28/07/2011
6.30-8.30 am

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1. Analysis — Introduction / LTI systems (convolution)

2. Approximation — FS — T.FS / E.F.S

3. Transformation — C.T.FS / LT / DTFT / ZT / DFT

Reference

1. Signals & systems – Oppenheim & Nawab
IES → 2. Signals & systems – Haykins & vaoveen.
3. Signals & systems – schaum's series.
4 Signals & systems – Nagoorkani.

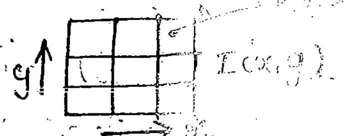
Signal → Whatever unknown to us, from which extracting new information. Func. of more than one vb
Function → Familiar to us, No new info.
→ only one indep. variable.

"Signal is an indication about which some amount of information is conveyed."

To retain the org. s/l.
1. Enhancing 2. Extracting 3. storing 4. Filtering
eg: Amplified

Characteristics of signal.

1. More than one indep. variable.
eg: speech – single dimensional (time)
Image – two dimensional.



Electromagnetics – two dimensional
TV picture – 3D – $I(x,y,t)$.

Room temperature $o(x, y, z, t)$ – 4 D.
(means in SAS)
Here, signal & function is considered as same.
eg: step function & step signal (only t/f is varying Here)

2. Randomness.

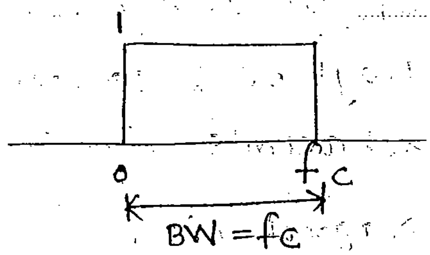
As Randomness ↑ → Information ↑ → signal strength ↑

$$I = \log_2 \frac{1}{P_i} \Rightarrow \text{as } P_i \uparrow \rightarrow \frac{1}{P} \downarrow \rightarrow I \downarrow$$

More the randomness more the info. content

3. Bandwidth.

Min BW required to txmitt the s/l must be known.

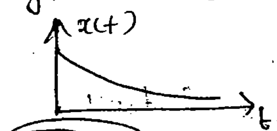


if 3 chara is satisfied then it is s/l otherwise function.

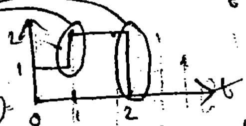
Types of Signal.

1. Continous s/l

→ Continous flow of signal.
eg: voice s/l.
→ Occurs for continous values of time. ($t: -\alpha$ to $+\alpha$)
* → Continous both in time & amplitude.
eg: $x(t) = e^{-3t}u(t)$.



discontinuity (piecewice s/l, descontinous s/l)



→ Amplitude at 1 & 2 is not taking is SAS. Mathematical convinience → avg value.

2. Descrete s/l

→ Reason:- To process the s/ls in a computer is easy
* Multiplexing
→ * Continous in amplitude, Descrete in time. (intiger values of time index)

$f_s \geq 2f_m$. $y(n) = x[nT_s]$

x(t) continous s/l. → samples → descrete s/l.

$t = nT_s$

$n = 0, \pm1, \pm2, \pm3 \dots$

T_s = Sampling time.

$x(t) = e^{-3t}u(t)$

$x[nT_s] = e^{-3[nT_s]}u[nT_s]$

if $T_s = 1sec$

$x[n] = e^{-3n}u(n)$.

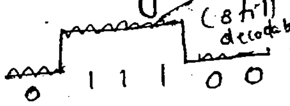
3. Digital s/l.

→ A signal which is descrete both in time & amplitude

→ Continous Amp 1. quantized → Digital Amp

5.6V → 5V (truncated value), 6V (round of value)

2. Coding



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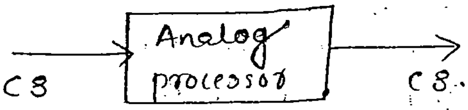
→ Except at the pt of discontinuity continous & precewise s/ls are same.

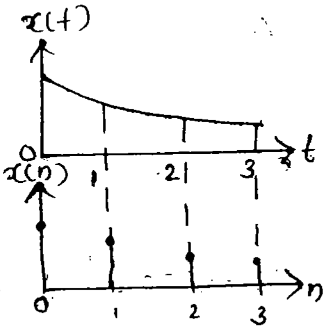
→ Also called as Analog s/ls.

Analog - similar (Amplitude is similar to time).

before 1965



Multiplexing becomes easy.



→ value exists in other time but for rep. this math info is only needed.

→ Adv:- Chance of reducing the band width.

→ * Right reppresentation of descrete s/l $x[nT_s]$, $x[nT_s - 1]$

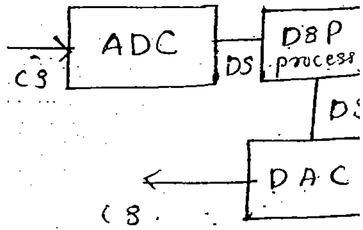
T_s is decided by the eqn $f_s \geq 2f_m$.

→ Some of desrete s/ls are predefined eg: temp vs time. No need of sampling

* Regenerative repeater (Not available in ~~dg~~ conti s/m)

eg: lumped inductor at telephone post

After 1965



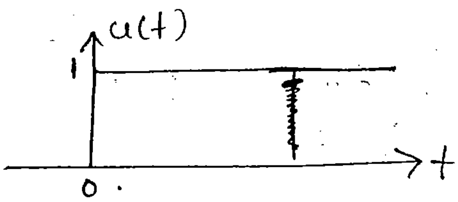
fastest A-D C

— σ — Δ

— desined by Z-T

Standard Signals.

1. Unit step Function.



$$u(t) = \begin{cases} 1 & ; t > 0 \\ 0 & ; t < 0 \end{cases}$$

at $t = 0$; $u(t)$ is not defined.

avg value $u(0) = \frac{1}{2}$ for math convenience

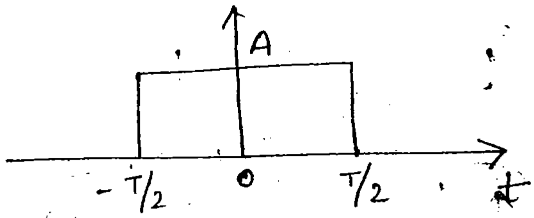
eg. ~~App~~ starting of automobile.

Imp : * transient response

* stability of s/m → boundness (related with Amp)

(megnitude) $|u(t)| = 1$

2. Rectangular / Gate function.

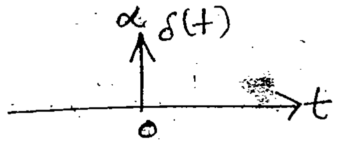


$A\,\text{rect}(t/T)$ or $A\,\Pi(t/T)$

$$x(t) = \begin{cases} A; & -T/2 < t < T/2 \\ 0; & \text{otherwise} \end{cases}$$

practical form of impulse ⟵ Rect s/l.

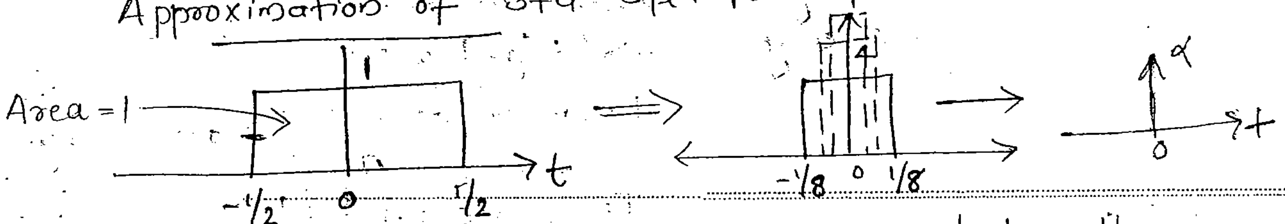
3. Continous. Impulse. / Dirac- Delta function.



$$\delta(t) = \begin{cases} \alpha; & t = 0 \\ 0; & \text{otherwise.} \end{cases}$$

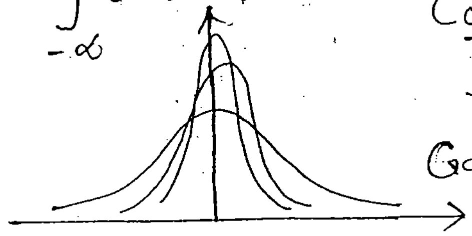
physically impossible
Mathematically possible.

sifting ppty → Any s/l can be rep. using imp. s/l.
Approximation of std s/l. to impulse signal.



$$\int_{-\alpha}^{\alpha} \delta(t)\,dt = 1$$

$\delta(t)$ means area/strength.
Continous impulse s/l is always rep. using area concept.



Gaussian s/l.

Standard s/ls can be approximated to impulse function.

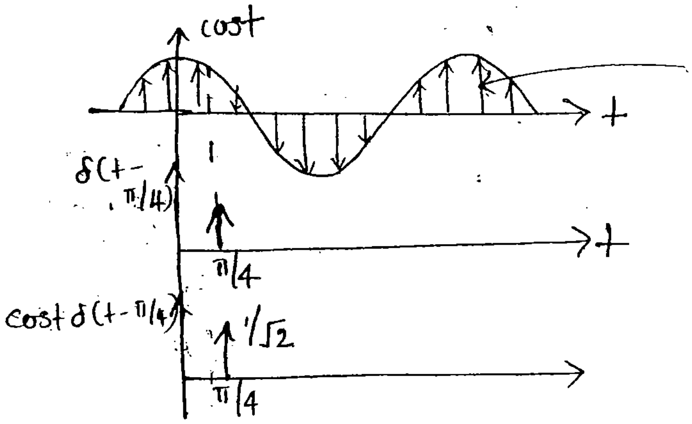
Property

1. Area under impulse function is one ⇒ $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$
2. Impulse is an even function
   $\delta(-t) = \delta(t)$.

Imp 3. Scalling ppty: $\delta(at) = \frac{1}{|a|}\delta(t)$.
$|a|$ → a may be +ve/-ve but impulse is even.

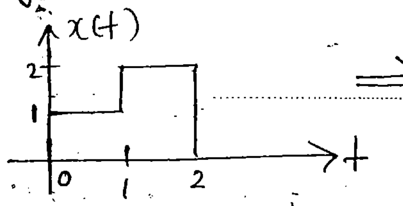
Imp 4. Product ppty:
Time shifting: $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$
if s/l is continous at $t = t_0$.

eg: $\cos t \, \delta(t-\frac{\pi}{4}) = \frac{1}{\sqrt{2}} \delta(t-\frac{\pi}{4})$ 3



Any gen. s/l is sum of impulse s/l (basis of shifting ppty)

eg 2; $t\,\delta(t) = 0$; $\Rightarrow \delta(t) = 0$



$\Longrightarrow$ $x(t)\,\delta(t-2)$ = No answer. at $t=2$ Not defined the value of x.

Imp. 5. Sampling Property.

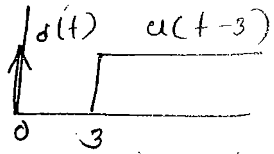
1^{st} observation
~~* s/l is shifted~~ within the limit / not
* substitute t_0 (shifted value) at t.

$$\int_{t_1}^{t_2} x(t)\,\delta(t-t_0)\,dt = x(t_0) \; ; \quad t_1 \le t_0 \le t_2$$

eg: $\int_{-1}^{2} (t+t^2)\,\delta(t-4)\,dt = 0$ (shift is outside the limit).

$$\int_0^{\alpha} [t+\cos\pi t]\,\delta(t-1)\,dt = 1+\cos\pi = \underline{\underline{0}}.$$

$$\int_0^{\alpha} \cos t\,[u(t-3)\,\delta(t)]\,dt = 0$$



No overlap b/w $\delta(t)$ & $u(t-3)$

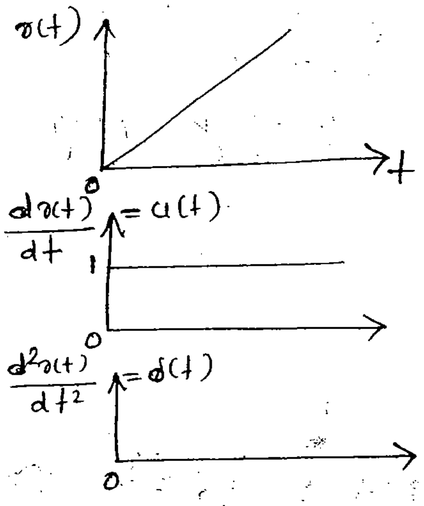
$$\int_0^{\alpha} e^{(t-3)}\,\delta(2t-6)\,dt$$

$$\delta(2t-6) = \delta(2(t-3)) = \frac{1}{2}\delta(t-3).$$

$$\int_0^{\alpha} e^{(t-3)} \frac{1}{2}\,\delta(t-3)\,dt = \frac{1}{2} e^{(3-3)} = \frac{1}{2}e^0$$

$$= \underline{\underline{\frac{1}{2}}}$$

4. unit ramp function.

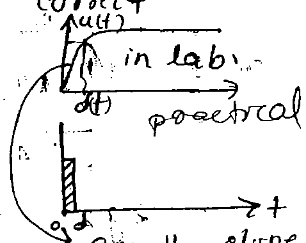


$$r(t) = \begin{cases} t \; ; & t \geq 0 \\ 0 \; ; & t < 0 \end{cases}$$

$$\boxed{\delta(t) = \frac{d\,u(t)}{dt}}$$

$$\boxed{u(t) = \int_{-\infty}^{t} \delta(t)\, dt}$$

Mathematically wrong, still correct.



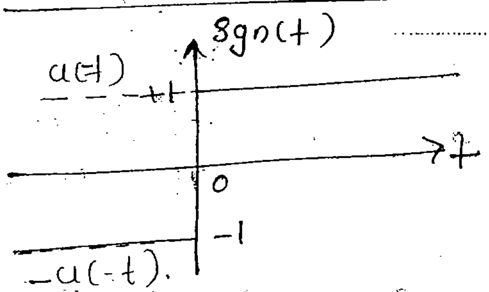
small slope looks like ramp at small int of time dt

Singularity Function.

Function Doesn't have higher derivatives.

eg: ramp signal, unit step function.

5. Signum function.



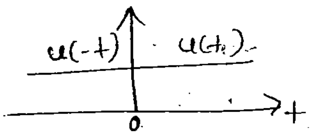
Bipolar Amplitude.

purpose: to reduce the amplitude.

odd function → Amplitude is always '0' at origin

Antisymmetric signal.

$$\boxed{\begin{aligned} sgn(t) &= u(t) - u(-t) \\ &= 2u(t) - 1 \\ [&= u(t) - (1 - u(t)) \\ &= 2u(t) - 1] \end{aligned}}$$



$u(t) + u(-t) = 1$

$u(t)|_{t=0} = \frac{1}{2}$

$u(-t)|_{t=0} = \frac{1}{2}$

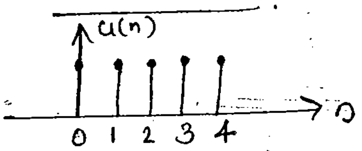
$u(0) = \frac{1}{2}$ (avg value)

$\therefore u(t) = 1 - u(-t)$

Predefined s/ls

Descrete s/l.

1. unit step sequence.



$$u(n) = \begin{cases} 1 \; ; & n \geq 0 \\ 0 \; ; & n < 0 \end{cases}$$

predefined s/l.

Not sampled version of u(t)

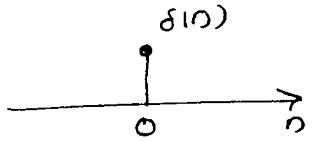
because u(t) at t=0 Not defined

u(n) at n=0 is '1'

2. Descrete Impulse / kronecher delta.

$$\delta(n) = \begin{cases} 1 \; ; & n=0 \\ 0 \; ; & n \neq 0 \end{cases}$$

4



Relation b/w $\delta(n)$ & $u(n)$

$$\delta(t) = \frac{d}{dt} u(t) \longrightarrow \frac{d}{dt} \longrightarrow \nabla$$

$$\delta(n) = \nabla u(n) \qquad \nabla x(n) = x(n) - x(n-1)$$

$$\therefore \boxed{\delta(n) = u(n) - u(n-1)}$$

$$u(t) = \int_{-\alpha}^{t} \delta(\tau)\, d\tau \qquad \int \Rightarrow \Sigma$$

$$t \Rightarrow n.$$

$$\tau \Rightarrow k.$$

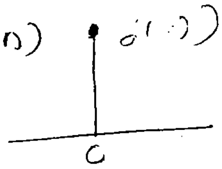
$$u(n) = \sum_{-\alpha}^{n} \delta(k)$$

put $n = k + m$ $\quad k \to -\alpha \Rightarrow m \Rightarrow \alpha$

$k \to n \Rightarrow m = 0.$

$$\boxed{u(n) = \sum_{m=+\alpha}^{0} \delta(n-m)}$$

$$\boxed{\delta(kn) = \delta(n)}$$



Let $k = 3$

$\delta(3n) = \delta($

Scaling pptg

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Transformation of signal.

1. Time scaling $x(at) \,|\, x[mn]$

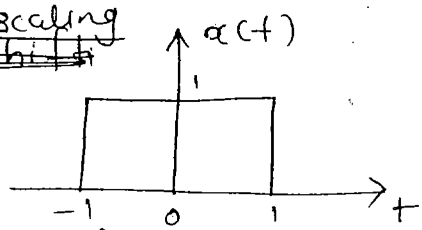
$x(t) \xrightarrow{\text{scaled}} x(at)$; $x(n) \xrightarrow{\text{scaled}} x(mn)$.

Appn: Compression, expansion.

2. Time shifting $(x(t \pm t_0)) \,|\, x[n-n_0]$

3. Time reversal $x(-t) \,|\, x[-n]$

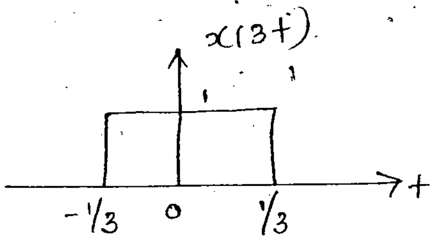
eg: ~~shift~~ scaling



$x(3t)$, $x(t/5)$

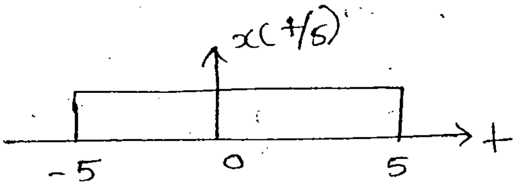
$$x(t) = \{ -1 \le t \le 1$$

$$x(3t) = \{ -1 \le 3t \le 1 \Rightarrow -\tfrac{1}{3} \le t \le \tfrac{1}{3}$$



Multiplication $\longrightarrow$ Division in time axis

Division $\longrightarrow$ Multiplication in time axis.



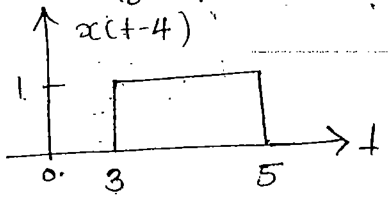
$a > 1$	– compression
$a < 1$	– expansion

Shifting

$x(t-4)$

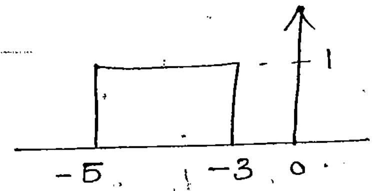
$\downarrow$

$t_0 = 4$



$x(t+4)$

$\downarrow t_0 = -4$



$t_0 < 0$

Left shift:

$x(t-t_0) \Rightarrow t_0 > 0$

Right shift.

+ve $t_0 \rightarrow$ Time delay.

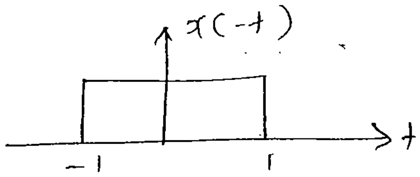
-ve $t_0 \rightarrow$ time advance – Not possible.

before applying i/p o/p is expecting.

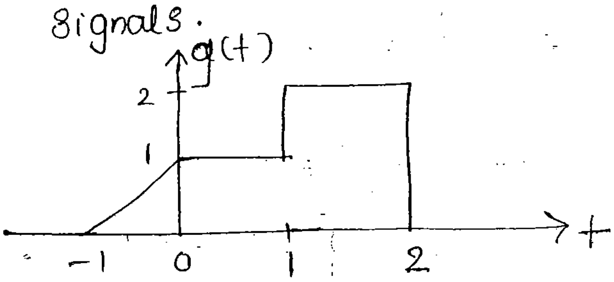
off-line s/l processing

Reversal

Rotation w.r.p to 'y' axis.



Q. For the s/l shown in fig, draw the following signals.



$g(t-4)$

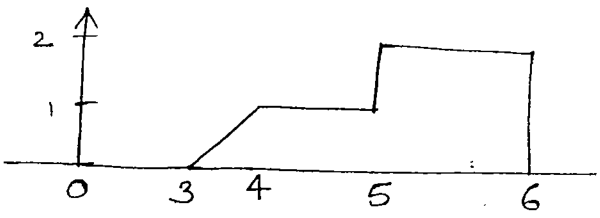
$g(2t+1)$

$g(5-t)$

$g(-t-2)$

$[x(t)+x(-t)]\,u(t)$

(i) $g(t-4)$



(ii) $g(2t+1)$

$x(\alpha t+\beta)$

5

Method 1 :

1)st scaling.

$x(\alpha t+\beta) \rightarrow x(\alpha(t+\frac{\beta}{\alpha}$

Method 2 :

1)st shifting

assume $x(t+\beta)$.

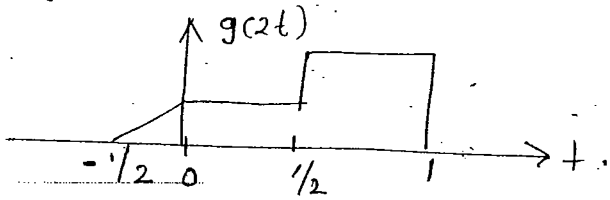
then shift

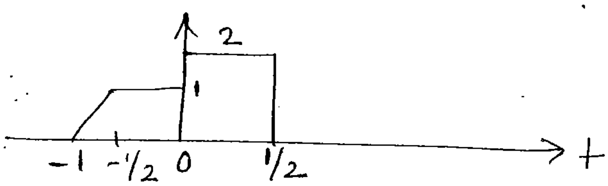
the scale $x(\alpha t+\beta)$

$g(2t+1)$.

(i) Method 1

$g(2t+1) \Longrightarrow g(2(t+1/2))$

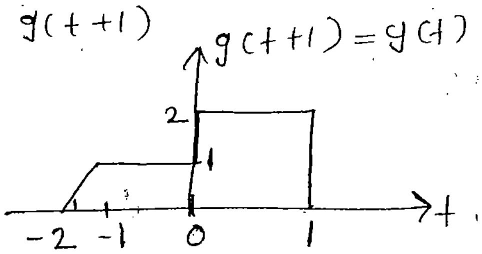




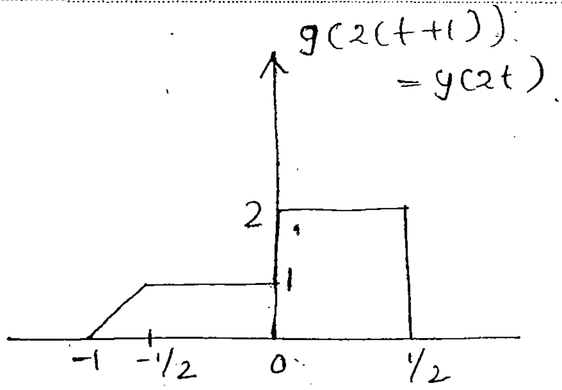
(ii) Method 2

$g(2t+1)$

$g(t+1)$

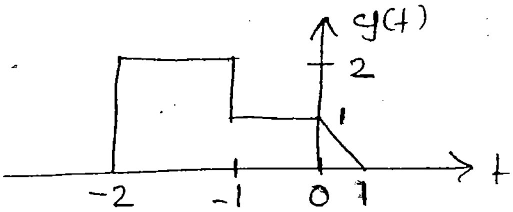


$\Longrightarrow$

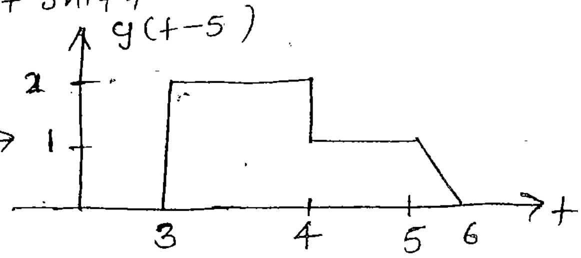


(iii) $g(5-t) = g(-(t-5))$

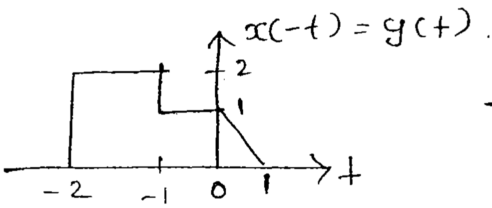
Time reversal $\longrightarrow$ Right shift.



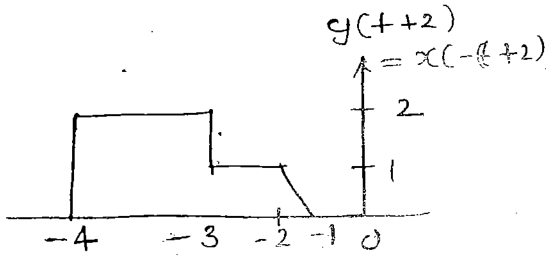
$\longrightarrow$



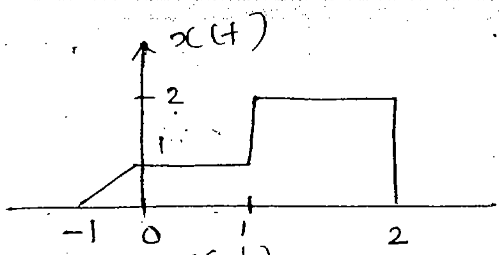
(iv) $x(-t-2) = x(-(t+2))$



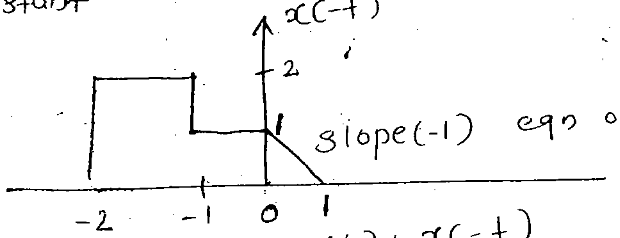
$\longrightarrow$



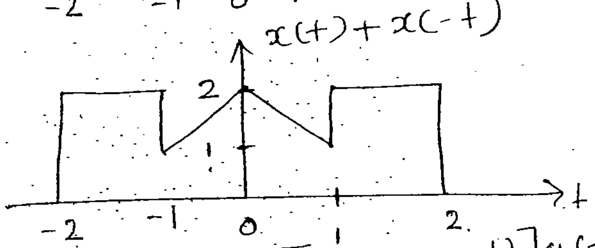
(iv)

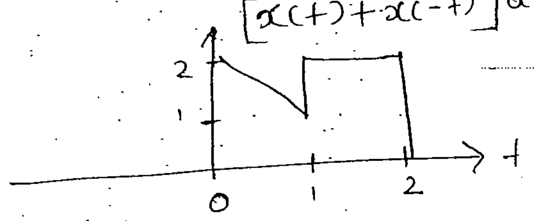


ramp + constant
= ramp



slope(-1) eqn of ramp = (-1) t

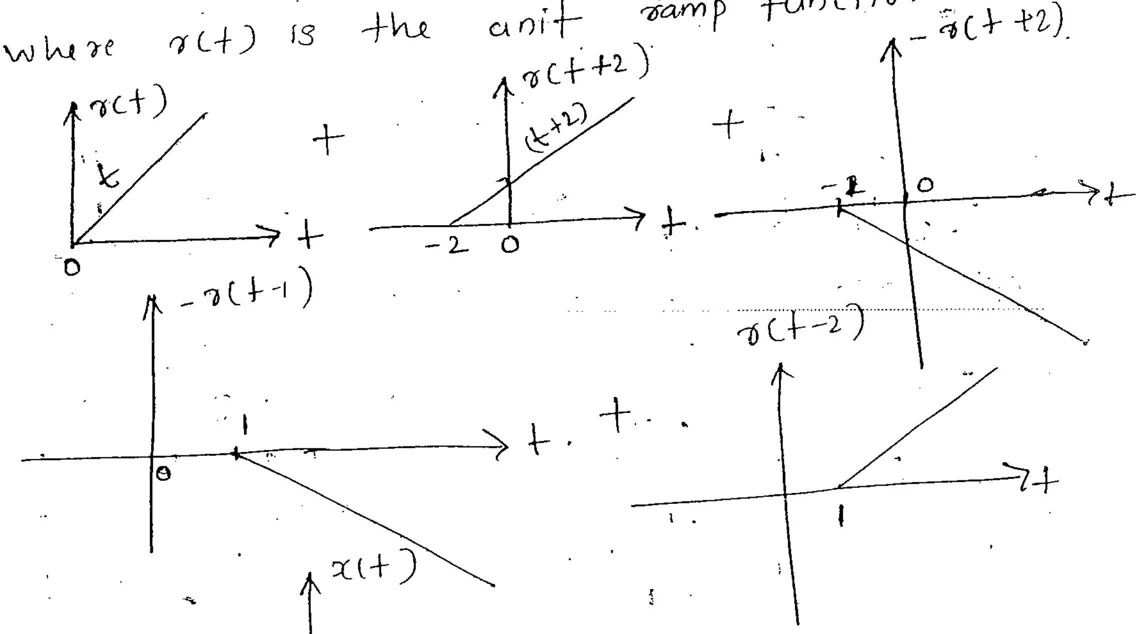


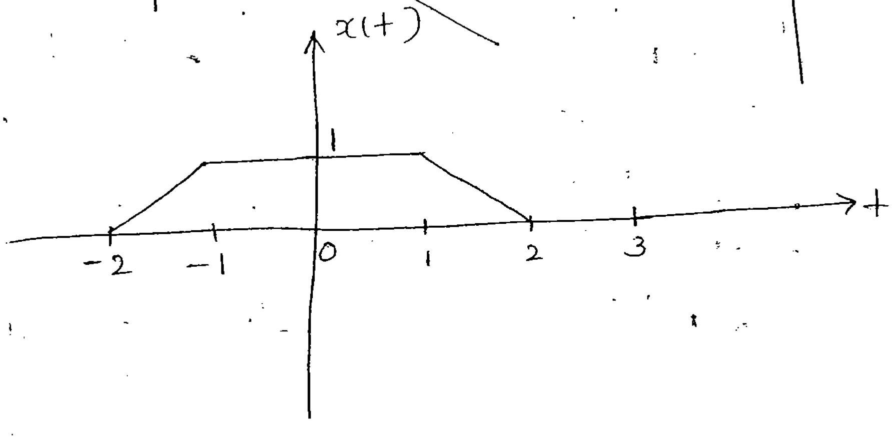


Q). Draw the s/l.

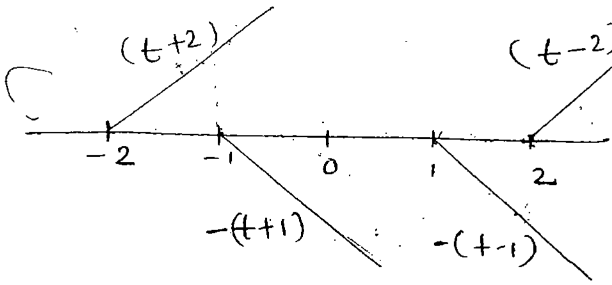
$x(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$.

where $r(t)$ is the unit ramp function.

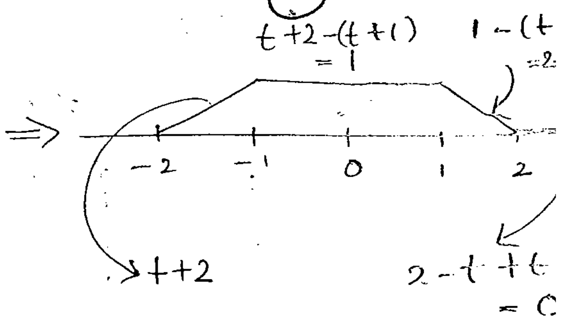




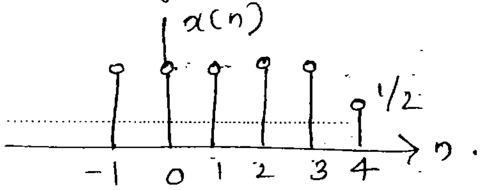
(6)



$\Rightarrow$



Q) For the descrete s/l shown in fig. Draw the following s/ls?:



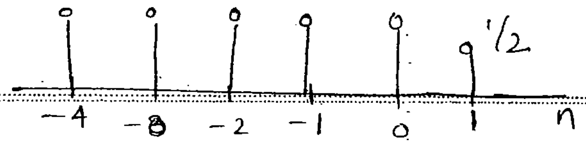
(i) $x(n+3)$

(ii) $x[6-n]$

(iii) $x[3n+1]$

(iv) $x[n/2]$

(i) $x(n+3)$



(ii) $x[6-n] = x[-(n-6)]$

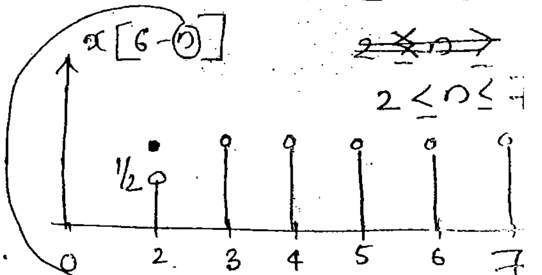
$x[n] : -1 \le n \le 4$

$x[6-n] : -1 \le 6-n \le 4$

$: -7 \le -n \le -2$

$7 \not\le n \not\le 2$

$2 \le n \le 7$



substitute value to n & get the amp. from the original s/l x

(iii) $x[3n+1] = x[3(n+1/3)]$

$n_0 = -1/3$; not possible technique failed. n is defined for only integer values.

$x[3n+1] : -1 \le 3n+1 \le 4.$

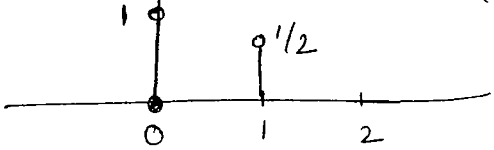
$: -2 \le 3n \le 3.$

$: -2/3 \le n \le 1$

$\Downarrow$

$0 \le n \le 1.$

defined for only two values of 1.



$x(3n+1)$

$x(3\times0+1) = x(0) = 1$

$x(3\times1+1) = x(4) = 1/2$

$x[mn]$ eg: $x[3n]$

if $m > 1$.

decimation / down sampling

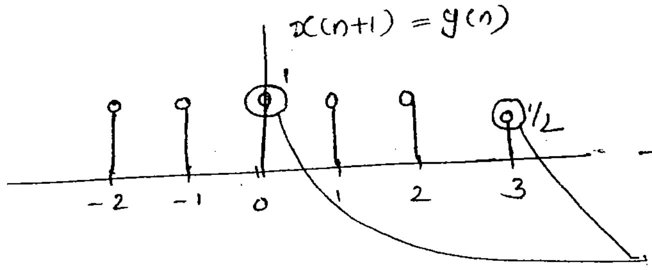
red. no of samples

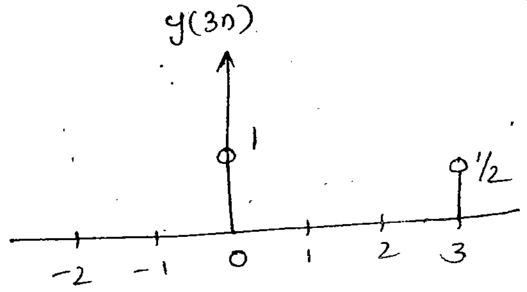
if $m < 1$

up sampling / interpolation

increase the no of samples

shifting - scaling

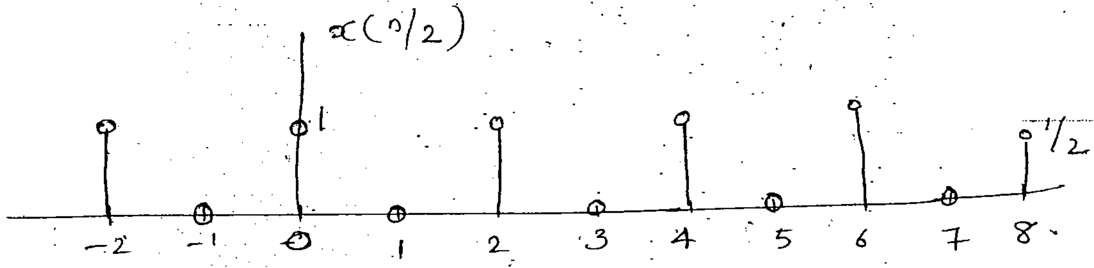




Samples which gives integer value while scaling with factor 3.

(iv) $x[n/2]$

$x[n/2] = -1 \leq n/2 \leq 4 \implies -2 \leq n \leq 8$.



Zero Interpolation ⟶ Most of the samples are zero.

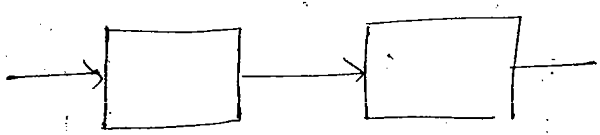
Interpolation - Increase the no of samples.

Vaidyanadhan

Multirate DSP -

CD : f_s = 44.1 KHZ.

DAT : f_s = 48 KHZ.



44100 ⟶ 48000.

⟶ Need to increase the no of samples.

$x\left(\frac{2n}{3}\right) \longrightarrow x(n/3)$ then $x(2n)$

Interpolation decimation.

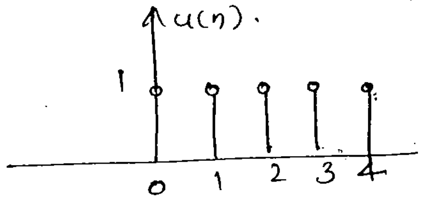
Whenever there is need for both interpolation & decimation.

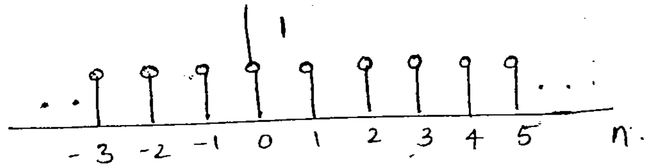
Compression ⟶ Decimation.

Q) Given $x[n] = 1 - \sum_{k=3}^{\alpha} \delta[n-1-k]$ such that

$x[n] = u[mn - n_0]$ find m, n_0?

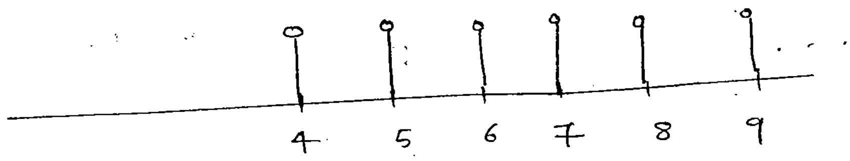
Ans:

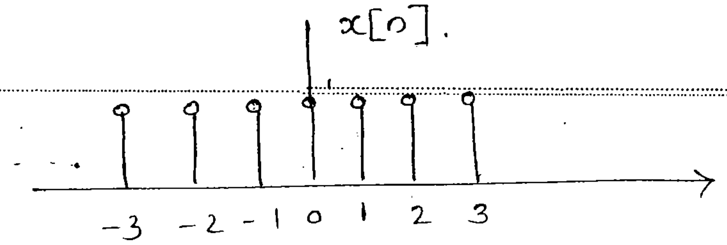




$\delta(n-1-3)$ to $\delta(n-1-\alpha)$.

$\delta(n-4)$ to $\delta(n-\alpha)$. $= \delta(n-4) + \delta(n-5) + \ldots \delta(n-\alpha)$



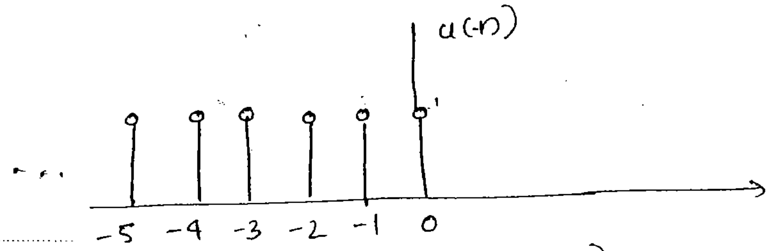


$x(n) = 1;\ n \leq 3$

$: n-3 \leq$

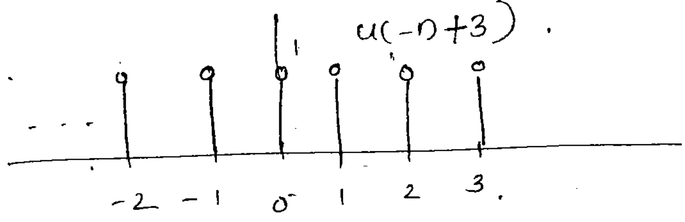
$3-n \geq$

$= u(3-n$



$m = 1$

$n_0 = -3$



Classification of signals.

① Energy & Power Signals:

Largness / Max capacity of the signal

avg value ⟶ Doesn't give correct info. (eg: sine)

Energy s/l $\longrightarrow$ $0 \le E < \infty$. (finite)

Power s/l $\longrightarrow$ $0 \le P \le \infty$ (finite)

Energy Content of $x(t)$

* $$E_{x(t)} = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt$$

$$P_{av}\, x(t) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} |x(t)|^2 dt$$

$| \; | \rightarrow$ for complex s/l.

No complex s/l is physically possible.
But mathematical convinience over other s/l we go for exponential term

$$|x(t)|^2 = x(t)\, x(t)^*$$

Appn: SNR in commn s/ms.

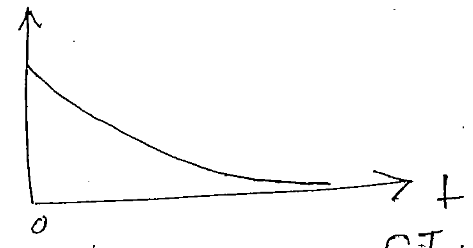
Energy in a Descrete s/l.

$$E_{x[n]} = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$$

$$\text{Power} \quad P_{av} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

$n = 0$ $\rightarrow$ seperate sample ~~at~~

eg: $x(t) = e^{-t}u(t)$



As $|t| \longrightarrow \infty$, amp $= 0$ $\Big\}$ Energy s/l.

for physically possible s/l.

$$E_{x(t)} = \lim_{t\to\infty} \int_0^T e^{-2t} dt.$$

$$= \lim_{t\to\infty} \left[\frac{e^{-2t}}{-2}\right]_0^T$$

$$= \lim_{t\to\infty} \left[\frac{1 - e^{-2T}}{2}\right] = \frac{1}{2}$$

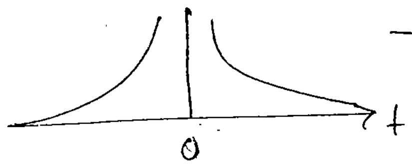
Energy is finite $\therefore$ energy s/ls.

Avg Power in s/l.

$$P_{avg} = \underset{T \to \infty}{Lt} \frac{E}{2T} = \underline{\underline{0}}$$

* Energy s/l has average power is zero. (power is computed over infinite time)

$$g(t) = \left|\frac{1}{t}\right|$$



→ Not physically possible.
∴ Not energy signals/power s/l

eg: 2 $x(t) = A\,u(t)$.

* A s/l which maintains a constant amplitude over infinite time is power s/ls.

verification

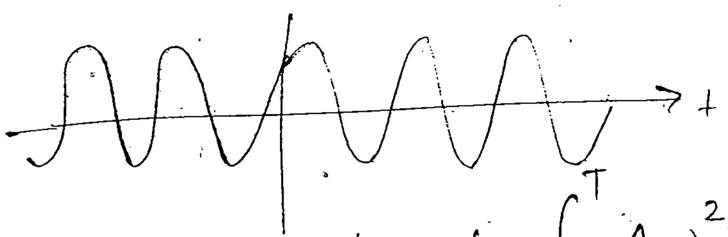
$$P_{avg} = \underset{T \to \infty}{Lt} \frac{1}{2T} \int_0^T |A^2|\, dt = \underset{T \to \infty}{Lt} \frac{A^2}{2T} \left[t \right]$$

$$= \underline{\underline{\frac{A^2}{2}}}$$

$$\boxed{\text{The Energy of power s/l} = \infty}$$

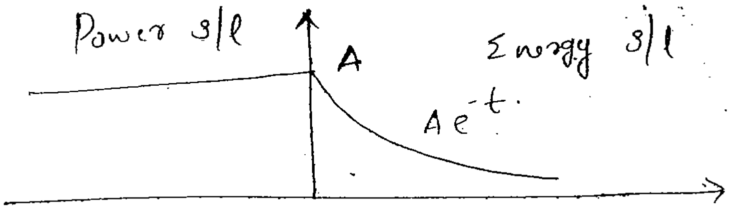
eg: 3 $A\cos(\omega t + \phi)$ - Power s/ls

* All periodic s/ls are power s/ls. but not viceversa



$$P_{avg} = \underset{T \to \infty}{Lt} \frac{1}{2T} \int_{-T}^{T} \left(\frac{A}{\sqrt{2}}\right)^2 dt = \underline{\underline{\frac{A^2}{2}}}.$$

eg: 4



$$P_{av} = \underset{T\to\infty}{Lt} \frac{1}{2T}\left[\int_{-T}^{0} A^2 dt + \int_0^T A^2 e^{-2t} dt\right]$$

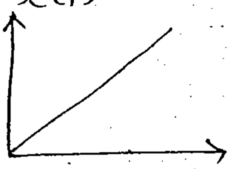
$$= \underline{\underline{A^2/2}}$$ $\therefore$ It is a power s/l.

Power s/l + Energy s/l $\longrightarrow$ Power s/l.

08/08/2011
2 - 5.30 pm

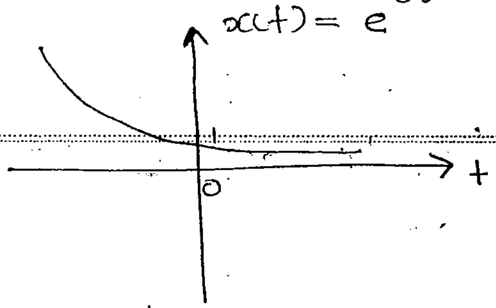
$x(t) = t\, r(t) =$

Neither energy nor power



$x(t) = e^{-3t}$

$t \to \infty$; $x(t) \to 0$
$t \to -\infty$; $x(t) \to \infty$ } Neither energy / power.



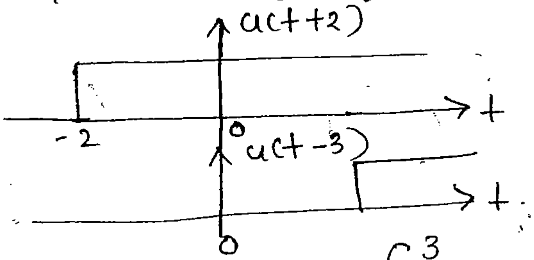
Energy s/l : — at $\pm\infty$; $x(t) = 0$

Q) Given $x(t) = \delta(t+2) - \delta(t-3)$ and $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$. Find the energy in $y(t)$?

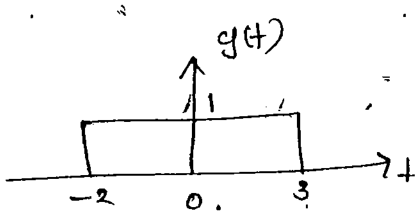
Ans: $\left[u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau \right]$

$$y(t) = \int_{-\infty}^{t} [\delta(\tau+2) - \delta(\tau-3)]\, d\tau$$

$$y(t) = u(t+2) - u(t-3).$$



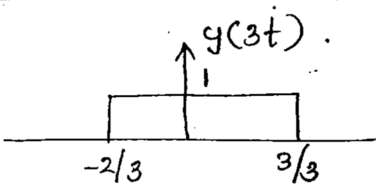
$\Rightarrow$



$$\text{Energy}_{y(t)} = \int_{-2}^{3} |y(t)|^2 dt = \int_{-2}^{3} 1\, dt = 3-(-2)$$
$$= \underline{\underline{5}}$$

* All finite duration s/ls of finite amp. are 'E' signals.

Q. $y(3t) \Rightarrow$



$$\text{Energy} = \int_{-2/3}^{1} (1)\,dt = 1+\frac{2}{3} = \frac{5}{3} = \frac{E_{y(t)}}{3}$$

If the s/l is compressed in time the energy is scaled by that unit

Q. $y(2-\frac{t}{5}) \longrightarrow 5\,E_{y(t)}$.

shifting will not alter energy.

$$\therefore \boxed{y(at) \longrightarrow \frac{E}{|a|} \text{ (Energy)}}$$

Q $A\,y(t) \longrightarrow A^2 E_{y(t)}$.

$A + y(t) \longrightarrow \infty$ Energy (power s/l has ∞ energy)

Power signal + Energy signal $\longrightarrow$ Power signal.

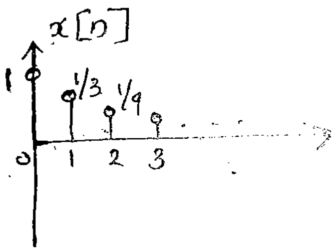
(constant is power s/l)

Descrete Signal

Q. (1) $x[n] = a^n u[n]$

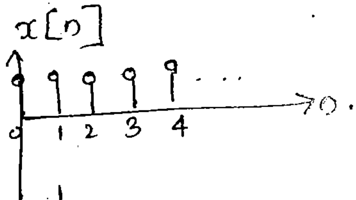
$|a| < 1$

Let $a = 1/3 \Rightarrow x[n] = (1/3)^n u[n]$



at $n = \pm\infty$ $x[n]$ = finite (0).

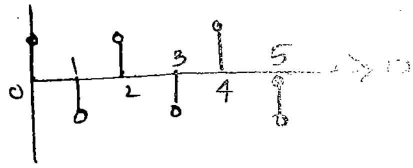
$\therefore$ Energy s/l.



(2) $|a| = 1 \Rightarrow x[n] = u[n]$

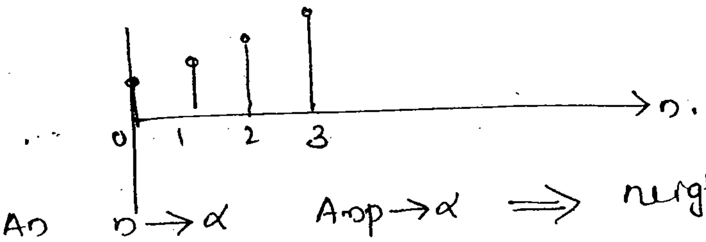
$a = -1 \Rightarrow x[n] = (-1)^n u(n)$.

Constant Amp. $\Rightarrow$ Power s/l.



③ $|a| > 1$

Let $a = 2 \Rightarrow x[n] = 2^n u(n)$

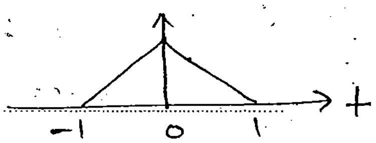


As $n \to \infty$ $A_{mp} \to \infty \Rightarrow$ neighter energy nor power.

② Even & ODD signals.

$x(t) = x(-t) \longrightarrow$ Even s/l.

eg:



$x(t) = -x(-t) \longrightarrow$ odd signal.

eg: $\sin\omega_0 t$.

Conditions are correct for real s/l

For Complex s/ls:

Even Conjugate (EC)	ODD Conjugate (OC)
$x(t) = x^*(-t)$	$x(t) = -x^*(-t)$.
eg: $x(t) = t^2 e^{j\omega t}$	eg: $te^{j\omega t}$.

$x(-t) = t^2 e^{-j\omega t}$

$x^*(-t) = t^2 e^{j\omega t}$.

a: signal $\longrightarrow$ odd component + even component

$x(t) \longrightarrow x_e(t) + x_o(t)$. ——— ①

Replacing t by -t

$x(-t) = x_e(-t) + x_o(-t)$

$= x_e(t) - x_o(t)$ ——— ②

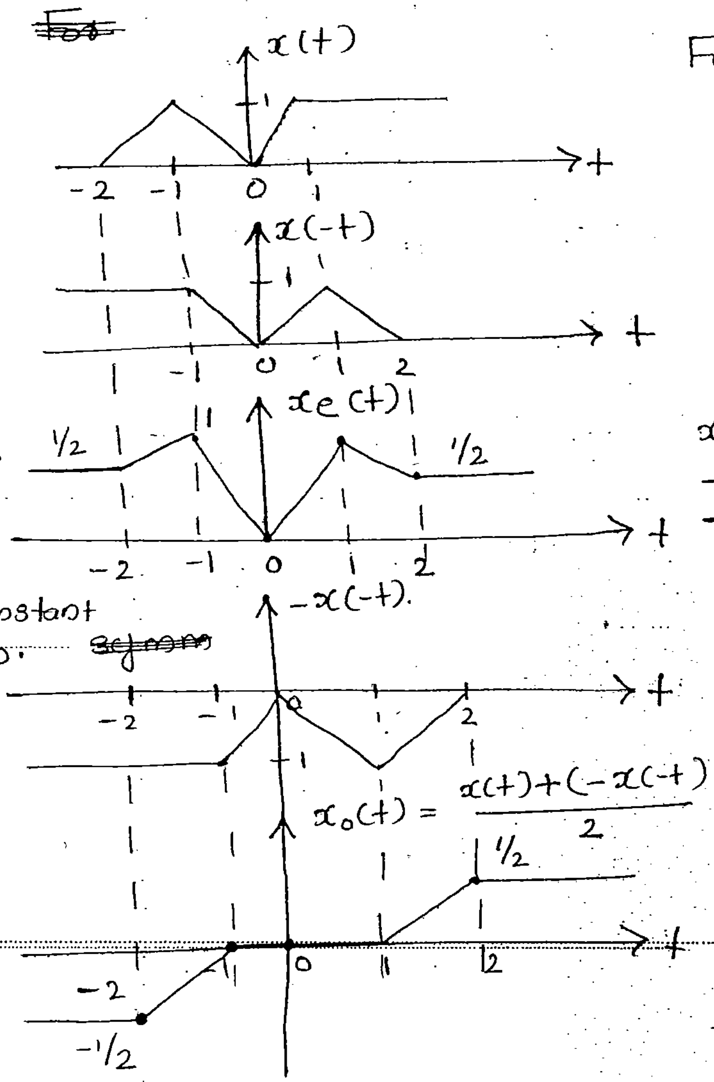
$$x_e(t) = \frac{x(t) + x(-t)}{2}$$; Even component

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$; odd component

Note: For Complex signals put * (conjugate)

Q ~~For~~ Find even & odd component of x(t)?

(10)



look for discontinous pt.

ramp + Constant = ramp. ~~Symm~~

x(t) = x(-t).
- If we know left side
- portion automatically the right side can be drawn
  symmetry - even
  Antisymmetry - ODD.

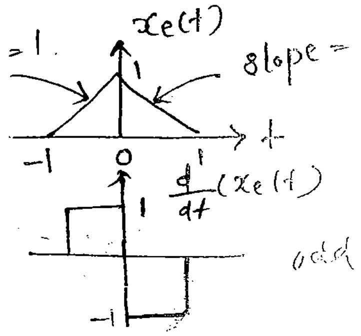
x(t) = -x(-t).
ODD signals are antisymmetric about Y axis.

even s/l + even s/l = even.
odd s/l + odd s/l = odd
even s/l + odd s/l = Neither even nor odd

even × even = even
odd × odd = even
even × odd = odd

Area under odd s/l = '0'
Area under even s/l = 2 × one sided area.
(+ve area
-ve area)

$\frac{d(\text{even signal})}{dt}$ = odd



"Diff. & integration of even s/l is odd s/l & vice versa."

Note ① The conj. antisymmetric part of $x(n) = -$

$\{1+j3, \underset{\uparrow}{2}, j5\}$ $\qquad x_o[n] = \dfrac{x[n] - x^*[-n]}{2}$

$-1 \quad 0 \quad +1$

$$x_o(-1) = \frac{x(-1) - x^*(1)}{2} = \frac{1+j3-(-j5)}{2} = \frac{1+j8}{2}$$

$$x_o(0) = \frac{x(0) - x^*(-0)}{2} = 0$$

$$x_o(1) = \frac{x(1) - x^*(-1)}{2} = \frac{j5-(1-j3)}{2} = \frac{-1+j8}{2}$$

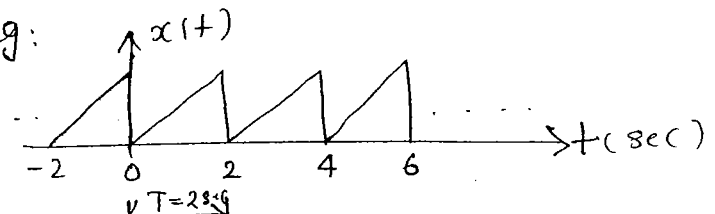
$$x_o(n) = \left\{\frac{1+j8}{2}, \underset{\uparrow}{0}, \frac{-1+j8}{2}\right\}$$

③ Periodic & Non-periodic Signal.
{ A periodic }.

Periodic :- obeys the integer value of periodic $(n\omega_0)$

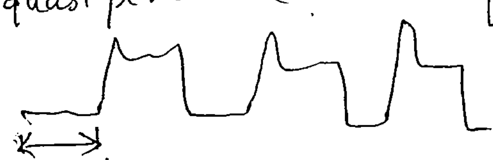
$$\boxed{x(t) = x(t+T)}$$ T = least time period of $x(t)$.

eg:



exception $\longrightarrow$ DC : $\Longrightarrow$ $a_0 + \Sigma \cos + \Sigma \sin$.
$\downarrow$
Can have any value.
if T = 2 ; dc = 2.

eg: electricity,
clock., quasi periodic (Noise) [Initially aperiodic then periodic)



Note: Steps for finding time period

1. Identify the individual time periods (T_1, T_2, T_3 . . .)

2. Calculate $\frac{T_1}{T_2}, \frac{T_1}{T_3}, \frac{T_1}{T_4} \ldots$

3. If the ratios ~~are~~ of second step is a rational number, then the over all signal is periodic.

11

4. Calculate LCM of denominators of 2nd step.

5. T is LCM x T1.

procedure is valid for only ~~multip~~ addition of signals.

