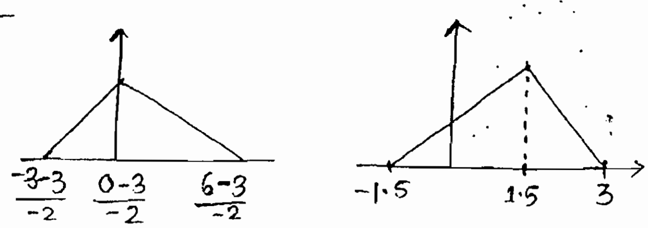


2nd method →

$x(t) \longrightarrow x(t+3) \longrightarrow x(2t+3)$



3rd method →



Chapter-01
Signal defination & Classifications

<u>Signal</u> → A signal is a f^n which contains some information.

<u>System</u> → A sys. is interconnection of devices (or) componants which converts signal from one form to another form.

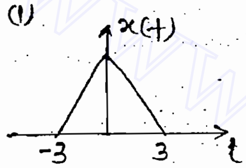
Classification of signals →

1) Even & odd signals →

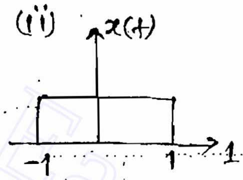
* <u>Even</u> → This are <u>symmetrical</u> (or) <u>mirror image</u> about y-axis.

i.e. → $\boxed{x(t) = x(-t)}$ → time reversal

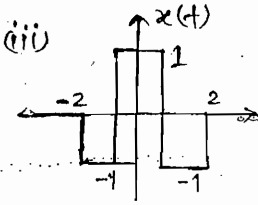
<u>Eg</u>:- (i)



(ii)



(iii)



(iv) $x(t) = \cos\omega_0 t$ (even)

$t = -t$

$x(-t) = \cos\omega_0(-t)$

$= \cos\omega_0 t$

$x(-t) = x(t)$

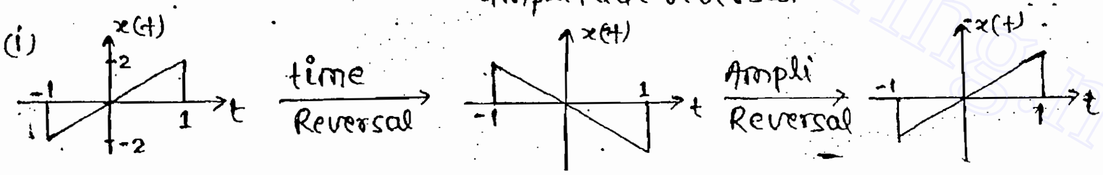
* <u>Odd</u> → This are <u>antisymmetrical</u> about y-axis.

i.e. $\boxed{\begin{matrix} x(-t) = -x(t) \\ (or) \\ x(t) = -x(-t) \end{matrix}}$

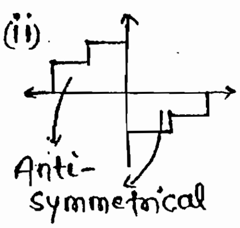
time reversal

amplitude reversal

<u>Eg</u>:- (i)



(ii)



(iii) $x(t) = \sin\omega_0 t$ → odd signal.

$(t = -t)$

$x(-t) = \sin\omega_0(-t)$

$x(-t) = -\sin\omega_0 t$

$\boxed{x(-t) = -x(t)}$

* The avg. value of an odd signal is 0; but converse of this statement is not true.

<u>Important points</u> →

Important points →

(1.) Even × Even = Even ; $t^2 \times t^4 = t^6$

(2.) Even × odd = odd ; $t^2 \times t^3 = t^5$

(3.) Odd $\times$ odd = Even ; $t^3 \times t^5 = t^8$

(4.) Even ± Even = Even

$x(t) = t^2 + \cos t$

$x(-t) = t^2 + \cos t = x(t)$

(5.) Odd + Odd = Odd

$x(t) = \sin t + t^3$

$x(-t) = -\sin t - t^3$

$\boxed{x(-t) = -x(t)}$

(6.) Even + odd = Neither even nor odd.

$x(t) = t^2 + \sin t$

$x(-t) = t^2 - \sin t$

$\boxed{x(-t) \neq x(t)}$

* Any signal can be devided into 2 part in which one part will be even & the other part will be odd.

i.e. $\boxed{x(t) = x_E(t) + x_O(t)}$

where;

$x_E(t)$ = even part of $x(t)$ $= \dfrac{x(t) + x(-t)}{2}$

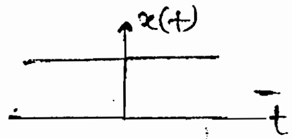
$x_O(t)$ = Odd part of $x(t)$ $= \dfrac{x(t) - x(-t)}{2}$

Eg.→ $x(t) = 2$ = dc signal

$\downarrow$ $t = -t$

$x(-t) = 2 = x(t)$ [Even signal]

dc signal is a Even signal.



(2.) $f(k) = \sin(k^2)$

$\downarrow k = -k$

$f(-k) = \sin(k^2) = f(k)$ [Even signal]

(3.) $f(r) = \sin \pi/2$

$= 1$

$f(r) = f(-r)$ [Even signal]

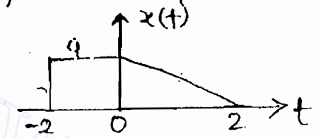
(4.) Find $x_E(t)$ & $x_O(t)$ of the signal.

$$x(t) = 3 - \frac{t^2}{\sin t} + \frac{\cos t}{t} - \frac{\sin^2 t}{t^4} + t^3 \sin^3 t$$

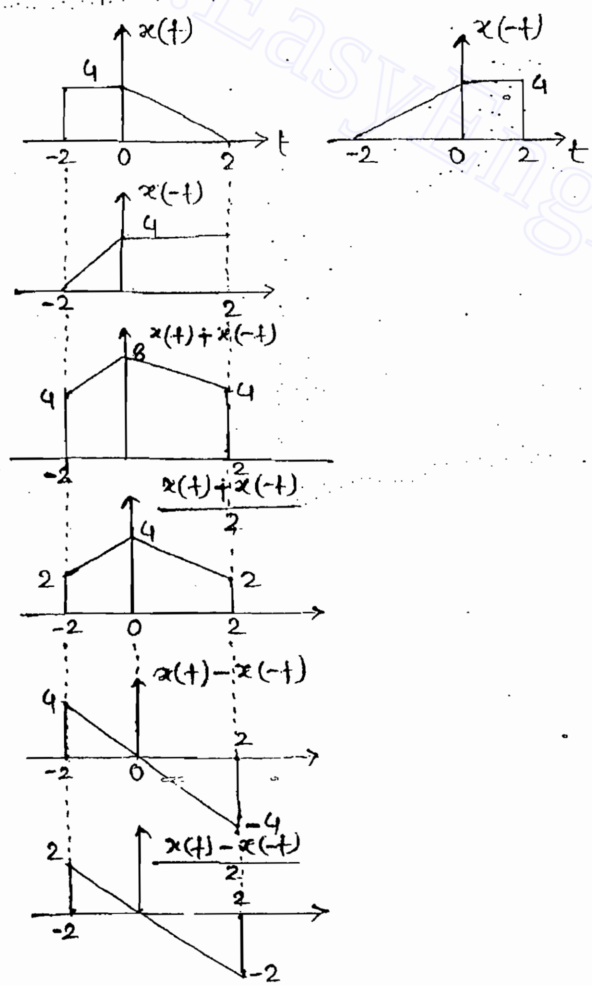
$$\begin{array}{ccccc} E - \frac{E}{O} & + \frac{E}{O} & - \frac{E}{E} & + O \times O \\ \downarrow \quad \downarrow & \downarrow & \downarrow & \downarrow \\ E \quad O & O & E & E \end{array}$$

$$x_e(t) = 3 - \frac{\sin^2 t}{t^4} + t^3 \sin^3 t \quad , \quad x_O(t) = \frac{-t^2}{\sin t} + \frac{\cos t}{t}$$

Que. → Draw $x_E(t)$ & $x_O(t)$ of



Sol^n → for even part of $x(t)$



(2) Conjugate symmetric (CS) & Conjugate antisymmetric (CAS) signal →

★ Conjugate symmetric (CS)

$$\boxed{x(t) = x^*(-t)}$$

$x(t) = a(t) + jb(t)$ ——— (i)

$(t = -t)$

$x(-t) = a(-t) + jb(-t)$

$x^*(-t) = a(-t) - jb(-t)$ ——— (ii)

From eq^n (i) & (ii)

$a(t) = a(-t) \longrightarrow$ Even

$b(t) = -b(-t) \longrightarrow$ Odd

Eg:- $x(t) = t^2 + \sin t$

$\underbrace{t^2}_{E} \quad \underbrace{\sin t}_{O}$

★ Conjugate antisymmetric (CAS)

$$\boxed{x(t) = -x^*(-t)}$$

$x(t) = a(t) + j\,b(t)$

$a(t) = -a(-t) \longrightarrow$ Odd

$b(t) = b(-t) \longrightarrow$ Even

Eg:- $x(t) = \sin t + jt^2$

$\underbrace{\sin t}_{O} \quad \underbrace{t^2}_{E}$

(3) Halfwave symmetric signal (HWS) →

For Half-wave symmetry (HWS)

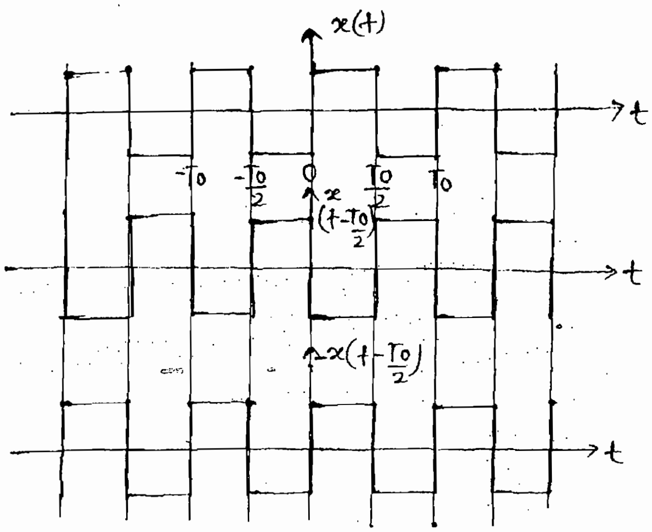
$$\boxed{x(t) = -x\left(t \pm \frac{T_0}{2}\right)}$$

time shifting

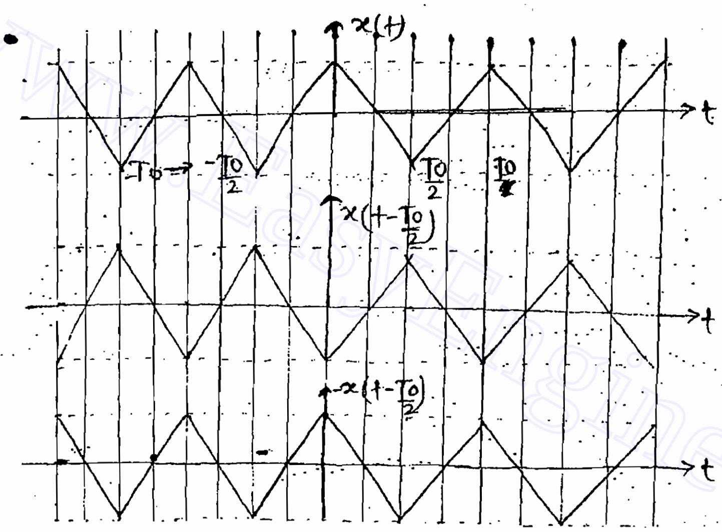
amp. reversal

Eg →

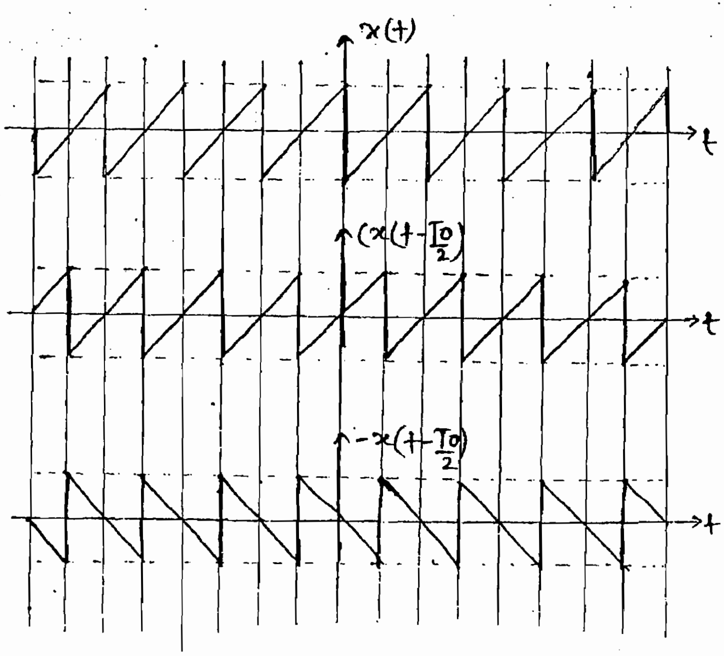
(1.)



(2.)



(3.)



so; sawtooth wave doesn't follow the HWS.

* The avg. value of a HWS is 0. but converse of this statement is not true.

$\boxed{\text{DATE-10/10/14}}$

(4.) Periodic & non-periodic signal →

Periodic → A signal repeats itself after some time period, the signal is said to be periodic.

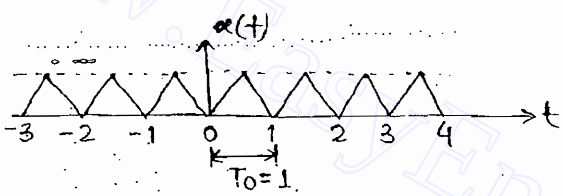
i.e. $\boxed{x(t) = x(t \pm nT_0)}$

where, n = an integer

T_0 = Fundamental time period. $\left\{ \begin{matrix} T_0 \neq 0 \\ T_0 \neq \infty \end{matrix} \right\}$

FTP → It is the smallest, +ve & fixed value of the time for which signal is periodic.

Eg →



FTP = 1

Q → Find FTP of signal $x(t)$

$$x(t) = A_0 e^{j\omega_0 t}$$

Sol^n → Let 'T_0' be the FTP of the signal

i.e.

$$x(t) = x(t+T_0)$$

$$x(t+T_0) = A_0 e^{j\omega_0(t+T_0)}$$

$$A_0 e^{j\omega_0 t} = A_0 e^{j\omega_0(t+T_0)}$$

$$\cancel{A_0 e^{j\omega_0 t}} = \cancel{A_0 e^{j\omega_0 t}} e^{j\omega_0 T_0}$$

$$e^{j\omega_0 T_0} = 1 = e^{j2\pi K} \quad \text{(where } K = \text{an integer)}$$

$$j\omega_0 T_0 = j2\pi K$$

$$\underset{\text{(smallest)}}{T_0} = \frac{2\pi \cancel{K}^{\,1 \text{(least integer)}}}{\omega_0}$$

$$\boxed{T_0 = \frac{2\pi}{\omega_0}}$$

Q.→ Find FTP of following signal →

(i.) $x_1(t) = A_0 \sin(2\pi t)$

$\omega_0 = 2\pi$

$T_0 = \frac{2\pi}{2\pi} = 1$

(ii) $x_2(t) = A_0 \sin(2\pi t + 30^\circ)$

$\omega_0 = 2\pi$

$T_0 = 1$

(iii) $x_3(t) = -x_1(t)$

$= -A_0 \sin(2\pi t)$

$\omega_0 = 2\pi$, $T_0 = 1$

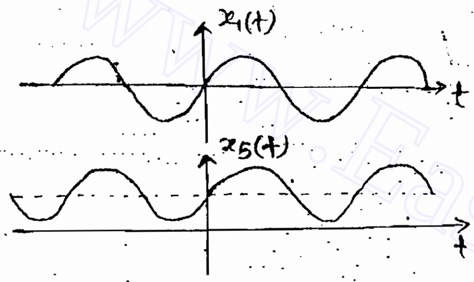
(iv) $x_4(t) = x_1(-t)$

$= -A_0 \sin 2\pi t$

$\omega_0 = 2\pi$, $T_0 = 2\pi$

(v.) $x_5(t) = A_0 + x_1(t)$

$= A_0 + A_0 \sin(2\pi t)$



(vi) $x_6(t) = x_1(t - t_0)$

$= A_0 \sin[2\pi(t - t_0)]$

$\omega_0 = 2\pi$

$T_0 = 1$

* Time period of signal is unaffected by time shifting, time reversal, amp. reversal, amp. shifting & change in phase of signal.

(vii) $f(t) = \sin^2(4\pi t)$

$= \frac{1 - \cos 8\pi t}{2}$

$\omega_0 = 8\pi$

$T_0 = \frac{2\pi}{8\pi} = \frac{1}{4}$

* The sum of 2 (or) more than 2 periodic signal will be periodic if ratios of there fundamental time period (or) freq. are rational.

i.e. $x(t) = x_1(t) + x_2(t)$

$x_1(t) \rightarrow T_1, f_1, \omega_1$

$x_2(t) \rightarrow T_2, f_2, \omega_2$

→ $\frac{T_1}{T_2}$ (or) $\frac{\omega_1}{\omega_2}$ (or) $\frac{f_1}{f_2}$ (Rational no.)

→ $T_0 = \text{LCM}[T_1, T_2]$

→ $f_0 = \text{HCF}[f_1, f_2]$

Q.→ Find FTP of signal if it is periodic :-

(i) $x(t) = \sin 2t + \cos 3\pi t$

Soln → $\omega_1 = 2$ $\quad \frac{\omega_1}{\omega_2} = \frac{2}{3\pi}$ (Irrational no)

$\omega_2 = 3\pi$

Hence it is non-periodic

(ii) $x(t) = \sin 2\pi t + \cos\sqrt{2}\pi t$

Soln → $\omega_1 = 2\pi, \quad \omega_2 = \sqrt{2}\pi$

$\frac{\omega_1}{\omega_2} = \frac{2\pi}{\sqrt{2}\pi} = \sqrt{2}$ (Irrational no.)

Hence it is Non-periodic

(iii) $x(t) = \sin 4\pi t + \sin 7\pi t$

Soln → $\omega_1 = 4\pi, \quad \omega_2 = 7\pi$

$\frac{\omega_1}{\omega_2} = \frac{4\pi}{7\pi} = \frac{4}{7}$ (Rational no.)

Hence it is periodic. Then calculate To.

1st method :-

$\omega_0 = \text{HCF}[\omega_1, \omega_2] = \text{HCF}[4\pi, 7\pi]$

$\omega_0 = \pi$

$T_0 = \frac{2\pi}{\omega_0} = 2$

*** $\text{HCF}\left[\frac{P_1}{q_1}, \frac{P_2}{q_2}\right] = \frac{\text{HCF}[P_1, P_2]}{\text{LCM}[q_1, q_2]}$ $\quad \text{LCM}\left[\frac{P_1}{q_1}, \frac{P_2}{q_2}\right] = \frac{\text{LCM}[P_1, P_2]}{\text{HCF}[q_1, q_2]}$

2nd method →

$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{4\pi} = \frac{1}{2}$ $\quad T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{7\pi} = \frac{2}{7}$

$T_0 = \text{LCM}[T_1, T_2] = \text{LCM}\left[\frac{1}{2}, \frac{2}{7}\right]$

$= \frac{\text{LCM}[1,2]}{\text{HCF}[2,7]} = \frac{2}{1} = 2$

* Area & avg. value of signal →

Area of $x(t)$:-

$$\text{Area} = \int_{-\infty}^{\infty} x(\tau)\,d\tau$$

Area of $x(t)$ over Range (t_1, t_2)

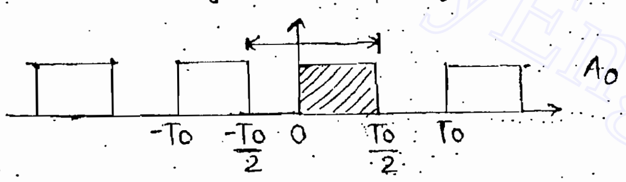
$$\text{Area} = \int_{t_1}^{t_2} x(\tau)\,d\tau$$

Avg. value of $x(t)$:

$$\text{Avg.} = \begin{cases} \dfrac{1}{T_0}\displaystyle\int_{-T_0/2}^{T_0/2} x(\tau)\,d\tau, & \text{For periodic sig.} \\ \displaystyle\lim_{T\to\infty} \dfrac{1}{T}\int_{-T/2}^{T/2} x(\tau)\,d\tau, & \text{for Non-periodic sig.} \end{cases}$$

Que.→ Find the avg. value of sig.

(i)



Sol^n →

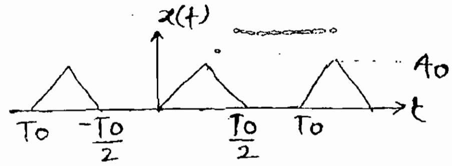
$$\text{avg.} = \frac{\int_{-T_0/2}^{T_0/2} x(\tau)\,d\tau}{T_0}$$

$$= \frac{\text{Area of } x(t) \text{ over '}T_0\text{'}}{T_0}$$

$$= \frac{A_0 \times \frac{T_0}{2}}{T_0}$$

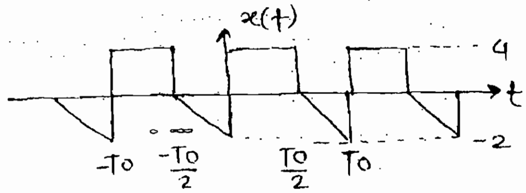
$$= \frac{A_0}{2}$$

(2.)



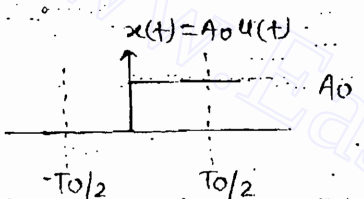
Soln →
$$\text{avg.} = \frac{\text{Area over } T_0}{T_0} = \frac{1/2 \times A_0 \times T_0/2}{T_0} = \frac{A_0}{4}$$

(3.)



Soln →
$$\text{Avg.} = \frac{\text{Area over } T_0}{T_0} = \frac{-1/2 \times \frac{T_0}{2} \times 2 + 4 \times \frac{T_0}{2}}{T_0} = \frac{3}{2}$$

(iv)



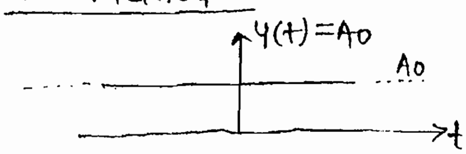
Soln →
$$\text{Avg.} = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(\tau)\, d\tau$$

$$= \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{0}^{T_0/2} A_0 \, d\tau$$

$$= \lim_{T_0 \to \infty} \frac{A_0 \times T_0/2}{T_0}$$

$$= \frac{A_0}{2}$$

2nd method →



$$\text{avg } y(t) = A_0$$

$$\text{avg } x(t) = \frac{\text{avg } y(t)}{2}$$

$$= \frac{A_0}{2}$$

(5.) Energy & power signal →

* Energy of $x(t) = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$$\boxed{E = \int_{-\infty}^{\infty} |x(t)|^2 dt}$$

* Power of $x(t)$

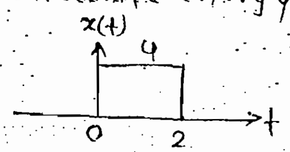
$$P = \begin{cases} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x(t)|^2 dt \; ; & \text{For periodic sig.} \\ \lim_{T_0 \to \infty} \int_{-T_0/2}^{T_0/2} |x(t)|^2 dt ; & \text{Non periodic sig.} \end{cases}$$

* For an energy sig., energy should be finite & power should be zero.
* Energy signals are absoulately integrable signal.

i.e. $\boxed{\int_{-\infty}^{\infty} |x(t)| dt < \infty}$

Q.→ Calculate energy of sig.

(i)



Sol^n→ $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

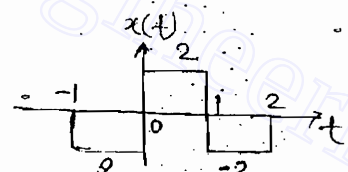
$= \int_0^2 4^2 dt = 32$

2nd method →

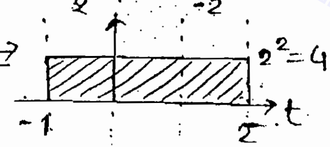
$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$= 16 \times 2 = 32$

(ii)



Sol^n→

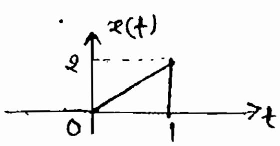


$E_{x(t)}$ = area of $|x(t)|^2$

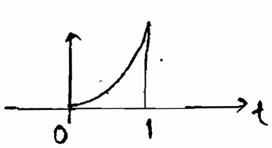
$= 4 \times 3$

$= 12$

(iii)



Sol^n→

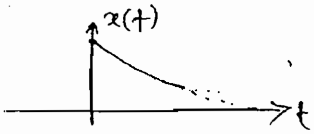


$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$= \int_0^1 (2t)^2 dt = \frac{4}{3}$

Q.→ Cal. area & energy of signal:-

(i) $x(t) = e^{-at}u(t),\ a>0$



Sol^n→

$$\text{area} = \int_{-\infty}^{\infty} x(t)\,dt$$

$$= \int_{0}^{\infty} e^{-at}\,dt$$

$$= \left(\frac{e^{-at}}{-a}\right)_0^{\infty} = \frac{e^{-a\infty} - e^{0}}{-a}$$

$\because e^{-a\infty} = 0,\ a>0\ \ (a=2)$

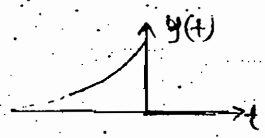
$e^{-2\infty} = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$

$$= \frac{0-1}{-a} = \frac{1}{a}$$

$$\text{Energy} = \int_{-\infty}^{\infty} |x(t)|^2\,dt$$

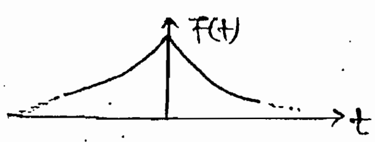
$$= \int_{0}^{\infty} e^{-2at}\,dt = \left(\frac{e^{-2at}}{-2a}\right)_0^{\infty} = \frac{e^{-2a\infty} - e^{0}}{-2a} = \frac{1}{2a}$$

(ii) $y(t) = x(-t) = e^{at}u(-t),\ a>0$



Sol^n→ $\text{Area} = \frac{1}{a}$, $\text{Energy} = \frac{1}{2a}$

(iii) $f(t) = x(t) + y(t) = e^{-a|t|},\ a>0$



Sol^n→

$$f(t) = e^{-a|t|},\ a>0$$

$$= \begin{cases} e^{at}, & t<0 \\ e^{-at}, & t>0 \end{cases}$$

$*\ |t| = \begin{cases} -t, & t<0 \\ t, & t>0 \end{cases}$

$$\text{area} = \frac{1}{a} + \frac{1}{a} = \frac{2}{a}$$

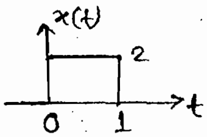
Energy = 1 . 1 . 1

2.→ $x(t) \longrightarrow E$

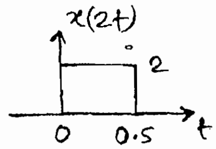
$x(2t) \longrightarrow ?$

(a) $\frac{E}{4}$ (b) $\frac{E}{2}$ (c) $2E$ (d) E

Solⁿ→



$E \rightarrow 4$



$E \rightarrow 2 = \frac{E}{2}$

$x(t) \longrightarrow E$

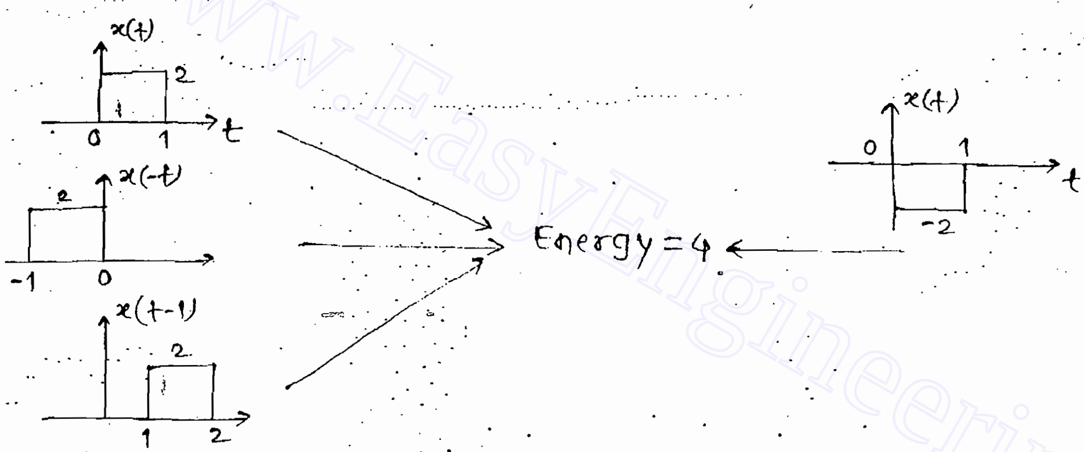
$x(2t) \longrightarrow \frac{E}{2}$

$x(-2t) \longrightarrow \frac{E}{2}$

***

$$\boxed{x(at)_{a\neq 0} \longrightarrow \frac{E}{|a|}}$$

*



* Energy of signal is independent of amp. reversal, time reversal, time shifting.

*** Power Signal →** * For this signal power should be finite & energy should be ∞.

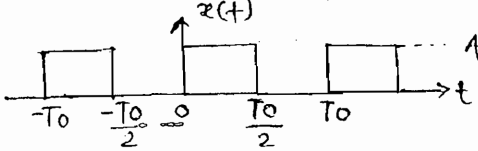
* Periodic power signals are absolutely integrable over there time period.

i.e. $\boxed{\int_{T_0} |x(t)|\,dt < \infty}$ periodic power sig.

$$P = \begin{cases} \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt & ;\ \text{for periodic signal.} \\ \lim_{T_0\to\infty} \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt & ,\ \text{for Non-periodic} \end{cases}$$

Q→ Calculate power of signal:-

(i.)



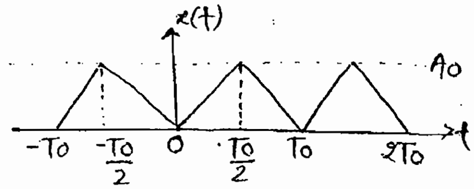
Sol^n→

$$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt$$

$$= \frac{1}{T_0}\int_{0}^{T_0/2} A_0^2 dt$$

$$\boxed{P = \frac{A_0^2}{2}}$$

(ii)



Sol^n→

$$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt$$

$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt$$

$$= \frac{2}{T_0}\int_{0}^{T_0/2} |x(t)|^2 dt$$

$$x(t) = mt = \left(\frac{2A_0}{T_0}\right)t \quad \left(\because m = \frac{A_0}{T_0/2}\right)$$

$$= \frac{2}{T_0}\int_{0}^{T_0/2} \left(\frac{2A_0}{T_0}\right)^2 t^2 dt$$

$$= \frac{8\times A_0^2}{T_0^3}\int_{0}^{T_0/2} t^2 dt$$

$$= \frac{8A_0^2}{\cancel{T_0^3}} \times \frac{\cancel{T_0^3}}{\cancel{8}\times 3}$$

$$\boxed{P = \frac{A_0^2}{3}}$$

(iii) $x(t) = A_0 \sin\omega_0 t$

Sol^n→

$$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} (A_0 \sin\omega_0 t)^2 dt$$

$$P = \frac{A_0^2}{T_0}\int_{-T_0/2}^{T_0/2} \left(\frac{1-\cos 2\omega_0 t}{2}\right) dt$$

$$P = \frac{2A_0^2}{2T_0}\int_{0}^{T_0/2} (1-\cos 2\omega_0 t)\, dt$$

$$= \frac{A_0^2}{T_0}\left[\frac{T_0}{2} - \left(\frac{\sin 2\omega_0 t}{2\omega_0}\right)\Bigg|_0^{\frac{T_0}{2}}\right]$$

$$= \frac{A_0^2}{T_0}\left[\frac{T_0}{2} - \frac{\sin \cancel{2}\omega_0 \frac{T_0}{\cancel{2}}}{2\omega_0}\right]$$

$(\because \omega_0 T_0 = 2\pi)$
$$= \frac{A_0^2}{T_0}\left[\frac{T_0}{2} - \frac{\cancel{\sin\omega_0 T_0}^{\,0}}{2\omega_0}\right]$$

$$= \frac{A_0^2}{T_0} \times \frac{T_0}{2}$$

$$\boxed{P = \frac{A_0^2}{2}}$$

∴ Rms of the Given signal is $\frac{A_0}{\sqrt{2}}$

*** $\boxed{Rms^2 = \frac{A_0^2}{2} = P}$

* Power is also known as mean square value of signal.

Q.→ Calculate power of signal

(i.) $x_1(t) = A_0 \sin \omega_0 t$

(ii) $x_2(t) = x_1(t-t_0) = A_0 \sin[\omega_0(t-t_0)]$

(iii) $x_3(t) = x_1(2t) = A_0 \sin 2\omega_0 t$

(iv) $x_4(t) = A_0 \sin(\omega_0 t + \phi)$

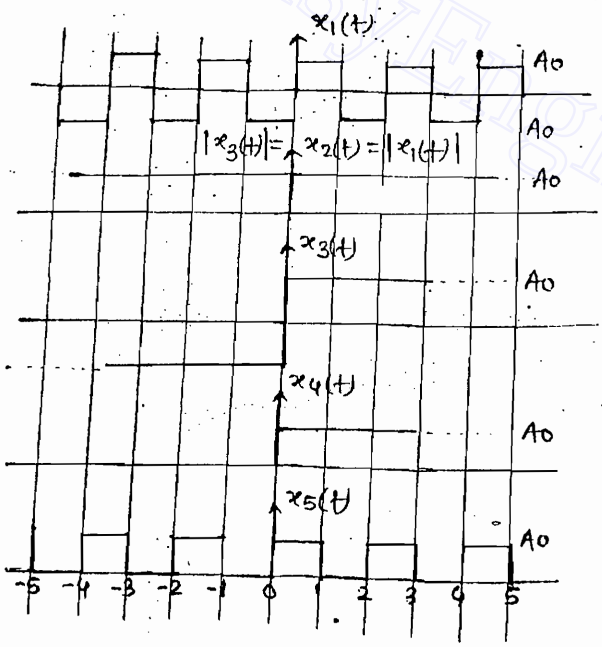
Soln→ For above all signals

$$Rms = \frac{A_0}{\sqrt{2}}$$

$$power = \frac{A_0^2}{2}$$

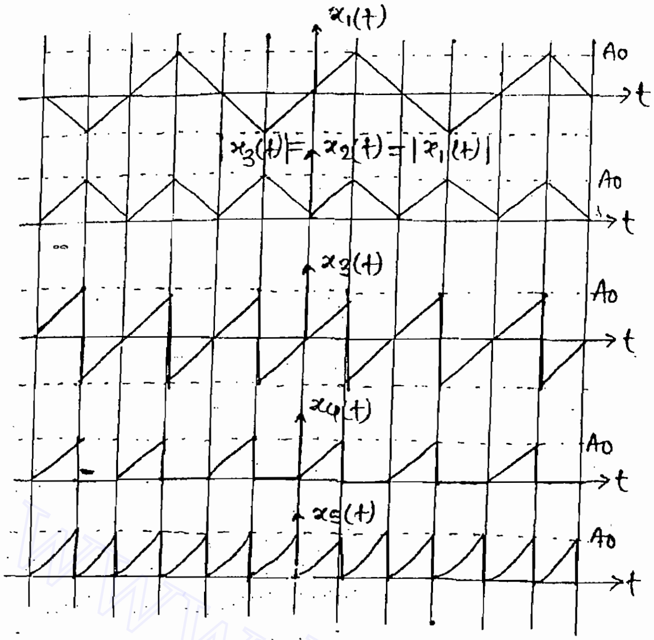
* Power calculation is independant of time shifting, time scaling, change in freq. (or) time period & change in phase of signals.

Q.→



Power	Avg.
$P_1 = A_0^2$	0
$P_2 = A_0^2$	A_0
$P_3 = A_0^2$	0
$P_4 = \frac{P_2}{2} = \frac{A_0^2}{2}$	$\frac{A_0}{2}$
$P_5 = \frac{P_2}{2} = \frac{A_0^2}{2}$	$\frac{A_0}{2}$

Q.→



	Power	Avg.
$P_1 =$	$\frac{A_0^2}{3}$	0
$P_2 =$	$\frac{A_0^2}{3}$	A_0
$P_3 =$	$\frac{A_0^2}{3}$	0
$P_4 =$	$\frac{A_0^2}{6}$	$\frac{A_0}{4}$
$P_5 = 2P_4 =$	$\frac{A_0^2}{3}$	$\frac{A_0}{2}$

DATE-13/10/14

***Concept of Orthogonality →** 2 signals $x_1(t)$ & $x_2(t)$ are said to be orthogonal if

* $\int_{-\infty}^{\infty} x_1(t) . \cancel{x_2(t) dt} = 0$, For non-periodic sig.

* $\int_{T_0} x_1(t) . x_2(t) dt = 0$; For periodic sig.

Use of orthogonality for energy & power calculation →

If $x_1(t)$ & $x_2(t)$ are orthogonal & $z(t) = x_1(t) \pm x_2(t)$ then;

$$P_z = P_{x_1} + P_{x_2} \quad \{\text{If } x_1 \text{ \& } x_2 \text{ are power signal}\}$$

(or)

$$E_z = E_{x_1} + E_{x_2} \quad (\text{If } x_1 \text{ \& } x_2 \text{ are energy signals})$$

Important trignomatrical results →

(1.) $\int_{T_0} \sin(m\omega_0 t + \phi)\, dt = 0$, (m = an integer, $T_0 = \frac{2\pi}{\omega_0}$)

(2.) $\int_{T_0} \cos(m\omega_0 t + \phi)\, dt = 0$

(3.) $\int_{T_0} \sin^2(m\omega_0 t + \phi)\, dt = \frac{T_0}{2}$

* (4.) $\int_{T_0} \cos^2(m\omega_0 t + \phi)\, dt = \frac{T_0}{2}$

* (5.) $\int_{T_0} \sin(m\omega_0 t + \phi_1) \cdot \sin(n\omega_0 t + \phi_2)\, dt = 0$; ($m \neq n$ & both are integer)

Q→ $z(t) = 2\sin(3\pi t + 30^\circ) - 4\sin(7\pi t + 40^\circ)$

Sol^n→ In the above sig. the freq. of the signals are diff. ($m \neq n$). So that they are orthognal.

$$P_z = P_{x_1} + P_{x_2}$$

$$P_{x_1} = \frac{2^2}{2} = 2 \qquad P_{x_2} = \frac{4^2}{2} = 8$$

$$P_z = 10$$

Q→ $z(t) = 2\sin 3\pi t + 4\sin(7\pi t + 30) + 5\sin(10\pi t + 45^\circ)$

Sol^n→ $P_z = P_1 + P_2 + P_3$

$$= \frac{2^2}{2} + \frac{4^2}{2} + \frac{5^2}{2}$$

* (6.) $\int_{T_0} \cos(m\omega_0 t + \phi_1) \cdot \cos(n\omega_0 t + \phi_2) = 0 \quad \{m \neq n\}$

Q→ $z(t) = 3\cos(3\pi t + 70^\circ) + 4\cos(7\pi t + 85^\circ)$

Sol^n→ $P_z = \frac{3^2}{2} + \frac{4^2}{2}$

* (7.) $\int_{T_0} \cos(m\omega_0 t + \phi_1) \cdot \sin(n\omega_0 t + \phi_2)\, dt = 0$ → ($m \neq n$) / → ($m = n, \phi_1 = \phi_2$)

Q→ $z(t) = 2\sin(3\pi t + 40^\circ) + 3\cos(7\pi t)$

Sol^n→ $P = \frac{2^2}{2} + \frac{3^2}{2}$

Q→ $z(t) = 2\sin(2\pi t + 45^\circ) + 3\cos(2\pi t + 45^\circ)$

Sol^n→ $P = \frac{2^2}{2} + \frac{3^2}{2}$

* (8.) $\int_{T_0} A_0 \sin(m\omega_0 t + \phi) = 0$

A_0 ↓ AC $\sin(m\omega_0 t + \phi)$ ↓ sinusoidal (sin, cos)

Q→ $z(t) = 2 + 4\sin(3\pi t + 45^\circ)$

Sol^n→ $P_z = P_1 + P_2$

$= 2^2 + \frac{4^2}{2}$

* Harmonics of diff. freq. are orthogonal.
* Sine & cosine f^n of same freq. & same phase are also orthogonal.
* DC & sinusoidal f^n are also orthogonal.

Q.→ $z(t) = A_1\sin(\omega_0 t + \phi_1) + A_2\sin(\omega_0 t + \phi_2)$ where $\phi_1 - \phi_2 \neq \frac{n\pi}{2}$; (n = integer)

Sol^n→ $P = \frac{1}{T_0}\int_{T_0} z^2(t)\,dt$

$$P = \frac{1}{T_0}\int_{T_0}\left[A_1\sin(\omega_0 t + \phi_1) + A_2\sin(\omega_0 t + \phi_2)\right]^2 dt$$

$$= \frac{1}{T_0}\int_{T_0}\left[A_1^2\sin^2(\omega_0 t + \phi_1) + \frac{A_2^2}{2}\sin^2(\omega_0 t + \phi_2) + 2A_1A_2\sin(\omega_0 t + \phi_1)\cdot\sin(\omega_0 t + \phi_2)\right]$$

$$= \frac{1}{T_0}\int_{T_0}\left[A_1'\left(1 - \cos(2\omega_0 t\right.\right.$$

$$\boxed{P = \frac{A_0^2}{2}, \quad RMS = \frac{A_0}{\sqrt{2}} \qquad A_0 = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_1 - \phi_2)}}$$

Q.→ $z(t) = 2\sin 3\pi t + 3\cos(3\pi t + \frac{\pi}{3})$

Sol^n→ $A_0 = \sqrt{2^2 + 3^2 + 2\times 2\times 3\cos(0 - \pi/3)}$

$= \sqrt{13 + 12\times\frac{1}{2}}$

$= \frac{\sqrt{19}}{\sqrt{2}}$

Above calculation is wrong because sin & cos is present.

$z(t) = 2\sin 3\pi t + 3\sin(3\pi t + \frac{\pi}{3} + \frac{\pi}{2})$

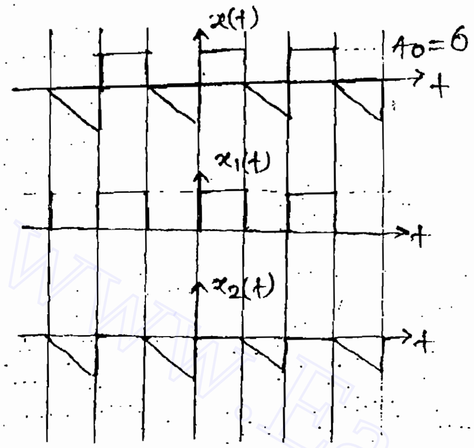
$= 2\sin 3\pi t + 3\sin(3\pi t + \ldots)$

$$\phi_1 - \phi_2 = 150^\circ$$

$$A_0 = \sqrt{2^2 + 3^2 + 2 \times 2 \times 3 \cos(150^\circ)}$$

$$RMS = \frac{A_0}{\sqrt{2}} = 1.14$$

Q.→



RMS = ?

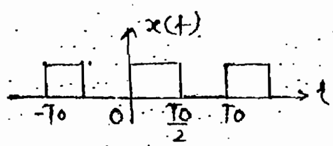
Sol^n→

For this 1st check that are they orthogonal (or) not.

$$P_z = P_1 + P_2 = \frac{A_0^2}{2} + \frac{A_0^2}{6} = \frac{6^2}{2} + \frac{6^2}{6} = 24$$

$$RMS = \sqrt{24} = 2\sqrt{6}$$

Q.→



Sol^n→ $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

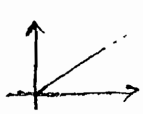
$= \underbrace{\text{no. of pulses}}_{\infty} \times \underbrace{\int_{T_0} |x(t)|^2 dt}_{\text{finite}} = \infty$

Note→

* Periodic signals are not energy signals because there energy content is ∞.

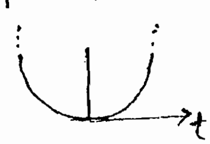
*(1.) If magnitude of sig. is ∞ at any instant of time then signal will be <u>neither energy nor power.</u>

eg.→ (a.)



$x(t) = r(t)$ [Ramp sig]

(b.)



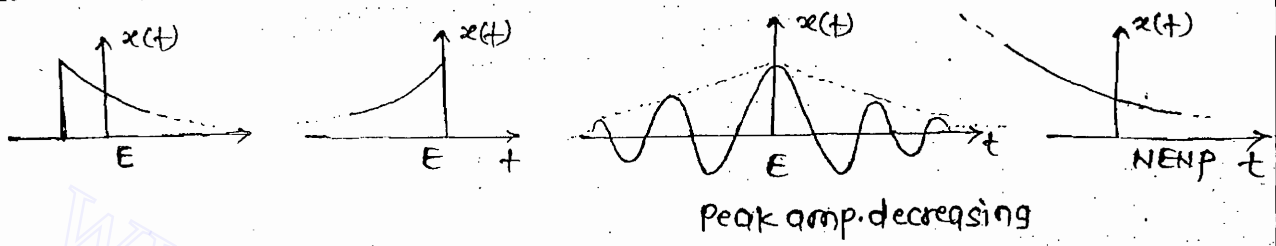
(c.) $x(t) = \tan t$

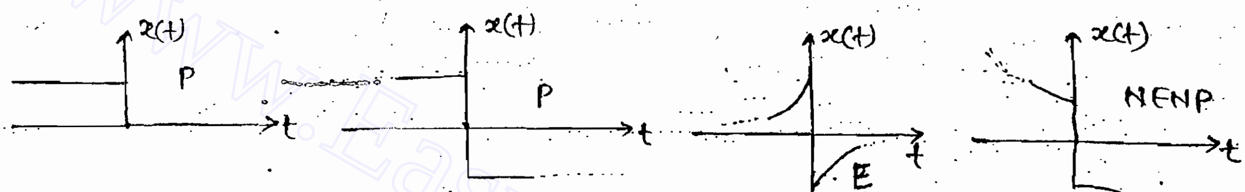
(d.) $x(t) = \frac{1}{t}$ (because $t = 0$, $x(t) = \infty$)

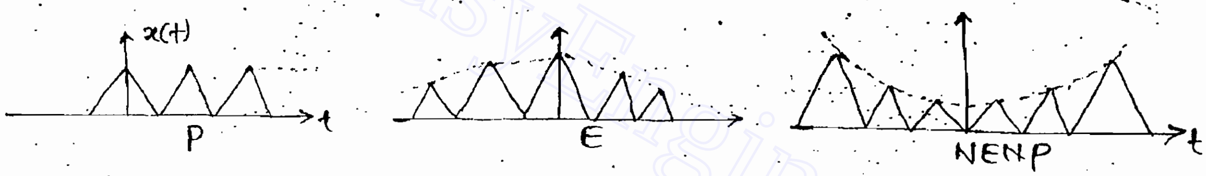
*(2.) Energy signals are:-

(i) Finite duration signals having finite amp. for each & every instant of time.

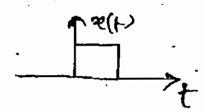
(ii) ∞ extension signals with amp. (or) peak amp decreasing in nature.

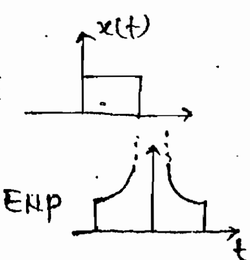


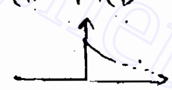




* Periodic Signals → P → sint ; NENP → tant

* Non-Periodic → E →  ; P → u(t) ; NENP → r(t) = tu(t)

* Finite duration → E  ; NENP

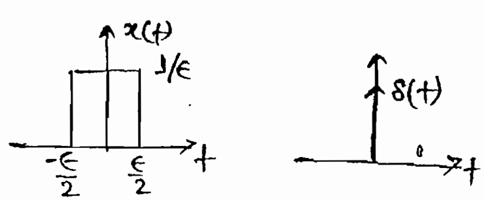
* ∞ extension → E  ; P → u(t) ; NENP → r(t)

*Basic Signals →

(1.) Unit-impulse :- $\delta(t)$

$$\delta(t) = \lim_{\epsilon \to 0} x(t)$$

$$= \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$$



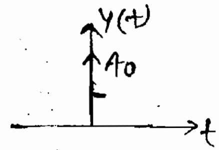
Properties →

* (1.) $\delta(t)$ is an even signal.
* (2.) It is a NENP signal.
* (3.) Area under impulse :-

$$= \int_{-\infty}^{\infty} \delta(t)\,dt = \int_{-\infty}^{\infty} \left[\lim_{\epsilon \to 0} x(t)\right] dt = \lim_{\epsilon \to 0} \int_{-\infty}^{\infty} x(t) \cdot dt = 1$$

* (4.) Weight/ strength of impulse :-

$$y(t) = A_0 \delta_0(t)$$



Area of weighted impulse $y(t)$

$$= \int_{-\infty}^{\infty} y(t)\,dt = A_0 \int_{-\infty}^{\infty} \delta(t) \cdot dt = A_0 = \text{weight of impulse.}$$

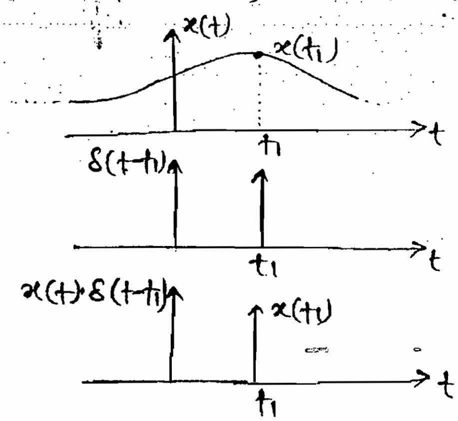
* (5.) Scaling property of impulse :-

$$\delta[a(t-t_1)]_{a \neq 0} = \frac{1}{|a|} \delta(t)$$

Eg. → (1.) $\delta(-2t) = \frac{1}{2} \delta(t)$

(2.) $\delta(2t-3) = \delta[2(t-3/2)] = \frac{1}{2} \delta(t-3/2)$

* (6.) $x(t) \times \delta(t-t_1) = ? = x(t_1) \cdot \delta(t-t_1)$



Eg:- (1.) $y(t) = 2\sin t \cdot \delta(t - \frac{\pi}{2})$

$$= 2\sin\left(\frac{\pi}{2}\right) \delta\left(t - \frac{\pi}{2}\right)$$

$$= 2\delta(t - \pi/2)$$

(2.) $y(t) = e^{-2t^2} \cdot \delta(2t-1)$

$$= e^{-2t^2} \delta\left[2\left(t - \frac{1}{2}\right)\right]$$

$$= e^{-2t^2} \cdot \frac{1}{2} \delta\left(t - \frac{1}{2}\right)$$

$$= \frac{1}{2} \cdot e^{-2 \times \frac{1}{4}} \delta\left(t - \frac{1}{2}\right)$$

$$= \frac{1}{2} e^{-1/2} \delta\left(t - \frac{1}{2}\right)$$

★(7.) $\int_{-\infty}^{\infty} x(t)\cdot\delta(t-t_1)\,dt = ?$

$= \int_{-\infty}^{\infty} x(t_1)\,\delta(t-t_1)\,dt$

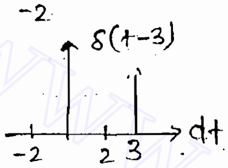
$= x(t_1)\int_{-\infty}^{\infty} \cancel{\delta(t-t_1)\,dt}^{\,1}$

$= x(t_1)$

Q.→ Calculate the value of

(i) $I = \int_{-2}^{2} \delta(t-3)\,dt$

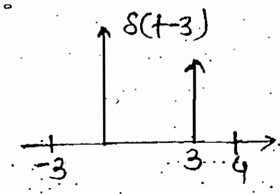
Sol^n→



$\boxed{I=0}$

(ii) $I = \int_{-3}^{4} \delta(t-3)\,dt$

Sol^n→



$\boxed{I=0}$

(iii) $I = \int_{-\infty}^{\infty}\left[2\cos\left(\frac{t}{2}\right)+t^2\right]\delta(t-\pi)\,dt$

Sol^n→ $I = \int_{-\infty}^{\infty}\underbrace{\left[2\cos\left(\frac{t}{2}\right)+t^2\right]}_{x(t)}\ \underbrace{\delta(t-\pi)}_{\delta(t-t_1)}\,dt$

$= x(t_1)$

$= \left[2\cos\frac{\pi}{2}+\pi^2\right]$

$= \pi^2$

★(8.) $\int_{-\infty}^{\infty} x(t)\cdot\frac{d^n\delta(t-t_1)}{dt^n}\cdot dt = (-1)^n\left.\frac{d^n x(t)}{dt^n}\right|_{t=t_1}$

Sol^n→

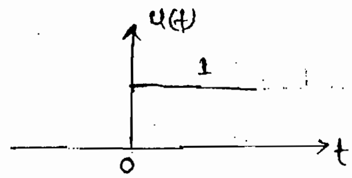
Eg.→ $\int_{-\infty}^{\infty}(t^2+3t)\,\delta'(t-2)\,dt$

$= (-1)^1\frac{d}{dt}(t^2+3t)\Big|_{t=2}$

$= -(2t+3)\Big|_{t=2}$

$= -(4+3) = -7$

(2) Unit-step signal → $u(t)$



* $u(t)$ is discontineous at $t=0$.

Gibb's phenomenon → At the point of discontinuity signal value is given by the avg. of signal value taking just before & after the point of discontinuty.

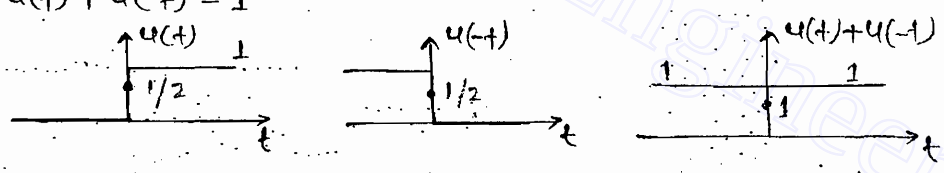
$$u(0) = \frac{u(0^-)+u(0^+)}{2}$$

$$= \frac{0+1}{2}$$

$$\boxed{u(0) = \frac{1}{2}}$$

*** Properties →**

(1) $u(t) + u(-t) = 1$

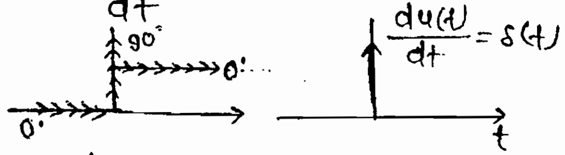


(2) $u(t)$ is a power signal.

Power $= \frac{1}{2}$, Rms $= \frac{1}{\sqrt{2}}$; avg. $= \frac{1}{2}$

(3) Derivative of $u(t)$

$$\frac{du(t)}{dt} = \delta(t)$$

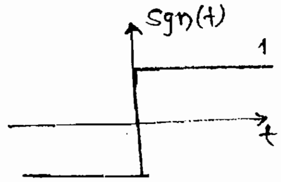


$\left\{ \frac{dx(t)}{dt} = \text{slope of } x(t) \text{ wrt 't'} \right.$

* And $\int_{-\infty}^{t} \delta(t)\,dt = u(t)$

$$\begin{tikzcd} \delta(t) \arrow[r, "I"] & u(t) \arrow[l, bend right, "D"'] \arrow[r, "I"] & r(t) \arrow[l, bend right, "D"'] \arrow[r, "I"] & P(t) \arrow[l, bend right, "D"'] \end{tikzcd}$$

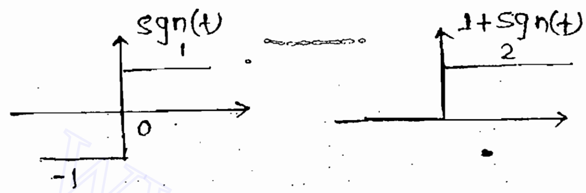
(3.) Signum function →



* This is a power signal.

P=1, RMS=1, Avg=0

*

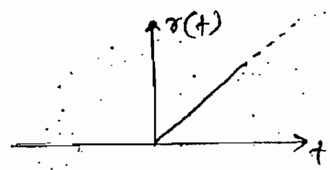


$1+sgn(t) = 2u(t)$

*** $\boxed{u(t) = \frac{1+sgn(t)}{2}}$

(4.) Ramp signal → r(t)

$$r(t) = \int_{-\infty}^{t} u(t)\,dt = t\,u(t)$$



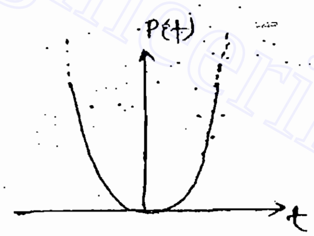
* This is NENP sig.

(5.) Parabolic signal →

$$P(t) = \int_{-\infty}^{t} r(t)\,dt$$

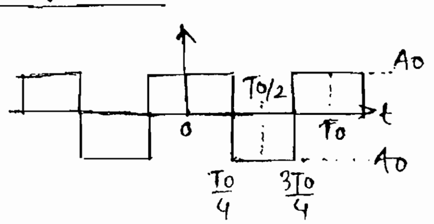
$$= \int_{-\infty}^{t} t\,u(t)\,dt$$

$$= \frac{t^2}{2} u(t)$$



* This is NENP signal.

(6.) Square signal →

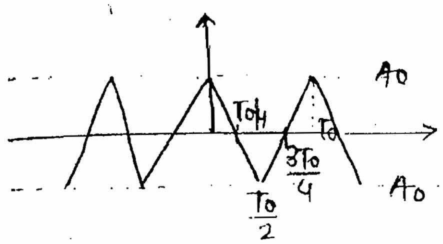


$P = A_0^2$

$RMS = A_0$

$Avg = 0$

(7.) Triangular wave →



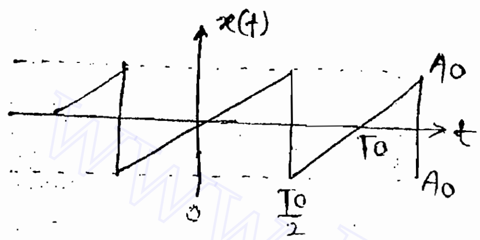
$P = A_0^2/3$

$Rms = A_0/\sqrt{3}$

avg. = 0

HWS = yes

(8.) Sawtooth wave →



$P = A_0^2/3$

$Rms = A_0/\sqrt{3}$

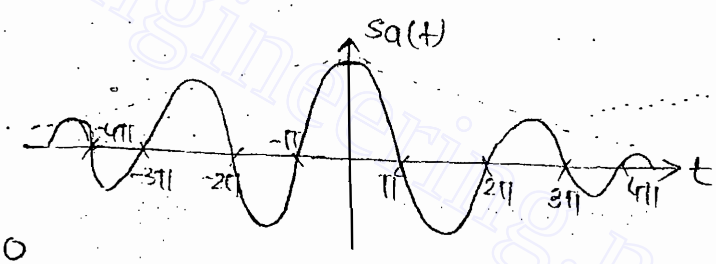
Avg. = 0

HWS = No.

(9.) Sampling signal →

$$Sa(t) = \frac{\sin t}{t}$$

* $Sa(0) = \lim_{t \to 0} \frac{\sin t}{t} = 1$



* $Sa(\infty) = \frac{\sin \infty}{\infty} = \frac{(-1,1)}{\infty} = 0$

* If $Sa(t) = 0$, then $\frac{\sin t}{t} = 0$

so $\sin t = 0$, $\boxed{t = n\pi, n \neq 0}$

* This is a energy signal.

$$E = x(t) = \int_{-\infty}^{\infty} \frac{\sin^2 t}{t^2} dt$$

$$= \int_{-\infty}^{\infty} \left(\frac{1 - \cos 2t}{2t^2}\right) dt$$

$$= \frac{1}{2}\left[\left(\frac{1}{-2t^2}\right)_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \frac{\cos 2t}{t^2} dt\right]$$

$$\boxed{E = \pi}$$

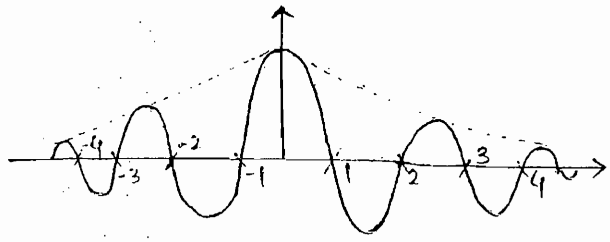
(10.) Sinc function →

$$\text{sinc}\,t = \frac{\sin \pi t}{\pi t} = Sa(\pi t)$$

* $\text{sinc}(0) = \dfrac{\sin \pi t}{\pi t} = 1$

* $\text{sinc}(\infty) = 0$

* If $\text{sinc}(t) = 0$, $\dfrac{\sin(\pi t)}{\pi t} = 0$



$\sin \pi t = 0$

$\cancel{\pi} t = n \cancel{\pi}$, $n \neq 0$

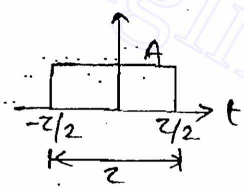
$\boxed{t = n, n \neq 0}$

* Energy = 1

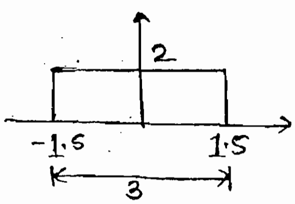
$x(t) \longrightarrow E$	$Sa(t) = \pi$
$x(at) \longrightarrow \dfrac{E}{\lvert a \rvert}$	$\text{sinc}(t) = Sa(\pi t) = 1$

(11.) Rect function →

$$x(t) = A\,\text{rect}\left(\frac{t}{\tau}\right)$$

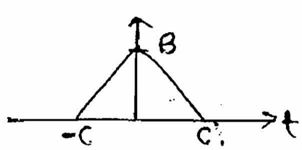


$$x(t) = 2\,\text{rect}\left(\frac{t}{3}\right)$$

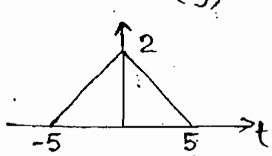


(12) Tri-function →

$$x(t) = B\,\text{tri}\left(\frac{t}{c}\right)$$



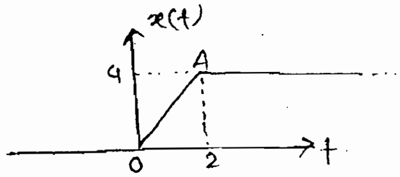
$$x(t) = 2\,\text{tri}\left(\frac{t}{5}\right)$$



DATE-14/10/14

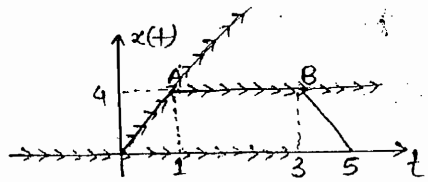
Mathematical representation of waveform →

(1.)



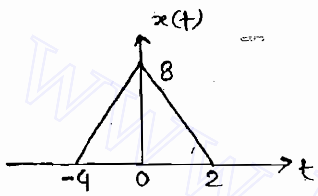
$$* \; x(t) = 0 + 2r(t-0) - 2r(t-2)$$

(2.)



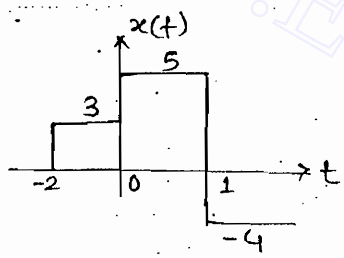
$$* \; x(t) = 0 + 4r(t-0) \cancel{+} - 4r(t-1) - 2r(t-3) + 2r(t-5)$$

(3.)



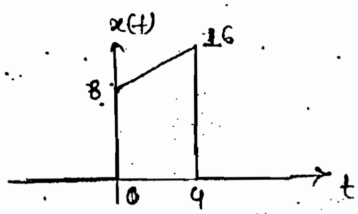
$$* \; x(t) = 0 + (+2)r(t+4) - 2r(t-0) - 4r(t-\cancel{8}) + 4r(t-2)$$

(4.)



$$* \; x(t) = 0 + 3u(t+2) + 2u(t-0) - 9u(t-1)$$

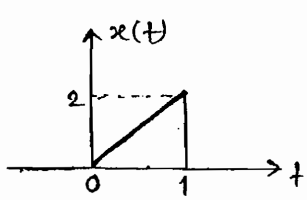
(5.)



$$* \; x(t) = 0 + 8u(t-0) + 2r(\cancel{t-0}) - 2r(t-4) \cancel{+} - 16u(t-4)$$

$$= 8u(t) + 2r(t) - 2r(t-4) - 16u(t-4)$$

(6.)



$$* \; x(t) = 2r(t) - 2r(t-1) - 2u(t-1)$$

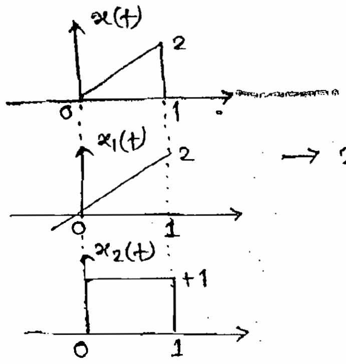
$$= 2tu(t) - 2(t-1)u(t-1) - 2(\cancel{t-1})u(t-1)$$

$$= 2tu(t) - 2tu(t-1) + 2\cancel{u}(t-1) - 2\cancel{u}(t-1)$$

$$= 2t[u(t) - u(t-1)]$$

$$= 2t[u(t) - u(t-1)]$$

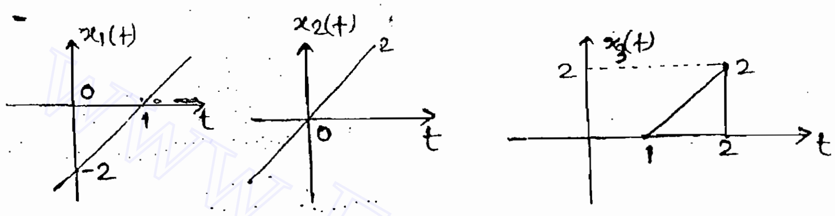
2nd method →



$\rightarrow 2t[u(t) - u(t-1)]$

$$\because x(t) = x_1(t)\cdot x_2(t)$$
$$= 2t[u(t) - u(t-1)]$$

Q. →



Ans. → $x_2(t) = 2t$

$x_1(t) = 2(t-1)$

$x_3(t) = 2(t-1)[u(t-1) - u(t-2)]$

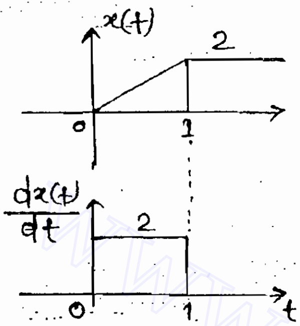
Chapter-02
Different operations of signal

(1.) Differentiation →

$$x(t) = \frac{dx(t)}{dt} = \text{slope of } x(t) \text{ wrt 't'}$$

* Graphical diff is applicable for triangular & rectangular type signal.

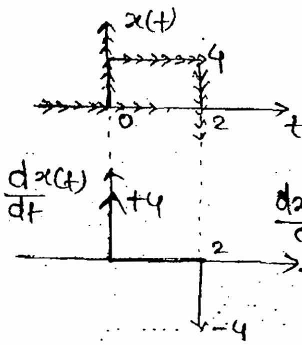
(1.)



$x(t) = 2r(t) - 2r(t-1)$

$\frac{dx(t)}{dt} = 2u(t) - 2u(t-1)$

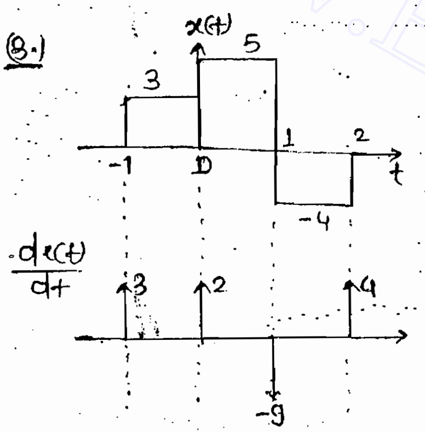
(2.)



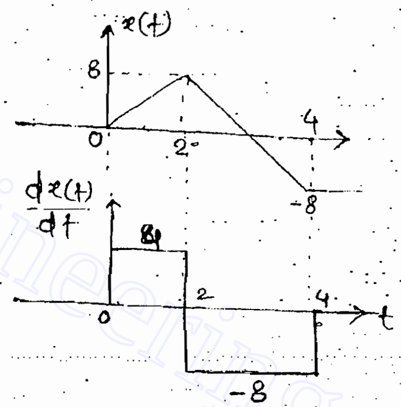
$x(t) = 4[u(t) - u(t-2)]$

$\frac{dx(t)}{dt} = 4[\delta(t) - \delta(t-2)]$

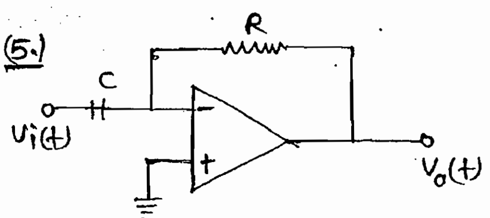
(3.)



(4.)

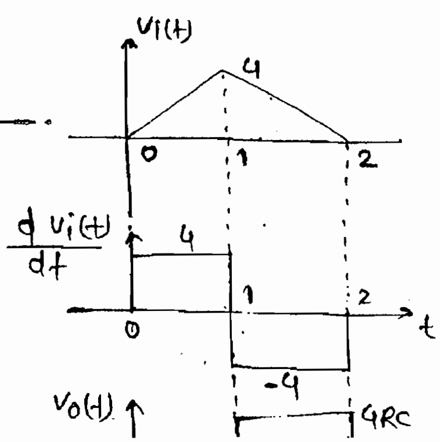


(5.)



$$V_0(t) = -RC\frac{dv_i(t)}{dt}$$

amp reversal



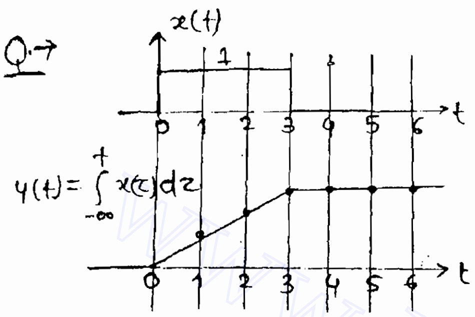
(2.) Integration →

$$x(t) = y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$$

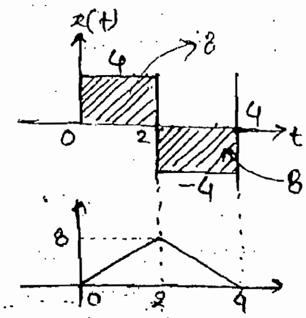
$$= \text{area of signal } x(t) \text{ wrt } 't'$$

* Graphical integration is applicable only for rectangular type waveform.

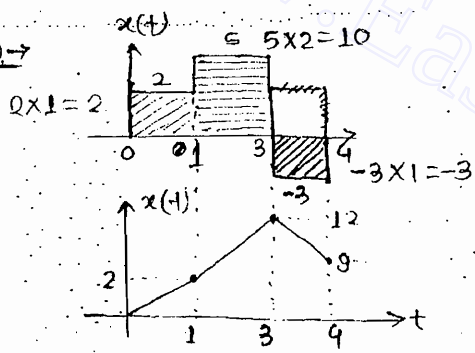
Q.→



Q.→

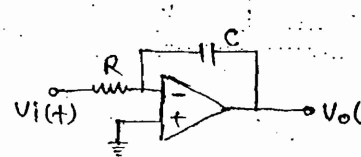


Q.→

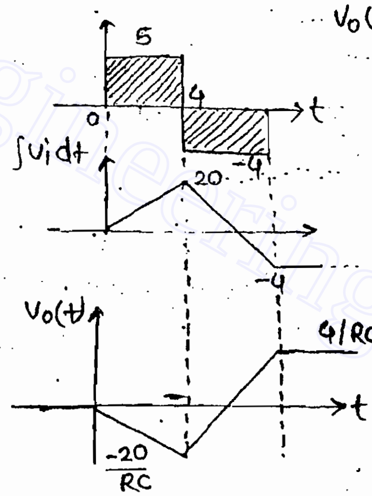


Total area $= 2 + 10 - 3 = 9$

Q.→



$$V_o(t) = \frac{-1}{RC}\int v_i\,dt$$



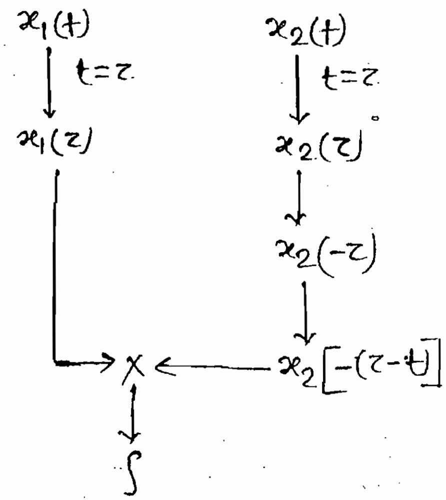
(3.) Convolution →

It is a mathematical operator & it is used for calculation of responce of LTI system.

$$y(t) = x_1(t) * x_2(t)$$

$$= \int_{-\infty}^{\infty} x_1(\tau) \cdot x_2(t-\tau)\,d\tau$$

$$\boxed{x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)\cdot x_2(t-\tau)\,d\tau}$$



Steps→

(1.) Folding

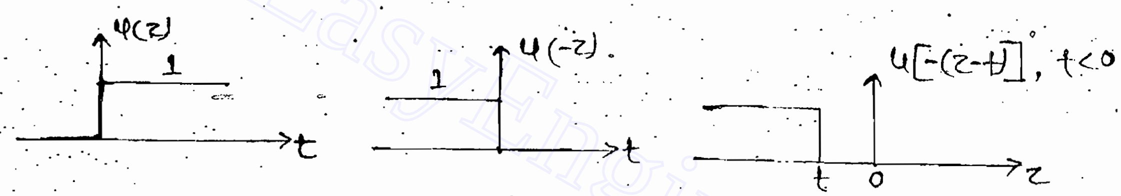
(2.) shifting

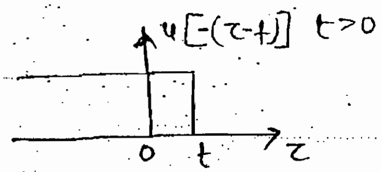
(3.) multiplication

(4.) Integration.

Q→ $y(t) = u(t) * u(t)$

solⁿ→ $= \int_{-\infty}^{\infty} u(\tau)\cdot u(t-\tau)\, d\tau$





$$y(t) = \int_{-\infty}^{\infty} u(\tau)\cdot u(t-\tau)\, d\tau$$

$$= \begin{cases} 0 & ; t<0 \\ \int_0^t dt & ; t>0 \end{cases}$$

$$= \begin{cases} 0 & , t<0 \\ t & , t>0 \end{cases}$$

$$= r(t)$$

2nd method→

$$y(t) = x_1(t) * x_2(t)$$

$$Y(s) = X_1(s)\cdot X_2(s)$$

$$Y(s) = \frac{1}{s}\cdot\frac{1}{s} = \frac{1}{s^2}$$

$$\boxed{y(t) = r(t)}$$

* Properties of Convolution →

(1.) Commutative →

$$x_1(t) * x_2(t) = x_2(t) * x_1(t)$$

$$\int_{-\infty}^{\infty} x_1(\tau) \cdot x_2(t-\tau)\, d\tau = \int_{-\infty}^{\infty} x_2(\tau)\, x_1(t-\tau)\, d\tau$$

(2.) Associative →

$$x_1(t) * [x_2(t) * x_3(t)] = [x_1(t) * x_2(t)] * x_3(t)$$

(3.) Distributive →

$$x_1(t) * [x_2(t) + x_3(t)] = x_1(t) * x_2(t) + x_1(t) * x_3(t)$$

(4.) Impulse Responce →

$$x(t) * \delta(t-t_1) = x(t-t_1)$$

$$\downarrow t_1 = 0$$

$$x(t) * \delta(t) = x(t)$$

eg:- (1.) $u(t-1) * \delta(t+2) = u[(t+2)-1] = u(t+1)$

(5.) Derivative →

$$y(t) = x_1(t) * x_2(t)$$

$$\frac{dy(t)}{dt} = \frac{dx_1(t)}{dt} * x_2(t) = x_1(t) * \frac{dx_2(t)}{dt}$$

eg:- Find $y(t) = ?$

$$y(t) = \frac{d}{dt}[r(t) * u(t)]$$

Soln →

$$y(t) = \frac{d}{dt} r(t) * u(t) \qquad \text{(OR)} \quad y(t) = r(t) * \frac{d}{dt} u(t)$$

$$= u(t) * u(t) \qquad\qquad = r(t) * \delta(t)$$

$$= r(t) \qquad\qquad = r(t)$$

(6.) Step Responce →

$$y(t) = x(t) * u(t) = ?$$

$$y(t) = \int_{-\infty}^{t} \frac{dy(t)}{dt}\, dt = \int_{-\infty}^{t} \left[x(t) * \frac{du(t)}{dt}\right] dt$$

Eg → (1) $u(t) * u(t) = \int_{-\infty}^{t} u(t)\,dt = r(t)$

(2) $r(t) * u(t) = \int_{-\infty}^{t} r(t)\,dt = p(t)$

(7) Time scaling →

If $x_1(t) * x_2(t) = y(t)$ then;

$$x_1(at) * x_2(at) = \frac{1}{|a|} y(at) \quad (a \neq 0)$$

(8) Area →

If $x_1(t) * x_2(t) = y(t)$ then;

$$\text{Area}_{y(t)} = \text{Area}_{x_1(t)} \times \text{Area}_{x_2(t)}$$

(9) Time delay →

$$x_1(t) * x_2(t) = y(t)$$

$$x_1[(t-t_1)] * x_2(t-t_2) = y[t-(t_1+t_2)]$$

Eg:- (1) $u(t-1) * u(t-2) = r(t-3)$

$= (t-3)\,u(t-3)$

(2) $r(t-1) * u(t+3) = p(t+2)$

$= \frac{(t+2)^2}{2} u(t+2)$

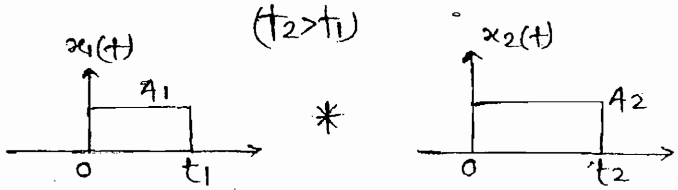
*** (10) Duration →

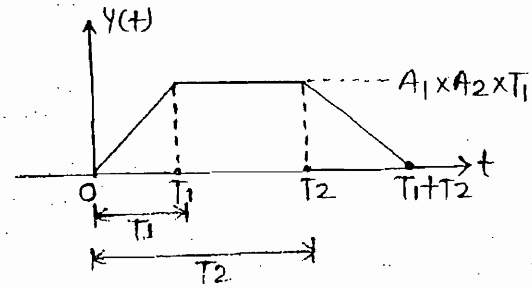
$$y(t) = x_1(t) * x_2(t)$$

signal	extension
$x_1(t)$	$t_1 \le t \le t_2$
$x_2(t)$	$t_3 \le t \le t_4$
$y(t)$	$t_1+t_3 \le t \le t_2+t_4$

* Convolution of 2 rectangular pulses of equal duration will be a triangle.
* Convolution of 2 rectangular pulses of unequal duration will be a Trapezoid.