Q. $x(t) = \cos 50\pi t + \sin 60\pi t$.

$$\omega_1 = 50\pi \qquad \omega_2 = 60\pi$$

$$\frac{2\pi}{T_1} = 50\pi \qquad \frac{2\pi}{T_2} = 60\pi$$

1 step. $T_1 = 1/25 \qquad T_2 = 1/30$

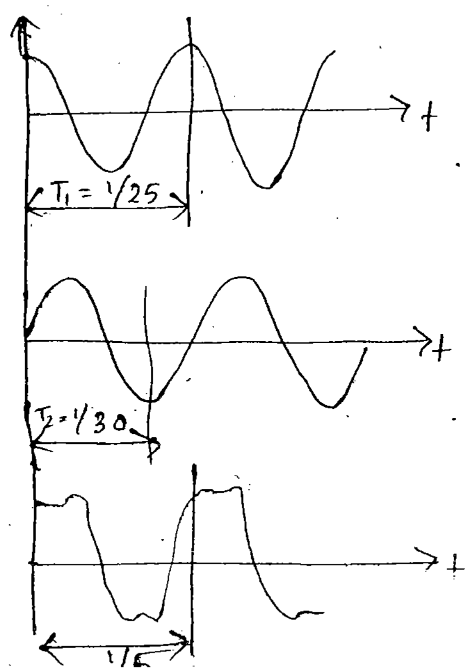
2nd step: $T_1/T_2 = 30/25 = 6/5$

3rd : rational ∴ periodic

4th : LCM $= 5$

5 : $T = \text{LCM} \times T_1 = 5 \times \frac{1}{25} = \frac{1}{5}$

Physical Meaning



Easy Method

$50\pi \Rightarrow 5(10\pi)$
$60\pi \Rightarrow 6(10\pi)$.
$n\,\omega_0$

GCD → Greatest Common division

$\omega_0 = 10\pi \Rightarrow \frac{2\pi}{T} = 10\pi$

$T = \frac{2}{10} = 1/5$

LCM → With respect to 'T'
GCD → with respect to 'f'

Q). $x(t) = \sin\left(\frac{2\pi t}{3}\right) \cos\left(\frac{4\pi t}{5}\right)$

$$= \frac{1}{2}\left[\sin\left(\frac{2\pi}{3}t + \frac{4\pi t}{5}\right) + \sin\left(\frac{2\pi}{3}t - \frac{4\pi}{5}\right)t\right]$$

$$= \frac{1}{2}\left[\sin\frac{22\pi}{15}t - \sin\frac{2\pi}{15}t\right]$$

$$\omega_1 = \frac{22\pi}{15} \rightarrow 11\left(\frac{2\pi}{15}\right)$$

$$\omega_2 = \frac{2\pi}{15} \rightarrow 1\left(\frac{2\pi}{15}\right)$$

$n(\omega_0)$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{15} \Longrightarrow T = 15$$

Q) $x_3(t) = \sin 13t + \cos 17\pi t$.

$$T_1 = \frac{2\pi}{13} \quad T_2 \rightarrow \frac{2}{17}$$

$$T_1/T_2 = \frac{17\pi}{3} \rightarrow \text{Irrational} \therefore \text{Non periodic}$$

*Q) $x(t) = je^{j10t} = e^{j\pi/2}\, e^{j10t}$

$$\omega_0 = 10 \Longrightarrow T = \frac{2\pi}{\omega_0} = \frac{\pi}{5} \text{ (periodic)}$$

$$x\left(t+\frac{\pi}{5}\right) = je^{j10(t+\pi/5)}$$

$$= je^{j10t} \cdot e^{j10\pi/5}$$

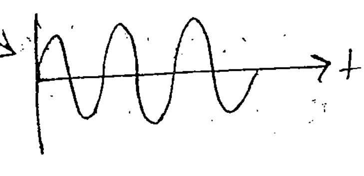
$$= je^{j10t} \cdot e^{j2\pi} = je^{j10t}$$

* Extra phase angle will not change the periodicity of the signal.

Q) $x(t) = \sin\left(\frac{\pi t}{4}\right)u(t)$.

Non periodic → because the signal starts from '0'.

* All periodic s/ls are everlasting s/ls. They should extend from $-\alpha$ to $+\alpha$.



switched sine form

semi periodic

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Q) $x(t) = \text{Even}\{\cos 3\pi t\, u(t)\}$

$x(t) = x_e(t) + x_o(t)$

$x_e(t) = \frac{x(t)+x(-t)}{2}$

$x_o(t) = \frac{x(t)-x(-t)}{2}$

$$= \frac{\cos 3\pi t\, u(t) + \cos(-3\pi t) u(-t)}{2}$$

$$= \frac{\cos 3\pi t\,[u(t)+u(-t)]}{2} = \underline{(\cos 3\pi t)/2} \quad \text{(periodic)}$$

$$\omega_0 = \frac{3\pi}{1} = \frac{2\pi}{T} = \quad T = \frac{2}{3}$$

Conclusion.

* Three forms of periodicity $\left\{ \begin{array}{l} \cos\omega_0 t \\ \sin\omega_0 t \\ e^{j\omega_0 t} \end{array} \right\}$ Periodic individually for all values of ω_0.

Multiple s/ls → GCD / LCM concept.

Q) $x[n] = \sin\left[\frac{3\pi n}{5}\right]$

Note: Continous sinusoids & Complex exponentials are periodic for any value of ω_0, whereas the equivalent descrete terms are periodic if $\frac{\omega_0}{2\pi}$ is a rational number (It must be ratio of 2 integers)

$$\boxed{\frac{\omega_0}{2\pi} = \frac{m}{N}}$$

$$\frac{3\pi/5}{2\pi} = \frac{m}{N}$$

$$\frac{3}{10} = \frac{m}{N}$$

$$\Rightarrow N = \frac{10m}{3};$$

m = how many integer cycles the signal is repeating.

$x[n] = x[n+N]$

$x[n] = \sin\omega_0 n.$

$x[n+N] = \sin\omega_0(N+n)$

$= \sin n\omega_0 \underbrace{\cos\omega_0 N}_{1}$

$+ \cos n\omega_0 \underbrace{\sin\omega_0 N}_{0}$

$\cos\omega_0 N = 1 = \cos 2\pi n$

$\omega_0 N = 2\pi n$

$\frac{\omega_0}{2\pi} = \frac{n}{N}.$

descrete ⟷ Periodic

in one domain — in another domain.

Continous ⟷ Non periodic

in one domain. — in another domain.

Q) * $x[n] = \cos\left[\frac{n}{6} - \frac{\pi}{4}\right]$

$= \cos[\omega_0 n + \theta]$

$\frac{\omega_0}{2\pi} = \frac{1/6}{2\pi} = \frac{1}{12\pi} = \frac{m}{N}$ (irrational)

Non periodic

Q). $x[n] = e^{j5\pi n}$

$\frac{5\pi}{2\pi} = \frac{m}{N} = \frac{5}{2} \Rightarrow N = \frac{2m}{5}$ (It will be integer for multiples of 5).

ie $N = 2$

Q) $x(n) = (j)^{n/2} = \left(e^{j\pi/2}\right)^{n/2}$

$= e^{jn\pi/4}$

$\omega_0 = \pi/4 \Rightarrow \frac{\omega_0}{2\pi} = \frac{\pi/4}{2\pi} = \frac{1}{8}$ (periodic)

$N = 8$

Q) $x[n] = \sin\left[\frac{\pi n}{3}\right] + \cos\left[\frac{\pi n}{4}\right]$

$\downarrow$ $\qquad\qquad$ $\downarrow$

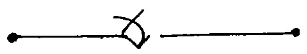
$N_1 = 6/1$ $\qquad$ $N_2 = 8$

$N = \text{LCM}[6, 8]$

$= 24$

Q. A signal $x(t) = 2\cos(150\pi t + 45°)$ is sampled at 200 Hz. Find the time period of discrete signal?

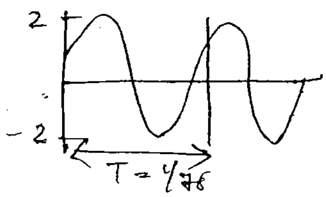
$f_s = 200\text{Hz}$ $\qquad$ $x[n] = 2\cos\left(\frac{150\pi n}{200} + 45°\right)$



$\omega_0 = \frac{150\pi}{200} = \frac{3}{4}\pi$

$t = nT_s = \frac{n}{200\text{Hz}}$.

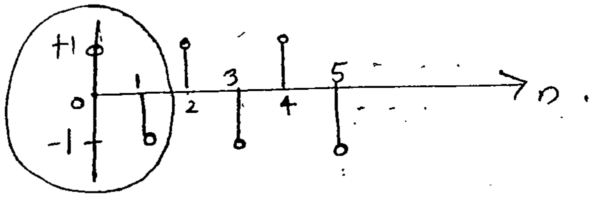
$\frac{\omega_0}{2\pi} = \frac{3}{8} = \frac{m}{N}$



$N = 8$ samples

Q) $x[n] = (-1)^{n^2}$

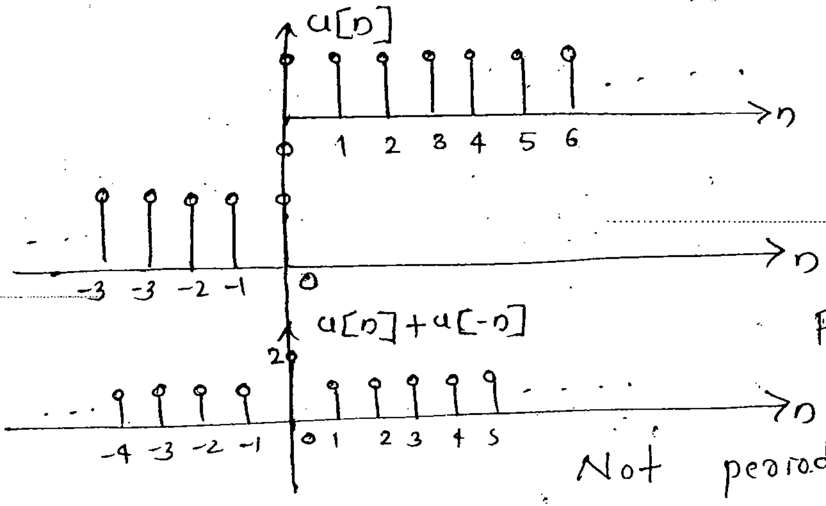
draw the shape of sig. if it is not given in std form.



(13)

$N = 2$

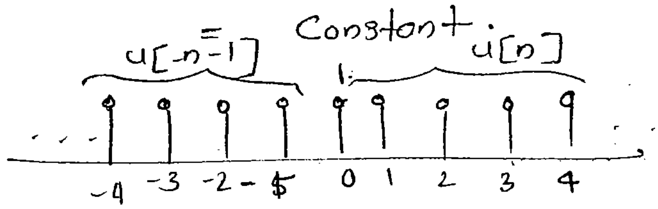
Q) $x[n] = u[n] + u[-n]$



Periodicity is gone because of the value at $n=0$.

Not periodic

Q) $x[n] = u[n] + u[-n-1]$ is periodic

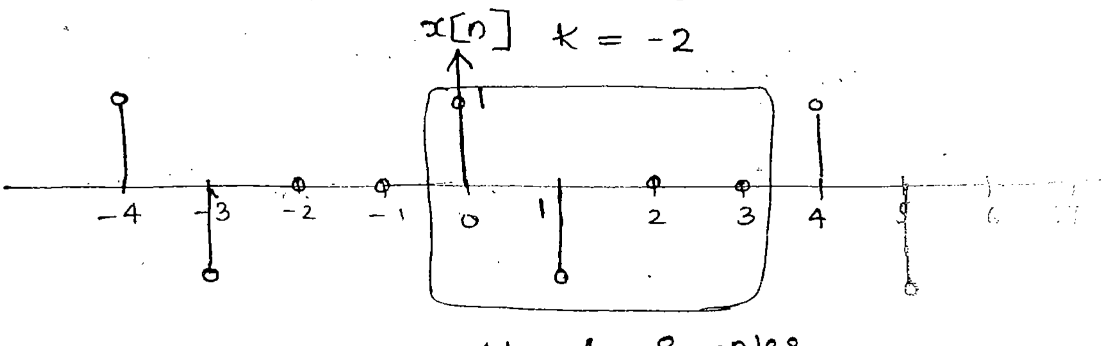


Q) $x[n] = \sum_{k=-\alpha}^{+\alpha} \delta[n-4k] - \delta[n-1-4k]$

$k=-1$ $\Rightarrow \underbrace{\delta[n+4] - \delta[n+3]}_{k=-1} + \underbrace{\delta[n] - \delta[n-1]}_{k=0}$

$+ \delta[n-4] - \delta[n-5] + \cdots$

$k=-2$



$N = 4$ Samples.

Conclusion

Three basic forms.

$\left.\begin{array}{l}\cos\omega_0 n \\ \sin\omega_0 n \\ e^{j\omega_0 n}\end{array}\right\}$ use the equation. $\dfrac{\omega_0}{2\pi} = \dfrac{m}{N}$

(4) Causal & Non causal Signal

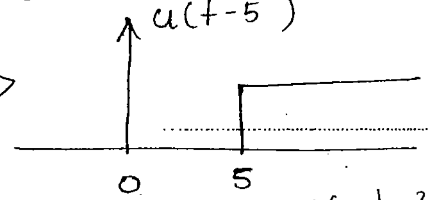
If any signal is defined only for +ve time then the signal is causal.

for $t \geq 0$, $n \geq 0$ .. $x[n] / x(t)$ is defined.

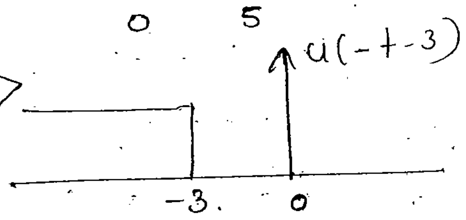
$x(t) = 0$; $t < 0$

$x[n] = 0$; $n < 0$.

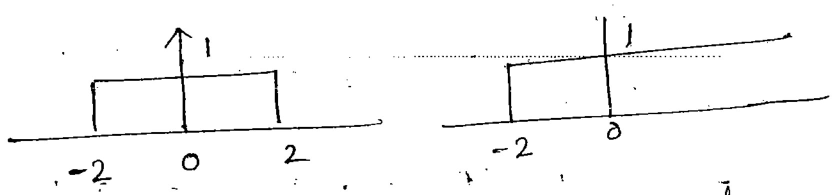
eg: $u(t-5) \Rightarrow$
causal.



eg2: $u(-t-3) \Rightarrow$
Anticausal
signal



two sided s/l $\longrightarrow$ Non-causal s/l.



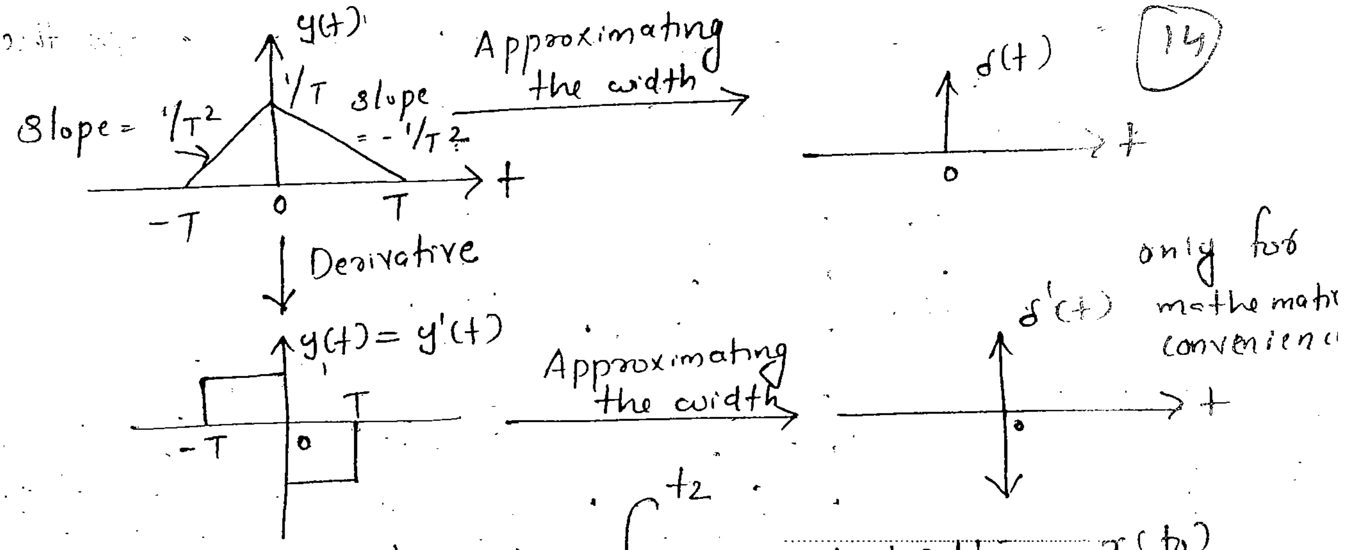
Periodic s/ls $\longrightarrow$ Non causal.

(5) Deterministic signals & Random s/ls.

Deterministic $\longrightarrow$ know the shape
eg: Voice.

Random $\longrightarrow$ Don't know the shape.
eg: Noise.

* Doublet $\longrightarrow$ Derivative of Continous Impulse.



Sampling ppty $\rightarrow$ $\displaystyle\int_{t_1}^{t_2} x(t)\,\delta(t-t_0)\,dt = x(t_0)$

$+ t_1 < t_0 < t_2.$

$$\boxed{\int_{t_1}^{t_2} x(t)\,\delta^{(n)}(t-t_0)\,dt = (-1)^n \frac{d^n x(t)}{dt^n}\Bigg|_{t=}}$$

$$\int_{-\infty}^{\infty} \delta'(t)\,dt = 0 \qquad \left[\begin{array}{ll} \delta(t) & = \text{Even} \\ \delta'(t) & = \text{odd} \\ \int \text{odd} & = 0. \end{array}\right]$$

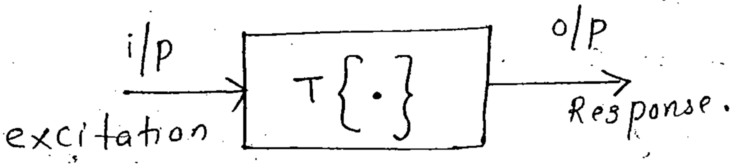
Q) $\displaystyle\int_0^{\infty} e^{-3t^2}\cdot\delta'(t-10)\,dt = (-1)^1 \frac{d\,e^{-3t^2}}{dt}\Bigg|_{t=10}$

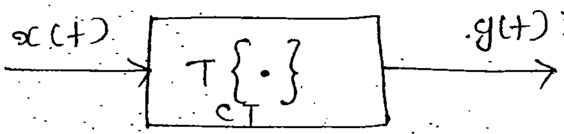
$= -e^{-3t^2} \times -3\times 2t\times 1 \Big|_{t=10}$

$= 6\times 10\times e^{-3\times 10^2} = \underline{60\times e^{-300}}$

System

system is an operator which maps the relation between i/p and o/p





$y(t) = T\{x(t)\}$. For continous-time signals.

Getting the o/p $\longrightarrow$ Analysis.
Designing the s/m $\longrightarrow$ system.

Representation:

① $y(t) = x^2(t)$; $y(t) = e^{-x(t)}$.

② Differential / Difference equation.

$$\frac{dy(t)}{dt} + 3x(t) = 2x(t)$$

$$y[n] + y[n-1] = x[n]$$

Transfer function $\rightarrow$ $H(s)$ | $H(z)$. (LTI s/m)

Impulse Response $\longrightarrow$ $h(t)$ | $h(n)$.

state-space.

Classification of systems.

① Linear & Non linear s/m.

superposition theorem.

scaled version of i/p = scaled version of o/p

ⓐ Additivity

$x_1(t) \xrightarrow[\text{s/m operation}]{T} y_1(t)$

$x_2(t) \longrightarrow y_1(t)$.

$$x_1(t) + x_2(t) \longrightarrow y_1(t) + y_2(t)$$

(b) Scaling / homogeneity.

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$$c\,x(t) \longrightarrow c\,y(t).$$

$$a x_1(t) + b x_2(t) \longrightarrow a y_1(t) + b y_2(t).$$

If both (a) & (b) are obeyed s/m is linear.

Q) $y(t) = x(t)\,x(t-4)$? unknown s/l product → Makes s/m non linear

$$x_1(t) \Longrightarrow x_1(t)\,x_1(t-4) = y_1(t)$$

$$x_2(t) \Longrightarrow x_2(t)\,x_2(t-4) = y_2(t);$$

$$y(t) = \{x_1(t) + x_2(t)\}\{x_1(t-4)\,x_2(t-4)\}$$

$$\neq y_1(t)\,y_2(t)$$ Non linear. Aditivity is not satisfied.

Q) $y(t) = x(t)\,\sin 6t$ → predefined s/l

linear s/m.

* "Unknown signal product makes the system Non linear"

Q) $y(t) = \sin\{x(t)\}$? Non linear

* Any trignometric function of i/p is non linear"

Q) $y(t) = \int_{-\alpha}^{t} x(t)\,dt$? Linear s/m.

Q) $y(t) = 3x(t) + 2$? Non linear.

* Addition of constant makes the s/m Non linear

Incrementally Linear s/m :-

Q) $y(t) = |x(t)|$? Non linear.

$$c\,x(t) \longrightarrow c\,y(t).$$

$$-3x(t) \longrightarrow |-3x(t)| = 3|x(t)| \neq -3|x(t)|$$

Q) $y[n] = e^{x[n]}$? ∴ Non linear

Non linear curve.

Q) $y[n] = \dfrac{x[n]}{x[n-1]}$? Non linear

$$\frac{a+b}{c+d} \neq \frac{a}{c} + \frac{b}{d}$$

* unknown signal division is non linear

05/08/2011
9 - 1 pm

Q) $y[n] = x^*[n]$

o/p due to $c\,x[n] \longrightarrow c\,y[n]$

$(1+j5) \longrightarrow (1+j5)\,x^*[n]$

$= \{(1+j5)\,x[n]\}^* = (1-j5)\,x^*[n]$

Non linear

Q) $y[n] = Re[x(n)] \longrightarrow$ Non linear

Real part, imag. part scaling is not obeyed.

Q) $y(t) =$ Sampling $\{x(t)\} \longrightarrow$ done with respect to time.

∴ linear.

Time variant operation.

Q) $y[n] = \text{median}\{x[n]\}$

i/p 1 ⇒ 1 2 3 4 5

i/p 2 ⇒ 1, 1, 2, 1, 2 ⇒ 1, 1, 1, 2, 2

i/p 3 ⇒ 1, 2, 3, 4, 5, 1, 1, 2, 1, 2

⇒ 1, 1, 1, 1, 2, 2, 2, 3, 4, 5

$\frac{2+2}{2} = 2$

Combined median is not equal to the addition of median

∴ non linear

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + \underbrace{2}_{N.L} = x(t)$$

Addition of Constant makes the s/m Non linear

Constant $\longrightarrow$ Initial condition.

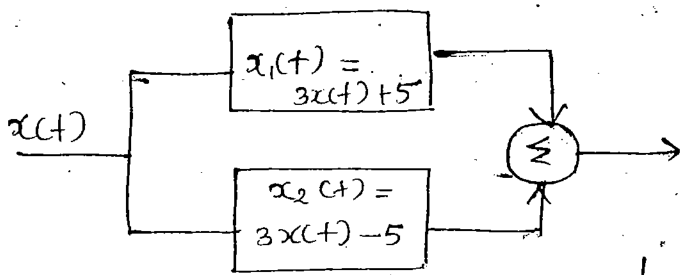
2 non linear s/ms are connected in series is the overall s/m will be linear?

$x(t) \rightarrow$ [exp] $\xrightarrow{e^{x(t)}}$ [ln] $\rightarrow y(t) = x(t) \Rightarrow$ linear

It depends on the s/m blocks.

$x(t) \rightarrow$ [exp] $\rightarrow$ $[\;]^2$ $\rightarrow$ NL

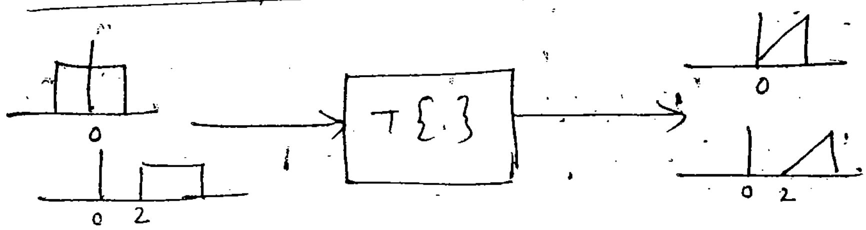
* When both s/ms are opposite to each other then the over s/m is linear



$y(t) = 6x(t)$
linear.

* Whether the s/m is linear / Non linear depends on the s/m operations.

(2) Time-invariant | shift invariant &
Time-variant | shift dependent.



If $x(t) \longrightarrow y(t)$

then $x(t-t_0) \xrightarrow{T} y(t-t_0)$ eg: Ohm's law eg: time - varying : human body.

checking time invariance. $\boxed{\left. y(t) \right|_{x(t-t_0)} \longrightarrow \left. y(t) \right|_{t=t-t_0}}$

Q) $y(t) = t\,x(t)$

Delay i/p $\Rightarrow$ $y_1(t) = t\,x(t-t_0)$ —— ①

Delay o/p $\Rightarrow$ $y(t-t_0) = (t-t_0)\,x(t-t_0)$ —— ②

① $\neq$ ② s/m : time variant

* if i/p or o/p is externally the function of time then it is a varying s/m.

Q) $y(t) = e^{-x(t)}$ time invariant.

Q) $y(t) = x^2(t)$ time invariant.

Q) * $y(t) = x(t)\cos\omega_c t \rightarrow$ ~~[illegible]~~ variant s/m. Modulation.

$y(t-t_0) = x(t-t_0)\cos\omega_c(t-t_0) \neq x(t-t_0)\cos\omega_c t$

Q) $y(t) = \frac{d\,x(t)}{dt} \rightarrow$ Time invariant.

i/p $\rightarrow \frac{d}{dt}x(t-t_0)$

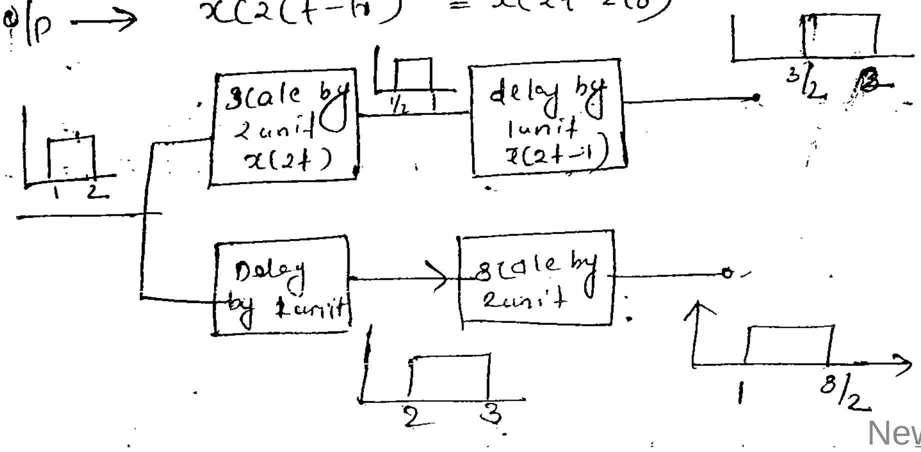
o/p $\rightarrow \frac{d}{dt}x(t-t_0)$

Q). $y(t) = x(2t)$

* Scaling operation is time varying.

i/p $\rightarrow x(2t-t_0)$.

o/p $\rightarrow x(2(t-t_0)) = x(2t-2t_0)$



Q) $x(t) = x(-t)$

* Time reversal is time - variant

In general : $\boxed{\begin{array}{l} x(\alpha t) = \text{Time variant} \\ \alpha \neq 1 \end{array}}$

Q) $y(t) = \begin{cases} x(t) ; & t > 0 \\ 0 ; & t < 0 . \end{cases}$

Variant s/m : o/p is externally function of time.

Q) $y[n] = g[n]\, x[n]$

Time variant s/m.

i/p : $g[n]\, x[n-n_0]$; o/p : $g[n-n_0]\, x[n-n_0]$

Q) $t \dfrac{dy(t)}{dt} + 2y(t) = x^2(t)$

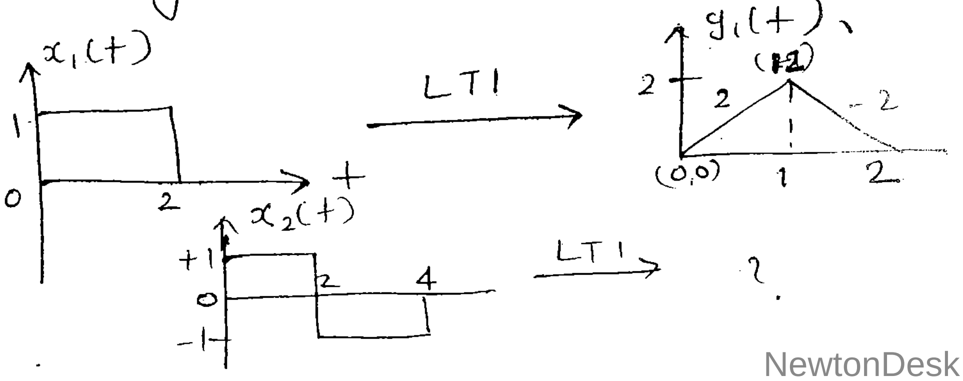
time variant, Non linear (square operation)
(function of time)

Q) $\left.\dfrac{dy(t)}{dt} + 4y(t) = 3x(t)\right\}$ LTI system.

Time invariant , linear

Note : In any differential equation, if all the coefficie are fixed with linear elements, that is LTI (Linear Time invariant) system.

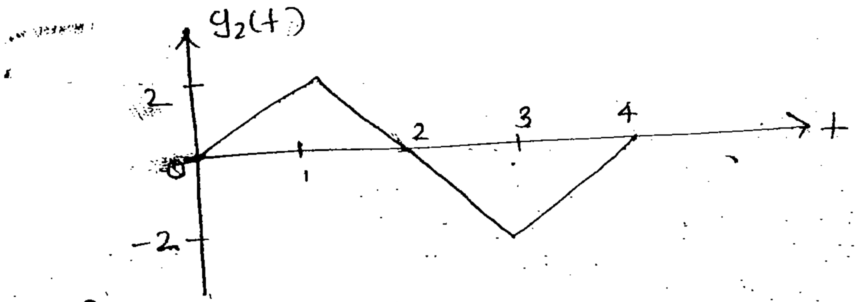
Q) An LTI s/m with i/p $x_1(t)$ & o/p $y_1(t)$ are shown in fig. Find the o/p due to following i/ps.



$$x_2(t) = x_1(t) - x_1(t-2)$$

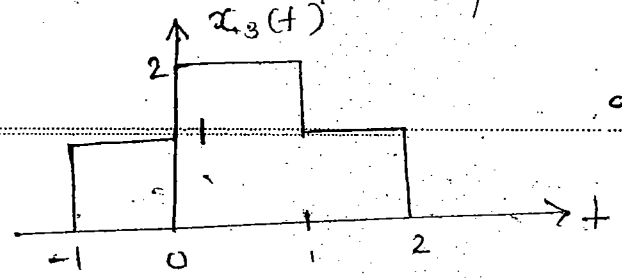
$$\Big\downarrow \text{LTI}$$

$$y_2(t) = y_1(t) - y_1(t-2)$$



If the s/m is linear whatever happens to i/p, it will happen to o/p & we can decide the o/p even though we don't know the s/m operation.

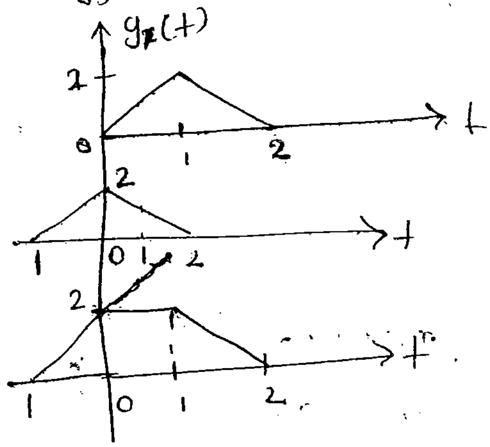
Q)



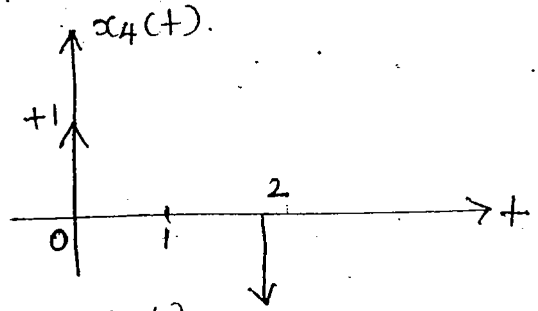
don't use time scalling

$$x_3(t) = x(t) + x(t+1)$$

$$y_3(t) = y(t) + y(t+1)$$



Q

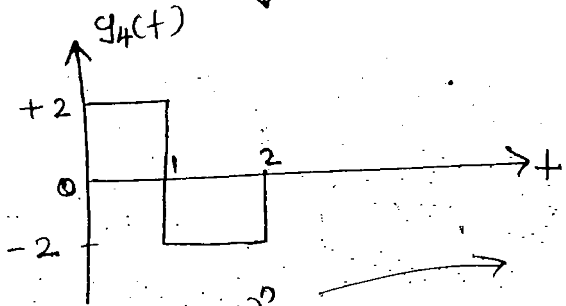


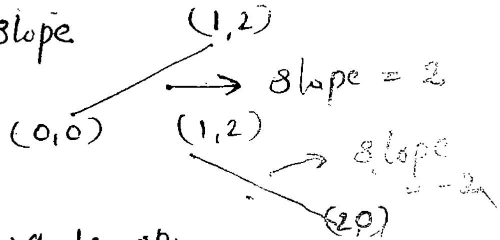
$$x_4(t) = \frac{d}{dt} x_1(t)$$

$$y_4(t) = \frac{d}{dt} y_1(t).$$

(18)

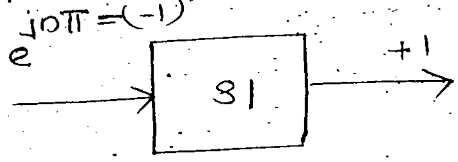
to differentiate find the slope

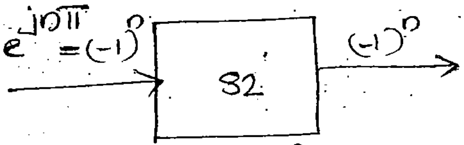




square | Mode op both are non linear
∴ Not an LTI s/m.

Q)





i/p = o/p → LTI s/m.

3. Causal & Non-causal s/m.

Causality → Present o/p should not depend on future o/p.

* A s/m is causal if the present o/p depends on present i/p, & past values of the i/p but not on future values. ie Causal s/ms ar non-anticipative.

Q) $y(t) = (3t+2)\,x(t)$ Causal.

relation b/w i/p time & o/p time, not external time

Q) $y(t) = |x(t)|$ Causal.

Q) $y(t) = \sin\{x(t)\}$ Causal.

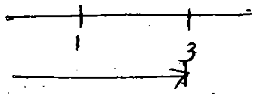
Q) $y(t) = x\{\sin t\}$ Non Causal.

$y(-\pi) = x(0)$

Q) $y(t) = e^{-3(t+1)} x(t)$ Causal.

Q) $y(t) = \int_{-\infty}^{3t} x(\tau)\, d\tau$. Non Causal.

$y(1) = \int_{-\infty}^{3} x(\tau)\, d(\tau)$



Q) $y(n) = \sum_{k=n_0}^{n} x(k)$; n_0 is finite

It is Causal depends on the value of n or n_0.

Case 1 $n_0 > n$

$3 > 2$

$y(2) = \sum_{k=3}^{2} x(k)$. Non causal.

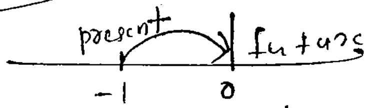
Case 2. $n_0 \leq n$.

$y(n)$ is causal.

The s/m is Conditionally ~~~~ causal

Q) $y[n] = \sum_{k=0}^{n} x[k]$ Non causal.

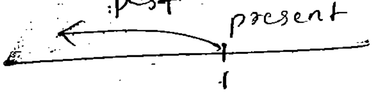
$y(-1) = \sum_{k=0}^{-1} x[k] = x(0) + x(-1)$



Accumulator = Addition.

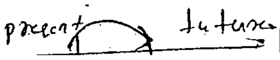
Q) $y[n] = \sum_{k=-\infty}^{n} x[k]$ Causal s/m

$y(1) = \sum_{k=-\infty}^{1} x(k) = x(1) + x(0) + x(-1) + \cdots$



Q) $y(n) = \begin{cases} x[n] ; & n < 0 \\ 0 ; & n = 0 \\ x(n+1) ; & n > 0 \end{cases}$ Non Causal

$y(1) = x(2)$



Q) $y[n] = 2x[n] + 3u[n+1]$ Causal s/m

(19)

Q) $\boxed{y[n] = \frac{1}{2m+1} \sum_{k=-m}^{m} x[n-k]}$ Non Causal

∴ summation covers both +ve & -ve values of m.

$y[1] = \frac{1}{3}\left[x[n+1] + x[0] + x[n-1]\right]$

→ Moving Average s/m (Low pass filter).

Note: Non Causal s/ms are physically cannot be designed when the independent Vble is time. (~~s~~ 't', 'n')

Q) $\dot{y}(t+4) + 2y(t) = x(t+2)$ Causal.

$\tau = t+4$

$\dot{y}(\tau) + 2(\tau-4) = x(\tau-2)$

w.r.to $t+4$, t & $t+2$ are past.

4. Static (Memoryless / instantaneous) & Dynamic (With memory)

Q) $y(t) = (t+1)^2 x(t)$ static

Q) $y(t) = e^{-4t} x(t)$ static

Q) $y(t) = x(-t)$ dynamic

Q) $y(t) = \frac{d\,x(t)}{dt} \longrightarrow$ Dynamic

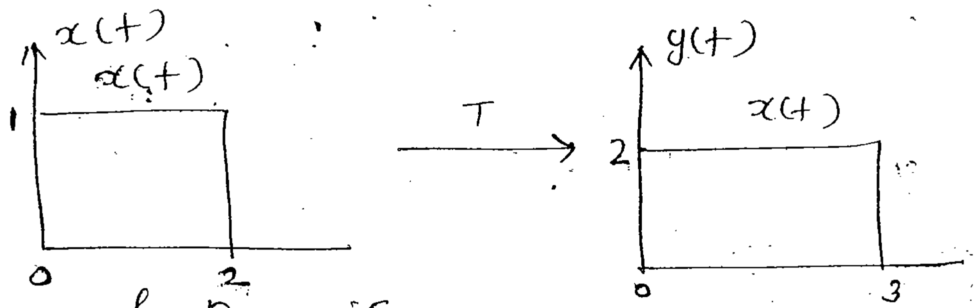
Rate of change → two instant.

diff. term ⟶ Energy storing element (s/m be dynamic ...)

Q) $y[n] = g[n+1]\,x[n]$ static

* All static s/ms are Causal but Not vice versa"

eg:



Causal & Dynamic

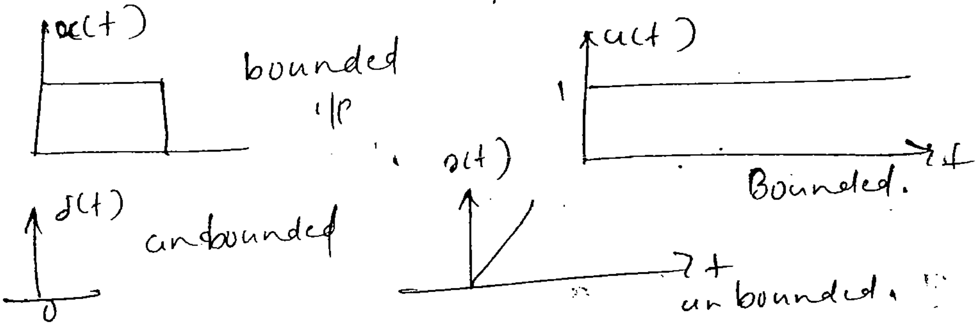
<u>Causality</u> → Before application of i/p o/p doesn't start.

Dynamic → The width of i/p & o/p are not same(May due to <u>scalling</u> & shifting)

5. <u>stable & unstable s/m</u>

A small i/p doesn't cause large change in o/p.

Boundedness → We can't take more then the maximum Amplitude/Value.



$$\boxed{\begin{array}{l} \text{if } |x(t)| \le m_x < \alpha \\ \text{then } |y(t)| \le m_y \le \alpha \end{array}}$$

(Q) $y(t) = x^2(t)$ <u>stable.</u>

$x(t) = u(t)$; Give bounded i/p

$y(t) = u^2(t) = u(t)$ o/p bounded

Alternate Method.

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$|y(t)| = |x(t)|^2 = |\text{finite}|^2 = \frac{\text{finite}}{\text{stable.}}$

(Q) $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$ unstable.

if $x(t) = u(t) \Longrightarrow \int u(t) = r(t)$.

$y(t)$ = ramp = $\underline{\text{unstable}}$

(Q) $y(t) = x(3t)$ stable.

Compl expand doesn't affect stability

Amp: is constant.

(Q) $y(t) = x(t-5)$ stable.

Time shifting & time scalling are stable operations.

(Q) $y(t) = \frac{d x(t)}{dt}$ unstable.

If $x(t) = u(t)$; $y(t) = \delta(t)$.

If the order of i/p derivative is more than the o/p derivative then the s/m is unstable.

(Q) $y[n] = e^{x[n]}$ stable

$|y[n]| = e^{|x[n]|} = e^{|\text{finite}|} = \text{finite} \therefore$ stab

(Q) $y[n] = x[n]\, x[n-3]$ Stable.

$|y[n]| = |x[n]|\, |x[n-3]|$

$= |\text{finite}|\, |\text{finite}| = \underline{\underline{\text{finite}}}$

$\boxed{Q}$ $y[n] = \sum_{k=n_0}^{n} x[k]$ n_0 is finite.

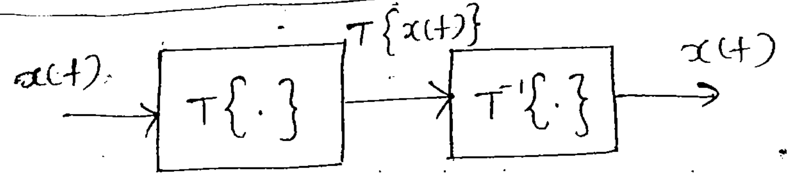
unstable s/m.

$n_0 = 1$ $x(k) = 1$

$y[n] = \sum_{1}^{n} (1) =$

6. Invertable & Inverse System.

A s/m is invertable if different i/p leads to diff. o/ps. ie two diff. i/p for a given s/m should not produce the same output.



$$T \cdot T^{-1} = I$$

eg 1: $y(t) = x^2(t)$.

if $x(t) = u(t) \Rightarrow y(t) = u(t)$.

if $x(t) = -u(t) \Rightarrow y(t) = u(t)$

Non- invertability.

eg 2: $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

if $x(t) = \delta(t) \Rightarrow y(t) = \int_{-\infty}^{t} \delta(t)\, dt = u(t)$.

$x(t) = -\delta(t) \Rightarrow y(t) = -u(t)$

$x(t) = u(t) \Rightarrow y(t) = r(t)$

$\therefore$ Invertable.

Inverse operation $\frac{dy(t)}{dt}$.

eg 3: $y(t) = \frac{d}{dt} x(t)$. Non invertable.

* Diff. of any Constant is zero.

$\frac{d}{dt} \xrightarrow{\text{Inverse}} \int_{0}^{t}$ or $\int_{-\infty}^{t}$ not unique solution

eg 4: $y(t) = x(t-4)$ delayed by 4 units

$\delta(t) \longrightarrow \delta(t-4)$

$-\delta(t) \longrightarrow -\delta(t-4)$

$u(t) \longrightarrow u(t-4)$

$t\, u(t) \longrightarrow (t-4)\, u(t-4)$

Invertable.

Inverse $\longrightarrow y(t+4)$ advanced by 4 units eg 5: $y[n] = n\,x[n]$

$\delta[n] \longrightarrow n\,\delta[n] = 0$ $\left[\delta(0) = 1 \text{ at } n=0, \quad 0 \times 1 = 0 \right]$

$-\delta[n] \longrightarrow -n\,\delta[n] = 0$

$\therefore$ Non invertable.

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eg 6: $y[n] = x[n]\,x[n-3]$

$\left.\begin{array}{l} x[n] \\ -x[n] \end{array}\right\rangle +ve$ $\therefore$ Non-invertable.

Gate eg 7: $y[n] = \sum_{k=-\infty}^{n} x[k]$ Invertable.

If $x[n] = \delta(n) \Rightarrow \sum_{k=-\infty}^{n} \delta(k)$

$-\delta[n] \Rightarrow -u[n]$

$\Rightarrow \sum_{k=0}^{n} (1) = (n+1)\,u[n]$

Invertable.

* Inverse $y[n] - y[n-1]$ Non invertable.

$= \sum_{k=-\infty}^{n} x(k) - \sum_{k=-\infty}^{n-1} x(k)$

$= x[n] + \sum_{k=-\infty}^{n-1} - \sum_{k=-\infty}^{n-1} x(k))$

$= x(n)$

* Integration & Summation are invertable. But differentiation & difference are non invertible

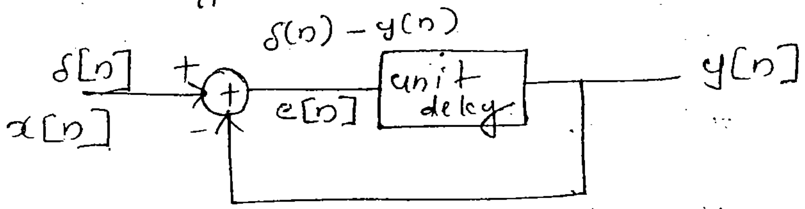
eg 8: $y(n) = x[n] \sin\left(\frac{\pi n}{3}\right)$

$\delta(n) \Rightarrow \delta[n] \sin\left(\frac{\pi n}{3}\right) = 0$.

$-\delta(n) \Rightarrow 0$. Non invertible.

Only two steps are required 1. Calculate impulse respo

Q) For the feed back s/m shown in fig: assume $y[n] = 0$; $n < 0$. Find $y[n]$ when the i/p is $x[n] = \delta[n]$?



$$\cancel{y[n]} = \cancel{0}$$

$$y[n] = e[n-1]$$

$$e[n] = x[n] - y[n]$$

$$y[n] = x[n-1] - y[n-1]$$

$n=1$ $\quad y(1) = x(0) - y(0) = 1 - 0 = 1$

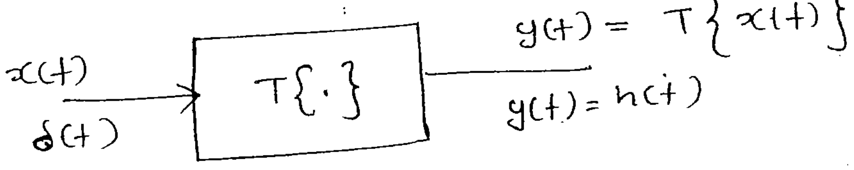
$n=2$ $\quad y(2) = x(1) - y(1) = 0 - 1 = \underline{\underline{-1}}$

$n=3$ $\quad y(3) = x(2) - y(2) = 0 - (-1) = \underline{\underline{1}}$

$n=4$ $\quad y(4) = x(3) - y(3) = 0 - 1 = \underline{\underline{-1}}$

II LTI or LSI SYSTEM

Impulse Response ($h[n]$ | $h(t)$)



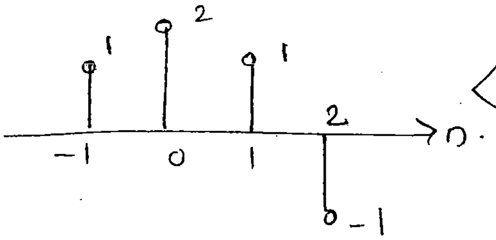
i/p		o/p	
$\delta[n]$	$\longrightarrow$	$h[n]$	
$\delta[n-k]$	$\longrightarrow$	$h[n-k]$	TI
$x(k)\,\delta[n-k]$	$\longrightarrow$	$x(k)\,h(n-k)$	scaling } linearity
$\sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$	$\longrightarrow$	$\sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$	additivity }

1. An LTI system is always considered with respect to impulse response. (if i/p is impulse output is impulse response).

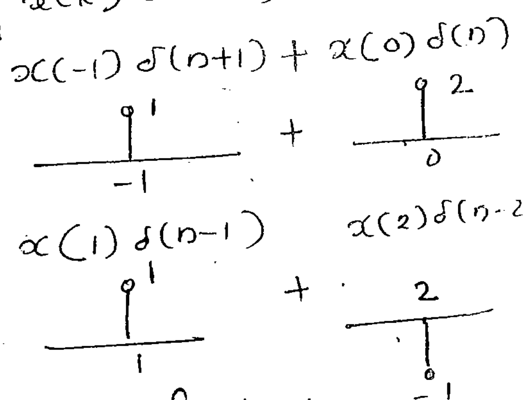
2. Sifting ppty states that any signal is produced as combination of impulses.

$$\sum_{k=-1}^{2} x(k)\,\delta(n-k)$$

eg:



$\Longleftrightarrow$



3. By using convolution we are finding the zero state response (all initial conditions are zero.) for a given i/p & s/m.

Continous time formula for convolution.

$$y(t) = x(t) * h(t)$$

$$= \int_{\tau=-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

folding ①

shifting ②

Multiplication ③

Integration ④

$$= \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau$$

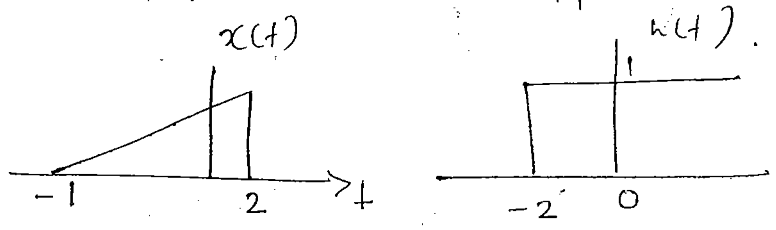
Convolution (infinite length of data) = Multiplication (finite length of data)

Steps in Convolution.

1. Obtain the limits of $y(t)$

Sum of lower limits $< t <$ Sum of upper limits.

eg:



$$-3 < t < \alpha$$
$$(-1+-2) \qquad (2+\alpha)$$

2. Change of axis from t to τ

$x(t) \longrightarrow x(\tau)$

$h(t) \longrightarrow h(\tau)$

3. Folding (Flipping)

$x(-\tau) \mid h(-\tau)$

• Shifting

$x(t-\tau) \mid h(t-\tau)$

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$h(t) = \{1; t>0$

$h(t-\tau) = \{1; t-\tau>2$
$t-2 > \tau$
$\tau < t-2$

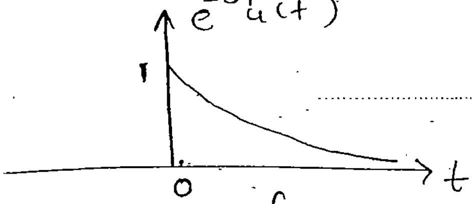
4. Multiplication:

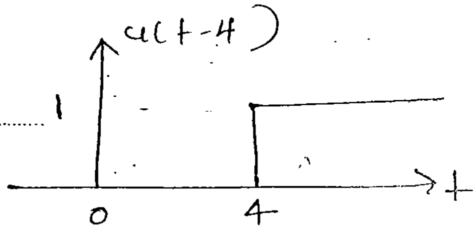
$x(\tau)\, h(t-\tau)$ $\quad$ $x(t-\tau)\, h(\tau)$

5. Performing integration.

* same for anticausal also

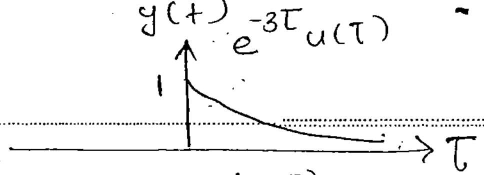
Q) Find the convolution of $x(t) = e^{-3t}u(t)$ & $h(t) = u(t-4)$?

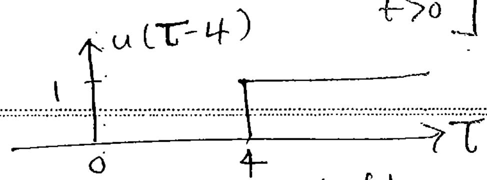




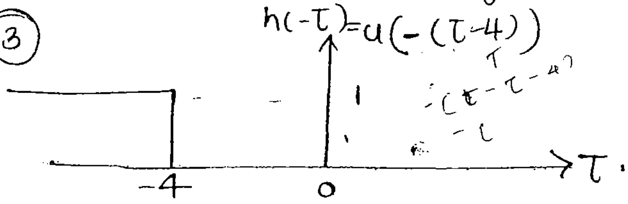
① limits of $y(t)$: $4 \leq t \leq \alpha$ [o/p remains for $t>0$].

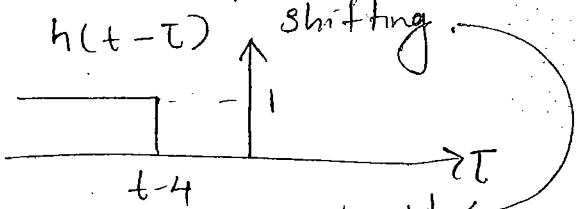
②





③



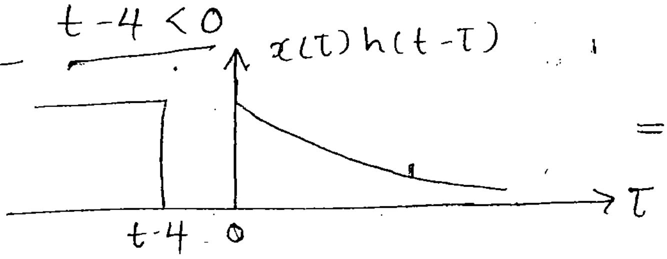


just add t to limits.

④

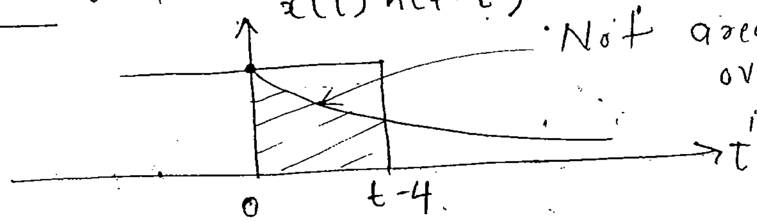
Convolution of a causal signals give causal signal only.

Case 1: $t-4<0$



$= g(t) = 0;\ t-4<0$
$t<4$

Case 2: $t-4>0$



Not area. It is overlap over which integrant is non zero.

$$y(t) = \int_0^{t-4} \underbrace{x(\tau)\,h(t-\tau)}_{\text{Integrent is not zero.}}\,d\tau = \int_0^{t-4} (1)\cdot e^{-3\tau}\,d\tau = \left[\frac{e^{-3\tau}}{-3}\right]_0^{t-4}$$

$$= -\frac{1}{3}\left[e^{-3(t-4)} - 1\right] = \frac{1}{3}\left[1 - e^{-3(t-4)}\right];\ t>4$$

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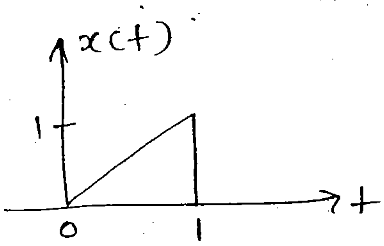
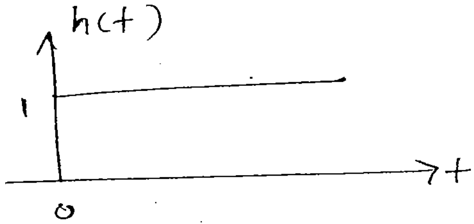
06/08/2011
9-1pm

Q)

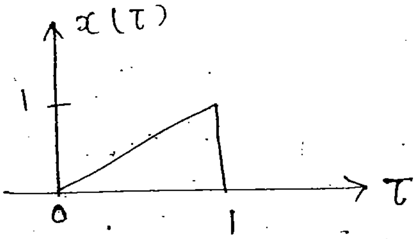


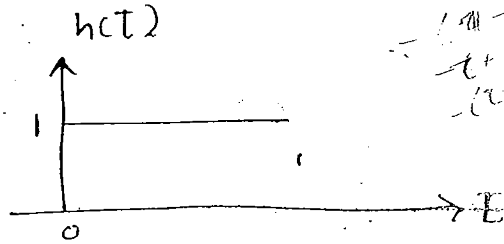


$$x(t) = \begin{cases} t & ; 0 < t < 1 \\ 0 & ; \text{otherwise.} \end{cases} \qquad h(t) = \{ 1 \; ; t > 0$$

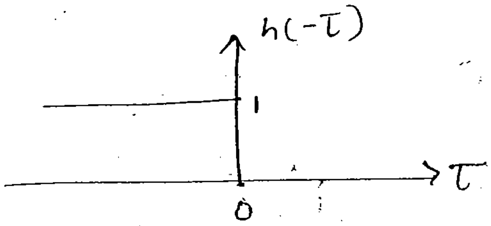
(i) limits of o/p : $0 < t < \alpha$

(ii)

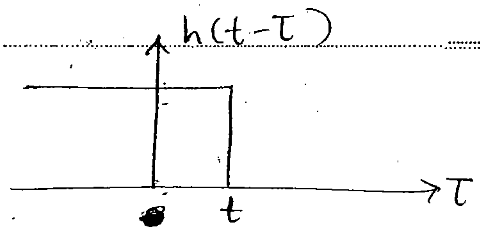




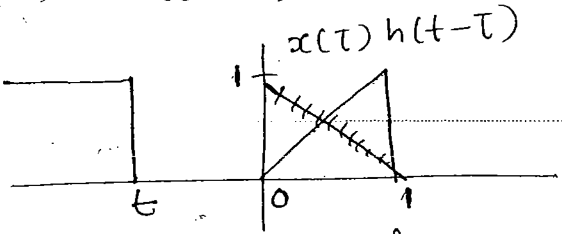
(iii) Folding



(iii) shifting

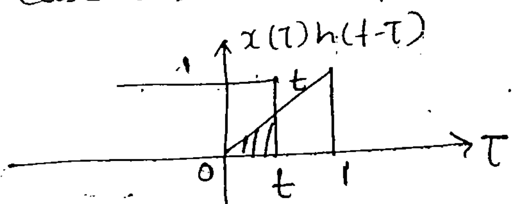


(iv) Case 1 ; $t < 0$.