Trapezoid → $(t_2 > t_1)$

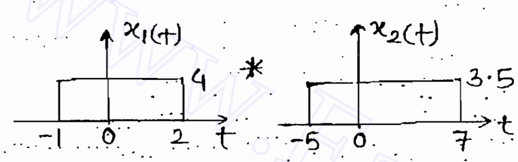




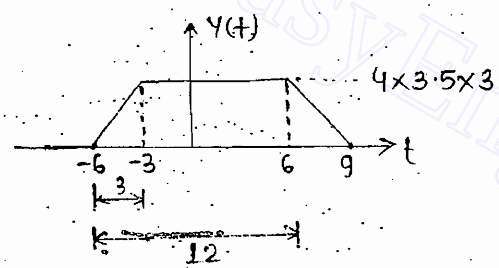
T_1 = smaller duration

T_2 = larger duration

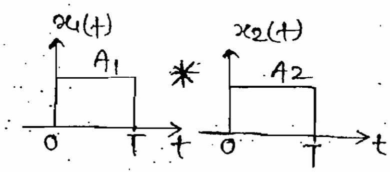
Que →



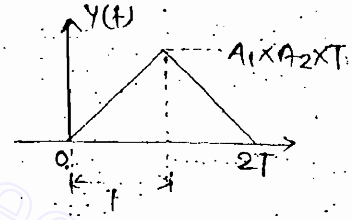
Soln →



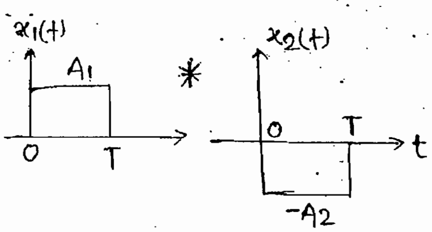
Que →



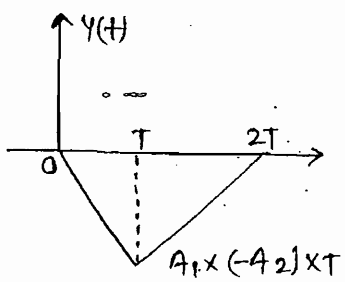
Soln →



Que →

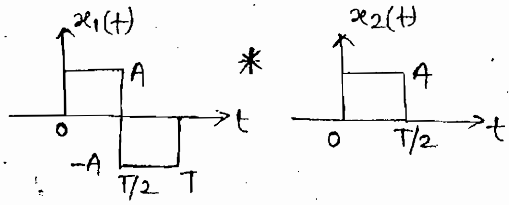


Soln →

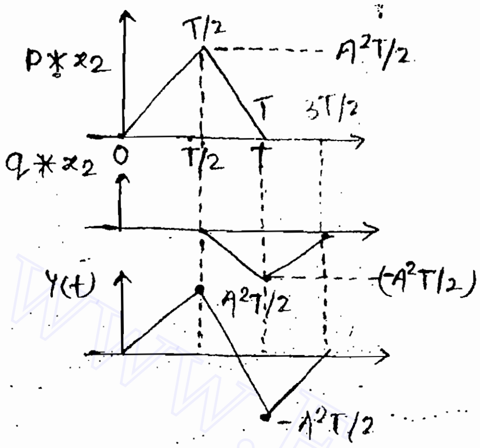


Because $(-A_2)$ Δ will be -ve.

Q.→

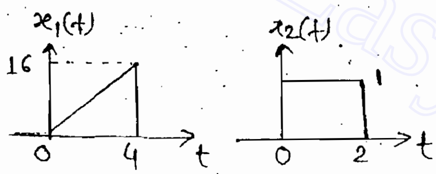


Sol^n→



$$y(t) = x_1(t) * x_2(t)$$
$$= [p(t) + q(t)] * x_2(t)$$
$$= [p(t) * x_2(t)] + [q(t) * x_2(t)]$$

Q.→



$y(t) = x_1(t) * x_2(t)$

Find value of $y(2)$

(a) 4 (b) 8 (c) 16 (d) 32

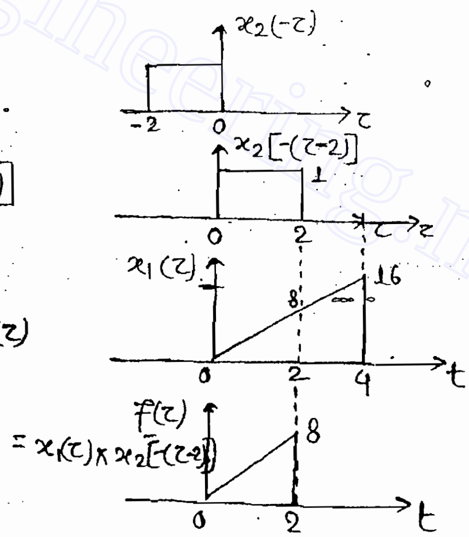
Sol^n→

$$y(t) = x_1(t) * x_2(t)$$
$$= \int_{-\infty}^{\infty} x_1(\tau) \cdot x_2(t-\tau)\, d\tau$$

$\rightarrow x_2[-(\tau-2)]$

$$y(2) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(2-\tau)\, d\tau$$
$$= \int_{-\infty}^{\infty} f(\tau)\, d\tau = \text{area of } f(\tau)$$

where $f(\tau) = x_1(\tau) \cdot x_2[-(\tau-2)]$



$$\text{Area} = \frac{1}{2} \times 2 \times 8 = 8$$

$$\boxed{\text{Area} = 8}$$

DATE-15/10/14

Chapter-03
Basic system properties

$$i/p \rightarrow x(t) \longrightarrow \boxed{\text{sys.}} \longrightarrow y(t) \rightarrow o/p$$

$y(1)\quad y(t) \begin{cases} x(t-1) = x(0) \rightarrow \text{Past} \\ x(t) = x(1) \rightarrow \text{Present} \\ x(t+1) = x(2) \rightarrow \text{Future} \end{cases}$

(1.) Static & dynamic sys. →

Static → If o/p of sys. depands only on present values of i/p at each & every instant of time then sys. will be static.

* These sys. are also known as memoryless system.

Dynamic → * If o/p of sys. depands on past (or) future values of i/p at any instant of time then sys. will be dynamic.

* This sys are also known as sys. with memory.

Q. → Check static/dynamic sys.

(1.) $y(t) = x(t) + x(t-1)$

(2.) $y(t) = x(-t)$

(3.) $y(t) = x(\sin t)$

(4.) $y(t) = x(t-1)$

(5.) $y(t) = \text{Even}[x(t)]$

(6.) $y(t) = \text{Real}[x(t)]$

(7.) $y(t) = \int_{-\infty}^{t} x(z)\,dz$

(8.) $y(t) = e^{-(t+1)}x(t)$

Ans. → (1.) Dynamic.

(2.) Dynamic.

(3.) $y(t) = x(\sin t)$

$y(-\pi) = x(0)$

$-3.14\,\text{sec} = x(0)$ Future — system is dynamic.

(4.) Dynamic

(5.) $y(t) = \dfrac{x(t) + x(-t)}{2}$

$(t=1)$

$y(1) = \dfrac{x(1) + x(-1)}{2}$ past — system is dynamic.

(6.) $y(t) = \dfrac{x(t) + x^*(t)}{2}$ system is static

(7.) ... system is dynamic

Note →

(1.) Integral & derivative sys. are dynamic sys.

(2.) In case of time scaling (or) time shifting system will be dynamic.

(2.) Causal & Non-Causal system →

* causal → * If o/p of sys. is independant of future values of i/p at each & every instant of time then sys. will be causal.

* This sys. are practical (or) physicaly relisable sys.

Eg:- (1.) $y(t) = x(t)$

(2.) $y(t) = x(t-1)$

(3.) $y(t) = x(t) + x(t-1)$

* Non-Causal system → * If o/p of sys. depands on future values of i/p at any instant of time then sys. will be non-causal.

Eg:- (1.) $y(t) = \underline{x(t+1)}$

(2.) $y(t) = x(t) + \underline{x(t+1)}$

(3.) $y(t) = x(t-1) + \underline{x(t+1)}$

(4.) $y(t) = x(t) + x(t-1) + \underline{x(t+1)}$

* Anti Causal System → * If o/p of sys. depands only on future values of i/p then sys. will be anticausal.

Eg:- $y(t) = x(t+1)$

* All anti-causal systems are non-causal but converse of this statement is not true.

Que. → Check Causal & Non-Causal system.

(1.) $y(t) = x(2t)$

(2.) $y(t) = x(-t)$

(3.) $y(t) = x(\sin t)$

(4.) $y(t) = \begin{cases} x(2t) \; ; & t<0 \\ x(t-1) \; ; & t \geq 0 \end{cases}$

(5.) $y(t) = \text{odd}[x(t)]$

(6.) $y(t) = \sin(t+2) \cdot x(t-1)$

(7.) $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

(8.) $y(t) = \int_{-\infty}^{t+1} x(\tau)\, d\tau$

(9.) $y(t) = \int_{-\infty}^{2t} x(\tau)\, d\tau$

Sol$^n \rightarrow$ (i.) $y(t) = x(2t)$

$(t=1)\downarrow$

$y(t) = x(2)$ (system is non-causal)

(ii) $y(t) = x(-t)$

$(t=-1)$

$y(-1) = x(t)$ (system is Non-causal)

(iii) $y(t) = x(\sin t)$

$(t=-\pi)$

$y(-\pi) = x(0)$

$-3.14 = x(0)$ (system is non-causal)

(iv) $y(t) = \begin{cases} x(2t), & t<0 \longrightarrow \text{past} \\ x(t-1), & t\geq 0 \longrightarrow \text{past} \end{cases}$

(system is causal)

(v.) $y(t) = \text{odd}\, x(t)$

$= \dfrac{x(t)-x(-t)}{2}$

$(t=-1)$ future

$y(-1) = \dfrac{x(-1)-x(1)}{2}$ (system is non-causal)

(vi) $y(t) = \sin(t+2)\cdot x(t-1)$

(Coefficient) past

(system is causal.)

(vii) $y(t) = \int_{-\infty}^{t} x(\tau)d\tau \rightarrow x(t)$

$= \int_{-\infty}^{t} x(t)$ (system is causal)

viii) $y(t) = \int_{-\infty}^{(t+1)} x(\tau)d\tau \rightarrow x(t+1)$

(system is non-causal)

(ix) $y(t) = \int_{-\infty}^{2t} x(\tau)d\tau \rightarrow x(2t)$

(system is non-causal)

(3.) Linear & Non-linear system →

linear → * A linear sys. follows the law of superposition.

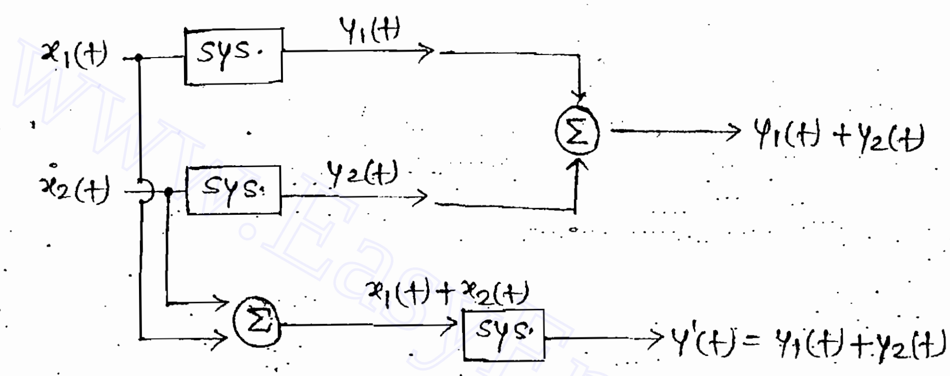
* This law is necessary & sufficient to prove linearity of system.

* It is a combination of two laws:-

(i) Law of additivity.

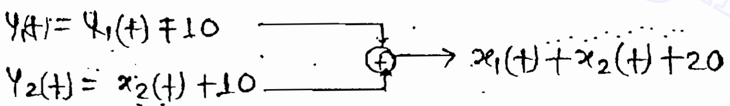
(ii) Law of Homogenity.

(1.) Law of additivity →



Eg:- $y(t) = x(t)+10$

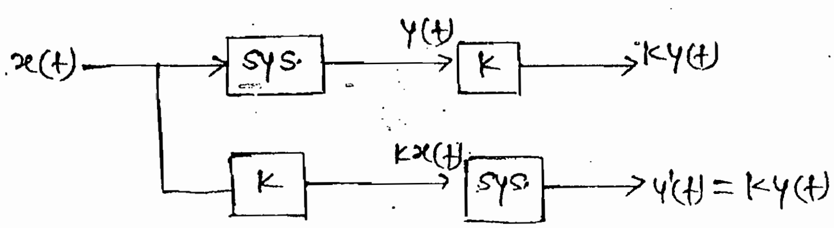
o/p = i/p + 10



$y'(t) = x_1(t) + x_2(t) + 10$

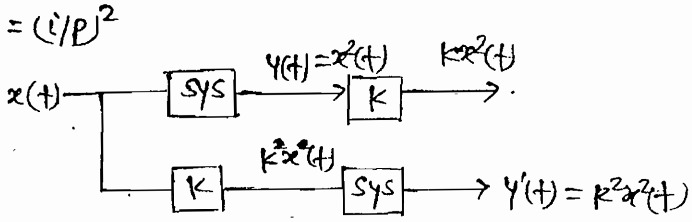
$y(t) \neq y'(t)$ [sys. is NL]

(2.) Law of Homogenity →



Eg:- $y(t) = x^2(t)$

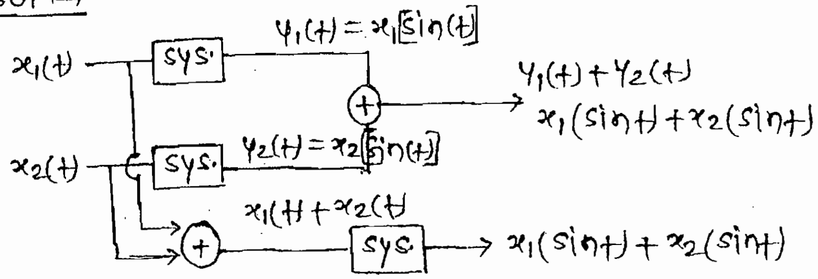
$o/p = (i/p)^2$



Que. → Check linear/Non-linear sys.

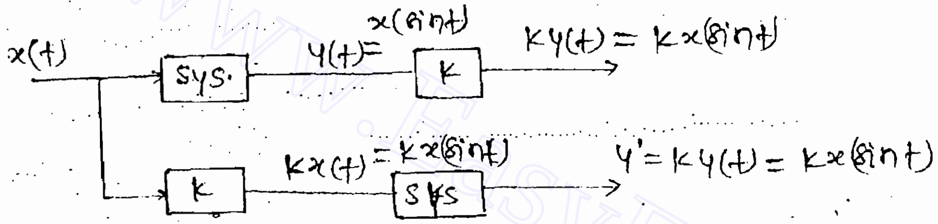
(1.) $y(t) = x(\sin t)$ (2.) $y(t) = x(\tan t)$ (3.) $y(t) = x(t^2)$

Sol^n →



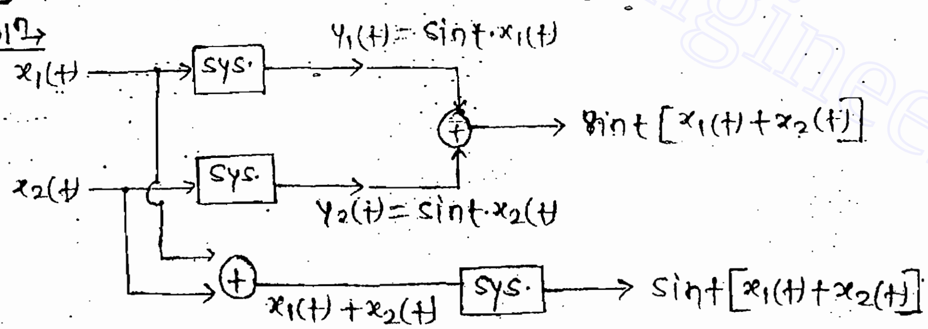
Note →

Linearity of sys. is independant of time scaling.

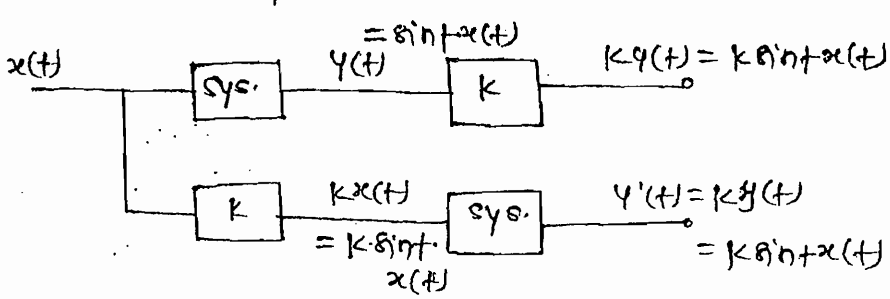


(2.) $y(t) = \sin t \cdot x(t)$ (3.) $y(t) = \log t \cdot x(t)$

Sol^n →



Note :- linearity of sys. is independant of cofficient used in sys. relationship.



2nd method →

for linearity :-

(i) O/p should be 0 for 0 i/p.

(ii) There should not be any 'NL' operation.

eg:- sin, cos, tan, sec, cosec, cot ....
log, exponential, modulus, sq, cubes......
.... root, ... sq( ), sinc( ) ... sgn( ) etc
either on 'x' or 'y'.

Q.→ Check linear/NL sys.

(i) $y(t) = x(t) + 2$ ⟶ put $(t=0)$ then $y(0) \neq x(0)+2$ ⟶ NL

(ii) $y(t) = e^{x(t)}$ ⟶ Because of $e^{x(t)}$ it is NL & also both condn not satisfying.

(iii) $y(t) = x(\tan t)$ ⟶ Linear

(i). If $(t=0)$, then $y(0) = x(0)$ means No i/p no o/p

(ii) above fn is not operating on 'x', it is operating on the 't'.

(iv) $y(t) = \tan[x(t)]$
system is NL

(v.) $y(t) = x(t-1) + x(t+1)$

i/p ⟶ $\boxed{\text{sys.}}$ ⟶ o/p = past i/p + Future i/p

No any NL fn so this is linear

(vi) $y(t) = \text{even}[x(t)]$

$$y(t) = \frac{x(t) + x(-t)}{2}$$

No non-linear operator so linear

(vii) $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$

$$y(t) = \int_{-\infty}^{t} x(0)\,d\tau$$ linear

(viii) $y(t) = \begin{cases} x(t-1), & t<0 \\ x(t+1), & t \geq 0 \end{cases} = \begin{cases} \text{past i/p}, & t<0 \\ \text{future i/p}, & t \geq 0 \end{cases}$ linear

Note →

(1.) Integral & derivative operators are linear.

(2.) Even & odd operators are linear.

(ix) $y(t) = \int_{-\infty}^{t} x^2(\tau)\,d\tau$

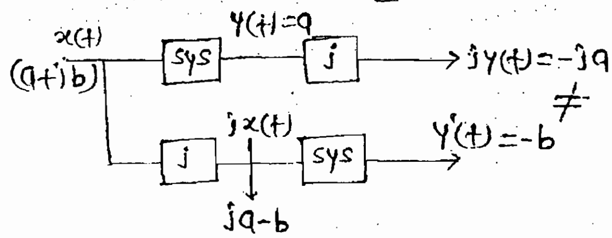
$y(t) = \int_{-\infty}^{t} x^{2}(\tau)\,d\tau$ NL

(x.) $y(t) = e^t x(t)$

$y(t)^0 = e^t x(t)^0$ · linear

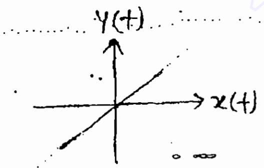
(xi) $y(t) = \mathrm{Real}[x(t)]$

$y(t) = \frac{x(t) + x^*(t)}{2}$ NL



Note → Real & imaginary operators are NL.

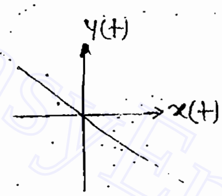
(xii)



$y(t) = m\,x(t)$

system is linear

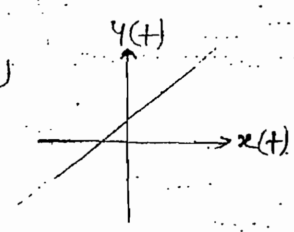
(xiii)



$y(t) = -m\,x(t)$

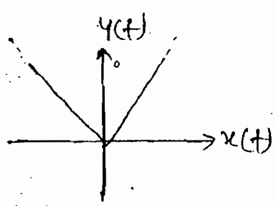
system is L

(xiv)



when i/p 0 then we got o/p NL

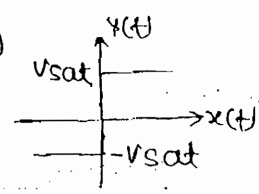
(xv)

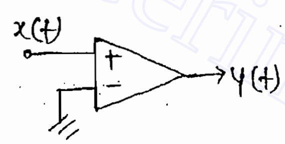


$y(t) = |x(t)|$

NL

(xvi)

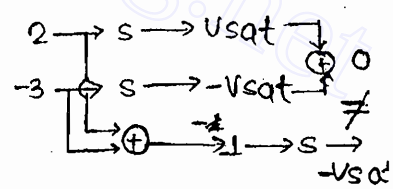




$y(t) = V_{sat}\,\mathrm{Sgn}[x(t)]$

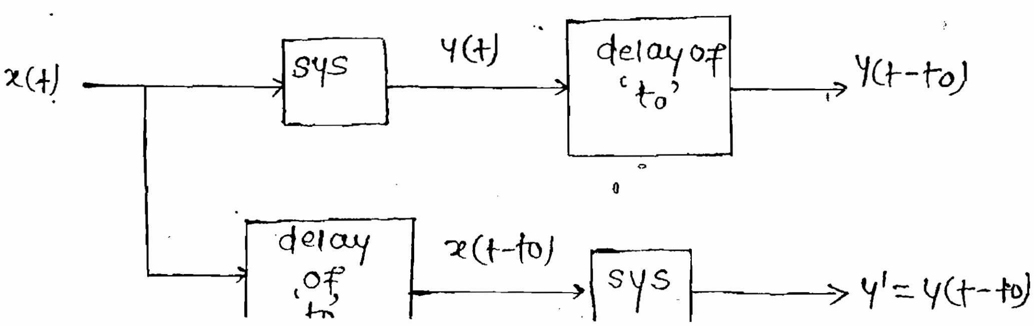
NL

(or)



(4.) Time invarient & time varient sys. →

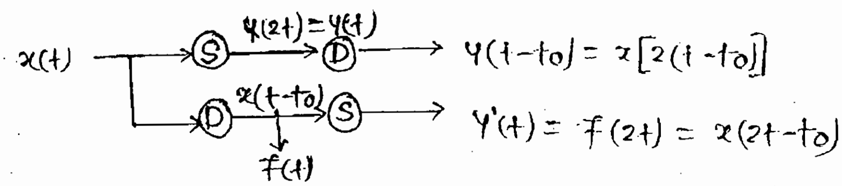
Time invarient →



Note:- Any delay provided in i/p must be reflected in o/p for a time invarient system.

Que.→ check time invarient/varient sys.

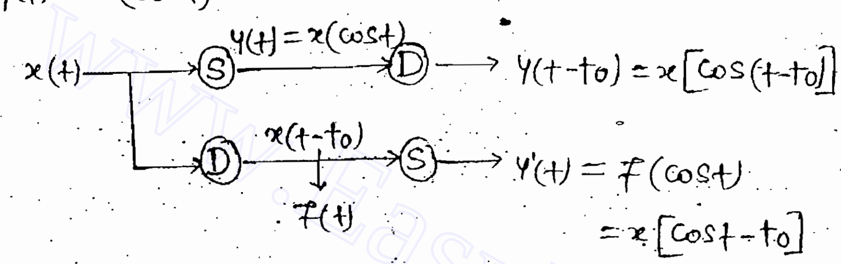
(1.) $y(t) = x(2t)$



system is (TV)

(2.) $y(t) = x(\cos t)$

Sol^n→

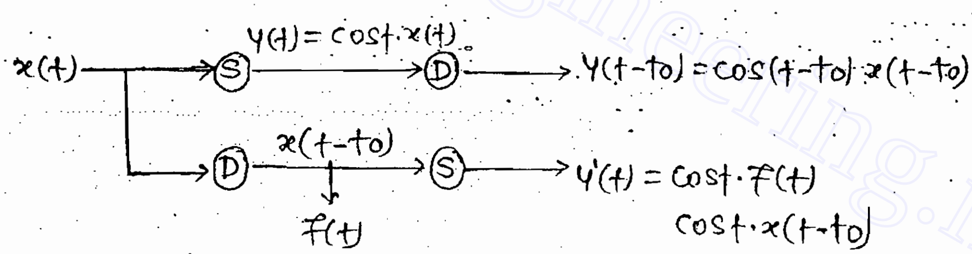


System is (TV)

Note:- In case of time scaling sys. will be time varient.

(3.) $y(t) = \cos t \cdot x(t)$ (4.) $y(t) = \log t \cdot x(t)$

Sol^n→



system is (TV)

Note:- If coefficient in sys. relationship is f^n of time then sys. will be time varient.

(5.) $y(t) = \text{odd}[x(t)]$

Sol^n→ $y(t) = \dfrac{x(t) - x(-t)}{2}$ → Time scaling (TV)

(6.) $y(t) = CS[x(t)]$

$y(t) = \dfrac{x(t) + x^*(-t)}{2}$ → Time scaling (TV)

(vii) $y(t) = x(t-1) + x(t+1)$

Sol^n → $y(t) = x(t-1) + x(t+1)$

No scaling

coefficient are independant of time so TIV

(viii) $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$

Sol^n → $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau \rightarrow x(t)$

TIV

(ix) $y(t) = \int_{-\infty}^{3t} x(\tau)\,d\tau$

Sol^n → $y(t) = \int_{-\infty}^{3t} x(\tau)\,d\tau \rightarrow x(3t)$ (TV)

(x) $y(t) = \int_{-\infty}^{t} \cos\tau \cdot x(\tau)\,d\tau$

Sol^n → $y(t) = \int_{-\infty}^{t} \cos\tau\, x(\tau)\,d\tau \rightarrow \cos t\, x(t)$ (TV)

(xi) $y(t) = \begin{cases} x(t-1), & t \le 0 \\ x(t+1), & t \ge 0 \end{cases}$

Sol^n → $= a(t)\, x(t-1) + b(t)\, x(t+1)$

split systems are time varient system.

(5.) Stable / Unstable sys. → finite / bounded in amplitude

Stable → Bounded i/p bounded o/p (BIBO) criteria.

BIBO → For stable system, o/p should be bounded (or) finite for finite (or) bounded i/p at each & every instant of time.

Eg:- Bounded i/p are u(t), dc-signal, sin t, cos t, sgn(t)

Que. → Check stable/unstable sys.

(1.) $y(t) = x(t) + 2$

$x(t)$	$y(t)$
10	12

(Stable)

(2.) $y(t) = t\,x(t)$

$x(t)$	$y(t)$
10	$10t$

$\infty = \infty$

$-\infty = \infty$

(Unstable)

(3.) $y(t) = \dfrac{x(t)}{\sin t}$

$x(t)$	$y(t)$
2	$\dfrac{2}{\sin t}$

$(t = 0, \pi)$

(Unstable)

(4.) $y(t) = \sin t \cdot x(t)$

Sol^n → $y(t) = \sin t \; x(t)$

$\sin t \rightarrow (-1, 1)$

$y(t) \rightarrow -x(t), x(t)$

$x(t)$	$y(t)$
2	$(-2, 2)$

(stable)

(5.) $y(t) = \sin[x(t)]$

Sol^n → $y(t) = \sin[x(t)]$

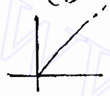
$\rightarrow (-1, 1)$

(stable)

(6.) $y(t) = \int_{-\infty}^{t} x(\tau) d\tau$

Sol^n → $x(t) = u(t) \longrightarrow$ bounded

$y(t) = r(t) \longrightarrow t\,u(t)$



(Unstable)

(7.) $y(t) = \frac{d\,x(t)}{dt}$

Sol^n → $x(t) = u(t) \longrightarrow$ bounded.

$y(t) = \delta(t) \longrightarrow$ Unbounded

(Unstable).

(8.) $\int_{-\infty}^{t} \cos\tau \cdot x(\tau) d\tau = y(t)$

Sol^n → $y(t) = \int_{-\infty}^{t} \cos\tau \cdot x(\tau) d\tau$

$x(t) \longrightarrow \cos t =$ bounded sig.

$$y(t) = \int_{-\infty}^{t} \cos\tau \cdot \cos\tau \, d\tau$$

$$= \int_{-\infty}^{t} \cos^2\tau \, d\tau$$

$$= \int_{-\infty}^{t} \left(\frac{1+\cos 2\tau}{2}\right) d\tau$$

$$= \frac{1}{2}\left[\underbrace{(\tau)_{-\infty}^{t}}_{\text{(Unbounded)}} + \overbrace{\left(\frac{\sin 2\tau}{2}\right)_{-\infty}^{t}}^{\text{finite}} \right]$$

(Unstable)

Note:- so the integration & differentiation signals are unstable sys.

Static | Dynamic – D → I, D, TS & TS

linear / NL – L → I, D, E, O

NL → R & I

* Linear time invarient (LTI) system →

$$x(t) \longrightarrow \boxed{\text{LTI sys.}} \longrightarrow y(t)$$

$h(t) \rightarrow$ Impulse-Responce of sys.

$H(\omega)$ (or) $H(s) \longrightarrow$ TF of sys.

* Impulse Responce & TF terms are used only for LTI system.
* Impulse Responce is used for defining LTI sys. in time domain & TF is used for defining LTI sys. in freq. domain.

Impulse Responce →

$$\delta(t) \longrightarrow \boxed{\text{LTI}} \longrightarrow h(t)$$

* If i/p to LTI sys. is unit impulse then o/p of sys is known as impulse Responce.

Transfer function →

* It is the ratio of laplace Xform of o/p to Laplace Xform of i/p when all initial condn are assumed to be 0.

$$H(s) = \left.\frac{Y(s)}{X(s)}\right|_{\text{zero intial cond}^n}$$

Total o/p = Zero i/p responce + zero state responce

$$\text{Total o/p} = \underbrace{ZIR}_{\substack{\text{due to}\\ \text{initial cond}^n/\\ \text{states}}} + \underbrace{ZSR}_{\substack{\text{due to applied}\\ \text{i/p}}}$$

* for linearity of sys., initial condn are assumed to be zero, because non-zero initial condn make the sys. non-linear.

Convolution →

$$v = x(t) \rightarrow \underset{h(t) = v}{\boxed{\text{LTI}}} \longrightarrow y(t) \qquad y(t) = x(t) * h(t)$$

* Convolution is a linear time invarient operator & it is used only for LTI system.

$$y(t) = x(t) * h(t)$$

$$y(t) = \int_{-\infty}^{t} h(\tau)\cdot x(t-\tau)\,d\tau$$

* The above relation is both linear & TIV. so it is LTI system.

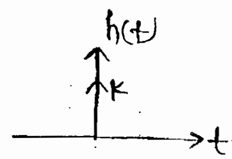
Condition for LTI system to be static →

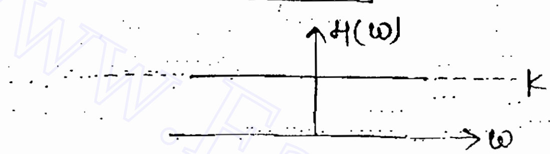
$$y(t) = k\,x(t)$$

$h(t) \downarrow y(t) \qquad \downarrow x(t) = \delta(t)$

$$\boxed{h(t) = k\,\delta(t)}$$

$$\boxed{H(\omega) = k}$$

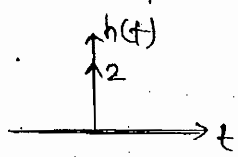




* Impulse $\delta(t)$ is the f^n whose all xform is one.
* For static LTI system, impulse responce should be impulse at origin & TF should be independant of freq.

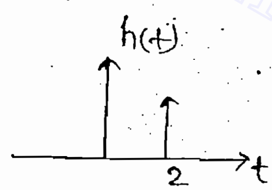
Q.→ Check S/D LTI sys.

(i)



(static)

(ii)

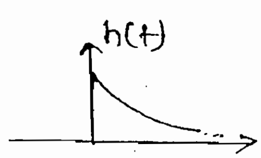


Implulse is not at origin so dynamic.

(iii) $h(t) = u(t)$

(Dynamic)

(iv)



(dynamic)

(v) $H(s) = 2$

static (free of freq.)

(vi) $H(s) = \dfrac{1}{s+1}$

(dynamic)

* Filters are dynamic system because there TF depends on fm.

DATE-16/10/14

$*$ Cond^n for LTI system to be causal $\rightarrow$

$$y(t) = x(t) * h(t) \rightarrow \text{Future i/p} \neq 0$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau) \cdot x(t-\tau)\, d\tau$$

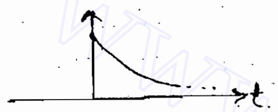
$$h(\tau) = 0 ; \ \tau < 0$$

$$\downarrow \tau = t$$

$$\boxed{h(t) = 0 . t < 0}$$

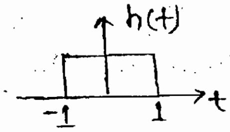
Que $\rightarrow$ Check C/NC LTI system.

(1.) $h(t) = e^{-2t} u(t)$



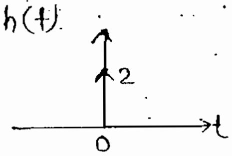
(Causal)

(2.) $h(t) = u(t+1) - u(t-1)$



(Non-causal)

(3.) $h(t)$

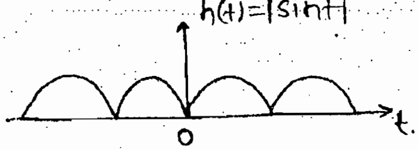


(causal)

(4.) $h(t) = e^{-(t+1)} u(t)$

(causal)

(5.) $h(t) = |\sin t|$



(Non-causal)

$*$ Cond^n for LTI sys. to be stable $\rightarrow$ $*$ If impulse Responce of LTI sys. is absolutely integrable then sys. will be stable. i.e.

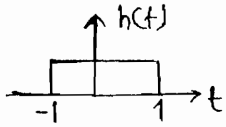
$$\boxed{\int_{-\infty}^{\infty} |h(t)|\, dt < \infty}$$

$*$ A sign If impulse responce of LTI sys. is represented by energy signal (or) unit impulse f^n, then sys. will be stable.

i.e. $h(t) \rightarrow \text{Energy} / \delta(t) \longrightarrow \text{Stable}$

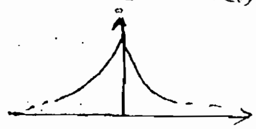
Que → Check s/us system.

(1.) $h(t) = u(t+1) - u(t-1)$



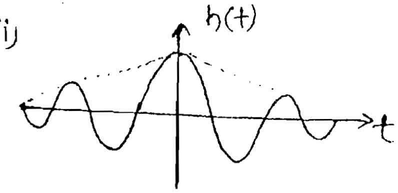
(Stable)

(2.) $h(t) = e^{-2|t|}$



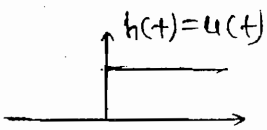
(Stable)

(iii)



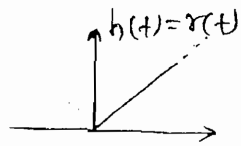
(Stable)

(iv)



(Unstable)

(v)



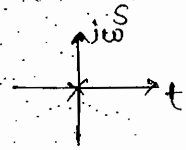
(Unstable)

(vi) $H(s) = \frac{1}{s^2+1}$

Poles $= s = \pm j$

* Because of imaginary axis lying so it is marginary stable
* $h(t) = \sin t \, u(t)$
↓
(Unstable)

(vii) $H(s) = \frac{1}{s}$



Pole → $s = 0$

* marginaly stable
* $h(t) = u(t)$ → power sig.
(Unstable)

$$H(s) = \frac{1}{s} = \frac{Y(s)}{X(s)}$$

$$Y(s) = \frac{X(s)}{s}$$

Inverse LT.

$$y(t) = \left.\int_{-\infty}^{t} x(t)\,dt\right] \text{ Integrator}$$

According to BIBO criteria:–

$x(t) = u(t) =$ bounded i/p

$$y(t) = \int_{-\infty}^{t} u(t)\,dt = r(t)$$

$r(t) =$ Unbounded sig.

So it is Unstable.

Note →

* All marginaly stable LTI sys. are BIBO Unstable.

* Distortions in LTI systems →

Types:- (i) Magnitude/Amplitude distortion

(ii) Delay/Phase distortion.

Note:-

(1.) $x(t) \longrightarrow \boxed{\text{NL sys.}} \longrightarrow y(t) = x(t) + x^2(t)$

If $x(t) = \sin\omega_0 t$ then $y(t) = \sin\omega_0 t + \sin^2\omega_0 t$

$= \sin\omega_0 t + \frac{1-\cos 2\omega_0 t}{2}$

ω_0 $\qquad$ $\omega_0, 2\omega_0$

(2.) $x(t) \longrightarrow \boxed{\text{TV sys}} \longrightarrow y(t) = x(t) + x(2t)$

$x(t) = \sin\omega_0 t$ $\qquad$ $y(t) = \sin\omega_0 t + \sin 2\omega_0 t$

ω_0 $\qquad$ $\omega_0, 2\omega_0$

*** For production of harmonics, nature of sys should be either NL (or) TV.

(1.) Magnitude/Amplitude distortion → If sys. provides unequal amount of amplification (or) attneuation two ~~to~~ diff freq. componants present in i/p sys, then sys. is having magnitude distortion.

$x(t) \longrightarrow \boxed{\text{LTI sys.}} \longrightarrow y(t) = A_1\sin\omega_1 t + A_2\sin\omega_2 t$

$= \sin\omega_1 t + \sin\omega_2 t$ $\qquad$ $A_1 \neq A_2$

$\omega_1 \neq \omega_2$

(2.) Delay (or) phase distortion → If sys. provides unequal amount of time delays to diff freq. componants present in i/p signal then sys. is having delay (or) phase distortion.

$x(t) = \sin\omega_1 t + \sin\omega_2 t$ $(\omega_1 \neq \omega_2)$ $\longrightarrow$ [LTI sys.] $\longrightarrow$ $y(t) = \sin[\omega_1(t-t_1)] + \sin[\omega_2(t-t_2)]$ $(t_1 \neq t_2)$

* Cond^n for LTI sys. to be distortionless →

$x(t) = \sin\omega_1 + \sin\omega_2$ $\longrightarrow$ [LTI sys.] $\longrightarrow$ $y(t) = k\,x(t-t_0)$
$= k\sin[\omega_1(t-t_0)] + k\sin[\omega_2(t-t_0)]$

$$\boxed{y(t) = k\,x(t-t_0)}$$

So laplace transform of above eq^n

$$Y(s) = k \times X(s)\, e^{-st_0}$$

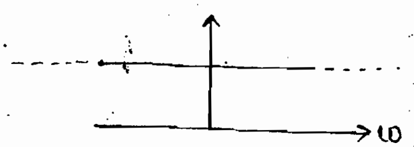
$$H(s) = \frac{Y(s)}{X(s)}$$

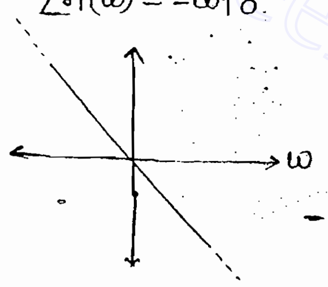
$$= k e^{-st_0}$$

$(s = j\omega)$

$$H(j\omega) = k e^{-j\omega t_0}$$

$|H(\omega)| = K$ $\qquad\qquad$ $\angle H(\omega) = -\omega t_0$





*** For distortionless LTI sys., magnitude of TF should be independant of freq. & phase of TF should be linear.

* Differential eq^n for LTI sys. →

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots + a_0 y(t)$$
$$= b_m \frac{d^m x(t)}{dt^m} + b_{m-1}\frac{d^{m-1} x(t)}{dt^{m-1}} + \cdots + b_0 x(t)$$

For linearity →

All initial condⁿ should be zero.

Time-invariance →

Coefficients $a_n, a_{n-1}, \dots a_0, b_m, b_{m-1} \dots, b_0$ should be indepandant of time.

Que. → Check time invariance & linearity of sys. (initial condⁿ are zero).

(1.) $\frac{2d^2y(t)}{dt^2} + \frac{3dy(t)}{dt} + y(t) = x(t)$

(2.) $\frac{2d^2y(t)}{dt^2} + 3t \cdot \frac{dy(t)}{dt} + y(t) = x(t)$

(3.) $2\left[\frac{dy(t)}{dt}\right]^2 + \frac{3dy(t)}{dt} + y(t) = x(t)$

Ans. → (1.) L, TIV

(2.) L, T.V

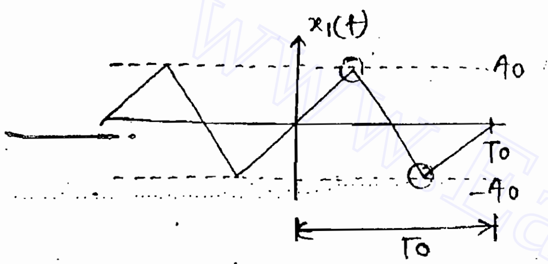
(3.) NL, TIV

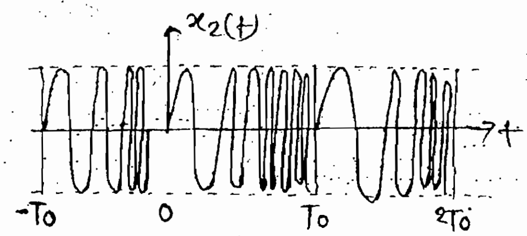
Chapter-04
Fourier Series

* FS expansion is used only for periodic signal.
* In FS sig. is expanded in terms of its harmonics which are sinusoidal & orthognal to one another.

<u>Cond^n for existance of FS expansion</u> → (Dirichlet cond^n)

*(1.) signal should have finite no. of maxima & minima over its time-period.