$y(t) = 0 ; t < 0$.

Case 2 : $t > 0$ & $t < 1$ $\quad 0 < t < 1$

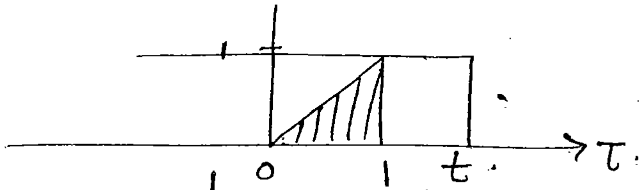


$$y(t) = \int_0^t x(\tau)\, h(t-\tau)\, d\tau = \int_0^t \tau \cdot 1 \, d\tau = \left[\frac{\tau^2}{2}\right]_0^t$$

$$= \frac{t^2}{2} \; ; \quad 0 < t < 1$$

Case 3 : $t > 1$

(24)

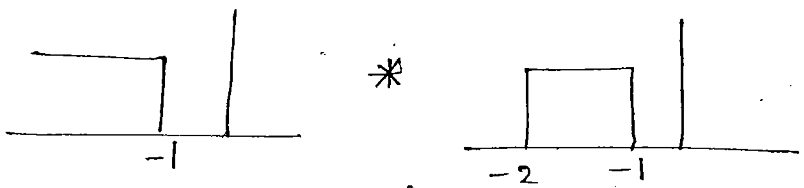


$$y(t) = \int_{-\infty}^{1} x(\tau)\, h(t-\tau)\, d\tau = \int_{0}^{1} \tau \cdot 1 \, d\tau$$

$$= \left[\frac{\tau^2}{2}\right]_0^1 = \frac{1}{2} \; ; \quad t > 1.$$

$$y(t) = \begin{cases} 0 \; ; & t < 0 \\ \frac{t^2}{2} \; ; & 0 < t < 1 \\ \frac{1}{2} \; ; & t > 1 \end{cases}$$

* Convolution of two causal signals resulted in causal o/p.



P2.1.3
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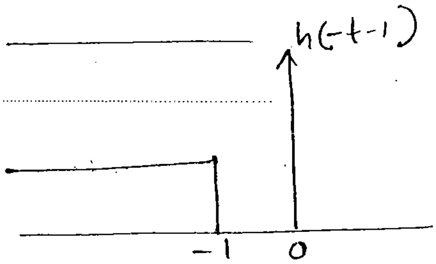
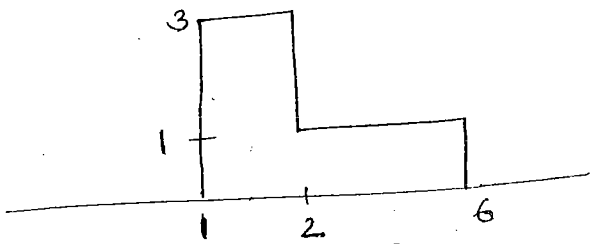
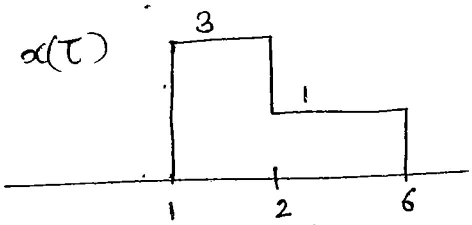
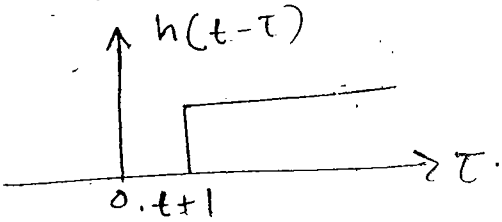


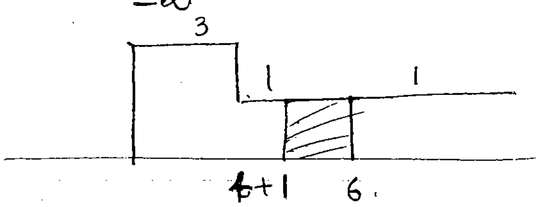
$x(t)$.







$$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau$$

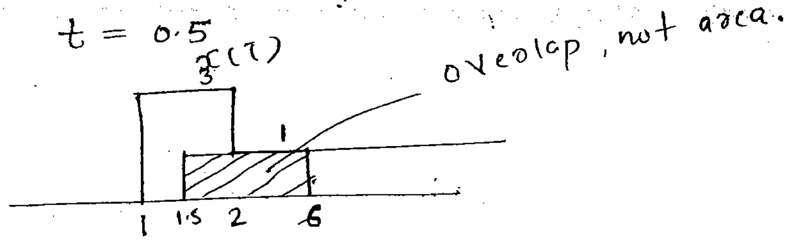


$$y(4) = \int_5^6 x(\tau)\, h(4-\tau)\, d\tau$$

$$= \int_5^6 1 \cdot 1 \, d\tau$$

$$= 1$$

at $t = 0.5$



$$y(0.5) = \int_{-\infty}^{\infty} x(\tau) \cdot h(0.5-\tau)\, d\tau$$

$$y(0.5) = \int_{1.5}^{2} 3 \cdot 1 \, d\tau + \int_{2}^{6} 1 \cdot 1 \, d\tau$$

$$= 3[2-1.5] + 1[6-2]$$

$$= 3 \times 0.5 + 4 = \underline{5.5}$$

overlapp concept.

P2.1.4

$$z(t) = \int_{-\infty}^{\infty} x(-\tau+a) \cdot h(t+\tau)\, d\tau$$

Express $z(t)$ in terms of $y(t) = x(t) * h(t)$

Ans:

$$z(t) = \int_{-\infty}^{\infty} x(-\tau+a)\, h(t+\tau)\, d\tau$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

put $t+\tau = \lambda$ $\quad \tau = \lambda - t$.

$$z(t) = \int_{-\infty}^{\infty} x(a-\lambda+t)\, h(t+\lambda-t)\, d\lambda$$

$$y(t+a) = \int_{-\infty}^{\infty} h(\lambda)\, x((t+a)-\lambda)\, d\lambda$$

$$= \underline{y(t+a)}$$

Convolution ppty of Continious impulse

$$x(t) * \delta(t-t_0) = x(t-t_0)$$

eg: $x(t+5) * \delta(t-9) = x(t-9+5)$

$$= \underline{x(t-4)}$$

eg 2: $x(t) * \delta(2t+1) = x(t) * \delta(2(t+\frac{1}{2}))$

$= x(t) * \frac{1}{2}\delta(t+1/2)$

$= \frac{1}{2} x(t+1/2)$

25

by convolution

$x(t) * \delta(t-t_0)$

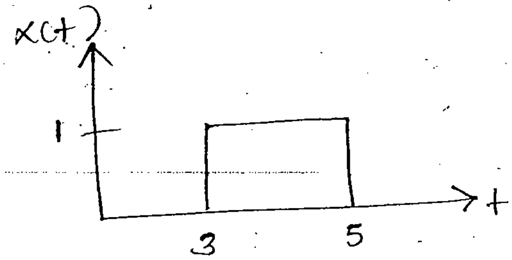
$= x(t-t_0)$

P2.1.6 : (a) Product ppty

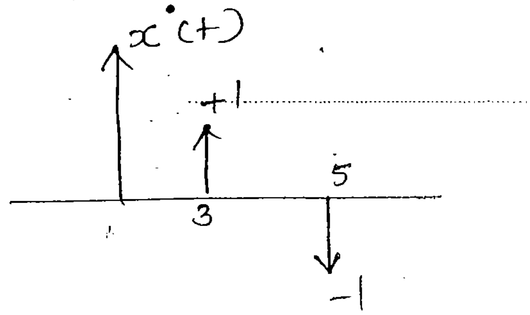
(b) Sampling ppty

(c) Convolution ppty.

P2.1.7



$\Longrightarrow$



$\frac{d x(t)}{dt} * h(t) \Longrightarrow [\delta(t-3) - \delta(t-5)] * e^{-3t}u(t)$

Same relation valid for i/p o/p of LTI s/m $\Longrightarrow e^{-3(t-3)}u(t-3) - e^{-3(t-5)}u(t-5)$

if $y(t) = x(t) * h(t)$

then $\frac{d\, x(t)}{dt} * h(t) = \frac{d\, y(t)}{dt}$ (since it is LTI s/m)

$$x^{(m)}(t) * h^{(n)}(t) = y^{(m+n)}(t).$$

m & n, - order of differentiation.

$$x(t-\alpha) * h(t-\beta) = y(t-\alpha-\beta)$$

$x(t) \longrightarrow A_x$

$h(t) \longrightarrow A_h$

$y(t) \longrightarrow A_y = ?\ A_x \cdot A_h$ (Multiplication)

$y(t) = \int x(\tau) h(t-\tau)\, d\tau$

$\int y(t)dt = \iint x(\tau) h(t-\tau)\, d\tau\, dt$

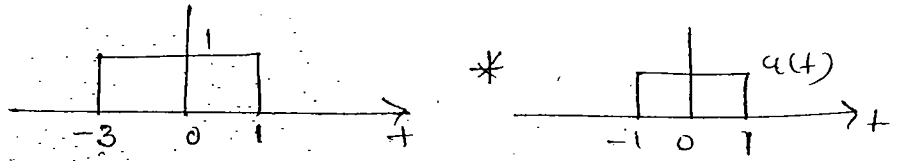
A_y $\int x(\tau)d\tau \int h(t-\tau)dt = A_x \cdot A_h$

$$\boxed{x(at) * h(at) = \frac{1}{|a|}\, y(at)}$$

$$\boxed{y(-t) = x(-t) * h(-t)}$$

P.2.1.9

$$[u(t+3) - u(t-1)] * [u(t+1) - u(t-1)]$$



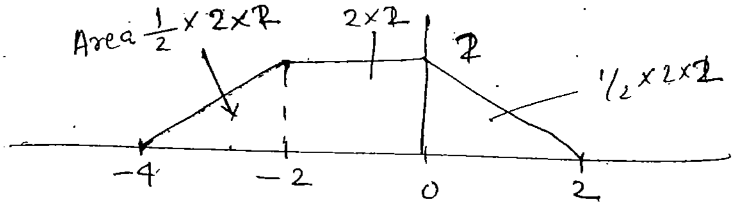
Applying shifting ppty of convolution

$$u(t+3) * u(t+1) - u(t+3) * u(t-1) - u(t-1) * u(t+1) + u(t-1) * u(t-1)$$

4+2

$$* \; [u(t) * u(t) = r(t)]$$

$$\Rightarrow \; r(t+4) - r(t+2) - r(t) + r(t-2)$$



Convolution of two unequal duration rectangular functions is a trapezium if the duration is same, then it is a triangle.

2.1(A) :

$$x_3(t) = x_1(t) * x_2(t)$$

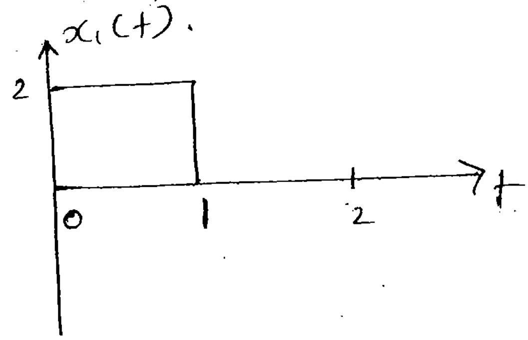
$$\frac{d\,x_3(t)}{dt} = \frac{d\,x_1(t)}{dt} * x_2(t)$$

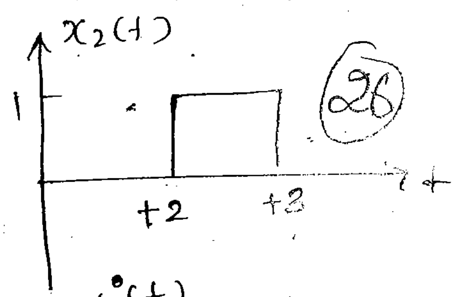
$$\text{or} \quad x_1(t) * \frac{d\,x_2(t)}{dt}$$

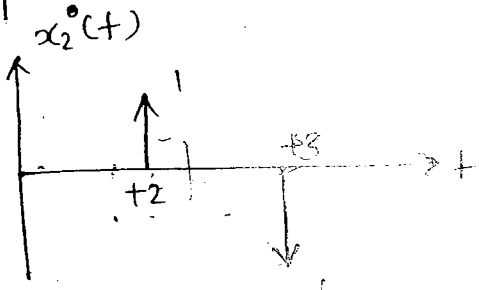
$$x_3(t) = \int_{-\alpha}^{t} x_1(t) * \frac{d\,x_2(t)}{dt}\, dt$$

(26)

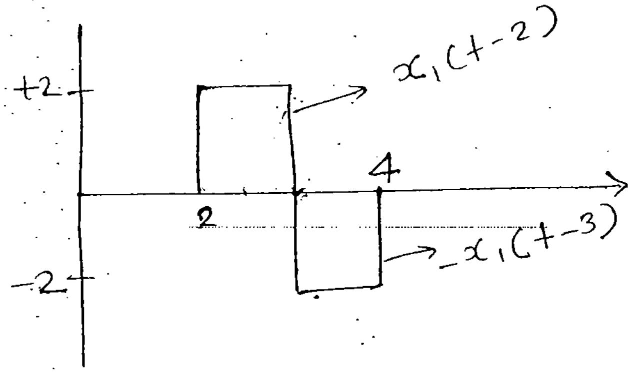
eg:



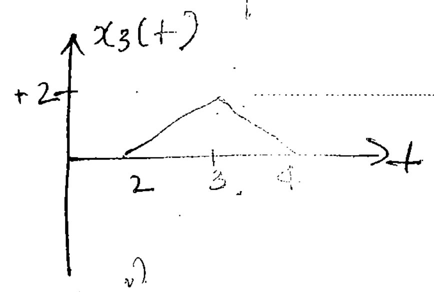




$x_1(t) * x_2'(t) = x_3'(t)$



$\Rightarrow$



Case (i) $2 < t < 3$

$$x_3(t) = \int_2^t 2\,dt = 2(t-2) \; ; \quad 2 < t < 3$$

case (ii) $3 < t < 4$

$$x_3(t) = 2 + \int_3^t -2\,dt = -2t + 8 \; ; \quad 3 < t < 4$$

P 2.1.10

Ans: (c)

Assume $f(t) = u(t+1) * r(t-2)$

Let $g(t) = u(t) * r(t)$

$$\frac{dg(t)}{dt} = \frac{du(t)}{dt} * r(t)$$

$$= r(t)$$

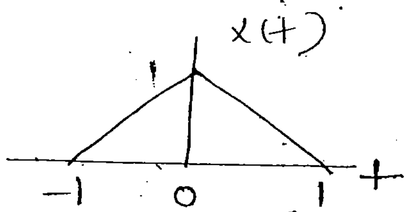
$$= t\,u(t).$$

$$g(t) = \int_0^t t\,u(t)\,dt = \frac{t^2}{2} \; ; t > 0.$$

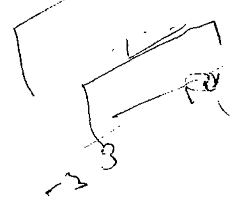
$$f(t) = g(t-2+1) = g(t-1) = \frac{(t-1)^2}{2} \; ; \; t-1 > 0$$

$t > 1$

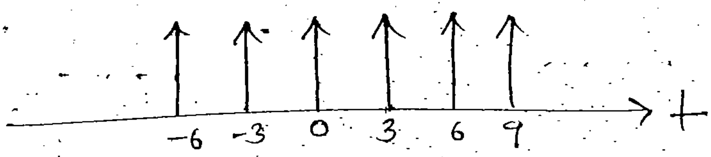
P2.1.8



$$x(t) * \delta(t-t_0) = x(t-t_0)$$

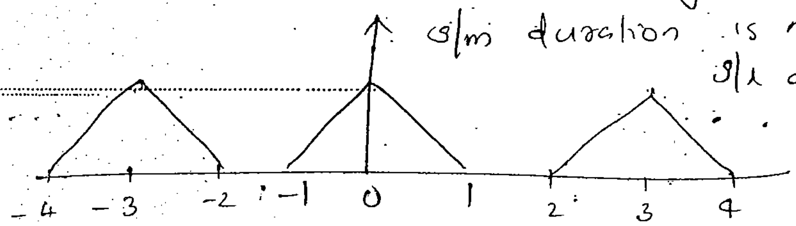


$$h(t) = \sum_{n=-\infty}^{\infty} \delta(t-3n)$$

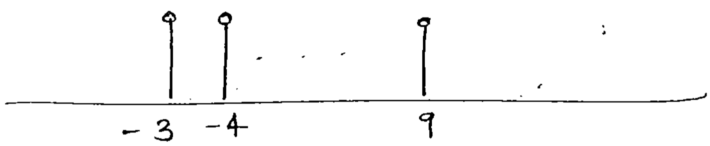


Conv. of gen. s/l with periodic s/l is the periodic repetition of general s/l

statement is valid only when the s/m duration is more than s/l duration.



P.2.1.11



$$h(n) = \left[\frac{1}{2}\right]^{n-1} \quad ; \; -3 < n < 9$$

$$\left[\frac{1}{2}\right]^{(n-k)-1} \quad ; \; -3 \le n-k \le 9$$

$$-3 - n \le -k \le 9 - n$$

$$3 + n \ge k \ge n - 9$$

$$\underline{n - 9 < k < n + 3}$$

$A = \underline{n-9}$

$B = \underline{n+3}$

P2.1.12

$$y(n) = \sum_{k=-\infty}^{+\infty} x(k)\, g(n-2k)$$

$g(n) = u(n) - u(n-4)$; $y(n)$ when $x(n) = \delta(n-2)$?

$x(n) = \delta(n-2)$. $x(n)$ is defined only at '2'

(27)

$\therefore$ substitute $t = 2$,

O/P $\Rightarrow x(2) \cdot g(n-4)$

then $y(n) = g(n - 2 \times 2) = \underline{g(n-4)}$

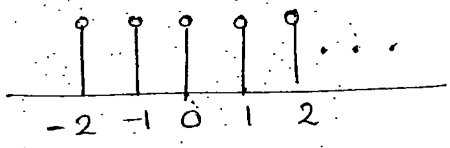
Where ever the sample occurs replace the value with time.

& sim as timevariant also

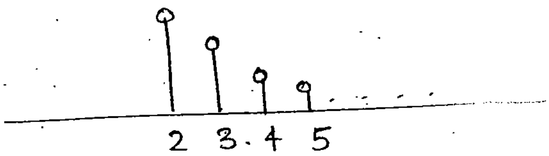
Q) $x[n] = \left(\frac{1}{2}\right)^{n-2} u(n-2)$ & $h(n) = u(n+2)$.

P.2.1.13

$h[n] = u(n+2)$



$x[n] = \left(\frac{1}{2}\right)^{n-2} u(n-2)$

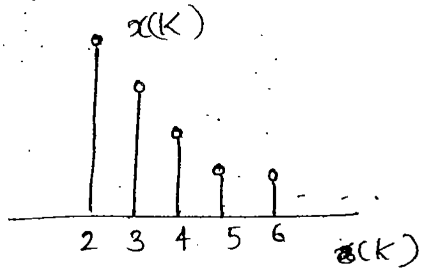


(i) length of o/p: $0 < n < \alpha$.

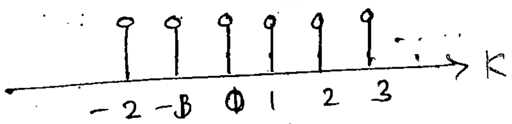
(ii) $n \longrightarrow k$.

zero value also to be noted

$x(K)$

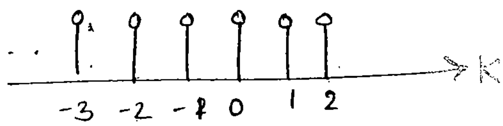


$h[k]$



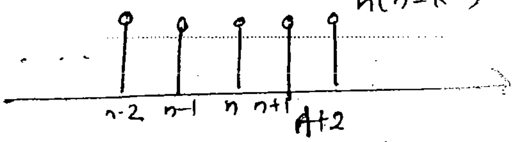
folding

$h[-K]$



shifting.

$h(n-K)$

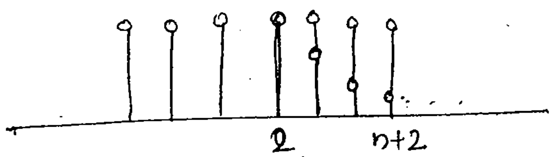


Case (1): $n+2 < 2$; $n < 0$;

$y[n] = 0$; $n < 0$.

Case (2) ; $n+2 > 2$; $n > 0$;

$x(K)\, h(n-K)$



$$y[n] = \sum_{k=-\infty}^{\infty} x(K)\, h(n-K)$$

$$= \sum_{K=2}^{n+2} \left(\frac{1}{2}\right)^{K-2} (1)$$

put $m = K - 2$

$$y[n] = \sum_{m=0}^{n} \left(\frac{1}{2}\right)^{m}$$

$\frac{1 - (1/2)^{n+1}}{1/2}$

$n \geq 0$

P2.1.14

$x(n) = \{1, 2, 3, 4\}$

$h(n) = \{1, 2, 1, -1\}$

$u(t) * u(t) = t\,u(t)$

$u(n) * u(n) = (n+1)\,u(n)$

Add & shift Method

	1	2	3	4
1	1	2	3	4
2	2	4	6	8
1	1	2	3	4
-1	-1	-2	-3	-4

If
i/p length = m
s/m length = n
o/p s/l length = m+n-1

$y(n) = \{1, 4, 8, 11, 9, 1, -4\}$

$x(n) = \{1, \underset{\uparrow}{2}, 3, 4\}$

$h(n) = \{1, 2, 1, -1\}$

No change in Method

limits: $(-1 < n < 5)$

$y(n) = \{1, \underset{\uparrow}{4}, 8, 11, 9, 1, -4\}$

Sum by Column Method

$x[n] \longrightarrow$	1	2	3	4			
$h[n] \longrightarrow$	1	2	1	-1			
	1	2	3	4			
		2	4	6	8		
			1	2	3	4	
				-1	-2	-3	-4
	1	4	8	11	9	1	-4

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i/p → m samples

s/m → n samples

o/p → n+m−1 samples.

eg: $x(n) = \overset{w^0\ w^1\ w^2\ w^3}{\{1, 2, 3, 4\}}$.

$g(n) = \overset{w^0\ w^1\ w^2\ w^3\ w^4\ w^5\ w^6}{\{1, 4, 8, 11, 9, 1, -4\}}$.

$h(n) = ?$

Deconvolution/ s/m identification.

$$
\begin{array}{lccccccc}
x[n] = & 1 & 2 & 3 & 4 & & & \\
h[n] = & a & b & c & d & & & \\
\hline
 & a & 2a & 3a & 4a & & & \\
 & & b & 2b & 3b & 4b & & \\
 & & & c & 2c & 3c & 4c & \\
 & & & & d & 2d & 3d & 4d \\
\hline
 & \cancel{a} & \cancel{2a+b} & & & & & \\
 & 1 & 4 & 8 & 11 & 9 & 1 & -4
\end{array}
$$

$a = 1$; $b = 4 - 2 = \underline{\underline{2}}$ $c = 8 - 4 - 3 = \underline{\underline{1}}$

$d = 11 - 2 - 6 - 4 = \underline{\underline{-1}}$

$h[n] = \underline{\underline{[1 \quad 2 \quad 1 \quad -1]}}$

Convolution − Multiplication

Deconvolution − Division.

$$
\begin{array}{r|l}
 & w + 2w^1 + w^2 - w^3 \\
\hline
1 + 2w^1 + 3w^2 + 4w^3 & w^0 + 4w^1 + 8w^2 + 11w^3 + 9w^4 + w^5 - 4w^6
\end{array}
$$

Another Method

Recursion Division.

Periodic Convolution.

fast convolution.

$x(n)$ & $h(n)$ should be periodic

ordinary conv $\rightarrow$ slow convolution.

ordinary	periodic
s/m $\rightarrow$ n samples	$x_p(n) \longrightarrow$ m samples
i/p $\rightarrow$ m	$h_p(n) \longrightarrow$ n
o/p $\rightarrow$ m+n-1 samples	$y_p(n) \longrightarrow$ Max (m, n) maximum of the m10

$$x_p[n] = 1 \quad 2 \quad 3 \quad 4$$

$$h_p[n] = 1 \quad 2 \quad 1 \quad -1$$

ordinary conv.

$$\begin{array}{ccccccc}
1 & 2 & 3 & 4 & & & \\
 & 2 & 4 & 6 & 8 & & \\
 & & 1 & 2 & 3 & 4 & \\
 & & & -1 & -2 & -3 & -4 \\
\hline
1 & 4 & 8 & 11 & 9 & 1 & -4 \\
9 & 1 & -4 & & & & \\
\hline
10 & 5 & 4 & 11 & & &
\end{array}$$

Donot change

wraparound the last part of linear conv. with the first max(m, n) samples

$$\boxed{\text{linear Convolution + Aliasing} \longrightarrow \text{Periodic Convolution.}}$$

Matrix Multiplication Method.

1 2 3 4 1 2 3 4 1 2 3 4

$$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1+8+3-2 \\ 2+2+4-3 \\ 3+4+1-4 \\ 4+6+2-1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 4 \\ 11 \end{bmatrix}$$

Circulent Matrix

last sample of the first column is the first semple of next column.

P2.1.17

$x_g = \{a, b, c, d\}$

~~o/p y = {x, x, x~~ . . N times}

$$\underbrace{a\,b\,c\,d}_{\delta(n)}\ \underbrace{a\,b\,c\,d}_{\delta(n-4)}\ \underbrace{a\,b\,c\,d}_{\delta(n-8)}\ \underbrace{a\,b\,c\,d}_{\delta(n-12)}$$

$$h(n) = \sum_{i=0}^{N-1} \delta(n-i4)$$

Ppties of LTI s/m.

$$\text{L.T.I s/m} \begin{cases} h(t) \\ h(n) \end{cases}$$

Causality

Before application of i/p as impulse at $t=0$, we can't expect the o/p as impulse response before $t=0$. ie

$h(t) = 0 ; \ t < 0$

$h(n) = 0 ; \ n \leq 0$..

Stability

$$\int_{-\infty}^{\infty} |h(\tau)|\, d\tau < \infty$$

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$|y(t)| = \left| \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \right|$$

$$= \int_{-\infty}^{\infty} \underbrace{|x(\tau)|}_{\text{finite}}\; |h(t-\tau)|\, d\tau$$

$$= \text{finite} \underbrace{\int_{-\infty}^{\infty} |h(t-\tau)|\, d\tau}_{\text{should be finite}}$$

or $\int_{-\infty}^{\infty} |h(t)|\, dt \to$ finite

Static & Dynamic

$h(t) = k\,\delta(t)$

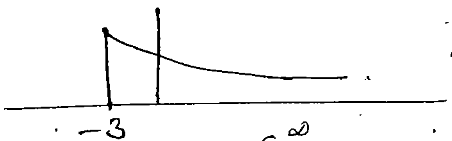
$h(t) = 0$ for $t \neq 0$.

eg: $y(t) = 5x(t)$

$h(t) = 5\,\delta(t)$.

If the impulse response is impulse then only an LTI S/m is static

eg: $h(t) = e^{-2t}\, u(t+3)$



Non causal
two sided.

Stable $\int_{-\infty}^{\infty} e^{-2t} u(t+3) =$ finite.

eg: $h[n] = 2^n u[n-2]$

Causal, unstable. ($2^n = \infty$ as $n \to \infty$)

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P2.2.4

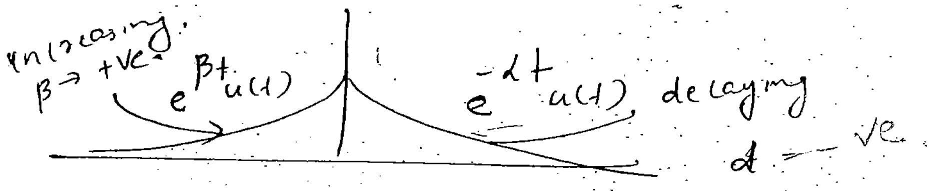
$$h(n) = \begin{cases} a^n ; & n \geq 0 \\ b^n ; & n < 0 \end{cases}$$

if $|a| < 1$ $a^n \leq 1$ (finite) for any value of n, +ve

$|b| > 1$ b^n = finite.

P2.2.5

$$h(t) = e^{\alpha t} u(t) + e^{\beta t} u(-t).$$

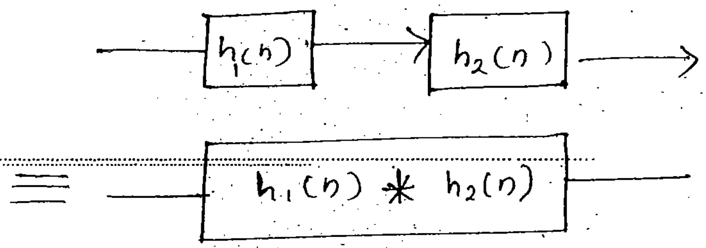


$\alpha < 0, \beta > 0.$

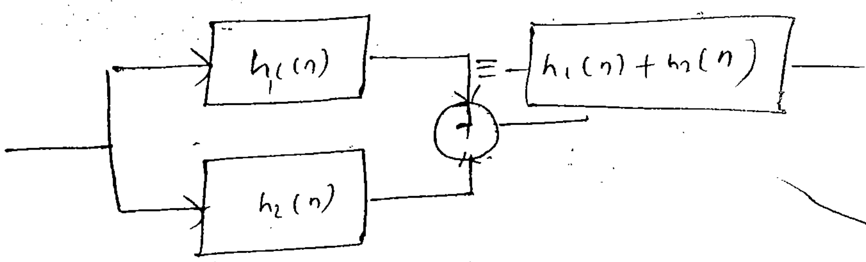
for +ve time exponent should be for -ve time ,, ,,

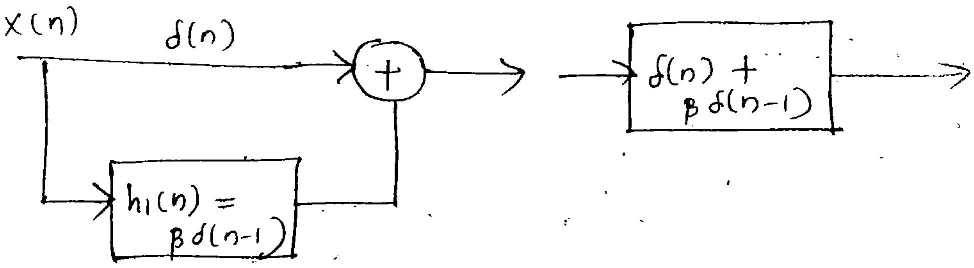
P2.2.3

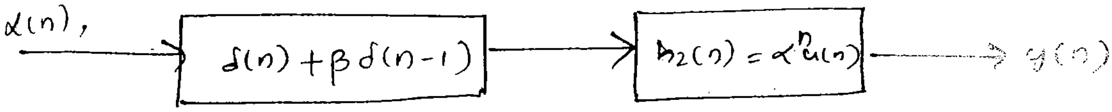
Series.



parallel







$$\underbrace{\alpha^n u(n)}_{n \geq 0} + \beta \underbrace{\alpha^{n-1} u(n-1)}_{n \geq 1} \implies \text{causal.}$$

for stability ...

2.2.6

$x(n) \rightarrow \boxed{y_1(n) = x(n) - \tfrac{1}{2}x(n-1)} \rightarrow \boxed{h_2(n) = (\tfrac{1}{2})^n u(n)} \rightarrow y(n)$

$x(n) = \delta(n)$

the $y_1(n) = h_1(n)$

$h_1(n) = \delta(n) - \frac{1}{2}\delta(n-1)$

$$h_{overall} = h_1(n) * h_2(n)$$

$$= \left(\frac{1}{2}\right)^n u(n) - \frac{1}{2}\left(\frac{1}{2}\right)^{n-1} u(n-1)$$

$$= \left(\frac{1}{2}\right)^n [u(n) - u(n-1)]$$

$$= \left(\frac{1}{2}\right)^n [\delta(n)] = \delta(n)$$

$$\boxed{h(n) * h_{inv}(n) = \delta(n)}$$ Invertability of LTI S/m.

$$y(n) = \sum_{k=-\infty}^{n} x(k)$$

Inverse s/m is $y(n) - y(n-1)$

$$h(n) = \sum_{k=-\infty}^{n} \delta(k) = u(n)$$

$$u(n) * \underbrace{[\delta(n) - \delta(n-1)]}_{u(n)} = \delta(n)$$

P2.2.7

(1) False.

(2) False.

(3) Correct

$$\int_{-\infty}^{\infty} |h(\tau)|\, d\tau = \text{infinity}$$

curtable.

11/08/2011
8.30-?pm

Step Response of LTI S/m.

(51)

$$u(t) \longrightarrow \boxed{T\{\cdot\}} \longrightarrow s(t)$$

Total S/m behaviour - h(t)

(Transient) sudden change in S/m i/p behaviour - s(t)

$$\boxed{\begin{aligned} \delta(t) &= \frac{d\,u(t)}{dt} \\ h(t) &= \frac{d\,s(t)}{dt}. \\ s(t) &= \int_{-\infty}^{t} h(t)\,dt. \end{aligned}}$$

Differentiation of Step response = Impulse response

- Impulse Response seen is valid only for LTI S/ms.

eg: $x(t) = u(t-3)$, $h(t) = e^{-2t}u(t)$, step response

$\downarrow$

o/p $= y(t) = s(t-3)$

$$s(t) = \int_{-\infty}^{t} h(\tau)\,d\tau = \int_{-\infty}^{t} e^{-2\tau}u(\tau)\,d\tau = \frac{1-e^{-2t}}{2} \; ; \; t>0$$

$$\therefore \quad y(t) = s(t-3) = \frac{1-e^{-2(t-3)}}{2} \; ; \; t>3.$$

$$\boxed{\text{Step Response} = \text{Area under impulse response.}}$$

eg: $s(t) = (1-e^{-2t})\,u(t)$. $h(t)$?

$$h(t) = \frac{d\,s(t)}{dt} = \frac{d}{dt}(1-e^{-2t})\,u(t)$$

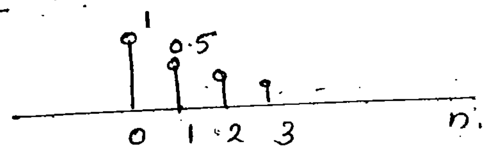
$$= 2e^{-2t}$$

Practically not using - Conc. on high frequency in the differentiation. (Noise S/l) unnecessarily adding

$$\boxed{\begin{aligned} s(n) &= \sum_{k=-\infty}^{n} h(k) \\ h(n) &= s(n) - s(n-1). \end{aligned}}$$

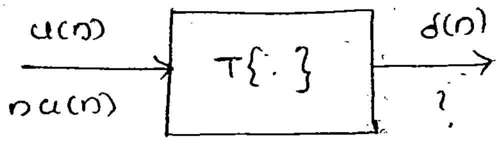
differentiation will amplify the high freq components of a S/l which generally represents noise [Wide band spectrum] eg: $h[n] = (0.5)^n u(n)$.

area under step / Impulse



$$s(n) = \sum_{k=0}^{n} (0.5)^k = \frac{1-(0.5)^{n+1}}{1-(0.5)} ; \ n \geq 0$$

P.2.3.3



$(n-1)u(n-1) - nu(n)$

$n u(n-1) - u(n-1)$

$- nu$

s/m $\longrightarrow$ 1st difference.

o/p = ~~nu(n)~~ $nu(n) - (n-1)u(n-1)$

$= n u(n) - n u(n-1) + u(n-1)$

$= n[u(n) - u(n-1)] + u(n-1)$

$= n\,\delta(n) + u(n-1)$

$= u(n-1)$ [i/p is externally function of time then it is decided by s/m operation]

if i/p $5u(n)$.

o/p $= 5u(n-1) - 5u(n)$

$= 5\delta(n)$ 'scaling'

eg: $y[n] = e^{x[n]}$

$$\boxed{x(n) = \delta(n) \Longrightarrow y(n) = h(n)}$$

Condition is valid only for LTI s/m
$e^{x(n)}$ is non-linear

$h(n) = e^{\delta(n)}$. $\quad \sum_{n=-\infty}^{\infty} |h(n)| < \infty$

$= \cdots 1 + 1 + e + 1 + \cdots$

ustable due to wrong concept

LTI s/m $\longrightarrow$ Impulse respons

another eg: $y(n) = \cos\{x(n)\}$

Fourier Series

one period $\Longrightarrow$ Many frequencies.

$\omega_0 \longrightarrow n\omega_0$.

approximation

Non sinusoidal $\Longrightarrow$ sinusoidal

Note: * It is an approximation process where a non sinusoidal waveform is converted to sinusoidal waveform such that all the s/ls are represented in unique form.

$x(t) = \sin t$

error

$g(t) \cong C\, x(t)$; $t_1 < t < t_2$

$$C = \frac{\int_{t_1}^{t_2} g(t)\, x(t)\, dt}{\int_{t_1}^{t_2} x^2(t)\, dt}$$

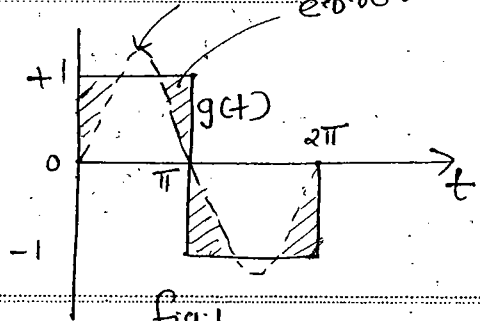


fig:1

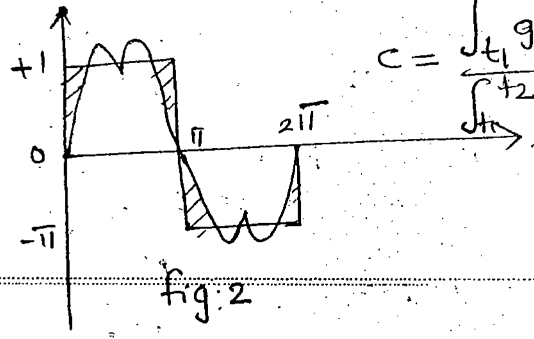


fig:2

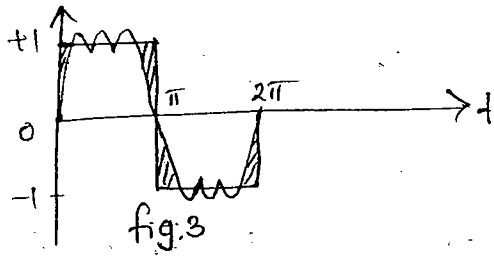


fig:3

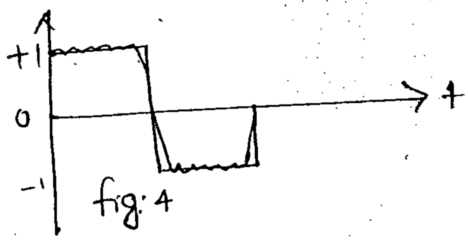


fig:4

* When we approximate a square wave form in terms of more number of sinusoidal component at every stage of approximation, error is minimized & we are recovering the original shape.

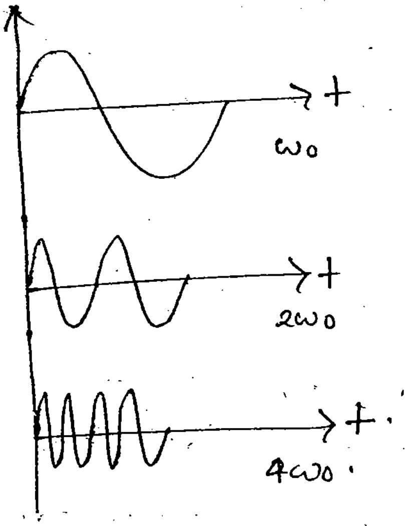
- Reason for using sin | Cos | exponential $\longrightarrow$ Orthogonality Property

$\int \sin t \cos t = 0$.

$$\int_{t_1}^{t_2} x_1(t)\, x_2^*(t)\, dt = 0$$

90° phase shift

* For a s/l to have fourier series, orthogonality is a must condition (two signals $x_1(t)$ and $x_2(t)$ are orthogonal if $\int x_1(t)\, x_2^*(t)\, dt = 0$)



* Making s/l freq are multiple freq.

$\omega_0 \Longrightarrow n\omega_0$

Trignometric Fourier Series

$$\boxed{g(t) = a_0 + \underbrace{\sum_{n=1}^{\infty} a_n \cos n\omega_0 t}_{} + \underbrace{\sum_{n=1}^{\infty} b_n \sin n\omega_0 t}_{}}$$

In both sides of the eqn, the fns are in time t. so they are still in time domain

but, a_0, a_n, b_n $\longrightarrow$ Amp. in freq domain (trigonometric FS coefficients)

for eg: a_1. ~~amplitude of~~ magnitude at ω_1.

Amplitude Spectrum $\begin{cases} \text{Mag. spectrum} \\ + \\ \text{Phase spectrum.} \end{cases}$

Fourier series spectrum - Descrete.

* Fourier series spectrum is descrete where as Continous fourier transform spectrum is continous.

P3.2:1

$x(t) = 3\sin(4t+30) - 4\cos(12t-60^\circ)$

$\omega_0 = n\omega_0$

$\omega_0 = GCD(4, 12) = \underline{\underline{4}}$

4 - 1st harmonic $\quad$ 12 = 3×4 = 3rd harmonic

2nd harmonic $= \underline{\underline{0}}$.

The frequency team for 2nd harmonic is not present in the exp. so it is evident that its amp is zero.

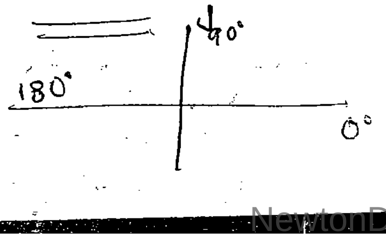
Phase of 3rd harmonic $= -60 \pm 180$??

$\underline{\underline{\pm 180 - 60}}$.

$\theta = \tan^{-1}(Y/X)$

=

	Angle
$1 + j0^\circ \Rightarrow$	0
$-1 + j0^\circ \Rightarrow$	$\pm 180^\circ$
$\pm j \Rightarrow$	$\pm 90^\circ$



P3.2.2   d) – Non periodic

Gate   dc can have any freq.

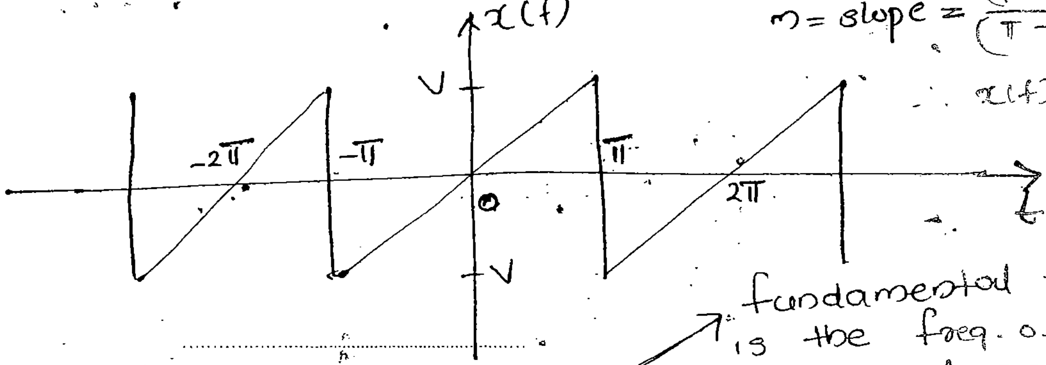
(33)

Q) Find the TFS representation of the periodic waveform shown below.

$y = mx$

$m = \text{slope} = \frac{(V-0)}{(\pi-0)} = \frac{V}{\pi}$

$\therefore x(t) = \frac{V}{\pi} t$



fundamental frequency is the freq. of the non-sinusoidal periodic sl. itself

$T = 2\pi \Rightarrow \omega_0 = 1$

odd symmetry $\longrightarrow a_0, a_n = 0$

$$b_n = \frac{2}{2\pi}\int_{-\pi}^{2\pi} \frac{Vt}{\pi} \sin n\omega_0 t \, dt \quad \left[b_n = \frac{2}{T}\int_0^T g(t) \sin n\omega_0 t \, dt \right]$$

odd × odd = even $\left[b_n = \frac{4}{T}\int_0^{T/2} g(t) \sin n\omega_0 t \right]$

$$\therefore b_n = 2 \times \frac{2}{2\pi}\int_0^{2\pi/2} \frac{Vt}{\pi} \sin n\omega_0 t \, dt$$

$$= \frac{2V}{(\pi)^2}\int_0^{\pi} t \sin n\omega_0 t \, dt \quad [\omega_0 = 1]$$

$$= \frac{2V}{(\pi)^2}\int_0^{\pi} t \sin nt \, dt = \frac{2V}{(\pi)^2}\left[t \cdot \frac{-\cos nt}{n} - \int 1 \cdot \frac{-\cos nt}{n} \right.$$

$$= \frac{2V}{(\pi)^2}\left[t \cdot \frac{-\cos nt}{n} + \frac{\sin nt}{n^2} \right]_0^{\pi}$$

$$= \frac{2V}{(\pi)^2}\left[\frac{-(-1)^n \cdot \pi}{n} - 0 \right] \Rightarrow (-1)\frac{2V}{\pi n}(-1)^n$$

$$= \cancel{-\frac{2V \cdot (-1)^{n-1}}{\pi n}} = \frac{2V}{\pi n}(-1)^{n+1}$$

$$b_n = \begin{cases} \frac{-2V}{\pi n} & ; \text{even } n \\ \frac{2V}{\pi n} & ; \text{odd } n \end{cases}$$

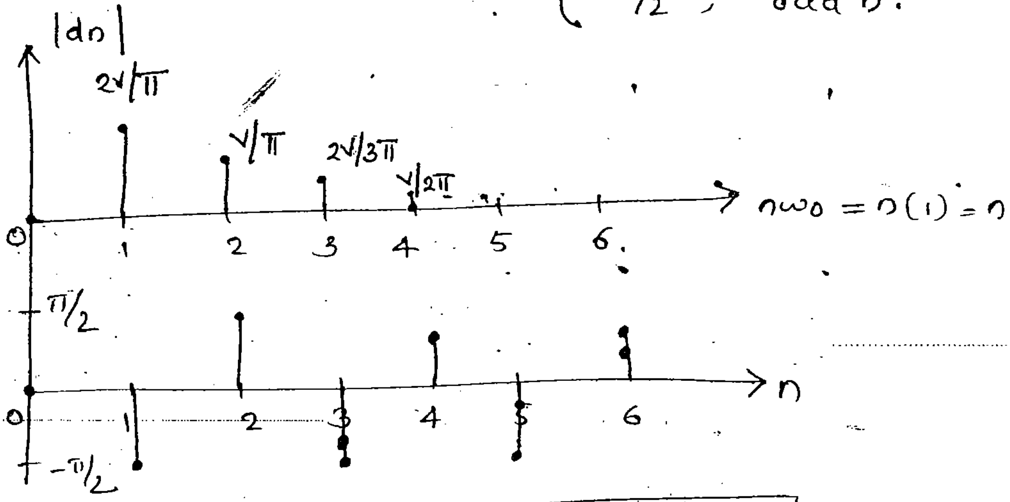
$$x(t) = \sum_{n=1,3,5}^{\infty} \frac{2V}{\pi n} \sin nt + \sum_{n=2,4,6}^{\infty} \frac{-2V}{\pi n} \sin nt$$

Polar form (to know the power contained in individual frequency (harmonics). Based on this info. filters can be designed.

$d_0 = a_0 = 0$

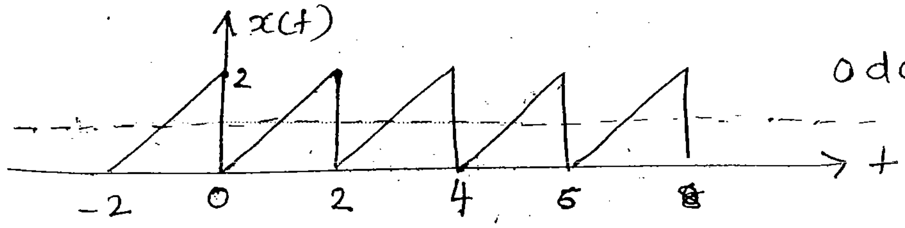
$d_n = \sqrt{a_n^2 + b_n^2} = \left| \frac{2V}{n\pi} \right|$

$\theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right) = \begin{cases} \pi/2 \; ; & \text{even } n \\ -\pi/2 \; ; & \text{odd } n. \end{cases}$

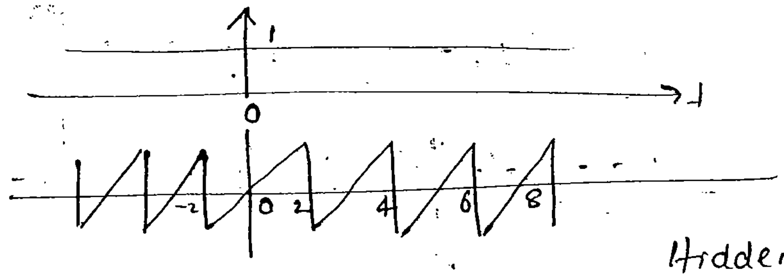


$$\boxed{\begin{array}{l} \text{real \& even} \Rightarrow 0^\circ \text{ (or) } \pm 180^\circ \\ \text{odd} \Rightarrow \pm 90^\circ \end{array}}$$

Q) For the periodic waveform shown in fig. which components are there in the TFS expansion.



odd symmetry



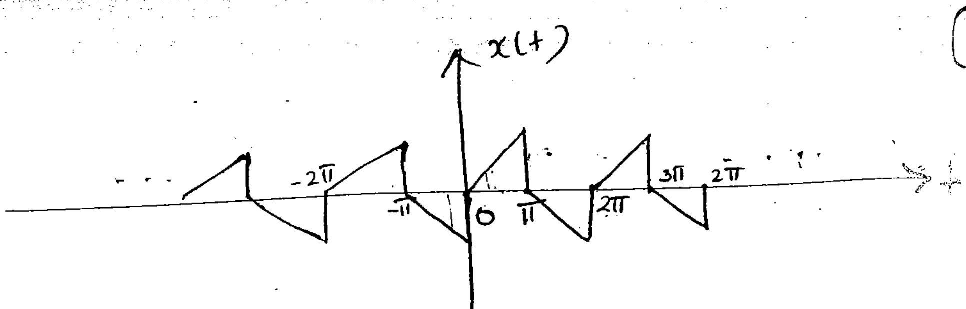
Hidden odd symmetry with dc value of +1V

$a_0 =$ Area over a period

$= \dfrac{\oint \frac{1}{2}\cdot 2\times 2}{2} = \underline{\underline{1}}$

Both dc & sin terms

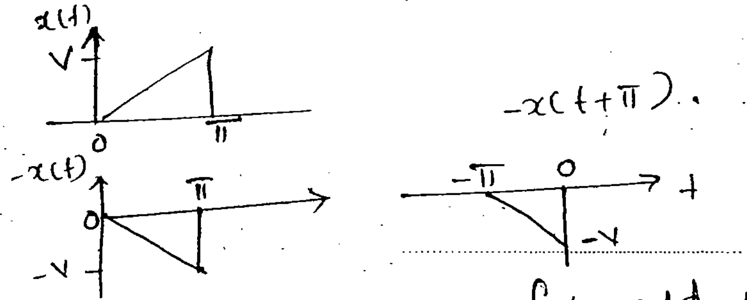
Q)



Half wave symmetry.

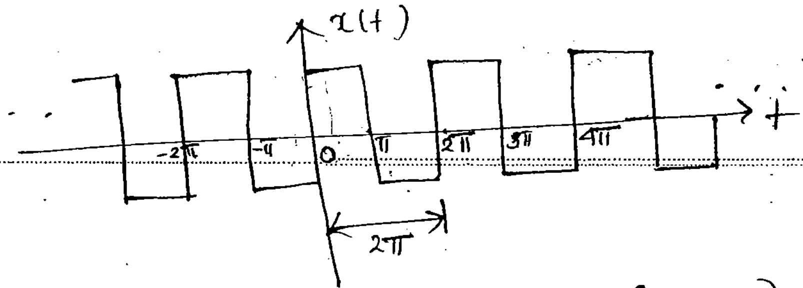
$$x(t) = -x(t + T/2) = -x(t+\pi).$$

$-x(t+\pi).$

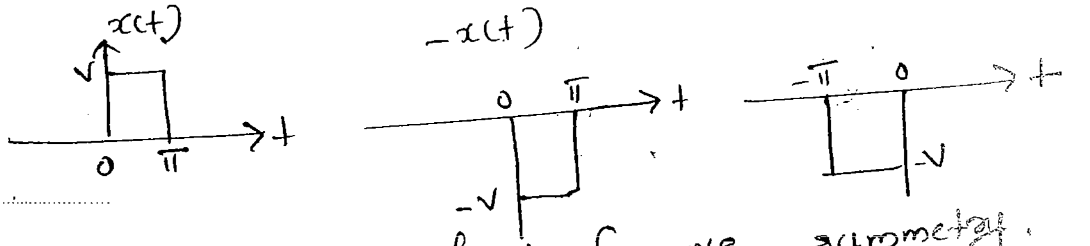


a_n & b_n <u>exists</u> for odd n.

Q)



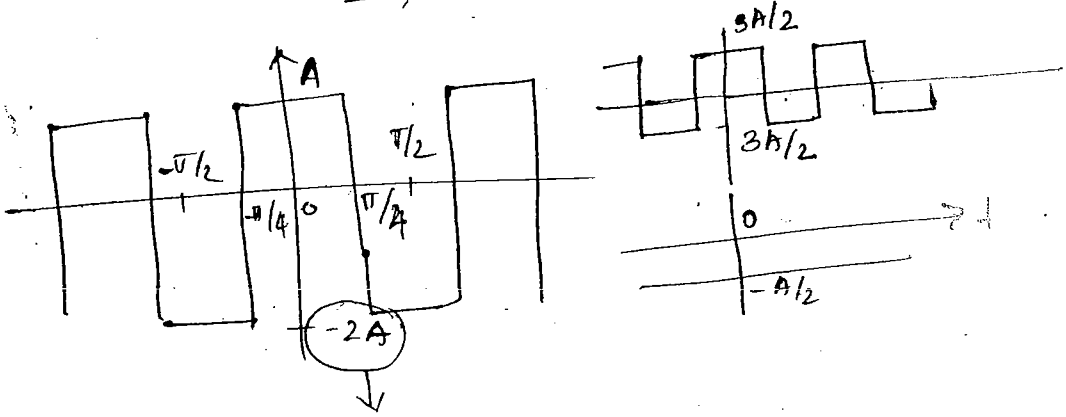
$$x(t) = -x(t + T/2) = -x(t+\pi).$$



∴ Both odd & half wave symmetry.

~~bn~~, a_n, b_n for odd n

Q)



dC

even s/m ⟶ Cosine terms. with −ve dC.

P3.2.5

Ans = 8W

Power in sinusoidal $= \frac{A^2}{2}$

$\cancel{10\pi} = \cancel{2(5\pi)}$; $\omega_0 \neq 5\pi$

$\cancel{30\pi} = \cancel{6(5\pi)}$

$\omega = 10\pi$ rad/sec $\Rightarrow f = \frac{\omega}{2\pi} = \frac{10\pi}{2\pi} = \underline{5 \text{ Hz}}$ (Not in the required band)

$\omega = 30\pi$ rad/sec $\Rightarrow f = \frac{\omega}{2\pi} = \frac{30\pi}{2\pi} = \underline{15 \text{Hz}}$ (within the required band $10<f<20$

$\therefore$ Power $= \frac{4^2}{2} = \underline{8 \text{ W}}$

P3.2.6

Ans: $\pi/4$

$$x(t)\Big|_{\pi/2} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

$$= \underbrace{1 - 0.33 + 0.2 - 0.13}_{\text{Approximation}}$$

$\Downarrow$

$\Rightarrow \pi/4$

$\omega = 2\pi f$

$$\text{Tan}^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$$

P3.2.7

(b)

$$\sin^2 t + \cos 2t = \frac{1-\cos 2t}{2} + \cos 2t$$

$$= \frac{1}{2} + \frac{1}{2}\cos 2t$$

$$f = 0 \quad f = 1/\pi$$

P3.2.8

$0.2 \Rightarrow 4(0.05)$

$0.25 \Rightarrow 5(0.05)$

$0.3 \Rightarrow 6(0.05)$

$n(\omega_0)$

$\omega_0 = \underline{0.05}$ rad/s

1+1+2

P3.2.9 (a) $5/6$.

P3.2.10 ν $a_0 = 0$ half wave.

P3.2.11

Ans (d)

$T = 4$ msec

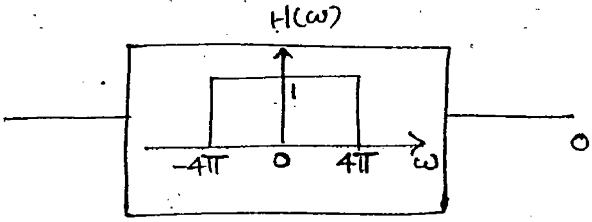
$\frac{1}{T} = 250$ Hz

5th harmonic $= 250 \times 5 = \underline{1250 \text{ Hz}}$.

fundamental frequency 'ω_0' is the frequency of the periodic, non-sinusoidal s/l itself, where $\omega_0 = \frac{2\pi}{T_0}$

$f_0 = \frac{1}{T} = \frac{1}{4 \text{msec}} = 250$Hz. fifth Harmonic $= 5 \times f_0 = \underline{1250 \text{ Hz}}$

8.2.12



o/p filtered

ideal LPF

$$T = 2 \Rightarrow \omega_0 = \frac{2\pi}{T} = \pi$$

- odd & half wave symmetry.

↓ $b_n = 2$.   ↓ a_0.

$$a_0 = \frac{(10)(1)}{2} = 5$$

$$b_n = \frac{2}{T}\int_0^T g(t) \cdot \sin n\omega_0 t \, dt = \frac{2}{2}\int_0^1 10 \sin \pi n t \, dt$$

$$= 10\left[\frac{-\cos \pi n t}{\pi n}\right]_0^1 = \frac{-10}{\pi n}\left[\cos \pi n - \cos 0\right]$$

$$= \frac{+10}{\pi n}\left[1 - (-1)^n\right] = \frac{20}{\pi n} \text{ (for n odd)}$$

$$\therefore x(t) = 5 + \sum_{n=1}^{\infty} \frac{20}{n\pi} \sin n\pi t.$$

$$= 5 + \frac{20}{1\pi}\sin \pi t + \underbrace{\cancel{\frac{20}{2\pi}\sin 2\pi t}}_{=0} + \frac{20}{3\pi}\sin 3\pi t +$$

o/p contains dc, 1st harmonic & 3rd harmonic.

Exponential (or) Complex FS

EFS → Compact / standard form of F.S.

FS → EFS.

$$y(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

$$= a_0 + \sum_{n=1}^{\infty} a_n\left[\frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2}\right] + b_n\left[\frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2j}\right]$$

$$= a_0 + \sum_{n=1}^{\alpha} e^{jn\omega_0 t}\left[\frac{a_n - jb_n}{2}\right] + e^{-jn\omega_0 t}\left[\frac{a_n + jb_n}{2}\right]$$

c_0 $\qquad$ c_n : $\qquad$ c_{-n}.

$$= c_0 + \sum_{n=1}^{\alpha} c_n e^{jn\omega_0 t} + \sum_{n=1}^{\alpha} c_{-n} e^{-j\omega_0 t}$$

put $-n = m$

$$= c_0 + \sum_{m=-1}^{-\alpha} c_m e^{jn\omega_0 t} + \sum_{n=1}^{\infty} c_n e^{jn\omega_0 t}$$

put

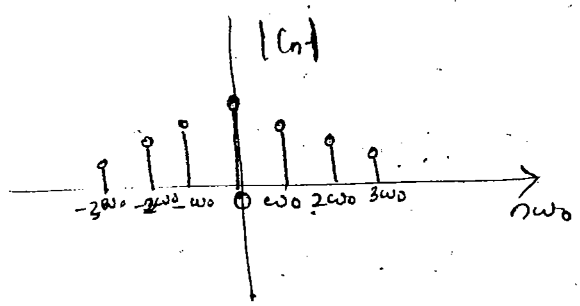
$$\boxed{g(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}}$$

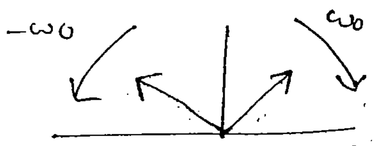
two sided spectrum.

$$c_0 = a_0, \quad c_n = \frac{a_n - jb_n}{2}$$

$$= \frac{1}{T}\int_0^T x(t)\, e^{-jn\omega_0 t}\, dt.$$

$$c_{-n} = \frac{a_n + jb_n}{2}$$





+ve & -ve frequency

* +ve & -ve frequencies denote opposite direction of rotation. ie opposite phase angle.
reason: to reproduce both sine, cosine term.

EFS to TFS $\Longrightarrow$

$$\boxed{\begin{aligned} a_0 &= C_0 \\ a_n &= C_n + C_{-n} \\ j\, b_n &= C_{-n} - C_n . \end{aligned}}$$

$y = mx$

$m = \frac{0-V}{\pi - 0} = -\frac{V}{\pi}$

P3.2.13 Even symmetry, $b_n = 0$.

$$a_0 = \text{Average} = \frac{2 \times \frac{1}{2} \times \pi \times V}{2\pi} = \frac{V}{2}$$

$T = 2\pi$

$\omega_0 = \frac{2\pi}{T} = 1$

$$a_n = \frac{2 \times 2}{2\pi} \int_0^{\pi} \left(-\frac{V}{\pi} + V\right) \cos n\omega_0 t \, dt + -\frac{2V}{\pi} \int_0^{\pi} \cos n\omega_0 t \, dt$$

$$= \frac{-2V}{(\pi)^2} \int_0^{\pi} \cos nt \, dt = \frac{-2V}{(\pi)^2} \left[\frac{-\sin nt}{n} \right]_0^{\pi}$$

$$= \frac{2V}{n(\pi)^2} \left[\sin n\pi - \sin 0 \right]$$

$$= \frac{2V}{n(\pi)^2} \sin n\pi$$

$$a_n = \begin{cases} \frac{2V}{n(\pi)^2} ; \\ 0 ; \end{cases}$$

$$a_n = 2 \times \frac{2}{2\pi} \int_0^{\pi} \left(-\frac{Vt}{\pi} + V \right) \cos n\omega_0 t \, dt$$

$$= -\frac{2V}{(\pi)^2} \int_0^{\pi} t \cos nt \, dt + \frac{2V}{\pi} \underbrace{\int_0^{\pi} \cos nt \, dt}_{=0} .$$

$$= \frac{-2V}{(\pi)^2} \left[t \times \frac{\sin nt}{n} + \cos nt \times \frac{t^2}{2} \right]_0^{\pi}$$

$$= \frac{-2V}{(\pi)^2} \left[0 - \cos n\pi \frac{(\pi)^2}{2} \right]$$

$$= \frac{2V}{(\pi)^2} \cdot \frac{(\pi)^2}{2} \cos n\pi = V \cdot (-1)^n .$$

$$\text{Ans} = \frac{2V}{\pi^2 n^2} \left[1 - (-1)^n \right]$$

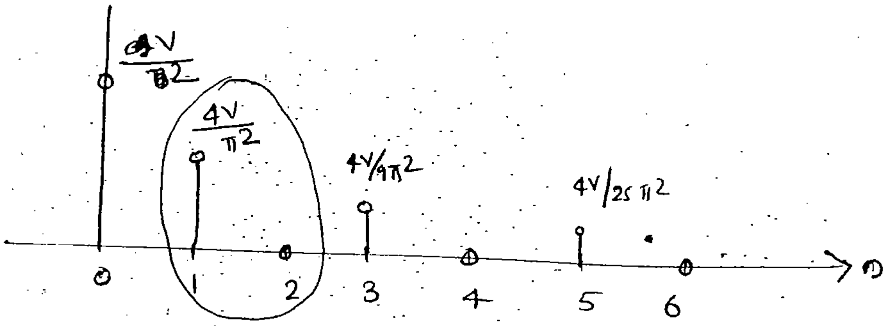
$$a_n = \begin{cases} 0 & ; \text{ even} \\ \frac{4V}{(n\pi)^2} & ; \text{ odd} . \end{cases}$$

Polar form

$$d_0 = a_0 = \frac{V}{2}$$

$$|d_n| = \sqrt{a_n^2 + b_n^2}$$

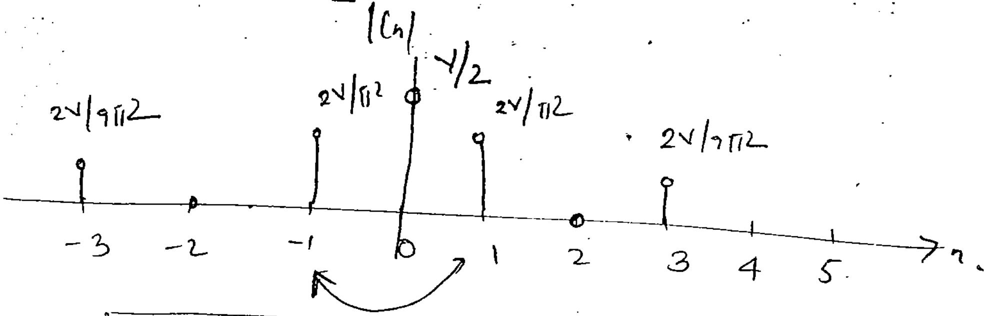
$$= |a_n| = \frac{4V}{\pi^2 n^2} \text{ (odd } n)$$



EFS:

$$c_0 = a_0 = \frac{V}{2}$$

$$c_n = \frac{a_n - jb_n}{2} = \frac{2V}{\pi^2 n^2} \text{ (odd } n)$$



$$\boxed{\begin{array}{l} |C_n| = |C_{-n}| \Rightarrow \text{Even.} \\ \cancel{|C_n| = -|C_{-n}| \Rightarrow} \\ \angle C_n = -\angle C_{-n} \Rightarrow \text{odd.} \end{array}}$$

Conjugate Symmetry.

$C_3 = 3+j4$

$C_{-3} = 3-j4$

P3.2.14

$$x(t) = 2 + \cos\left[\frac{2\pi t}{3}\right] + 4\sin\left[\frac{5\pi t}{3}\right]$$

$$\omega_0 = \frac{\pi}{3} \quad \left[\omega_0 = GCD\left(2\left(\frac{\pi}{3}\right), 5\left(\frac{\pi}{3}\right)\right)\right]$$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \quad \text{expecting.}$$

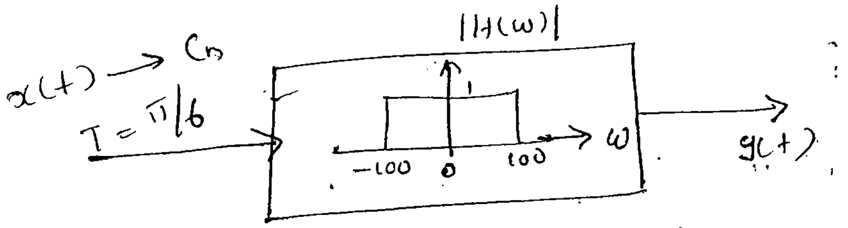
$$x(t) = 2 + \left(\frac{e^{j\frac{2\pi}{3}t} + e^{-j\frac{2\pi}{3}t}}{2}\right) + 4\left(\frac{e^{j5\frac{\pi}{3}t} - e^{-j\frac{5\pi}{3}t}}{2j}\right)$$

$$= 2 + \frac{1}{2}e^{-j(2)\left(\frac{\pi}{3}\right)t} + \frac{1}{2}e^{j(2)\left(\frac{\pi}{3}\right)t} + 2j\,e^{j(5)\left(\frac{\pi}{3}\right)t} - 2j\,e^{-j(5)\left(\frac{\pi}{3}\right)t}.$$

$$\left.\begin{array}{lll} C_0 = 2, & C_2 = 1/2 & C_5 = -2j \\ & C_{-2} = 1/2 & C_{-5} = +2j \end{array}\right\} C_{-n} = C_n^*$$

General Relation $\boxed{C_{-n} = C_n^*}$

P3.2.15



$$\omega_0 = \frac{2\pi}{T} = \underline{\underline{12}}$$

$\omega_0 = 12\,n.$

$= 12 \times 9 = 108$ (above the cut off frequency.

$\therefore\ n \geq 9 \quad \underline{\underline{C_n = 0}}$

P3.2.16

Fundamental frequency $GCD = 4. = \omega_0.$

$0(4) = 0$

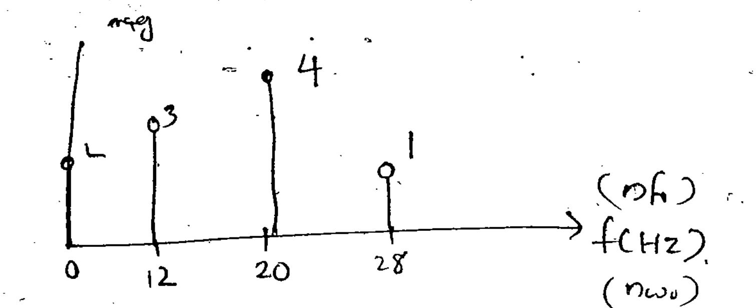
$3(4) = 12$

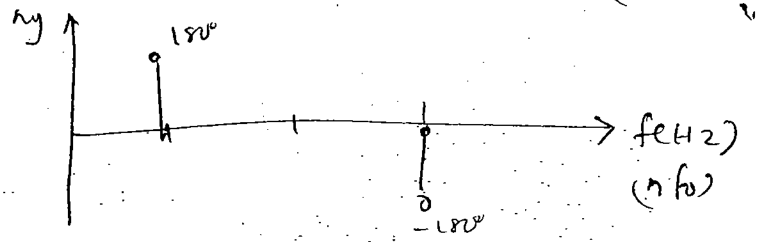
$5(4) = 20$

$7(4) = 28.$

$$x(t) = d_0 + \sum_{n=1}^{\infty} d_n \cos\left[(n\omega_0)t + C_n\right]$$

$$= 2 + 6\cos\left[2\pi(12)t + 180^\circ\right] + 8\cos\left[2\pi(20)t\right] + 2\cos\left[2\pi(28)t - 180^\circ\right]$$





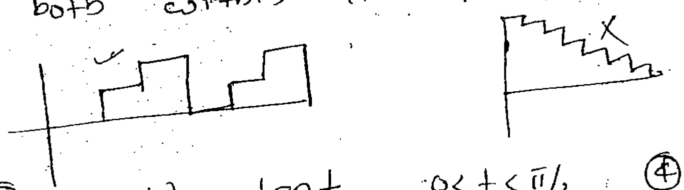
Convergance of F.S (Dirichlet's Condition)

① $\int_0^T |g(t)|\,dt < \infty$ = Absolutely integrable

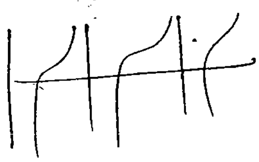
but $x(t) = \frac{1}{t}$ doesn't obecy this condition.

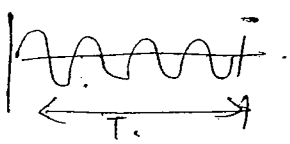
② g(n) may have finite no of discontinuity (Maxima & Minima)

both within Time period T



③ $x(t) = \tan t \quad 0 < t < \pi/2$ ④ $x(t) = \sin\left(\frac{2\pi}{t}\right)$





practically ③, ④, ⑤ are not used in

our labs.

13/08/2011
9 – 12.30

Properties of F.S

1. Linearity

$$x_1(t) \xrightarrow[\text{coeff}]{F.S} C_n$$

$$x_2(t) \longrightarrow d_n.$$

$$\alpha x(t) + \beta y(t) \longleftrightarrow \alpha C_n + \beta d_n$$

2. Time shift

$$x(t - t_0) \longleftrightarrow e^{-jn\omega_0 t_0} C_n.$$

3. Frequency shift

$$x(t) e^{j\omega_0 M t} \longleftrightarrow C_{n-M}.$$

$$C_n = \frac{1}{T}\int_0^T x(t) e^{-jn\omega_0 t}\, dt$$

F.S coeff of $x(t) e^{j\omega_0 M t}$

$$= \frac{1}{T}\int_0^T x(t) e^{j\omega_0 M t} e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{T}\int_0^T x(t) e^{-j\omega_0 (n-M)t}\, dt$$

$$= C_{n-M}$$

shifting in one domain = Multiplication by exponential in the other domain.

P.3.3.2

$$y(t) = x(t - t_0) + x(t + t_0)$$

$$\downarrow F.S$$

$$c^y = e^{-jn\omega_0 t_0} c^x + e^{+jn\omega_0 t_0} c^x$$

$$= c^x [e^{jn\omega_0 t_0} + e^{-jn\omega_0 t_0}] = 2c^x \cos n\omega_0 t_0$$

At $t_0 = \frac{T}{4}$ $\quad \cos n\omega_0 \frac{T}{4} = \cos n\left(\frac{2\pi}{T}\right)\left(\frac{T}{4}\right) = 0$.

* $y(t) = x(t) - x(t - \frac{T}{2})$

$= c_n^x - e^{-jn\omega_0 \frac{T}{2}} c_n^x$

$= c_n [1 - e^{-jn(\frac{2\pi}{T})\frac{T}{2}}]$

$= c_n [1 - (-1)^n]$

$= 0$ for even harmonics

$= 2c_n$ for odd 'n'

half wave symmetry is obeyed.

(4) Time scaling

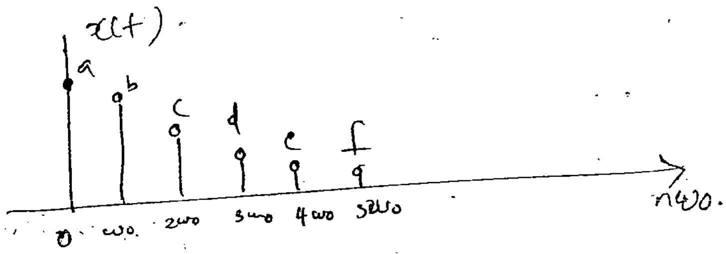
$x(t) \xrightarrow{F.S.C} c_n$ period $= T$

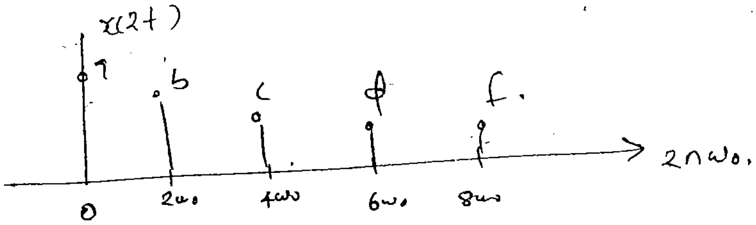
$x(\alpha t) \xrightarrow[F.S.C]{} c_n$ period $= T' = \frac{T}{\alpha}$

$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$

$x(\alpha t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0(\alpha t)}$

eg: $x(t) \longrightarrow c_n$.





Compression in time domain = Expansion in freq domain.

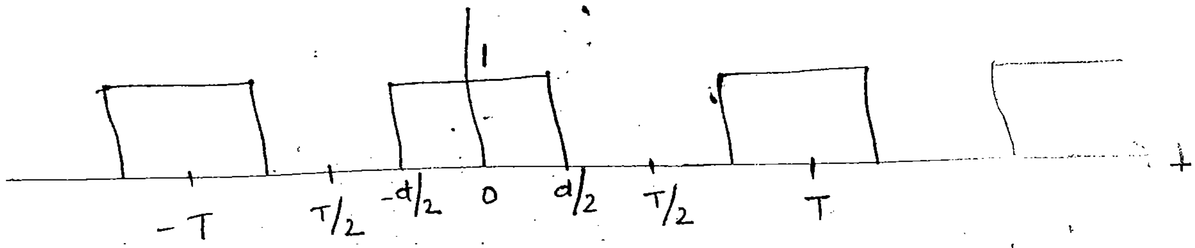
(5) Differentiation in time

$x(t) \xleftrightarrow{FSC} c_n$

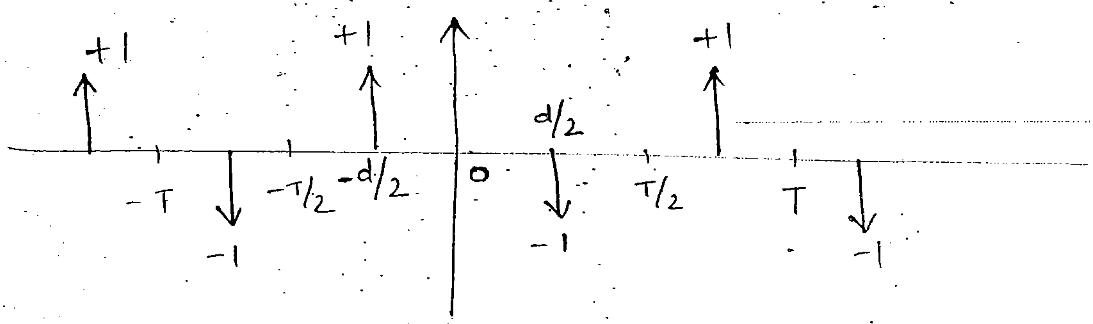
$\frac{d\,x(t)}{dt} \longleftrightarrow jn\omega_0 c_n$.

$$\frac{d^k x(t)}{dt^k} \longleftrightarrow (jn\omega_0)^k C_n.$$

$$x(t) \longrightarrow C_n.$$



$$\frac{d}{dt}x(t) = g(t) \longrightarrow d_n.$$



$$g(t) = \frac{d}{dt}(x(t))$$

$$d_n = (j\omega_0 n)\, C_n \Rightarrow C_n = \frac{d_n}{j\omega_0 n}$$

$$d_n = \frac{1}{T}\int_{-T/2}^{T/2} g(t)\, e^{-jn\omega_0 t}\, dt. = \frac{1}{T}\int_{-T/2}^{T/2} \left[\delta(t+d/2) - \delta(t-d/2)\right] e^{-jn\omega_0 t}\, dt$$

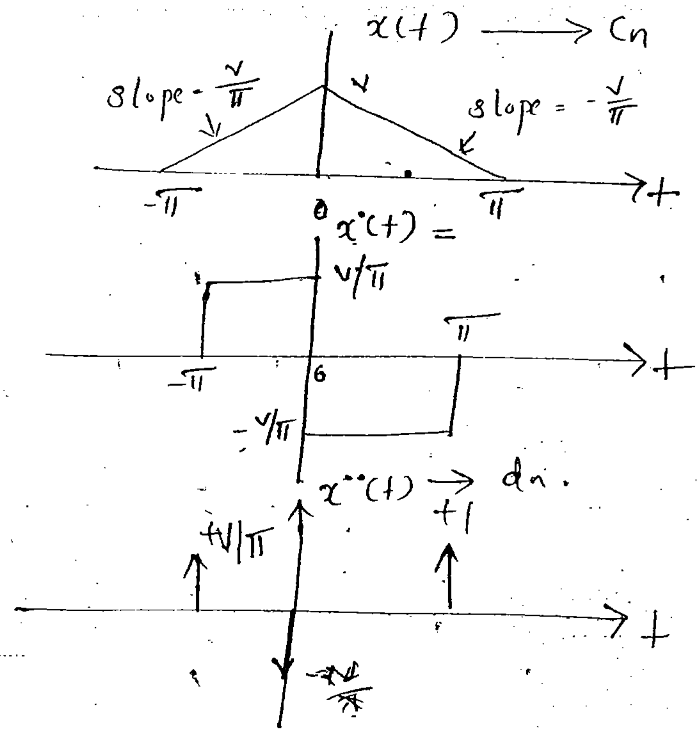
use Sampling ppty $\int z(t)\,\delta(t-t_0)\,dt = z(t_0)$.

$$d_n = \frac{2j}{T}\left[\frac{e^{-jn\omega_0(-d/2)} - e^{-jn\omega_0(d/2)}}{2j}\right]$$

$$= \frac{2j}{T}\sin\left(\frac{n\omega_0 d}{2}\right)$$

$$C_n = \frac{d_n}{j\omega_0 n} = \frac{2}{T\cdot\omega_0 n}\sin\left(\frac{n\omega_0 d}{2}\right)$$

$$C_n = \frac{2}{T\cdot\frac{2\pi}{T}n}\sin\left(\frac{n\omega_0 d}{2}\right) = \frac{1}{\pi n}\sin\left(\frac{n\omega_0 d}{2}\right)$$



$$d_n = (jn\omega_0)^2 C_n.$$

$$C_n = \frac{d_n}{(jn\omega_0)^n}.$$

$$= \frac{2V}{\pi n^2} \quad (\text{odd } n)$$

P3.3.4 (c) $\frac{1}{n}$ & $\frac{1}{n^2}$

P3.3.5 $x(t) \xleftrightarrow{F.S,\pi} C_n = -n\,2^{-|n|}$

π - fundamental frequency.

(a) $x(3t) \longrightarrow C_n$; $\omega_0' = 3\pi$

(b) $\frac{d\,x(t)}{dt} \longrightarrow \cancel{e^{-jn\omega_0 t}C_n}$ $(jn\pi)C_n$
$\cancel{e^{-jn(\pi)(1)}C_n}$.

(c) $x(t-1) \longrightarrow e^{-jn(\pi)(1)} C_n.$

(d) $\text{Re}\{x(t)\} = 0$

$$x(t) = x_R(t) + j\,x_I(t)$$

$$x^*(t) = x_R(t) - j\,x_I(t)$$

$$x_R(t) = \frac{x(t) + x^*(t)}{2}$$

$$x_e(t) \longrightarrow \frac{c_n + c_{-n}^*}{2} \quad \left\{ \text{Even part of } c_n \right\}$$

(e) $y(t) = x(t) \cos 4\pi t$.

$$= x(t) \, \frac{e^{j4\pi t} + e^{-j4\pi t}}{2}$$

$$\left[x(t) e^{j\omega_0 m t} \longrightarrow c_{n-m} \right]$$

$$\Rightarrow y(t) = \frac{1}{2}\left[x(t) e^{j4\pi t} + x(t) e^{-j4\pi t} \right]$$

$$= \frac{1}{2}\left[c_{n-4} + c_{n+4} \right]$$

6) Parseval's Power Theorem

$$x(t) \longrightarrow c_n.$$

Total power in the period waveform

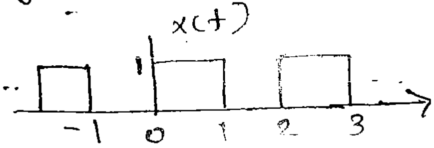
$$\frac{1}{T}\int_0^T |x(t)|^2 dt = \sum_{n=-\infty}^{\infty} |c_n|^2$$

sum of squared amplitude of each harmonic

Practically in any type of s/l, most of the energy is conc. in low frequency components.

P. 3.3.6 :-

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi.$$



Ref:
P. 3.2.12

amp = 1

$$c_0 = a_0 = \frac{1}{2} \; ; \quad c_n = \frac{a_n - jb_n}{2} = \frac{-j}{2}\left(\frac{2}{n\pi}\right)$$

$$= \frac{-j}{n\pi} \quad (\text{odd } n)$$

$$\text{Power upto } II^{nd} \text{ harmonic} = \sum_{n=-2}^{+2} |c_n|^2$$

$$= |\cancel{c_{-2}}|^2 + |c_{-1}|^2 + |c_0|^2 + |c_1|^2 + |\cancel{c_2}|^2$$

$$= \left(\frac{1}{\pi}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\pi}\right)^2 = \underline{0.45 \text{ Watts}}$$

Total power can be obtained from time domain.

$$P_{total} = \frac{1}{2}\int_0^T (1)^2 \, dt = \underline{\underline{0.5 \text{ Watts}}}.$$

$\frac{0.45}{0.5} \times 100 = 90\%$ of total power is upto the IInd harmonic.

- 99% of power – upto 9th harmonic

* Max Energy | Power is in the low frequency region.

Obs: → Total power is the representation of zero freq. component.

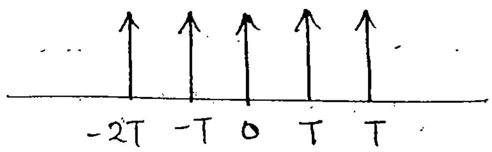
P.3.3.7 (a) wrong ; all periodic s/l are power s/ls.

(b) wrong : $e_0 = \pm j2$

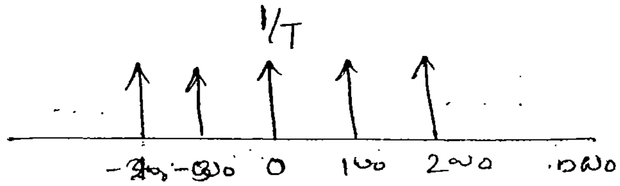
(c) Correct

(d) wrong: Area under even s/l is not zero.

P3.3.8 (A) Impulse train. – (b)



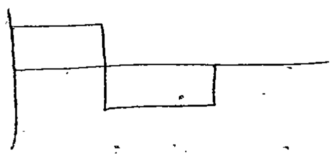
$$C_n = \frac{1}{T}\int_0^T \delta(t)\, e^{-jn\omega_0 t} = \underline{\frac{1}{T}}$$



(B) – (1)

(C) – (8)

(D)



odd & half wave s/m

= bn is present

(4)

system with periodic Input.

i/p $x(t) = e^{j\omega t} \Longrightarrow y(t) = e^{j\omega t} \underbrace{H(\omega)}_{\text{freq. Response}}$

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} e^{j\omega(t-\tau)}\, h(\tau)\, d\tau$$

$$= e^{j\omega t} \int_{-\infty}^{\infty} e^{j\omega\tau}\, h(\tau)\, d\tau$$

$$= e^{j\omega t}\, H(\omega)$$

i/p $x(t) = \sum C_n e^{jn\omega_0 t}$

$\downarrow$

o/p $y(t) = \sum C_n H(n\omega_0)\, e^{jn\omega_0 t}$.

Obs: If the input s/l coefficient $= C_n$

then o/p s/l coefficient $= C_n H(n\omega_0)$

(extracting frequencies)

S/l Analysis $\Longrightarrow$ Fourier Series & Fourier Transf

S/m Analysis $\Longrightarrow$ LT & ZT

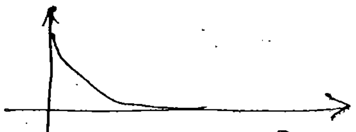
Fourier - Transform.

Extension of F.S for non periodic s/l.

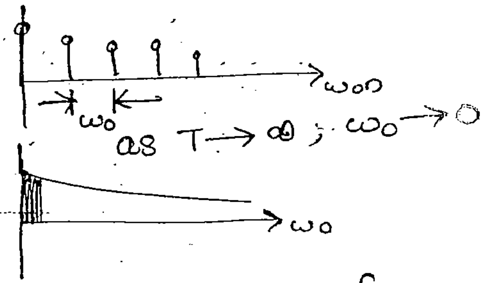
FT is applicable to both periodic & non periodic s/l

spectrum of FT is continous.

eg: $x(t) = e^{-t}u(t)$



$\left.\begin{array}{l} T \to \infty \\ \omega_0 \to 0 \end{array}\right\}$



F.T eqn obtained by ~~multiplying~~ modifying FS eqn.

F.S.C $\quad C_n = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-jn\omega_0 t}\, dt$

$$T C_n = \int_{-T/2}^{T/2} x(t)\, e^{-jn\omega_0 t}\, dt$$

$$\left.\begin{array}{l} T \to \infty \\ n\omega_0 \to \omega \end{array}\right\} \longrightarrow F.S \Longrightarrow FT$$

spectrum of the signal $x(t)$.

$$\boxed{\lim_{T\to\infty} T C_n = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = X(\omega)}$$

Analysis Equation

FT $\quad X(\omega) = \int_{-\infty}^{\infty} x(t)\, \underbrace{e^{-j\omega t}}_{\text{Kernal / basis}}\, dt$

synthesis equation.

IFT $\quad x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$

No information is lost when transforming from time domain to freq domain.

$$x(t) \longleftrightarrow X(\omega)$$
$$ \longleftrightarrow X(j\omega)$$
$$ \longleftrightarrow X(f)$$

$\omega = 2\pi f$

$d\omega = 2\pi df \implies df = \dfrac{d\omega}{2\pi}$

$$\boxed{\begin{aligned} X(f) &= \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt \\ x(t) &= \int_{-\infty}^{\infty} X(f)\, e^{j2\pi ft}\, df \end{aligned}}$$

$$\boxed{\omega = 2\pi f \qquad 2\pi\,\delta(\omega) = \delta(f)}$$

$$2\pi\,\delta(\omega) = 2\pi\,\delta(2\pi f) = 2\pi\,\frac{1}{2\pi}\,\delta(f) = \delta(f)$$

$$\boxed{\begin{aligned} \int_{-\infty}^{\infty} x(t)\,dt &= X(0) \\ \int_{-\infty}^{\infty} X(\omega)\,d\omega &= 2\pi x(0) \end{aligned}}$$

P 4.1.1

$x(t)$ = voltage s/l - volts.

integration done w.r.t time - sec

unit volt - sec (or) volt / Hz

P.4.1.2

$X(0) = \int_{-\infty}^{\infty} x(t)\,dt$; Area under the s/l

$= (1\times1) + (1\times1) + \frac{1}{2}\times1\times1 + \frac{1}{2}\times1\times1 + 2\times1$

$= 1 + 1 + \frac{1}{2} + \frac{1}{2} + 2 = \cancel{5}$

$= (4\times2) - (2\times\frac{1}{2}\times1\times1)$

$= 8 - 1 = \underline{7}$

$\int_{-\infty}^{\infty} X(\omega)\,d\omega = 2\pi x(0) = 2\pi(2) = \underline{4\pi}$

Area in one domain, look for signal at '0' in other domain

P4.1.3 $\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) = x(0) = e^{-t}\Big|_{t=0} = \underline{\underline{1}}$

Convergance of F.T

1. F.T is defined for stable & energy s/ls.
2. F.T of power s/l is defined as approximation to energy signals
3. F.T is not defined for NENP s/ls.

$$X(\omega) < \infty$$

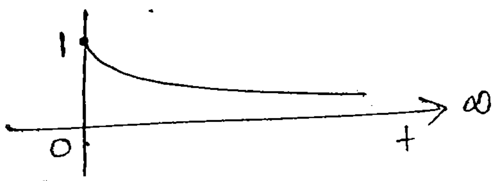
$$\underbrace{\int_{-\infty}^{\infty} |x(t)|\, dt < \infty}_{\text{sufficient}}$$

eg: Cost , u(t) power s/ls
energy $\rightarrow \infty$.

Physically Possible $\Longrightarrow$ FT exists.
(Energy s/ls)
Power s/ls are hardly not physically possible.
No s/l can have ∞ energy.

F.T of standard s/ls.

1. $x(t) = e^{-at} u(t)$.



$$X(\omega) = \int_{-\infty}^{\infty} e^{-at} u(t)\, e^{-j\omega t}\, dt$$

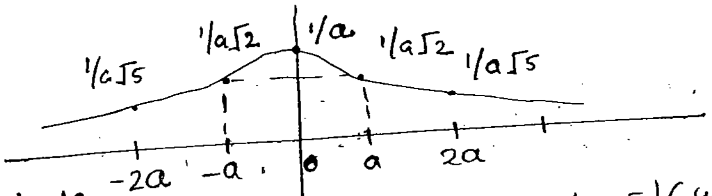
$$= \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \frac{1}{a+j\omega}.$$

$$\boxed{FT\left[e^{-at} u(t)\right] = \frac{1}{a+j\omega}}$$

$$|X(\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$$



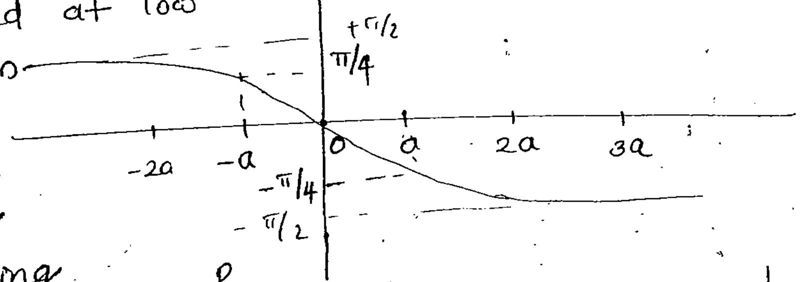
even function

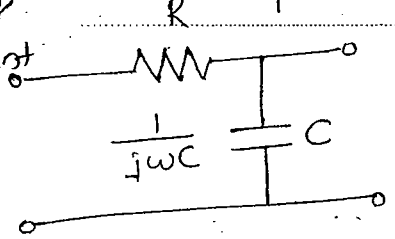
$$\angle X(\omega) = -\tan^{-1}(\omega/a)$$

odd function

conjugate s/m.

Max amplitude is provided at low freq. region for that value of 'a' is properly adjusted using R, L, C Component





$$H(\omega) = \frac{1}{1+j\omega RC}$$

$$= \frac{1}{RC\left(j\omega + \frac{1}{RC}\right)}$$

(Amplitude real No $0 \to \infty$)

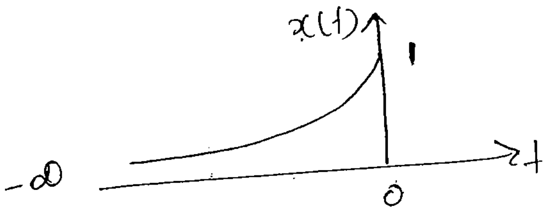
The FT of a real valued s/l exhibits conjugate symmetry (Magnitude Spectrum - Even.
Phase " - odd)

?. $X(\omega) = \frac{1}{\sqrt{a^2+\omega^2}} e^{-j\tan^{-1}(\omega/a)}$

$$|X(\omega)| = \frac{1}{\sqrt{a^2+\omega^2}} \qquad \theta(\omega) = \angle X(\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right)$$

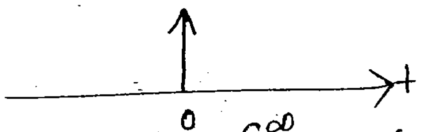
2. $x(t) = e^{at}u(-t)$; $a>0$.

Time reversal $t \to \omega$
$-t \to -\omega$.



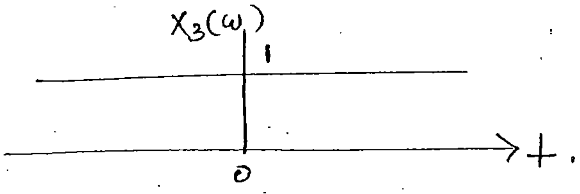
$$F.T\left[e^{at}u(-t)\right] = \frac{1}{a-j\omega}.$$

3. $x_3(t) = \delta(t)$



$$X_3(\omega) = \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t}\, dt = e^{-j\omega t}\Big|_{t=0} = \underline{\underline{1}}$$

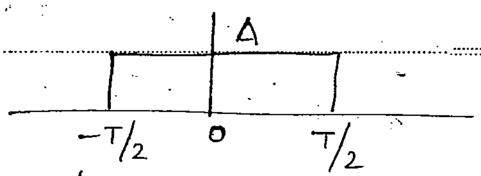
$$F.T\left[\delta(t)\right] = 1$$



Contains all frequency equally.
eg: white Noise.

Any transform of impulse is '1'

4. $x_4(t) = A\, \mathrm{rect}\left(t/T\right)$ (or) $A\,\Pi(t/T)$



$$X_4(\omega) = \int_{-T/2}^{T/2} A \cdot e^{-j\omega t}\, dt = A\left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-T/2}^{T/2}$$

$$= \frac{A}{j\omega}\left[e^{j\omega \frac{T}{2}} - e^{-j\omega\frac{T}{2}}\right]$$

$$X_4(\omega) = \frac{2A}{\omega}\sin\left(\omega\frac{T}{2}\right)$$

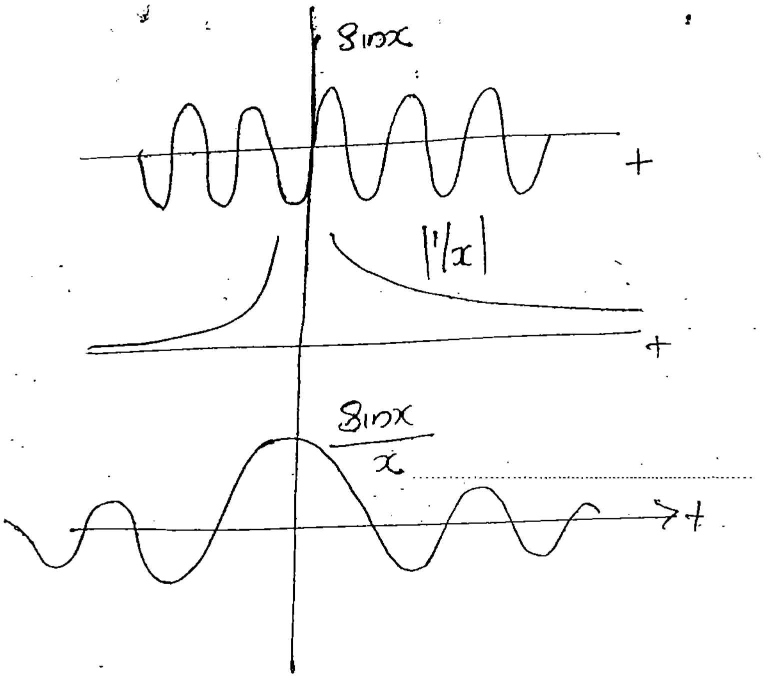
Sin over Argument

$$Sa(x) = \frac{\sin x}{x} \quad \Big| \quad \mathrm{sinc}(x) = \frac{\sin \pi x}{\pi x}.$$

$$\mathrm{sinc}\, x = Sa(\pi x)$$

$$X_4(\omega) = \frac{2A \sin\left(\frac{\omega T}{2}\right)}{\frac{\omega}{2}\cdot\frac{2}{T}} = AT\, Sa\left(\frac{\omega T}{2}\right) \text{ or } AT\, \mathrm{sinc}\left(\frac{\omega T}{2\pi}\right)$$

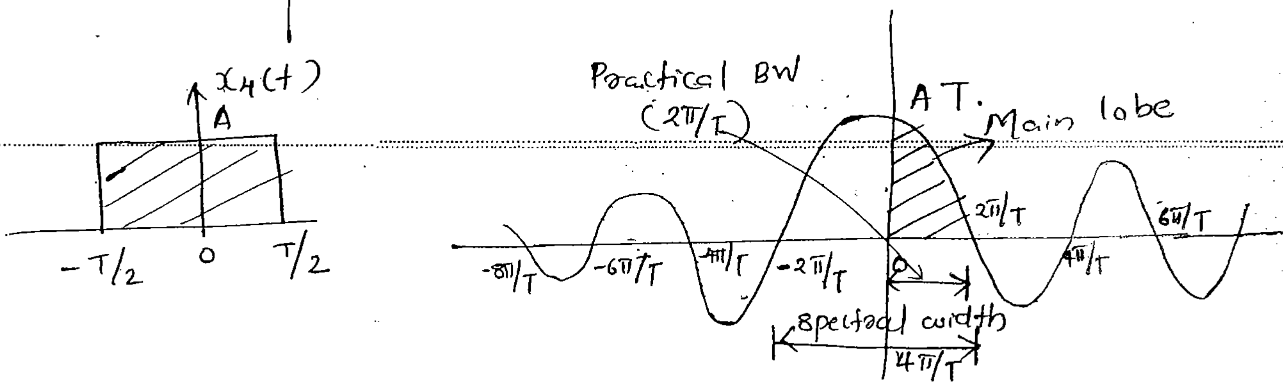
"Rectangle in time domain Corresponds to Sinc function in frequency domain"



$$\sin\frac{\omega T}{2} = 0$$

$$\frac{\omega T}{2} = \pm n\pi$$

$$\omega = \frac{\pm 2n\pi}{T}$$



Area under the s/l = spectrum at zero

= $\underline{\underline{AT}}$.

Ideal BW = infinity

practical BW = $4\pi/T$

Null to Null BW = Zero crossing BW

$= 4\pi/T - 2\pi/T = \underline{\underline{2\pi/T}}$

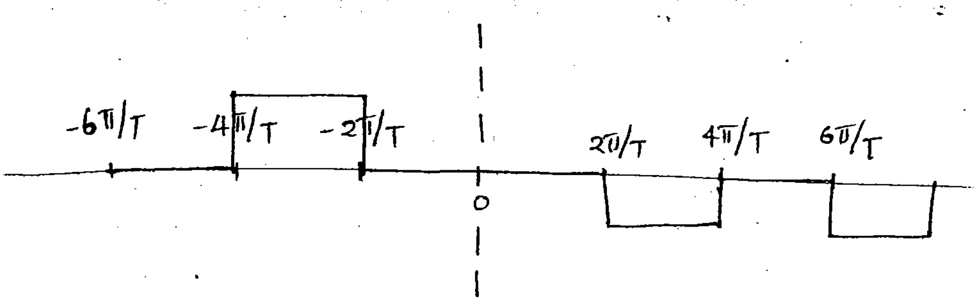
If the s/l is real & even $\rightarrow$ Phase $0°$ & $\pm 180°$.

+ve real $\longrightarrow$ ~~180°~~ $0°$

−ve real $\longrightarrow$ $-180°$

−ve lobe $\longrightarrow$ $\pm 180°$ phase.

+ve lobe $\longrightarrow$ $0°$.



A s/l can't be time limited & band limited simultaneously. $BW \propto \frac{1}{T}$

Narrow band in one domain = wide band in the other domain.

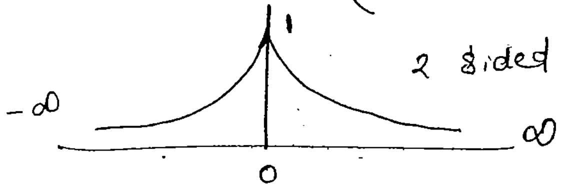
Properties of FT

Linearity

$$a x_1(t) \pm b x_2(t) \longleftrightarrow a X_1(\omega) + b X_2(\omega)$$

Q. * $x_1(t) = e^{-a|t|}$

$$= \begin{cases} e^{-at} & ; t>0 \\ e^{at} & ; t<0 . \end{cases}$$



2 sided exponential

$$x_1(t) = e^{-at}u(t) + e^{at}u(-t)$$

$\downarrow$ FT

$$X_1(\omega) = \frac{1}{a+j\omega} + \frac{1}{a-j\omega} = \frac{2a}{a^2+\omega^2}$$

$$\boxed{FT\left\{e^{-a|t|}\right\} = \frac{2a}{a^2+\omega^2}}$$

IInd Alternative

$$\boxed{Ev\{x(t)\} \longleftrightarrow Re\{X(\omega)\}}$$

$$x(t) = e^{-at}u(t)$$

$$X(\omega) = \frac{1}{a+j\omega} = \frac{a-j\omega}{a^2+\omega^2}$$

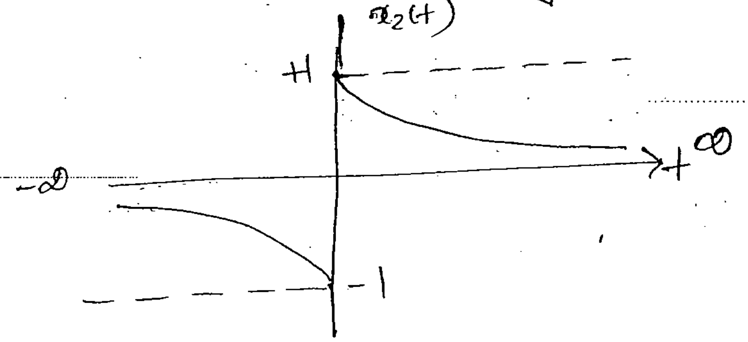
$$x_e(t) = \frac{e^{-at}u(t) + e^{at}u(-t)}{2} \xleftrightarrow{FT} \frac{a}{a^2+\omega^2}$$

$$e^{-a|t|} \longleftrightarrow \frac{2a}{a^2+\omega^2} \qquad x_o(t) = \frac{x(t) - x(-t)}{2}$$

Q. $\rightarrow$ $x_2(t) = e^{-at}u(t) - e^{at}u(-t)$

$$\Big\downarrow FT$$

$$X_2(\omega) = \frac{1}{a+j\omega} - \frac{1}{a-j\omega} = \frac{-2j\omega}{a^2+\omega^2}$$



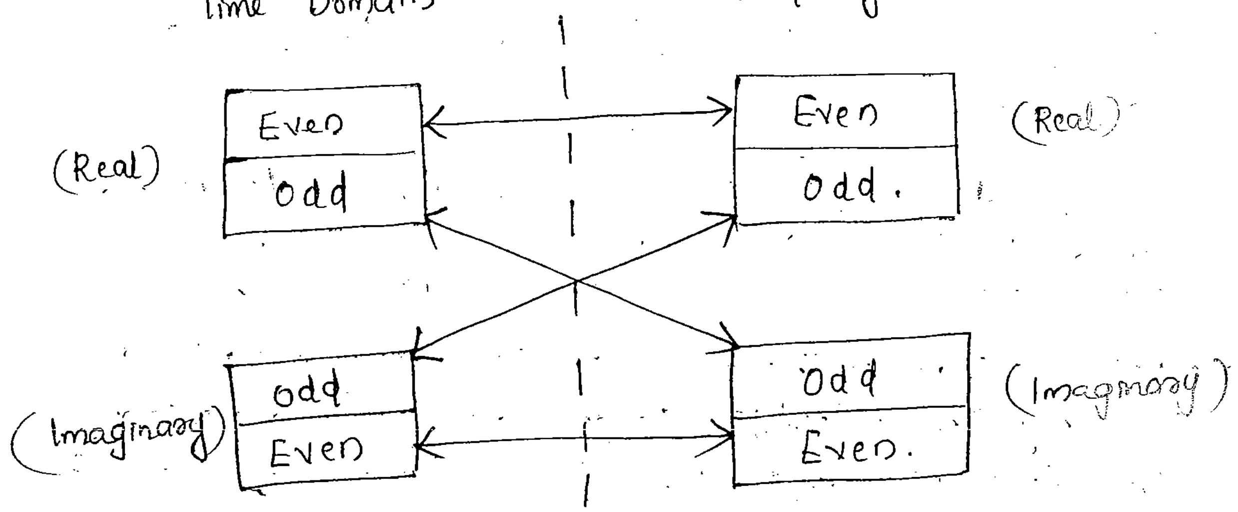
$$\lim_{a\to 0} x_2(t) = \underbrace{u(t) - u(-t)}_{sgn(t)}$$

$$\lim_{a\to 0} X_2(\omega) = \frac{-2j\omega}{\omega} = \frac{2}{j\omega}$$

$$\boxed{sgn(t) \longleftrightarrow \frac{2}{j\omega}}$$

Hilbert transform
90° phase shift

Time Domain — Frequency Domain.



Q. $G(\omega) = \dfrac{\omega^2+21}{\omega^2+9}$

proper s/l $\rightarrow$ powers are equal.
strictly proper $\rightarrow$ Den. should norm numerator.

$\downarrow$ I.F.T

$$\omega^2+9 \overline{\big) \omega^2+21} \big(1$$
$$\underline{\omega^2+9}$$
$$12$$

strictly proper.

Nr power	>	Dr power	$\Rightarrow$	Improper
Nr power	=	Dr power	$\Rightarrow$	Proper
Dr power	>	Nr power	$\Rightarrow$	strictly proper.

$$G(\omega) = 1 + 2\left[\frac{2(3)}{\omega^2+3^2}\right]$$ strictly proper

$\downarrow$ IFT

$$\underline{g(t) = \delta(t) + 2e^{-3|t|}}$$

?. A s/l $x(t)$ is having coefficient C_n with time period T. Find the F.S coefficient of $x(3t-1)$

?. Why the magnitude response tends to '0' for most practical s/ms as $\omega \rightarrow \infty$.

My assumption: Most of the practical s/ms use capacitive coupling. The output is taken across a capacitor, Capacitive reactance $X_C = \frac{1}{\omega C}$; as $\omega \rightarrow \infty$ $X_C \rightarrow 0$
$\therefore$ the o/p magnitude drops to zero.

16/08/2011
8.30 - 1 pm.

Duality

$x(t) \longleftrightarrow X(\omega)$

$X(t) \longleftrightarrow 2\pi x(-\omega)$

Proof:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega) e^{j\omega t} d\omega$$

Replace t by '$-t$'

$$2\pi x(-t) = \int X(\omega) e^{-j\omega t} d\omega$$

$$t \longleftrightarrow \omega$$

$$2\pi x(-\omega) = \int X(t) e^{-j\omega t} dt.$$

eg: $\dfrac{1}{3-jt}$

$$\frac{1}{3-j\omega} \longleftrightarrow e^{3t} u(-t)$$

$$\text{ie } \frac{1}{3-jt} \longleftrightarrow 2\pi e^{3(-\omega)} d\omega.$$

eg: $\dfrac{2}{t^2+1} \longleftrightarrow$

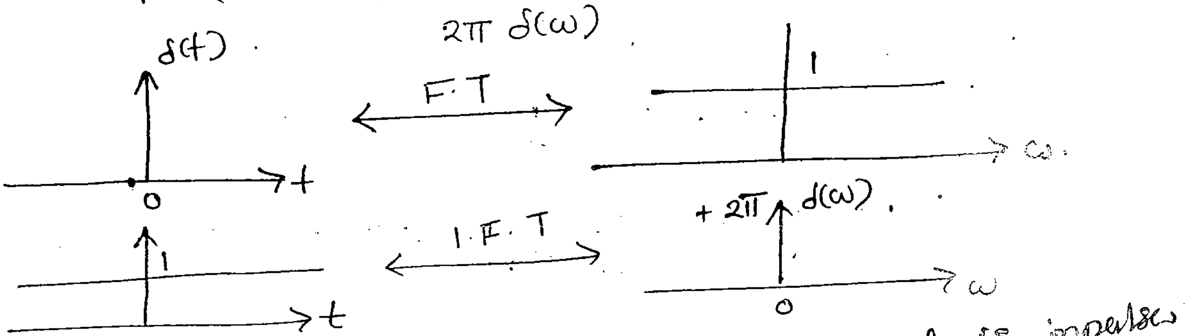
$$e^{-|t|} \longleftrightarrow \frac{2}{\omega^2+1}$$

$$\frac{2}{t^2+1} \xleftrightarrow{\text{Duality}} 2\pi . e^{-|-\omega|}.$$

eg: $\delta(t) \longleftrightarrow 1$

$$1 \xleftrightarrow{\text{Duality}} 2\pi\,\delta(-\omega)$$

$$2\pi\,\delta(\omega)$$



The fourier Transform of of a dc signal is impulse DC occupy zero frequency.

eg: $\operatorname{sgn} t \longleftrightarrow \frac{2}{j\omega}$

dual.

$$\frac{2}{jt} \longleftrightarrow 2\pi \operatorname{sgn}(-\omega)$$

$$\frac{1}{jt} \longleftrightarrow \pi \operatorname{sgn}(-\omega)$$

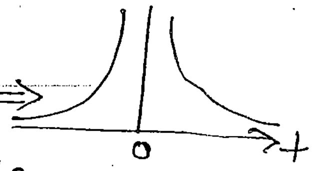
$$\frac{1}{\pi t} \longleftrightarrow -j \operatorname{sgn}(\omega)$$

Hilbert function, provide 90°

$\frac{1}{\pi t} \longrightarrow$ Impulse response of Hilbert transform produces 90° phase shift.

Hilbert Transform

Linear, Time Invariant, Non causal $\Longleftrightarrow$



(since it is LTI s/m response)

(since it is a two sided function)

$\Longrightarrow$ $\operatorname{sgn} t = 2u(t) - 1$

$$u(t) = \frac{1 + \operatorname{sgn} t}{2}$$

$$u(t) \xleftrightarrow{FT} \frac{2\pi\delta(\omega) + 2/j\omega}{2}$$

$$\boxed{u(t) \longleftrightarrow \frac{1}{j\omega} + \pi\delta(\omega).}$$

$\rightarrow$ IFT of $u(\omega)$?

$$u(t) \longleftrightarrow \frac{1}{j\omega} + \pi\delta(\omega)$$

$$2\pi\, u(-\omega) \xleftrightarrow{\text{duality}} \frac{1}{jt} + \pi\delta(t)$$

At $-t \longrightarrow -\omega$.

$$\left[\frac{1}{j(-t)} + \pi\delta(-t)\right]\frac{1}{2\pi} \longleftrightarrow u(\omega)$$

$$\boxed{u(\omega) \longleftrightarrow \frac{1}{2\pi}\left[\frac{1}{j(-t)} + \pi\delta(-t)\right]}$$

Time scaling

$$x(\alpha t) \longleftrightarrow \frac{1}{|\alpha|} X\left(\frac{\omega}{\alpha}\right)$$

Compression $\longleftrightarrow$ Expansion.

$\Rightarrow$ FT of $x_1(t) = A\,\text{rect}\left(\frac{2t}{T}\right)$

$$\because A\,\text{rect}(t/T) \longleftrightarrow AT\,\text{sa}\left(\frac{\omega T}{2}\right)$$

$$x(t) \longleftrightarrow X(\omega)$$

$$A\,\text{rect}\left(\frac{2t}{T}\right) = x(2t)$$

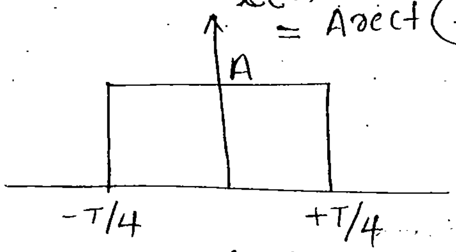
$$\Big\downarrow \text{F.T}$$

$$X_1(\omega) = \frac{1}{2} X\left(\frac{\omega}{2}\right)$$

$$= \frac{AT}{2}\,\text{sa}\left(\frac{\omega \frac{T}{2}}{2}\right)$$

$$= \frac{AT}{2}\,\text{sa}\left(\frac{\omega T}{4}\right) = \frac{AT}{2}\,\frac{\sin\frac{\omega T}{4}}{\frac{\omega T}{4}}$$

$x(2t) = A\,\text{rect}\left(\frac{2t}{T}\right)$



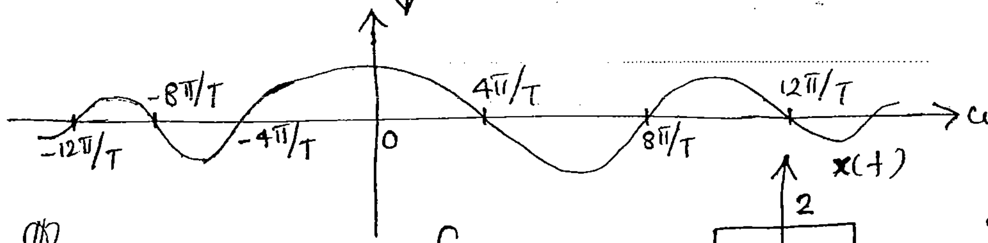
Compression in time domain

$$\sin\frac{\omega T}{4} = 0$$

$$\Rightarrow \frac{\omega T}{4} = \pm n\pi$$

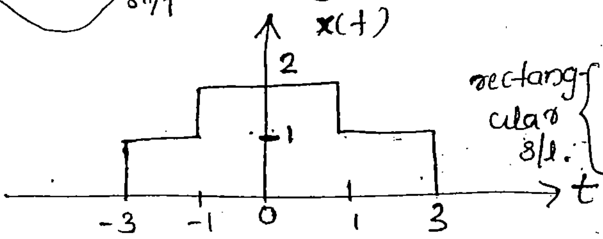
$$\omega = \pm \frac{n4\pi}{T}$$

$$\Big\updownarrow \text{FT}$$



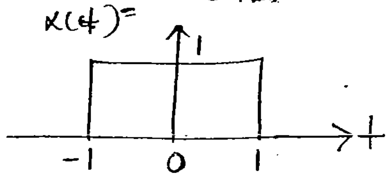
Exponsion in freq. domain.