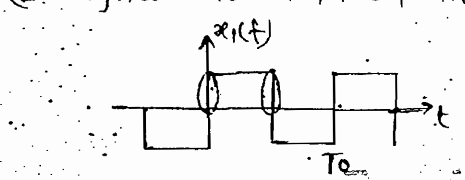




$x_1(t)$ → FS expansion is possible

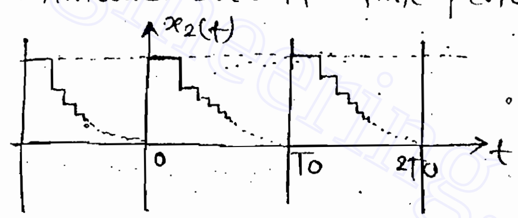
$x_2(t)$ → FS expansion is not possible.

*(2.) Signal should have finite no. of discontinuties over its time-period.



$x_1(t)$ → FS expansion is possiple

No. of discontinuities = 2

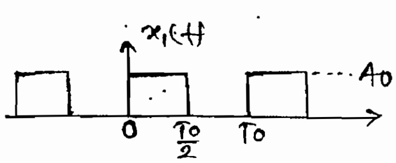


$x_2(t)$ → FS expansion is not possible

*(3.) Signal should be absolutely integrable over its time-periods.
i.e.

$$\boxed{\int_{T_0} |x(t)|\,dt < \infty}$$

Eg:- (1)



(FS is possible)

(2.) $x_2(t) = \tan(t)$

(FS is not possible)

* Types of FS expansion →

(1.) Trignometrical FS exp. →

$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos n\omega_0 t + b_n \sin n\omega_0 t]$$

where, $a_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt$ = Represents avg. (or) dc value of signal.

$$a_n = \frac{2}{T_0}\int_{T_0} x(t)\cos n\omega_0 t\cdot dt = a_{(-n)} \text{ [even } f^n \text{ wrt 'n']}$$

$$b_n = \frac{2}{T_0}\int_{T_0} x(t)\cdot \sin n\omega_0 t\, dt = -b_{(-n)} \text{ [Odd } f^n \text{ wrt 'n']}$$

(2.) Exponential FS exp. →

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

where; C_n = Complex exponential FS coefficient

$$= \frac{1}{T_0}\int_{T_0} x(t)\cdot e^{-jn\omega_0 t}\, dt \text{ ------ (i)}$$

$$C_{(-n)} = \frac{1}{T_0}\int_{T_0} x(t)\, e^{jn\omega_0 t}\, dt$$

$$\Big\downarrow * \text{(conjugate)}$$

$$C^*_{(-n)} = \frac{1}{T_0}\int_{T_0} x^*(t)\, e^{-jn\omega_0 t}\, dt \text{ ------ (ii)}$$

For conjugate symmetry (CS) C_n :-

$$C_n = C^*_{-n}$$

from eq^n (i) & (ii)

$\boxed{x(t) = x^*(t)}$ If $x(t)$ is real.

* If time domain signal is real then its exponential FS coefficient will be conjugate symmetry.

$$C_n = |C_n| e^{j\angle C_n} \text{ ------ (3.)}$$

where; $|C_n|$ = magnitude of n^{th} harmonic $(n\omega_0)$

from eq^n (3) $n = -n$

$$C_{(-n)} = |C_{(-n)}| e^{j\angle C_{(-n)}}$$

$$\downarrow *$$

$$C^*_{(-n)} = |C_{(-n)}| e^{-j\angle C_{(-n)}} \quad \text{------} (4)$$

for CS C_n:-

$$C_n = C^*_{-n}$$

from eq^n (3) & (4)

$$|C_n| = |C_{(-n)}| \longrightarrow \text{Even}$$

$$\angle C_n = \cancel{\angle C_{(-n)}} \; -\angle C_{(-n)} \longrightarrow \text{Odd}$$

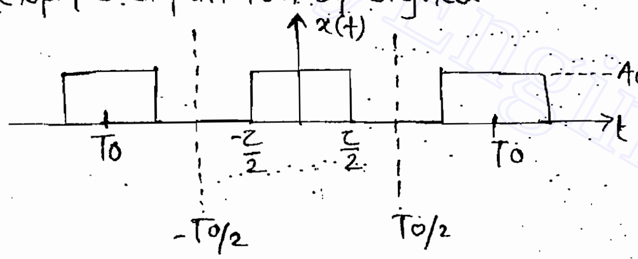
*** Note:-

For Real signal:- $(\overset{E}{\uparrow}{R}m \overset{O}{\uparrow}{I}P)$

(i) Real part of C_n will be even & imaginary part of C_n will be odd.

(ii) magnitude of C_n will be even & phase of C_n will be odd.

Que→ Find exp. FS expansion of signal.



Sol^n→

$$C_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\, e^{-jn\omega_0 t}\, dt$$

$(x(t) = A_0)$

$$= \frac{A_0}{T_0}\int_{-\tau/2}^{\tau/2} e^{-jn\omega_0 t}\, dt$$

$$= \frac{A_0}{T_0}\left(\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right)_{-\tau/2}^{\tau/2}$$

$$= \frac{A_0}{T_0(jn\omega_0)}\left[-e^{-jn\omega_0 \frac{\tau}{2}} + e^{jn\omega_0 \tau/2}\right]$$

$$= \frac{A_0}{T_0(jn\omega_0)} \times 2j \sin\left(\frac{n\omega_0\tau}{2}\right)$$

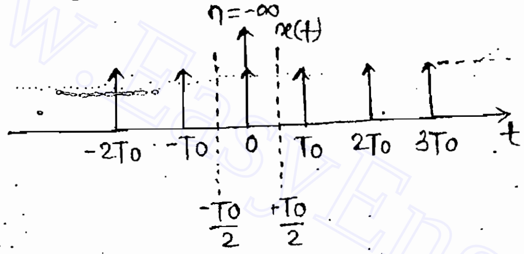
$$c_n = \frac{A_0}{T_0(n\omega_0 j)}(2j) \times \left[\frac{\sin\left(\frac{n\omega_0 \tau}{2}\right)}{\left(\frac{n\omega_0\tau}{2}\right)}\right] \times \left(\frac{n\omega_0 \tau}{2}\right)$$

$$\boxed{C_n = \frac{A_0 \tau}{T_0} \, Sa\left(\frac{n\omega_0 \tau}{2}\right)}$$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} \left[\frac{A_0 \tau}{T_0} Sa\left(\frac{n\omega_0 \tau}{2}\right)\right] \cdot e^{jn\omega_0 t}}$$

Que.→ Find C_n for sig. $x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$



Sol^n →

$$C_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t) \cdot e^{-jn\omega_0 T} dt$$

$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \delta(t) \cdot e^{-jn\omega_0 t} dt$$

$$\because f(t) \cdot \delta(t) = f(0) \cdot \delta(t)$$

$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \underbrace{\delta(t)}_{\text{area}=1} \cdot e^{0} dt$$

$$\boxed{C_n = \frac{1}{T_0}}$$

Que.→ The sig. $x(t)$ has '$T_0 = 2$' & following FS coefficients

$$C_k = \begin{cases} (1/2)^k, & k \geq 0 \\ 0, & k < 0 \end{cases}$$

The value of $x(0)$ will be.

Sol^n →

$$C_n \neq \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} (1/2)^k e^{-jk\omega_0 t}\, dt$$

$$z/\frac{1}{2}$$

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

$$\Big\downarrow t=0$$

$$x(0) = \sum_{k=-\infty}^{\infty} C_k$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k$$

$$x(0) = 1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \cdots\cdots$$

$$= \frac{1}{1-\frac{1}{2}} = \frac{1}{1/2} = 2$$

Que:→ The sig. $x(t)$ has FTP '$T_0=1$' & the following Fourier coefficients

$$c_k = \begin{cases} \left(\frac{-1}{3}\right)^k, & k \geq 0 \\ 0, & k < 0 \end{cases}$$

Find $x(t) = ?$

(a.) $\dfrac{1}{1-\frac{1}{3}e^{j2\pi t}}$ (b.) $\dfrac{1}{1+\frac{1}{3}e^{j2\pi t}}$ (c.) $\dfrac{1}{1-\frac{1}{3}e^{-j2\pi t}}$ (d.) $\dfrac{1}{-1+\frac{1}{3}e^{-j2\pi t}}$

Sol^n →

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

$$= \sum_{k=0}^{\infty} \left(\frac{-1}{3}\right)^k e^{jk\omega_0 t}$$

$$= \sum_{k=0}^{\infty} \left(\frac{-1}{3} e^{j\omega_0 t}\right)^k$$

$$= 1 + \left(\frac{-1}{3}e^{+j\omega_0 t}\right) + \left(\frac{-1}{3}e^{j\omega_0 t}\right)^2 + \cdots\cdots$$

$$= \frac{1}{1-\left(\frac{-1}{3}e^{j\omega_0 t}\right)}$$

$$= \frac{1}{1+\frac{1}{3}e^{j\omega_0 t}}$$

$$= \frac{1}{1+\frac{1}{3}e^{+j2\pi t}}$$

Note:-

$$\cancel{C_n} = \frac{1}{\cancel{T_0}}\int_{-T_0/2}^{T_0/2}$$

(*) $C_n = \frac{1}{T_0}\int_{T_0} x(t)e^{-jn\omega_0 t}dt$

$\downarrow n=0$

$$C_0 = \frac{1}{T_0}\int_{T_0} x(t)dt$$

$C_0 = a_0 = dc/avg.$ value of $x(t)$.

(*) $x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$

$$x(t) = \cdots\cdots + C_{(-1)}e^{-j\omega_0 t} + C_0 + C_1 e^{j\omega_0 t} + \cdots\cdots$$

Que.→ Consider the periodic sig.

$$x(t) = 1 + \sin\omega_0 t + 2\cos\omega_0 t + \cos\left(2\omega_0 t + \frac{\pi}{4}\right)$$

Determine C_n.

Soln→

$$x(t) = 1 + \left(\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}\right) + \left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right) + \frac{1}{2}\left[e^{j(2\omega_0 t + \frac{\pi}{4})} + e^{-j(2\omega_0 t + \frac{\pi}{4})}\right]$$

$$x(t) = 1 + \left(1+\frac{1}{2j}\right)e^{j\omega_0 t} + \left(1-\frac{1}{2j}\right)e^{-j\omega_0 t} + \left(\frac{1}{2}\right)e^{j(2\omega_0 t + \frac{\pi}{4})} + e^{-j(2\omega_0 t + \frac{\pi}{4})}(1/2)$$

$$= 1 + \left(1+\frac{1}{2j}\right)e^{j\omega_0 t} + \left(1-\frac{1}{2j}\right)e^{-j\omega_0 t} + \left(\frac{e^{j\pi/4}}{2}\right)\times e^{j2\omega_0 t} + \left(\frac{e^{-j\pi/4}}{2}\right)\times e^{-j2\omega_0 t}$$

$$= C_0 + C_1 e^{j\omega_0 t} + C_{(-1)}\cdot e^{-j\omega_0 t} + C_2 e^{j2\omega_0 t} + C_{(-2)}e^{-j2\omega_0 t}$$

$C_0 = 1$ $\qquad C_2 = \frac{e^{j\pi/4}}{2} = \frac{1+j}{2\sqrt{2}}$

$C_1 = \left(1+\frac{1}{2j}\right)$ $\qquad C_{(-2)} = \frac{e^{-j\pi/4}}{2} = \frac{1-j}{2\sqrt{2}}$

$C_{-1} = \left(1-\frac{1}{2j}\right)$

Que.→ Consider a periodic sig. $x(t)$ with '$T_0=8$' & FS coefficients.

$$C_1 = C_{-1} = 2$$
$$C_3 = 4j$$
$$C_{-3} = -4j$$

Find $x(t)$

Solⁿ→

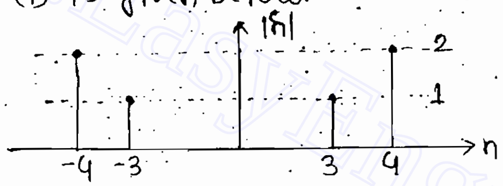
$$x(t) = C_1 e^{j\omega_0 t} + C_{-1} e^{-j\omega_0 t} + C_3 e^{j3\omega_0 t} + C_{-3} e^{-j3\omega_0 t}$$

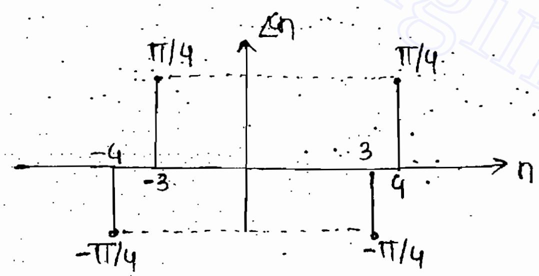
$$= 2\left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right] + 4j\left[e^{j3\omega_0 t} - e^{-j3\omega_0 t}\right]$$

$$= 2\times 2\cos\omega_0 t + 4j \times 2j\sin 3\omega_0 t$$

$$= 4\cos\omega_0 t - 8\sin 3\omega_0 t \qquad \left(\omega_0 = \frac{\pi}{4}\right)$$

Que.→ C_n for Sig. $x(t)$ is given below.
$(\because \omega_0 = \pi)$





Find $x(t)$ $\qquad$ (d) $4\cos\left(4\pi t + \frac{\pi}{4}\right) + 2\cos\left(3\pi t - \frac{\pi}{4}\right)$

Solⁿ→

$$C_3 = |C_3| e^{j\angle C_3} = 1\times e^{-j\pi/4}$$

$$C_{(-3)} = e^{j\pi/4}$$

$$C_4 = 2e^{j\pi/4} \qquad C_{(-4)} = 2e^{-j\pi/4}$$

$$x(t) = C_3 e^{j3\omega_0 t} + C_{(-3)} e^{-j3\omega_0 t} + C_4 e^{j4\omega_0 t} + C_{-4} e^{-j4\omega_0 t}$$

$$= e^{-j\pi/4} e^{3j\omega_0 t} + e^{j\pi/4} e^{(-j3\omega_0 t)} + 2e^{j\pi/4} e^{j4\omega_0 t} + 2e^{-j\pi/4} e^{-j4\omega_0 t}$$

$$= \cancel{e^{j3\pi/4 t} + e^{j(-\frac{3\pi}{4})t} + 2e^{j5\pi/4 t} + 2e^{-j5\pi/4 t}}$$

$$= \cancel{2\cos\left(\frac{3\pi}{4}\right)}\; 2\cos\left(\frac{3\pi}{4}t - \frac{\pi}{4}\right) + 4\cos\left(4\pi t + \frac{\pi}{4}\right)$$

$x(t) - C_n$ pairs →

	$x(t)$	C_n
(1.)	$x(t)$	C_n
(2.)	Real	CS
(3)	CS	Real
(4.)	Img.	CAS
(5.)	CAS	Img.
(6.)	Real + even	Real + Even
(7.)	Img. + Even	Img. + Even
(8.)	Read + Odd	Img. + odd
(9.)	Img + odd	Real + Odd

Que.→ $f(t) = \sum_{n=-\infty}^{\infty} \cos n\pi \, e^{jn\pi t}$ $f(t)$ will be ?

Sol^n→ $C_n = \cos n\pi$

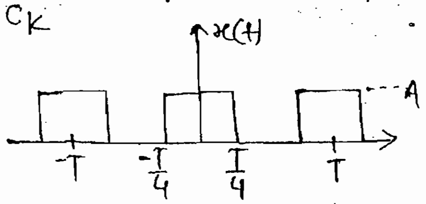
$= R + E$

Que.→ $\sum_{n=-\infty}^{\infty} j \sin \frac{n\pi}{2} e^{jn\pi t}$ $f(t)$ will be?

Sol^n→ $C_n = I + O = j \sin \frac{n\pi}{2}$

$f(t) = R + O.$

Que.→ find C_K



(a.) $\frac{A}{j\pi k} \sin \frac{\pi}{2} k$ (b.) $\frac{A}{j\pi k} \cos\left(\frac{\pi}{2} k\right)$ (c.) $\frac{A}{\pi k} \sin\left(\frac{\pi}{2} k\right)$ (d.) $\frac{A}{\pi k} \cos\left(\frac{\pi}{2} k\right)$

Sol^n→ $x(t) = R + E$

$C_K = R + E$ (A)

$\frac{\sin\left(\frac{\pi}{2}k\right)}{(\pi k)}$ $\frac{O}{O} = E$ $\qquad$ $\frac{\cos(\pi/2 k)}{(\pi k)} = \frac{E}{O} = O$

$f(k) = \pi k$

$f(-k) = -\pi k$

Ans.(d)

Que.→ $x(t) = c_n = \begin{cases} 2 & , n=0 \\ j(\frac{1}{2})^{|n|} & , \text{otherwise} \end{cases}$

which of the following is true?

(a) $x(t)$ is a real sig.
(b) $x(t)$ is an even sig.
(c) $\frac{dx(t)}{dt}$ is an even sig.
(d) both (b) & (c)

Sol^n→

$c_n \rightarrow n = -n$

so $c_n = \epsilon(n)$ (Even signal)

& c_n = imag.

$c_n = E + I$

Ans. (b)

$\frac{dx(t)}{dt}$ is always odd. (derivative of even is odd)

Relation between a_n, b_n & c_n →

* $a_0 = c_0$
* $c_n = \frac{1}{2}[a_n - jb_n]$

} → valid for any type of signal $x(t)$

* $a_n = 2\,\text{Real}[c_n]$
* $b_n = 2\,\text{Img.}[c_n]$

} → valid only for real signal $x(t)$

Que.→ FS expansion of real sig $f(t)$ is

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{3}{4+(3n\pi)^2} e^{jn\pi t}$$

Determine:-

(i) To

(ii) A term in that expansion is $A_0 \cos 6\pi t$, calculate the value of 'A_0'

(iii) Repeat (ii) for '$A_0 \sin 6\pi t$'.

Sol^n→

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$c_n = \frac{3}{4+(3n\pi)^2} = R + E$$

$\omega_0 = \pi$

(i.) $T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{\pi} = 2$

(ii) $A_0 \cos 6\pi t = a_n \cos n\omega_0 t = a_n \cos n\pi t$

$n = 6$

$A_0 = a_6$

$a_n = 2\,\text{Real}[C_n] = 2C_n = \frac{6}{4+(3n\pi)^2}$

$A_0 = a_6 = \frac{6}{4+(3n\pi)^2}$

$a_6 = \frac{6}{4+(18\pi)^2}$

(iii) $A_0 \sin 6\pi t = b_n \sin n\omega_0 t = b_n \sin n\pi t$

$(n=6)$

$A_0 = b_6 = 0$

$b_n = 2\,\text{Img}[C_n] = 0$

*** Symmetrysity is FS →**

(1.) Even symm. →

$$x(t) = \underset{\text{Even}}{\underbrace{a_0}} + \sum_1^{\infty}[\underset{\text{even}}{\underbrace{a_n \cos n\omega_0 t}} + \underset{\text{odd}}{\underbrace{b_n \sin n\omega_0 t}}]$$

* FS expansion of an even signal does not contain sine terms.

(2.) Odd symm. →

* Odd symm. signal contains only any- sine terms in the FS expansion.

(3.) Half wave symm. →

$$\underset{C_n}{\underbrace{x(t)}} = \underset{-C_{(+n)} = -C_n}{\underbrace{-x\left(t+\frac{T_0}{2}\right)}}$$

~~$C_n = C_{(-n)}$~~

$$C_n = -C_n e^{jn\frac{\omega_0 T_0}{2}}$$

$$1 = -e^{jn\pi} \qquad \left(\because \frac{\omega_0 T_0}{2} = \pi\right)$$

time shifting
$x(t) = C_n$
$x(t-t_0) = C_n e^{-jn\omega_0 t_0}$

$$1 + e^{jn\pi} = 0$$

$$1 + (-1)^n = 0$$

$$e^{jn\pi} = (e^{j\pi})^n = (\cos\pi + i\sin\pi)^n = (-1)^n$$

The above relation will be satisfied only when

$n \rightarrow$ odd-integer

$\hookrightarrow n\omega_0$ } odd-integer.

∴ FS expansion of any HWS signal contains only odd harmonics.

(4.) Even + HWS →

* Contains only odd harmonics. } Because of HWS
* Avg/dc value is zero ($a_0 = 0$) }
* Does not contain sine terms. → Because of even.

Note → FS expansion of an even HWS signal contains odd harmonics of cos.

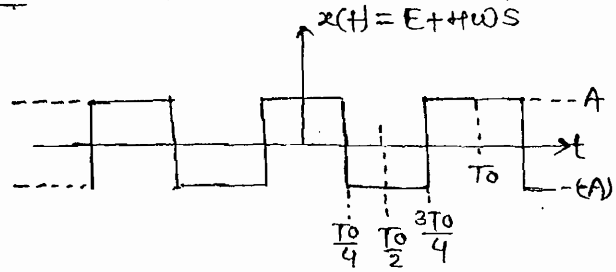
5.) Odd + HWS →

* Contains only sine terms.
* Contains only odd harmonics.

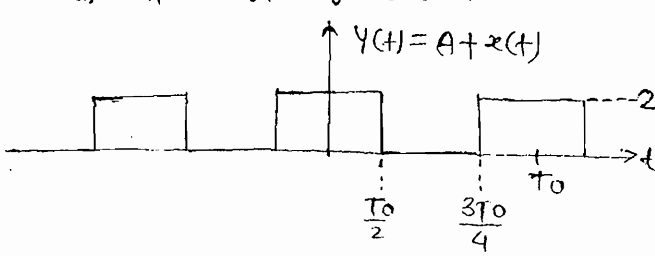
Note → FS expansion of an odd HWS signal contains sine terms without harmonics.

*

6.) Hidden Symm. →

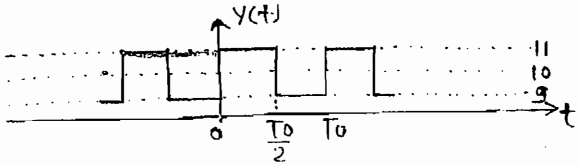


$$x(t) = a_1\cos\omega_0 t + a_3\cos 3\omega_0 t + a_5\cos 5\omega_0 t + \cdots$$

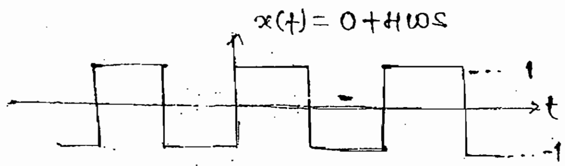


$$y(t) = A + x(t)$$
$$= A + a_1 \cos\omega_0 t + 3\cos 3\omega_0 t + \cdots$$
$$= dc + \text{odd harmonics of cos.}$$

Que.→

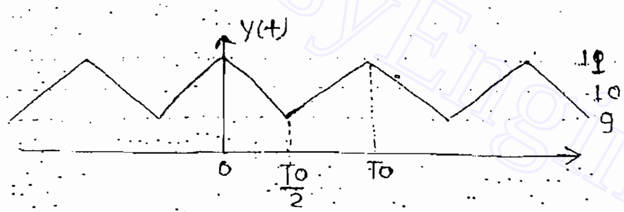


Soln→

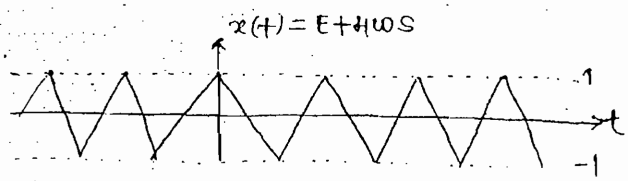


$$y(t) = 10 + x(t)$$
$$= 10 + b_1 \sin\omega_0 t + b_3 \sin 3\omega_0 t + \cdots$$
$$= dc + \text{odd harmonics of sine.}$$

Que.→

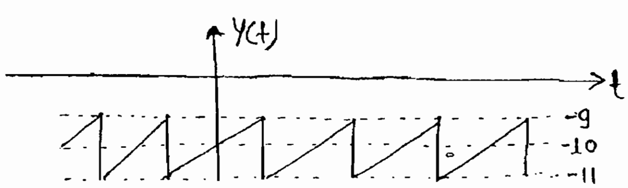


Soln→

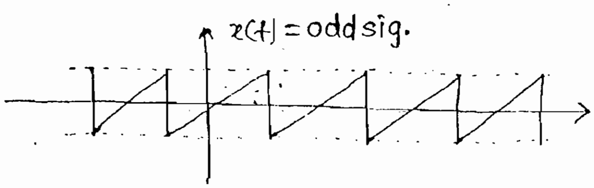


$$y(t) = 10 + x(t)$$
$$= dc + \text{odd harmonics of cos}$$

Que.→



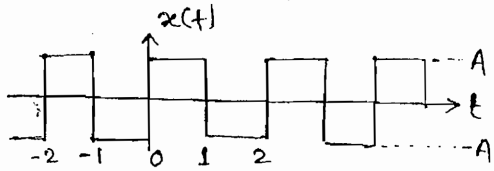
Soln→



HWS ≠ sawtooth

$$y(t) = -10 + x(t)$$

Que→



(a.) $\frac{A}{n\pi}\left[1-(-1)^n\right]$ (c.) $\frac{A}{jn\pi}\left[1-(-1)^n\right]$

(b.) $\frac{A}{n\pi}\left[1+(-1)^n\right]$ (d.) $\frac{A}{jn\pi}\left[1+(-1)^n\right]$

$C_n = ?$

Soln→

$x(t) = R + O$

So; $C_n = I + O$

$x(t) = HWS$

↓

Only odd harmonics are present.

$$C_n = \begin{cases} \neq 0, & n = \text{odd} \\ = 0, & n = \text{even} \end{cases}$$

from option (c.) $\frac{A}{jn\pi}\left[1-(-1)^n\right] \longrightarrow 0, n = \text{even}$

$\neq 0, n = \text{odd}$

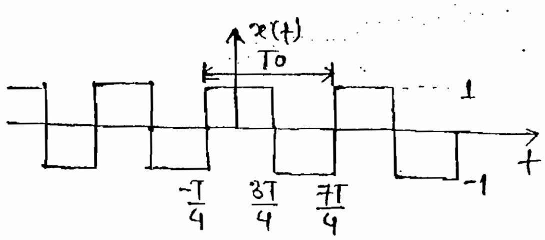
Que→ A sig. x(t) is given by

$$x(t) = \begin{cases} 1, & -T/4 < t < 3T/4 \\ -1 & 3T/4 < t \leq 7T/4 \\ -x(t+T) \end{cases}$$

Which of the following gives the fundamental Fourier term of x(t)?

(a.) $\frac{\pi}{4}\cos\left(\frac{\pi t}{T} - \frac{\pi}{4}\right)$ (b.) $\frac{\pi}{4}\cos\left(\frac{\pi t}{2T} + \frac{\pi}{4}\right)$ (c.) $\frac{4}{\pi}\sin\left(\frac{\pi t}{T} - \frac{\pi}{4}\right)$ (d.) $\frac{\pi}{4}\sin\left(\frac{\pi t}{2T} + \frac{\pi}{4}\right)$

Soln→



$T_0 = \frac{T}{4} + \frac{7T}{4} = 2T$

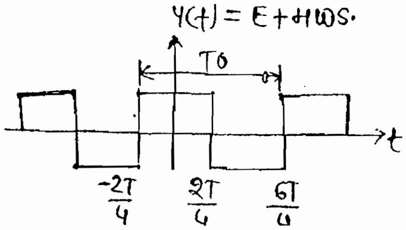
$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2T} = \frac{\pi}{T}$

for HWS→

$x(t) = -x\left(t + \frac{T_0}{2}\right) = -x(t+T)$

* The above signal is NENO (Neither even nor odd)

y(t) = E + HWS.



$$y(t) = a_1 \cos\omega_0 t + a_3 \cos 3\omega_0 t + \text{--------}$$

$$x(t) = y\left(t - \frac{T}{4}\right)$$

$$= a_1 \cos\omega_0\left(t - \frac{T}{4}\right) + a_3 \cos 3\omega_0\left(t - \frac{T}{4}\right) + \text{-------}$$

$$= a_1 \cos\left[\frac{\pi}{T}\left(t - \frac{T}{4}\right)\right]$$

$$\boxed{x(t) = a_1 \cos\left[\frac{\pi t}{T} - \frac{\pi}{4}\right]} \quad \underline{\text{Ans.}}$$

Fundamental Fourier term

$$= a_1 \cos\omega_0 t + b_1 \sin\omega_0 t$$

$$\text{\&} = c_1 e^{j\omega_0 t} + c_{-1} e^{-j\omega_0 t}$$

Note → Polarity of periodic signal at any time instant is decided by polarity of its fundamental fourier term ~~which~~ because this term is dominant as compare to all other terms in the expansion of periodic signal (This rule is applicable for those periodic signals in which ∞ no. of harmonics are present).

from the above options :-

$$\text{(a)} \ \frac{\pi}{4} \cos\left(\frac{\pi t}{T} - \frac{\pi}{4}\right) \xrightarrow{t=0} +ve$$

$$\cancel{\text{(b)}} \ \frac{4}{\pi} \sin\left(\frac{\pi t}{T} - \frac{\pi}{4}\right) \xrightarrow{t=0} -ve$$

* Properties of FS →

(1.) Linearity →

$$a_1 x_1(t) + a_2 x_2(t) \rightleftharpoons a_1 c_{1n} + a_2 c_{2n}$$

where; $x_1(t) = c_{1n}$

$x_2(t) = c_{2n}$

(2.) Time-reverse →

$$x(-t) \rightleftharpoons c_{(-n)}$$

(3.) Conjugation →

$$x^*(t) \rightleftharpoons c^*_{-n}$$

(4.) Time-shifting →

$$x(t-T_0) \rightleftharpoons C_n e^{-jn\omega_o t_0}$$

(5.) Freq. shifting →

$$e^{+jm\omega_o t} x(t) \rightleftharpoons C_{n-m}$$

(6.) Convolution in time →

$$x_1(t) * x_2(t) \rightleftharpoons T_0[C_{1n} * C_{2n}]$$

(7.) Multiplication in time →

$$x_1(t) \cdot x_2(t) \rightleftharpoons [C_{1n} * C_{2n}]$$

(8.) Diffenetiation →

$$\frac{d^m x(t)}{dt^m} \rightleftharpoons (jn\omega_o)^m C_n$$

(9.) Integration in time →

$$\int_{-\infty}^{t} x(t)\,dt \rightleftharpoons \frac{C_n}{jn\omega_0}$$

(10.) Passeval's power theorem →

$$P = \sum_{n=-\infty}^{\infty} |C_n|^2$$

Que → find C_n' in term of C_n
where; $y(t) \rightleftharpoons C_n'$
$x(t) \rightleftharpoons C_n$

(i) $y(t) = e^{-j2\omega_o t} \cdot x(t)$
(ii) $y(t) = x(t-t_0) + x(t+t_0)$
(iii) $y(t) = \frac{d^2 x(t)}{dt^2}$
(iv) $y(t) = \text{even}[x(t)]$
(v) $y(t) = \text{Real}[x(t)]$

Soln → (i) $C_n' = C_{n+2}$

(ii) $C_n' = C_n e^{-jn\omega_o t_0} + C_n e^{jn\omega_o t_0}$

$$= C_n[e^{jn\omega_o t_0} + e^{-jn\omega_o t_0}]$$

$$C_n' = 2C_n \cos(n\omega_o t_0)$$

(iii) $c_n' = (jn\omega_0)^2 c_n = -n^2\omega_0^2 c_n$

(iv) $y(t) = \dfrac{x(t) + x(-t)}{2}$ (time reversal)

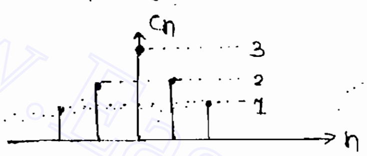
$c_n' = \dfrac{c_n + c_{-n}}{2}$

(v) $y(t) = \text{Real}[x(t)]$

$= \dfrac{x(t) + x^*(t)}{2}$ (conjugation)

$c_n' = \dfrac{c_n + c_{-n}^*}{2}$

Que.→ Calculate power of signal $x(t)$



Soln→ $P = \sum_{n=-\infty}^{\infty} |c_n|^2$

$= 1^2 + 2^2 + 3^2 + 2^2 + 1^2$

$= 19$

Que.→ Let $x(t)$ be the periodic signal with 'T_0' &

$y(t) = x(t-t_0) + x(t+t_0)$

$y(t) = b_k$
$x(t) = c_k$ } exp. FS coefficients.

If $b_k = 0$ for odd integer 'k' then 't_0' can be equal to

(a) $\frac{T}{8}$ (b) $\frac{T}{4}$ (c) $\frac{T}{2}$ (d) $2T$.

Soln→

$y(t) = x(t-t_0) + x(t+t_0)$

$b_k = 2c_k \cos k\omega_0 t_0$ --------(i)

Given that;

$b_k = 0$; K = an odd-integer

$= 2m+1$

$m = \text{int}$

$b_{2m+1} = 0$

$$\Rightarrow 2C_{2m+1} \cdot \cos\left[(2m+1)\,\omega_0 t_0\right] = 0 \text{ ------ from (i)}$$

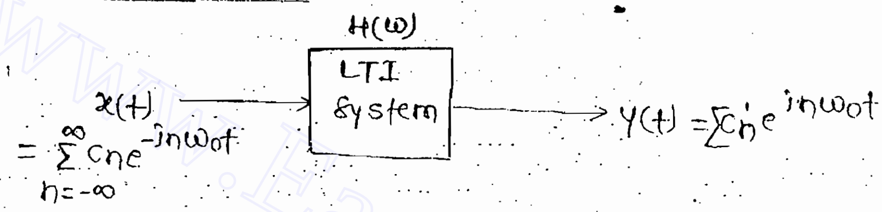
$$\cos\left[(2m+1)\,\omega_0 t_0\right] = 0$$

$$\cancel{\cos}(2m+1)\,\omega_0 t_0 = \cancel{\cos}(2m+1)\frac{\pi}{2}$$

$$\frac{2\pi}{T} t_0 = \frac{\pi}{2}$$

$$\boxed{t_0 = \frac{\pi}{4}}$$

* FS for LTI system →



$$\boxed{c_n' = c_n H(n\omega_0)}$$

Que.→ Consider a contineous time LTI sys. whose i/p $x(t)$ & o/p $y(t)$ are related by the following DE.

$$\frac{dy(t)}{dt} + 4y(t) = x(t)$$

find c_n' for o/p $y(t)$ if i/p $x(t) = \cos\omega_0 t$, $\omega_0 = 2\pi$

Soln→

$$sY(s) + 4Y(s) = X(s)$$

$$Y(s)\left[s+4\right] = X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{1}{(s+4)}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+4}$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{1}{j\omega+4}$$

$$\because x(t) = \cos\omega_0 t$$

$$= \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2}e^{-j\omega_0 t}$$

$C_1 = 1/2$ & $C_{-1} = 1/2$

$C_n' = H(n\omega_0) C_n$

$n = 1,\ C_1' = H(\omega_0) C_1$

$= \frac{1}{4 + j\omega_0} \times \left(\frac{1}{2}\right)$

$= \frac{1}{4 + j \times 2\pi} \times \left(\frac{1}{2}\right)$

$n = -1,\ C_{-1}' = H(-\omega_0) C_{-1}$

$= \frac{1}{4 - j2\pi} \times \left(\frac{1}{2}\right)$

$C_1' = \frac{1}{4 + j2\pi}\left(\frac{1}{2}\right) \quad C_{-1}' = \frac{1}{4 - j2\pi}\left(\frac{1}{2}\right)$

Que.→ Suppose we have given following information about a sig. $x(t)$

(1.) $x(t)$ is real & odd.

(2.) $x(t)$ is periodic with $T_0 = 2$

(3.) fourier coefficients

$C_n = 0, \ |n| > 1$

(4.) $\frac{1}{2}\int_0^2 |x(t)|^2 dt = 1$

The sig. that satiesfy this condⁿ is.

(a.) $\sqrt{2}\sin\pi t$ & unique

(b.) $\sqrt{2}\sin\pi t$ & ~~but~~ not unique

(c.) $2\sin\pi t$ & unique

(d.) $2\sin\pi t$ & but not unique.

Solⁿ→

$T_0 = 2, \quad \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2} = \pi$

The avg. of odd signal is 0.

$C_n = 0, \ |n| > 1$

C_1 & $C_{-1} \neq 0$

$C_0 = 0 \quad \leftarrow x(t)$ is odd.

$x(t) = A_0 \sin\omega_0 t = A_0 \sin\pi t$

$\frac{A_0^2}{2} = 1, \quad A_0 = \pm\sqrt{2}$ (Because of power signal)

Ans. (B)

Chapter-05
Fourier Transform

* FT is a mathematical tool for freq. analysis of sig. where as LT is a convenient mathematical tool for ckt analysis.
* FT exists for energy & power signals where as LT also exists for NENP signals. (upto certain extent only)
* In the category of NENP signal unit impulse is the only f^n for which FT also exists.

$$u(t) \xrightarrow{LT} \frac{1}{s} \quad s = j\omega \text{ (FT)} \quad \rightarrow \frac{1}{j\omega} + \pi\delta(\omega)$$

$$e^{2t}u(t) \xrightarrow{LT} \frac{1}{s-2} \quad \text{(FT does not exists)}$$

* The replacement $(s = j\omega)$ is used for Laplace to fourier conversion only for absolutely integrable signal.
* Impulse f^n & energy signals are absolutely integrable signals.

Fourier Xform →

$$x(t) \rightleftharpoons X(\omega) \quad (\omega \rightarrow \text{rad/sec})$$

$$\boxed{X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt}$$

$$\boxed{x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega}$$

Conditions for existance of FT :- (Dirichet's cond^n)

(1.) Sig. should have finite no. of maxima & minima over finite interval.

(2.) Sig. should have finite no. of discontinuties over finite interval.

(3.) Sig. should be absolutely integrable

i.e. $\boxed{\int_{-\infty}^{\infty} |x(t)|dt < \infty}$ → Impulse sig. → Energy sig.

* Dirichelt's condⁿ are sufficient but not necessary.

Que.→ Cal. FT for sig. $x(t) = e^{-at}u(t)$, $a>0$

Solⁿ→

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} e^{-at} e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} e^{-(a+j\omega)t} dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t} dt$$

$$= \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right]_0^{\infty}$$

$$* = \frac{e^{-(a+j\omega)\infty} - e^{0}}{-(a+j\omega)}$$

$$e^{-(a+j\omega)\infty} = e^{-a\infty} \cdot e^{-j\omega\infty}$$

$$= \cancel{e^{-a\infty}}^{\,0} \cdot \cancel{e^{-j\infty}}^{\,\text{(undefined)}}$$

$e^{-j\infty} = \cos\infty - j\sin\infty$

These cos & sin are not defined in the given Range.

$$e^{-a\infty} = 0, \quad a>0$$

$$* = \frac{0-1}{-(a+j\omega)}$$

$$\boxed{X(\omega) = \frac{1}{a+j\omega}}$$

* At $t = \pm\infty$, complex exponantials & sinusoidal fⁿ are undefined.

Properties of FT →

* (1.) Linearity → $a_1x_1(t)+a_2x_2(t) \rightleftharpoons a_1X_1(\omega)+a_2X_2(\omega)$
* (2.) Time reversal → $x(-t) \rightleftharpoons X(-\omega)$
* (3.) Conjugation → $x^*(t) \rightleftharpoons X^*(-\omega)$
* (4.) Time shifting → $x(t-t_0) \rightleftharpoons X(\omega)e^{-j\omega t_0}$
* (5.) Time scaling → $x(at) \rightleftharpoons \frac{1}{|a|}X\left(\frac{\omega}{a}\right)$, $a \neq 0$
* (6.) Freq. shifting → $e^{-j\omega_0 t}x(t) \rightleftharpoons X(\omega+\omega_0)$
* (7.) Diff. in time → $\frac{d^n x(t)}{dt^n} \rightleftharpoons (j\omega)^n X(\omega)$
* (8.) Integration in time → $\int_{-\infty}^{t} x(t)\,dt \rightleftharpoons \frac{X(\omega)}{j\omega}+\pi X(0)\cdot\delta(\omega)$

  where; $X(0) = X(\omega)\Big|_{\omega=0}$
* (9.) Convolution in time → $x(t) * x_2(t) \rightleftharpoons [X_1(\omega)\cdot X_2(\omega)]$
* (10.) multiplication in time → $x_1(t)\cdot x_2(t) \rightleftharpoons \frac{1}{2\pi}[X_1(\omega) * X_2(\omega)]$

  $x_1(t)\cdot x_2(t) \rightleftharpoons X_1(f) * X_2(f)$
* (11.) Diff. in freq. → $t^n x(t) \rightleftharpoons (j)^n \frac{d^n X(\omega)}{d\omega^n}$
* (12) Parseval's energy theorem → $E = \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(\omega)|^2 d\omega$.
* (13.) Modulation → $x_1(t)\cdot\cos\omega_0 t \rightleftharpoons \frac{1}{2}[X(\omega+\omega_0)+X(\omega-\omega_0)]$

  $x(t)\cdot\sin\omega_0 t \rightleftharpoons \frac{j}{2}[X(\omega+\omega_0)-X(\omega-\omega_0)]$
* (14) Area of time-domain → $X(\omega) = \int_{-\infty}^{\infty} x(t)\,e^{-j\omega t}dt$

  $\Big\downarrow \omega = 0$

  $X(0) = \int_{-\infty}^{\infty} x(t)\cdot dt$

Eg:- $x(t) = e^{-at}u(t),\ a>0 \rightleftharpoons X(\omega) = \frac{1}{a+j\omega}$

$$\text{area of } x(t) = X(\omega)\Big|_{\omega=0} = \frac{1}{a}$$

* (15.) Area under freq. domain →

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

$$x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, d\omega$$

$$\int_{-\infty}^{\infty} X(\omega)\, d\omega = 2\pi x(0)$$

$$\boxed{\text{Area under } X(\omega) = 2\pi\, x(t)\Big|_{t=0}}$$

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Que.→ $x(t) = e^{at}u(-t) \rightleftharpoons X(\omega) = ?\ a>0$

Solⁿ→ $e^{-at}u(t) \rightleftharpoons X(\omega)$

$\downarrow (t = -t) \qquad \downarrow (\omega = -\omega)$ → time reversal.

$$e^{at}u(-t) \rightleftharpoons \frac{1}{a-j\omega}$$

Que→ $y(t) = e^{-a|t|},\ a>0 \rightleftharpoons Y(\omega) = ?$

Solⁿ→

$$y(t) = e^{-a|t|}$$

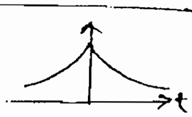
$$= \begin{cases} e^{at}, & t<0 \\ e^{-at}, & t>0 \end{cases}$$

$$= e^{at}u(-t) + e^{-at}u(t)$$

$$Y(\omega) = \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$Y(\omega) = \frac{2a}{a^2+\omega^2}$$

$$\boxed{e^{-a|t|},\ a>0 \rightleftharpoons \frac{2a}{a^2+\omega^2}}$$



* Property of duality →

(1.)

$$x(t) \rightleftharpoons X(\omega)$$

$(\omega = t)$ $\quad$ $(t = -\omega)$

$$X(t) \rightleftharpoons 2\pi\, x(-\omega)$$

$$x(t) \rightleftharpoons X(f)$$

$(f = t)$ $\quad$ $(t = -f)$

$$X(t) \rightleftharpoons x(-f)$$

Q.→ $x(t) = \dfrac{1}{a+jt} \rightleftharpoons X(\omega) = ?$

Sol^n→

$$x(t) = \frac{1}{a+jt}$$

$(t = \omega)$

$$e^{-at}u(t), a>0 \rightleftharpoons \frac{1}{a+j\omega}$$

$(\omega = t)$ $\quad$ $(t = -\omega)$

$$\boxed{\frac{1}{a+jt} \rightleftharpoons 2\pi e^{a\omega} u(-\omega),\ a>0}$$

Q.→ $x(t) = \dfrac{2a}{a^2+t^2} \rightleftharpoons X(\omega) = ?$

Sol^n→

$$x(t) = \frac{2a}{a^2+t^2}$$

$(t = \omega)$

$$e^{-a|t|}, a>0 \rightleftharpoons \frac{2a}{a^2+\omega^2}$$

$(\omega = t)$ $\quad$ $(t = -\omega)$

$$\frac{2a}{a^2+t^2} \rightleftharpoons 2\pi e^{-a|-\omega|}, a>0$$

$$\boxed{\frac{2a}{a^2+t^2} \rightleftharpoons 2\pi e^{-a|\omega|},\ a>0}$$

***

$$\boxed{A_0\,\delta(t) \rightleftharpoons A_0}$$

Q.→ $x(t) = A_0 \rightleftharpoons X(\omega) = ?$

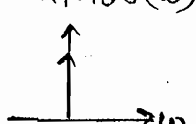
Sol^n→

$$A_0\,\delta(t) \rightleftharpoons A_0$$

$\omega = t$ $\quad$ $t = -\omega$

$$A_0 \rightleftharpoons \cancel{2\pi A_0 [\delta(\omega)]}\ \ 2\pi A_0\, \delta(-\omega)$$

$$\boxed{A_0 = \text{dc signal} \rightleftharpoons 2\pi A_0\, \delta(\omega)}$$



Q → Find $Y(\omega)$ in terms of $X(\omega)$

Soln →

$$x(t) = X(\omega)$$
$$y(t) = Y(\omega)$$

(i) $y(t) = e^{j2t} x(t)$

Soln → $Y(\omega) = X(\omega - 2)$ } freq. shifting property

(ii) $y(t) = x(-2t)$

soln → $Y(\omega) = \frac{1}{2} X\left(\frac{-\omega}{2}\right)$ } time scaling

Q →

(iii) $y(t) = x(2t-3)$

Soln → $y(t) = x(2t-3) = x\left[2\left(t-\frac{3}{2}\right)\right]$

$$x(t) \longrightarrow x(2t) \longrightarrow y(t) = x\left[2(t-1.5)\right] = f(t-1.5)$$

$$X(\omega) \quad F(\omega) = \frac{1}{2} X\left(\frac{\omega}{2}\right) \quad Y(\omega) = F(\omega) e^{-j1.5\omega}$$

$$= \frac{1}{2} X\left(\frac{\omega}{2}\right) e^{-j1.5\omega}$$

(OR)

$$y(t) = x(2t-3) = x\left[2(t-1.5)\right]$$

$$= \frac{1}{2} X\left(\frac{\omega}{2}\right) e^{-j\omega 1.5}$$

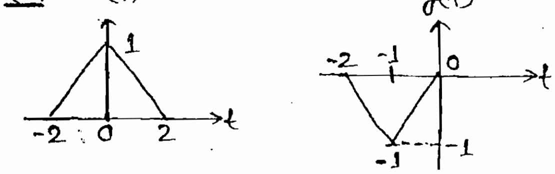
(iv) $y(t) = x(-2t-4)$

soln → $y(t) = x\left[-2(t+2)\right]$

$t_0 = 2$

$$Y(\omega) = \frac{1}{2} X\left(\frac{-\omega}{2}\right) e^{j\omega \cdot 2}$$

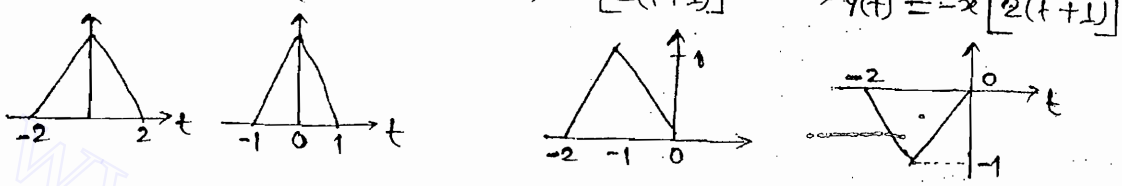
(V.) $x(t)$ $\quad y(t)$ $\quad Y(\omega) = ?$



Soln →

$T_{01} = 4 \quad T_{02} = 2$

$x(t) \longrightarrow x(2t) \longrightarrow x[2(t+1)] \longrightarrow y(t) = -x[2(t+1)]$



$y(t) = -x[2(t+1)]$

$Y(\omega) = -\frac{1}{2} X\left(\frac{\omega}{2}\right) e^{j\omega} \qquad (t_0 = 1)$

Q.→ $y(t) = x(t) * h(t)$ ------ (i)

$g(t) = x(3t) * h(3t)$ ------ (ii)

If $g(t) = Ay(Bt)$ then calculate values of A & B.

Soln → From eqn (i)

$Y(\omega) = X(\omega) H(\omega)$ ------ (iii)

From eqn (ii)

$G(\omega) = \left[\frac{1}{3} X\left(\frac{\omega}{3}\right)\right]\left[\frac{1}{3} H\left(\frac{\omega}{3}\right)\right]$

$G(\omega) = \frac{1}{9}\left[X\left(\frac{\omega}{3}\right) H\left(\frac{\omega}{3}\right)\right]$

$= \frac{1}{9}\left[Y\left(\frac{\omega}{3}\right)\right]$ from eqn (iii)

$= \frac{1}{3}\left[\frac{1}{3} Y\left(\frac{\omega}{3}\right)\right]$

$g(t) = \frac{1}{3} y(3t)$

$g(t) = Ay(Bt) = \frac{1}{3} y(3t)$

$\boxed{A = \frac{1}{3},\ B = 3}$

2nd method →

$x(t) * h(t) = y(t)$

$x(at) * h(at) = \frac{1}{|a|} y(at)$

$\downarrow (a = 3)$

$x(3t) * h(3t) = \frac{1}{3} y(3t)$

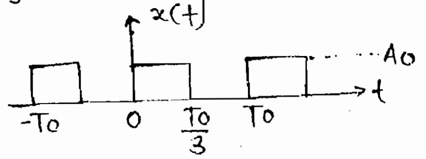
$g(t) = Ay(Bt)$

By comparision;

$\boxed{A = \frac{1}{3} \text{ (or) } B = 3}$

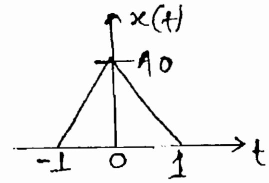
Avg. Value →

(1.)



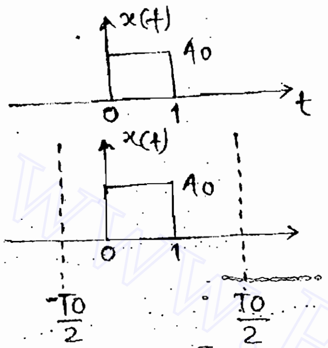
$$\boxed{avg. = \frac{Ao}{3}}$$

(3.)



$$\boxed{avg = Ao}$$

(2.)

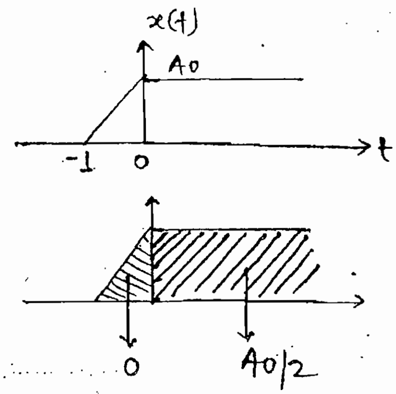


$$Avg. = \lim_{To\to\infty} \frac{1}{To}\int_{-To/2}^{To/2} x(t)\,dt$$

$$= \lim_{To\to\infty} \frac{1}{To}\int_{0}^{1} Ao\,dt = \frac{Ao}{To\to\infty\, To} = 0$$

$$\boxed{Avg. = 0}$$

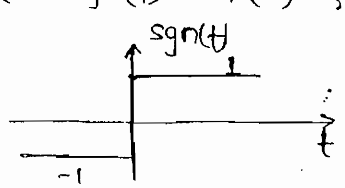
(4.)



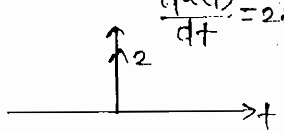
$$avg = \frac{Ao}{2}$$

★★★ **For any finite duration pulse avg. value will be = 0**

Q.→ $x(t) = sgn(t) \rightleftharpoons X(\omega) = ?$



Sol^n→ $\frac{dx(t)}{dt} = 2\delta(t)$

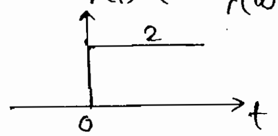


$$\frac{dx(t)}{dt} = 2\delta(t)$$

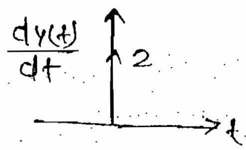
$\Big\downarrow$ FT.

$$\boxed{x(t) = Sgn(t) \rightleftharpoons X(\omega) = \frac{2}{j\omega}}$$

Que.→ $y(t) \rightleftharpoons Y(\omega) = ?$



Soln→



$\longrightarrow avg = \frac{2}{2}$
$= 1$
$= 2\pi\delta(\omega)$

$\frac{dy(t)}{dt} = 2\delta(t)$

$[j\omega]\, Y(\omega) = 2$

$\boxed{Y(\omega) = \frac{2}{j\omega}}$ X

2nd method →

$y(t) = 1 + x(t)$

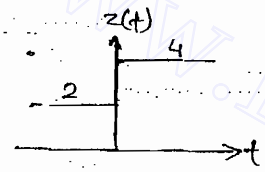
$\downarrow$ (FT)

$Y(\omega) = 2\pi\delta(\omega) + X(\omega)$

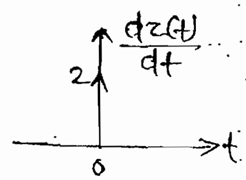
$= \frac{2}{j\omega} + 2\pi\delta(\omega)$

$\boxed{Y(\omega) = \frac{2}{j\omega} + 2\pi\delta(\omega)}$

Que.→



Soln→



$\frac{dz(t)}{dt} = 2\delta(t)$

$j\omega z(\omega) = 2$

$\boxed{Z(\omega) = \frac{2}{j\omega}}$ X

$Avg. = \frac{4+2}{2} = 3$

$\downarrow$

$3 \times 2\pi\delta(\omega)$

$6\pi\delta(\omega)$

$\boxed{Z(\omega) = \frac{2}{j\omega} + 6\pi\delta(\omega)}$

2nd method →

$z(t) = 3 + x(t)$

$\downarrow$ FT

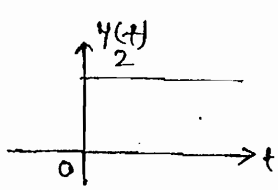
$Z(\omega) = 6\pi\delta(\omega) + \frac{2}{j\omega}\, X(\omega)$

$= X(\omega) + 6\pi\delta(\omega)$

$Z(\omega) = \frac{2}{j\omega} + 6\pi\delta(\omega)$

$\boxed{Z(\omega) = \frac{2}{j\omega} + 6\pi\delta(\omega)}$

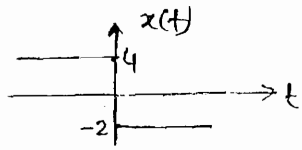
**



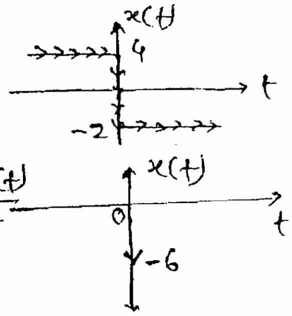
$2u(t) = \left[\frac{2}{j\omega} + 2\pi\delta(\omega)\right]$

$\frac{2u(t)}{2} = \frac{1}{2}\left[\frac{2}{j\omega} + 2\pi\delta(\omega)\right]$

Q.→



Sol^n→



$\frac{dx(t)}{dt}$

$$\frac{dx(t)}{dt} = -6\delta(t)$$

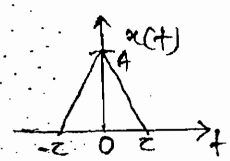
$$j\omega X(\omega) = -6$$

$$X(\omega) = \frac{-6}{j\omega}$$

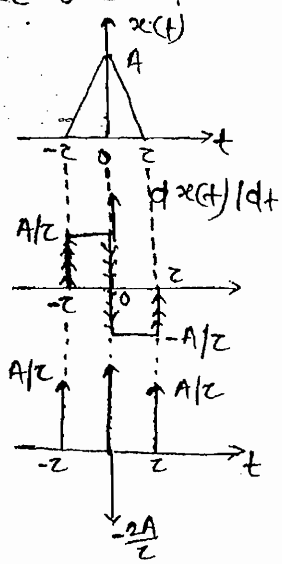
$$Avg. = \frac{4-2}{2} = 1$$

$$\boxed{X(\omega) = \frac{-6}{j\omega} + 2\pi\delta(\omega)}$$

Q.→



Sol^n→



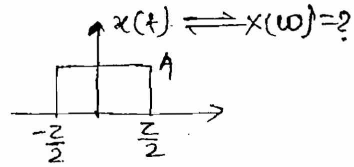
$\frac{d^2x(t)}{dt}$

$$\frac{d^2x(t)}{dx} = \frac{A}{\tau}\delta(t+\tau) + \frac{A}{\tau}\delta(t-\tau) - \frac{2A}{\tau}\delta(t)$$

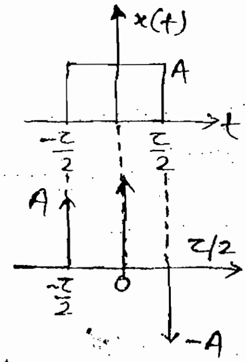
$\downarrow$ FT

$$(j\omega)^2 X(\omega) = A\,e^{j\omega\tau}\ A\,e^{-j\omega\tau} \ldots$$

Que.→ $x(t) \rightleftharpoons X(\omega) = ?$



Sol^n→



$\frac{dx(t)}{dt}$

$$\frac{dx(t)}{dt} = A\delta\left(t+\frac{\tau}{2}\right) - A\delta\left(t-\frac{\tau}{2}\right)$$

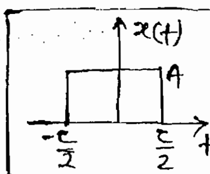
$$j\omega X(\omega) = Ae^{j\omega\tau/2} - Ae^{-j\omega\tau/2}$$

$$X(\omega) = \frac{A}{j\omega} \times \left[e^{j\omega\tau/2} - e^{-j\omega\tau/2}\right]$$

$$= \frac{A}{j\omega} \times 2j \times \sin\left(\frac{\omega\tau}{2}\right)$$

$$= \frac{2A}{\omega}\left[\frac{\sin\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)}\right] \times \left(\frac{\omega\tau}{2}\right)$$

$$= A\tau\, Sa\left(\frac{\omega\tau}{2}\right)$$



$$x(\omega) \rightleftharpoons A\tau\, Sa\left(\frac{\omega\tau}{2}\right)$$

$$X(\omega) = \frac{A}{\tau(-\omega^2)}\left[\left(e^{j\omega\tau} + e^{-j\omega\tau}\right) - 2\right]$$

$$X(\omega) = \frac{A}{-\tau\omega^2}\left[2\cos\omega\tau - 2\right]$$

$$X(\omega) = \frac{2A}{+\tau\omega^2}\left(1 - \cos\omega\tau\right)$$

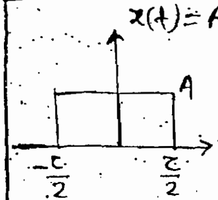
$$= \frac{2A}{\tau\omega^2}\,2\left[\sin^2\left(\frac{\omega\tau}{2}\right)\right]$$

$$= \frac{2A}{\tau\omega^2} \times 2\,\frac{\sin^2\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)^2} \times \left(\frac{\omega\tau}{2}\right)^2$$

$$\boxed{X(\omega) = A\tau\, sa^2\left(\frac{\omega\tau}{2}\right)}$$

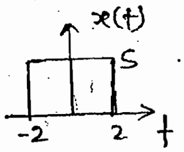
***

$x(t) = A\,rect\left(\frac{A}{2}\right)$



$$\Longrightarrow X(\omega) = A\tau\, sa\left(\frac{\omega\tau}{2}\right) = Area\ sa\left(\frac{\omega\, duration}{2}\right)$$

Q.→

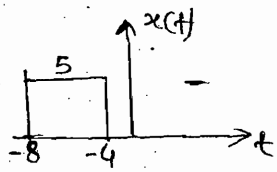


Sol^n→ $X(\omega) = 20\, sa\left(\omega\frac{4}{2}\right)$

$= 20\, sa(2\omega)$

$$\boxed{X(\omega) = 20\, sa(2\omega)}$$

Q.→



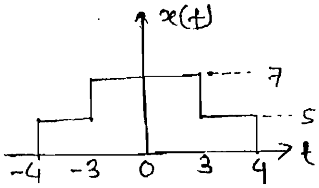
Sol^n→ $y(t) = x(t+6)$

$Y(\omega) = X(\omega)e^{j\omega 6}$

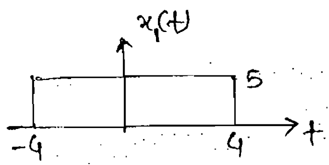
$= 20\, sa(2\omega)\, e^{j\omega 6}$

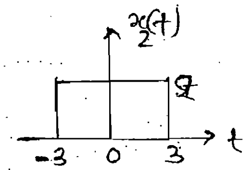
$$\boxed{Y(\omega) = 20\, sa(2\omega)\, e^{j\omega 6}}$$

Q.→



Soln→





$X_1(\omega) = \cancel{40 Sa}(4\omega)$ $\qquad$ $X_2(\omega) = 12 Sa(3\omega)$

$x(t) = x_1(t) + x_2(t)$

$X(\omega) = X_1(\omega) + X_2(\omega)$

$X(\omega) = 40 Sa(4\omega) + 12 Sa(3\omega)$

Q.→ $x(t) = rect\left(t - \frac{1}{2}\right)$

$y(t) = x(t) + x(-t) \rightleftharpoons Y(\omega)$ $\quad$ Find $Y(\omega) = ?$

(a.) $sinc\left(\frac{\omega}{2\pi}\right)$ $\qquad$ (c.) $2 sinc\left(\frac{\omega}{2\pi}\right) cos\left(\frac{\omega}{2}\right)$

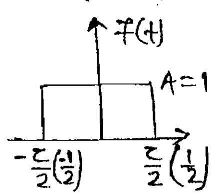
(b.) $2 sinc\left(\frac{\omega}{2\pi}\right)$

Soln→ $\quad x(t) = rect\left(t - \frac{1}{2}\right)$

$x(-t) = rect\left(-t - \frac{1}{2}\right)$

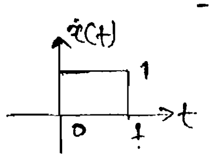
$y(t) = x(t) + x(-t) = rect\left(t - \frac{1}{2}\right) + rect\left(-t - \frac{1}{2}\right)$

$f(t) = rect(t) = A\, rect\left(\frac{t}{\tau}\right)$



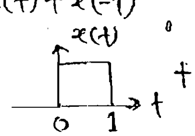
$\because Sa(k) = sinc\left(\frac{k}{\pi}\right)$

$x(t) = f\left(t - \frac{1}{2}\right)$

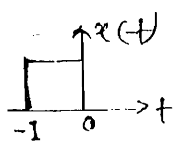


$y(t) = x(t) + x(-t)$

$=$



$+$



$$Y(\omega) = 2Sa(\omega) = 2\,\text{sinc}\left(\frac{\omega}{\pi}\right)$$

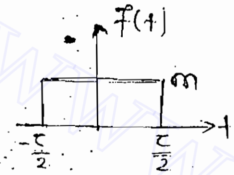
$$= \frac{2\sin\omega}{\omega} = 2\times\left(\frac{\sin\frac{\omega}{2}}{\omega/2}\right)\times\cos\frac{\omega}{2} = 2Sa\left(\frac{\omega}{2}\right)\cos\left(\frac{\omega}{2}\right)$$

$$= \cancel{\frac{2\times\sin\frac{\omega}{2}\cos\frac{\omega}{2}}{\omega}} \qquad = 2\,\text{sinc}\left(\frac{\omega}{2\pi}\right)\cdot\cos\left(\frac{\omega}{2}\right)$$

$$\boxed{Y(\omega) = 2\,\text{sinc}\left(\frac{\omega}{2\pi}\right)\cdot\cos\left(\frac{\omega}{2}\right)}$$

Q.→ $x(t) = A_0 Sa(t) \rightleftharpoons$ Draw FT $X(\omega)$

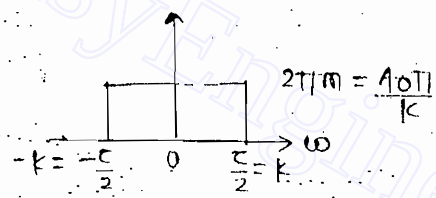
Sol^n→



$\rightleftharpoons m\tau\, Sa\left(\frac{\omega\tau}{2}\right)$

$(\omega = t)$ $\quad (t = -\omega)$

$$m\tau\, Sa\left(\frac{t\tau}{2}\right) \rightleftharpoons 2\pi f(-\omega)$$



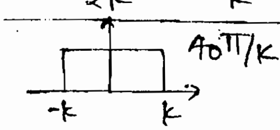
$$m\tau\, Sa\left(\frac{t\tau}{2}\right) = A_0 Sa(kt)$$

$$m\tau = A_0, \quad k = \frac{\tau}{2}$$

$$2\pi m = 2\pi\cdot\frac{A_0}{\tau} = \frac{2\pi\times A_0}{2k} = \frac{A_0\pi}{k}$$

★★★

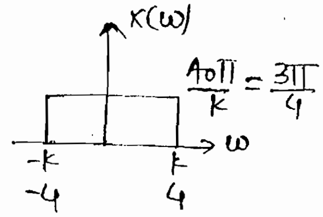
$A_0 Sa(kt) \rightleftharpoons$



Q.→ $x(t) = 3\,Sa(4t) \rightleftharpoons X(\omega)$

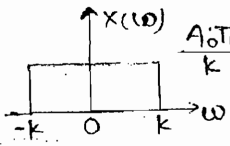
Sol^n→ $A_0 = 3$ $\quad A_0 Sa(kt)$

$k = 4$



Q→ Calculate area & energy of $x(t) = Sa(t)$

Sol^n→ $x(t) = Sa(t)$



$\frac{A_0\pi}{k} = \pi$ $= \cancel{A_0 Sa(kt)}$ $A_0 Sa(kt)$

$x(t) = A_0 Sa(kt) = Sa(t)$

$A_0 = 1, k = 1$

area under time-domain →

area of $x(t) = X(\omega)|_{\omega=0}$

$\boxed{A = \pi}$

parseval's energy theorem →

$$E = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

$$= \frac{1}{2\pi}\int_{-1}^{1} \pi^2 d\omega$$

$\boxed{E = \pi}$

Q.→ $h(t) \rightleftharpoons H(\omega) = \frac{2\cos\omega \cdot \sin 2\omega}{\omega}$

Find $h(0) = ?$

(a) 1/4 (b) 1/2 (c) 1 (d) 2

Sol^n→ $H(\omega) = \frac{2\cos\omega \cdot \sin 2\omega}{\omega}$

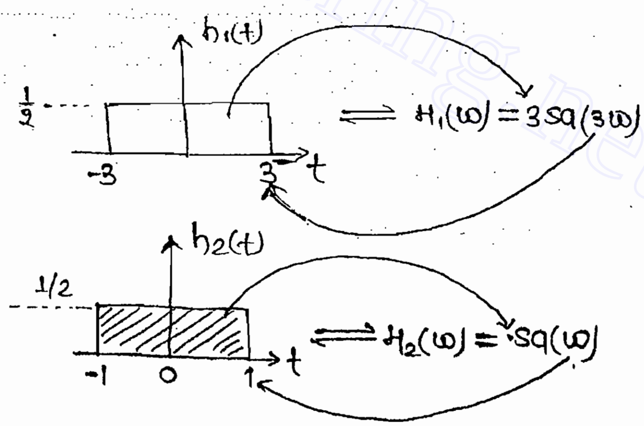
$$= \frac{\sin(3\omega) + \sin\omega}{\omega}$$

$$= \frac{\sin(3\omega)}{\omega} + \frac{\sin\omega}{\omega}$$

$$= \frac{3\sin(3\omega)}{3\omega} + \frac{\sin\omega}{\omega}$$

$H(\omega) = 3Sa(3\omega) + Sa(\omega)$

$= H_1(\omega) + H_2(\omega)$



$h(t) = h_1(t) + h_2(t)$

$\big\downarrow t=0$

$h(0) = h_1(0) + h_2(0)$

$= \frac{1}{2} + \frac{1}{2}$

$= 1$

Area

$x(t) \longrightarrow A$

$x(kt) \longrightarrow \frac{A}{k}$

$k\,x(t) \longrightarrow kA$

$Sa(\omega) \longrightarrow \pi$

$Sa(3\omega) \longrightarrow \pi/3$

$3\,Sa(3\omega) \longrightarrow 3 \times \frac{\pi}{3} = \pi$

2nd method →

Area under freq. domain

$2\pi h(0) = \text{Area of } H(\omega)$

$$h(0) = \frac{\text{Area of } H(\omega)}{2\pi}$$

$$h(0) = \frac{2\pi + \pi}{2\pi} = 1$$

$\boxed{h(0) = 1}$

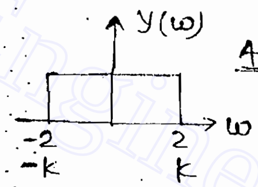
Q.→ $y(t) = x(t)\cos t \rightleftharpoons Y(\omega) = \begin{cases} 2, & |\omega| \le 2 \\ 0 & \text{otherwise.} \end{cases}$

Find $x(t)$

(a) $\frac{4}{\pi}\frac{\sin t}{t}$ (b) $\frac{2\sin t}{t}$ (c) $4\frac{\sin t}{t}$ (d) $2\pi \cdot \frac{\sin t}{t}$

Soln→ 1st method →

$y(t) = A_0 Sa(kt) \Longleftrightarrow$



$\frac{A_0\pi}{k} = 2$

$= \frac{4}{\pi} Sa(2t)$

$k = 2, A_0 = 4/\pi$

$= \frac{4}{\pi} \cdot \frac{\sin 2t}{2t} = \left[\frac{4}{\pi} \cdot \frac{\cancel{2}\sin t}{\cancel{2}t}\right] \cdot \cos t$

$= \left[\frac{4}{\pi} \cdot \frac{\sin t}{t}\right] \cdot \cos t$

$= x(t) \cdot \cos t$

2nd method →

area under freq. domain

$2\pi Y(0) = \text{area of } Y(\omega)$

$2\pi Y(0) = 8$

$$Y(0) = \frac{8}{2\pi} = \frac{4}{\pi} = x(0)$$

$Y(0) = x(0)$ at ...

Q.→ Find FT of $\cos\omega_0 t$

$$x(t) = \cos\omega_0 t \Longleftrightarrow X(\omega) = ?$$

Solⁿ→

$$x(t) = \cos\omega_0 t$$

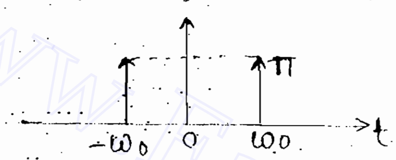
$$x(t) = \frac{1}{2}\left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right]$$

$$X(\omega) = \frac{1}{2}\left[2\pi\delta(\omega-\omega_0) + 2\pi\delta(\omega+\omega_0)\right]$$

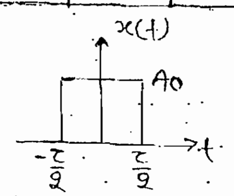
$$X(\omega) = \pi\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\pi$$

$$A_0 = 2\pi A_0\delta(\omega)$$
$$\downarrow A_0 = 1$$
$$1 = 2\pi\delta(\omega)$$
$$1.e^{j\omega_0 t} = 2\pi\delta(\omega-\omega_0)$$
$$e^{-j\omega_0 t} = 2\pi\delta(\omega+\omega_0)$$

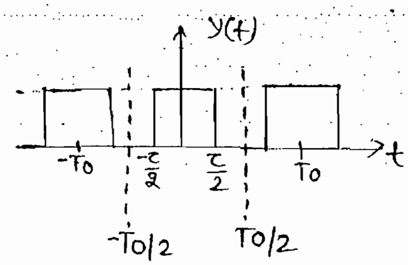
$$\cos\omega_0 t = \pi\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\pi$$



✱ Calculation of c_n by using FT→



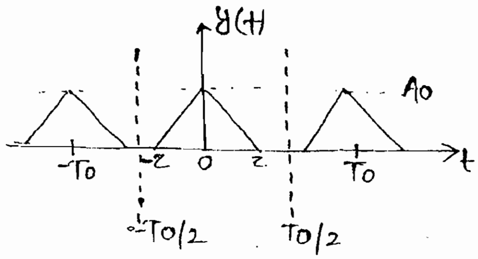
$$X(\omega) = A_0\tau Sa\left(\frac{\omega\tau}{2}\right)$$



✱✱✱
$$\boxed{C_n = \frac{X(n\omega_0)}{T_0}}$$

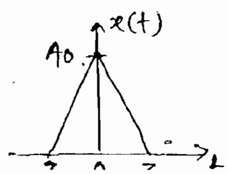
$$X(\omega) = \frac{A_0\tau}{T_0} Sa\left(\frac{n\omega_0\tau}{2}\right)$$

Q.→



$C_n = ?$

Solⁿ→



$$X(\omega) = A_0\tau Sa^2\left(\frac{\omega\tau}{2}\right)$$

$$C_n = X(n\omega_0) \ldots A_0\tau \ldots (n\omega_0\tau)$$

* FT for periodic Signal →

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \qquad X(\omega) = ?$$

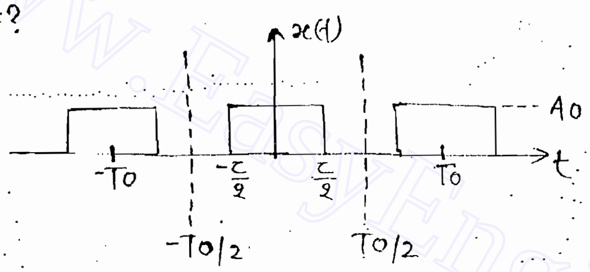
$$1 = 2\pi\,\delta(\omega)$$

$$C_n = 2\pi C_n\,\delta(\omega)$$

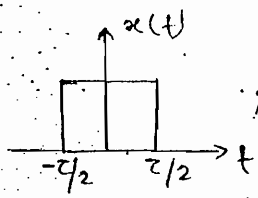
$$C_n e^{jn\omega_0 t} = 2\pi C_n\,\delta(\omega - n\omega_0) \quad \text{(Freq. shifting)}$$

$$\boxed{\sum C_n e^{jn\omega_0 t} = 2\pi \sum_{n=-\infty}^{\infty} C_n\,\delta(\omega - n\omega_0)}$$

Que.→ $X(\omega) = ?$



Soln→



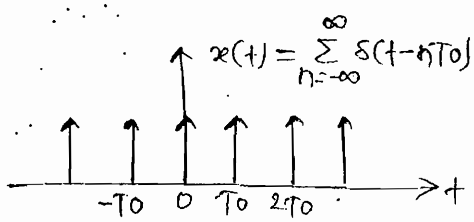
$$X(\omega) \longrightarrow A_0\tau\, Sa\left(\frac{\omega\tau}{2}\right)$$

$$C_n = \frac{A_0\tau}{T_0}\, Sa\left(\frac{n\omega_0\tau}{2}\right)$$

$$X(\omega) = 2\pi \sum C_n\,\delta(\omega - n\omega_0)$$

$$= 2\pi \sum_{n=-\infty}^{\infty} \left[\frac{A_0\tau}{T_0}\, Sa\left(\frac{n\omega_0\tau}{2}\right)\right] \delta(\omega - n\omega_0)$$

Que.→ $X(\omega) = ?$

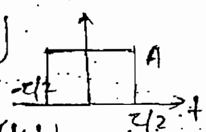
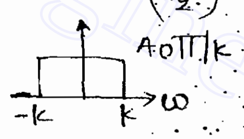
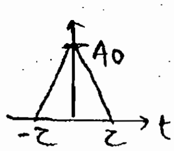


Soln→

$$X(\omega) = 2\pi \sum C_n\,\delta(\omega - n\omega_0)$$

$$= 2\pi \sum \frac{1}{T_0}\,\delta(\omega - n\omega_0)$$

*Important signal →

	$x(t)$	$X(\omega)$
(1.)	$\delta(t)$	1
(2.)	$u(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$
(3.)	$sgn(t)$	$\frac{2}{j\omega}$
(4.)	A_0	$2\pi A_0 \delta(\omega)$
(5.)	$e^{-at}u(t)$, $a>0$	$\frac{1}{a+j\omega}$
(6.)	$e^{-a\|t\|}u(t)$, $a>0$	$\frac{2a}{a^2+\omega^2}$
(7.)	$\cos\omega_0 t$	$\pi[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)]$
(8.)	$\sin\omega_0 t$	$\pi j[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)]$
(9.)	$A\,rect(\frac{t}{\tau})$ 	$A\tau Sa\left(\frac{\omega\tau}{2}\right)$
(10.)	$A_0 Sa(kt)$	
(11.)	Periodic sig.	$2\pi \sum_{n=-\infty}^{\infty} C_n \delta(\omega - n\omega_0)$
(12.)	$\sum \delta(t-nT_0)$	$\omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$
(13.)		$A\tau Sa^2\left(\frac{\omega\tau}{2}\right)$
~~(14.)~~	~~$e^{j\omega_0 t}$~~	$2\pi\delta(\omega-\omega_0)$
(15.)	$e^{-j\omega_0 t}$	$2\pi\delta(\omega+\omega_0)$

$\boxed{\text{DATE}-21/10/14}$

* $x(t)-X(\omega)$ pairs →

$x(t)$	$X(\omega)$
Real	CS
CS	Real
Img.	CAS
CAS	Img.
R+E	R+E

R+O	I+O
I+O	R+O

x(t)	X(ω)
Continues	Non-periodic
Non-periodic	contineous
Discrete	periodic
periodic	discrete

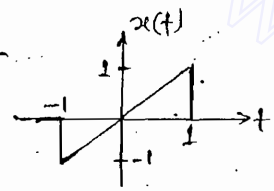
C + P ⟶ D + NP

C + NP ⟶ C + NP

D + P ⟶ D + P

D + NP ⟶ C + P

Q.)



$X(\omega) = ?$

(a.) $4\pi j\left[\frac{\cos\omega}{\omega} - \frac{\sin\omega}{\omega}\right]$

(c.) $2j\left[\frac{\cos\omega}{\omega} - \frac{\sin\omega}{\omega^2}\right]$

(b.) $4\pi j\left[\frac{\cos\omega}{\omega^2} - \frac{\sin\omega}{\omega}\right]$

(d.) $2j\left[\frac{\cos\omega}{\omega^2} - \frac{\sin\omega}{\omega}\right]$

Sol^n → For sol^n go through the option. ans. (c)

I + O

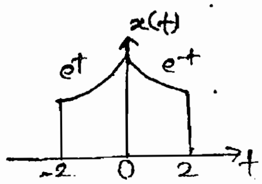
$x(t) \rightarrow R + O$

$X(\omega) \rightarrow I + O$

$\frac{\sin\omega}{\omega} = E$

$$X(\omega) = 2j\left[\frac{\cos\omega}{\omega} - \frac{\sin\omega}{\omega^2}\right]$$

Q.→



$\Rightarrow X(\omega) = ?$

(a) $2 - 2e^{-2}\sin 2\omega + 2\omega e^{-2}\sin 2\omega$

(b) $2 + 2e^{-2}\cos 2\omega - 2\omega e^{-2}\cos 2\omega$

(c.) $2 + 2e^{-2}\cos 2\omega - 2\omega e^{-2}\sin 2\omega$

(d.) $2 - 2e^{-2}\cos 2\omega + 2\omega e^{-2}\sin 2\omega$

Sol^n → $x(t) = R+E$; $X(\omega) = R+E$

so; option (a) & (b) $\neq$ R+E

Area under time domain;

$$X(0) = \int_{-\infty}^{\infty} x(t)\,dt$$

$$= \int_{-2}^{2} x(t)\,dt = 2\int_{0}^{2} x(t)\,dt$$

$$= 2\int_{0}^{2} e^{-t}\,dt$$

$$= \frac{2\left[e^{-t}\right]_0^2}{-1} = 2(1-e^{-2})$$

$$= 2-2e^{-2}$$

Now; put $x(0)$ in the option (c) & (D)

$\boxed{\text{Ans. (d)}}$

Q.→ $f(t) \rightleftharpoons F(\omega)$

$$g(t) = \int_{-\infty}^{\infty} F(u)\,e^{-jut}\,du$$

what is the relationship bet^n $f(t)$ & $g(t)$?

(a.) g(t) would always be proportional to f(t).

(b.) g(t) would always be proportional to f(t) is f(t) is an even sig.

(c.) g(t) would proportional to f(t) only if f(t) is sinusoidal f^n

(d.) g(t) would ~~area~~ never to be proportional to f(t).