Q.no. P4.2.4 $\Rightarrow$ FT of

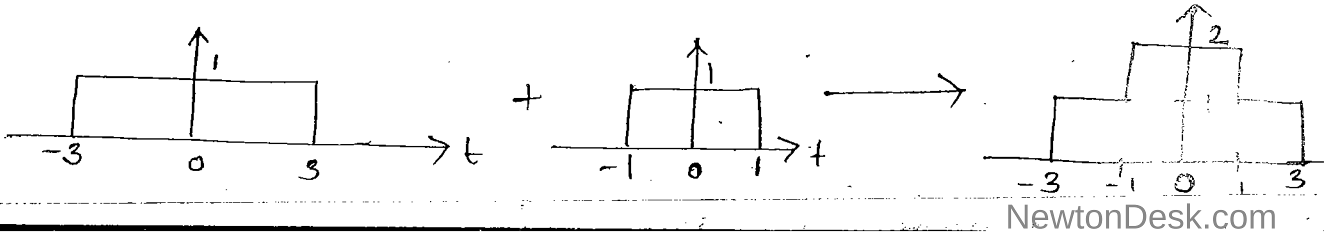


rectangular s/l. { length is finite, Amp. is finite

$x(t) = \text{rect}(t/2)$



$$\longleftrightarrow X(\omega) = 2\,\text{sa}\left(\frac{2\omega}{2}\right)$$



pptg $\longrightarrow$ linearity + scaling.

$$g(t) = x(t) + x(t/3)$$

$$\downarrow F.T$$

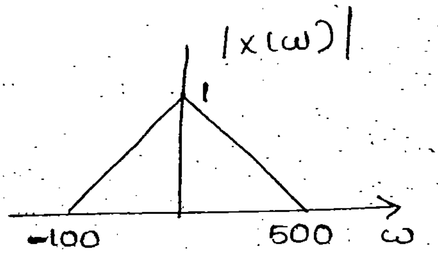
$$Y(\omega) = X(\omega) + \frac{1}{|1/3|} X\left(\frac{\omega}{1/3}\right)$$

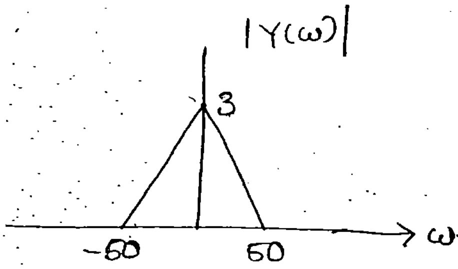
$$= X(\omega) + 3\,X(3\omega)$$

$$= 2\,Sa(\omega) + (3\times2)\,Sa(3\omega)$$

P4.2.5 $\Longrightarrow$

#2.





$$Y(\omega) = 3\,X(3\omega).$$

$$= \frac{1}{|1/3|} X\left(\frac{\omega}{1/2}\right) = \frac{3}{2}\,\frac{1}{|1/2|} X\left(\frac{\omega}{1/2}\right)$$

$$\downarrow IFT$$

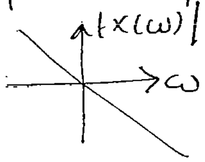
$$g(t) = \frac{3}{2}\,x(t/2)$$

Time - shifting

$$x(t-t_0) \longleftrightarrow e^{-j\omega t_0} X(\omega).$$

Application $\longrightarrow$ Distortionless txn.

linear phase spectrum.



$*$ $\quad g_1(t) = e^{3t} u(-t+1)$

$$g_1(t) = e^{3t} u(-(t-1))$$

$$x(t) = e^{at} u(t) \longleftrightarrow \frac{1}{a+j\omega}$$

$$x(t-t_0) = e^{a(t-t_0)} u(t-t_0)$$

$$g_1(t) = e^{3t} u(-(t-1))$$

$$= e^{3(t-1)} \cdot e^{3}\, u(-(t-1))$$

$$= e^{3}\, e^{3(t-1)}\, u(-(t-1))$$

$$\xleftrightarrow{F.T} e^{3}\left[\frac{e^{-j\omega(1)}}{3-j\omega}\right]$$

* $y_2(t) = rect\left(\frac{t+3}{4}\right)$ (only shifting)

$$x(t) = A\,rect(t/T) \xleftrightarrow{F.T} X(\omega) = AT\ Sa\left(\frac{\omega T}{2}\right)$$

$$x(t-t_0)\ A\,rect\left(\frac{t-t_0}{T}\right) \xleftrightarrow{F.T} e^{-j\omega t_0}\ AT\ Sa\left(\frac{\omega T}{2}\right)$$

$$rect\left(\frac{t-(-3)}{4}\right) \xleftrightarrow{F.T} e^{-j\omega(-3)}(1)(4)\ Sa\left(\frac{\omega(4)}{2}\right)$$

$$= e^{j3\omega}\,4 \cdot Sa(2\omega)$$

$$= 4\,Sa(2\omega)\,e^{j3\omega}.$$

* $y_3(t) = \delta(2t+3)$

$$= \delta(2(t+3/2)) = \frac{1}{2}\delta(t+3/2)$$

$$\xleftrightarrow{F.T} \frac{1}{2}e^{-j3\frac{\omega}{2}}$$

Alternate $y_3(t) = \delta(2(t+3/2))$

$$= \frac{1}{2}\delta(t+3/2)$$

$$\xleftrightarrow{F.T} \frac{1}{2}e^{+j\,3/2(\omega)}(1)$$

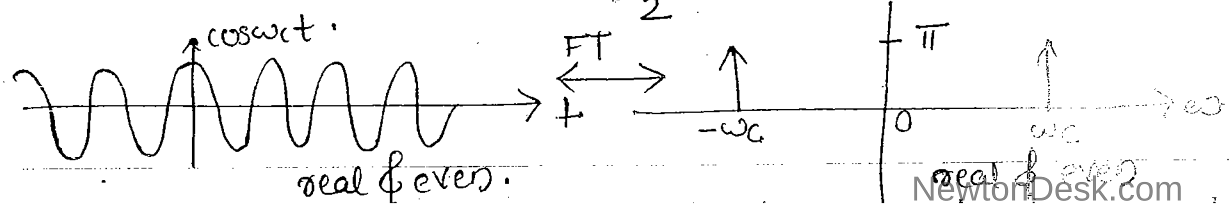
$$= \frac{e^{+\frac{j3\omega}{2}}}{2}$$

Frequency shifting / Modulation Property.

$$\boxed{x(t)e^{j\omega_c t} \longleftrightarrow X(\omega-\omega_c).}$$

* $\cos\omega_c t = 1.\dfrac{(e^{j\omega_c t} + e^{-j\omega_c t})}{2}$ $\quad 1 \longleftrightarrow 2\pi\delta(\omega)$

$$\xleftrightarrow{FT} \frac{2\pi\delta(\omega-\omega_c) + 2\pi\delta(\omega+\omega_c)}{2}$$



* $\sin\omega_c t = \frac{e^{j\omega_c t} - e^{-j\omega_c t}}{2j}$

$$\Big\downarrow F.T$$

$$= \frac{\pi}{j}\left[\delta(\omega-\omega_c) - \delta(\omega+\omega_c)\right]$$

* $e^{-3t}\sin 6t\, u(t)$. damped sinusoidal waveform.

  $ref = e^{-3t}u(t)$.

* $rect\left(\frac{t}{4}\right)\cos 10t$ RF pulse.

Any signal multiplied by cos, sin or exponential apply freq. shift operation.

* $y(t) = \frac{\cos 2t}{t^2+1}$

$$\underset{x(t)}{\frac{2}{t^2+1}} \xleftrightarrow{F.T} \underset{X(\omega)}{2\pi e^{-|\omega|}}.$$

$$y(t) = x(t)\left(\frac{e^{j2t}+e^{-j2t}}{2}\right)$$

$$\Big\downarrow F.T$$

$$= \frac{X(\omega-2)+X(\omega+2)}{2} = \frac{2\pi}{2}\left[e^{-|\omega-2|} + e^{-|\omega+2|}\right]$$

$$y(t) = \pi\left[e^{-|\omega-2|} + e^{-|\omega+2|}\right].$$

* I.F.T of $X(4\omega+3)$?

$$X(4\omega+3) = X(4(\omega+3/4)) = X\left(\frac{(\omega+3/4)}{1/4}\right)$$

$$= \frac{1}{4(1/4)}X\left(\frac{(\omega+3/4)}{1/4}\right)$$

$$\Big\downarrow I.F.T$$

$$= \frac{1}{4}x(t/4)$$

$$= \frac{1}{4}e^{-j3/4t}x(t/4)$$

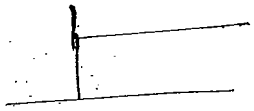
Differentiation in Time.

$$\frac{d\,x(t)}{dt} \longleftrightarrow j\omega\, X(\omega).$$

DC Component is lost.

$$- \quad X(\omega) \underset{\text{Not guaranteed}}{\neq} \frac{F\left\{\frac{d\,x(t)}{dt}\right\}}{j\omega}.$$

eg: $x(t) = u(t)$



$$\frac{d\,x(t)}{dt} = \delta(t)$$

$u(0) = 1/2$. while differentiation it is considered to be

$\downarrow$ F.T

$$j\omega X(\omega) = 1$$

$$X(\omega) = \frac{1}{j\omega} \quad \text{(Not Correct).}$$

discontinuity is not Considered.

eg: $sgn(t)$

$$x(t) = sgn(t) = 2u(t) - 1$$

$$\frac{d\,x(t)}{dt} = 2\delta(t)$$

$\downarrow$ F.T

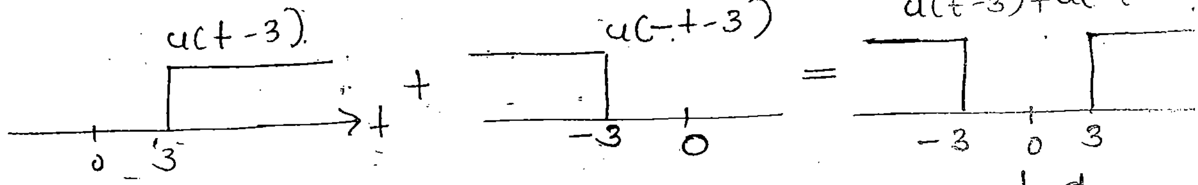
$$j\omega X(\omega) = 2(1)$$

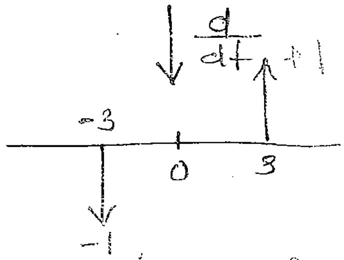
$$X(\omega) = 2/j\omega \quad \text{(Correct)},$$

At discontinuit value is already $\underline{\underline{sgn(0) = 0}}$.

Differentiation will miss the DC Component.

* Find the F.T $y(t) = \frac{d}{dt}\left[u(t-3) + u(-t-3)\right]$





$\delta(t-t_0)$
$\downarrow$ F.T
$e^{-j\omega t_0}(1)$

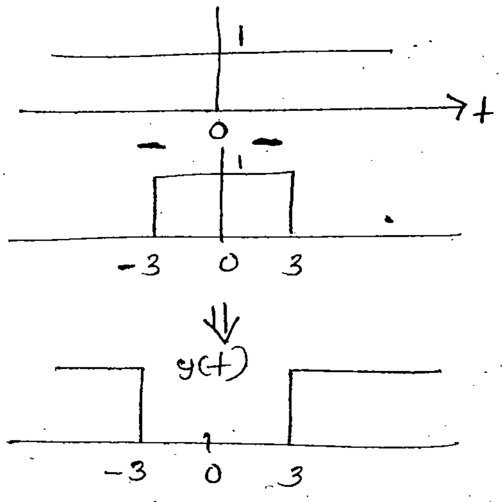
$$y(t) = \delta(t-3) - \delta(t+3)$$

$\downarrow$ F.T

$$Y(\omega) = e^{-j3\omega}(1) - e^{j3\omega}(1)$$

$$= -\left(\frac{e^{j3\omega} - e^{-j3\omega}}{2j}\right) \times \frac{1}{2j} = -2j\sin(3\omega) = \frac{-2j}{1}\sin(3\omega)$$

Alternate Method :



$$y_1(t) = 1 - \mathrm{rect}(t/6)$$

$$y(t) = \frac{d}{dt} y_1(t)$$

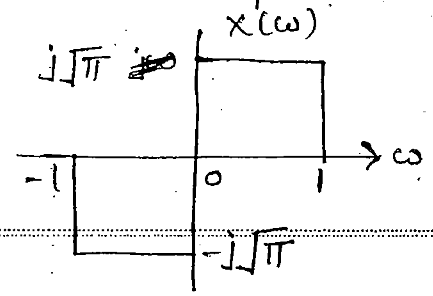
$$= \frac{d}{dt}\left[1 - \mathrm{rect}(t/6)\right]$$

$$= -\frac{d}{dt}\left[\mathrm{rect}(t/6)\right]$$

$$= -j\omega\,(1)(6)\,Sa\left(\frac{6\omega}{2}\right)$$

$$= \underline{-j6\omega\, Sa(3\omega)}$$

* For the spectrum shown in fig find $\left.\frac{dx(t)}{dt}\right|_{t=0}$



One domain at origin = Area at the other domain

$$x(t) = \frac{1}{2\pi}\int X(\omega)\, e^{j\omega t}\, d\omega$$

$$\Rightarrow \frac{dx(t)}{dt} = \frac{1}{2\pi}\int j\omega X(\omega)\, e^{j\omega t}\, d\omega$$

$$\left.\frac{dx(t)}{dt}\right|_{t=0} = \frac{1}{2\pi}\int_{-1}^{1} j\omega X(\omega)\, d\omega$$

$$= \frac{j}{2\pi}\left[\int_{-1}^{0} \omega \cdot -j\sqrt{\pi}\, d\omega + \int_{0}^{1} \omega \cdot j\sqrt{\pi}\, d\omega\right]$$

$$= \frac{-\sqrt{\pi}}{2\pi}\left[-\int_{-1}^{0} \omega\, d\omega + \int_{0}^{1} \omega\, d\omega\right]$$

$$= -\frac{1}{2\sqrt{\pi}}\left[-\left[\frac{\omega^2}{2}\right]_{-1}^{0} + \left[\frac{\omega^2}{2}\right]_{0}^{1}\right]$$

$$= -\frac{1}{2\sqrt{\pi}}\left[-\frac{1}{2}\left[0-1\right] + \frac{1}{2}\left[1-0\right]\right]$$

$$= -\frac{1}{2\sqrt{\pi}}\left[\frac{1}{2} + \frac{1}{2}\right] = \underline{-\frac{1}{2\sqrt{\pi}}}$$

Differentiation in Frequency

$$-jt\,x(t) \longleftrightarrow \frac{d}{d\omega}X(\omega)$$

$$\frac{d}{dt} \longrightarrow j\omega \quad , \quad \frac{d}{d\omega} \longrightarrow -jt$$

Differentiation w.r.p to one variable is multiplication in other domain by oposite variable.

* $y(t) = t\,e^{-at}u(t)$

$= t\,x(t)$

$\downarrow$ F.T

$$Y(\omega) = j\frac{d}{d\omega}\left[\frac{1}{a+j\omega}\right] = j\,\frac{-1\cdot(0\times j(1))}{(a+j\omega)^2} = \frac{1}{(a+j\omega)^2}$$

* I.F.T of $Y(\omega) = j\frac{d}{d\omega}\left[\frac{e^{j2\omega}}{1+j\omega/3}\right]$. $y(t) = ?$

$y(t) = t\cdot x(t)$

$= \cancel{t\,e^{(\frac{a}{3}+\frac{3}{2})t}u(t)} = \cancel{t\,e^{(\frac{3a}{3}+2)t}u(t)}$

$$Y(\omega) = j\frac{d}{d\omega}\left[\frac{e^{j2\omega}}{\frac{1}{3}(3+j\omega)}\right] = 3j\frac{d}{d\omega}\left[\frac{e^{j2\omega}}{3+j\omega}\right]$$

$$= 3j\frac{d}{d\omega}[Z(\omega)]$$

$$Z(\omega) = \frac{e^{j2\omega}}{3+j\omega} = e^{j2\omega}\frac{1}{3+j\omega}$$

$$= e^{j2\omega}X_1(\omega).$$

$$X_1(\omega) = \frac{1}{3+j\omega} \Longrightarrow x_1(t) = e^{-3t}u(t)$$

$$e^{j2\omega}X_1(\omega) \xrightarrow{\text{I.F.T}} x_1(t+2) = e^{-3(t+2)}u(t+2)$$

$$z(t) \longrightarrow \text{I.F.T}(Z(\omega)) = e^{-3(t+2)}u(t+2)$$

$$Y(\omega) = 3j\frac{d}{d\omega}[Z(\omega)] \longleftrightarrow 3t\,z(t).$$

$$y(t) = \text{IFT}[Y(\omega)] = 3t\,e^{-(t+2)3}u(t+2)$$

* Find the F.T of following signals in terms of $X(\omega)$?

a) $x_1(t) = x(2-t) + x(-t-2)$

$= x(-(t-2)) + x[-(t+2)]$

$X_1(\omega) = [e^{-j2\omega} + e^{j2\omega}]\, X(-\omega)$

b) $x_2(t) = x(3t-6) = x[3(t-2)]$

$X_2(\omega) = \frac{1}{3} X\left(\frac{\omega}{3}\right) e^{-j\omega(2)}$

c) $x_3(t) = \frac{d^2}{dt^2} x(t+2)$

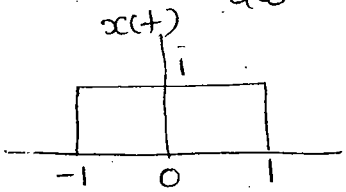
$X_3(\omega) = (j\omega)^2 e^{-j\omega(-2)} X(\omega)$

d) $x_4(t) = t \frac{d}{dt} x(t)$

$= t\, y(t)$

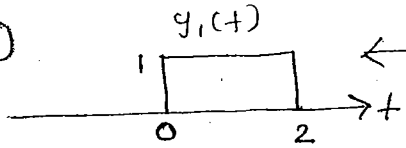
$X_4(\omega) = j \frac{d}{d\omega} Y(\omega) = j \frac{d}{d\omega} [(j\omega) X(\omega)]$

*



$\longleftrightarrow \frac{2 \sin \omega}{\omega}$

①



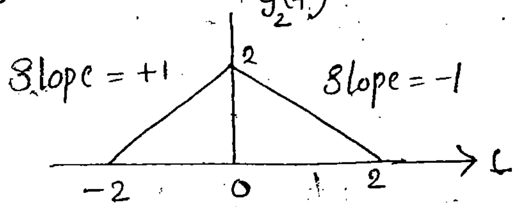
$\longleftrightarrow$

$y_1(t) = x_1(t-1)$

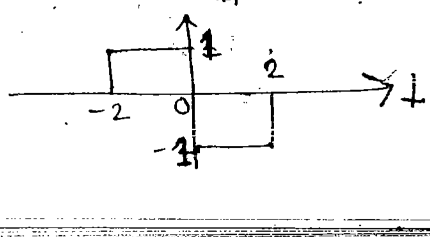
$\downarrow$ FT

$Y_1(\omega) = e^{-j\omega(1)} X(\omega) = e^{-j\omega(1)} \left[\frac{2 \sin \omega}{\omega}\right]$

②



$\downarrow \frac{d}{dt}$



$\frac{dy_2(t)}{dt} = y_1(-t) - y_1(t)$

$\downarrow$ F.T

$j\omega Y_2(\omega) = Y_1(-\omega) - Y(\omega)$

$Y_2(\omega) = \frac{1}{j\omega}\left[e^{j\omega} - e^{-j\omega}\right] \frac{2 \sin \omega}{\omega}$

$$\Rightarrow Y_2(\omega) = 2\frac{\sin\omega}{\omega} \cdot \frac{2\sin\omega}{\omega}$$

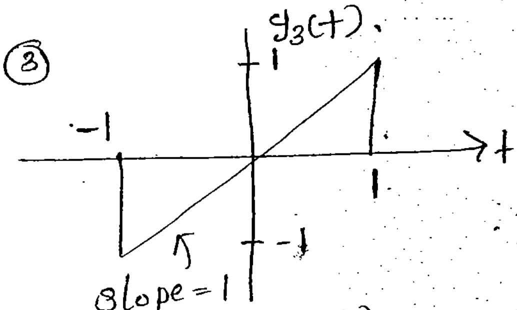
$$= \left(\frac{2\sin\omega}{\omega}\right)^2$$

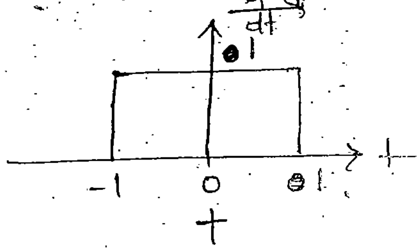
Fourier Transform of triangular function

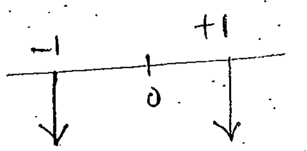
$$= (\operatorname{sinc}())^2$$

The high frequency components are concentrated by the differentiation. So more energy will be there in the high freq. component & it is not possible to avoid the high freq components.

* (3)







Impulse due to discontinuity at $t=-1$ and $t=+1$

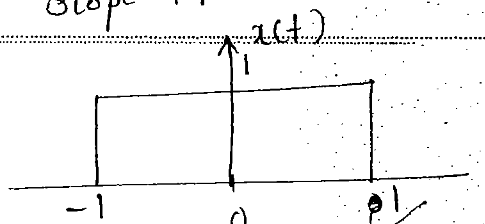
*
?

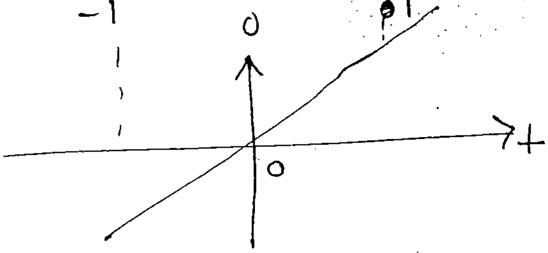
$\frac{d}{dt} y_3(t) = x(t)$

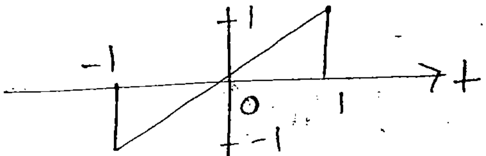
$\downarrow$ F.T

$j\omega \cdot Y_3(\omega) = X(\omega) = \frac{2\sin\omega}{\omega}$

$Y_3(\omega) = \frac{1}{j\omega} \cdot \frac{2\sin\omega}{\omega}$



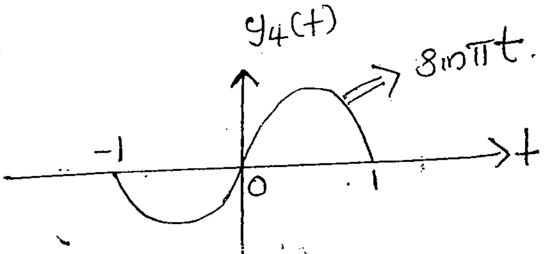


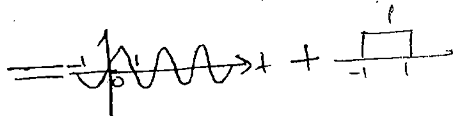


$$y_3(t) = t\, x(t) \xleftrightarrow{F.T} Y_3(\omega) = j\frac{d}{d\omega}X(\omega)$$

$$Y_3(\omega) = j\frac{d}{d\omega}\left[\frac{2\sin\omega}{\omega}\right]$$

(4)

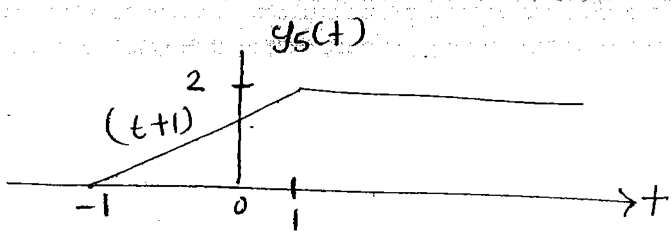




$$y_4(t) = x(t)\sin\pi t = x(t)\left[\frac{e^{j\pi t} - e^{-j\pi t}}{2j}\right]$$

$$\xleftrightarrow{X(\omega)} \frac{1}{2j}\left[X(\omega-\pi) - X(\omega+\pi)\right]$$

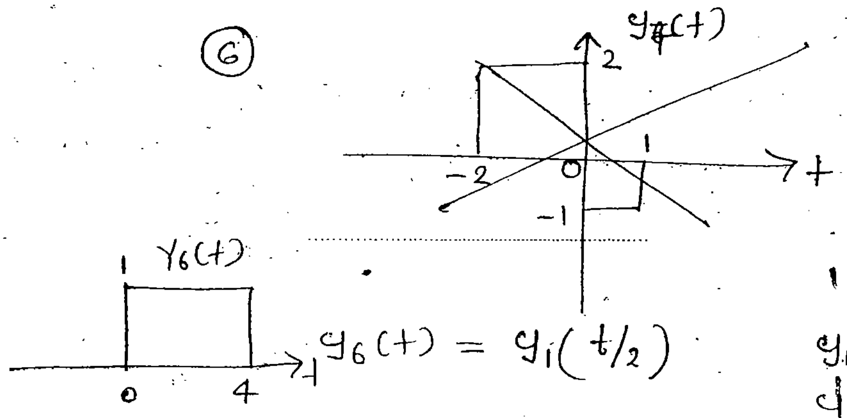
(5)



$$y_5(t) = (t+1)x(t) + 2u(t-1)$$

due to diff. the dc component will be lost.
so diff. technique is not used here.

(6)

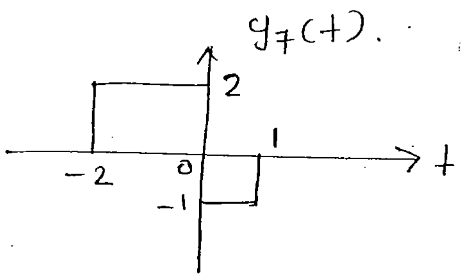


$$y_6(t) = y_1(t/2) \qquad y_1(t) = x(t-1)$$
$$y_1(t/2) = x(t/2 - 1)$$

$$Y_6(\omega) = \frac{1}{|1/2|} Y_1\left(\frac{\omega}{1/2}\right)$$

$$= 2\,Y_1(2\omega)$$

(7)

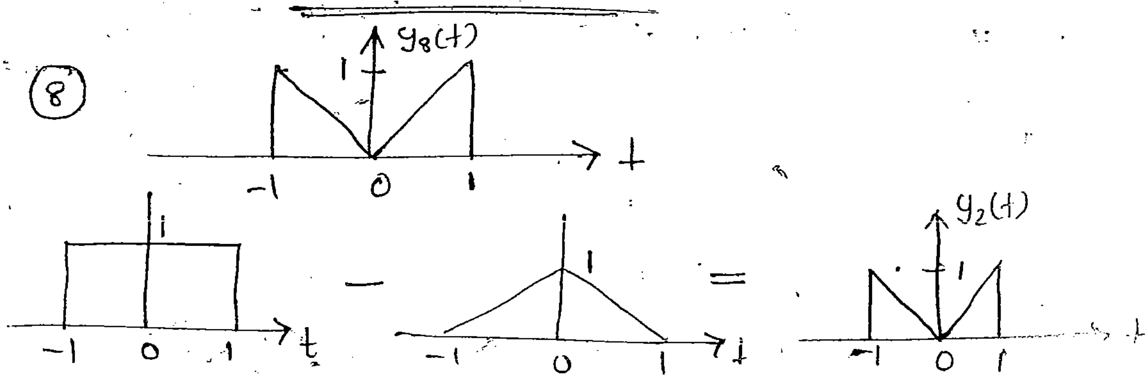


$$Y_7(t) = y_1(-t) - y_1(2t)$$

$\downarrow$ F.T

$$Y_7(\omega) = Y_1(-\omega) - \frac{1}{2} Y_1\left(\frac{\omega}{2}\right)$$

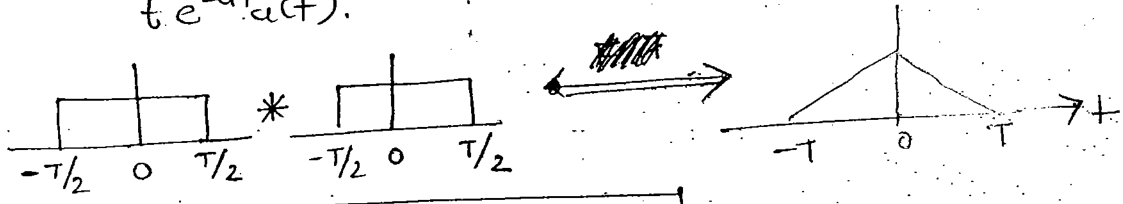
(8)



Convolution in Time

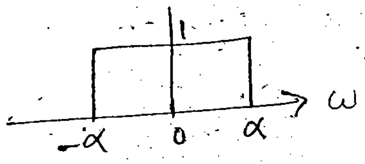
$$\boxed{x(t) * h(t) \longleftrightarrow X(\omega)\, H(\omega).}$$

eg: $\underbrace{e^{-at}u(t) * e^{-at}u(t)}_{t\,e^{-at}u(t).} \xleftrightarrow{F.T} \dfrac{1}{(a+j\omega)^2}$



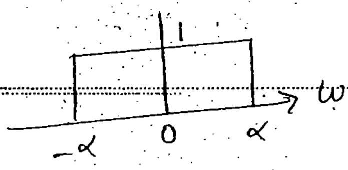
$$\boxed{\frac{\sin\alpha t}{\pi t} * \frac{\sin\alpha t}{\pi t} = \frac{\sin\alpha t}{\pi t}.}$$

$\dfrac{\sin\alpha t}{\pi t} \xleftrightarrow{F.T}$

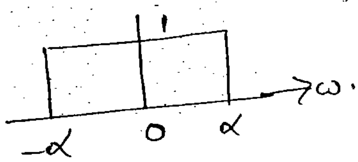


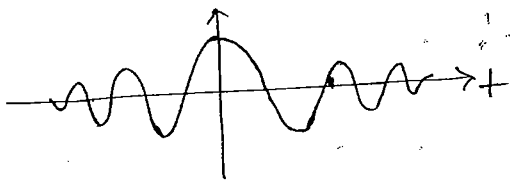
$*$

$\dfrac{\sin\alpha t}{\pi t} \xleftrightarrow{F.T}$

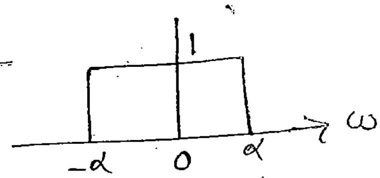


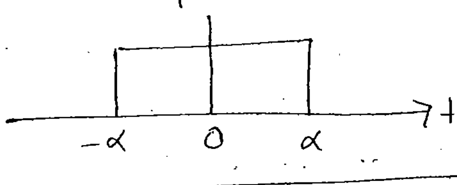
$\dfrac{\sin\alpha t}{\pi t.} \xleftarrow{I.F.T}$



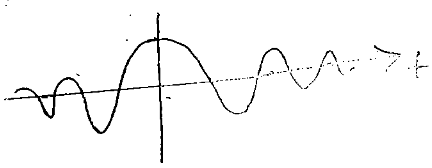


$\xleftrightarrow{\text{Duality}}$





$\longleftrightarrow$



$$\boxed{\begin{aligned} \delta(t) * \delta(t) &= \delta(t) \\ e^{-at} * e^{-at} &= e^{-at} \end{aligned}}$$

* Given $y(t) = x(t) * h(t)$ and $g(t) = x(2t) * h(2t)$ such that $g(t) = A\,y(Bt)$. Find A and B

$$y(t) \longleftrightarrow Y(\omega)$$

$$Y(\omega) = X(\omega)\,H(\omega)$$

$$= \frac{1}{2} X(\omega/2) \cdot \frac{1}{2} H(\omega/2)$$

$$= \frac{1}{4} X(\omega/2)\, H(\omega/2)$$

$$g(t) = \frac{1}{2} y(2t) = A\,y(Bt)$$

$$A = \frac{1}{2}; \quad B = 2$$

* An LTI system is having an impulse response $h(t) = \frac{\sin 3\pi t}{\pi t}$. For which the i/p applied is $\cos \pi t + \sin 4\pi t$. Find the output?

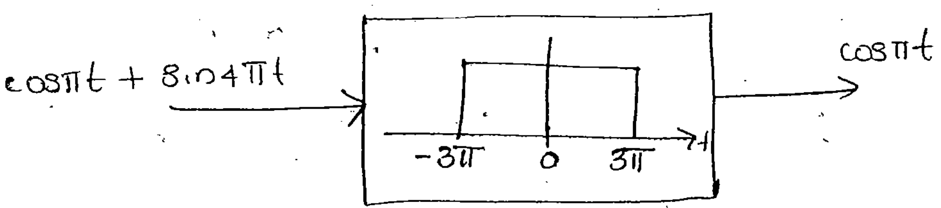
The fourier transform of sinc is rectangular.

$$y(t) = x(t) * h(t).$$

$$Y(\omega) = X(\omega) \cdot H(\omega).$$

$$h(t) = \frac{\sin \omega t}{\omega}$$

$$\omega = 3\pi$$



i/p is sinusoidal = o/p is sinusoidal.

sinc in time domain = rectangle in frequency domain.

* Given the i/p signal spectrum is $X(\omega) = \delta(\omega) + \delta(\omega - \pi) + \delta(\omega - 5)$ & the impulse response $h(t) = u(t) - u(t-2)$.

(i) Is $x(t)$ periodic?

(ii) Is $x(t) * h(t)$ periodic?

Solution:

(i)

$$1 \longleftrightarrow 2\pi\,\delta(\omega)$$

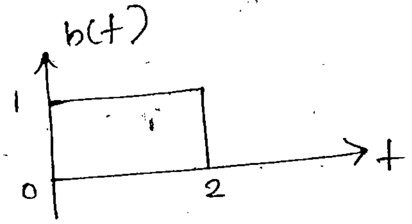
$$e^{j\omega_0 t} \longleftrightarrow 2\pi\,\delta(\omega - \omega_0)$$

$1, + \sin\pi t \cdot \sin 5t$

$$x(t) = \frac{1}{2\pi} + \frac{1}{2\pi} e^{j\pi t} + \frac{1}{2\pi} e^{j5t}$$

$\omega_0 = GCD(1, \pi, 5)$ ∴ Non periodic (Not possible to get integer value of fundamental)

(ii)



$$\xleftrightarrow{F.T} \frac{2\sin\omega}{\omega} e^{-j\omega}$$

spectrum $= 0$; $\omega = \pm n\pi$

$$y(t) = x(t) * h(t)$$

$\downarrow$ FT

$$Y(\omega) = X(\omega) H(\omega)$$

$X(\omega)\,\delta(\omega-\omega_0)$

$= X(\omega_0)\,\delta(\omega-\omega_0)$

$$= \frac{2\sin\omega}{\omega} e^{-j\omega}\left(\delta(\omega) + \delta(\omega-\pi) + \delta(\omega-5)\right)$$

$$= 2\delta(\omega) + \frac{2\sin 5}{5} e^{-j5}\,\delta(\omega-5) + \underbrace{\frac{2\sin\pi}{\pi} e^{j\pi}\,\delta(\omega-\pi)}_{\text{is zero}}$$

$$= 2\delta(\omega) + \frac{2\sin 5}{5} e^{-j5}\,\delta(\omega-5)$$

$\delta(\omega) \rightarrow \omega = 0$, $\delta(\omega-5) \rightarrow \omega = 5$

$\omega_0 = GCD(0, 5) =$ periodic

Convolution two non periodic signals may become periodic.

Convolution in Frequency

$$\boxed{x_1(t)\, x_2(t) \xleftrightarrow{F.T} \frac{1}{2\pi}\left[X_1(\omega) * X_2(\omega)\right]}$$

Application :→
* Can be used as modulators
* Get samples in frequency.

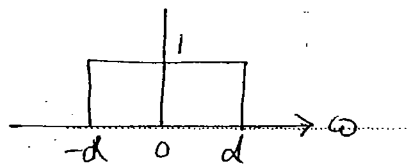
eg: $x(t)\cos\omega_c t \longleftrightarrow \frac{1}{2\pi}\left[X(\omega) * \pi\left[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)\right]\right]$

$$\longleftrightarrow \frac{X(\omega-\omega_c)+X(\omega+\omega_c)}{2}$$

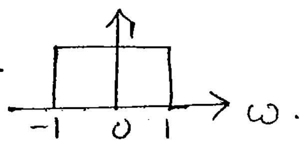
* Find the FT of $\frac{\sin t \,\sin(t/2)}{\pi t^2}$

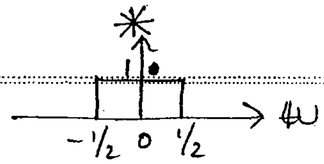
$$\frac{\sin t\,\sin(t/2)}{\pi t^2} = \left[\frac{\sin t}{\pi t}\right]\left[\frac{\sin(t/2)}{\pi t}\right]\pi$$

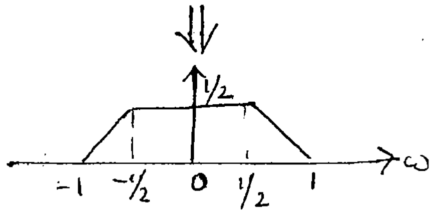
$$\frac{\sin\alpha t}{\pi t} \longleftrightarrow$$



$$\left[\frac{\sin t}{\pi t}\right]\left[\frac{\sin(t/2)}{\pi t}\right]\pi \longleftrightarrow$$







* A signal spectrum satisfy the relation $\ln(X(\omega)) = -|\omega|$. Find the corresponding s/l if (i) $x(t)$ is real and even? (ii) $x(t)$ is real and odd?

two sided exponential formula

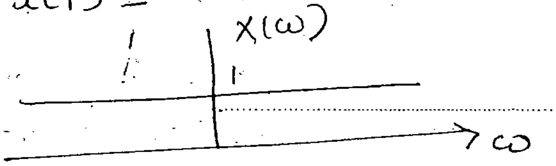
Integration in time.

$$\int_{-\infty}^{t} x(\tau)\, d\tau \longleftrightarrow \frac{X(\omega)}{j\omega} + \pi X(0)\,\delta(\omega) \quad \text{if } X(0) \neq 0$$

$$x(t) * \int_{-\infty}^{t} \underbrace{\delta(\tau)\, d\tau}_{u(t)} \xleftrightarrow{F.T} X(\omega)\left[\frac{1}{j\omega} + \pi\delta(\omega)\right]$$

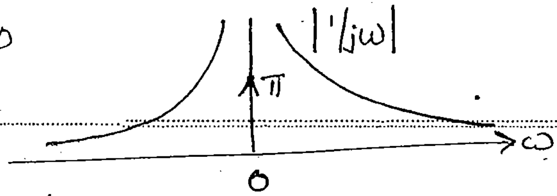
$$\frac{X(\omega)}{j\omega} + \frac{\pi X(\omega)\delta(\omega)}{\pi X(0)\,\delta(\omega)}$$

* $x(t) = \delta(t)$

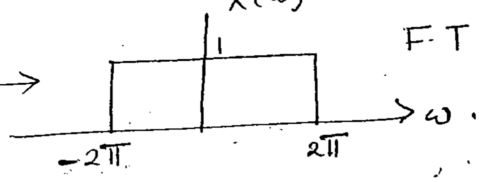


- spectrum at zero should be defined
- Low frequency Compo... are concentrated integration. (Max amplitudes a... in low freq. regio...

$$\int_{-\infty}^{t} \delta(\tau)\, d\tau \longleftrightarrow \frac{1}{j\omega} + \pi(1)\,\delta(\omega)$$

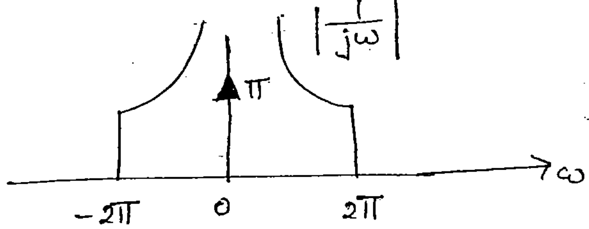


* $$\int_{-\infty}^{t} \frac{\sin 2\pi\tau}{\pi\tau}\, d\tau \longleftrightarrow$$



F.T of $\frac{\sin 2\pi\tau}{\pi\tau}$ (s... f...

$$\therefore \int_{-\infty}^{t} \frac{\sin 2\pi\tau}{\pi\tau}\, d\tau \xleftrightarrow{F.T} \frac{\text{rect}(\omega/4\pi)}{j\omega} + \pi(1)\,\delta(\omega).$$



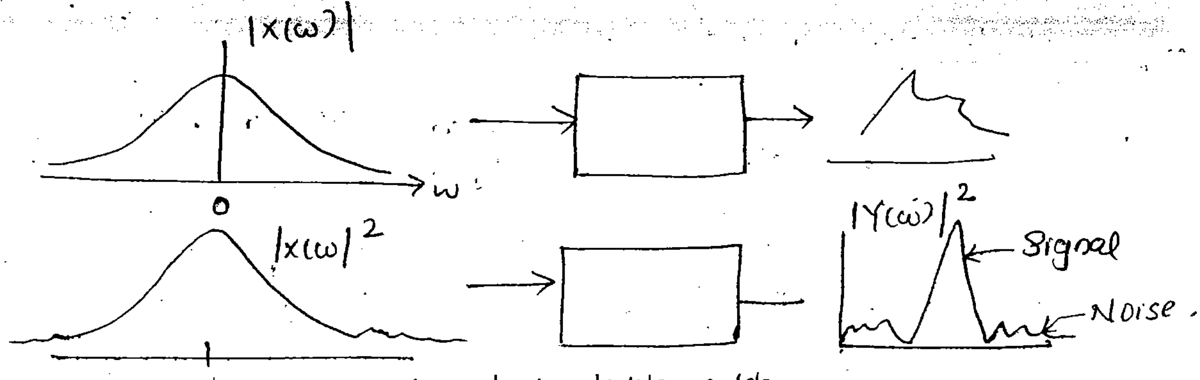
Band limited to -2π to $+2\pi$.

Rayleigh's Energy theorem (or) Parseval's theorem.

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \underbrace{|X(\omega)|^2}_{\text{spectral density}} d\omega$$

E.S.D / P.S.D

Area under spectral density energy / power



Amplitude levels match

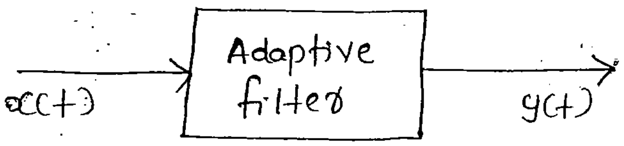
Peak Amplitude $\longrightarrow$ Represents Signal

Amp. levels will not match.

less Amplitude $\longrightarrow$ Respresents Noise.

Adaptive Filter $\rightarrow$ Internal filter Coefficients are changing based on the desired o/p. It is using in all practical application.

Introduced by Wiener / kalman

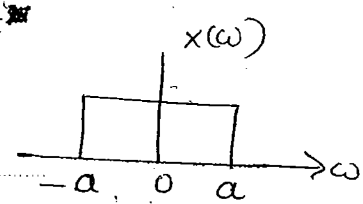


Where the i/p is to be matched, filter coefficien.. are made one. otherwise it made zero.

Q.
P4.2.29

Energy $x(t) = \dfrac{\sin at}{\pi t}$ (Peak Amp $= \frac{a}{\pi}$)

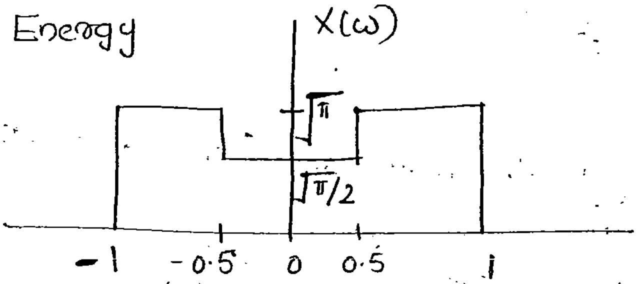
$\dfrac{\sin at}{\pi t} \overset{F.T}{\longleftrightarrow}$



$$E_{x(t)} = \frac{1}{2\pi}\int_{-a}^{a} 1\, d\omega = \frac{a}{\pi}.$$

P4.2.30
Q.

Energy

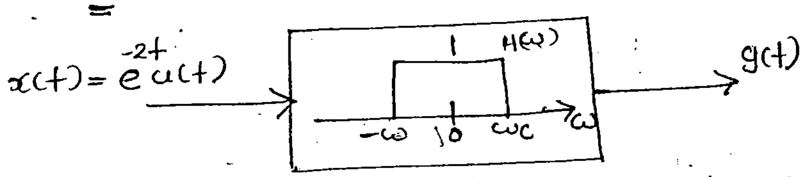


$$E_{x(t)} = 2 \times \frac{1}{2\pi}\int_0^1 |X(\omega)|^2 d\omega = \frac{1}{\pi}\left[\int_0^{0.5} \frac{\pi}{4}\, d\omega + \int_{0.5}^{1} \pi\, d\omega\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi}{4} \times 0.5 + \pi \times 0.5\right] = 5/8$$

4.2.31

Q.



$$E_{g(t)} = \frac{1}{2} E_{x(t)}.$$ Calculate ωc?

$$x(\omega) = \frac{1}{2+j\omega}$$

Solution ✓ $Y(\omega) = X(\omega) \cdot H(\omega)$.

$$= \frac{1}{2+j\omega} \cdot 1 = \frac{1}{2+j\omega} ; \; |\omega| < \omega c.$$

✓ $$E_{x(t)} = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_0^{\infty} |e^{-2t}|^2 dt = \left[\frac{e^{-4t}}{-4}\right]_0^{\infty}$$

$$= -\frac{1}{4}\left[e^{-\infty} - e^0\right] = \frac{1}{4} \text{ Watts.}$$

✓ $$E_{g(t)} = \frac{1}{2} E_{x(t)} = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}.$$

$$|Y(\omega)| = \frac{1}{\sqrt{4+\omega^2}}$$

✓ $$E_{g(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} |Y(\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\omega c}^{\omega c} \frac{1}{(4+\omega^2)} d\omega = \frac{1}{8}.$$

$$\int_{-\omega c}^{\omega c} \frac{1}{4+\omega^2} = \frac{\pi}{4}. \qquad \left[\int \frac{1}{a^2+b^2x^2} dx = \frac{1}{ab} \tan^{-1}\left(\frac{bx}{a}\right)\right.$$

even function

$$2 \times \int_0^{\omega c} \frac{1}{4+\omega^2} = \frac{\pi}{4}.$$

$$\Rightarrow \int_0^{\omega c} \frac{1}{4+\omega^2} = \frac{\pi}{8}.$$

$$\Rightarrow \left[\frac{1}{2} \tan^{-1}\left(\frac{\omega}{2}\right)\right]_0^{\omega c} = \frac{\pi}{8}.$$

$$\Rightarrow \left[\tan^{-1}\left(\frac{\omega}{2}\right)\right]_0^{\omega c} = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{\omega c}{2}\right) - \tan^{-1}(0) = \frac{\pi}{4}$$

$$\Rightarrow \frac{\omega c}{2} = \tan\left(\frac{\pi}{4}\right) = 1$$

$$\omega c = 2 \text{ rad/sec}$$

P4.2.32 $x(t) = 2.$

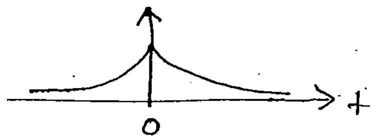
$$F^{-1}\left((1+j\omega)\,X(\omega)\right) = Ae^{-2t}u(t).$$

$$X(\omega) = \frac{F(Ae^{-2t}u(t))}{(1+j\omega)}$$

P4.2.33 find $\displaystyle\int_{-\infty}^{\infty} \frac{8}{(\omega^2+4)^2}\,d\omega$.

$$= \frac{1}{2}\int \left[\frac{2(2)}{\omega^2+(2^2)}\right]^2 d\omega \qquad \left[e^{-a|t|} \longleftrightarrow \frac{2a}{\omega^2+a^2}\right]$$

$$= \frac{1}{2}(2\pi)\int_{-\infty}^{\infty} \left|e^{-2|t|}\right|^2 dt$$



$$= 2\pi\int_0^{\infty} e^{-4t}\,dt = \frac{\pi}{2}$$

Gate 5 marks.

P4.2.34 *

$$g(t) = \frac{2a}{a^2+t^2} \xleftrightarrow{\text{Duality}} G(\omega) = 2\pi e^{-a|\omega|}$$

Essential BW 0 to B Hz.

$$\boxed{E_B = 0.99\, E_{g(t)}.}$$

$$E_{g(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} |G(\omega)|^2\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \left|\left(2\pi e^{-a|\omega|}\right)\right|^2 d\omega = 2\pi\times 2\int_0^{\infty} e^{-2a\omega}\,d\omega.$$

$$= \frac{2\pi}{a}$$

E_B = Energy within the bandwidth "B"

$$E_B = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left|2\pi e^{-a|\omega|}\right|^2 d\omega = \frac{1}{\pi}\int_0^{B} 4\pi^2 e^{-2a\omega}\,d\omega = 0.99\,\frac{2\pi}{a}$$

$$= 4\pi\left[\frac{e^{-2a\omega}}{-2a}\right]_0^B = 0.99\,\frac{2\pi}{a}.$$

$$\Rightarrow \frac{2\pi}{a}\left[1-e^{-2aB}\right] = 0.99\left(\frac{2\pi}{a}\right)$$

$$\Rightarrow B = \frac{3.02}{a} \text{ rad/sec}$$

$$\boxed{\begin{aligned} \omega &= 2\pi f \\ 2\pi\delta(\omega) &= \delta(f) \end{aligned}}$$

Applications of Fourier Transform.

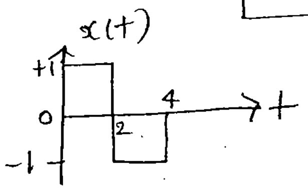
1. Distortionless Transmission:

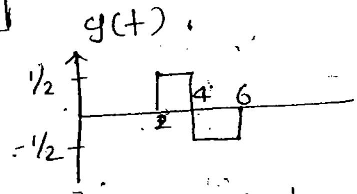
Distortion $\longrightarrow$ deviation from the shape.

Difference b/w distortion & Noise:

1. Noise is external, distortion is internal
2. Noise is random, distortion is deterministic

$x(t) \longrightarrow$ [Distortion less Txn] $\longrightarrow y(t) = A\,x(t-t_0)$



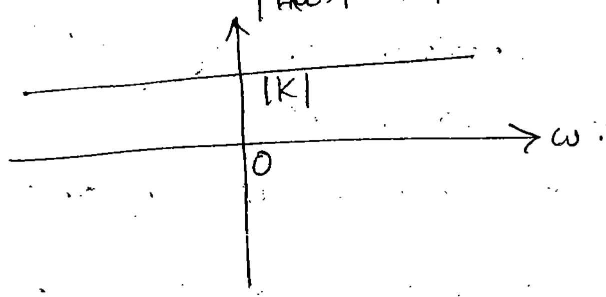


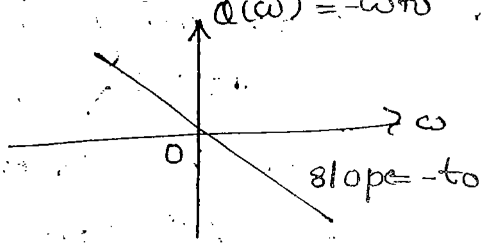
$$y(t) = A\,x(t-t_0) \xleftrightarrow{F.T} A e^{-j\omega t_0} X(\omega).$$

Frequency Response

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = A e^{-j\omega t_0} = |H(\omega)|\, e^{j\theta(\omega)}.$$

$|H(\omega)| = |K|$ $\quad |H(\omega)| = \text{constant}$ $\quad \theta(\omega) = -\omega t_0$





Magnitude spectrum is Constant

Application:- Audio Amplifier
Ear Can't recognise

Phase spectrum is linear function of frequency

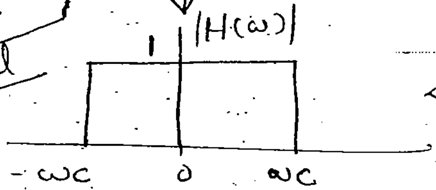
Application:- Video of digital Communication s/m

Note: To maintain distortionless condition, output is exact replica of the input with scaling in its amplitude & possible timedelay. The equivalent Condition in the frequency domain is magnitude spectrum is constant whereas phase spectrum is linear function of frequency.

$$E_{H(\omega)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} K^2 d\omega = \infty$$

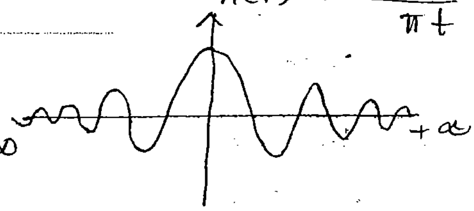
It is not possible to select entire band.

↓ truncate.



⟷ IFT

$h(t) = \frac{\sin\omega_c t}{\pi t}$



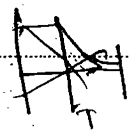
$-\infty < h(t) < \infty$

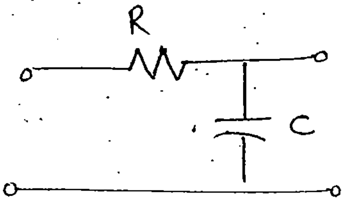
Non- Causal.

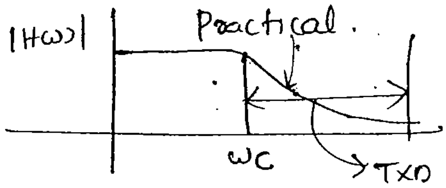
Can't use practical.

$(h(t) = 0$
$t<0)$

Ideal LPF.
[Not practical]







→ Txn band ⇒ ↓ order ↑

Note: → According to Rayleigh's theorem we require ∞ energy to maintain distortionless condition which is impractical so we limited the frequency range from 0 to ω_c that is ideal filter
← (IFT of rectangular spectrum is ~~syn~~ sinc function which coveres from $-\infty$ to $+\infty$)

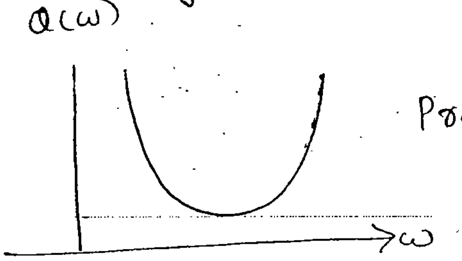
→ All ideal filters are non- Causal & unstable

→ Phase delay is the delay that is occuring at single frequency which is due to Carrier wave form.

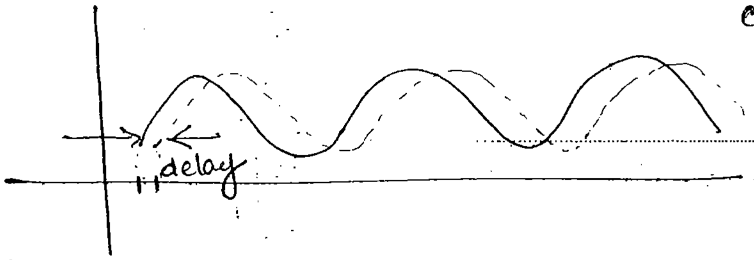
$t_p(\omega) = -\frac{\theta(\omega)}{\omega}$; −ve sign represents any phase lag present in the s/m.

→ Group delay is the delay that is occuring at a group ~~of~~ or narrow band of frequencies which is due to envolope of the msge signal.

$$t_g(\omega) = -\frac{d\,\theta(\omega)}{d\omega}.$$



Practical phase response within the voice frequency



$\cos(\omega t+\theta) = \cos\omega(t+\frac{\theta}{\omega})$

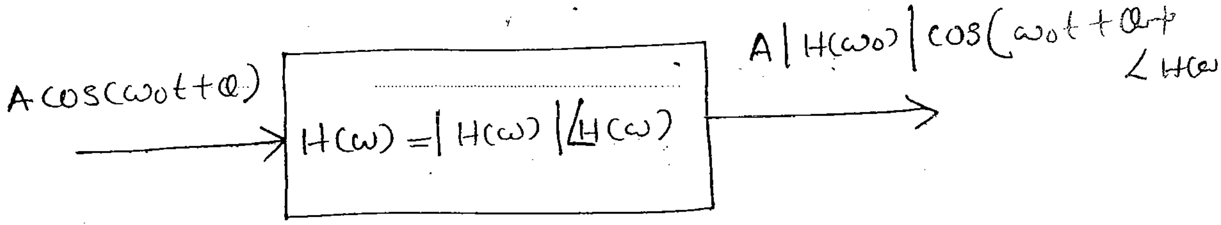
↓

phase delay

The phase response can be make linear by differentiating the parabolic phase response.

P 4.3.1

Q. No.



i/p $x(t) = 2\cos 10\pi t + \sin 26\pi t$.

o/p $y(t) = (2)(2)\cos(10\pi t - \frac{\pi}{6}) + (1)\sin(26\pi t - \frac{13\pi}{30})$

$30\pi \rightarrow \pi/2$; $10\pi \rightarrow \frac{\pi}{6}$; $26\pi \rightarrow \frac{13\pi}{30}$.

P 4.3.2

Q. No.

$\theta(\omega) = \omega$

$t_p(\omega) = \frac{-\theta(\omega)}{\omega} = \underline{\underline{-1}}$

$t_g(\omega) = \frac{-d\omega}{d\omega} = \underline{\underline{-1}}$

$t_p = t_g =$ Constant.

$$\boxed{\text{linear phase} = \text{Phase spectrum is constant}}$$

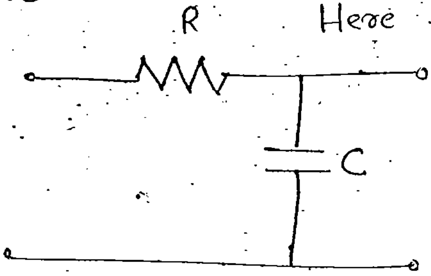
P4.3.3
Gate
Mo

(a) $R = 1k\Omega, \quad C = 1\mu F$

$H(f)$ = frequency Response of RC LPF
(Need to find the transfer function)

$$\frac{|H(f_1)|}{|H(0)|} \geq 0.95 \; ; \; 0 \leq |f| \leq f_1$$

[$\frac{1}{2\pi RC}$ is the frequency at -3dB gain
Here the magnitude is not 0.707
so it can't be used.]



(transfer function) $H(f) = \dfrac{1}{1+j2\pi fRC}$; $|H(f)| = \dfrac{1}{\sqrt{1+(2\pi fRC)^2}}$

$$|H(0)| = 1$$

$$\Longrightarrow |H(f_1)| \geq 0.95$$

$$\frac{1}{\sqrt{1+(2\pi f_1 RC)^2}} \geq 0.95$$

$$\underline{f_1 = 52.2 \text{ Hz}}$$

(b)
$$t_g(f) = \frac{-d\theta(f)}{2\pi df}$$
$$= -\frac{1}{2\pi}\frac{d}{df}\left[-\tan^{-1}(2\pi fRC)\right]$$
$$= -\frac{1}{2\pi}\left[\frac{-(-2\pi RC)}{1+(2\pi fRC)^2}\right]$$

$$t_g(100) = \frac{10^{-3}}{1+4\pi^2(100)^2(10^{-3})^2}$$
$$= \underline{0.717 \text{ msec}}$$

3.5 Q.

$t_g = 10^{-8}$ $\quad t_p = 1.56 \times 10^{-6}$

$$y(t) = \frac{1}{100} \underbrace{\cos(100t - 10^{-6})}_{\text{Envolope}} \underbrace{\cos(10^6 t - 1.56)}_{\text{Carrier}}$$

$$= \frac{1}{100} \cos 100\big(t - \underbrace{10^{-8}}_{t_g}\big) \cos 10^6 \big(t - \underbrace{1.56 \times 10^{-6}}_{t_p}\big)$$

3.6 Q.

No amp distortion = 20 to 50

No phase distortion = 0 to 30

Combinly no distortion = $\underline{\underline{20 \text{ to } 30 \text{ range}}}$.

Q. For a distortionless LTI s/m, If the i/p applied is $x(t) = 3\cos 12\pi t + \sin 15\pi t$. The output is observed to be $y(t) = 3\cos\left[12\pi t - \pi/3\right] + 4\sin\left[15\pi t - \pi/6\right]$

What kind of distortion has occured ?

Ans: Amplitude distortion is there :- Not scaled by same factor.

$$y(t) = 3\cos 12\pi\big(t - \underbrace{1/36}_{t_0}\big) + 4\sin 15\pi\big(t - \underbrace{\tfrac{1}{90}}_{t_0}\big)$$

$\theta(\omega) = -\omega t_0$

ω can change

t_0 can't change

t_0 is not constant.

f can vary.

∴ Phase distortion is also there.

Hilbert Transform. (HT)

$$x(t) \longrightarrow \boxed{h(t) = \frac{1}{\pi t}} \longrightarrow$$

$\hat{x}(t) = $ HT of $x(t)$.

$$\hat{x}(t) = x(t) * \frac{1}{\pi t}$$

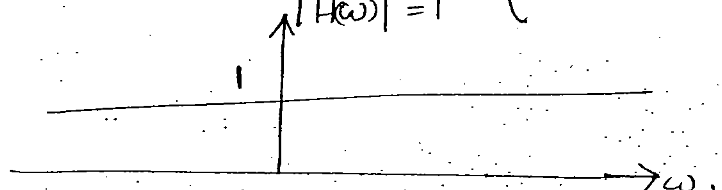
$\downarrow$ F.T

$$\hat{X}(\omega) = X(\omega) \cdot [-j\,\mathrm{sgn}(\omega)]$$

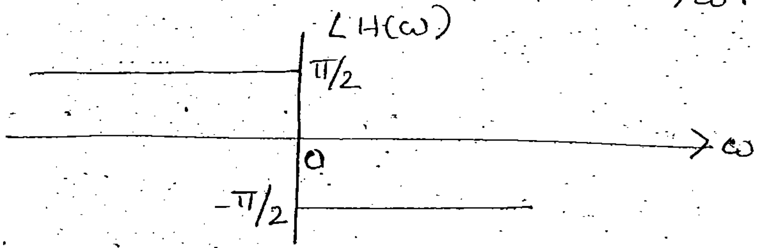
Frequency Response

$$H(\omega) = \frac{\hat{X}(\omega)}{X(\omega)} = -j\,\mathrm{sgn}(\omega)$$

$$= \begin{cases} -j & ; \omega > 0 \\ j & ; \omega < 0. \end{cases}$$



All pass filter
Magnitude Respons is constant



$\therefore$ HT $\longrightarrow$ Not changing domain
~~only changing~~ phase angle.

s/l & its HT are orthogonal to each other

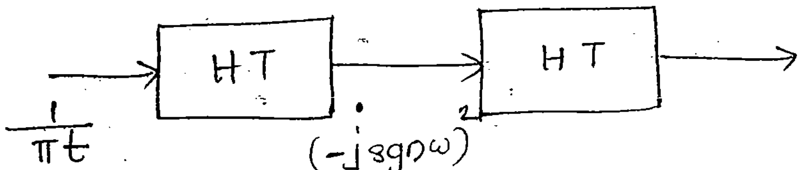
Area $\boxed{\int x(t)\,\hat{x}(t)\,dt = 0}$ Orthogonality Condition.

H.T. of $\cos\omega_c t = \cos(\omega_c t \mp \pi/2) = \sin\omega_c t$.

" " $\sin\omega_c t = \sin(\omega_c t - \pi/2) = -\cos\omega_c t$.

" $e^{j\omega_c t} = -j\, e^{j\omega_c t}$ $\left[e^{j(\omega_c t \mp \pi/2)} = e^{j\omega_c t} \times e^{-\pi/2} = -j\, e^{j\omega_c t} \right]$

Q. HT of $\frac{1}{\pi t}$ is $\underline{-\delta(t)}$



$(-j\,\mathrm{sgn}\,\omega)^2$

$j^2\,\mathrm{sgn}^2\omega$

$= -1$

$= -\delta(t)$.

* HT of $x(t)\cos\omega_c t = x(t)\sin\omega_c t$

$\downarrow$

$BW = B$

$B <<< \omega_c$

? ✓ * HT of $\frac{1}{1+t^2}$ is ______

Do is same.

Correlation :

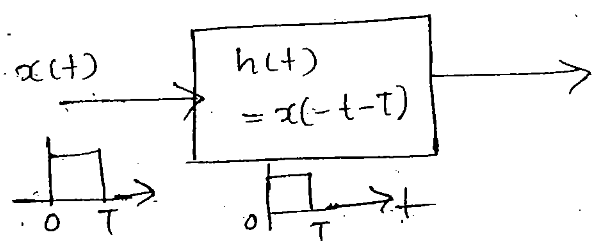
Correlogram $\rightarrow$ used to compare s/ls.

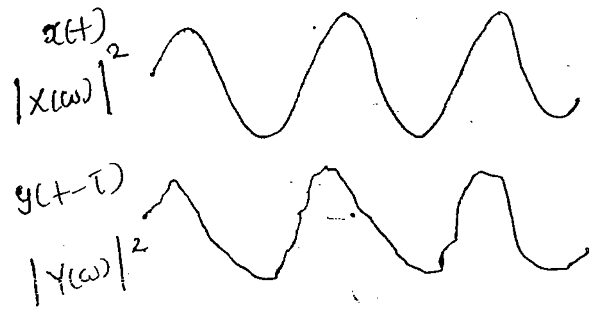
* Delayed version of signal is same as the origin signal.
* No folding in correlation compared to convolution.
* Auto Correlation: $\rightarrow$ $x(t)$, $x(t-T)$

Correlation.

- Auto Correlation Function. $(x(t), x(t-T))$
  - Energy
  - Power
- Cross Correlation function. $(x(t), y(t-T))$
  - Energy
  - Power

Matched filter





hidden periodicity

A.C.F of an Energy signal $x(t)$ is

$$R_x(\tau) = \int_{-\infty}^{\infty} x(t)\, x(t-\tau)\, dt$$

$$= \int_{-\infty}^{\infty} x(t+\tau)\, x(t)\, dt$$

$\tau \rightarrow$ lag / searching parameter

A.C.F of a power s/l.

$$R_x(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)\, x(t-\tau)\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t+\tau)\, x(t)\, dt$$

Max amp. is always at $=0$.

Energy / Power $=$ Max size of the s/l.

— Study the properties —

$$R_x(\tau) = x(\tau) * x(-\tau)$$

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$x(\tau) * x(-\tau) = \int_{-\infty}^{\infty} x(t)\, x(-(\tau - t))\, dt.$$

$$\boxed{R_x(\tau) \xleftrightarrow{F.T} S_x(\omega)}$$

A.C.F ESD / PSD

$$R_x(\tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S_x(\omega)\, e^{j\omega\tau}\, d\omega$$

$$\boxed{R_x(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_x(\omega)\, d\omega}$$

Area under spectral density is Energy / power.

$$S_x(\omega) = |X(\omega)|^2$$

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$y(t) = x(t) * h(t)$$

$$\Big\downarrow \text{FT}$$

$$Y(\omega) = X(\omega)\, H(\omega)$$

$$|Y(\omega)|^2 = |X(\omega)|^2 \, |H(\omega)|^2$$

✓ $$S_Y(\omega) = S_X(\omega)\, |H(\omega)|^2$$

Important.

$$\boxed{\begin{array}{l}\text{Output Spectral density} \\ \quad = [\text{Input spectral density}]\,[\,|H(\omega)|^2\,]\end{array}}$$

Q.

P.4.5.1

$x(t) = 6\cos\left(6\pi t + \frac{\pi}{4}\right)$ (power signal)

$$R_X(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)\cdot x(t-\tau)\, dt$$

$$R_X(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} 6\cos\left(6\pi t + \frac{\pi}{4}\right)\cdot 6\cos\left(6\pi t - 6\pi\tau + \frac{\pi}{4}\right) dt$$

$2\cos A \cos B$

$$= \lim_{T\to\infty} \frac{18}{2T}\int_{-T}^{T} \cos\left(12\pi t - 6\pi\tau + \frac{\pi}{2}\right) + \cos(6\pi\tau)$$

$$= \lim_{T\to\infty} \frac{18\cos(6\pi\tau)}{2T}\int_{-T}^{T} dt$$

$$R_X(\tau) = 18\cos(6\pi\tau)$$

$$\frac{6^2}{2} = 18$$

Auto correlation

$$\boxed{\left.\begin{array}{c} A\sin(\omega t + \theta) \\ \text{(or)} \\ A\cos(\omega t + \theta)\end{array}\right\} = \frac{A^2}{2}\cos\omega\tau}$$

P4.5.2

$x(t) = e^{-3t}u(t)$ Energy s/l.

$$R_X(\tau) = \int_{-\infty}^{\infty} x(t)\, x(t-\tau)\, dt = \int_{0}^{\infty} e^{-3t} \cdot e^{-3(t-\tau)}$$

$$= \int_0^{\infty} e^{-6t} e^{3\tau} dt$$

$$= e^{3\tau} \left[\frac{e^{-6t}}{-6} \right]_0^{\infty}$$

$$= \frac{e^{3\tau}}{6}$$

$$= \int_{-\infty}^{\infty} e^{-3t} u(t)\, e^{-3(t-\tau)} u(t-\tau)\, dt$$

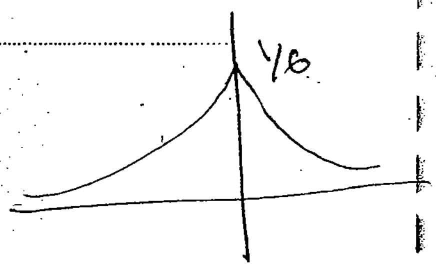
$$R_x(\tau) = x(\tau) * x(-\tau)$$

$$e^{-3\tau} u(\tau) * e^{3\tau} u(-\tau)$$

$$\Big\downarrow \text{FT}$$

$$S_x(\tau) = \left[\frac{1}{3+j\omega} \right] \left[\frac{1}{3-j\omega} \right]$$

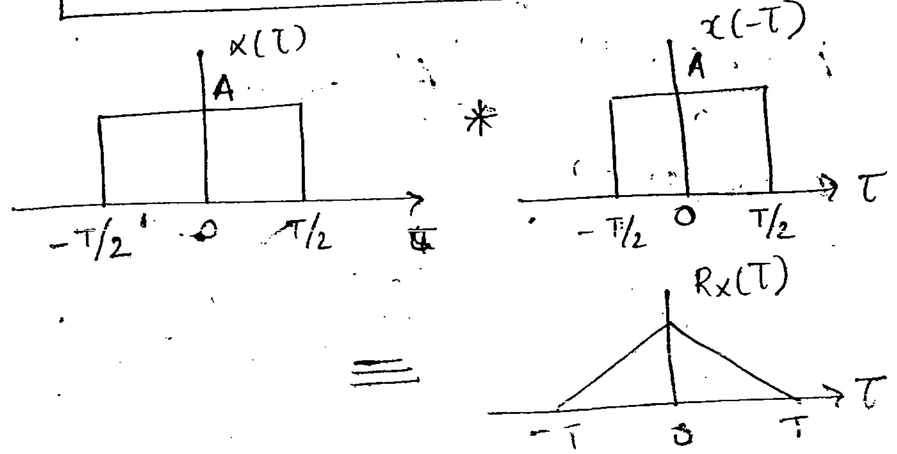
$$= \frac{1}{6} \left[\frac{2(3)}{\omega^2 + (3)^2} \right]$$



$$\Big\downarrow \text{IFT}$$

$$R_x(\tau) = \frac{1}{6} e^{-3|\tau|}$$

$$\boxed{\begin{array}{l} \text{Power S/l} \Rightarrow \text{Normal technique. (use integral)} \\ \text{Energy S/l} \Rightarrow \text{Convolution Method} \end{array}}$$



4.5.3 (a) $x(t) = e^{-2t}u(t)$

$$\text{o/p SD} = \text{i/p SD} \cdot |H(\omega)|^2$$
$$= |X(\omega)|^2 |H(\omega)|^2$$
$$= \left|\frac{1}{2+j\omega}\right|^2 \left|\frac{1}{1+j\omega}\right|^2$$

$$\text{ESD of o/p} = \frac{1}{(4+\omega^2)(1+\omega^2)}$$

(b)

$$\text{ESD of o/p} = \frac{1}{(4+\omega^2)(1+\omega^2)} = \frac{-1/3}{4+\omega^2} + \frac{1/3}{1+\omega^2}$$

Apply partial fraction.

$$\text{o/p Energy} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \text{o/p SD} \, d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\infty}^{\infty} \frac{-1/3}{4+\omega^2}\, d\omega + \int_{-\infty}^{\infty} \frac{1/3}{1+\omega^2}\, d\omega\right]$$

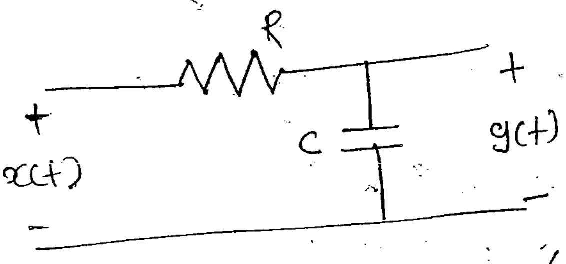
$\frac{1}{a}\tan^{-1} x/a$ $\quad \frac{1}{x^2+a^2}$

$$= \frac{1}{12}$$

$$= \frac{1}{3} E_{x(t)}.$$

where $E_{x(t)} = \int_{0}^{\infty} e^{-4t}\, dt = \frac{1}{4}$

Q. A power s/l $x(t)$ with input spectral density $S_x(\omega) = K$ is applied to RC low pass filter. Find the mean square value of output?



m.s.v of o/p = o/p power

$$o/p \ SD = i/p \ SD \times |H(\omega)|^2$$

$$|H(\omega)| = \frac{1}{\sqrt{1+(2\pi f RC)^2}}$$

$$|H(\omega)|^2 = \frac{1}{1+(\omega RC)^2}$$

$$o/p \ SD = K \cdot \frac{1}{1+(\omega RC)^2}$$

$$= \frac{K}{1+(\omega RC)^2}$$

$$o/p \ power = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{K}{1+(\omega RC)^2}\, d\omega$$

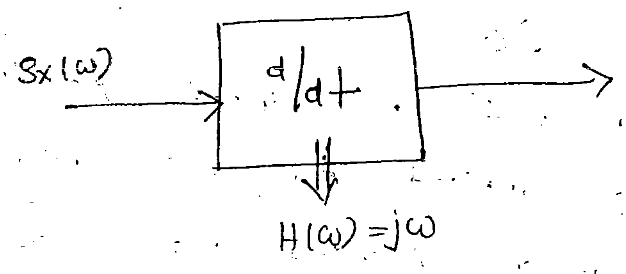
$$= \frac{K}{2\pi (RC)^2}\int_{-\infty}^{\infty} \frac{1}{\omega^2+\left(\frac{1}{RC}\right)^2}\, d\omega$$

$$= \frac{K}{2\pi (RC)^2}\,\frac{1}{\frac{1}{RC}}\left[\tan^{-1}\left(\frac{\omega}{1/RC}\right)\right]_{-\infty}^{\infty}$$

$$= \frac{K}{2\pi RC}\left[\frac{\pi}{2}+\frac{\pi}{2}\right]$$

$$= \frac{K}{2RC}$$

P 4.5.4



P 4.5.5

$$x(t) = e^{-t}u(t) \qquad y(t) = e^{-3t}u(t).$$

$$R_{XY}(\tau) = x(\tau) * y(-\tau)$$

$$R_{YX}(\tau) = y(\tau) * x(-\tau)$$

$$R_{XY}(\tau) = R_{YX}(-\tau)$$

page : 64

F.T of periodic signals.

$$x_p(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

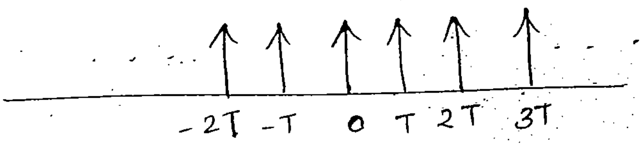
$1 \longleftrightarrow 2\pi\delta(\omega)$

$1.e^{j\omega_0 t} \longleftrightarrow 2\pi\delta(\omega - k\omega_0)$

$\Big\downarrow$ F.T

$$\boxed{X_p(\omega) = \sum_{n=-\infty}^{\infty} 2\pi C_n \delta(\omega - n\omega_0)}$$

$$C_n = \frac{1}{T}\int_0^T x_p(t)\, e^{-jn\omega_0 t}\, dt$$

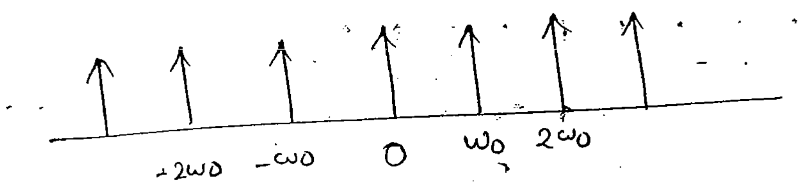


$$C_n = \frac{1}{T}\int_0^T \delta(t)\, e^{-jn\omega_0 t}\, dt = \frac{1}{T}$$

$$X_p(\omega) = \sum_{n=-\infty}^{\infty} \frac{2\pi}{T_s} \delta(\omega - n\omega_0)$$

(or)

$$= \frac{1}{T_s} \sum_{n=-\infty}^{\infty} \delta(f - nf_0)$$



$\Rightarrow$ Periodic & sampling in one domain Correspond to periodic & sampling in other domain.

$$\cos \omega_0 t = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

$$= \frac{1}{2} e^{j(1)\omega_0 t} + \frac{1}{2} e^{j(-1)\omega_0 t} \quad \Big| \; n=1 \text{ assume}$$

$$2\pi\left(\frac{1}{2}\right)\delta(\omega-\omega_0)$$

$$2\pi\left(\frac{1}{2}\right)\delta(\omega+\omega_0)$$

$$\Longleftrightarrow \pi\left[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right]$$

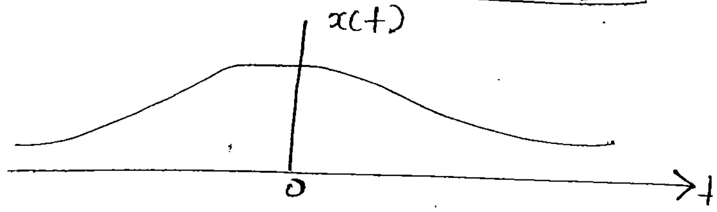
22/08/2011
9 - 1pm.

Sampling Theorom.

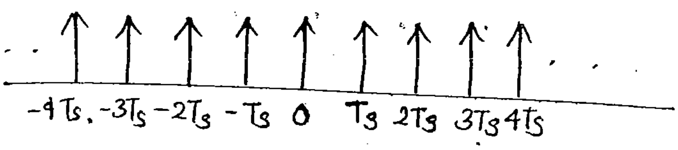
- Ideal
- Practical.
  - Natural
  - Flat top.

Sampling theorom $\Longrightarrow \omega_0 \geq 2\omega_m$.

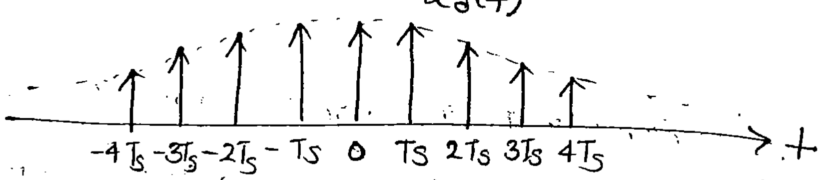
Ideal / Instantaneous Sampling.



$\delta_{Ts}(t)$



$x_\delta(t)$



$x_\delta(t) \longrightarrow$ Ideally sampled Signal.

$$x_\delta(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$\Big\downarrow FT$$

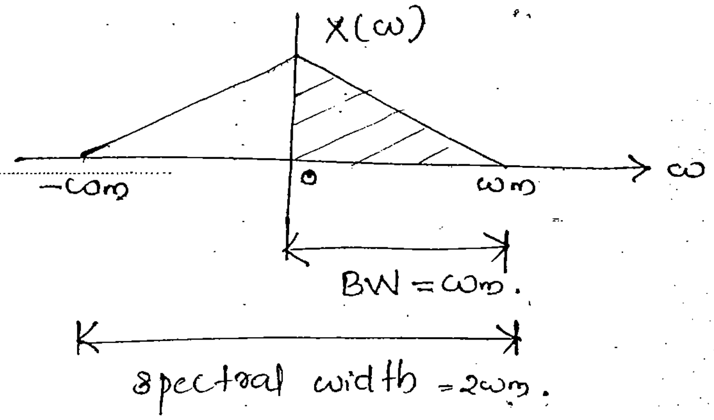
$$X_\delta(\omega) = \frac{1}{2\pi}\left[X(\omega) * \frac{2\pi}{T_s} \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)\right]$$

$$\boxed{X_\delta(\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(\omega - n\omega_0)}$$

ω_0 = Sampling Rate.

Proof of theorem:

Assume band limited spectrum.



Max energy is in low freq.

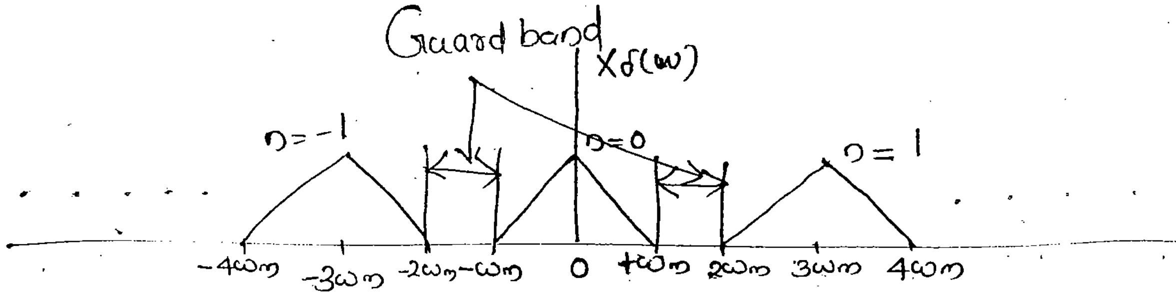
$\omega_0 > 2\omega_m \Longrightarrow$ over sampling

$= 2\omega_m \Longrightarrow$ Critical "

$< 2\omega_m \Longrightarrow$ under "

Case (1): $\omega_0 > 2\omega_m$

Let $\omega_0 = 3\omega_m \Longrightarrow X_\delta(\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(\omega - 3n\omega_m)$



Descrete in one domain = Periodic in other domain.

The repetition is for multiple s/e. not ~~the~~ same spectrum is repeating.

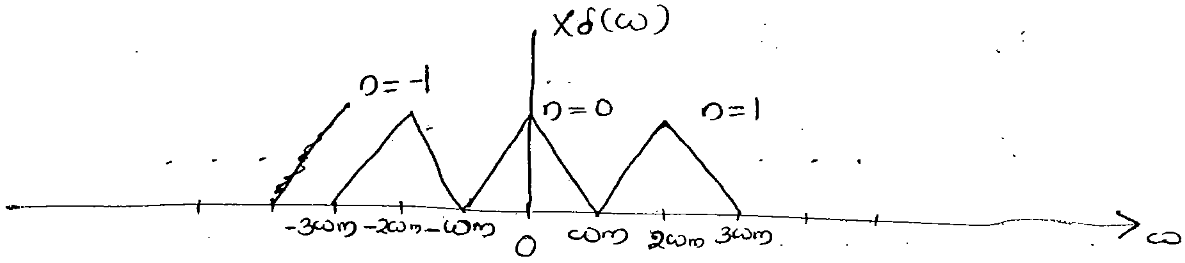
Images do not overlap under the condition. } $\omega_0 - \omega_m > \omega_m$

$\omega_0 > 2\omega_m$.

Case II

$\omega_0 = 2\omega_m$

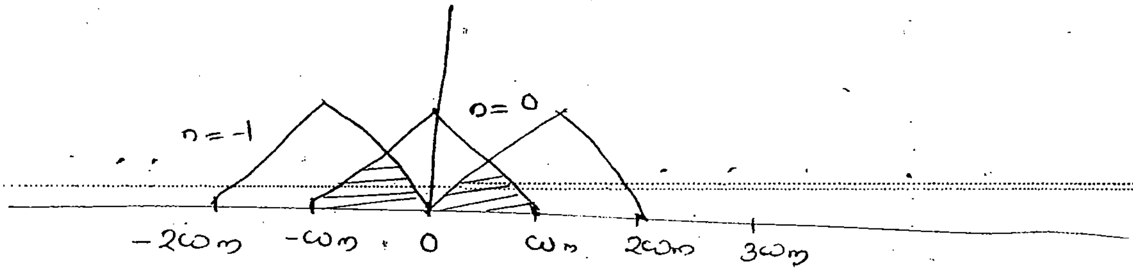
$$X_\delta(\omega) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(\omega - 2\omega_m n)$$



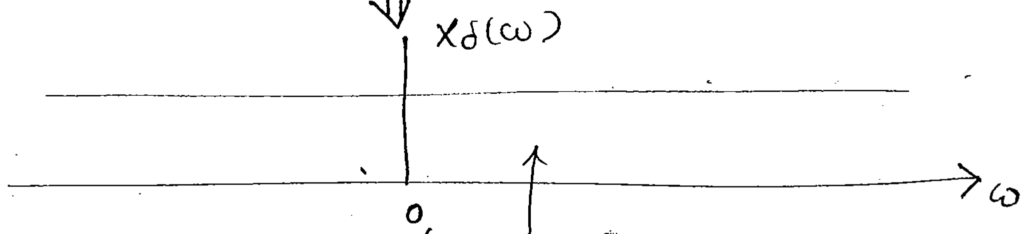
Case III

$\omega_0 < 2\omega_m$

Let $\omega_0 = \omega_m \implies X_\delta(\omega) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(\omega - \omega_m n)$



$\Downarrow$



(Aliasing (or) spectral folding)

$\Downarrow$

original info. is replaced by new information (dc).

eg: Movies

To avoid aliasing

1. $\omega_0 \geq 2\omega_m$

Minimum sampling rate Required = Nyquist rate = $2 \times$ Highest frequency component of the sl spectrum.

$(\omega_0)_{min} = 2\omega_m$.

Page 65
P4.6.1

(a) $x(t) = \dfrac{\sin 200\pi t}{\pi t}$

$(\omega_0)_{min} = 2\omega_m = 2 \times 200\pi = \underline{\underline{400\pi}}$ rad

Nyquist Interval $= \dfrac{1}{T_s}$

(b) $x_2(t) = \left(\dfrac{\sin 200\pi t}{\pi t}\right)^2 = \dfrac{1 - \cos 400\pi t}{2(\pi t)^2}$

$(\omega_0)_{min} = 2\omega_m = 2 \times 400\pi = \underline{\underline{800\pi}}$ rad

(c) $x_3(t) = 5\cos(1000\pi t)\cos 4000\pi t$

$5000\pi \qquad 3000\pi$

$NR = 2(5000\pi) = 10,000\pi$ rad

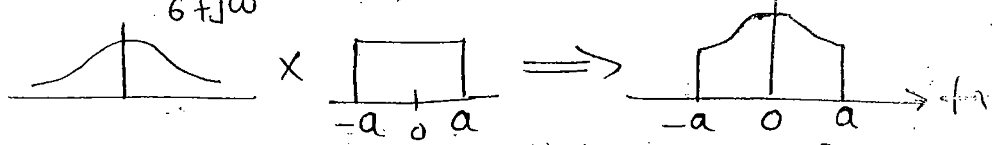
$f_s = \underline{\underline{5KHZ}}$

(d) $x_4(t) = e^{-6t}u(t) * \dfrac{\sin at}{\pi t}$

$NR = 2\omega_m = \underline{\underline{2a}}$

FT $\downarrow \dfrac{1}{6+j\omega}$ FT $\downarrow$

$X_4(\omega)$



$NR = \underline{\underline{2 \times a}}$

(e) $x_5(t) = \text{sinc}(100t) + 3\,\text{sinc}^2(60t)$

$100\pi \qquad 120\pi$

$\text{sinc}\,x = \dfrac{\sin \pi x}{\pi x}$

$NR = 2 \times 120\pi = \underline{\underline{240\pi \text{ rad}}}$

$= \underline{\underline{120 \text{ HZ}}}$

$2\omega_m$

P4.6.2

(a) $x(t) \longleftrightarrow \omega_0$ (Nyquist rate)

$x(t) + x(t-1) \longrightarrow \omega_0$ (No change in NR)

(b) $j\omega X(\omega) = \omega_0$.

(c) $x(3t) \longleftrightarrow \frac{1}{3} X\left(\frac{\omega}{3}\right)$

$NR = 3\omega_0$

(d) $x(t)\cos\omega_0 t \longrightarrow 3\omega_0$.

$$\omega_0 = 2\omega_m \Longrightarrow \omega_m = \frac{\omega_0}{2}.$$

$$\underset{\downarrow\ \frac{\omega_0}{2}}{x(t)}\ \underset{\downarrow\ \omega_0}{\cos\omega_0 t} \Rightarrow 2\left(\frac{\omega_0}{2}+\omega_0\right) = \underline{3\omega_0}\ (NR).$$

Multiplication – Addition & subtraction of frequency.

$*$ $x(t) * x(t) \longrightarrow \omega_0$ (No change)

P4.6.3 (a) $x_1(t) \longrightarrow 2\,KHz\ (\omega_m)$. $\quad x_2(t) \longrightarrow 3\,KHz\ (\omega_m)$

$x_1(2t) \longrightarrow 8\,KHZ$.

$x(\omega/3)$

(b) $x_2(t-3) \longrightarrow 6\,KHZ$.

(c) $x_1(t) + x_2(t) \longrightarrow 6\,KHz$.

(d) $x_1(t)\,x_2(t) \longrightarrow 10\,KHz$

(e) $x_1(t) * x_2(t) \longrightarrow 4\,KHz$.

(f) $x_1(t)\cos(1000\pi t) \longrightarrow 5\,KHz$

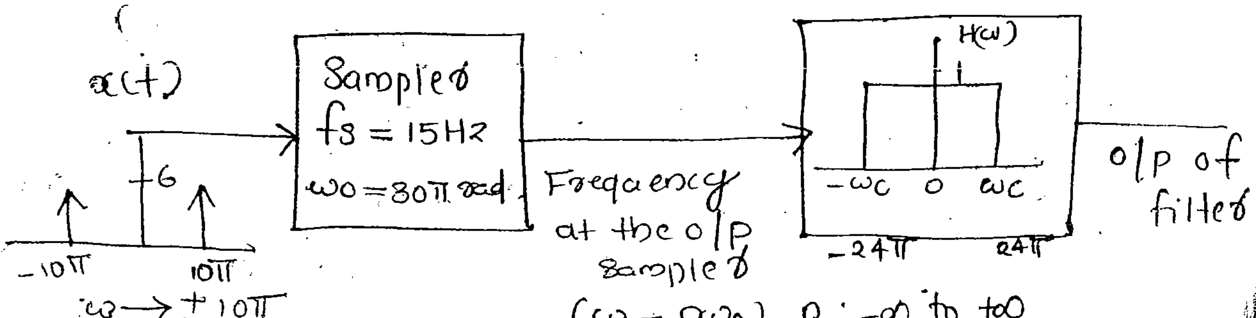
Imp

$*$ A signal $x(t) = 6\cos(10\pi t)$ is ideally sampled at 15 Hz & is passed through an ideal LPF with cut off frequency 12 Hz. What frequencies will ~~be~~ appear at the o/p of the filter?

Solution: $x(t) = 6\cos(10\pi t)$

$$f_s = 15\,HZ = 30\pi\ rad$$

$$f_c = 12\,HZ = 24\pi\ rad$$



$(\omega - n\omega_0)$ $n: -\infty$ to $+\infty$

$n = 0$

ω → 10π, -10π

$n = 1$

$\omega - \omega_0$

$\pm 10\pi - 30\pi$

→ -40π, -20π

$n = -1$

$\omega + \omega_0$

→ 20π, 40π

$n = 2$

$\omega - 2\omega_0$

$\pm 10\pi - 60\pi$

→ -50π, -20π

over sampling

P4.6.4

$f_m = 200$ HZ

$f_s = 300$ HZ under sampling

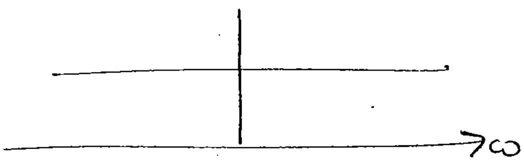
freq. at the o/p of sampler

$f_m \pm n f_s$, $n = 0, 1, 2, 3 \ldots$

at $n = 1$

$f_m \pm n f_s$ → -100 HZ passed, 500 HZ

P4.6.5

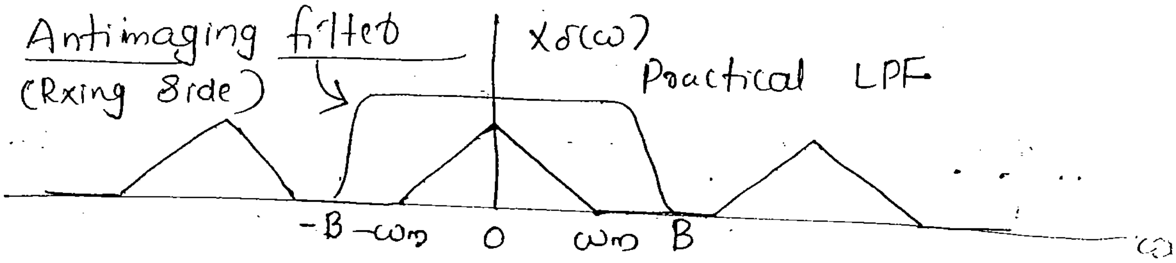


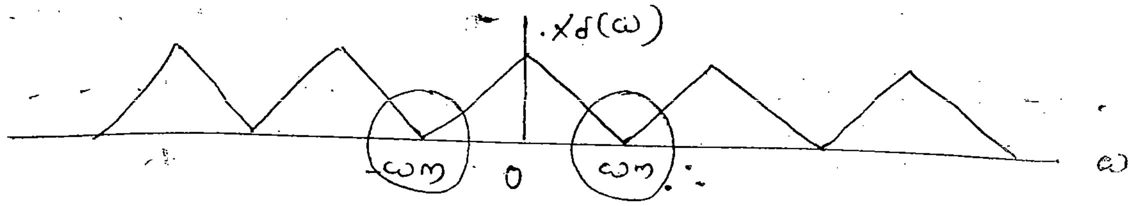
BW = 1K HZ. (ω_m)

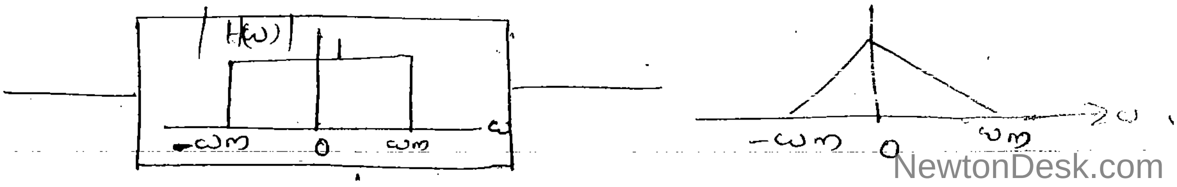
ω_0 = 1K Hz = $\frac{1}{\text{msec}}$

Signal Reconstruction

Antimaging filter
(Rxing side)







If not mentioned

$f_s \Rightarrow$ 5 to 10 times Pass Band frequency

$\Rightarrow$ 2.5 to 5 times the Nyquist rate.

Advantage of over sampling

1. Practical LPF is easy.
2. Improve S/N ratio.

for recovery of original s/l

$$\boxed{\omega_m < B < \omega_0 - \omega_m}$$

P4.6.7

$$\omega_m < B < \omega_0 - \omega_m.$$

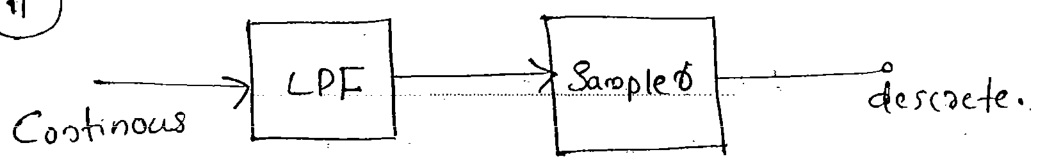
$$f_m = \frac{10\pi}{2\pi} = 5\ Hz$$

$$f_0 = 14\ Hz$$

$$\therefore 5 < B < \underset{9\ Hz}{(f_0 - f_m)}$$

To avoid aliasing.

(1)

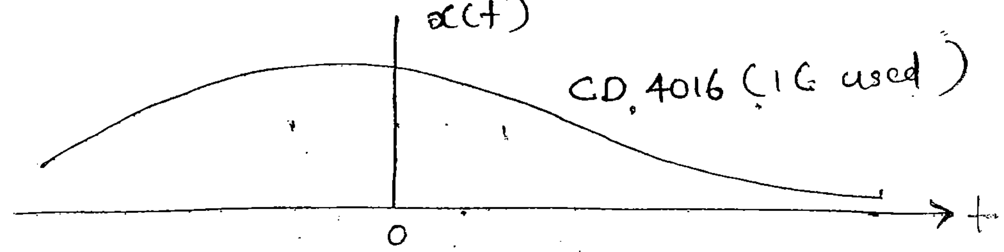


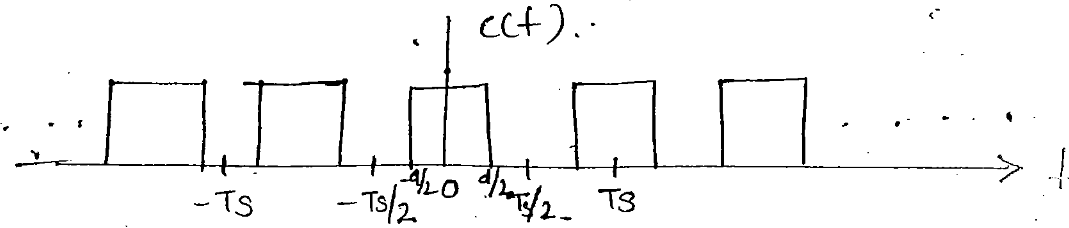
Antialiasing filter (txmitting side).

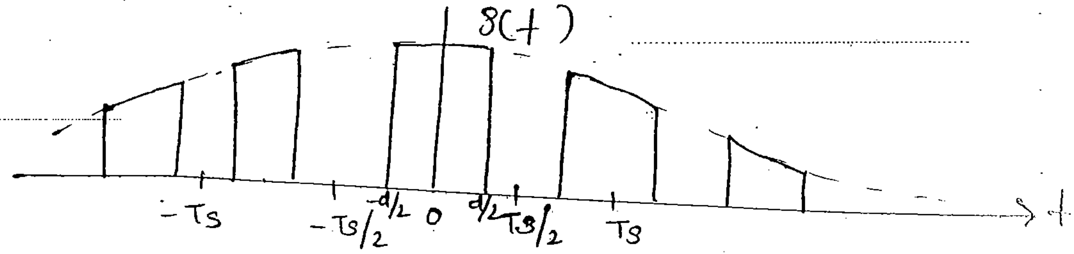
No s/l is of infinite duration in time. ∴ the spectrum of the s/l is ~~not~~ infinite (not band limited). This will contradict with sampling theorem assumption (s/ls are band limited). ∴ we use a LPF to band limit the s/l before giving to the sampler. & filter is called Anti-aliasing filter

Natural Sampling

CD 4016 (IC used)







Naturally sampled s/l

$$s(t) = x(t)\, c(t)$$

$$= x(t) \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

F.T

$$\boxed{S(\omega) = \sum_{n=-\infty}^{+\infty} C_n\, X(\omega - n\omega_0)}$$

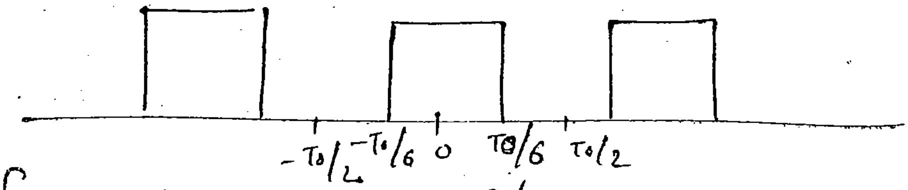
$$C_n = \frac{1}{T_s} \int_{-d/2}^{d/2} (1)\, e^{-jn\omega_0 t}\, dt = \frac{\sin\left(\frac{n\omega_0 d}{2}\right)}{n\omega_0 T_s}$$

Note: The frequencies at the o/p of sampler is decided by $(\omega - n\omega_0)$. In ideal sampling we can take 'n' value from $-\infty$ to $+\infty$, but in natural sampling N is decided by decided by fourier series Coefficient C_n.

P4.6.8

0.4 K Hz　　　　0.7K Hz
400 Hz　　　　700 Hz

$$x(t) = 2\cos(800\pi t) + \cos(1400\pi t)$$



$f_s = 1\text{KHz}$

$$C_n = \frac{1}{T_0}\int_{-T_0/6}^{T_0/6} 3 \cdot e^{-jn\omega_0 t}\,dt = \frac{\sin n\pi/3}{n\pi}$$

$n = 0, 1, 2, 4, 5,$

frequencies at the o/p of sampled

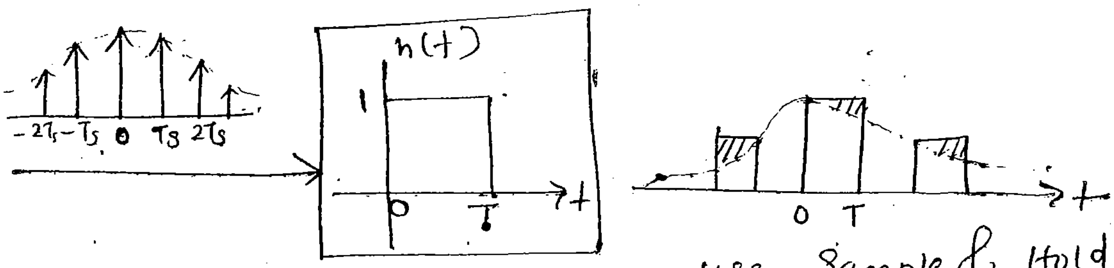
$$\left.\begin{matrix} f_{m1} \pm nf_s \\ f_{m2} \pm nf_s \end{matrix}\right\} n = 0, 1, 2, 4, 5, 7, 8$$

$$\left.\begin{matrix} 0.4K \pm n(1K) \\ 0.7K \pm n(1K) \end{matrix}\right\} n = 0, 1, 2, 4, 5, 7, 8$$

Ans: 2.7, 3.3 (d)

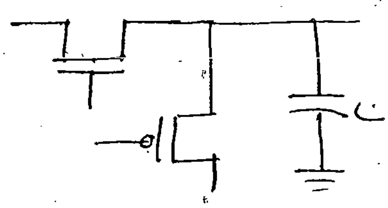
Flat top Sampling.

Zero-order Hold (Z.O.H)



$x(t) * \delta(t-t_0)$
$= x(t-t_0)$

use Sample & Hold Circuit



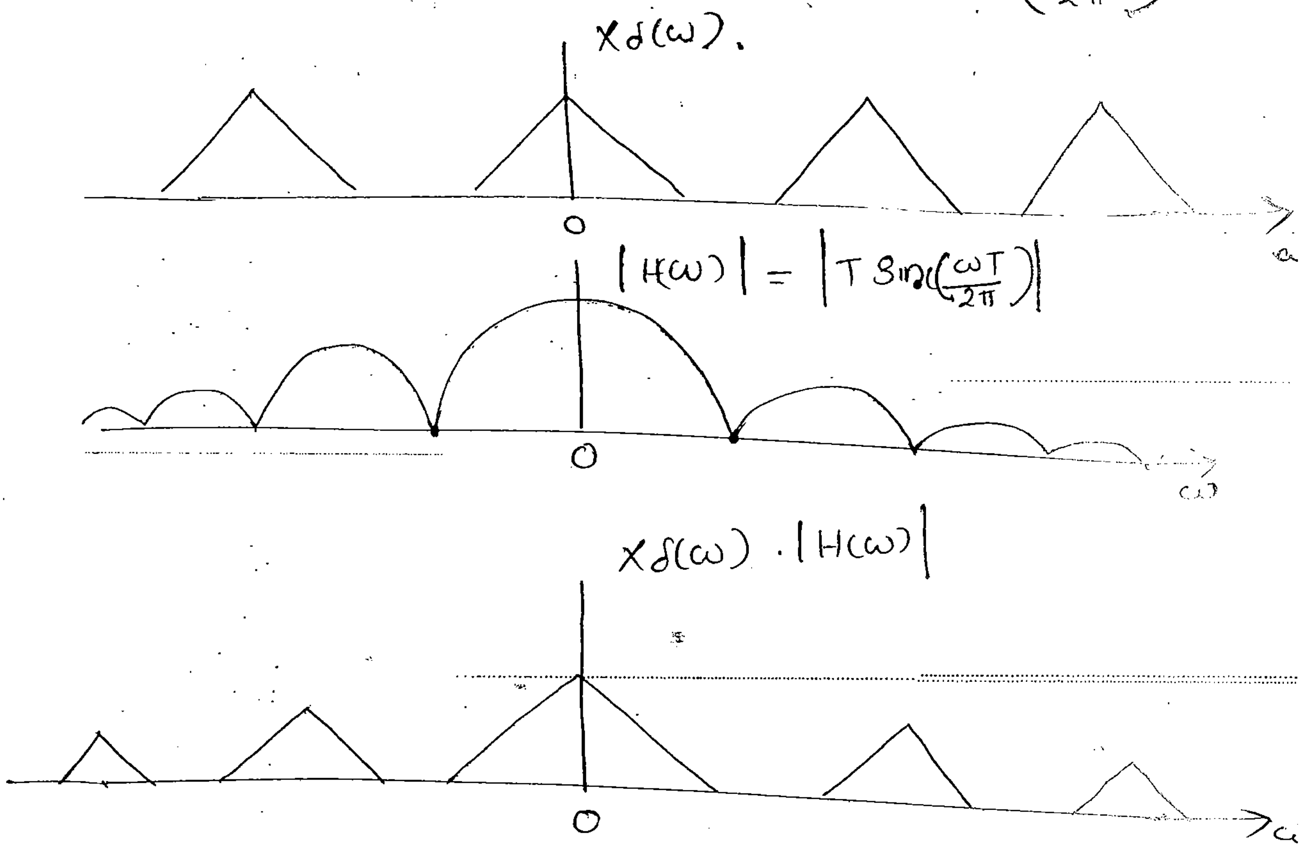
Distortion present in flat top

Aperture effect

$$s(t) = x_\delta(t) * h(t) \xleftrightarrow{FT} S(\omega) = X_\delta(\omega) H(\omega)$$

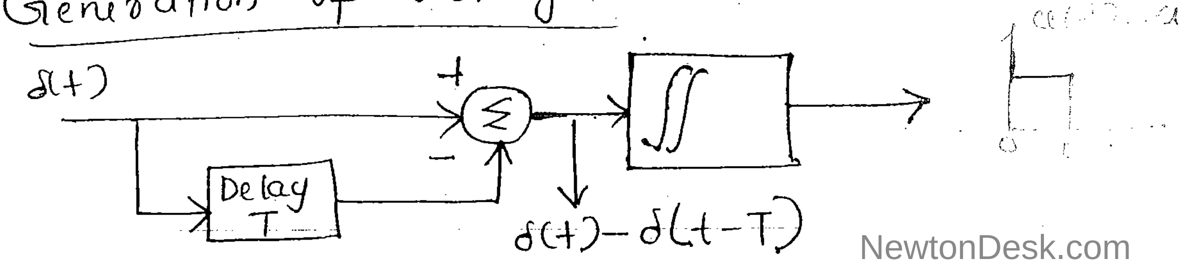
$$S(\omega) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(\omega - n\omega_0) H(\omega) \quad (6)$$

Where $H(\omega) = T \, \text{sinc}\left(\frac{\omega T}{2\pi}\right) e^{-j\omega T/2}$



Note: Because of maintaining constant amplitude level, we have introduced amplitude distortion of $T \, \text{sinc}\left(\frac{\omega T}{2\pi}\right)$ & phase delay of $-\frac{\omega T}{2}$ which is known as aperture effect. To cancel this flat top signal is applied to an equalizer $\left(\frac{1}{|H(\omega)|}\right)$

Generation of rectangle



(Q) T.F of zero-order Hold Circuit?

Disadvantage of FT

- Not defined for unstable
- Not considering initial Condition.

Laplace Transform.

66

Generalization of FT :- Fourier Transform is defined only for stable systems. But LT can be defined for stable as well as unstable s/m

Integro- differential Equation $\longrightarrow$ Algebraic Equat

if i/p $x(t) = e^{st}$ $(s = \sigma + j\omega) \Longrightarrow y(t) = e^{st} H(s)$

Complex variable.

transfer function of the s/m

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau = \int_{-\infty}^{+\infty} e^{s(t-\tau)} h(\tau)\, d\tau$$

$$= e^{st} \int_{-\infty}^{\infty} h(\tau)\, e^{-s\tau} d\tau$$

$$= \underline{e^{st} H(s)}.$$

Laplace transform of impulse response h(t) is the transfer function H(s)

$$L\{x(t)\} = X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st} dt$$

$$X(\sigma + j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-(\sigma + j\omega)t}\, dt$$

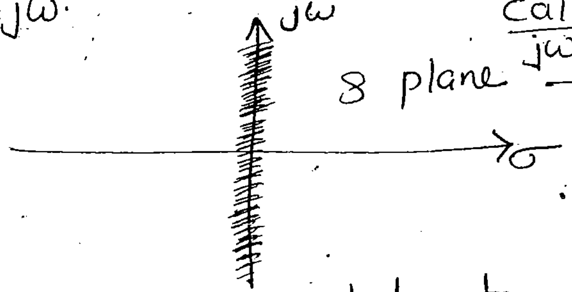
$$\boxed{X(s) = F\left\{x(t)\, e^{-\sigma t}\right\}}.$$

When $\sigma = 0$ $\quad X(s) = X(\omega)$.

$\Downarrow$

$s = j\omega$. $\quad LT \underset{j\omega}{=} FT$

Laplace transform calculated on the $j\omega$ axis is the FT



1. unstable s/l is converted to stable s/l by extra multiplier $e^{-\sigma t}$.

LT

$-\infty$ to $+\infty$ — B.L.T.

0 to ∞ — U.L.T

B.L.T. $\Downarrow$ Adv:

directly from TF (pole zero plot) { Causality, Stability, freq Response ($s=j\omega$) }

U.L.T $\Downarrow$ Adv:

* used to solve differential equation with Initial Conditions.

Disadvantage of Laplace.

- freq. Content is invisible.
  (use bode plot to get mag & phase response at various frequencies $s=j\omega$)

Region of Convergance of LT (ROC)

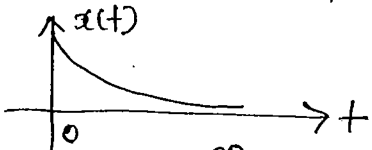
What values of 's' LT is finite.

- $X(s) < \infty$

$$\int_{-\infty}^{\infty} \left| x(t)\, e^{-\sigma t} \right| dt < \infty \;:\; \text{Compulsary Condition.}$$

→ Laplace value is decided by the values of 'σ'. Hence the name.

* $x(t) = e^{-at} u(t)$; $Re\{a\} > 0$



$$X(s) = \int_0^{\infty} e^{-at} e^{-st} dt = \int_0^{\infty} e^{-(s+a)t} dt \rightarrow \infty$$

$$= \left[\frac{e^{-(s+a)t}}{-(s+a)} \right]_0^{\infty} = \left[\frac{e^{-(\sigma+a+j\omega)t}}{-(\sigma+a+j\omega)} \right]_0$$

X(s) finite should be than zero.

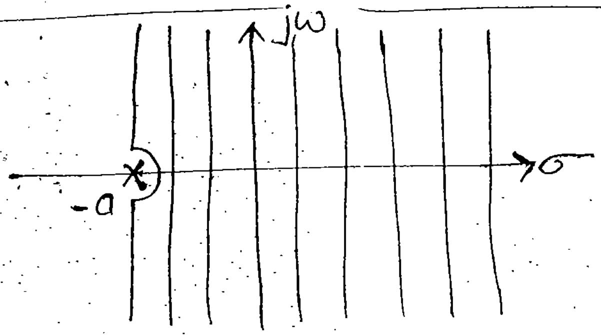
(69)

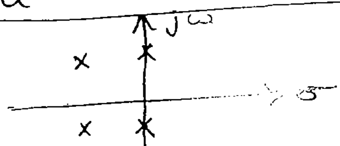
$$= \frac{1}{s+a} \quad ; \quad \sigma + a > 0$$

$$\sigma > -a$$

$$Re\{s\} > Re\{-a\}$$

$$\boxed{e^{-at}u(t) \xleftrightarrow{LT} \frac{1}{s+a} \quad ; \quad \sigma > Re\{-a\}}$$





While considering R.O.C Imaginary part is not considering because Complex poles occurs as pairs as shown above. They cancel each other (even though $\omega \to \infty$)

R.O.C of LT $\rightarrow$ Lines $||^{el}$ to $j\omega$ axis.

$LT = FT$

$s = j\omega$

(1) If ROC including $j\omega$ axis, $\Rightarrow$ FT exists.

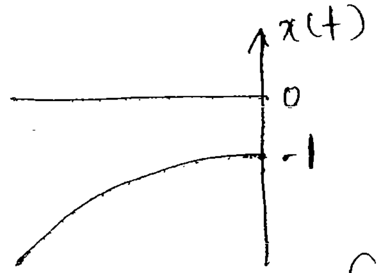
(2) ROC should n't contain any pole.

Note: Properties

1. R.O.C of L.T. Consists of lines parellel to $j\omega$ axis.
2. R.O.C should not contain any pole.

Note: 2. For the stability in laplace domain R.O.C must include imaginary axis

* $x(t) = -e^{-at}u(-t)$; $Re\{a\} > 0$.



unstable s/m
No FT.

As $t \to -\infty$
$x(t) \to -\infty$
this can be verified by considering R.O.C

$$L.T = X(s) = \int_{-\infty}^{0} -e^{-at}e^{-st}dt = -\int_{-\infty}^{0} e^{-(s+a)t}dt$$

$$= +\left[\frac{e^{-(s+a)t}}{(s+a)}\right]_{-\infty}^{0}$$

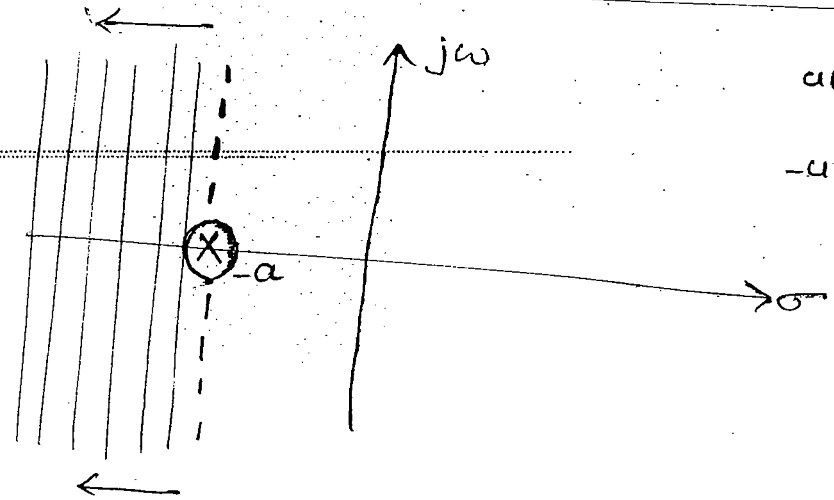
$$= \left[\frac{e^{-(\sigma+a+j\omega)t}}{(s+a)}\right]_{-\infty}^{0}$$

$$= \frac{1}{s+a} \quad ; \quad \sigma+a<0$$

$\sigma<-a \; ; \; \sigma< Re\{-a\}$

At $j\omega$ axis ($\sigma=0$) the LT becomes fourier transform. If the R.O.C contains $j\omega$ axis then only FT exists. If R.O.C does'nt contain $j\omega$ axis, then for that particular s/l No FT can be defined & it is unstable. (FT is defined only for stable s/ms) NoFT means No ...

$$\boxed{-e^{-at}u(t) \longleftrightarrow \frac{1}{s+a} \; ; \; \sigma< Re\{-a\}}$$



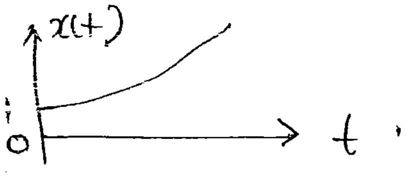
$u(t) \rightarrow$ Right sided

$-u(-t) \rightarrow$ left sided

R.O.C doesn't include $j\omega$ axis ∴ the s/l is unstable & no FT exists.

⇒✱ Solution of $\mathcal{L}$T is unique only when R.O.C is given. If R.O.C is not given LT of $e^{-at}u(t)$ & $-e^{-at}u(-t)$ are the same.

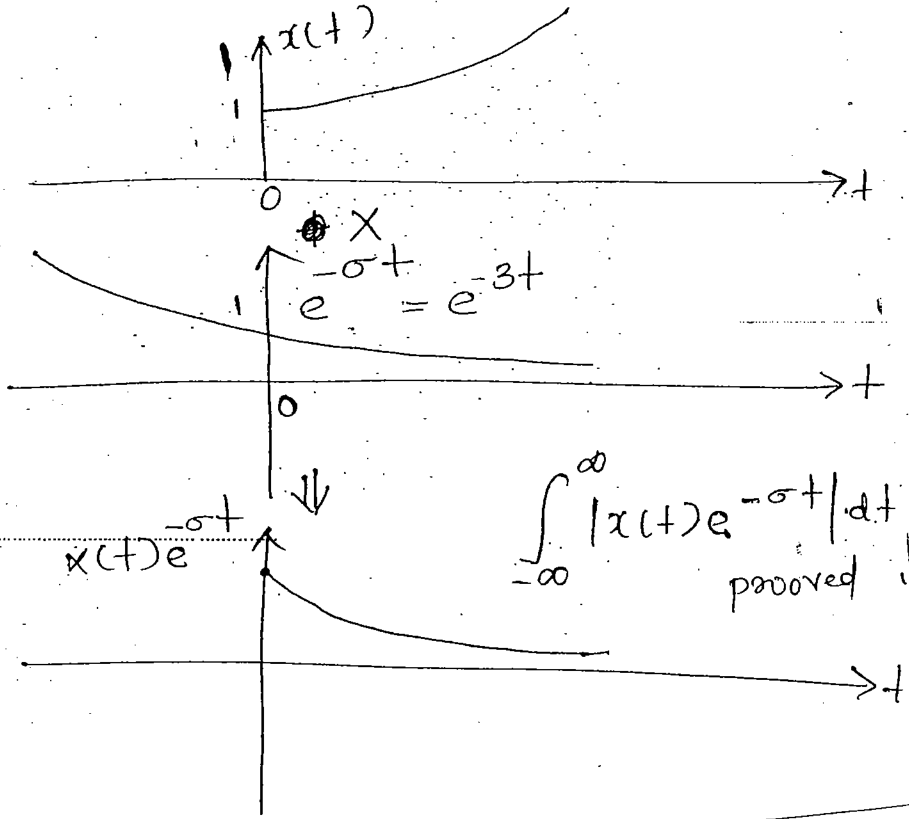
✱ $x(t) = e^{at}u(t)$; $Re\{a\}>0$.