Sol^n→ IFT (inverse FT)

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\,e^{j\omega t}\,d\omega$$

$$2\pi f(t) = \int_{-\infty}^{\infty} F(\omega)\,e^{j\omega t}\,d\omega$$

$\downarrow (\omega = u)$

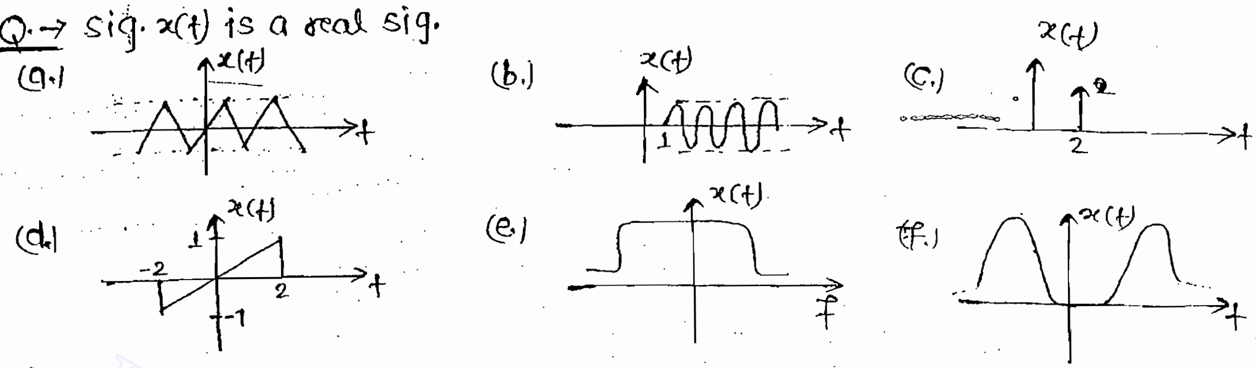
$$2\pi f(t) = \int_{-\infty}^{\infty} F(u)\,e^{jut}\,du$$

$\downarrow (t = -t)$

$$2\pi f(-t) = \int_{-\infty}^{\infty} F(u)\,e^{-jut}\,du$$

$\boxed{g(t) = 2\pi f(t)}$ If $f(t)$ is even
$f(-t) = f(t)$

Q.→ sig. $x(t)$ is a real sig.



(i) $Re[X(\omega)] = 0$

(a.) a,d (b) e,f (c) b,c (d) b,d.

Sol^n→

$x(t) \longrightarrow$ Real

$X(\omega) \longrightarrow CS_o$ (given)

$= Real[X(\omega)] + j\,Img[X(\omega)]$

even   odd

$\therefore X(\omega) \rightarrow CS$ & nature is imag. odd.

So $x(t) \rightarrow R+O$.

$\boxed{\text{ans (a.)}}$

(ii) $\int_{-\infty}^{\infty} X(\omega)\,d\omega = 0$

(a) e (b) a,b,c,d,f (c) b,c (d) a,d,e,f.

Sol^n→ Area under freq. domain

$$2\pi x(0) = \int_{-\infty}^{\infty} X(\omega)\,d\omega$$

$$2\pi x(0) = 0$$

$$\boxed{x(0) = 0}$$

$\boxed{\text{ans.(b)}}$

(iii) $\int_{-\infty}^{\infty} \omega X(\omega)\, d\omega = 0$

(a.) a, b, c, d, f (b.) b, c, e, f (c.) e (d.) b, c

Soln →

$$x(t) \rightleftharpoons X(\omega)$$

$$\frac{dx(t)}{dt} = (j\omega) X(\omega)$$

$$\frac{1}{j}\frac{dx(t)}{dt} = \omega X(\omega)$$

$$y(t) = Y(\omega)$$

area under Freq. domain

$$2\pi y(0) = \int_{-\infty}^{\infty} Y(\omega)\, d\omega$$

$$= \int_{-\infty}^{\infty} \omega X(\omega)\, d\omega$$

$$2\pi y(0) = 0$$

$$y(0) = (0)$$

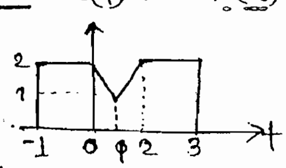
$$y(t)\Big|_{t=0} = 0$$

$$\frac{1}{j}\frac{dx(t)}{dt}\Big|_{t=0} = 0$$

$$\boxed{\frac{dx(t)}{dt} = 0}$$ (Slope is zero at the origin)

$\boxed{\text{ans.(b)}}$

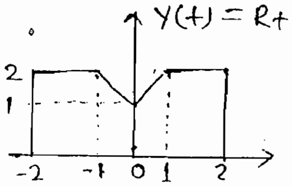
Q. → $x(t) \rightleftharpoons X(\omega)$



Find $\angle X(\omega) = ?$

Soln →

$Y(t) = R+E \rightleftharpoons Y(\omega) = R+E$



$\angle X(\omega) = \angle Y(\omega) + (-\omega)$

$x(t) = y(t-1)$

$\because Y(\omega) = R+E$ & $\angle Y(\omega) = 0$

where; $f(0^-) = |f(t)|_{t=0^-}$

$$f'(0^-) = \left.\frac{df(t)}{dt}\right|_{t=0^-} \qquad f''(0^-) = \left.\frac{d^2f(t)}{dt^2}\right|_{t=0^-} \quad \ldots$$

(8.) Integration in time →

$$\int_{-\infty}^{t} f(t)\,dt = \begin{cases} \dfrac{F(s)}{s}, & \text{Bilateral FT.} \\[2ex] \dfrac{F(s)}{s} + \dfrac{\int_{-\infty}^{0^-} f(t)\,dt}{s}, & \text{unilateral LT.} \end{cases}$$

(9.) Convolution in time →

$$f_1(t) * f_2(t) = F_1(s) \cdot F_2(s)$$

(10.) Multiplication in time →

$$f_1(t) \cdot f_2(t) = \frac{1}{2\pi j}\left[F_1(s) * F_2(s)\right]$$

(11.) Differentiation in freq. →

$$t^n f(t) = (-1)^n \frac{d^n F(s)}{ds^n}$$

(12.) Integration in freq. →

$$\frac{f(t)}{t} = \int_{s}^{\infty} F(s)\,ds$$

(13.) Initial value theorem →

$$x(0) = \lim_{s\to\infty}\left[sX(s)\right]$$

Condition:- It is applicable only for causal type signals.

i.e. $x(t) = 0;\ t < 0$

(14.) Final value theorem →

$$x(\infty) = \lim_{s\to 0}\left[sX(s)\right]$$

Condition →

(i) It is applicable only for causal type signals.

i.e.; $x(t) = 0,\ t<0$

*** (ii) "$sX(s)$" should have only LHS poles in s-plane

Que → $F(s) = \frac{1}{s^2+1} \rightleftharpoons f(t)$. Calculate $f(\infty) = ?$

(a) -1 (b) 0 (c) 1 (d) $-1 \le f(\infty) \le 1$.

Sol^n → $sF(s) = \frac{s}{s^2+1}$

Poles:- $s = \pm j \ne$ LHS poles

* FVT is not applicable because $s = \pm j$ poles)

$f(t) = \sin t\, u(t)$

$f(\infty) = \sin\infty\, u(\infty)$

$(-1, 1)$

$f(\infty) = -1 \le f(\infty) \le 1$

Que → $F(t) = F(s) = \frac{1}{s(s-1)}$; $f(\infty) = ?$

(a) 0, (b) ∞ (c) -1 (d) 1

Sol^n → (1.) signal is causal because depand on past $(s-1)$

(2.) $sF(s) = \frac{s}{s(s-1)} = \frac{1}{s-1}$

Pole: $s=1 \ne$ LHS plane

FVT is not applicable.

$$F(s) = \frac{1}{s(s-1)} = -\frac{1}{s} + \frac{1}{(s-1)}$$

~~$f(t) = e^t u(t) + e$~~ $\quad f(t) = e^t u(t) - u(t)$

$$f(\infty) = \infty - 1 + \infty$$

$$\boxed{F(\infty) \simeq \infty}$$

Q→ $f(t) = -e^{-at}u(-t) \rightleftharpoons F(s) = ?\ ,\ ROC = ?$

Sol^n→

$$f(t) = e^{-at}u(-t)$$

$$e^{-at}u(t) \rightleftharpoons \frac{1}{s+a} \ ;\ (\sigma > -a)$$

$$-e^{-at}u(t) \rightleftharpoons \frac{-1}{s+a} \ ;\ (\sigma > -a)$$

$\downarrow (t=-t) \quad \downarrow (s=-s) \quad \downarrow$ (By time Reversal) $(\sigma = -\sigma)$

$\because s = \sigma + j\omega$

$-s = -\sigma - j\omega$

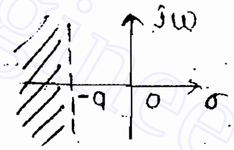
$$-e^{at}u(-t) \rightleftharpoons \frac{-1}{-s+a} \ ;\ (-\sigma > -a)$$

$\downarrow (a=-a) \qquad \downarrow (a=-a)$

$$-e^{-at}u(-t) \rightleftharpoons \frac{-1}{-s-a} \ ;\ (-\sigma > a)$$

$$-e^{-at}u(-t) \rightleftharpoons \frac{1}{s+a} \ ;\ (\sigma < -a)$$

$$-e^{-at}u(-t) \rightleftharpoons \frac{1}{s+a} \ ;\ (\sigma < -a)$$



$(a=0)$

& also;

$$e^{-at}u(t) \rightleftharpoons \frac{1}{s+a} \ ;\ (\sigma > -a)$$

$(a=0)$

$$u(-t) \longrightarrow \frac{1}{s} \ ;\ \sigma < 0$$

$$u(t) \longrightarrow \frac{1}{s} \ ;\ \sigma > 0$$

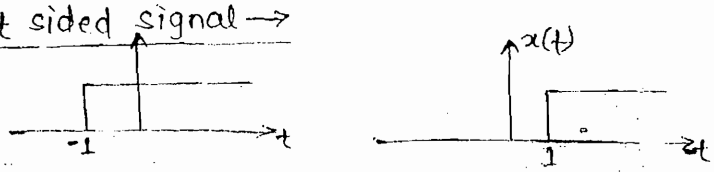
$$\boxed{\begin{array}{l} e^{-at}u(t) \rightarrow \frac{1}{s+a} \ ;\ (\sigma > -a) \\ -e^{-at}u(-t) \longrightarrow \frac{1}{s+a} \ ;\ (\sigma < -a) \end{array}} \qquad \boxed{\begin{array}{l} u(-t) \longrightarrow \frac{1}{s} \ ;\ \sigma < 0 \\ u(t) \longrightarrow \frac{1}{s} \ ;\ \sigma > 0 \end{array}}$$

Region of convergance (ROC) → It is defined as the range of complex variable as in s-planes for which LT of signal is convergant (or) finite.

Properties →

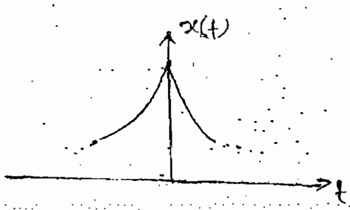
(i) ROC doesn't include any pole.

Right sided signal →



Left sided signal →

Both sided signal →



(ii) For right side signal ROC will be right side to the right most pole.
For left sided signal ROC will be left side to the left most pole.

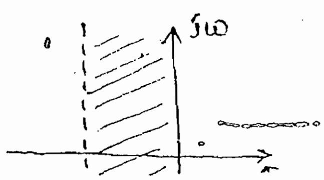
(iii) For stability ROC includes imaginary axis in s-plane.

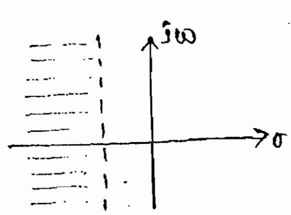
(iv) For both sided sig. ROC is a strip in s-plane.

(v) For ∞ duration signal ROC is entire s-plane excluding possibly $s=0$ or $\pm\infty$.

Q.→ Check stability of LTI sys. of & comment about extention of $h(t)$.

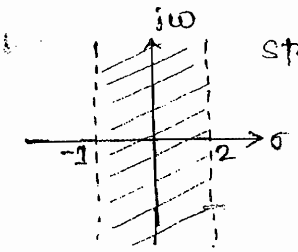
(i) ROC: $\sigma > -1$ (ii) ROC: $\sigma < -2$





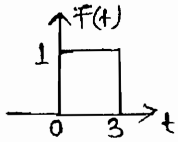
$h(t)$ = Unstable + LS

(3.) ROC: $-1<\sigma<2$



strip type $\quad h(t) =$ stable + Both side.

Que.→



$\rightleftharpoons F(s) = ?\ ;\ ROC = ?$

Soln→ $F(s) = \int_{-\infty}^{\infty} f(t)\, e^{-st} dt$

$$= \int_0^3 e^{-st} dt = \int_0^3 e^{-st} dt = \frac{(e^{-st})_0^3}{-s} = \frac{1-e^{-3s}}{s}$$

At $(s=0)$ $\quad F(0) = \frac{1-e^{-3s}}{s} = \frac{0}{0}$ then use L-Hospital Rule

diff wrt. $s = 3e^{-3s}\Big|_{s=0} = 3$

at $(s=\infty)$

$F(\infty) = 0$

at $(s=-\infty)$ $\quad F(-\infty) = 3e^{-3s}\Big|_{(s=-\infty)} = 3e^{\infty} = \infty$

ROC : Entire s-plane excluding $s=-\infty$

$e^{2t}u(-t+3)$ $\qquad$ $e^{2t}u(-t-4)$

ROC: $\sigma<2$ $\qquad$ ROC: $\sigma<2$

Que.→ $f(t) = e^{-2t}u(-t) + e^{3t}u(t)$

Soln→ $f(t) = \underbrace{e^{-2t}u(-t)}_{ROC:\ \sigma<-2} + \underbrace{e^{3t}u(t)}_{\sigma>3}$

LT of $f(t)$ will not exist because of no common ROC

Que.→ $f(t) = -e^{-2t}u(-t) + e^{-3t}u(t)$

Soln→ $f(t) = -e^{-2t}u(-t) + e^{-3t}u(t)$

common ROC: $-3<\sigma<-2$ → strip

Q.→ $f(t) = e^{-3|t|}$; ROC = ?

Soln→ $f(t) = \begin{cases} e^{3t} & ; t<0 \\ e^{-3t} & ; t>0 \end{cases}$

ROC :- $-3<\sigma<3$ $\quad f(t) = \underbrace{e^{3t}u(-t)}_{\sigma<3} + \underbrace{e^{-3t}u(t)}_{\sigma>-3}$

Que.→ $f(t) = e^{r(t)} = e^{t}u(t)$; ROC = ?

Soln→ $f(t) = \begin{cases} 1 & ; t<0 \\ e^t & ; t>0 \end{cases}$ (No common ROC)

$= \underbrace{u(-t)}_{\sigma<0} + \underbrace{e^t u(t)}_{\sigma>1}$

Que.→ $f(t) = e^{-at}u(t)$

Soln→ where $a = b+jc$

$f(t) = e^{-(b+jc)t}u(t)$

for existance of LT

$= \int_{-\infty}^{\infty} |f(t)\, e^{-\sigma t}|\, dt < \infty = \int_{-\infty}^{\infty} |e^{-(b+jc)t} e^{-\sigma t}|\, dt < \infty = \int_{-\infty}^{\infty} |e^{-(b+\sigma)t}|\,|e^{-jct}|\,dt < \infty$

$|e^{-jct}| = 1$

$= \int_0^{\infty} e^{-(\sigma+b)t} dt < \infty \quad = (\sigma+b) > 0 \quad = \sigma > -b \quad = \sigma > -Re(a)$

Que.→ $f(t) = \cos\omega_0 t\, u(t) \Longleftrightarrow F(s) = ?$, ROC = ?

Soln→ $\cos\omega_0 t = \dfrac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$

$f(t) = \frac{1}{2}\left[e^{j\omega_0 t}u(t) + e^{-j\omega_0 t}u(t)\right]$

$e^{-at}u(t) \Longleftrightarrow \dfrac{1}{s+a}$; $\sigma > -Re(a)$

$a = 0+j\omega_0$

$e^{-j\omega_0 t}u(t) \Longleftrightarrow \dfrac{1}{s+j\omega_0}$; $\sigma > 0$

$a = 0-j\omega_0$

$e^{j\omega_0 t}u(t) \Longleftrightarrow \dfrac{1}{s-j\omega_0}$; $\sigma > 0$

Important signals →

F(t)	F(s)	ROC
$\delta(t)$	1	entire s-plane
$u(t)$	$1/s$	$\sigma > 0$
$e^{-at}u(t)$	$\frac{1}{s+a}$	$\sigma > -a$
$e^{-at}u(-t)$	$\frac{1}{s+a}$	$\sigma < -a$
$t\,u(t)$	$\frac{1}{s^2}$	
$e^{-at}t \cdot u(t)$	$\frac{1}{(s+a)^2}$	$\sigma > -a$
$\cos\omega_0 t\, u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\sigma > 0$
$-te^{-at}u(-t)$	$\frac{1}{(s+a)^2}$	$\sigma < -a$
$\sin\omega_0 t\, u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\sigma > 0$
$t^n u(t), n \geq 0$	$\frac{\lfloor n}{s^{n+1}}$	$\sigma > 0$

Que.→ $r(t) = t\,u(t) \rightleftharpoons R(s) = 1/s^2$

Que.→ $f(t) = t\,u(t-1) \rightleftharpoons [(t-1)+1]\cdot u(t-1) = (t-1)\,u(t-1) + u(t-1)$

$$= \frac{1}{s^2}e^{-s} + \frac{e^{-s}}{s}$$

$$y(t) = r(t-1) \rightleftharpoons Y(s) = R(s)e^{-s} = \frac{1}{s^2}e^{-s}$$

$$= (t-1)\,u(t-1)$$

Que.→ $f(t) = (t^2+5t-2)\,u(t-1)$

Sol^n→ $[(t-1)^2 + 7t - 3]\; u(t-1)$

$$= [(t-1)^2 + 7(t-1) + 4]\; u(t-1)$$

$$= (t-1)^2 u(t-1) + 7(t-1)u(t-1) + 4u(t-1)$$

$$F(s) = \frac{2}{s^3}e^{-s} + \frac{7}{s^2}e^{-s} + \frac{4}{s}e^{-s}$$

Que.→ $f(t) = (t^3 + 5t^2 + 3t + 1)\; u(t-1)$

Sol^n→ $(t = t+1)$

$$f(t+1) = [(t+1)^3 + 5(t+1)^2 + 3(t+1) + 1]\; u(t)$$

$$f(t+1) = [t^3 + 8t^2 + 16t + 10]\; u(t)$$

$$\downarrow \text{LT}$$

$$F(s) = \left(\frac{6}{\ } + \frac{16}{\ } + 16 + 10\right)e^{-s}$$

DATE-28/10/14

Q.→ $f(t) = F(s) = \log\left(\frac{s+5}{s+6}\right)$

Find $f(t)$:- (a) $\frac{1}{t}(e^{-6t} - e^{-5t})u(t)$ (c.) $t(e^{-6t} - e^{-5t})u(t)$

(b.) $\frac{1}{t}(e^{-5t} - e^{-6t})u(t)$ (d.) $t(e^{-5t} - e^{-6t})u(t)$

Solⁿ→ Diff. in freq. domain;

$$f(t) \rightleftharpoons F(s)$$

$$t f(t) \rightleftharpoons \frac{-dF(s)}{ds} = -\left[\frac{1}{s+5} - \frac{1}{s+6}\right]$$

$$t f(t) = \frac{1}{s+6} - \frac{1}{s+5}$$

$$t f(t) = e^{-6t}u(t) - e^{-5t}u(t)$$

$$f(t) = \frac{1}{t}(e^{-6t} - e^{-5t})u(t)$$

Q.→ $f(t) = \frac{(1-e^{t})u(t)}{t}$; $F(s) = ?$

(a.) $\log\left(\frac{s}{s-1}\right)$ (c.) $\log\left(\frac{s-1}{s+1}\right)$

(b.) $\log\left(\frac{s-1}{s}\right)$ (d.) $\log\left(\frac{s+1}{s-1}\right)$

Solⁿ→ Integration in freq. domain :-

$$y(t) = (1-e^{t})u(t) \rightleftharpoons Y(s)$$

$$f(t) = \frac{y(t)}{t} \rightleftharpoons F(s) = \int_s^\infty Y(s)\cdot ds$$

$$F(s) = \int_s^\infty Y(s)\,ds = \int_s^\infty \left(\frac{1}{s} - \frac{1}{s-1}\right) ds$$

$$F(s) = \left[\log(s) - \log(s-1)\right]_s^\infty = \left[\log\left(\frac{s}{s-1}\right)\right]_s^\infty$$

$$= \log\left[\lim_{s\to\infty}\left(\frac{s}{s-1}\right)\right] - \log\left(\frac{s}{s-1}\right)$$

$$= \log(1) - \log\left(\frac{s}{s-1}\right)$$

$$= 0 - \log\left(\frac{s}{s-1}\right)$$

Que→ Find inverse LT of $F(s) = \dfrac{s^2+2s+5}{(s+3)(s+5)^2}$

for (i) $\sigma > -3$ (ii) $\sigma < -5$ (iii) $-5 < \sigma < -3$

Soln→

$$F(s) = \frac{A}{(s+3)} + \frac{B}{(s+2)} + \frac{C}{(s+2)^2}$$

$$A = 2,\ B = -1,\ C = -10$$

$$F(s) = \frac{2}{s+3} - \frac{1}{s+5} - \frac{10}{(s+5)^2}$$

Poles:- $s = -3, -5$

(i) $\sigma > -3$; f(t) will be Right sided.

$$f(t) = 2e^{-3t}u(t) - e^{-5t}u(t) - 10te^{-5t}u(t)$$

(ii) $\sigma < -5$; f(t) will be left sided.

$$f(t) = -2e^{-3t}u(-t) - [-e^{-5t}u(-t)] - 10[-te^{-5t}u(-t)]$$

(iii) $\underbrace{-5 < \sigma < -3}_{\text{strip}}$; f(t) will be both sided.

ROC; $\sigma > -5$ → Right sided

ROC; $\sigma < -3$ → left sided

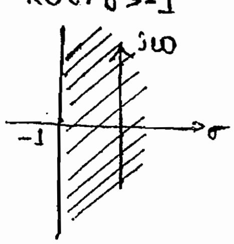
$$f(t) = -2e^{-3t}u(-t) - e^{-5t}u(t) - 10te^{-5t}u(t).$$

*Causal system→

(1.) $h(t) = 0$; $t < 0$

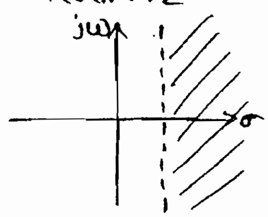
For causal sys. ROC will be right side to the right most pole.

Eg:- ROC: $\sigma > -1$



(Stable)

ROC: $\sigma > 2$



(Unstable)

$H(s) = \dfrac{1}{s+1}$, $\sigma > -1$

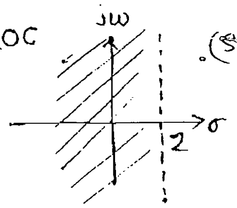
$h(t) = e^{-t}u(t)$ (energy)

$H(s) = \dfrac{1}{s-2}$; $\sigma > 2$

$h(t) = e^{2t}u(t)$ (NENP)

Note→ For stability of causal sys., poles of TF should lie in the LHS of s-plane.

Eg:- ROC



~~(stable)~~

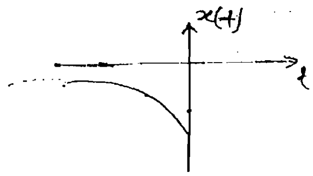
$H(s) = \frac{1}{(s-2)}$; $\sigma < 2$

$h(t) = -e^{2t} u(-t)$

(Energy)

↓

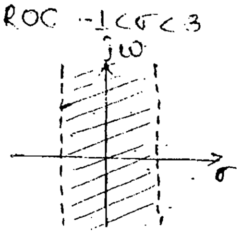
stable.



* For anticausal sys. ROC will be left side to the left most pole.

* For stability of anticausal sys., poles of TF should lie in the RHS of s-plane.

Eg:-(4) ROC $-1 < \sigma < 3$



(stable) + NC

Que→ Consider a contineous time LTI sys. whose i/p x(t) & o/p y(t) are related by the differential eqn.-

$$\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = x(t)$$

determine h(t) of sys.

(a) when the sys. is causal.

(b) when the sys. is ~~NC~~ stable.

(c) Neithe stable nor causal.

Soln→

$$s^2Y(s) - sY(s) - 2Y(s) = X(s)$$

$$Y(s)[s^2 - s - 2] = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2 - s - 2}$$

$$= \quad 1$$

$$H(S) = \frac{\left(\frac{1}{3}\right)}{(S-2)} + \frac{\frac{1}{(-3)}}{(S+1)}$$

Poles:- $-1, 2$

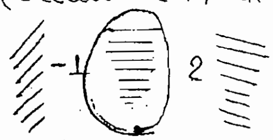
(i) when the sys is causal h(t) will be Right sided.

$$h(t) = -\frac{1}{3}e^{-t}u(t) + \frac{1}{3}e^{2t}u(t)$$

(ii) When sys is stable. h(t) will be both sided.

ROC:- $-1<\sigma<2$ (because img axis is included)

Poles:-



$S+1 \longrightarrow$ Right sided.

$S-2 \longrightarrow$ Left sided.

$$h(t) = -\frac{1}{3}e^{-t}u(t) - \frac{1}{3}e^{2t}u(-t)$$

(iii) ROC: $\sigma<-1$; h(t) will be left sided.

$$h(t) = \left(\frac{1}{3}\right)e^{-t}u(-t) - \frac{1}{3}e^{2t}u(-t)$$

Que.→ For diff. eq^n

$$\frac{d^2y(t)}{dt^2} + \frac{6dy(t)}{dt} + 8y(t) = 0$$

with initial cond^n $y(0^-)=1$, $y'(0^-)=0$; the sol^n of $y(t)$ is;

(a) $2e^{-2t}-e^{-4t}$ (b) $2e^{-6t}-e^{-2t}$ (c) $-e^{-6t}+2e^{-4t}$ (d) $e^{-2t}+2e^{-4t}$

Sol^n→

$$\cancel{s^2Y(S)+6Y(S)s+8Y(S)=0}$$

$$\cancel{Y(s)(s^2+6s+8)=0}$$

By applying LT on given diff eq^n

$$[s^2Y(s) - sy(0^-) - y'(0^-)] + 6[sY(s) - y(0^-)] + Y(s) = 0$$

$$s^2Y(s) - s + 6[\underbrace{sY(s)}_{-1}] + Y(s) = 0$$

$$Y(S)[s^2+6s+1] = s+6$$

$$Y(s) = \frac{s+6}{s^2+6s+1} = \frac{s+6}{(s+2)(s+4)}$$

$$Y(s) = \frac{2}{(s+2)} - \frac{1}{(s+4)}$$

$$\boxed{y(t) = 2e^{-2t} - e^{-4t}}$$

Que.→ For DE $\frac{d^2y(t)}{dt^2} + \frac{2dy(t)}{dt} + y(t) = \delta(t)$

with initial condn; $y(0^-) = -2$, $y'(0^-) = 0$

the value of $\left.\frac{dy(t)}{dt}\right|_{t=0^+}$ is

(a) -1 (b) 1 (c) 0 (d) 2

Soln→

$$s^2Y(s) - sy(0^-) - y'(0^-) + 2[sY(s) - y(0^-)] \cancel{- y'(0^-)} + Y(s) = 1$$

$$Y(s)[s^2+2s+1] - (-2)s + 2 \times (2) = 1$$

$$Y(s) = \frac{-2s-3}{s^2+2s+1}$$

$$Y(s) = \frac{-2s-3}{(s+1)^2} = \frac{-2(s+1)-1}{(s+1)^2}$$

$$Y(s) = \frac{-2}{(s+1)} - \frac{1}{(s+1)^2}$$

$$y(t) = -2e^{-t} - te^{-t}$$

$$\frac{dy(t)}{dt} = 2e^{-t} - (e^{-t} - te^{-t})$$

$$\left.\frac{dy(t)}{dt}\right|_{t=0^+} = 2(1) - (1-0)$$

$$\boxed{\text{Ans.} = 1}$$

Que.→ $y(t) = \sum_{n=0}^{\infty} f(t-nT_0)$ Find $Y(s)$ in terms of $F(s)$

Soln→

$$y(t) = \sum_{n=0}^{\infty} f(t-nT_0)$$

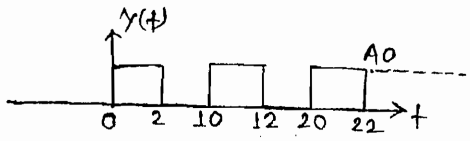
$$y(t) = f(t) + f(t-T_0) + f(t-2T_0) + \cdots$$

$\big|$ LT

$$Y(s) = F(s)\left[1 + e^{-sT_0} + (e^{-sT_0})^2 + \cdots\right]$$

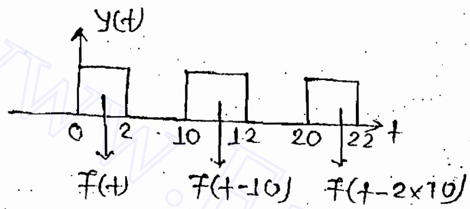
$$\boxed{Y(s) = \frac{F(s)}{1 - e^{-sT_0}}}$$

Que.→



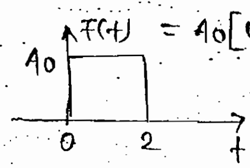
Find $Y(s)$

Sol^n→



$$y(t) = f(t) + f(t-10) + f(t-2\times10) + \cdots$$

$$= \sum_{n=0}^{\infty} f(t - nT_0) \; ; \; T_0 = 10$$



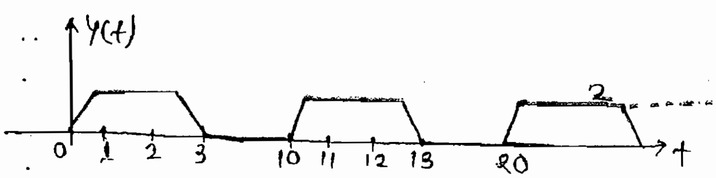
$$f(t) = A_0[u(t) - u(t-2)]$$

$$F(s) = \frac{A_0}{s}(1 - e^{-2s})$$

$$Y(s) = \frac{F(s)}{(1 - e^{-sT_0})}$$

$$\boxed{Y(s) = \frac{A_0(1 - e^{-2s})}{s(1 - e^{-10s})}}$$

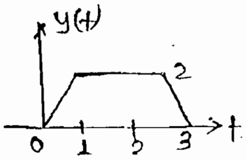
Que→



Find LT of the signal.

Sol^n→

$$y(t) = \sum_{n=0}^{\infty} f(t - nT_0)$$



$$f(t) = 2r(t) - 2r(t-1) - 2r(t-2) + 2r(t-3)$$

$$F(s) = \frac{2}{s^2} - \frac{2e^{-s}}{s^2} - \frac{2e^{-2s}}{s^2} + \frac{2e^{-3s}}{s^2}$$

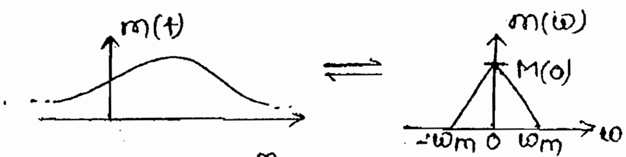
$$F(s) = \frac{2}{s^2}\left[1 - e^{-s} + e^{-2s} + e^{-3s}\right]$$

$$Y(s) = \frac{F(s)}{1 - e^{-sT_0}}$$

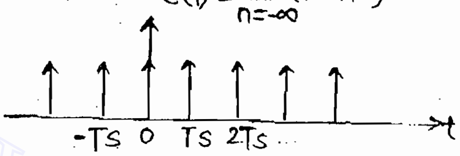
$$Y(s) = \frac{\frac{2}{s^2}\left(1 - e^{-s} + e^{-2s} + e^{-3s}\right)}{1 - e^{-10s}}$$

Chapter-07
Sampling Theorem

Sampling theorem→

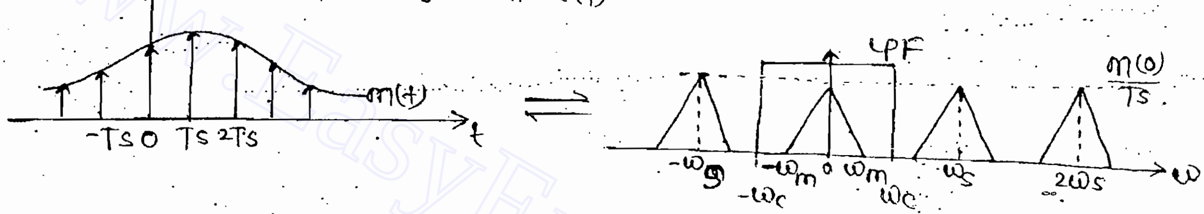


$c(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$



$\rightleftharpoons \quad C(\omega) = \omega_s \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)$

$s(t) = \text{sampled sig.} = m(t) \cdot c(t)$



$s(t) = m(t) \cdot c(t)$

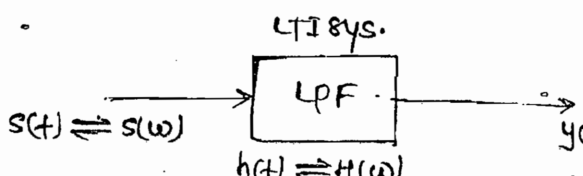
$S(\omega) = \frac{1}{2\pi}\left[M(\omega) * C(\omega)\right]$

$= \frac{1}{2\pi}\left[M(\omega) * \omega_s \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)\right]$

$= \frac{1}{T_s}\left[\sum_{n=-\infty}^{\infty} M(\omega - n\omega_s)\right]$

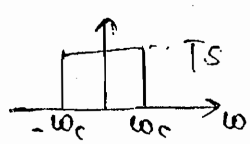
$= \frac{1}{T_s}\left[\text{-------} + M(\omega+\omega_s) + M(\omega) + M(\omega-\omega_s) + M(\omega - 2\omega_s) + \cdots\right]$

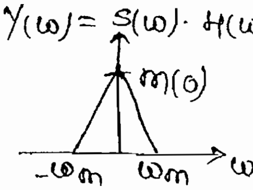
$\boxed{T_s = \text{sampling interval} = \frac{2\pi}{\omega_s}}$



$y(t) = s(t) * h(t)$

$Y(\omega) = S(\omega) \cdot H(\omega)$





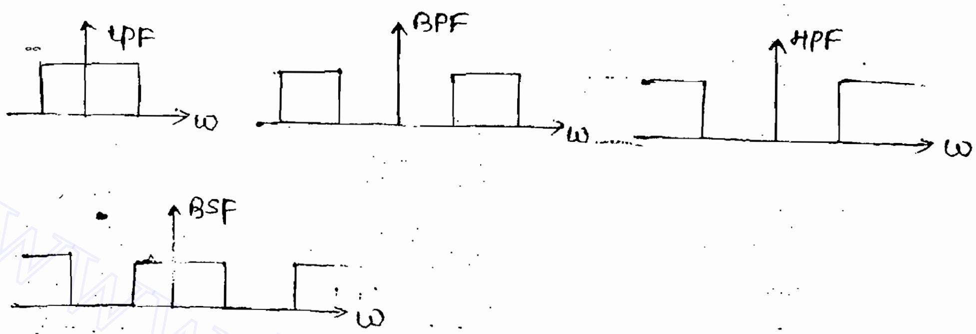
$$\boxed{\omega_m \le \omega_c \le \omega_s - \omega_m}$$

* To avoid overlapping in sampled sig. spectrum:-

$$\omega_m \le \omega_s - \omega_m$$

$$\boxed{\omega_s \ge 2\omega_m}$$

Note:-



Statement → A sig. can be represented by its samples (or) recovered back from its samples if sampling freq. is greater then (or) equal to twice of max^m freq. componant present in signal.

Nquist Rate →

$$F_{ny} = 2F_m$$

Nquist-interval →

$$T_{ny} = \frac{1}{F_{ny}} = \frac{1}{2F_m}$$

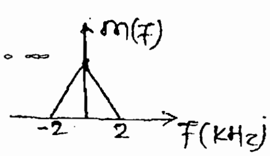
Over sampling →

* $F_s > 2F_m$
* Allowable case

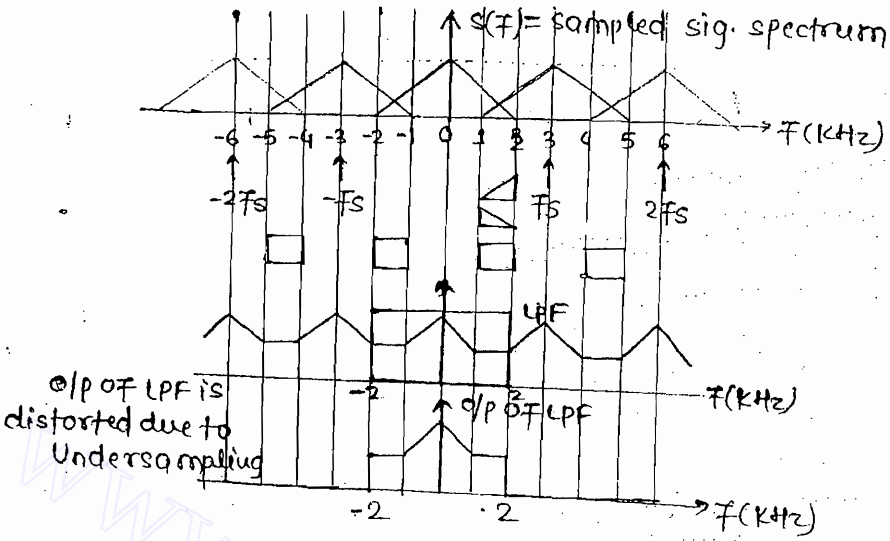
Under-sampling →

* $F_s < 2F_m$
* not allowable.

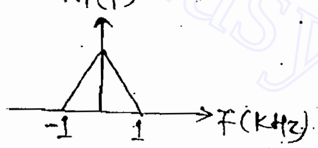
Eg:- $m(t) \rightleftharpoons$



* $f_m = 2KHz$
* $f_s = 3KHz < 2f_m$
* Undersampling



Que.→ $m(t) \rightleftharpoons M(f)$



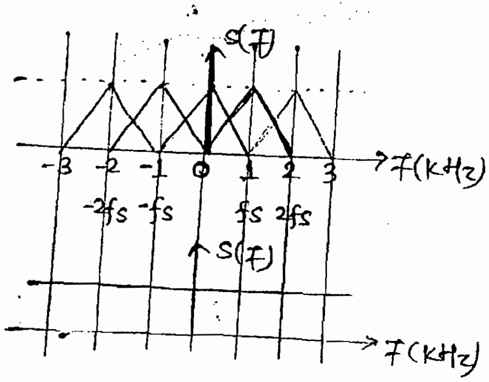
$T_s = 1ms$

draw sampled sig. spectrum.

Soln→

$f_m = 1kHz$

$f_s = \frac{1}{T_s} = 1kHz < 2f_m$ (Undersampling)



Que.→ Calculate Nquist Rate in (rad/sec)

(i) $m(t) = 2\sin 4\pi t \cdot \cos 2\pi t$.

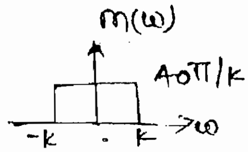
Soln→ $m(t) = 2\sin 4\pi t \cdot \cos 2\pi t$

$= \sin 6\pi t + \sin 2\pi t$

(ii) $m(t) = Sa(4\pi t)$

Sol^n→ $m(t) = Sa(4\pi t)$

$= A_0 \, Sa(kt)$



$\omega_m = k = 4\pi$

$\omega_{ny} = 2\omega_m = 8\pi$

(iii) $m(t) = Sa^3(5\pi t)$

Sol^n→ $\omega_m = 3 \times 5\pi$

$\omega_{ny} = 2\omega_m$

$= 30\pi$

$$\boxed{\begin{array}{l} y(t) = [x(t)]^n \\ \quad\quad\quad \downarrow \\ \quad\quad\quad \omega_m \\ \omega_m' = n\omega_m \end{array}}$$

(iv) $m(t) = Sa^2(4\pi t) \cdot Sa^4(3\pi t)$

Sol^n→ $\omega_{m1} = 2 \times 4\pi$

$\omega_{m2} = 4 \times 3\pi$

$\omega_m = \omega_{m1} * \omega_{m2} = 20\pi$

$\omega_{ny} = 2\omega_m = 40\pi$

$$\boxed{\begin{array}{l} m(t) = m_1(t) \cdot m_2(t) \\ \quad\quad\quad \downarrow \quad\quad \downarrow \\ \quad\quad\quad \omega_{m1} \quad \omega_{m2} \\ \omega_m = \omega_{m1} + \omega_{m2} \end{array}}$$

(v) $m(t) = m_1(t) * m_2(t)$

$\quad\quad \downarrow \quad\quad\quad \downarrow$

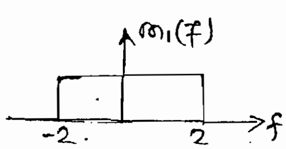
$f_{m1} = 2kHz \quad f_{m2} = 3kHz$

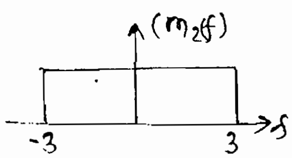
$f_{ny} = ?$

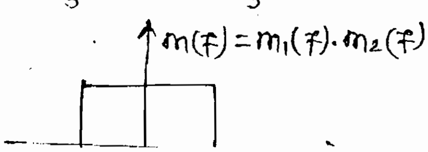
(a) 4kHz (b) 6kHz (c) 10kHz (d) 12kHz

Sol^n→

$m(f) = m_1(f) \cdot m_2(f)$





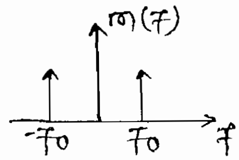


$f_m = 2kHz$

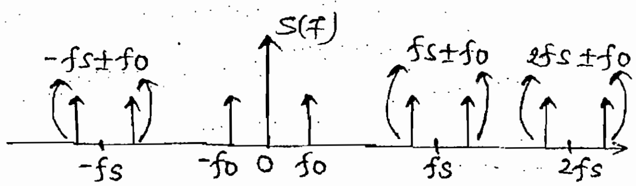
$f_{ny} = 2f_m = 4kHz$

Important points →

(1.) $m(t) = \cos 2\pi f_0 t$ $\rightleftharpoons$



$$c(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$



Freq. componants present in $S(f)$

$: \pm f_0, \; f_s \pm f_0, \; 2f_s \pm f_0, \ldots\ldots$

$\boxed{: \; nf_s \pm f_0}$

where; n = an integer

$= \pm 1, \pm 2, \pm 3, \ldots$

(2.) $m(t) = \cos 2\pi f_1 t + \cos 2\pi f_2(t)$

Freq. componant present in $S(f)$

$\boxed{: \; nf_s \pm f_1, \; nf_s \pm f_2}$

Que. → A sig. $m(t) = 100\cos(24\pi \times 10^3 t)$ is idealy sampled at $T_s = 50\,\mu s$ & passed through an LPF with $f_c = 15$kHz. which of the following freq. is/are present at the o/p of the LPF?