No FT, unstable.

$$x(s) = \int_0^{\infty} e^{at} e^{-st} dt = \int_0^{\infty} e^{-(s-a)t} dt$$

$$= \left[\frac{e^{-(\sigma - a + j\omega)t}}{-(s-a)} \right]_0^{\infty} = \frac{1}{s-a} \quad ; \quad \sigma - a > 0 \quad \sigma > a$$



$$\int_{-\infty}^{\infty} |x(t) e^{-\sigma t}| dt < \infty$$

prooved in time domain

$$\boxed{\begin{aligned} e^{at} u(t) &\longleftrightarrow \frac{1}{s-a} \; ; \; \sigma > Re\{a\}. \\ -e^{at} u(-t) &\longleftrightarrow \frac{1}{s-a} \; ; \; \sigma < Re\{a\}. \end{aligned}}$$

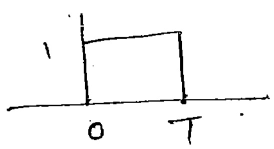
Pptg

Cond Imp

3 If $x(t)$ is of finite duration then the R.O.C is Complete s plane.

$x(t) = \delta(t) \Longrightarrow X(s) = 1$

R.O.C entire s plane.



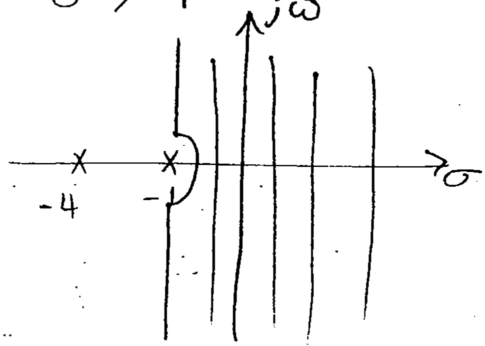
ROC entire s plane.

P5.1.1

$$X(s) = \frac{1}{(s+1)(s+4)} = \frac{1/3}{s+1} + \frac{-1/3}{s+4}$$

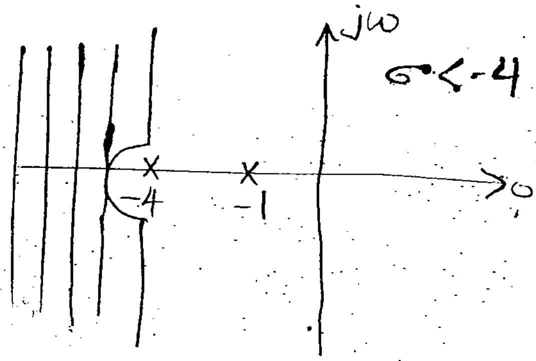
$x(t)$ is right sided

$\sigma > -1$



$$x(t) = \frac{1}{3}e^{-t}u(t) - \frac{1}{3}e^{-4t}u(t)$$

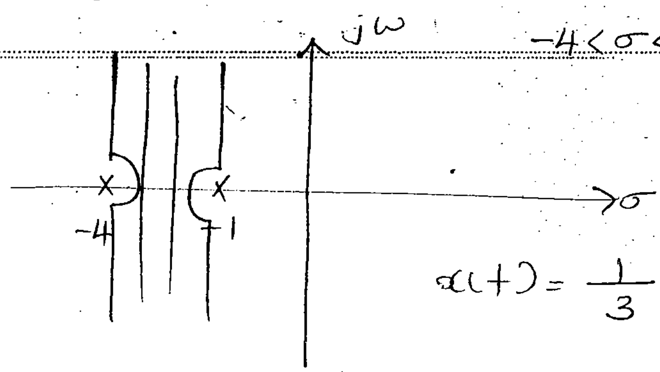
$x(t)$ is left sided



$$x(t) = \frac{1}{3}e^{-t}(-u(-t)) - \frac{1}{3}e^{-4t}(-u(-t))$$

$x(t)$ is two sided.

$-4 < \sigma < -1$



$$x(t) = \frac{1}{3}e^{-t}(-u(-t)) - \frac{1}{3}e^{-4t}u(t).$$

P5.1.2 ① $x(t) = e^{-t}u(t) + e^{-3t}u(t)$

$\downarrow LT$

$$X(s) = \frac{1}{s+1} + \frac{1}{s+3}$$

$(\sigma > -1)$ $(\sigma > -3)$

$$X(s) = \frac{1}{s+1} + \frac{1}{s+3}\ ; \quad \sigma > -1$$

② $x_2(t) = e^{-2t}u(t) + e^{4t}u(-t)$

$\updownarrow LT$

$\frac{1}{s+2}\ ; \ \sigma > -2$ $\qquad \frac{-1}{s-4}\ ; \ \sigma < 4$

$-2 < \sigma < 4$

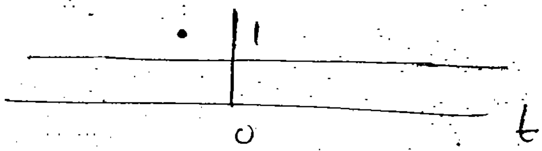
(3) $x_3(t) = e^{-t}u(-t) + e^{5t}u(t)$

$$\updownarrow LT \qquad\qquad \updownarrow$$

$$\frac{-1}{s+1};\ \sigma < -1 \qquad \frac{+1}{s-5};\ \sigma > 5$$

No LT (since no common ROC)

(4) $x_4(t) = 1 \quad \forall t$



$x(t) = u(t) + u(-t)$

$$LT \updownarrow \qquad LT \updownarrow$$

$$1/s;\ \sigma > 0 \qquad 1/s;\ \sigma \le 0.$$

No common ROC, No LT

(5) $x_5(t) = sgn(t)$.

$= u(t) - u(-t)$

$$LT \updownarrow \qquad \updownarrow LT$$

$$1/s;\ \sigma > 0 \qquad 1/s;\ \sigma < 0.$$

No common ROC $\Rightarrow$ No LT

FT is not perfect (obtained by mathemati approximation).

P5.1.3 $x(t) = \left[e^{-5t} + e^{-\beta t}\right]u(t) = e^{-5t}u(t) + e^{-\beta t}u(t)$.

Given ROC : $Re\{s\} > -3$

$$\updownarrow LT \qquad \updownarrow LT$$

$$\frac{1}{s+5}\ \sigma > -5 \qquad \frac{1}{s+\beta};\ \sigma > \ \text{Rea}$$

$Real\{\beta\} = 3$

To get region ~~of~~ convergence only real ~~part is~~ considered

P5.1.4 4. $[\sigma > 2,\ \sigma < -3,\ -3 < \sigma < -1,\ -1 < \sigma < 2]$

P5.1.5

$e^{(a+2)t+5}u(t) \quad — \quad e^{st+5}u(t)$

$Re\{s\} > a+2$

Time - shifting

$x(t-t_0) \longleftrightarrow e^{-st_0}X(s)$ with ROC same. (poles location is not changing)

* $y(t) = u(-t+4)$

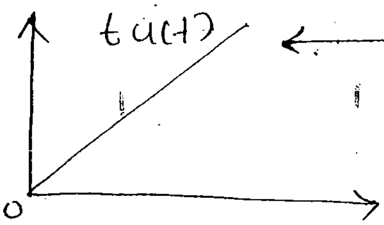
$= u[-(t-4)]$

$\downarrow LT \qquad t_0 = 4$

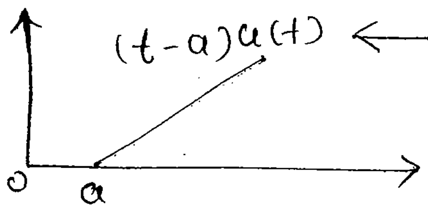
$= e^{-4s}\left[-\frac{1}{s}\right] \quad ; \sigma < 0$

---

* LT of ramp at $t = a$ is ________



$t\,u(t) \longleftrightarrow \frac{1}{s^2}$



$(t-a)u(t) \longleftrightarrow \frac{e^{-as}}{s^2} \quad ; \sigma > 0$

23/08/2011
9-1 pm

* $Y(s) = \frac{e^{-3s}}{(s+1)(s+2)} \quad ; \sigma > -1$

$= e^{-3s}X(s) \qquad X(s) = \frac{1}{(s+1)(s+2)}$

$\downarrow ILT \qquad = \frac{1}{s+1} - \frac{1}{s+2}$

$y(t) = x(t-3)$

$x(t) = e^{-t}u(t) - e^{-2t}u(t)$

pg: 77

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P5.1.8

ⓐ $x(t) = e^{-5t}u(t-1)$

$= e^{-5(t-1+1)}u(t-1)$

$= e^{-5} \cdot e^{-5(t-1)}u(t-1)$

$\downarrow LT$

$X(s) = e^{-5}\left[\frac{e^{-s(1)}}{s+5}\right]$; $\sigma > -5$

ⓑ $g(t) = A e^{-5t}u(-t-t_0)$

$= A e^{-5t}u(-(t-(-t_0)))$

$= \cancel{\frac{A}{s+5}}$; $\cancel{\sigma < -5}$

$= A e^{-5(t+t_0-t_0)}u(-(t+t_0))$

$= A e^{5t_0} e^{-5(t+t_0)}u(-(t+t_0))$

$\downarrow LT$

$G(s) = A e^{5t_0}\left[\frac{-e^{-s(-t_0)}}{s+5}\right]$; $\sigma < -5$

Equating $A = -1$, $t_0 = -1$

P5.1.9

$f(t) = u(t) - u(t-1) + 2[u(t-1) - u(t-2)]$

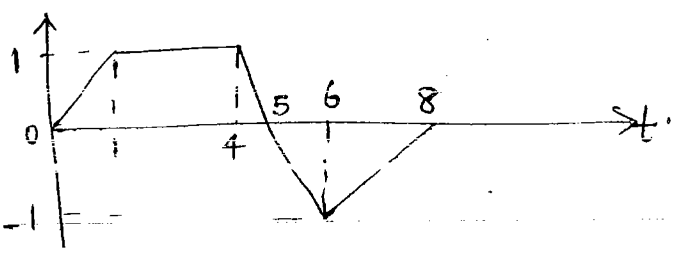
$= u(t) + u(t-1) - 2u(t-2)$

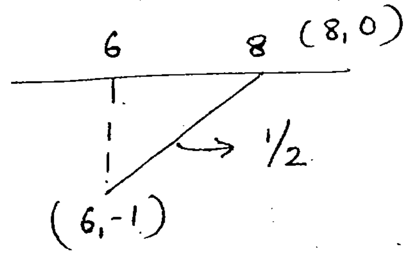
$\downarrow LT$

$F(s) = \frac{1}{s} + \frac{e^{-s}}{s} + -2\frac{e^{-2s}}{s}$

finit duration s/l $\Longrightarrow$ R.O.C is entire 's' pla

P5.1.10





$D = -\frac{1}{2}$

. to make a +ve ramp of slope ½ zero, a -ve value of ½ is added.

Shift in s domain.

$$x(t)e^{s_0 t} \longleftrightarrow X(s-s_0)$$

$$ROC = R + Re\{s_0\}.$$

eg:* $y(t) = \cos\omega_0 t\, u(t)$

$$= \left[\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right] x(t)$$

Let $x(t) = u(t)$

$X(s) = \frac{1}{s}$; $s>0$

$\downarrow$ LT

$$Y(s) = \frac{X(s-j\omega_0) + X(s+j\omega_0)}{2}$$

$$= \frac{1}{2}\left[\frac{1}{s-j\omega_0} + \frac{1}{s+j\omega_0}\right]$$

$$= \frac{s}{s^2+\omega_0^2} \; ; \; \sigma > 0+0$$

*eg: $y_1(t) = te^{-at}u(t)$

$= x(t)\, e^{-at}$

$x(t) = tu(t)$

$X(s) = \frac{1}{s^2}$; $\sigma > 0$

$\downarrow$ LT

$$Y_1(s) = X(s+a)$$

$$= \frac{1}{(s+a)^2} \; ; \; \sigma > 0 + Re\{-a\}$$

$$Y(s) = \frac{s^2 - s + 1}{(s+1)^2} \; ; \sigma > -1 \quad \text{find } g(t)?$$

(7.3)

proper fraction

$$Y(s) = 1 - \frac{3s}{s^2+2s+1}$$

$$s^2+2s+1 \overline{\left) \begin{array}{l} s^2 - s + 1 \\ s^2 + 2s + 1 \\ \hline -3s \end{array} \right.} \; 1$$

$$= 1 - \frac{3s}{(s+1)^2}$$

$$= 1 - \frac{3(s+1-1)}{(s+1)^2} = 1 - \frac{3(s+1)}{(s+1)^2} + \frac{3}{(s+1)^2}$$

$$= 1 - \frac{3}{(s+1)^2} + \frac{3}{(s+1)^2}$$

$$g(t) = \delta(t) - 3e^{-t}u(t) + 3te^{-t}u(t).$$

$$g(t) = \delta(t) - 3\frac{d}{dt}\{te^{-t}\} = \delta(t) - 3\left[t \times e^{-t} \times -1 + e^{-t} \times 1\right]$$

$$= \delta(t) + 3te^{-t}u(t) - 3e^{-t}u(t)$$

P5.1.12

$x(t) \longrightarrow$ 2 poles -1 & -3

$$g(t) = x(t)e^{2t}$$

$G(\omega)$ converges

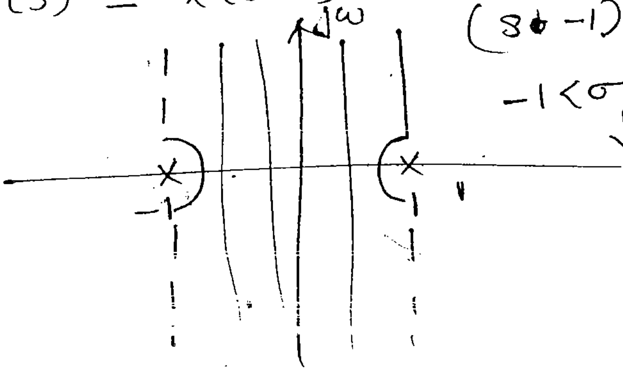
$\Downarrow$

F.T defined $\Rightarrow$ stability guaranteed

ie R.O.C of $G(s)$ includes $j\omega$ axis

$$X(s) = \frac{1}{(s+1)(s+3)}$$

$$G(s) = X(s-2) = \frac{1}{(s-1)(s+1)}$$



$-1 < \sigma < 1$

$\Downarrow$

2-sided.

Time Reversal

$$x(-t) = X(-s), \quad ROC = -R$$

P5.1.B

$$g(t) = x(t) + \alpha\, x(-t)$$

$$x(t) = \beta e^{-t} u(t)$$

$$G(s) = \frac{s}{s^2-1} \quad -1 < Re\{s\} < 1$$

α ? $\beta = ?$

L.T

$$G(s) = \frac{\beta}{s+1} + \frac{\alpha\beta}{-s+1}$$

$$\left[x(t) = \frac{\beta}{s+1} \qquad x(-t) = \beta e^{t} u(-t) \qquad X(-s) = \frac{+\beta\alpha}{-s+1} \right]$$

$$G(s) = \frac{\beta}{s+1} - \frac{\alpha\beta}{s-1} = \frac{s}{s^2-1}$$

$$\beta s - \beta - \alpha\beta s - \alpha\beta = s$$

$$s(\beta - \alpha\beta) - \beta - \alpha\beta = s$$

$$\beta + \alpha\beta = 0 \implies \beta(1+\alpha) = 0$$

$$\beta - \alpha\beta = 1 \implies \beta(1-\alpha) = 1$$

$$\alpha = -1, \quad \beta = 1/2$$

Differentiation in time.

$$\left.\begin{aligned} \frac{d}{dt} &\longrightarrow j\omega \longrightarrow s \\ \frac{d}{d\omega} &\longrightarrow -jt \\ \frac{d}{d(j\omega)} &\longrightarrow -t \\ \frac{d}{ds} &\longrightarrow -t \end{aligned}\right\}$$

ROC = R
same because the poles are not changing.

5.1.14

$$\frac{dx(t)}{dt} = -2y(t) + \delta(t)$$

$$\frac{dy(t)}{dt} = 2x(t) \quad \text{find } X(s) \text{ \& } Y(s).$$

(74)

$\downarrow$ LT

$$sY(s) = 2X(s) \Rightarrow Y(s) = \frac{2}{s}X(s).$$

LT

$$sX(s) = -2Y(s) + 1$$

$$= -2 \times \frac{2}{s} X(s) + 1$$

$$= \frac{-4}{s} X(s) + 1$$

$$\Rightarrow \left(s + \frac{4}{s}\right) X(s) = 1$$

$$X(s) = \frac{1}{\frac{s^2+4}{s}} = \frac{s}{s^2+4} \quad ; \sigma > 0$$

$$Y(s) = \frac{2}{s}X(s) = \frac{2}{s^2+4} \quad ; \sigma > 0$$

s^2+4
s^2+2^2
$(s+2j)(s-2j)$
$s^2-(2j)^2$
s^2+4

*g:

$$Y(s) = \log\left(\frac{s+2}{s+3}\right)$$

$$= \log(s+2) - \log(s+3)$$

LT

$$\frac{dY(s)}{ds} = \frac{1}{s+2} - \frac{1}{s+3}.$$

$\downarrow$ ILT

$$-t\,y(t) = e^{-2t}u(t) - e^{-3t}u(t) \quad \text{[Assuming right sided s/l]}$$

*g:

$$X(s) = \frac{4}{(s+2)(s+1)^3} =$$

$$= \frac{A}{s+2} + \frac{B}{(s+1)^3} + \frac{C}{(s+1)^2} + \frac{D}{(s+1)}$$

$$C = \frac{d}{ds}\left[(s+1)^3 X(s)\right]_{s=-1}$$

$$D = \frac{1}{2!}\frac{d^2}{ds^2}\left[(s+1)X(s)\right]_{s=-1}$$

$$A = -4 \quad ; \quad B = 4$$

substitate value for s & obtain eqn in terms of c & D the solve for c & D.

$$\frac{4}{(s+2)(s+1)^3} = \frac{-4}{s+2} + \frac{4}{(s+1)^3} + \frac{C}{(s+1)^2} + \frac{D}{(s+1)}$$

put $s = 0$

$$2 = -2 + 4 + C + D \implies \underline{\underline{C = -D}}$$

put $s = +1$

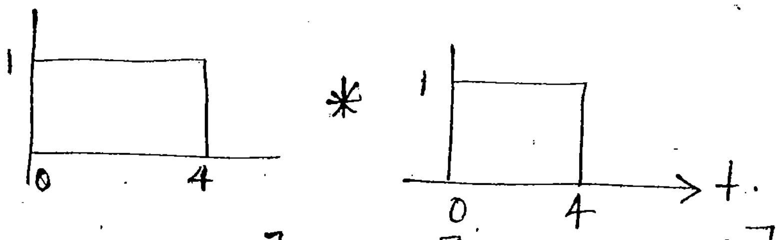
$$\frac{4}{3 \times 8} = \frac{-4}{3} + \frac{4}{8} + \frac{C}{4} + \frac{D}{2}$$

$$\implies C + 2D = 4 \implies \underline{D = 4}$$

$$\underline{\underline{C = -4}}$$

$$\boxed{\frac{t^{n-1}}{(n-1)!} e^{-at} u(t) \longleftrightarrow \frac{1}{(s+a)^n}}$$

Convolution in Time.



$$\left[u(t) - u(t-4)\right] * \left[u(t) - u(t-4)\right]$$

$$\left[\frac{1}{s} - \frac{e^{-4s}}{s}\right]^2$$

$$\frac{1}{s^2} + \frac{e^{-8s}}{s^2} - \frac{2e^{-4s}}{s^2}$$

$\Updownarrow$ ILT

$$tu(t) + (t-8)u(t) - 2(t-4)u(t) \implies$$

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Solve. $y(t) + \int_0^{\infty} y(\tau)\, x(t-\tau)\, d\tau = x(t) + \delta(t)$

$$\downarrow LT$$

$$Y(s) + Y(s)X(s) = X(s) + 1$$

$$Y(s)\left[1 + X(s)\right] = \left[1 + X(s)\right]$$

$$Y(s) = 1$$

$$\downarrow ILT$$

$$\underline{\underline{y(t) = \delta(t)}}$$

Assume both $y(t)$ & $x(t)$ are +ve sided s/l.
Otherwise start integral from $-\infty$.

P5.1.20

$$y(t) = x_1(t-2) * x_2(-t+3) = x_1(t-2) * x_2(-(t-3))$$

$$x_1(t) = e^{-2t}u(t) \qquad x_2(t) = e^{-3t}u(t)$$

LT

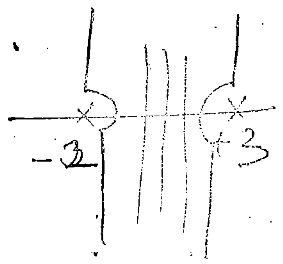
$$Y(s) = e^{-2s}X_1(s) \cdot e^{-3s}X_2(-s)$$

valid/invalid multiple
based on R.O.C

$$X_1(s) = \frac{1}{s+2} \; ; \sigma > -2$$

$$X_2(s) = \frac{1}{s+3} \; ; \sigma > -3$$

$$X_2(-s) = \frac{1}{-s+3} \; ; \sigma < 3$$



$$Y(s) = e^{-2s} \cdot \frac{1}{s+2} \cdot e^{-3s} \cdot \frac{1}{(-s+3)}$$

$$= \underline{\underline{\frac{e^{-5s}}{(s+2)(-s+3)}}}$$

P5.1.21

$$\frac{dy(t)}{dt} + 4y(t) + 3\int_{-\infty}^{t} y(\tau)\,d\tau = x(t)$$

$$sY(s) + 4Y(s) + \frac{3}{s}Y(s) = X(s).$$

$$Y(s)\left[s^2+4s+3\right] = s\,X(s)$$

Transfer function:

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s}{s^2+4s+3} = \frac{s}{(s+1)(s+3)}$$

$$= \frac{-1/2}{s+1} + \frac{3/2}{s+3}$$

$\downarrow$ ILT

$$h(t) = \frac{-1}{2}e^{-t}u(t) + \frac{3}{2}e^{-3t}u(t)$$

Right sided (since given that s/m is causal)

$$x(t) = u(t) + \delta(t).$$

$$X(s) = \frac{1}{s} + 1 = \frac{1+s}{s}$$

$$Y(s) = X(s)\cdot H(s)$$

$$= \frac{1}{s+3} \qquad \sigma > -3$$

$$y(t) = e^{-3t}u(t)$$; ~~Ans~~

P5.1.22

$$\int_{-\infty}^{t} g(\lambda)\,e^{-3(t-\lambda)}\,u(t-\lambda)\,d\lambda$$

$$= g(t) * e^{-3t}u(t).$$

P5.1.23

$$x(t) = e^{-2t}u(t) + \delta(t-6)$$

$$h(t) = u(t)$$

$u(t) * \delta(t-6) = u(t-6)$ term should be in answer

$$X(s) = \frac{1}{s+2} \qquad H(s) = \frac{1}{s}.$$

$$X(s)\,H(s) = \left(\frac{1}{s+2}\right)\left(\frac{1}{s}\right) = \frac{1/2}{s} - \frac{1/2}{s+2}$$

$$\Big\downarrow \text{ILT}$$

$$= \frac{1}{2}u(t) - \frac{1}{2}e^{-2t}u(t) + u(t-6)$$

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5.1.24

*2

$$f(t) \longleftrightarrow F_1(s)$$

$$f(t-\tau) \longleftrightarrow F_2(s) = e^{-s\tau}F_1(s)$$

$$G(s) = \frac{F_2(s)\,F_1^*(s)}{|F_1(s)|^2}$$

$$= \frac{e^{-s\tau}F_1(s)\,F_1^*(s)}{|F_1(s)|^2}$$

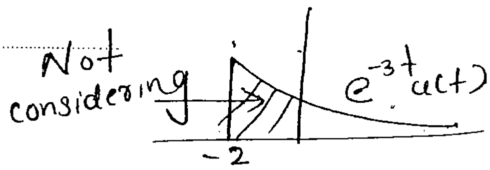
2.

$$g(t) = \delta(t-\tau)$$

Unilateral Laplace Transform

$$X(s) = \int_0^{\infty} x(t)\,e^{-st}\,dt$$

*g U.L.T of $e^{-3t}u(t+2)$ is ______



Ans: $\frac{1}{s+3}$.

*g U.L.T of $\delta(t+3)$ is $\underline{0}$

*g " $\delta(t-4)$ is $\underline{e^{-4s}(1)}$

U.L.T of $x(t)$ is B.L.T of $x(t)u(t)$

Differentiation in time.

$$\frac{d}{dt}x(t) \longleftrightarrow sX(s) - x(0)$$

$$\frac{d^2}{dt^2}x(t) \longleftrightarrow s^2X(s) - sx(0) - x'(0)$$

*g

Total o/p

SSR (or) Zero state Response (Z.S.R) (or) Forced Response

TR (or) Zero Input Response (Z.I.R) (or) Natural Response.

To get TR (s/m pole) ← **Z.S.R** :- All initial Conditions are zero but input is given (applied)

To get characteristics polenomeal (s/e pole) ← **Z.I.R** :- Input is not taken
All initial Conditions are taken.

P5.2.2

$$y''(t) + 5y'(t) + 6y(t) = 2x'(t) + x(t)$$

$$y'(0) = 2 \qquad y(0) = 1 \qquad x(t) = u(t)$$

Z.S.R

ULT

$$s^2Y(s) - sY(0) - Y'(0)$$

$$+5[sY(s) - Y(0)] + 6Y(s) = 2sX(s) - x(0) + x(s)$$

$$s^2Y(s) + 5sY(s) + 6Y(s) = 2sX(s) + X(s)$$

$$(s^2 + 5s + 6)Y(s) = (2s + 1)X(s).$$

$$Y(s) = \frac{(2s+1)}{(s^2+5s+6)}X(s).$$

$$X(s) = L\{u(t)\} = \frac{1}{s}$$

$$Y(s) = \frac{(2s+1)}{s(s^2+5s+6)} = \frac{2s+1}{s(s+2)(s+3)}$$

(77)

$$= \frac{1/6}{s} + \frac{3/2}{s+2} + \frac{-5/3}{s+3}$$

$\downarrow$ ILT

forced

$$y^{(f)}(t) = \frac{1}{6}u(t) + \frac{3}{2}e^{-2t}u(t) + -\frac{5}{3}e^{-3t}u(t)$$

Z.I.R

$$s^2Y(s) - sY(0) - Y'(0) + 5[sY(s) - Y(0)] + 6Y(s) = 0$$

$$Y(s)[s^2+5s+6] - s\times 1 - 2 - 5 = 0$$

$$Y(s) = \frac{s+7}{s^2+5s+6} = \frac{s+7}{(s+2)(s+3)}$$

$$= \frac{5}{s+2} + \frac{-4}{s+3}$$

$\downarrow$ ILT

$$y^{(n)}(t) = 5e^{-2t}u(t) - 4e^{-3t}u(t)$$

$$y(t) = y^{(f)}(t) + y^{(n)}(t)$$

$$= \frac{1}{6}u(t) + \frac{13}{2}e^{-2t}u(t) - \frac{17}{3}e^{-3t}u(t)$$

$$y(0) = \frac{1}{6} + \frac{13}{2} - \frac{17}{3} = 1$$

Initial & Final value theorem.

Initial value theorem: $x(0) = \lim_{s\to\infty} sX(s)$

Final value theorem: $x(\infty) = \lim_{s\to 0} sX(s)$.

P5.2.4 (a) $X(s) = \frac{2s+5}{s^2+5s+6} \Rightarrow sX(s) = \frac{s^2(2+5/s)}{s^2(1+5/s+6/s^2)}$

$$x(0) = \frac{2}{1} \qquad x(\infty) = 0$$

(b) $X(s) = \dfrac{4s+5}{2s+1}$

$sX(s) = \dfrac{s(4s+5)}{2s+1} = \dfrac{4s^2+5s}{2s+1}$

$\cancel{x(0) = \lim_{s\to\infty} sX(s) = \lim_{s\to\infty} \dfrac{s(4+5/s)}{s(2+1/s)}}$

$\cancel{2s+1 \overline{\big)\, 4s^2+5s\,}\; 2s \quad 4s^2+2s \quad +3s}$

$\cancel{\Rightarrow sX(s) = 2s + \dfrac{3s}{2s+1}}$

$2s+1 \overline{\big)\, 4s+5\,}\; 2$
$2s+2$ (sic)
3

$X(s) = 2 + \dfrac{3}{2s+1}$

$x(0) = \lim_{s\to\infty} s\left[\dfrac{3}{2s+1}\right]$

$= \underline{\underline{3/2}}$

$x(\infty) = \lim_{s\to 0} s\left[\dfrac{3}{2s+1}\right]$

$= \underline{\underline{0}}$

proper fraction should made strictly proper before applying initial value & final value.

Imp

(c) $X(s) = \dfrac{12(s+2)}{s(s^2+4)}$

$\cancel{x(0) = \lim_{s\to\infty} \dfrac{12s^2(1+2/s)}{s^3(1+4/s)}} = \underline{\underline{0}}$

$x(\infty) = 6$ (Apply theorom)

$x(0) =$ Indeterminate.

2. poles are on the $j\omega$ axis.

Imp Final value theorom of Laplace transfrom is valid only if all poles have -ve real parts except a simple pole at $s = 0$.

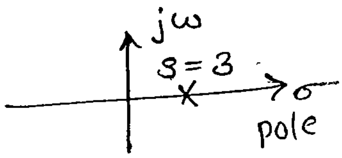
$$X(s) = \frac{s}{s^2+\omega_0^2}$$

$$x(\infty) = \lim_{s\to 0} s\left[\frac{s}{s^2+\omega_0^2}\right]$$

$$= \underline{\underline{0}}$$

$$x(t) = \cos\omega_0 t\, u(t)$$

$$x(t)\Big|_{t\to\infty} = \underline{\underline{\pm 1}}$$

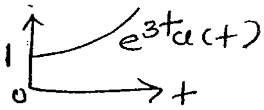


$$X(s) = \frac{1}{s-3}$$

$$x(\infty) = \lim_{s\to 0} \frac{s}{s-3} = \underline{\underline{0}}$$

pole is on +ve real axis. final value theorem can't be applied

$$x(t) = e^{3t}u(t) \qquad x(\infty) = \infty.$$



(d) $x(\infty) = 0$ $\quad x(\infty) = \underline{\underline{-1}}$

$$X(s) = e^{-s}\left[\frac{-2}{s(s+2)}\right]$$

$$x(0) = \lim_{s\to\infty} sX(s) = \underline{\underline{0}}$$

$$x(\infty) = \lim_{s\to 0} sX(s) = \underline{\underline{-1}}$$

PS 2.5 Poles at $s = -2$ & $s = -4$.

zeros at $s = -1$

$$x(t) = u(t)$$

$$y(t)\Big|_{t\to\infty} = 1 \qquad \text{I.R } h(t) = ?$$

$$H(s) = \frac{s+1}{(s+2)(s+4)} = \frac{-1/2}{s+2} + \frac{3/2}{s+4}$$

$$Y(s) = X(s)H(s)$$

$$y(\infty) = \lim_{s\to 0} sY(s)$$

$$= \lim_{s\to 0} s\left[\frac{1}{s}\right]\left[\frac{s+1}{(s+2)(s+4)}\right] = 1/8.$$

But given $y(t)\big|_{t\to\infty}$

Gain has to be adjusted.

$$H(s) = \frac{8(s+1)}{(s+2)(s+4)}$$

$$= \frac{-4}{s+2} + \frac{12}{s+4}$$

$\downarrow$ I.L.T

$$h(t) = -4e^{-2t}u(t) + 12e^{-4t}u(t)$$

P5.2.3

$$H(s) = \frac{s-2}{s^2+4s+4}$$

$$x(t) = 8\cos 2t \qquad y_{s.s}(t) = ?$$

$$Y(s) = X(s) \cdot H(s)$$

$$= \left[\frac{8s}{s^2+4}\right]\left[\frac{s-2}{s^2+4s+4}\right]$$

$$= \left[\frac{8s}{s^2+4}\right]\left[\frac{s-2}{(s+2)^2}\right]$$

$$= \frac{As+B}{s^2+4} + \frac{C}{(s+2)^2} + \frac{D}{s+2}.$$ Not follow this technique

Alternate Method

i/p: $x(t) = A\cos(\omega_0 t + \phi)$ if i/p is sinusoidal output is also sinus

Poles are on the $j\omega$ axis. $\leftarrow$ $H(s)\Big|_{s=j\omega_0} = K\angle\phi$

$$y_{ss}(t) = KA\cos(\omega_0 t + \theta + \phi)$$

$$A = 8;\ \omega_0 = 2;\ \theta = 0$$

$$H(s)\Big|_{s=j2} = \frac{j2-2}{(j2)^2+4(j2)+4}$$

$$= 0.3536\angle 45^\circ$$
$$K\angle\phi$$

if i/p $x(t) = u(t)$.

$\downarrow$ $X(s) = 1/s$

$y_{ss}(t) = u(t)\left[-\frac{2}{4}\right]$

output is scaled version of i/p. (79)

Q.

P5.2.6 2. (A) Correct

$s = j\omega$; $\omega = 1$ given

$H(s) = 0$.

P5.2.7 Imp. Final Value $\longrightarrow$ location of poles

one pole in right half of s plane

$\therefore$ Ans : unbounded. Before applying initial value & final value theorem, make sure that the poles are on the left half of $j\omega$ axis.

P5.2.9 (D) Correct.

F must be sine / cosine with equal freq.

Amp equal / unequal.

Causality

ROC : Right of rightmost pole

Stability :

ROC : $j\omega$ axis included.

$\Big\}$ For proper transfer function

Q: $H(s) = \frac{e^{s}}{s+2}$; $\sigma > -2$

Improper

$= e^{s}X(s)$

$x(t-t_0) \longleftrightarrow e^{-st_0}X(s)$

$\downarrow$ ILT

$h(t) = e^{-2(t+1)}u(t+1)$

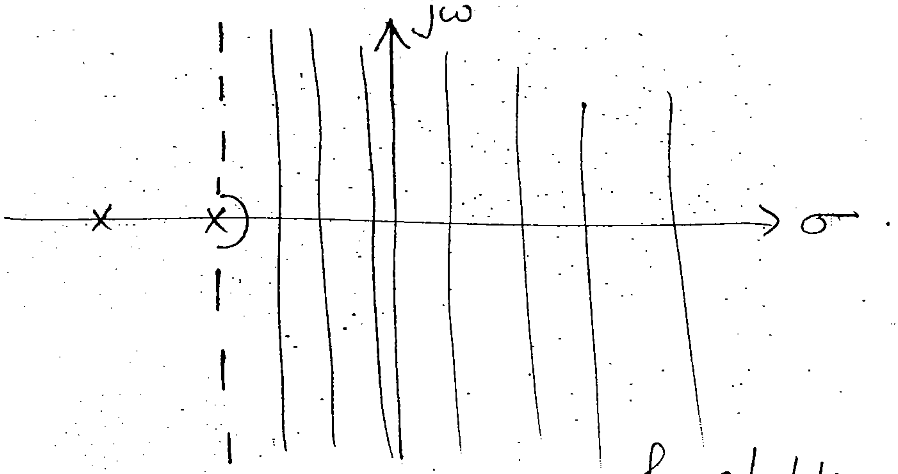
$\Downarrow$

Non Causal.

R.O.C doesn't guarantee Causality / Non causal

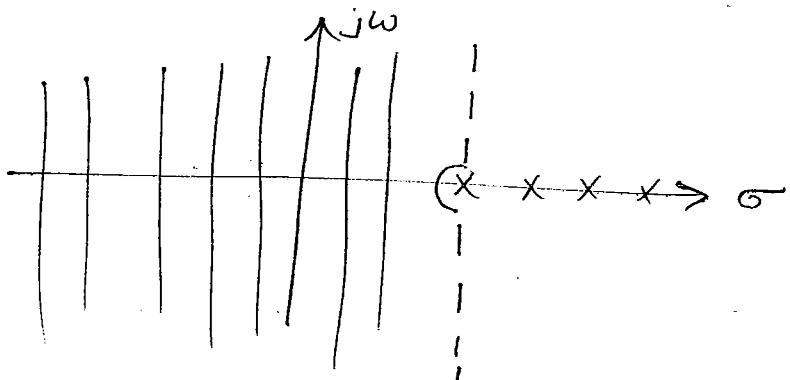
For causality TF should be strictly proper / atleast proper.

* For both causality & stability poles are on the left half of s-plane



$\sigma > -1 \Rightarrow$ Causal & stable

Anti Causality & Stability $\Longrightarrow$ All poles must lie in the right half of s plane

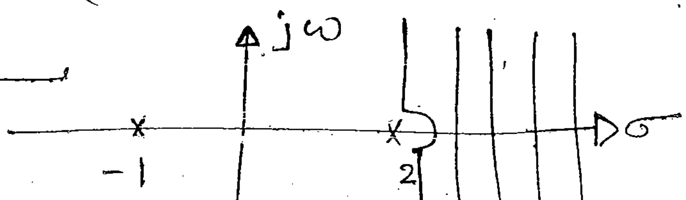


eg: $H(s) = \dfrac{1}{(s-2)(s-3)}$

P5.3.1

$$H(s) = \frac{s-1}{(s+1)(s-2)} = \frac{2/3}{s+1} + \frac{1/3}{s-2}$$

Possibility 1

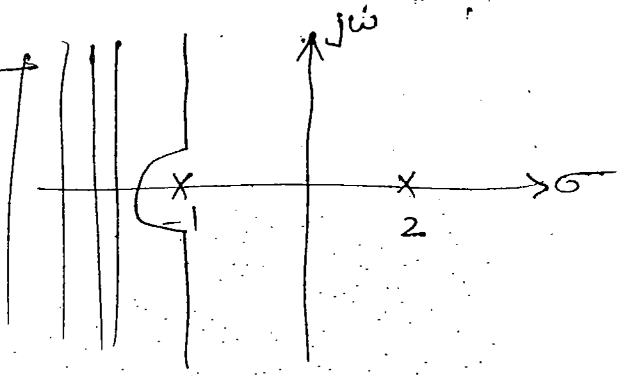


$\sigma > 2 \Rightarrow$ causal, unstable.

$$h(t) = \frac{2}{3} e^{-t} u(t) + \frac{1}{3} e^{2t} u(t)$$

(80)

Possibility 2

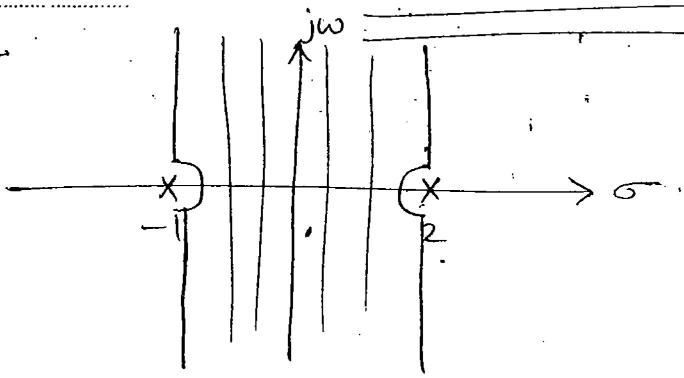


$\sigma < -1 \Rightarrow$ Anticausal, unstable.

$$h(t) = \frac{2}{3} e^{-t}(-u(-t)) + \frac{1}{3} e^{2t}(-u(-t))$$

$$= -\frac{2}{3} e^{-t} u(-t) - \frac{1}{3} e^{2t} u(-t)$$

Possibility 3



$-1 < \sigma < 2 \Rightarrow$ stable, non causal.

$$h(t) = \frac{2}{3} e^{-t} u(t) - \frac{1}{3} e^{2t} u(-t)$$

P5.3.2 stable.

Imp P5.3.3

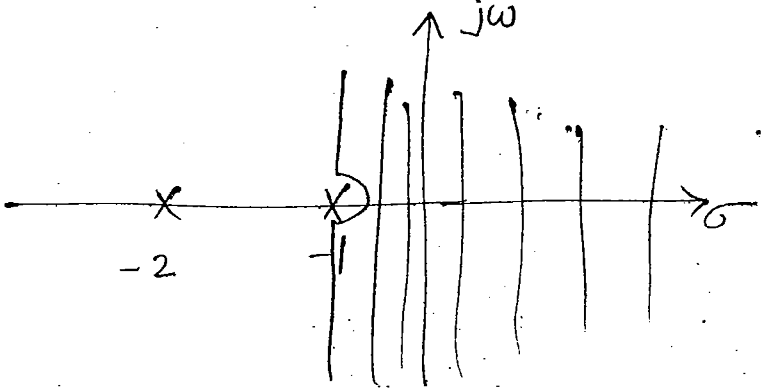
$$X(s) = \frac{s+2}{s-2}; \quad x(t) = 0,\ t > 0.$$ (Anticausal)

$$y(t) = -\frac{2}{3} e^{2t} u(-t) + \frac{1}{3} e^{-t} u(t)$$

$$Y(s) = +\frac{2}{3}\left[\frac{1}{s-2}\right] + \frac{1}{3}\left[\frac{1}{s+1}\right]$$

(left sided signal) $(\sigma < 2)$ $(\sigma > -1)$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s}{(s+1)(s+2)}$$

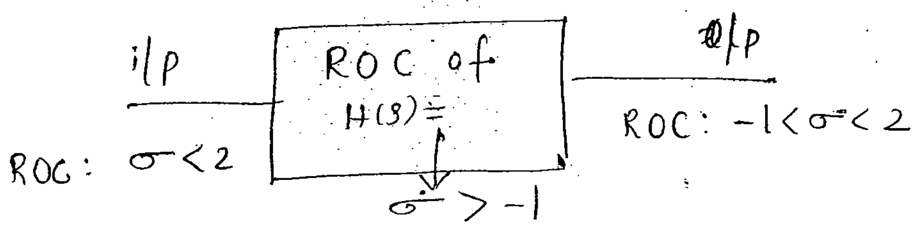


ROC : $\sigma > -1$

1) $\int_{-\infty}^{\infty} y(t)\,dt$ = finite (s/m stable)

∴ should contain jω axis.

2) $R_1 \cap R_2$



using i/p ROC & s/m ROC we can decide the o/p ROC.

(5) $x(t) = e^{3t} \;\forall t$?

$$\cancel{X(s) = \frac{1}{s-3}}$$

$$\cancel{Y(s) = X(s) \cdot H(s) = \frac{1}{(s-3)} \cdot \frac{s}{(s+1)(s+2)}}$$

i/p : $-\infty < t < \infty$

if $x(t) = e^{st} \Longrightarrow y(t) = e^{st} H(s)$.

$e^{3t} \Longrightarrow \quad = e^{3t} H(s)\Big|_{s=3}$

$= \frac{3}{(3+1)(3+2)} \cdot e^{3t}$

7. Convolution ppty can't be applied.
No LT for $x(t)$ since it contains all the time.

(8)

P5.3.4

$$H_{inv}(s) = \frac{1}{H(s)}$$

$$\frac{dy(t)}{dt} + 3y(t) = \frac{d^2x(t)}{dt^2} + \frac{dx(t)}{dt} - 2x(t)$$

$$sY(s) - Y(0) + 3Y(s) = s^2X(s) - sx(0) - x'(0) + sX(s) - 2X(s) - x(0)$$

$$Y(s)[s+3] = X(s)[s^2+s-2]$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s^2+s-2}{s+3}$$

$$H_{inv}(s) = \frac{1}{H(s)} = \frac{s+3}{s^2+s-2}$$

Minimum Phase System

All poles & zeros are on the left half of s-plane. eg: $H(s) = \frac{s+1}{(s+2)(s+3)}$

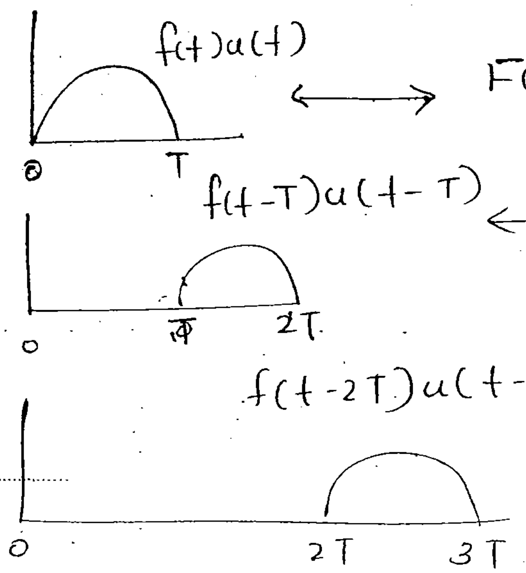
provide min group delay.

Non-minimum phase system.

One or more zeros may lie in the right half of s plane. But all the poles should lie in the left half of s-plane.

eg: $H(s) = \frac{s-2}{(s+2)(s+3)}$.

LT of switched periodic s/l



$$f(t)u(t) \longleftrightarrow F(s)$$

$$f(t-T)u(t-T) \longleftrightarrow e^{-sT}F(s)$$

$$f(t-2T)u(t-2T) \longleftrightarrow e^{-2sT}F(s)$$

$$\vdots$$

$$F(s)\left[1 + e^{-sT} + e^{-2sT} + \cdots\right]$$

$$X_p(s) = \frac{F(s)}{1-e^{-sT}} = \frac{\int_0^T f(t)\, e^{-st}\,dt}{1-e^{sT}}$$

Periodic ⟶ Discrete

Continous ⟶ Non periodic

	time Domain	Frequency Domain
C.F.S	Periodic & Continous	Descrete & Non-periodic
CTFT	Non-periodic & Continous	Continous & Non periodic.
DT.FT	Descrete & Non periodic	Periodic & Continous
DFT	Periodic & ~~dis continous~~ descrete	descrete & periodic

	i/p	o/p
CT FT	$e^{j\omega t}$	$e^{j\omega t} \underbrace{H(\omega)}_{\text{frequency Response}}$
LT	e^{st}	$e^{st} \underbrace{H(s)}_{\text{Transfer function}}$
DTFT	$e^{j\omega_0 n}$	$e^{j\omega_0 n} H(e^{j\omega_0})$
ZT	z^n	$z^n H(z)$

$$\underbrace{h(t)}_{\text{Impulse Response}} \xleftrightarrow{FT} \underbrace{H(\omega)}_{\text{Frequency Response}}$$

$$h(t) \xleftrightarrow{LT} \underbrace{H(s)}_{\text{Transfer function}}$$

$$h[n] \xleftrightarrow{DTFT} H(e^{j\omega})$$

$$h[n] \xleftrightarrow{ZT} H(z).$$

$A\cos(\omega_0 t + \phi)$

$$\boxed{\begin{array}{c} H(\omega) = |H(\omega)| \angle H(\omega) \\ H(e^{j\omega}) = |H(e^{j\omega})| \angle H(e^{j\omega}) \end{array}}$$

$y_{ss}(t) = A\,|H(\omega)|\cos(\omega_0 t + \phi + \angle$

$A\cos(\omega_0 n + \phi)$

$y_{ss}(n) = A\,|H(e^{j\omega})|$
$\cos(\omega_0 n + \phi + \angle H(e^{j$

$\underline{TF \longrightarrow FR}$

$s \longrightarrow j\omega_0$

$z \longrightarrow e^{j\omega}$

Analog frequency ω : $-\infty$ to

Digital frequency ω: $-\pi$ to
(0 to

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DTFT

CTFT	DTFT
$\omega : -\infty$ to $+\infty$	$\omega : -\pi$ to π
Non periodic Continous spectrum	Periodic, Continous spectrum

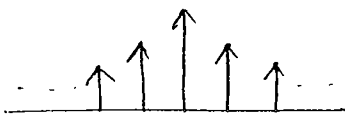
$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \Longrightarrow X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

∞
$X(\omega)$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega \Longrightarrow x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

IDFT.

obtained by the modification of sampling theorem



$$x_2(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$x(t)\,\delta(t - t_0) = x(t_0)\,\delta(t - t_0)$$

$$= \sum_{n=-\infty}^{\infty} x[nT_s]\,\delta(t - nT_s)$$

$$\text{FT of } x_2(t) = \int_{-\infty}^{\infty} x_2(t)\, e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x[nT_s]\,\delta(t - nT_s)\, e^{-j\omega t} dt$$

$$= \sum x[nT_s] \int_{-\infty}^{\infty} \delta(t - nT_s)\, e^{-j\omega t} dt$$

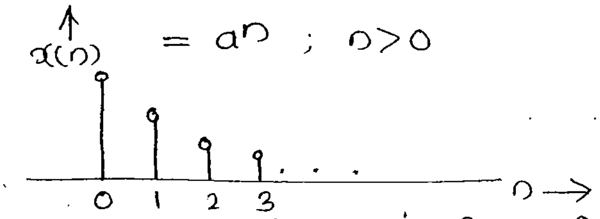
$$= \sum x[nT_s]\, e^{-j\omega n T_s}$$

Note: The spectrum of DTFT is periodically repeating for every 2π

FT of standard sequence

1. $x[n] = a^n u[n] \quad ; |a|<1$

$= a^n \; ; \; n>0$



$$X(\omega) = \sum_{n=0}^{\infty} a^n e^{-j\omega n} = \sum_{n=0}^{\infty} \left[a e^{-j\omega}\right]^n$$

P.6. 1.1

Gate 2m

$$= 1 + (a e^{-j\omega}) + (a + e^{-j\omega})^2$$

$$= \frac{1}{1 - a e^{-j\omega}}$$

$$\boxed{a^n u(n) \longleftrightarrow \frac{1}{1 - a e^{-j\omega}}}$$

$$= \frac{1}{1 - a\cos\omega + ja\sin\omega}$$

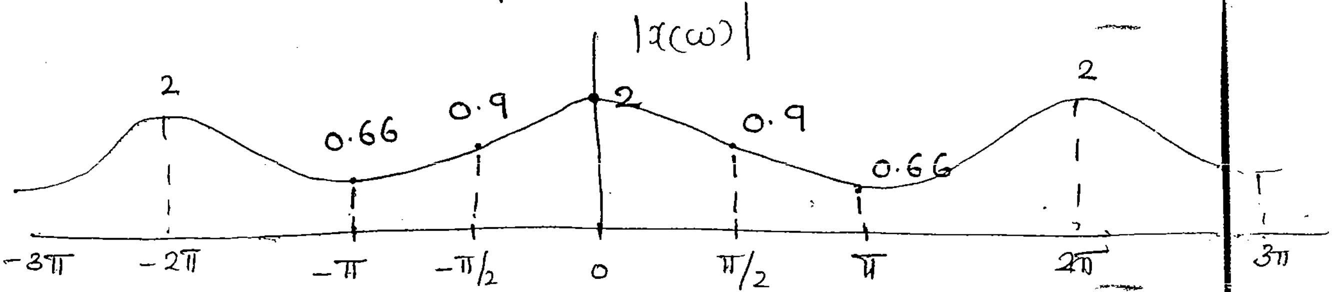
6.1

$$|X(\omega)| = \frac{1}{\sqrt{(1 - a\cos\omega)^2 + (a\sin\omega)^2}}$$

$$= \frac{1}{\sqrt{1 - 2a\cos\omega + a^2}}$$

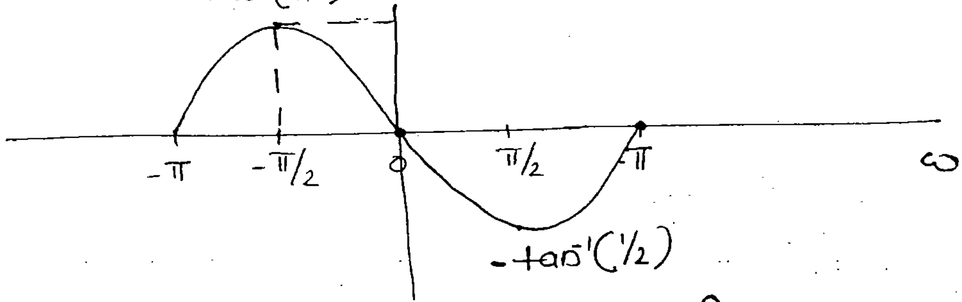
Let $a = \frac{1}{2} \quad (|a|<1)$

$$|X(\omega)| = \frac{1}{\sqrt{1.25 - \cos\omega}}$$



periodically repeating. Even function

$$\angle X(\omega) = -\tan^{-1}\left(\frac{\frac{1}{2}\sin\omega}{1-\frac{1}{2}\cos\omega}\right)$$



odd function.

Conjugate symmetry.

P. 6. 1.1
Gate 2 mark

$$X(\omega) = \sum_{n=-3}^{3} x(n)e^{-j\omega n}$$

$$= \frac{1}{2}\left[e^{-j\omega(3)} + e^{-j\omega(-3)}\right] + 1\left[e^{-j\omega(2)} + e^{-j\omega(-2)}\right]$$

$$+ \frac{3}{2}\left[e^{-j\omega(1)} + e^{-j\omega(-1)}\right] + 2$$

$$= \underline{\underline{\cos 3\omega + 2\cos 2\omega + 3\cos\omega + 2}}$$

6.1.2 (a)

$$x(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$y(n) = x^2(n) = \left(\frac{1}{4}\right)^n u(n) \longleftrightarrow \frac{1}{1-\frac{1}{4}e^{-j\omega}}$$

$$Y(e^{j0}) = \frac{1}{1-\frac{1}{4}} = \underline{\underline{\frac{4}{3}}}$$

$$\Downarrow$$

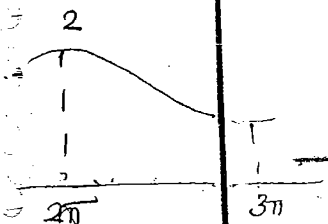
$$\sum_{n=-\infty}^{\infty} y(n) = \sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n =$$

(b)

$$X(e^{j\omega}) = \cos^3(3\omega) \longrightarrow \text{frequency } \pi$$

$$S = \sum_{n=-\infty}^{\infty} (-1)^n x(n)$$

$$X(\pi) = \cos^3(3\pi) = (-1)^3 = \underline{\underline{-1}}$$



..n function

dreally

Time - shift

$$x(n-n_0) \longleftrightarrow (e^{-j\omega n_0})\, X(e^{j\omega})$$

6.1.3 (i) $g_1(n) = \left(\frac{1}{3}\right)^n u(n+2)$

$$= \left(\frac{1}{3}\right)^{n+2-2} u(n+2)$$

$$= \left(\frac{1}{3}\right)^{-2} \cdot \left[\left(\frac{1}{3}\right)^{(n+2)} u(n+2)\right]$$

$$= 9\, e^{-j\omega(-2)} \frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

$$y_1(n) = \left(\frac{1}{4}\right)^n u(n-3)$$

$$= \left(\frac{1}{4}\right)^{n-3+3} u(n-3)$$

$$\Longleftrightarrow \frac{1}{4^3}\, e^{-j\omega(3)} \frac{1}{1-\frac{1}{4}e^{-j\omega}}$$

* $g(n) = \delta(6-3n) = \delta(-3(n-2))$

$$= \cancel{\frac{1}{3}\delta\left(n-\frac{2}{3}\right)}$$

$$= \cancel{\frac{1}{3}\delta(n-2)}$$

$$= \cancel{\frac{1}{3}}$$

$\delta(Kn) = \delta(n)$
no scaling factor

$$= \delta(n-2)$$

$$= e^{-j\omega(2)}(1) = e^{-j2\omega}$$

$$\delta(n) = \begin{cases} 1; & n = 0 \end{cases} \Longrightarrow \delta(6-3n) = \begin{cases} 1; & 6-3n = 0 \\ & 6 = 3n \\ & n = 2 \end{cases}$$

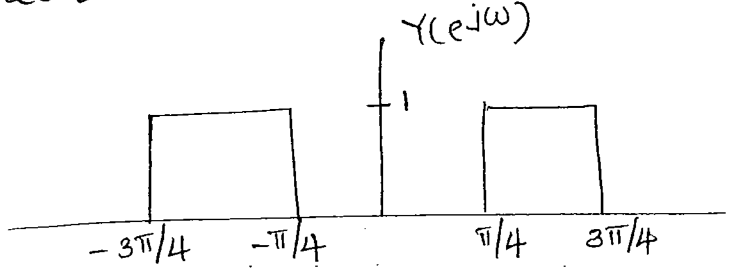
* $$Y(\omega) = \frac{e^{-j\omega} - \frac{1}{5}}{1-\frac{1}{5}e^{-j\omega}} = \frac{e^{-j\omega}}{1-\frac{1}{5}e^{-j\omega}} - \frac{\frac{1}{5}}{1-\frac{1}{5}e^{-j\omega}}$$

$$\xleftrightarrow{\text{IDFT}} g(n) = \left(\frac{1}{5}\right)^{(n-1)} u(n-1) - \left(\frac{1}{5}\right)^{n+1} u(n)$$

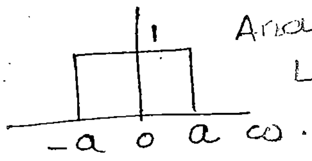
Frequency - shift / Modulation

$$x(n)\, e^{j\omega_0 n} \longleftrightarrow X\left[e^{j(\omega-\omega_0)}\right]$$

6.1.4

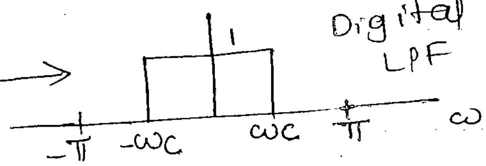


$$\frac{\sin at}{\pi t} \longleftrightarrow$$



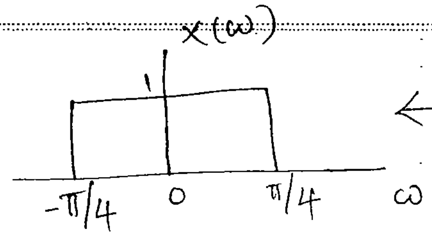
Analog LPF

$$\frac{\sin \omega_c n}{\pi n} \longleftrightarrow$$



Digital LPF

Assume



$$\longleftrightarrow \frac{\sin(\pi n/4)}{\pi n}.$$

$$Y(\omega) = X(\omega - \pi/2) + X(\omega + \pi/2) \xleftrightarrow{\text{IFT}} y(n) = x(n)\, e^{j\omega\pi/2} + x(n)\, e^{-j\omega\pi/2}$$

$$\Rightarrow y(n) = x(n)\left[e^{jn\pi/2} + e^{-jn\pi/2}\right]$$

$$= 2x(n)\cos\left(\frac{n\pi}{2}\right)$$

LPF — BPF → only by frequency shifting.