(a.) 8kHz (c) 8 & 10 kHz

(b.) 12kHz (d) 8 & 12 kHz

Sol. →

$f_0 = 12$kHz

$f_s = \dfrac{1}{T_s} = 20$kHz

LPF : $f_c = 15$kHz

Freq. compo. present in $S(f)$

$: f_0, \; f_s \pm f_0, \; 2f_s \pm f_0, \ldots$

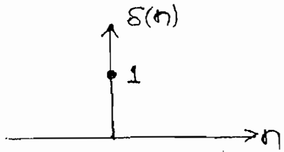
$: \pm 12, \; 20 \pm 12, \; 40 \pm 12, \ldots$

$: 12, 8, 32, 28, 52, \ldots$

Chapter-08
Discrete time signal

1) Unit-impulse signal $\delta(n) \rightarrow$

$$\delta(n) = \begin{cases} 1 ; & n=0 \\ 0 ; & n \neq 0 \end{cases}$$



properties →

(1.) $\delta(n)$ is an even signal.

(2.) $\delta(n)$ is an energy signal.

$\delta(t)$ is NENP sig.

(3.) $\delta(an) = \delta(n)$

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

(4.) $x(n)\cdot\delta(n-n_1) = x(n_1)\,\delta(n-n_1)$

(5.) $x(n) * \delta(n-n_1) = x(n-n_1)$

(6.) $\int_{-\infty}^{t} \delta(\tau)\,d\tau = u(t)$

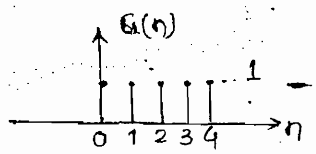
$$\downarrow$$

$\int = \Sigma$, $t = n$, $\tau = K$

$$\sum_{K=-\infty}^{n} \delta(K) = u(n)$$

(2.) Unit-step sig. →

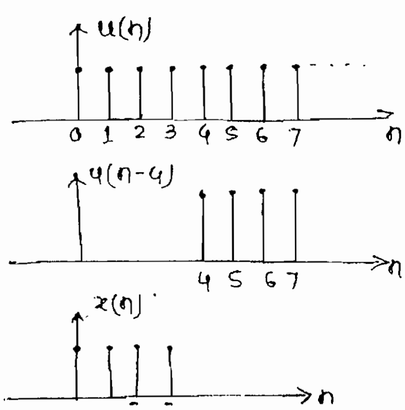
$$u(n) = \begin{cases} 1 , & n \geq 0 \\ 0 , & n < 0 \end{cases}$$



Q.→ Draw sig. $x(n)$

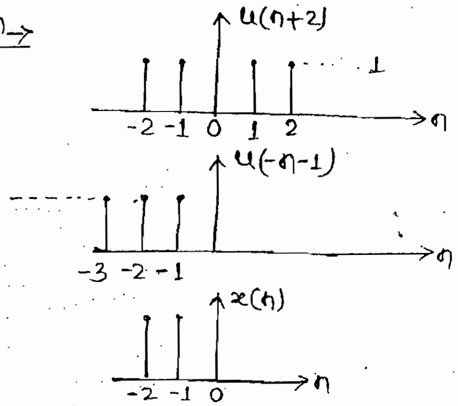
$$x(n) = u(n) - u(n-4)$$

Sol^n →



Que→ Draw the signal $x(n) = u(n+2)\cdot u(-n-1)$

Soln→



$-n-1 = 0$

$n = -1 \longrightarrow$ starting point

Because LS.

Operations on signal→

(1) Time-shifting→

$x(n) = \{5\ 3\ \underset{\uparrow}{7}\ 4\ 8\ 9\}$ ($n=-2$ … $n=3$)

axis not shifts only signal shifts

$x(n-1) = \{5\ \underset{\uparrow}{3}\ 7\ 4\ 8\ 9\}$ (RS)

$x(n+2) = \{5\ 3\ 7\ 4\ \underset{\uparrow}{8}\ 9\}$ (LS)

(2) Time Compression→(Decimation)→

$x(n) = \{5, 3, 7, \underset{\uparrow}{8}, -2, 4, 9\}$ ($n=-3$ … $n=3$)

$f(n) = x(2n) = \{3, \underset{\uparrow}{8}, 4\}$

$f(n) = x(3n) = \{5, \underset{\uparrow}{8}, 9\}$

$f(-2) = x(-4) = 0$
$f(-1) = x(-2) = 3$
$f(0) = x(0) = 8$
$f(1) = x(2) = 4$
$f(2) = x(4) = 0$

(3) Time expansion→(Interpolation)

$x(n) = \{4, \underset{\uparrow}{3}, 5\}$ ($n=-1$ … $n=1$)

$f(n) = x\left(\frac{n}{2}\right) = \{4, 0, 3, 0, 5\}$

$1 = 2-1$

$x\left(\frac{n}{3}\right) = \{4, 0, 0, 3, 0, 0, 5\}$

$2 = 3-1$

$f(-3) = x(-3/2) = 0$
$f(-2) = x(-1) = 4$
$f(-1) = x(-1/2) = 0$
$f(0) = 0 \quad = 3$
$f(1) = x(1/2) = 0$
$f(2) = x(1) = 5$
$f(3) = x(3/2) = 0$

Que.→ $x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$ Find $y(n)$

(i) $y(n) = x\left(\frac{2n}{3}\right)$

Solⁿ→ $x(n) \xrightarrow{Dec} x(2n) \xrightarrow{Int} x\left(\frac{2n}{3}\right)$

$\{1, 2, \underset{\uparrow}{3}, 4, 5\}$ $\{1, \underset{\uparrow}{3}, 5\}$ $\{1, 0, 0, \underset{\uparrow}{3}, 0, 0, 5\}$

(ii) $y(n) = x(-2n)$

Solⁿ→ $x(n) \longrightarrow x(2n) \longrightarrow x(-2n)$ (-ve - folding about arrow)

$\{1, 2, \underset{\uparrow}{3}, 4, 5\}$ $\{1, \underset{\uparrow}{3}, 5\}$ $(5, \underset{\uparrow}{3}, 1)$

(iii) $y(n) = x(-n-1)$

Solⁿ→ $x(n) \longrightarrow x(n-1) \longrightarrow x(-n-1)$

$(1, 2, \underset{\uparrow}{3}, 4, 5)$ $(1, \underset{\uparrow}{2}, 3, 4, 5)$ $(5, 4, 3, \underset{\uparrow}{2}, 1)$

(iv) $y(n) = x(2n-1)$

Solⁿ→ $x(n) \longrightarrow x(n-1) \longrightarrow x(2n-1)$

$(1, 2, \underset{\uparrow}{3}, 4, 5)$ $(1, \underset{\uparrow}{2}, 3, 4, 5)$ $(\underset{\uparrow}{2}, 4)$

(4) Convolution→

$$y(n) = x_1(n) * x_2(n)$$
$$= \sum_{k=-\infty}^{\infty} x_1(k)\, x_2(n-k)$$

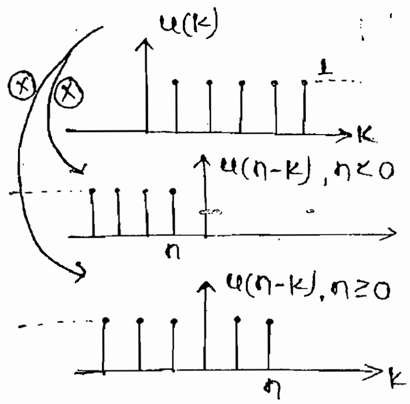
signal	extension	Length
$x_1(n)$	$n_1 \le n \le n_2$	L_1
$x_2(n)$	$n_3 \le n \le n_4$	L_2
$y(n)$	$n_1+n_3 \le n \le n_2+n_4$	L_1+L_2-1

length = no. of samples

Que.→ $y(n) = u(n) * u(n)$

Solⁿ→

$$y(n) = u(n) * u(n)$$
$$= \sum_{k=-\infty}^{\infty} u(k) \cdot u(n-k)$$



$n-k=0 \rightarrow -ve\,(LS)$

$n=k$

$$y(n) = u(n) * u(n)$$

$$= \sum_{-\infty}^{\infty} u(k)\cdot u(n-k)$$

$$= \begin{cases} 0 & , n<0 \\ \sum_{k=0}^{n} 1 & ; n\geq 0 \end{cases}$$

$$= \begin{cases} 0 & , n<0 \\ n+1 & , n\geq 0 \end{cases}$$

$$\boxed{y(n) = (n+1)\,u(n)}$$

Que.→ $x_1(n) = (1, \underset{\uparrow}{2}, -2)$; $x_2(n) = (2, 0, \underset{\uparrow}{1})$

$y(n) = x_1(n) * x_2(n) = ?$

Sol^n→ Tabular method →

$x_2(n)$ \ $x_1(n)$	1	2	-2
2	2	4	-4
0	0	0	0
→ 1	1	2	-2

$x_1(n) = (1, \underset{\uparrow}{2}, -2) = 2^{nd}$ element

$x_2(n) = (2, 0, \underset{\uparrow}{1}) = 3^{rd}$ element

$2+3=5$

$5-1=4$

(arrow point)

$y(n) = (2, 4, -3, \underset{\uparrow}{2}, -2)$

Que.→ $x_1(n) = (-1, \underset{\uparrow}{2}, 0, 1)$ $x_2(n) = (3, 1, \underset{\uparrow}{0}, -1)$

$y(n) = x_1(n) * x_2(n)$

Sol^n→

$x_2(n)$ \ $x_1(n)$	-1	2	0	1
3	-3	6	0	3
1	-1	2	0	1
→ 0	0	0	0	0
-1	1	-2	0	-1

$y(n) = (-3, 5, 2, \underset{\uparrow}{4}, -1, 0, -1)$

Que→

$$x(n) = (\underset{\uparrow}{1}, -1) \longrightarrow \boxed{\text{LTI sys.}}_{h(n)} \longrightarrow y(n) = (\underset{\uparrow}{1}, 0, 0, 0, -1)$$

Find $h(n) = ?$

(a.) $(\underset{\uparrow}{1}, 0, 0, 1)$ ~~(b) $(\underset{\uparrow}{1}, 0, 1)$~~

(c.) $(\underset{\uparrow}{1}, 1, 1, 1)$ (d) $(\underset{\uparrow}{1}, 1, 1)$

Sol^n→ $y(n) = x(n) * h(n)$

$x(n) = 2$

$y(n) = 5$

$5 = 2 + L_2 - 1$

$L_2 = 4$ {ans (c) or (a)}

Tabular method →

h(n) \ x(n)	1	-1
1	1	-1
1	1	-1
1	1	-1
1	1	-1

Que→ $y(n) = h(n) * g(n)$, $h(n) = (\frac{1}{2})^n u(n)$

$g(n)$ is causal sequence.

If $y(0) = 1$, $y(1) = 1/2$ then $g(1)$ is equal to

(a) 0 (b) 1/2 (c) 1 (d) 3/2

Sol^n→

g(n) \ h(n)	1	1/2	1/4	1/8 --------
1	1	1/2	1/4	1/8 -------
0	0	0	0	0 -------

ans-(a)

* Energy & power signal →

* Energy signal →

* $E =$ Finite ; $P = 0$
* This are absolutely summable signal i.e.

$$\boxed{\sum_{n=-\infty}^{\infty} |x(n)| < \infty} \quad \boxed{E = \sum_{n=-\infty}^{\infty} |x(n)|^2}$$

Que → Calculate energy of signal :-

(i) $x(n) = \delta(n)$ (ii) $x(n) = \left(\frac{1}{3}\right)^n u(n)$

Sol^n → $x(n) = \delta(n)$

$= (\cdots 0, \underset{\uparrow}{1}, 0 \cdots)$

$E = \sum |x(n)|^2 = 1$

Sol^n → $x(n) = \left(\frac{1}{3}\right)^n u(n)$

$E = \sum_{-\infty}^{\infty} |x(n)|^2 = \sum_{0}^{\infty} \left(\frac{1}{9}\right)^n$

$= 1 + \frac{1}{9} + \left(\frac{1}{9}\right)^n + \cdots$

$= \frac{1}{1 - 1/9} = \frac{9}{8}$

(iii) $x(n) = (\underset{\uparrow}{1+j}, 1-j, -2, 2)$

Sol^n → $E = \sum |x(n)|^2$

$= (\sqrt{2})^2 + (\sqrt{2})^2 + (2)^2 + (2)^2$

$= 12$

Que → calculate energy of $y(n)$

$x(n) = (\underset{\uparrow}{1}, 2, 3, 4, 5)$

(i) $y(n) = x(-n)$ (ii) $y(n) = x(n-1)$ (iii) $y(n) = x\left(\frac{n}{2}\right)$ (iv) $y(n) = -x(n)$ (v) $y(n) = x(3n)$

$= x(3n)$

Sol^n → (i) $y(n) = x(-n)$

$x(n) = E[x(n)] = 55$

$x(-n) = 55$

(ii) $y(n) = x(n-1)$

$E_{x(n-1)} = 55$

(iii) $y(n) = x\left(\frac{n}{2}\right)$

$= (\underset{\uparrow}{1}, 0, 2, 0, 3, 0, 4, 0, 5)$

$E_{x(n/2)} = 55.$

(iv) $y(n) = -x(n)$

$E_{y(n)} = 55$

(v) $y(n) = x(3n)$

$= (1, 4)$

$E_{y(n)} = 1^2 + 4^2 = 17$

Note:- Energy calculation is independent of time shifting, time reversal, amp. reversal & interpolation.

* Power signal →

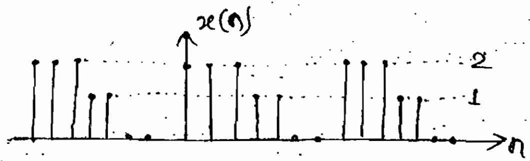
(1) P = finite, E = ∞

(2)
$$P = \begin{cases} \frac{1}{N} \sum_{n=N} |x(n)|^2 \text{; For periodic signal.} \\ \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 \text{; For Non-periodic signal.} \end{cases}$$

Que.→ Calculate power of signal.

(i) $x(n) = A_0 u(n)$

(ii)



Sol^n→ $x(n) = A_0 u(n)$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} A_0^2 (N+1)$$

$$= \lim_{N\to\infty} \frac{A_0^2(N+1)}{2N+1}$$

$$\boxed{P = \frac{A_0^2}{2}}$$

Sol^n→ $P = \frac{1}{N} \sum |x(n)|^2$

$$= \frac{2^2+2^2+2^2+1^2+1^2+0^2+0^2}{7}$$

$$= \frac{14}{7}$$

$$\boxed{P = 2}$$

Que→ Find even, odd, CS & CAS for sig. x(n)

$$x(n) = (-4-5j, \underset{\uparrow}{1+2j}, 4)$$

Sol^n→ * Even part → $\frac{x(n)+x(-n)}{2}$

$$x(-n) = (4, \underset{\uparrow}{1+2j}, -4-5j)$$

$$= \left(\frac{-5j}{2}, \underset{\uparrow}{1+2j}, \frac{-5j}{2}\right)$$

* Odd part → $\frac{x(n)-x(-n)}{2}$

$$= \left(-4-\frac{5j}{2}, 0, 4+\frac{5j}{2}\right)$$

* CAS part → $\frac{x(n)-x^*(-n)}{2}$

$$x^*(-n) = (4, 1-2j, -4+5j)$$

* Periodic Signal →

$$\boxed{x(n) = x(n \pm kN)}$$

where; k = an-integer

N = FTP = integer

$$x(n) = \underset{\displaystyle N_1}{\underset{\downarrow}{x_1(n)}} + \underset{\displaystyle N_2}{\underset{\downarrow}{x_2(n)}}$$

$$\rightarrow \frac{N_1}{N_2} = \frac{\text{integer}}{\text{integer}} = \text{Rational no. (always)}$$

Note:- The sum of 2 (or) more 2 periodic signals in case of discrete time system will be always periodic.

Complex-exponential → Complex exponential & sinusoidal signals are always periodic in case of contineous time signal.

Eg:- $x(n) = A_0 e^{j\omega_0 n}$

let N be the FTP of $x(n)$ i.e.

$$x(n) = x(n+N)$$

$$A_0 e^{j\omega_0 n} = A_0 e^{j\omega_0 (n+N)}$$

$$e^{j\omega_0 N} = 1 = e^{j2\pi k} \quad (k = \text{an integer})$$

$$\omega_0 N = 2\pi k$$

$$\boxed{\frac{2\pi}{\omega_0} = \frac{N}{k} = \text{Rational no.}}$$

In case of discrete time sys; complex exponantials & sinusoidal sig. will be periodic only if ratio $2\pi/\omega_0$ is rational no.

$$\boxed{N = \frac{2\pi}{\omega_0} k}$$

k is a least int. for which N is an integer.

Que → Calculate FTP of sig. if it is periodic

(i) $x(n) = e^{j2n}$.

$$\frac{2\pi}{\omega_0} = \frac{2\pi}{2} = \pi = \text{irrational no.}$$

(ii) $x(n) = \cos \frac{3\pi}{4} n$

$$\frac{2\pi}{\omega_0} = \frac{2\pi}{3\pi/4} = \frac{8}{3} \text{ (R. no.)}$$

(iii) $x(n) = \sin\left(\frac{3\pi}{4}n\right) + \cos\left(\frac{5\pi}{4}n\right)$

Sol^n $\Rightarrow$ $x(n) = \sin\left(\frac{3\pi}{4}n\right) + \cos\left(\frac{5\pi}{4}n\right)$

$\downarrow$ $N_1 = 8$ $\quad$ $\downarrow$ N_2

$N_2 = \frac{2\pi}{\omega_2}k_2 = \frac{2\pi}{5\pi/7}k_2 = \frac{14}{5}k_2 = 14$

$N = \text{LCM}(N_1, N_2)$

$= \text{LCM}(8, 14)$

$= 56.$

$e^{j2t} \longrightarrow$ periodic

$e^{j2n} \longrightarrow NP$

$\sin 4t \longrightarrow P$

$\sin 4n \longrightarrow NP$

$\underset{(P)}{\sin 2t} + \underset{(P)}{\cos 4t} \longrightarrow P$

$\underset{(NP)}{\sin 3n} + \underset{(NP)}{\cos 4n} \longrightarrow NP$

Chapter-09
Z-transform

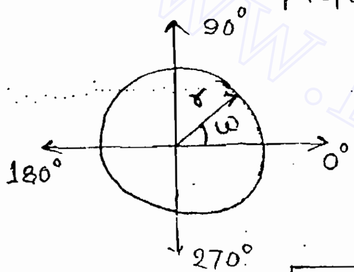
* Discrete time fourier transform (DTFT) exists for E & P signals whereas Z-TF also exist for NENP sig. (upto certain only).
* The replacement $z = e^{j\omega}$ is used for z-transform to DTFT conversion only for absolutely summable signal.

$$x(n) \rightleftharpoons X(z)$$

where; z = complex variable

$$= re^{j\omega}$$

r → damping factor; $e^{j\omega}$ → oscillation factor.



$z \to 2\pi$

$X(z) \to 2\pi$

$$\boxed{X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}}$$

Que. → $x(n) = a^n u(n) \rightleftharpoons X(z) = ?$

Soln →

$$X(z) = \sum_{n=-\infty}^{\infty} a^n z^{-n}$$

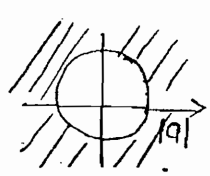
$$= \sum_{n=0}^{\infty} a^n z^{-n}$$

$$= \sum_{n=0}^{\infty} (az^{-1})^n$$

$$= (az^{-1})^0 + (az^{-1}) + (az^{-1})^2 + \dots\dots$$

$$X(z) = \frac{1}{1-az^{-1}} \; ; \; |az^{-1}| < 1$$

$$a^n u(n) \rightleftharpoons \frac{1}{1-az^{-1}} \; ; \; \text{ROC: } |z| > a$$



Que.→ $x(n) = -a^n u(-n-1)$ cal. z TF & ROC.

Sol^n→

$$X(z) = \sum_{-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{-\infty}^{\infty} -a^n u(-n-1) z^{-n}$$

$$= \sum_{-\infty}^{-1} (-a^n) z^{-n}$$

$$= -\sum_{-\infty}^{-1} (az^{-1})^n$$

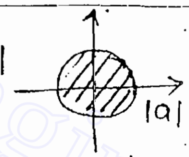
$$X(z) = -[(az^{-1})^{-1} + (az^{-1})^{-2} + \cdots]$$

$$= -[a^{-1}z + (a^{-1}z)^2 + \cdots]$$

$$= \frac{-a^{-1}z}{1-a^{-1}z} \; ; \; |a^{-1}z| < 1$$

$$X(z) = \frac{1}{1-az^{-1}} \; ; \; |z| < |a|$$

$$\boxed{-a^n u(-n-1) \rightleftharpoons \frac{1}{1-az^{-1}} \; ; \; |z| < |a|}$$



$$-a^n u(-n-1) \rightleftharpoons \frac{1}{1-az^{-1}} \; ; \; |z| < |a|$$

$$a^n u(n) \rightleftharpoons \frac{1}{1-az^{-1}} \; ; \; |z| > |a|$$

(a=1) (a=1)

$$u(n) \rightleftharpoons \frac{1}{1-z^{-1}} \; ; \; |z| > 1$$

$$-u(-n-1) \rightleftharpoons \frac{1}{1-z^{-1}} \; ; \; |z| < 1$$

Properties of z-transform →

1.) linearity :- $a_1x_1(n)+a_2x_2(n) \rightleftharpoons a_1X_1(z)+a_2X_2(z)$

2.) Time-reversal :- $x(-n) \rightleftharpoons X(z^{-1})$

3.) Conjugation :- $x^*(n) \rightleftharpoons X^*(z^*)$

4.) Time-shifting :- $x(n-n_0) \rightleftharpoons X(z).z^{-n_0}$

$$\downarrow (n_0=1)$$

$$x(n-1) \rightleftharpoons z^{-1}X(z)$$

5.) scaling of z :- $a^n x(n) \rightleftharpoons X(a^{-1}z)$

6.) Convolution in time :-

$$x_1(n) * x_2(n) \rightleftharpoons X_1(z).X_2(z)$$

7.) multiplication in time :-

$$x_1(n).x_2(n) \rightleftharpoons \frac{1}{2\pi j}\left[X_1(z) * X_2(z)\right]$$

8.) Successive diff./difference in time →

$$\frac{dx(n)}{dn} = \frac{x(n)-x(n-1)}{n-(n-1)}$$

$$= x(n)-x(n-1) \rightleftharpoons X(z)-z^{-1}X(z)$$

$$x(n)-x(n-1) \rightleftharpoons (1-z^{-1})X(z)$$

9.) Accumulation/Integration in time →

$$\sum_{k=-\infty}^{n} x(k) \rightleftharpoons \frac{X(z)}{1-z^{-1}}$$

10.) Differentiation in Freq. →

$$n.x(n) \rightleftharpoons -z\frac{dX(z)}{dz}$$

(11) Initial value theorem →

$$x(t)\Big|_{t=0} = \lim_{s\to\infty} sX(s)$$

Cond^n:- Applicable only for causal type signal i.e

$x(n)=0,\ n<0$

(12) Final value theorem →

$$x(t)\Big|_{t=\infty} = \lim_{s\to 0}\left[sX(s)\right]$$

$$x(n)\Big|_{n=\infty} = \lim_{z\to 1}\left[(1-z^{-1})X(z)\right]$$

Condⁿ :- (i) Applicable only for causal signals. i.e. $x(n)=0,\ n<0$

(ii) poles of term $[(1-z^{-1})X(z)]$ should lie inside unit circle in z-plane.

DATE-30/10/14

Region of Convergance (ROC) → It is defined as the range of complex variable z in z-plane for which z-transform of signal is convergant (or) Finite.

Properties of ROC →

(1) ROC does not include any pole.

(2) For right sided signal ROC will be outside circle in z-plane.

(3) For left sided signal ROC will be inside circle in z-plane.

RS LS
↑ ↑
(ROC)
outside inside

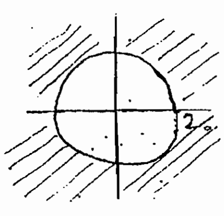
(4) For both sided signal ROC is a ring in z-plane.

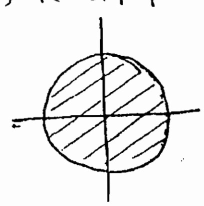
(5) For stability, ROC includes unit circle in z-plane.

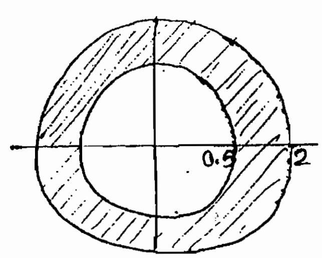
(6) For finite duration sig. ROC is intire z-plane excluding possibly $z=0$ &/or $\pm\infty$.

Que → Check stability of sys. & comment about extension of $h(n)$.

(i) ROC: $|z|>2$ (ii) ROC: $|z|<2$ (iii) ROC: $0.5<|z|<2$







Solⁿ → $h(n) = RS + US$ $h(n) = LS + S$ $h(n) = BS + S$

Que.→ $x(n) = \{2, \underset{\uparrow}{5}, 3, 7, 8\}$ $X(z) = ?$, ROC = ?

Sol^n→

$$X(z) = \sum_{-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{-1}^{3} x(n) z^{-n}$$

$$= x(-1) z^{1} + x(0) z^{0} + x(1) z^{-1} + x(2) z^{-2} + x(3) z^{-3}$$

$$= 2z + 5 + 3z^{-1} + 7z^{-2} + 8z^{-3}$$

ROC→ Entire z-plane excluding

$z = 0, +\infty, -\infty$ (excluded) [Because they give ∞ sol^n in $X(z)$]

Que.→

$x(n) \rightleftharpoons X(z) \longrightarrow$ [LTI sys.] $\longrightarrow y(n)$

$h(n) = 2\delta(n-3)$

$X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$. Find $y(4)$

(a) -6 (b) 0 (c) 2 (d) -4

Sol^n→

$$X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$$

$$x(n) = \{1, 0, 1, -2, 2, 0, 0, 0, -3\}$$

$\delta(n) \rightleftharpoons 1$

$2\delta(n) \rightleftharpoons 2$

$2\delta(n-3) \rightleftharpoons 2z^{-3}$

$$y(n) = x(n) * h(n)$$

$$Y(z) = X(z) \cdot H(z)$$

$$= 2z^{-3}(z^4 + z^2 - 2z + 2 - 3z^{-4})$$

$$Y(z) = 2z + 2z^{-1} - 4z^{-2} + 4z^{-3} - 6z^{-7}$$

$$y(n) = \{2, \underset{\uparrow}{0}, 2, -4, 4, 0, 0, 0, -6\}$$

$$y(4) = 0$$

$$\boxed{y(4) = 0}$$

Que.→ $X(z) = 1 - 3z^{-1} \rightleftharpoons x(n) \neq$ i/p

$Y(z) = 1 + 2z^{-2} \rightleftharpoons y(n) \neq$ o/p

An LTI sys. has impulse responce $h(n)$ defined as $h(n) = x(n-1) * y(n)$. The o/p of sys. for i/p $\delta(n-1)$ ~~has ZT~~.

soln → $i/p \rightarrow \delta(n-1) = z^{-1}$

$$p(n) = P(z) \longrightarrow \boxed{\text{LTI system}} \longrightarrow q(n) = Q(z)$$

$P(z) = z^{-1}$ $h(n) \rightleftharpoons H(z)$

$$h(n) = x(n-1) * y(n)$$

$$H(z) = z^{-1}X(z) \cdot Y(z)$$

$$q(n) = p(n) * h(n)$$

$$Q(z) = P(z) \cdot H(z)$$

$$= (z^{-1})\left[z^{-1}X(z) \cdot Y(z)\right]$$

$$Q(z) = z^{-2}X(z) \cdot Y(z)$$

$$= z^{-2}(1-3z^{-1})(1+2z^{-2})$$

$$= z^{-2}(1-3z^{-1}+2z^{-2}-6z^{-3})$$

$$Q(z) = z^{-2}-3z^{-3}+2z^{-4}-6z^{-5}$$

$$q(n) = \delta(n-2) - 3\delta(n-3) + 2\delta(n-4) - 6\delta(n-5)$$

ans (b).

2nd method →

$$q(n) = p(n) * h(n)$$

$$= \delta(n-1) * h(n)$$

$$= h(n-1)$$

$$h(n) = x(n-1) * y(n)$$

$$X(z) = 1-3z^{-1} \rightleftharpoons x(n) = (\underset{\uparrow}{1}, -3)$$

$$x(n-1) = (\underset{\uparrow}{0}, 1, -3)$$

$$Y(z) = 1+2z^{-2} \rightleftharpoons y(n) = (\underset{\uparrow}{1}, 0, 2)$$

$$h(n) = (\underset{\uparrow}{0}, 1, 2) * (\underset{\uparrow}{1}, 0, 2)$$

Que.→ $x(n) = \left(-\frac{1}{2}\right)^n u(-n+1) + 3^n u(n)$

$X(z) = ?$

Sol^n→

$\cancel{|z| < \left|\frac{-1}{2}\right|}$ $\qquad$ $\cancel{|z| > |3|}$

$\cancel{|z| < (1/2)}$ $\qquad$ $\cancel{|z| > 3}$

$\cancel{\left(\frac{1}{2}\right) < |z| < 3}$

$(-2)^n u(-n-3)$ LS

$|z| < |-2|$

$|z| < 2$

$x(n) = \left(-\frac{1}{2}\right)^n u(-n+1) + 3^n u(n)$

$= \underbrace{(-2)^n u(-n+1)}_{|z|<2} + \underbrace{3^n u(n)}_{|z|>3}$

ZT of the $x(n)$ will not exist because no common ROC

Que.→ $x(n) = \left(\frac{1}{3}\right)^{|n|} - \left(\frac{1}{2}\right)^n u(n)$ $\quad$ ROC = ?

Sol^n→ $x(n) = \left(\frac{1}{3}\right)^n - \left(\frac{1}{2}\right)^n u(n)$

$$\left(\frac{1}{3}\right)^n = \begin{cases} \left(\frac{1}{3}\right)^{-n} ; & n<0 \\ \left(\frac{1}{3}\right)^n ; & n \geq 0 \end{cases}$$

$$= \begin{cases} 3^n & ; n<0 \\ (1/3)^n & ; n \geq 0 \end{cases}$$

$= 3^n u(-n-1) + \left(\frac{1}{3}\right)^n u(n)$

$= |z|<3 \qquad |z| > 1/3$

$x(n) = \left(\frac{1}{3}\right)^n - \left(\frac{1}{2}\right)^n u(n)$

$= \underbrace{3^n u(-n-1)}_{|z|<3} + \underbrace{\left(\frac{1}{3}\right)^n u(n)}_{|z|>1/3} - \underbrace{\left(\frac{1}{2}\right)^n u(n)}_{|z|>1/2}$

$|z| < 3 \qquad |z| > 1/3$

$\frac{1}{3} < |z| < 3$

$$\boxed{ROC = \frac{1}{3} < |z| < 3}$$

Que.→ $x(n) = 2^{r(n)}$ ZT=?

Sol^n→ $x(n) = 2^{r(n)}$

$$\cancel{X(z) = \sum_{-\infty}^{\infty} 2^{r(n)} z^{-n}}$$

$$x(n) = 2^{r(n)}$$
$$= 2^{nu(n)}$$
$$= \begin{cases} 1 & ; n<0 \\ 2^n & ; n\geq 0 \end{cases}$$
$$= u(-n-1) + 2^n u(n)$$
$$|z|<1 \qquad |z|>2$$

ZT will not exists because no common ROC.

Que.→ $x(n) = \cos\omega_0 n \cdot u(n)$ $X(z) = ?$, ROC=?

Sol^n→ $x(n) = \cos\omega_0 n\, u(n)$

$$\cos\omega_0 n = \frac{e^{j\omega_0 n} + e^{-j\omega_0 n}}{2}$$

$$x(n) = \frac{1}{2} e^{j\omega_0 n} u(n) + \frac{1}{2} e^{-j\omega_0 n} u(n)$$

$$a^n u(n) \rightleftharpoons \frac{z}{z-a}, \; |z|>|a|$$

$a = e^{j\omega_0}$

$$e^{j\omega_0 n} u(n) \longrightarrow \frac{z}{z-e^{j\omega_0}} \quad |z|>1$$

$a = e^{-j\omega_0}$

$$e^{-j\omega_0 n} u(n) \longrightarrow \frac{z}{z-e^{-j\omega_0}} \quad |z|>1$$

$$X(z) = \frac{1}{2}\left[\frac{z}{z-e^{j\omega_0}} + \frac{z}{z-e^{-j\omega_0}}\right] ; \; |z|>1$$

$$= \frac{1}{2}\left[\frac{z(z-e^{-j\omega_0}) + z(z-e^{j\omega_0})}{z^2 - z(e^{j\omega_0} + e^{-j\omega_0}) + 1}\right], \; |z|>1$$

$$= \frac{1}{2}\left[\frac{2z^2 - z(e^{j\omega_0} + e^{-j\omega_0})}{z^2 - 2z\cos\omega_0 + 1}\right], \; |z|>1$$

$$= \frac{1}{2}\left[\frac{2z^2 - 2z\cos\omega_0}{z^2 - 2z\cos\omega_0 + 1}\right], \; |z|>1$$

$$= \frac{z^2 - z\cos\omega_0}{z^2 - 2z\cos\omega_0 + 1} \quad |z|>1$$

Important signals →

$x(n)$	$X(z)$	ROC
$\delta(n)$	1	entire z-plane
$a^n u(n)$	$\frac{1}{1-az^{-1}}$	$\|z\|>\|a\|$
$-a^n u(-n-1)$	$\frac{1}{1-az^{-1}}$	$\|z\|<\|a\|$
$u(n)$	$\frac{1}{1-z^{-1}}$	$\|z\|>1$
$n.a^n u(n)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\|>\|a\|$
$-na^n u(-n-1)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\|<\|a\|$
$\cos\omega_0 n$	$\frac{z^2 - z\cos\omega_0}{z^2-2z\cos\omega_0+1}$	$\|z\|>1$
$\sin\omega_0 n$	$\frac{z\sin\omega_0}{z^2-2z\cos\omega_0+1}$	$\|z\|>1$

Que.→ $x(n) \rightleftharpoons X(z) = \frac{0.5}{1-2z^{-1}}$

It is given that ROC of $X(z)$ includes unit circle. The value of $x(0)$ is :-

(a) 0.5 (b) 0 (c) 0.25 (d) 0.5

Sol^n →

$$X(z) = \frac{0.5}{1-2z^{-1}}$$

Pole:- $1-2z^{-1}=0$

$z=2$

ROC:- $|z|<2$ (Given)

so inverse will be left sided, $x(n)$

$$x(n) = -0.5(2)^n u(-n-1)$$

$u(-n-1) \rightarrow (-\infty \text{ to } -1)$

Que.→ Find inverse ZT of

$$X(z) = \frac{z}{(z-1)(z-2)^2} \quad \text{if}$$

(i) $|z|>2$ (ii) $|z|<1$ (iii) $1<|z|<2$

Soln→

$$X(z) = \frac{z}{(z-1)(z-2)^2}$$

$$\boxed{\begin{aligned} f(n) &= a^n u(n) \rightarrow F(z) = \frac{z}{z-a} \\ f(n-1) &= a^{n-1} u(n-1) \\ z^{-1}F(z) &= \frac{1}{(z-a)} \end{aligned}}$$

$$X(z) = \frac{A}{(z-1)} + \frac{B}{(z-2)} + \frac{C}{(z-2)^2}$$

$$\frac{X(z)}{z} = \frac{A}{(z-1)} + \frac{B}{(z-2)} + \frac{C}{(z-2)^2}$$

$$X(z) = \frac{Az}{(z-1)} + \frac{Bz}{(z-2)} + \frac{Cz}{(z-2)} \times \frac{z^{-2}}{z^{-2}}$$

$$X(z) = \frac{A}{1-z^{-1}} + \frac{B}{1-2z^{-1}} + \frac{C}{2}\left[\frac{2z^{-1}}{(1-2z^{-1})^2}\right]$$

$A=1,\ B=-1,\ C=1$

$$X(z) = \frac{1}{1-z^{-1}} - \frac{z}{1-2z^{-1}} + \frac{1}{2}\left[\frac{2z^{-1}}{(1-2z^{-1})^2}\right]$$

Poles→ $z = 1, 2$

(i) ROC: $|z|>2$

* $x(n)$ will be RS.
* $x(n) = u(n) - 2^n u(n) + \frac{1}{2} n 2^n u(n)$

(ii) ROC: $|z|<1$

* $x(n)$ will be LS.
* $x(n) = -u(-n-1) - [-2^n u(-n-1)] + \frac{1}{2}[-n2^n u(-n-1)]$

(iii) ROC: $1<|z|<2$

* $x(n)$ will be both sided.
* $x(n) = u(n) - [-2^n u(-u-1)] + \frac{1}{2}[-n2^n u(-n-1)]$

Que.→ $x(n) = 4^n u(n) \rightleftharpoons X(z)$

$y(n) \rightleftharpoons Y(z) = X^2(z)$ Find $y(n)$

(a) $4^n u(n)$ (b) $(n+1) 4^n u(n+1)$ (c) $(n+1) 4^n u(n)$ (d) $n \cdot 4^n u(n+1)$

Soln→

$$X(z) = \frac{1}{1-4z^{-1}}$$

$$X^2(z) = \left(\frac{1}{1-4z^{-1}}\right)^2 = \frac{1}{(1-4z^{-1})^2} = \frac{1}{1+16z^{-2}-8z^{-1}}$$

$$Y(z) = X^2(z) = \frac{1}{(1-4z^{-1})^2}$$

$$Y(z) = \frac{z^2}{(z-4)^2} = \frac{z(z-4)+4z}{(z-4)^2}$$

$$= \frac{z}{(z-4)} + \frac{4z}{(z-4)^2} \times \frac{z^{-2}}{z^{-2}}$$

$$Y(z) = \frac{1}{1-4z^{-1}} + \frac{4z^{-1}}{(1-4z^{-1})^2}$$

$$y(n) = 4^n u(n) + n 4^n u(n)$$

$$\boxed{y(n) = (n+1) 4^n u(n)}$$

ans: (b) & (c)

$$(n+1) 4^n u(n+1) = (0, \underset{\uparrow}{1}, 8 \ldots\ldots)$$

$$(n+1) 4^n u(n) = (\underset{\uparrow}{1}, 8 \ldots\ldots)$$

Que.→ $X(z) = \log(1+az^{-1})$; $|z|>|a|$ Find $x(n) = ?$

Soln→ diff in freq:

$$x(n) \rightleftharpoons X(z)$$

$$n x(n) \rightleftharpoons -z\frac{dX(z)}{dz} = -z\left[\frac{1}{1+az^{-1}} (-az^{-2})\right]$$

$$n \cdot x(n) = \frac{az^{-1}}{1+az^{-1}} ; \quad |z|>|a|$$

$$n x(n) = 1 - \frac{1}{1+az^{-1}} \quad |z|>|a|$$

$$n x(n) = \delta(n) - (-a)^n u(n)$$

Que→ $X(z) = \dfrac{1+z^{-1}}{1+\frac{1}{3}z^{-1}} \rightleftharpoons x(n)$

(a) Assuming ROC to be $|z| < 1/3$. determine $x(0), x(-1), x(-2)$

(b.) Assuming ROC to be $|z| > 1/3$ determine $x(0), x(1)$ & $x(2)$

Soln→

$$X(z) = \frac{1+z^{-1}}{1+(\frac{1}{3})z^{-1}}$$

(a) $|z| < \frac{1}{3}$; $x(n)$ will be LS sig.

Arrange numerator & denominator polynomials in asending powers of 'z'.

$$X(z) = \frac{z^{-1}+z^{0}}{(\frac{1}{3})z^{-1}+z^{0}}$$

LED (LA)

$$\frac{1}{3}z^{-1}+1 \,\Big)\, z^{-1}+1 \,\Big(\, 3-6z+18z^2+\cdots$$

$$\begin{array}{l} z^{-1}+3 \\ \hline -2 \\ -2-6z \\ \hline 6z \\ 6z+18z^2 \\ \hline -18z^2 \\ \vdots \end{array}$$

$$X(z) = 3-6z+18z^2+\cdots\cdots$$
$$= x(0)+x(-1)z+x(-2)z^2+\cdots$$

$x(0) = 3, \; x(-1) = -6, \; x(-2) = 18$

(b.) $|z| > \frac{1}{3}$; $x(n)$ will be RS sig.

Arrange nume. & deno. poly. in decensing powers of 'z'.

$$X(z) = \frac{1+z^{-1}}{1+(\frac{1}{3})z^{-1}}$$

$$1+\frac{1}{3}z^{-1} \,\Big)\, 1+z^{-1} \,\Big(\, 1+\frac{2}{3}z^{-1}-\frac{2}{9}z^{-2}+\cdots$$

$$\begin{array}{l} 1+\frac{1}{3}z^{-1} \\ \hline \frac{2}{3}z^{-1} \\ \frac{2}{3}z^{-1}+\frac{2}{9}z^{-2} \\ \hline -\frac{2}{9}z^{-2} \end{array}$$

$$X(z) = 1 + \frac{2}{3}z^{-1} - \frac{2}{9}z^{-2} + \cdots\cdots$$

$$= x(0) + x(1)z^{-1} + x(2)z^{-2} + \cdots\cdots$$

$$x(0) = 1,\ x(1) = \frac{2}{3},\ x(2) = \frac{-2}{9}$$

Que.→ $x(n) \rightleftharpoons X(z) = \dfrac{z}{2z^2 - 3z + 1}$, $|z| < \frac{1}{2}$. Find $x(-2)$

(a) 0 (b) 1 (c) 2 (d) 3

Solⁿ→

$$X(z) = \frac{z}{1 - 3z + 2z^2}$$

$$1 - 3z + 2z^2 \overline{)\ z\ } \left(z + 3z^2 + 7z^3 + \cdots\cdots\right.$$

$$\underline{\cancel{z} - 3z^2 + 2z^3}$$

$$3z^2 - 2z^3$$

$$\underline{3z^2 - 9z^3 + 6z^4}$$

$$7z^3$$

$$\underline{7z^3 - 21z^4 + 14z^5}$$

$$\boxed{x(-2) = 3}$$

Que.→ $x(n) \rightleftharpoons X(z) = \dfrac{z + z^{-3}}{z + z^{-1}}$; $x(n)$ series has

(a) Alternate −1s (c) Alternate 1s

(b) Alternate 0s (d) Alternate 2s.

Sol→

$$X(z) = \frac{z + z^{-3}}{z + z^{-1}}$$

Here ROC is not given, so devide in the given form.

$$z + z^{-1} \overline{)\ z + z^{-3}\ } \left(1 - z^{-2} + 2z^{-4} - 2z^{-6} + 2z^{-8} - 2z^{-10}\right.$$

$$\underline{\cancel{z} + z^{-1}}$$

$$-z^{-1} + z^{-3}$$

$$\underline{-z^{-1} - z^{-3}}$$

$$2z^{-3}$$

$$\underline{2z^{-3} + 2z^{-5}}$$

$$-2z^{-5}$$

$$\underline{-2z^{-5} - 2z^{-7}}$$

$$\vdots$$

$$X(z) = 1 - z^{-2} + 2z^{-4} + 2z^{-6} + 2z^{-8} + \cdots$$

$$x(n) = (1, 0, -1, 0, 2, 0, -2, 0, 2 \cdots)$$

ans.(b).

*** Causal system →**

(1.) $h(n) = 0, \; n < 0$

(2.) $H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n} = \sum_{n=0}^{\infty} h(n) z^{-n}$

$$= h(0) + h(1)z^{-1} + h(2)z^{-2} + \cdots = \frac{N(z)}{D(z)}$$

Note → For causal sys., expansion of TF does not include +ve powers of z.

(3.) $\lim_{z \to \infty} H(z) = h(0) = 0$ (or) finite.

Note → *For causal sys.; order of numerator can't exceed order of denominator.

* for causal sys. ROC will be outside circle in z-plane.

* for stability of ~~dtf~~ discrete time causal sys., poles of TF should lie inside unit circle in z-plane.

*** Anticausal system →**

* For this sys. ROC will be inside circle in z-plane.

* For stability of this sys, poles of TF should lie outside unit circle in z-plane.

Que. → A causal LTI sys. is described by the difference eqn

$$2y(n) = \alpha y(n-2) - 2x(n) + \beta x(n-1)$$

The sys. is stable only if

(a.) $|\alpha| = 2, \; |\beta| < 2$

(b.) $|\alpha| > 2, \; |\beta| > 2$

(c.) $|\alpha| < 2$, for any value of 'β'

(d.) $|\beta| < 2$, for any value of 'α'

Soln → $2Y(z) = aY(z) \cdot z^{-2} - 2X(z) + \beta X(z) z^{-1}$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{-2+\beta z^{-1}}{2-az^{-2}}$$

Poles:- $2-az^{-2}=0$

$$z = \sqrt{\frac{a}{2}}$$

For stability of causal sys.

$$|pole| < 1$$

$$\sqrt{\frac{a}{2}} < 1$$

$$\boxed{|a|<2}$$

Que → $x(n) \rightleftharpoons X(z) = \frac{z^{-1}(1-z^{-4})}{4(1-z^{-1})^2}$

Find $x(\infty) = ?$ (a) 1/4 (b) 0 (c) 1 (d) ∞

Soln →

$$x(n) \rightleftharpoons X(z) = \frac{z^{-1}(1-z^{-4})}{4(1-z^{-1})^2}$$

(i) $\lim_{z\to\infty} X(z) = 0$

Condn for causality is satisfied.

(ii) $$(1-z^{-1})X(z) = \frac{z^{-1}(1-z^{-4})}{4(1-z^{-1})}$$

$$= \frac{z^{-1}(1+z^{-2})(1-z^{-2})}{4(1-z^{-1})}$$

$$= \frac{z^{-1}(1+z^{-2})(1+z^{-1})\cancel{(1-z^{-1})}}{4\cancel{(1-z^{-1})}}$$

$$= \frac{z^{-1}(1+z^{-2})(1+z^{-1})}{4}$$

Pole:- $z=0<1$

Both the condn are satisfied. so we can use final value theorem.

Final value theorem →

$$x(\infty) = \lim_{z\to 1}\left[(1-z^{-1})X(z)\right]$$

$$= \lim_{z\to 1}\left[\frac{z^{-1}(1+z^{-2})(1+z^{-1})}{4}\right]$$

$$= \frac{2\times 2}{4}$$

$$= 1$$

Que → A stable & causal sys. is described by the diff. eqⁿ

$$y(n) + \frac{1}{4}y(n-1) + \frac{1}{8}y(n-2) = -2x(n) + \frac{5}{4}x(n-1)$$

h(n) of the sys. is

(a.) $\left(\frac{1}{4}\right)^n u(n) + 3\left(\frac{-1}{2}\right)^n u(n)$ (b.) $\left(\frac{1}{4}\right)^n u(n) - 3\left(\frac{-1}{2}\right)^n u(n)$

(c.) $\left(\frac{1}{4}\right)^n u(n) - 3u(n)$ (d.) $\left(\frac{1}{4}\right)^{n-1} u(n-1) - 3u(n-1)$

Solⁿ →

$$Y(z) + \frac{1}{4}Y(z)z^{-1} - \frac{1}{8}Y(z)z^{-2} = -2X(z) + \frac{5}{4}X(z)z^{-1}$$

$$Y(z)\left[1 + \frac{1}{4}z^{-1} - \frac{z^{-2}}{8}\right] = X(z)\left[-2 + \frac{5}{4}z^{-1}\right]$$

$$H(z) = \frac{-2 + \frac{5}{4}z^{-1}}{1 + \frac{1}{4}z^{-1} - \frac{z^{-2}}{8}}$$

from option

(a.) $|z| > \frac{1}{4}$ $|z| > \frac{1}{2}$ common ROC $|z| > \frac{1}{2}$ (stable)

(b.) common ROC $|z| > \frac{1}{2}$ (stable)

(c.) common ROC $|z| > 1$ (US)

(d.) common ROC $|z| > 1$ (US)

Here sys. is causal so follow the Initial value theorem

$$h(0) = \lim_{z\to\infty} H(z) = -2$$

(a.) $h(0) = 1+3 = 4$ (b.) $h(0) = 1-3 = -2$

* Jury Test →

(1.) To check stability of contineous time causal LTI sys. Routh-hurwitz criteria is used.

(2.) Jury test is used to check stability of discrete time causal LTI sys.

$$H(z) = \frac{k_m z^m + k_{m-1} z^{m-1} + \cdots + k_0}{a_n z^n + a_{n-1} z^{n-1} + \cdots + a_0} = \frac{N(z)}{D(z)}$$

For causality :- $n \geq m$

c/s eqn :- $D(z) = 0$

necessary condn for stability →

(1.) $D(1) > 0$

(2.) $D(-1) \begin{cases} > 0 \;;\; n = \text{even} \\ < 0 \;;\; n = \text{odd} \end{cases}$

Jury table → no. of rows $= 2n-3$

	z^0	z^1	z^2 -----	z^n
1.	a_0	a_1	a_2 -----	a_n
2.	a_n	a_{n-1}	a_{n-2} -----	a_0
3.	b_0	b_1	b_2 ------	b_{n-1}
4.	b_{n-1}	b_{n-2}	b_{n-3} ----	b_0
5.	c_0	c_1	c_2 ------	c_{n-2}
			⋮	

$$b_i = \begin{vmatrix} a_0 & a_{n-i} \\ a_n & a_i \end{vmatrix} = a_0 a_i - a_n a_{n-i}$$

$$b_0 = a_0 a_0 - a_n a_{n-0} = a_0^2 - a_n^2$$

$$b_1 = a_0 a_1 - a_n \cdot a_{n-1}$$

$$c_0 = b_0^2 - b_{n-1}^2$$

$$c_1 = b_0 b_1 - b_{n-1} \cdot b_{n-2}$$

⋮

Sufficient cond^n →

(1.) $|a_n| > |a_0|$

(2.) $|b_{n-1}| < |b_0|$

(3.) $|c_{n-2}| < |c_0|$

(4.) $|d_{n-3}| < |d_0|$

Que.→ $H(z) = \dfrac{2z^3+2z^2+3z+1}{2z^4+3z^3+z^2-1}$. check stability of sys.

Sol^n→ $D(z) = 2z^4+3z^3+z^2-1$

Order → $n=4$, (even)

necessity cond^n:-

(i) $D(1) > 0$ ✓

(ii) $D(-1) > 0$, $n=$even ✗

Que.→ check stability of sys. having

$D(z) = 5z^3+2z^2+4z+1$

Sol^n→ $D(z) = 5z^3+2z^2+4z+1$

Order – odd ($n=3$)

Cond^n→ (i) $D(1) > 0$

(ii) $D(-1) < 0$; $n=$odd

$\downarrow$

-6

Jury table → No. of rows $2n-3 = 2\times3-3 = 3$

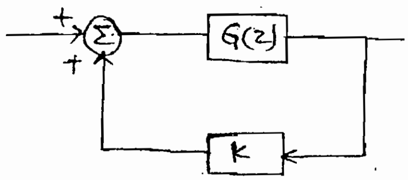
	z^0	z^1	z^2	z^3
a_0 1.	1	4	2	5 a_n
2.	5	2	4	1
b_0 3.	-24	-6	-18	0 b_{n-1}
4	~~0~~	~~-18~~	~~-6~~	~~-24~~

Sufficient cond^n:-

(1.) $|a_n| > |a_0|$ ✓ (2.) $|b_{n-1}| < |b_0|$ ✓

sys. is stable

Que.→



$g(n) \rightleftharpoons G(z)$

$g(n) = (\underset{\uparrow}{0}, 1, 1)$

The sys. is stable for range of ~~value~~ value of 'k'

(a) $(-1, 1/2)$ (b) $(-1, 1)$ (c) $(-1/2, 1)$ (d) $(-\frac{1}{2}, 2)$

Sol^n→

$$G(z) = z^{-1} + z^{-2}$$

$$H(z) = \frac{G(z)}{1 - kG(z)}$$

$$= \frac{z^{-1} + z^{-2}}{1 - k(z^{-1} + z^{-2})} \times \frac{z^2}{z^2}$$

$$= \frac{z+1}{z^2 - k(z+1)}$$

$$D(z) = z^{②} - kz + 1 \xrightarrow{} \text{even}$$

* $D(1) > 0$, $1 - 2k > 0 \Rightarrow k < \frac{1}{2}$ ---- (i)

* $D(-1) > 0$; $1 > 0$

* $|a_n| > |a_0|$

$1 > |k|$

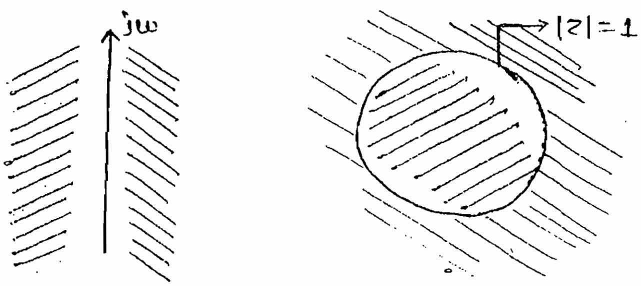
$(-1 < k < 1)$ ----- (ii)

from (i) & (ii)

$$\boxed{-1 < k < \frac{1}{2}}$$

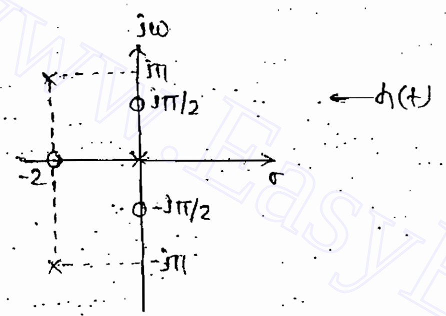
DATE-31/10/14

Mapping between s-plane & z-plane →



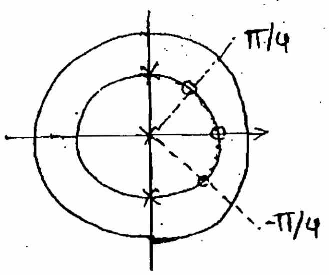
Pole mapping - $\boxed{z = e^{sT}}$ T - sampling time interval

Que.→

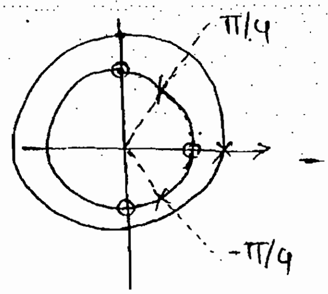


The impulse response of h(t) is sampled at 2kHz to get h(n). Which one of the following represents equivalent pole-zero plot of H(z) in z-plane? (The concentric circles are $|z|=1$, $|z|=\frac{1}{e}$)

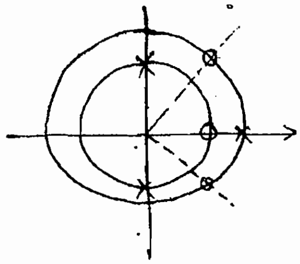
(a.)



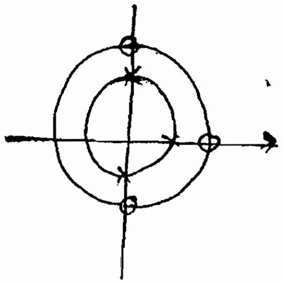
(b.)



(c.)



(d.)



Sol^n → S-plane

$s = 0,\ -2+j\pi,\ -2-j\pi$

Z-plane

$z = e^{sT} = e^{s/f} = e^{s/2}$

$= 1,\ e^{-1+j\pi/2},\ e^{-1-j\pi/2}$

$= 1e^{0j},\ \frac{1}{e}e^{j\pi/2},\ \frac{1}{e}e^{-j\pi/2}$

* DTFT →

$x(n) = X(e^{j\omega})$

$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

$\downarrow z = e^{j\omega}$

$$\boxed{X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}}$$

$\downarrow (\omega = 0)$

***

$$\boxed{X(e^{j0}) = \sum_{n=-\infty}^{\infty} x(n)}$$

$x(n) = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega.$

$\downarrow (n = 0)$

$x(0) = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega}) d\omega$

***

$$\boxed{2\pi x(0) = \int_{2\pi} X(e^{j\omega}) d\omega}$$

16/18

$$Y(e^{j0}) = \sum_{-\infty}^{\infty} y(n)$$

$$= \sum_{0}^{\infty} \left(\frac{1}{4}\right)^n$$

$$= 1 + \left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2 + \cdots$$

$$= \frac{1+}{1-\frac{1}{3}} = \frac{4}{3}$$

31/20

$$h(n) = \frac{1}{2}\left[\delta(n) + \delta(n-2)\right] \qquad |H(e^{j\omega})| = ?, \quad \omega = \Omega$$

$$H(z) = \frac{1}{2}(1 + z^{-2}) \quad z = e^{j\omega}$$

$$H(e^{j\omega}) = \frac{1}{2}(1 + e^{-j2\omega})$$

$$H(e^{j\omega}) = e^{-j\omega}\left[\frac{e^{j\omega} + e^{-j\omega}}{2}\right]$$

$$|H(e^{j\omega})| = |\cancel{e^{-j\omega}}|^{1}\,|\cos\omega|$$

$$|He^{j\omega}| = |\cos\omega|$$

45/31

$$x(n) = \sin(\omega_0 n + \phi) \longrightarrow \boxed{\text{LTI sys.}} \longrightarrow y(n) = Ax(n-n_0)$$

$$H(e^{j\omega})$$

$$\angle He^{j\omega_0} = ?$$

$$y(n) = A x(n-n_0)$$

$$Y(z) = A X(z) \cdot z^{-n_0}$$

$$H(z) = A z^{-n_0}$$

$$\downarrow (z = e^{j\omega})$$

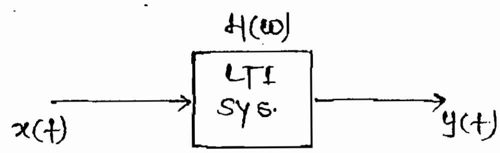
$$H(e^{j\omega}) = A \cdot e^{-j\omega n_0} \quad \downarrow \omega = \omega_0$$

$$H(e^{j\omega_0}) = A e^{-j\omega_0 n_0} \cdot e^{j2\pi k} \quad \to k = \text{an int.}$$

$$= A e^{j(2\pi k - n_0\omega_0)}$$

$$\angle H(e^{j\omega_0}) = 2\pi k - n_0\omega_0$$

**



$x(t) = A_0 \sin(\omega_0 t + \phi)$

$y(t) = A_0 \times |H(\omega_0)| \times \sin\{[(\omega_0 t + \phi)] + \angle H(\omega_0)\}$

(21/19)

$h(t) = e^{-2t} u(t)$

$x(t) = 2\cos(2t)$; $\omega_0 = 2$

$y(t) = ?$

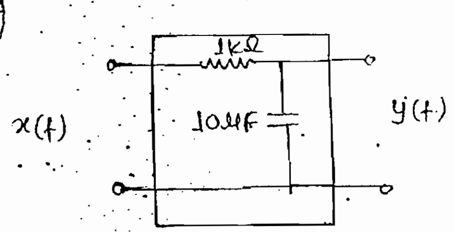
$$H(\omega) = \frac{1}{j\omega + 2}$$

$$H(\omega_0) = \frac{1}{2+2j} \qquad |H(\omega_0)| = \frac{1}{2\sqrt{2}}, \ \angle H(\omega_0) = -\pi/4$$

$$y(t) = 2 \times \frac{1}{2\sqrt{2}} \times \cos(2t - \pi/4)$$

$$= e^{-0.5} \cos(2t - 0.25\pi)$$

(11/22)



$x(t) = 3 + 4\sin 100t$

$\omega_0 = 100$

$$H(s) = \frac{1}{1 + sCR}$$

$(s = j\omega) \downarrow$

$$H(\omega) = \frac{1}{1 + j\omega RC}$$

$\downarrow (\omega = \omega_0)$

$$\left\{ \begin{aligned} \omega_0 RC &= 100 \times 10^3 \times 10 \times 10^{-6} \\ &= 1 \end{aligned} \right.$$

$$H(\omega_0) = \frac{1}{1 + j\omega_0 RC}$$

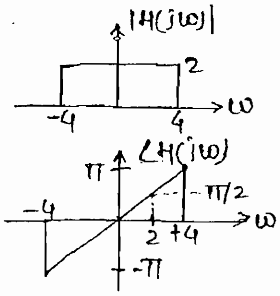
$$= \frac{1}{1+j}$$

$$|H(\omega_0)| = \frac{1}{\sqrt{2}}, \ \angle H(\omega_0) = -\frac{\pi}{4}$$

$$y(t) = 3 + 4 \times \frac{1}{\sqrt{2}} \times \sin[100t + (-\pi/4)]$$

$$= 3 + \frac{4}{\sqrt{2}} \sin(100t - \pi/4)$$

Que.→



$x(t) = 2\sin 2t$

$y(t) = ?$

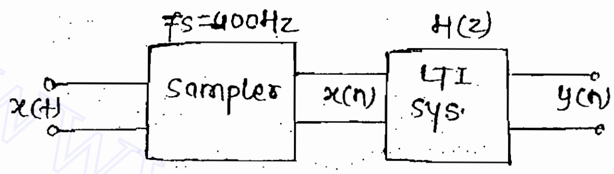
Soln→

$\omega_0 = 2 \quad x(t) = 2\sin 2t$

$y(t) = 2 \times 2 \times \sin(2t + \pi/2)$

$y(t) = 4\cos 2t$

Que.→



$x(t) = 2 + 5\sin(100\pi t)$

$H(z) = \frac{1}{N}\left[\frac{1-z^{-N}}{1-z^{-1}}\right]$ where N = no. of samples per cycle.

The o/p. y(n) of sys. under steady state is

(a) 0 (b) 1 (c) 2 (d) 5

Soln→ $y(\infty) = ?$; N = no. of samples per cycle = sampling freq.

$= 400$

$x(t) = 2 + 5\sin(100\pi t)$

$\Big\downarrow \; t = nT_s = \frac{n}{f_s} = \frac{n}{400}$

$x(n) = 2 + 5\sin\left(100\pi \times \frac{n}{400}\right)$

$= 2 + 5\sin\left(\frac{n\pi}{4}\right) \quad ; \omega_0 = \frac{\pi}{4}$

$(z = e^{j\omega})$ $\quad H(\omega) = \frac{1}{N}\left[\frac{1-e^{-j\omega N}}{1-e^{-j\omega}}\right]$

$\Big\downarrow \left(\omega = \omega_0 = \frac{\pi}{4}\right)$

$H(\omega_0) = \frac{1}{N}\left[\frac{1-e^{-j\frac{\pi}{4}N}}{1-e^{-j\pi/4}}\right]_{N=400}$ $\neq 1$ $\quad (\because e^{-j100\pi} = 1)$

$H(\omega_0) = 0$

$y(n) = 2 +$ o/p due to ~~sinusoidal~~ part

$= 2$

$\boxed{y(\infty) = 2}$

Que.→ $x(n) \rightleftharpoons X(z) = \sum_{n=0}^{\infty} \frac{3^n}{2+n} z^{2n}$

$x(n)$ is orthogonal to the signal.

(a.) $y_1(n) = Y_1(z) = \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n z^{-n}$

(c.) $y_3(n) = Y_3(z) = \sum_{n=-\infty}^{\infty} 2^{-|n|} z^{-n}$

(b.) $y_2(n) = Y_2(z) = \sum_{n=0}^{\infty} (5^n - n) z^{-(2n+1)}$

(d.) $y_4(n) = Y_4(z) = 2z^{-4} + 3z^{-2} + 1$

Sol^n→

$$\sum_{n=-\infty}^{\infty} x_1(n) \cdot x_2(n) = 0$$

(orthogonal)

$y_2(n)$ is available for odd instants of 'n'

ans. (b)

$x_1(n)\, y_2(n) = 0$ so $\sum_{-\infty}^{\infty} x(n)\, y_2(n) = 0$

so; they are orthogonal.

$\frac{3}{4}$ $3y^2(t) + 2y^2(t) + y(t) = x^2(t) + x(t)$

all values are present (static)

NL (sq. term)

$\frac{10}{4}$ anticipator – anticausal sys.

$\frac{11}{4}$ $h(n) = \delta(n+2) - \delta(n-2)$ (OR) $H(z) = z^2 - z^{-2}$

$y(n) = x(n+2) - x(n-2)$ (a.) $H(z) =$

$\downarrow (n = n-2)$

$y(n-2) = x(n) - x(n-4)$

$\frac{14}{5}$ $y(n) = x(n) * h(n)$

$= \sum x(k) \cdot h(n-k)$

$= \sum a^k u(k) \cdot b^{n-k} u(n-k)$

16.
5

$$x(n) = \begin{cases} 0, & n < -2 \text{ or } n > 4 \\ 1 & \text{otherwise.} \end{cases}$$

$(n-2)$

$x(n-2) = 0 \quad n-2 < -2 \cdot (-n-2) > 4$

$n > 0 \qquad n < -6$

22
6

$$\text{even part of } u(t) = \frac{u(t)+u(-t)}{2} = \frac{1}{2}$$

$$\text{Odd part of } u(t) = \frac{u(t)-u(-t)}{2}$$

$\because u(t)+u(-t) = 1$

$u(-t) = 1 - u(t)$

$$= \frac{u(t)-1+u(t)}{2}$$

$$= \frac{2u(t)-1}{2}$$

$\because u(t) = \frac{1+\operatorname{sgn}(t)}{2}$

$\operatorname{sgn}(t) = 2u(t)-1$

$$= \frac{\operatorname{sgn}(t)}{2}$$

$$= \frac{x(t)}{2}$$

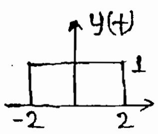
Ans. $\left(\frac{1}{2}, \frac{x(t)}{2}\right)$

27
7

$$x(t) = \delta(t+2) - \delta(t-2)$$

$$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$$

$$= u(t+2) - u(t-2)$$



$$E_y = \int_{-\infty}^{\infty} |y(t)|^2 dt$$

$$= \int_{-2}^{2} 1^2 dt = 4$$

28/7 scaling property of conv^n

$(t^2+t) \quad (t)$

$x_1(t) * x_2(t) = y(t)$

$x_1(at) * x_2(at) = \frac{1}{|a|} y(at)$

$(t = t+3)$

$\downarrow a=3$

$x_1(3t) * x_2(3t) = \frac{1}{3} y(3t)$

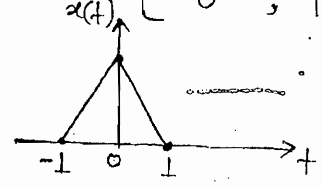
$(9t^2+3t) \quad (3t)$

32/8

$(-1 \le t \le 1)$

$$f(t) = \begin{cases} 1-|t| & |t| \le 1 \\ 0 & |t| \ge 1 \end{cases}$$

$$= \begin{cases} 1+t & ; -1 \le t \le 0 \\ 1-t & ; 0 \le t \le 1 \\ 0 & ; |t| > 1 \end{cases}$$



Chapter-02

1/9

$y(n+2) - 5y(n+1) + 6y(n) = x(n)$

$H(z) = \frac{1}{z^2 - 5z + 6}$

$H(z) = \frac{1}{(z-3)(z-2)}$

Poles:- 3, 2

9/9

$y(t) + \int_{0^-}^{\infty} y(\tau) x(t-\tau) d\tau = \delta(t) + x(t)$

$y(t) + y(t) * x(t) = \delta(t) + x(t)$

$\downarrow (LT)$

$Y(s) + Y(s) \cdot X(s) = 1 + X(s)$

$Y(s)[1 + X(s)] = 1 + X(s)$

1/1

$h(n) = (1, \frac{1}{2}, \frac{1}{4})$

$x(n) = (1, 0, 1)$

$y(n) = x(n) * h(n)$

Tabular method

15/10

$x(t) = u(t) \longrightarrow y(t) = 0.5(1 - e^{-2t}) u(t)$

$X(s) = \frac{1}{s} \qquad Y(s) = 0.5\left(\frac{1}{s} - \frac{1}{s+2}\right)$

$= \frac{1}{s(s+2)}$

$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+2}$

$h(t) = e^{-2t} u(t)$

17/10

(i) $y(t) = t x(t) \longrightarrow L$

(ii) $y(t) = t x^2(t) \longrightarrow NL$

(iii) $y(t) = x(2t) \longrightarrow L$

18/11

$y(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} x(\tau) d\tau$

$\downarrow y(t) = h(t) \qquad \downarrow x(t) = \delta(t)$

$h(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} \delta(\tau) d\tau$

$h(t) = \frac{1}{T} [u(\tau)]_{t-T/2}^{t+T/2}$

$= \frac{1}{T} [u(t+T/2) - u(t-T/2)]$

$h(t)$

18/11

$h(t) = \delta(t) + \delta(t-1) \rightleftharpoons H(s) = 1 + e^{-s}$

$y(t) = u(t) + u(t-1) \rightleftharpoons Y(s) = \frac{1}{s} + \frac{1}{s}e^{-s}$

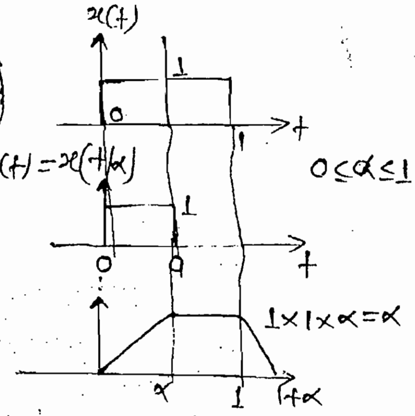
$= \frac{1}{s}(1 + e^{-s})$

$H(s) = \frac{Y(s)}{X(s)}$

$X(s) = \frac{1}{s}$

$x(t) = u(t)$

20/11



$0 \le \alpha \le 1$

21/12

LPF; $f_c = 100Hz$

$v(t) = 30\sqrt{2}\sin(1256t)$

$f_i = \frac{1256}{2\pi} \approx 200Hz$

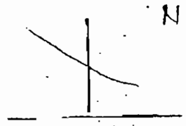
Chapter-03

9/14

$x_1(t) \to e^{j20t}$ (P)

$x_2(t) \to e^{(-2+j)t} = e^{-2t} e^{jt}$

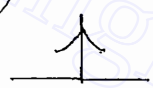
NP N P



13/14

$e^{-|t|}$ $\sin 25t$

NP P



13/15

$v(t) = (10\sin 2\pi t)\ 100t$

NP P NP

Chapter-04

4/16

C+P → D+NP

CTFS

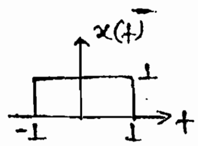
CTFT C+NP

D+P DTFS

Dirchelt cond → DTFT D+NP

NP

20/19



$X(\omega) = 2Sa(\omega)$

$X(\omega) = 0,\ \omega = ?$

$2Sa(\omega) = 0$

$Sa(\omega) = 0$

$\frac{\sin\omega}{\omega} = 0$

$\sin\omega = 0$

$\omega = n\pi;\ n \ne 0$

8/17

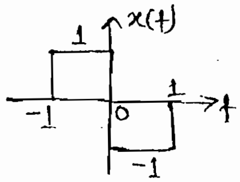
$y(n) - \frac{1}{2}y(n-1) = x(n) = k\delta(n)$

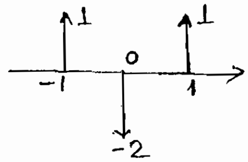
$Y(z) - \frac{1}{2}Y(z)z^{-1} = X(z) = k$

$Y(z) = \frac{k}{1 - \frac{1}{2}z^{-1}}$

$y(n) = k\left(\frac{1}{2}\right)^n u(n)$

24/19



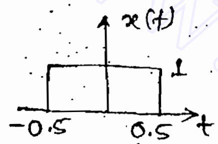


$$\frac{dx(t)}{dt} = \delta(t+1) + \delta(t-1) - 2\delta(t)$$

$$j\omega X(\omega) = (e^{j\omega} + e^{-j\omega}) - 2$$

$$X(\omega) = \frac{2\cos\omega - 2}{j\omega}$$

25/19



$\rightleftharpoons X(\omega) = Sa\left(\frac{\omega}{2}\right)$

$$h(t) = e^{j\omega_0 t} \rightleftharpoons H(\omega) = 2\pi\delta(\omega - \omega_0)$$

$$y(t) = 0,\ \omega_0 = ?$$

$$Y(\omega) = 0$$

$$X(\omega) \cdot H(\omega) = 0$$

$$Sa\left(\frac{\omega}{2}\right) \cdot 2\pi\delta(\omega - \omega_0) = 0$$

$$Sa\left(\frac{\omega_0}{2}\right) 2\pi\delta(\omega - \omega_0) = 0$$

$$Sa\left(\frac{\omega_0}{2}\right) = 0$$

$$\frac{\sin\left(\frac{\omega_0}{2}\right)}{\left(\frac{\omega_0}{2}\right)} = 0$$

$$\sin\left(\frac{\omega_0}{2}\right) = 0$$

$$\frac{\omega_0}{2} = n\pi,\ n \neq 0$$

$$\omega_0 = 2n\pi,\ n \neq 0$$

$\boxed{\text{Chapter-05}}$

2/21

$$x(t) = u(t) \rightleftharpoons y(t) = t^2 e^{-2t} u(t)$$

$$t^2 u(t) \rightleftharpoons \frac{2}{S^3}$$

$$e^{-2t} t^2 u(t) \rightleftharpoons \frac{2}{(S+2)^3}$$

$$H(S) = \frac{Y(S)}{X(S)} = \frac{\frac{2}{(S+2)^3}}{1/S} = \frac{2S}{(S+2)^3}$$

9/21

$$\frac{d^2y(t)}{dt^2} = x(t-2)u(t-2) + \frac{d^2x(t)}{dt^2}$$

$$H(S) \longrightarrow \text{LTI}$$

12/22

$$f(t) \rightleftharpoons F(S) = \frac{S+2}{S^2+1}$$

$$g(t) \rightleftharpoons G(S) = \frac{S^2+1}{(S+3)(S+2)}$$

$$h(t) = \int_0^t f(\tau) \cdot g(t-\tau)d\tau \rightleftharpoons H(S) = ?$$

$$= \int_{-\infty}^{\infty} f(\tau) \cdot g(t-\tau)d\tau$$

$$h(t) = f(t) * g(t)$$

let assump $\rightarrow$ $f(t)$ & $g(t) \rightarrow$ causal

$$H(S) = F(S)\, G(S) = \frac{1}{S+3}$$

23/24

$$x(t) = \sin\omega t$$

$$y(\infty) = 0,\ \omega = ?$$

$$y(t) = 0,\ \omega = ?$$

$$Y(S) = 0$$

$$H(S) \cdot X(S) = 0$$

$$H(S) = 0$$

$(S = j\omega)$

$$H(\omega) = 0,\ \omega = ?$$

26/24

$f(t) \rightleftharpoons F_1(S)$

$f(t-\tau) \rightleftharpoons F_2(S) = F_1(S)\, e^{-S\tau}$

$g(t) \rightleftharpoons G(S) = \dfrac{F_2(S) \cdot F_1^*(S)}{|F_1(S)|^2}$

?

$G(S) = \dfrac{F_2(S) \cdot F_1^*(S)}{F_1(S) \cdot F_1^*(S)} = e^{-S\tau}$

$g(t) = \delta(t-\tau)$

29/24

$y(t)=0,\ \omega = ?$

$Y(S)=0$

$\dfrac{1}{S} - \dfrac{3}{S+1} + \dfrac{3}{S+2} = 0$

$(S+1)(S+2) - 3S(S+2) + 3S(S+1) = 0$

$S^2 + 3S + 2 - 3S^2 - 6S + 3S^2 + 3S = 0$

$S^2 + 2 = 0$

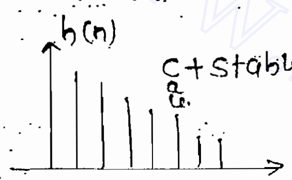
$(j\omega)^2 + 2 = 0$

$-\omega^2 + 2 = 0$

$\omega = \sqrt{2}$ rad/s.

$\boxed{\text{Chapter-06}}$

1/25



C + stable

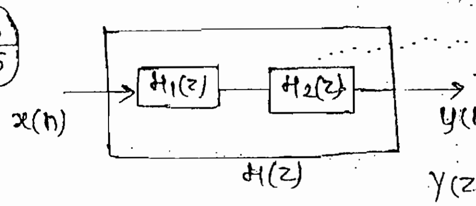
|Poles| < 1

7/26

$a^n u(n) \rightleftharpoons \dfrac{z}{z-a}$, $|z| > |a|$

$+a^n u(-n-1) \rightleftharpoons \dfrac{-z}{z-a}$ $|z| < |a|$

9/25



$y(n) = x(n-2)$

$Y(z) = X(z) z^{-2}$

$H(z) = z^{-2}$

$H(z) = H_1(z) \cdot H_2(z)$

$H_2(z) = \dfrac{H(z)}{H_1(z)}$

$H_2(z) = \dfrac{z^{-2}(z-0.8)}{z-0.5}$

$= \dfrac{z^{-1} - 0.8z^{-2}}{z-0.5} \times \dfrac{z^{-1}}{z^{-1}}$

$= \dfrac{z^{-2} - 0.8z^{-3}}{1-0.5z^{-1}}$

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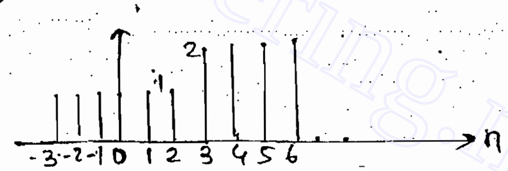
$h(n) = u(n+3) + u(n-3) - 2u(n-7)$



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$x(n)$ $y(n)$ LTI sys.

$u(n) \rightarrow s(n)$ = step responce

$\downarrow d/dn$

$\delta(n) \longrightarrow h(n) = \dfrac{ds(n)}{dn}$

$= s(n) - s(n-1)$

$h(n) = s(n) - s(n-1)$

$s(n) = (\underset{\uparrow}{1}, 1/2, 1/4, 1/8 \cdots)$

$s(n-1) = (\underset{\uparrow}{0}, 1, 1/2, 1/4 \cdots)$

$h(n) = (1, -1/2, -1/4, -1/8 \ \ )$

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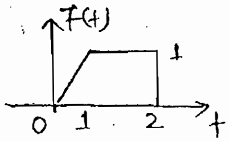
$G(S) \rightarrow$ Poles $\Rightarrow S = 0, 5$

$z = e^{ST} = 1, e^{5T}$

1/1)



$$f(t) = r(t) - r(t-1) - u(t-2)$$

$$= \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-2s}}{s}$$

$$= \frac{1}{s^2}\left(1 - e^{-s} - s e^{-2s}\right)$$

1/1)

$$\frac{d^2y}{dt^2} = x(t-2) + \frac{d^2x}{dt^2}$$

$$s^2 Y(s) = e^{-2s} X(s) + s^2 X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{s^2 + e^{-2s}}{s^2} = 1 + \frac{e^{-2s}}{s^2}$$

)

$$F(s) = \frac{27s+97}{s(s+33)} \Longrightarrow f(t)$$

$$\frac{27s+97}{s(s+33)} = \frac{A}{s} + \frac{B}{s+33} \qquad s=0,\ A = \frac{97}{33}$$

$$s = -33,\ B = \frac{-27\times 33 + 97}{-33\times 33} = \frac{794}{1089}$$

$$f(t) = \frac{97}{33}u(t) + \frac{794}{1089} e^{-33t} u(t)$$

$$f(0^+) = \frac{97}{33}u(0^+) + \frac{794}{1089} e^{-33(0^+)} = \frac{97}{33}$$

2)

$$x_1(t) = e^{k_1 t} u(t) \qquad x_2(t) = e^{-k_2 t} u(t)$$