Time scaling

$$x[n/K] \longleftrightarrow X(e^{j\omega K})$$

K should be intiger multiple of n

(n)
Factor

$6-3n=0$
$6=3n$
$n=2$

$\frac{1}{5}$
$-\frac{1}{5}e^{-j\omega}$
(n)

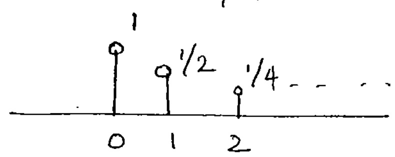
*Q. $Y(\omega) = \dfrac{1}{1 - \frac{1}{2} e^{-j10\omega}}$ IDFT?

$= X(e^{j10\omega})$

where $X(\omega) = X(e^{j\omega}) = \dfrac{1}{1 - \frac{1}{2} e^{-j\omega}}$

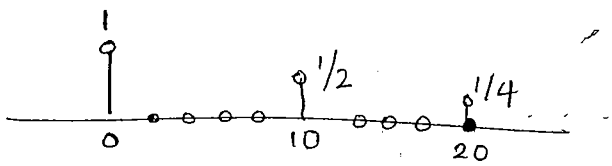
$\updownarrow$ IDFT

$x(n) = (1/2)^n u(n)$.



$y(n) = x(n/10)$

$= (1/2)^{n/10}$; $n = 0, 10, 20, 30 \ldots$



padding zeros interpolation.

*Q. F.T $x[2n+3]$

$= x[2(n + 3/2)]$

$\updownarrow$ DFT

$e^{j\omega(3/2)} X(e^{j\omega/2})$ X non integer shift

$\therefore$ not possible.

1^{st} shifting + II^{nd} scaling.

$x[n+3] \longleftrightarrow e^{-j\omega(-3)} X(\omega)$

$x[2n+3] \longleftrightarrow e^{+j\frac{\omega}{2}(3)} X(\omega/2)$

Frequency - Differentiation

$$-jn\,x(n) \longleftrightarrow \frac{d}{d\omega}\left(X(e^{j\omega})\right)$$

*Q. DTFT $y(n) = n a^n u(n)$

$= n u(n)$

$\downarrow$ FT

$$Y(\omega) = j\frac{d}{d\omega}\left[\frac{1}{1-ae^{-j\omega}}\right]$$

$$= \frac{ae^{-j\omega}}{(1-ae^{-j\omega})^2}$$

*Q. $\sum_{n=0}^{\infty} n(1/3)^n = X(e^{j(0)})$

$$= \frac{1/3\, e^{-j(0)}}{\left[1-1/3\, e^{-j(0)}\right]^2} = \frac{1/3}{(1-1/3)^2}$$

$$= \frac{3}{8}$$

lation.

6.1.10 FT of $x(n) = n e^{jn\pi/8} \alpha^{n-3} u(n-3)$

$$x_1(n) = \alpha^n u(n) = \frac{1}{1-\alpha e^{-j\omega}}$$

g shift

TS $x_1(n-3) \longleftrightarrow \dfrac{e^{-j3\omega}}{1-\alpha e^{-j\omega}}$

FS $e^{jn\pi/8} x_1(n-3) \longleftrightarrow \dfrac{e^{-j3(\omega-\pi/8)}}{1-\alpha e^{-j(\omega-\pi/8)}}$

FD $n e^{jn\pi/8} \alpha^{n-3} u(n-3) \longleftrightarrow j\dfrac{d}{d\omega}\left[\dfrac{e^{-j3(\omega-\pi/8)}}{1-\alpha e^{-j(\omega-\pi/8)}}\right]$

Convolution in time

$$x(n) * h(n) \longleftrightarrow X(e^{j\omega})\, H(e^{j\omega})$$

$$\underbrace{a^n u(n) * a^n u(n)}_{(n+1)a^n u(n)} \xleftrightarrow{F.T} \frac{1}{(1-ae^{-j\omega})^2}$$

$u(n) * u(n) = (n+1)u(n)$

(sample increments)

In Continous

$$\underbrace{e^{-at}u(t) * e^{-at}u(t)}_{t\,e^{-at}u(t)} \xleftrightarrow{FT} \frac{1}{(a+j\omega)^2}$$

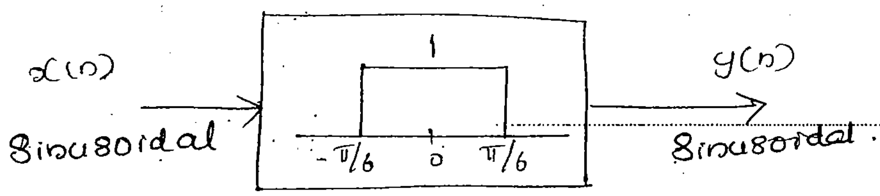
$u(t) * u(t) = \cancel{t\,u(t)}$

(powers of t)

6.1.11

$$x(n) = \sin\left(\frac{\pi n}{8}\right) - 2\cos\left(\frac{\pi n}{4}\right)$$

$$h(n) = \frac{\sin(n\pi/6)}{\pi n} \qquad y(n) = ?$$



$\omega_c = \pi/6$.

$$\therefore \quad y(n) = \sin\left(\frac{\pi n}{8}\right)$$

Imp
6.1.12
Gate 2004
2marks

$$h(n) = \begin{cases} 4\sqrt{2} & ; n = 2, -2 \\ -2\sqrt{2} & ; n = 1, -1 \\ 0 & ; \text{elsewhere.} \end{cases}$$

$$x(n) = e^{jn\pi/4} \; ?$$

$$x(n) = e^{j\omega_0 n} \Rightarrow y(n) = e^{j\omega_0 n} \underbrace{H(e^{j\omega_0})}_{\text{frequency response.}}$$

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n)e^{-j\omega n}$$

$$= 4\sqrt{2}\left[e^{-j\omega(2)} + e^{j\omega(2)}\right]$$

$$- 2\sqrt{2}\left[e^{-j\omega} + e^{j\omega}\right]$$

$$= 8\sqrt{2}\cos 2\omega + - 4\sqrt{2}\cos\omega$$

$$H(e^{j\pi/4}) = 8\sqrt{2}\cos 2\cdot\frac{\pi}{4} + - 4\sqrt{2}\cos\frac{\pi}{4}$$

$$= -4$$

$$y(n) = -4\cdot e^{j\left(\frac{\pi}{4}\right)n}$$

6.1.13 $h(n) = \{\alpha, \underset{\uparrow}{\beta}, \alpha\}$

$f = 1/3$ blocks $\Rightarrow$ magnitude $= 0$.

$f = 1/8 \Rightarrow$ gain unity

DC gain of filter?

Solution: $H(e^{j\omega}) = \sum_{n=-1}^{1} h(n)e^{-j\omega n}$

$$= \alpha\left[e^{-j\omega} + e^{j\omega}\right] + \beta$$

$$= 2\alpha\cos\omega + \beta$$

$$|H(e^{j\omega})| = \sqrt{(2\alpha\cos\omega+\beta)^2}$$

$$= 2\alpha\cos\omega + \beta$$

$f = \frac{1}{3} \Rightarrow \omega = 2\pi f = \frac{2\pi}{3} \Rightarrow |H(e^{j\omega})| = 0$.

$f = \frac{1}{8} \Rightarrow \omega = \frac{2\pi}{8} = 2\alpha\cos\left(\frac{2\pi}{3}\right) + \beta = 0$ ——— (1)

$2\alpha\cos\left(\frac{2\pi}{8}\right) + \beta = 1$ ——— (2)

(1) $\Rightarrow$ $-\alpha + \beta = 0 \Rightarrow \alpha = \beta$

(2) $\Rightarrow$ $\sqrt{2}\alpha + \beta = 1 \Rightarrow (\sqrt{2}+1)\alpha = 1$

$$\alpha = \beta = \frac{1}{1+\sqrt{2}}$$

DC gain $He^{j(0)} = 2\alpha\cos(0) + \beta$

$$= 2\alpha + \beta$$

$$= \frac{3}{1+\sqrt{2}}$$

High frequency gain

$$H(e^{j(+\pi)}) = 2\alpha\cos\pi+\beta$$
$$= -2\alpha+\alpha \quad (\text{since } \alpha=\beta)$$
$$= -\alpha$$
$$= \frac{-1}{1+\sqrt{2}}$$

6.1.14

① $y(n) = x(n) - x(n-1)$ filter 2.

$$\frac{d\,x(t)}{dt} \longrightarrow x(n)-x(n-1) \longrightarrow \text{HPF}$$

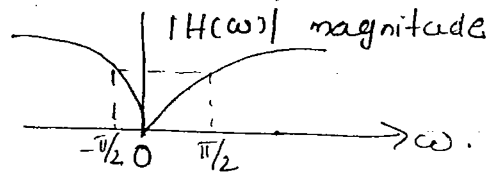
1st difference

FT ⟹

$$Y(\omega) = X(\omega) - e^{-j\omega}X(\omega)$$
$$= X(\omega)\left[1-e^{-j\omega}\right]$$
$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = (1-e^{-j\omega})$$

$H(0) = 0$; $H(\pi/2) = 1+j = \sqrt{2}\angle 45^\circ$

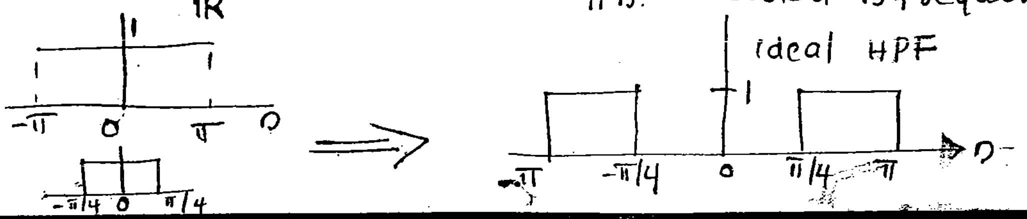
$H(\pi) = 2$; $2\angle 0^\circ$.



② $y(n) = x(n) + x(n-1) \longrightarrow$ LPF

* Addition of simultaneous samples LPF
subtraction of simultaneous samples HPF

③ $\underbrace{h[n]}_{IR} = \delta[n] - \frac{\sin(n\pi/4)}{\pi n}$ Impz in time domain constant vs frequency.



$\alpha = \beta)$

*Q. $\underbrace{h[n]}_{IR} = \delta[n] - \delta[n-8]$

$\Downarrow$

Comb filter

$\downarrow$ FT

$$H(\omega) = 1 - e^{-j8\omega}$$

$$\left.\begin{aligned} H(0) &= 1 - 1 = 0 \\ H(\pi) &= 1 - 1 = 0 \end{aligned}\right\} \text{Band-pass filter}$$

*Q. $h[n] = \delta(n) + \delta(n-2)$

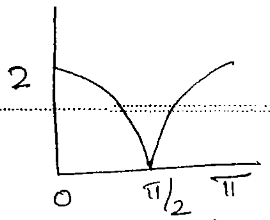
$\downarrow$ FT

$$H(\omega) = 1 + e^{-j2\omega}$$

$$H(0) = 1 + 1 = 2$$

$$H(\pi/2) = 1 + e^{-j\pi} = 0$$

$$H(\pi) = 1 + e^{-j2\pi} = 2$$



Band stop.

$\angle 45^\circ$

6.1.15 $y(n) = a\,y(n-1) + b\,x(n) + x(n-1)$

a & b are real $|H(e^{j\omega})| = 1 \;\forall \omega$.

$$Y(\omega) = a e^{-j\omega} Y + bX + e^{-j\omega} X$$

$$Y[1 - a e^{-j\omega}] = [b + e^{-j\omega}] X$$

$$H = \frac{Y}{X} = \frac{b + e^{-j\omega}}{1 - a e^{-j\omega}}$$

$$|H(e^{j\omega})| = 1 \Longrightarrow |H(e^{j\omega})|^2 = 1$$

$$H \cdot H^* = 1$$

$$\left[\frac{b + e^{-j\omega}}{1 - a e^{-j\omega}}\right]\left[\frac{b + e^{j\omega}}{1 - a e^{j\omega}}\right] = 1$$

3 LPF

28 HPF

ime domain

requency.

$\to n$

$$\frac{b^2+2b\cos\omega+1}{1-2a\cos\omega+a^2}=1$$

At $\omega = 0$

$H = 1$

$$\frac{b+1}{1-a} = 1 \Rightarrow \underline{\underline{b=-a}},$$

DRDO

6.1.16

$x(n) \longrightarrow 3$

$h(n) \longrightarrow 5$

$|x(n)| \leq B$ & $|h(n)| \leq L$

Max value of $Y(0)$?

$y(n) = x(n) * h(n)$

$\downarrow$ FT

$Y(\omega) = X(\omega)\, H(\omega)$

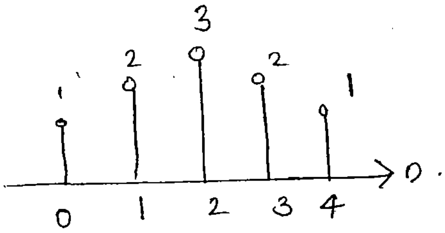
$|Y(0)| = X(0)\, H(0)$

$= \sum_{n=0}^{2} |x(n)| \sum_{n=0}^{4} |h(n)|$

$= \sum_{n=0}^{2} B \sum_{n=0}^{4} L$

$= \underline{\underline{15LB}}$

6.1.17



$$t_g(\omega) = -\frac{d\theta(\omega)}{d\omega}$$

$$H(\omega) = \sum_{n=0}^{4} h(n)\, e^{-j\omega n}.$$

FIR filter $\Rightarrow \theta(\omega) = -\alpha\omega$ (Linear phase condition)

$= -\left(\frac{N-1}{2}\right)\omega$

$= -\left(\frac{5-1}{2}\right)\omega$

$= \underline{\underline{-2\omega}}$

6.1.10

6.1.1

6.1.20

6.1.2

Linear phase

$$h[n] = h[N-1-n]$$

$$h[n] = h[4-n]$$

$N \rightarrow$ length of I.R

6.1.18 HPF

6.1.19 $h(n) = (1/3)^n u(n)$

$$x(n) = \underbrace{2}_{\omega=0} + \cos\underbrace{(\pi n + \pi/3)}_{\omega=\pi}$$

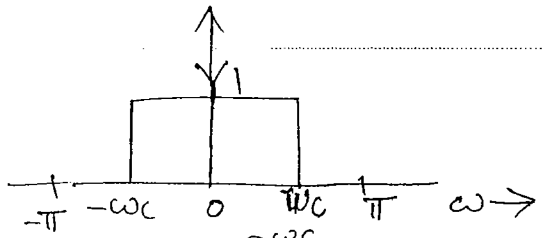
$$H(\omega) = \frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

$$Y(\omega) = 2(3/2) + \frac{3}{4}\cos\left[\pi n + \pi/3\right]$$

Parseval's relation

$$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi}\int_{\langle 2\pi \rangle} |X(e^{j\omega})|^2 d\omega.$$

6.1.20 $x(n) = \frac{\sin \omega_c n}{\pi n}$?



$$E_{x(n)} = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} (1)^2 d\omega. = \frac{\omega_c}{\pi}$$

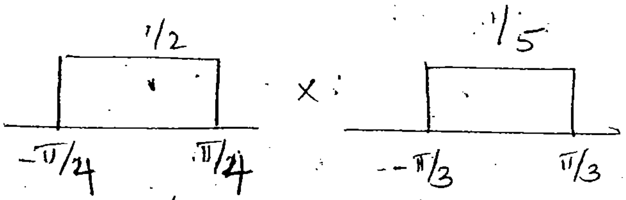
6.1.21 $$\sum_{n=-\infty}^{\infty} \frac{\sin\left(\frac{n\pi}{4}\right)}{2\pi n} \cdot \frac{\sin\left(\frac{n\pi}{3}\right)}{5\pi n}.$$

$$\sum_{n=-\infty}^{\infty} x(n) y^*(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, Y^*(\omega)\, d\omega$$

modified parseval's theorem.

$\frac{\theta(\omega)}{d\omega}$

...ear
...ase
...ndition)



$$= \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} \frac{1}{10}\, d\omega$$

6.1.22 (a) $X(e^{j0}) = \sum_{n=3}^{7} x[n] = 6$ 6.1.27

(b) $X(e^{j\pi}) = \sum x(n) e^{-jn\pi}$

$= \sum_{n=-3}^{7} x[n](-1)^n$

6.1.26

(c) $\int_{-\pi}^{\pi} X(e^{j\omega})\, d\omega$

$x[n] = \frac{1}{2\pi}\int X(e^{j\omega}) e^{j\omega n} d\omega$

when $n=0$

$\int X(e^{j\omega}) = 2\pi x[0]$

$= 2\pi x[0] = 2\pi(2)$

$= \underline{\underline{4\pi}}$

(d) $\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j2\omega}\, d\omega$ 6.1.23

$= 2\pi x[2]$

$= \underline{\underline{0}}$

(e) $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$

$= 2\pi \sum_{n=-3}^{7} |x(n)|^2$

$= \underline{\underline{28\pi}}$

6.1.24

(f) $\int_{-\pi}^{\pi} \left|\frac{d}{d\omega} X(e^{j\omega})\right|^2 d\omega$

$= 2\pi \sum_{n=-3}^{7} |-jn x(n)|^2$

$$= 2\pi \sum_{n=-3}^{7} |n x(n)|^2$$

(g) $\angle X(e^{j\omega})$

* Since the signal is symmetric w.r.t $n=2$, it may be delayed version of original even signal. ∴ $\angle X(e^{j\omega})$ is $-2\omega \pm 0^\circ$

$\frac{1}{2\pi}\int X(e^{j\omega}) e^{j\omega n} d\omega$

$= 2\pi x[0]$

6.1.27 $2\pi x[0]$
$= 10\pi$

6.1.26 $g(n) = A x[n-n_0]$

$\downarrow FT$

$Y(\omega) = A e^{-j\omega n_0} X(\omega)$

$H(\omega) = \frac{Y(\omega)}{X(\omega)} = A e^{-j\omega n_0}$

$\angle H(\omega) = -\omega_0 n_0 + 2\pi K$

6.1.23 $h[n] = [1, 2, 2]$

$f[n] = h[n] * h[n]$

$g[n] = h[n] * h[-n]$

Limits of $f[n] \longrightarrow 0 \leq n \leq 4 \Rightarrow$ causal.

Limits of $g[n] \longrightarrow -2 \leq n \leq 2 \Rightarrow$ non-causal.

$g(0) = E_{h(n)} = 1^2 + 2^2 + 2^2 = 9$.

6.1.24 $5\,KHz \leq f \leq 10\,KHz$.

$f_s = 40\,KHz$.

$\omega = \frac{2\pi f}{f_s}$

$-\pi \leq \omega \leq \pi \longrightarrow$ Digital frequency

$\frac{2\pi(5K)}{40K} \leq \omega \leq \frac{2\pi(10K)}{40K}$

$\pi/4 \leq \omega \leq \pi/2$

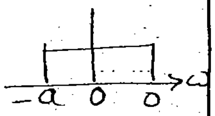
Comparison of Formulas

$e^{-at}u(t) \longleftrightarrow \frac{1}{a+j\omega}$

$\delta(t) \longleftrightarrow 1$

$1 \longleftrightarrow 2\pi\delta(\omega)$

$\frac{\sin at}{\pi t} \longleftrightarrow$



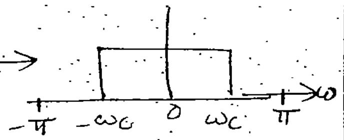
$a^n u(n) \longleftrightarrow \frac{1}{1-ae^{-j\omega}}$

$|a|<1$

$\delta[n] \longleftrightarrow 1$

$1 \longleftrightarrow 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega+2\pi k)$

$\frac{\sin \omega_c n}{\pi n} \longleftrightarrow$



Z - Transform

→ Inventors - Ragg$\boxed{z}$ini
$\boxed{Z}$adeh

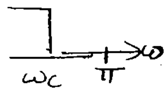
$\frac{1}{ae^{-j\omega}}$

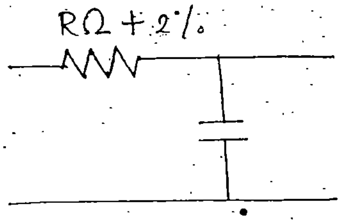
z - transform.

→ generalization of DTFT
→ descrete counterpart of LT.

$+2\pi k)$

• main disadvantage of LT is tolerance
→ In electrical n/w





$H(s) = \frac{1}{sRC+1}$

Since R varies due to tolerance H(s) also varies.

→ no such problems in z - transform by introducing a delay unit $\boxed{z^{-1}}$

→ delay unit a shift register - combination of flipflop.

In Laplace
$$x(t) = e^{st} \Rightarrow y(t) = e^{st}H(s)$$

In z transform
$$x[n] = z^n (z = re^{j\omega})$$

$$\Rightarrow y[n] = z^n H(z)$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[n-k]\, h[k]$$

$$= \sum_{k=-\infty}^{\infty} z^{n-k}\, h[k]$$

$$= z^n \sum_{k=-\infty}^{\infty} h[k] z^{-k} = z^n H(z)$$

$$z[x[n]] = X(z) = \sum_{n=-\infty}^{+\infty} x(n) z^{-n}$$

$$X(re^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) r^{-n} e^{-j\omega n}.$$

$$X(z) = F\left\{ r^{-n} x(n) \right\}$$

$X(z) = F\left\{x(n) r^{-n}\right\}$

$r = 1$

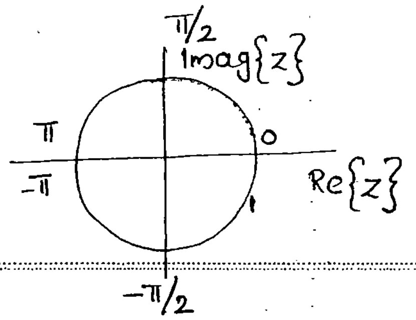
$\Downarrow$

$ZT = DTFT$

$X(s) = F\left\{x(t) e^{-st}\right\}$

$\sigma = 0 \Rightarrow s = j\omega$

$LT = FT$



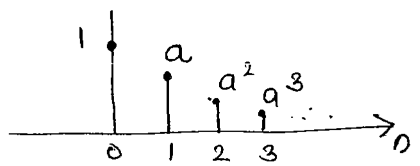
$-\pi \leq \omega \leq \pi$

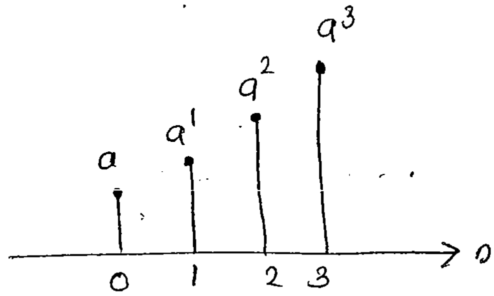
Laplace on imaginary axis is fourier.
z transform on real axis is DTFT

Applications of z transform.
→ Digital filter design.
→ sampled data control system.
(switched capacitor filter)

* $x[n] = a^n u[n]$
$= a^n \; ; \; n \geq 0$

$|a| < 1$





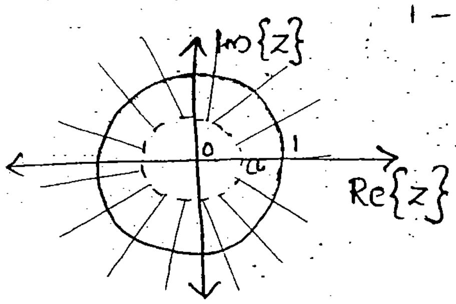
$$X(z) = \sum_{n=0}^{\infty} a^n z^{-n} = \sum_{n=0}^{\infty} (az^{-1})^n$$

$$= 1 + (az^{-1}) + (az^{-1})^2 + \cdots$$

$$= \frac{1}{1-az^{-1}} \; ; \; |az^{-1}| < 1 \Rightarrow |a/z| < 1$$

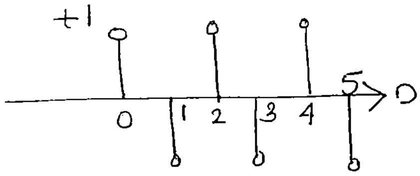
$$\Rightarrow \underbrace{|z| > |a|}_{R.O.C}$$

$$a^n u(n) \longleftrightarrow \frac{1}{1-az^{-1}} \text{ or } \frac{z}{z-a} \; ; |z| > a$$



When ever we are taking right sided sequence R.O.C will be at ~~outside~~ the circle and vice versa

* For the impulse response shown in figure where the pole is located in z-plane.

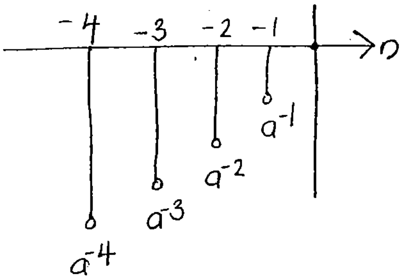


$$(-1)^n u(n) \longleftrightarrow \frac{z}{z+1}$$

pole at $z = -1$

* $x[n] = -a^n u[-n-1]$ left sided sequence

$$= -a^n \; ; \; n \leq -1$$



$$X(z) = \sum_{n=-\infty}^{-1} -a^n z^{-n}$$

put $-n = m$.

$$= -\sum_{m=\infty}^{1} a^{-m} z^{m}$$

$$= -\sum_{m=1}^{\infty} (a^{-1}z)^m$$

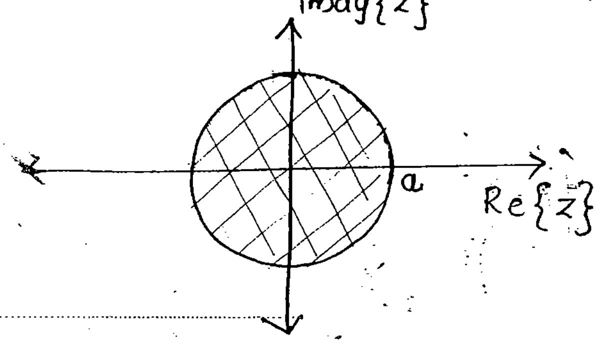
$$= -\left[a^{-1}z + (a^{-1}z)^2 + \cdots\right]$$

$$= \frac{-a^{-1}z}{1-a^{-1}z} \quad ; |a^{-1}z| < 1$$

$$|z| < |a|$$

30/08/201
9-1 pm

$$-a^n u(-n-1) \longleftrightarrow \frac{z}{z-a} \; ; |z| < |a|$$



Pg 96.

$*$ $x[n] = \delta[n]$

$; X(z) = 1$

$\Rightarrow \delta(n-1) \longleftrightarrow z^{-1} \cdot 1 = \underline{\underline{z^{-1}}}$ (pg 97)

R.O.C : z plane except at $z = 0$
or $|z| > 0$.

$\Rightarrow \delta(n+1) \longleftrightarrow z$

ROC: entire z plane except at $z = \infty$
or $|z| < \infty$

gate 2009

$x[n] = (1/3)^n u(n) + (1/2)^n u(-n-1)$

$|z| > 1/3 \qquad |z| < 1/2$

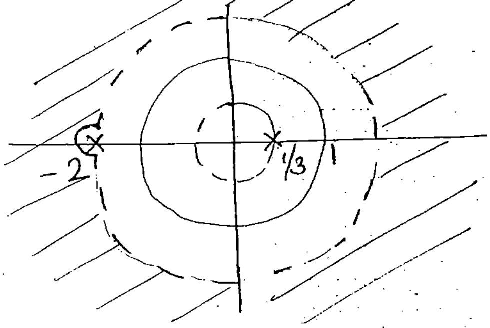
$1/3 < |z| < 1/2$ Common R.O.C

30/08/2011
9-1.pm

(Q) $$X(z) = \frac{1}{(1-\frac{1}{3}z^{-1})(1+2z^{-1})} = \frac{1/7}{1-\frac{1}{3}z^{-1}} + \frac{6/7}{1+2z^{-1}}$$

poles at $z = \frac{1}{3}, -2$

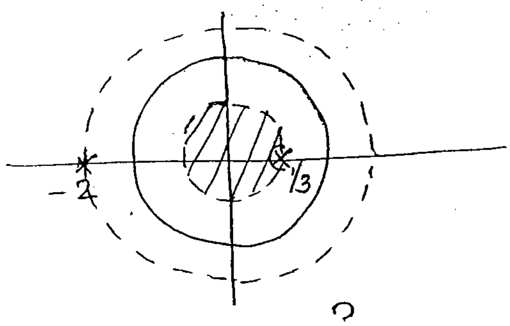
$x[n]$ is right sided.



Causal.
∴ R.O.C outside the circle with pole having largest magnitude

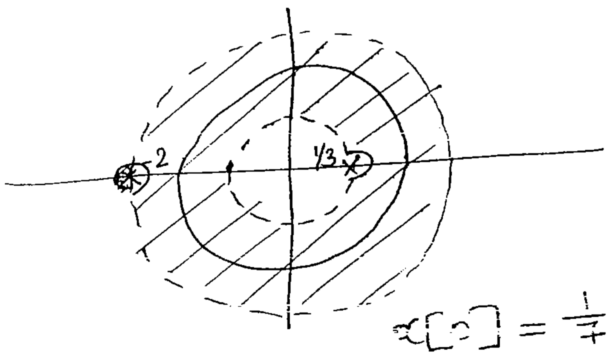
$$x(n) = \frac{1}{7}\left(\frac{1}{3}\right)^n u(n) + \frac{6}{7}(-2)^n u(n).$$

$x[n]$ is left-sided



$$x(n) = \frac{1}{7}\left(\frac{1}{3}\right)^n[-u(-n)] + \frac{6}{7}(-2)^n[-u(-n)]$$

$x[n]$ is 2-sided.



stable.
R.O.C include unit circle.

$$x[n] = \frac{1}{7}\left(\frac{1}{3}\right)^n u(n) + \frac{6}{7}(-2)^n[-(u(-n))]$$

P7.1.3 $\quad x(n) = (-1)^n u(n) + \alpha^n u(-n-n_0)$

ROC : $-1 < |z| < 2$

$|z| > |-1|$

> 1

$|z| < |\infty|$

$|\alpha| < 2$

$\alpha = \pm 2$

$-n_0$ is any where

$\alpha^n u[-n-1]$
$\alpha^n u[-n]$
$\alpha^n u[-n+2]$

7.2.1 $\quad g(n) = \alpha^{|n|}, \quad |\alpha| < 1 \quad$ ZT?

$$|n| = \begin{cases} n ; & n > 0 \\ -n ; & n < 0 \end{cases}$$

$$= \begin{cases} \alpha^n ; & n > 0 \\ \alpha^{-n} ; & n < 0 \end{cases}$$

$$g(n) = \alpha^n u(n) + \alpha^{-n} u(-n-1)$$

$$= \alpha^n u(n) + \frac{1}{\alpha^n} u(-n-1)$$

$|z| > |\alpha| \qquad |z| < \left|\frac{1}{\alpha}\right|$

Assume $\alpha = 1/2$

$|z| > \left|\frac{1}{2}\right| \qquad |z| < |2|$

$\therefore$ common R.O.C possible

$$|\alpha| < |z| < \left|\frac{1}{\alpha}\right|$$

if $\alpha > 1 \Rightarrow$ no common R.O.C $\Rightarrow$ No ZT

P.2.2

1 pole at $z = e^{j\pi/2}$

2 zeros at $z = 0$

$X(1) = 1$

(94)

$$x(z) = \frac{z^2}{(z-e^{j\pi/2})(z-e^{-j\pi/2})}$$

$$= \frac{z^2}{z^2 - z\left(e^{j\pi/2}+e^{-j\pi/2}\right)} = \frac{z^2}{z^2+1};$$

$$x(1) = 1 \implies x(z) = \frac{2z^2}{z^2+1}; \quad |z| > |\pm j|$$

$|z| > 1$

Right sided sequence.

Time shift

$$x(n-n_0) \longleftrightarrow z^{-n_0} x(z) \ ; \ ROC = R$$

P7.2.3 $\quad g(n) = \left(-\frac{1}{3}\right)^n u(-n-2)$

$x[n] = a^n u(-n-1)$; left sided sequence

$x[n+1] = a^{n+1} u[-(n+2)]$

ie $\quad g(n) = \left(\frac{-1}{3}\right)^{n+1-1} u[-(n+2)]$

$$= \left(-\frac{1}{3}\right)^{-1}\left(-\frac{1}{3}\right)^{n+1} \underbrace{u[-(n+2)]}_{n_0 = -1}$$

$$\xleftrightarrow{ZT} -3\left[\frac{-z^{-(-1)}}{1+\frac{1}{3}z^{-1}}\right] \ ; \ |z| < \frac{1}{3}$$

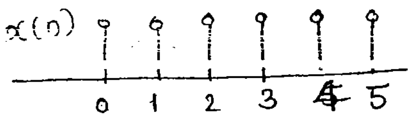
$$\frac{3z}{1+\frac{1}{3}z^{-1}} \ ; \ |z| < \frac{1}{3}$$

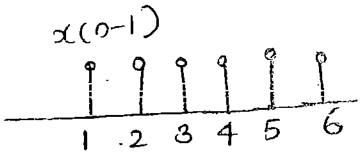
7.2.4

$$x(n) = \begin{cases} 1 \ ; & 0 \le n \le 5 \\ 0 \ ; & \text{else} \end{cases}$$

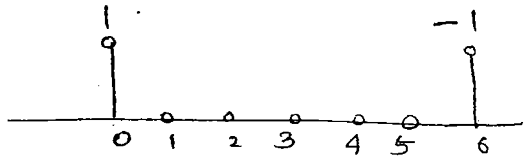
$g(n) = x(n) - x(n-1)$ $\quad$ find $G(z)$ with R.O.C?

Solution:





$y(n) = x(n) - x(n-1)$



$$G(z) = \cancel{1-z^{-6}}\ 1\cdot z^{0} - 1\cdot z^{-6}$$

$$= \underline{1 - z^{-6}} \ ; \ |z| > 0$$

7.2.5

$$X(z) = \frac{z^3 - 2z}{z-2} \ ; \ x[n] \text{ is left sided}$$

$$= \frac{z^3}{z-2} - \frac{2z}{z-2}$$

$$= z^2\left[\frac{z}{z-2}\right] - 2\left[\frac{z}{z-2}\right]$$

$$= z^2 Y(z) - 2Y(z)$$

$$Y(z) = \frac{z}{z-2} \ \xrightarrow{IZT} \ y(n) = 2^n[-u(-n-1)]$$

$\downarrow$ IZT

$$x(n) = y(n+2) - 2y(n)$$

$$x(n) = 2^{n+2}[-u[-n-3]] - 2 \cdot 2^n[-u(-n-1)]$$

Exponential Multiplication / scaling in z domain

$$a^n x[n] \longleftrightarrow X(z/a)$$

$$ROC = |a| R$$

$$y(n) = \cos\omega_0 n \ u[n]$$

$$= \left[\frac{e^{j\omega_0 n} + e^{-j\omega_0 n}}{2}\right] u[n]$$

Let $x[n] = u(n)$

$X(z) = \frac{1}{1-z^{-1}}$; $|z|>1$

$\downarrow$ ZT

$$Y(z) = \frac{X(z/e^{j\omega_0}) + X(z/e^{-j\omega_0})}{2}$$

DSP u ? MAT ?

Practical to DS

Applicat

7.2.1

DRDO

DSP using MATLAB } SK Mitra

Practical Approach to DSK } Ifeches

$$= \frac{1}{2}\left[\frac{1}{1-z^{-1}e^{j\omega_0}} + \frac{1}{1-z^{-1}e^{-j\omega_0}}\right]$$

$$\boxed{= \frac{1-z^{-1}\cos\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}}$$

$$|z| > (1)\left|e^{\pm j\omega_0}\right|$$

Digital Resonator

Application
- 5-band Graphic Equializer
- DTMF
- Gaertzel algorithm $\Longrightarrow$ DFT   echo cancellation

$$\boxed{\cos\omega_0 n\, u(n) \longleftrightarrow \frac{1-z^{-1}\cos\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}}$$

7.2.7

$$x(z) = \frac{(z+j)(z-j)}{z-1/2} = \frac{z^2+1}{z-1/2}$$

$$y(n) = \left(\tfrac{1}{2}\right)^n x(n).$$

$\downarrow$ ZT

$$Y(z) = x\left(z/_{1/2}\right) = x(2z)$$

$$= \frac{(2z)^2+1}{(2z)-1/2}$$

Scaling in time-Domain

DRDO

$$x\left[\frac{n}{K}\right] \longleftrightarrow X(z^K)$$

K is integer multiple of 'n'.

$$z = e^{j\omega}$$

Time Reversal

$$x[-n] \longleftrightarrow x(z^{-1}) \;;\; ROC = 1/R$$

Reflection   inversion.

sided

$\frac{2}{z-2}$

IZT

$2^n[-u(-n-1)]$

$(-n-1)]$

main

$2+$

$n]=u(n)$

$(z) = \frac{1}{1-z^{-1}}$

$|z|>1$

(Q) $y(n) = 4^n u(n)$

$= \left(\frac{1}{4}\right)^{-n} u[-n]$

$\downarrow ZT \quad = x[-n]$

$Y(z) = x(z^{-1})$

$= \frac{1}{1-\frac{1}{4}z} ; |z| < 4$

Assume

$x[n] = \left(\frac{1}{4}\right)^n u(n)$

$\downarrow ZT$

$x(z) = \frac{1}{1-\frac{1}{4}z^{-1}}$

$; |z| > \frac{1}{4}$.

Differentiation in z domain.

$n\, x(n) \longleftrightarrow -z \frac{d\, x(z)}{dz}; \quad ROC = R$

7.2.9

$x(z) = \log(1+az^{-1}); \; |z|>|a|$

$\frac{d\, x(z)}{dz} = \frac{1}{1+az^{-1}} \times (-az^{-2})$

$-z \frac{d x(z)}{dz} = \frac{az^{-1}}{1+az^{-1}}$

$\downarrow$

IZT

$n\, x(n) = a(-a)^{n-1} u(n-1)$.

$-jn \longleftrightarrow \frac{d}{d\omega}$

$z = e^{j\omega}$

$dz = e^{j\omega}(j)\, d\omega$

$= jz\, d\omega$

$d\omega = \frac{dz}{jz}$

$y(n-n_0) \longleftrightarrow z^{-n_0} Y(z)$

ZT multiplied by $z^{-1} \rightarrow$ shifting.

7.2.11

$\delta(n-n_0) \longleftrightarrow z^{-n_0}$

$x(n) \longleftrightarrow x(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$

$h(n) = 2\,\delta(n-3)$

$H(z) = 2 \cdot z^{-3}$

$Y(z) = X(z)\, H(z) = 2z + 2z^{-1} - 2z^{-2} + 4z^{-3} - 6z^{-7}$

o/p at. $n=4 \implies 0$.

7.2.1

7.2.12

$$y(n) = x_1(n+3) * x_2(-n+1)$$

$$x_1(n) = \left(\tfrac{1}{2}\right)^n u(n)$$

$\downarrow ZT$

$$X_1(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$$

$$x_1(n+3) \longleftrightarrow z^3 X_1(z)$$

$$\longleftrightarrow \frac{z^3}{1-\frac{1}{2}z^{-1}}$$

ROC : $|z| > 1/2$

$$x_2(n) = \left(\tfrac{1}{3}\right)^n u(n)$$

$\downarrow ZT$

$$X_2(z) = \frac{1}{1-\frac{1}{3}z^{-1}}$$

ROC: $|z| > 1/3$

$$x_2(-(n-1))$$

$$\longleftrightarrow z^{-1} X_2(z^{-1})$$

R.O.C : $|z| < 3$

$$Y(z) = z^3 X_1(z) \cdot z^{-1} X_2(z^{-1})$$

ROC : $\frac{1}{2} < |z| < 3$

Common ROC $\Longrightarrow$ valid product

$$Y(z) = \frac{z^3}{1-\frac{1}{2}z^{-1}} \cdot \frac{z^{-1}}{1-\frac{1}{3}z}$$

7.2.14

$$y(n) - \tfrac{1}{3}y(n-1) = x(n)$$

$$x(n) = \left(\tfrac{1}{2}\right)^n u(n). \qquad y(n),\ n \geq 0.$$

ZT

$$Y(z) - \tfrac{1}{3}z^{-1}Y(z) = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-\frac{1}{3}z^{-1}}$$

$$Y(z) = X(z) \cdot H(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \cdot \frac{1}{1-\frac{1}{3}z^{-1}}$$

$$= \frac{3}{1-\frac{1}{2}z^{-1}} - \frac{2}{1-\frac{1}{3}z^{-1}}$$

for $y(n)$ n greater than 0.

IZT

$$y(n) = 3\left(\tfrac{1}{2}\right)^n u(n) - 2\left(\tfrac{1}{3}\right)^n u(n)$$

7.2.15
EEE

$$G(z) = \alpha z^{-1} + \beta z^{-3}$$

DLPF

Assume $\alpha = \beta$ & $z = e^{j\omega}$

$$G(e^{j\omega}) = \alpha e^{-j\omega} + \alpha e^{-j3\omega}$$

$$= \alpha e^{-j2\omega} \underbrace{(e^{j\omega} + e^{-j\omega})}_{2\cos\omega}$$

$$\theta(\omega) = -2\omega$$

7.2.17
ECE

$h(2) = 1$ & $h(3) = -1$

$h(k) = 0$ elsewhere.

$$H(z) = \sum h(n) z^{-n}$$

$$= h[2] z^{-2} + h(3) z^{-3}$$

$$= z^{-2} - z^{-3}$$

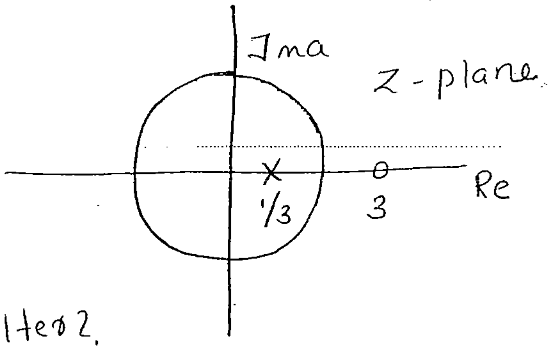
$$\text{linear phase} \leftarrow = z^{-2}(1 - z^{-1}) \rightarrow \text{HPF}$$

S1: false

S2: True.

7.2.11
oppenhe
EEE

(Q)



filter 2.

~~If the~~ In the z plane if poles & zeros are reciprocal to each other, then it represents an all pass filter. (or)

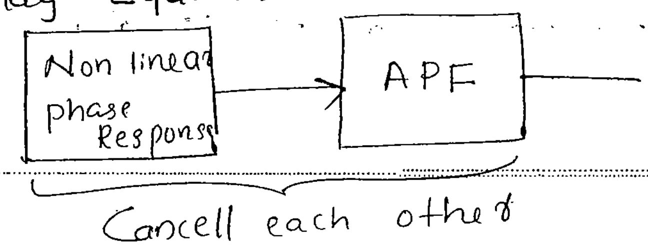
The Nr polynomeal is mirror image of Dr polenomeal

$$\boxed{H(z) = \frac{\alpha + z^{-1}}{1 + \alpha z^{-1}}}$$ zero $= \frac{-1}{\alpha}$

pole $= -\alpha$

$$Q(z) = 1 + \alpha z^{-1}$$
$$Q(z^{-1}) = 1 + \alpha z$$
$$z^{-1} Q(z^{-1}) = z^{-1} + \alpha$$
$$H(z) = \frac{z^{-1} Q(z^{-1})}{Q(z)}$$
$$\boxed{H_N(z) = \frac{z^{-N} Q_N(z^{-1})}{Q_N(z)}}$$

Delay Equializer



7.2.16

oppenham EEE

$$x(n) = (-2)^n \ \forall n \Rightarrow y(n) = 0 \ \forall n$$
$$x(n) = (1/2)^n u(n) \ \forall n \Rightarrow y(n) = d(n) + a(1/4)^n u(n) \ \forall n$$

$a = ?$

if $x[n] = z^n \Rightarrow y[n] = z^n H(z)$

$$(-2)^n \Rightarrow 0 = (-2)^n H(z)$$
$$H(z)\Big|_{z=-2} = 0.$$

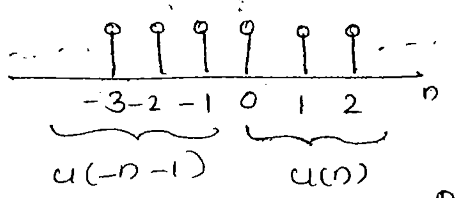
$$H(z) = \frac{Y(z)}{X(z)} = \frac{\left(1 + \frac{a}{1 - \frac{1}{4}z^{-1}}\right)}{\left(\frac{1}{1 - \frac{1}{2}z^{-1}}\right)}$$

But $H(z)\Big|_{z=-2} = 0.$

$$\Rightarrow \underline{a = \frac{-9}{8}}$$

(b) $x(n) = 1 \ \forall n \implies y(n) = ?$

$\implies$



No Common R.O.C

$\implies$ No ZT

$x(n) = (+1)^n \implies$

$y(n) = (1)^n H(z)\Big|_{z=1}$

If convolution can't be applied then use the concept if i/p is exponential output is also exponential

7.2.18 (a)

7.2.19 $f_s = 9KHz$.

$H(z) = 1 - z^{-6}$

$H(e^{j\omega}) = 1 - e^{-j6\omega}$

$= 1 - e^{-j\,6\,\frac{2\pi f}{16}}$

$= 1 - e^{-j\frac{(6)(2\pi)(f)}{9K}}$

if $f = 1.5K$

$H(e^{j\omega}) = 1 - e^{-j2\pi} = 1 - 1 = 0$

$f = 1.5K$ will be stopped

6.1.25 $z = e^{j\omega}$ $\omega = \frac{2\pi f}{f_s}$

Ans: (b) 6KHZ

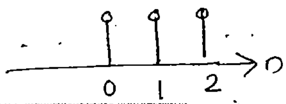
Accumalation.

$$\sum_{k=-\infty}^{\infty} x(k) \xleftrightarrow{ZT} \frac{X(z)}{1-z^{-1}} \; ; \quad ROC = R \cap |z|>1$$

(Q) eg: $x[n] = u[n]$

$$X(z) = \frac{1}{1-z^{-1}} \; ; |z|>1$$

$$\sum_{k=-\infty}^{\infty} u(k) \longleftrightarrow \frac{1/1-z^{-1}}{(1-z^{-1})}$$



$$\sum_{k=0}^{n} (1) \longleftrightarrow \frac{1}{(1-z^{-1})^2}$$

$(n+1)u(n)$

$\downarrow$

$u(n) * u(n)$. $\rightarrow$ Convolution of same signal in descrete is sample increments.

$$\frac{d\,x(t)}{dt} \xleftrightarrow{LT} sX(s)$$

$\Downarrow$

$$x[n] - x[n-1] \xleftrightarrow{ZT} X(z) - z^{-1}X(z)$$

$$s \rightarrow 1 - z^{-1}$$

$$\frac{1}{s} \rightarrow \frac{1}{1-z^{-1}}$$

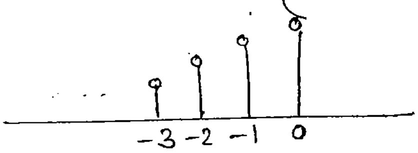
$$\boxed{u(n) * u(n) = (n+1)u(n)}$$

unilateral z- transform.

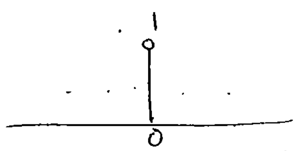
$$X(z) = \sum_{n=0}^{\infty} x[n]\, z^{-n}$$

(Q) U.Z.T of $3^n u[-n]$ is ___ ?

$$3^n u[-n] = \begin{cases} 3^n \; ; & n \leq 0 \end{cases}$$



$\equiv$

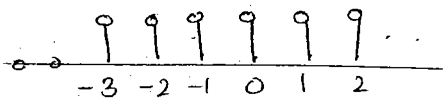


summation starts at 0

$$\therefore \; 3^n u[-n] \xrightarrow{UZT} \underline{\underline{1}}$$

(Q) U.Z.T of $\left(\frac{1}{2}\right)^n \underbrace{u(-n-3)}_{n\leq -3}$ is $\underline{\ 0\ }$

(Q) UZT of $u(n+3)$ is ____



$\xrightarrow{UZT} \frac{1}{1-z^{-1}}$

<u>Initial value</u>

$$x(0) = \underset{z\to\infty}{Lt}\ x(z)$$

<u>Final value</u>

$$x(\infty) = \underset{z\to 1}{Lt}\ (1-z^{-1})\,x(z)$$

$$= \underset{z\to 1}{Lt}\ (z-1)\ x(z)$$

$$x(z) = \sum_{n=0}^{\infty} x[n]\, z^{-n}$$

$$= x(0) + x(1)z^{-1} + x(2)z^{-2} + \cdots$$

(Q) $$x(z) = \frac{0.8}{1-1.8z^{-1}+0.8z^{-2}}$$

Initial & final value?

$$x(0) = \underset{z\to\infty}{Lt}\ x(z)$$

$$= \frac{0.8}{1} = \underline{\underline{0.8}}$$

$$x(\infty) = \underset{z\to 1}{Lt}\ (1-z^{-1})\,x(z)$$

$$= \underset{z\to 1}{Lt}\left[\frac{(1-z^{-1})\,(0.8)}{(1-z^{-1})(1-0.8z^{-1})}\right]$$

$$= \frac{0.8}{0.2} = \underline{\underline{4}}$$

(8) $X(z) = \dfrac{1}{(1-\frac{1}{4}z^{-1})(1+3z^{-1})}$ $x(0),\ x(\infty)$?

$x(0) = \lim_{z\to\infty} X(z) = \underline{\underline{1}}$

$x(\infty) =$ indeterminate

$x(\infty) = 0$ (Apply Th)

$X(z) = \dfrac{A}{1-\frac{1}{4}z^{-1}} + \dfrac{B}{1+3z^{-1}}$

$x(n) = A\left(\frac{1}{4}\right)^n u(n) + B(-3)^n u(n).$

$\underline{\underline{x(\infty) = \infty}}$. (contradiction)

one pole lies outside the unit circle.

∴ final value theorem can't be applied

7.2.22

$X(z) = \dfrac{0.5}{1-2z^{-1}}$ ROC: includes unit circle

$x(0) = ?$

IZT

ROC : $|z|<2$

$x(n) = 0.5\ 2^n\ u[-(n-1)]$

$\underline{\underline{\qquad}}\ n \le -1$

7.2.23

$x(n) = \begin{cases} 1\ ; & \text{even } n \\ 0\ ; & \text{odd } n \end{cases}$

$X(z) = \sum_{\substack{n=0 \\ (\text{even } n)}}^{\infty} 1\cdot z^{-n}$

$= 1 + z^{-2} + z^{-4} + z^{-6} + \cdots$

$= \dfrac{1}{1-z^{-2}} = \dfrac{1}{(1-z^{-1})(1+z^{-1})}$

$x(\infty) = \lim_{z\to 1} \dfrac{1}{1+z^{-1}} = \underline{\underline{1/2}}$

7.3.1

Causality & stability

$$\lim_{z\to\infty} X(z) \neq \infty \quad \text{causal.}$$

eg:
$$H(z) = \frac{z^3+z^2+1}{z^2+2z+1}$$
$$= z + [\quad]$$
$$\lim_{z\to\infty} H(z) = \infty$$

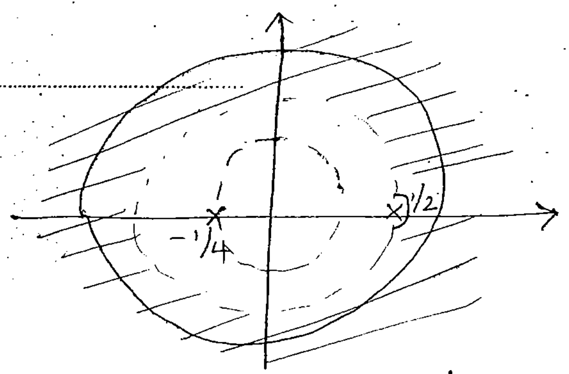
Nr power < Dr power.

All poles lies within the unit circle.

$\Longrightarrow$ both causal & stable.

Causal & stable

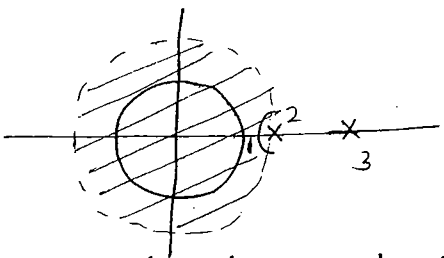
eg: $H(z) = \dfrac{1}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{4}z^{-1})}$



Anticausal & stable

* imp
7.3.2

$$H(z) = \frac{z}{(z-2)(z-3)}$$



poles must lie outside the unit circle.

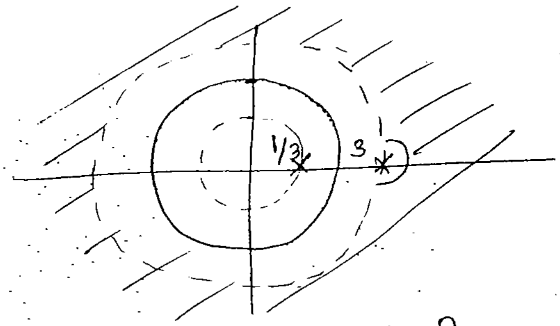
Transfer function

7.3.1

$$H(z) = \frac{3}{1 - \frac{10}{3}z^{-1} + z^{-2}} = \frac{3}{(1-\frac{1}{3}z^{-1})(1-3z^{-1})}$$

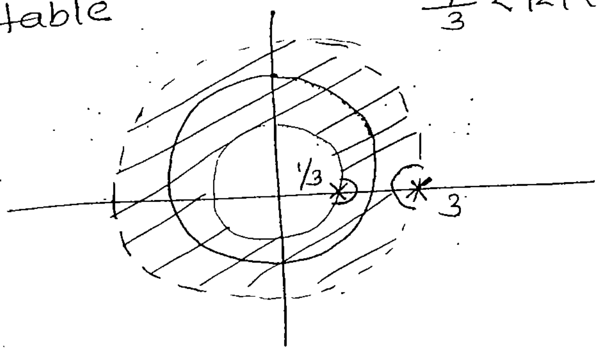
$$= \frac{-3/8}{(1 - 1/3\, z^{-1})} + \frac{27/8}{1-3z^{-1}}$$

Causal $|z| > 3$



$$h(n) = -\frac{3}{8}\left(\frac{1}{3}\right)^n u(n) + \frac{27}{8}(3)^n u(n)$$

stable $\frac{1}{3} < |z| < 3$



$$h(n) = -\frac{3}{8}\left(\frac{1}{3}\right)^n u(n) + \frac{27}{8}\, 3^n [-u(-n-1)]$$

* imp

7.3.2

$$2y[n] = \alpha y[n-2] - 2x[n] + \beta x[n-1]$$

$\downarrow$ ZT

$$2Y(z) = \alpha z^{-2} Y(z) - 2X(z) + \beta z^{-1} X(z)$$

$$(2 - \alpha z^{-2}) Y(z) = (-2 + \beta z^{-1}) X(z)$$

Transfer function; $H(z) = \frac{Y(z)}{X(z)} = \frac{-2+\beta z^{-1}}{2 - \alpha z^{-2}}$

poles at $z = \pm\sqrt{\frac{\alpha}{2}}$

For stability $\left|\pm\sqrt{\frac{\alpha}{2}}\right| < 1 \implies |\alpha| < 2$

β can have any value

$\beta = \text{any}$

Ans: C

7.3.3 (a) $0.5 < |z| < 2$

(b) Not possible.
(two poles outside $|z| = 1$)

(c) $|z| > 3$

$|z| < 0.5$

$0.5 < |z| < 2$

$2 < |z| < 3$

4 possible ∴ R.O.Cs

Note:

7.3.4 non-real – complex

Ans: d Anticausal & stable

$$H(z) = \frac{(z - \frac{3}{4}e^{j\omega})(z - \frac{3}{4}e^{-j\omega})}{z - 4/3}$$

Nr > Dr power ∴ non causal.

$$= z + [\quad]$$

$z \to \infty \implies H(z) = \infty$

Anticausal $|z| < 4/3$

& stable.

Imp (Q) EEE

(Q) A proper rational TF and its inverse are both causal and stable iff poles and zeros are lying inside the unit circle. correct / wrong ?

Imp

True

eg: $H(z) = \dfrac{1 - \frac{1}{2}z^{-1}}{1 + \frac{1}{3}z^{-1}}$ $|z| > 1/3$

$$H_{(inv)}(z) = \frac{1}{H(z)}$$

$$= \frac{1 + \frac{1}{3}z^{-1}}{1 - \frac{1}{2}z^{-1}}$$

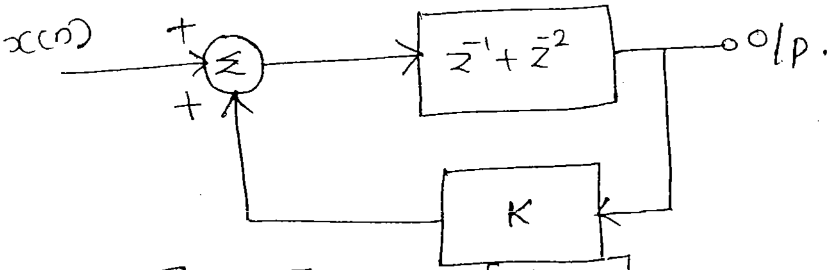
$|z| > 1/2$

$H(z)$, $H_{inv}(z) \longrightarrow$ Min phase.

<u>Note:</u> For more stability ~~and~~ (or) fast response in the z domain, place the pole very near to the origin. Where as in the s-plane locate the pole <u>away from the jω axis towards left half of the s-plane.</u>

<u>Imp</u>
(Q) For the causal filter structure shown below, the range of K for system stability?
EEE

power
causal.



inverse

(A) $[-1, 1/2]$ (B) $[\frac{1}{2}, 1]$

(C) $[1, 2]$ (D) $[1/2, 2]$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1} + z^{-2}}{1 - K(z^{-1} + z^{-2})}$$

$$= \frac{z^{-1} + z^{-2}}{1 - Kz^{-1} - Kz^{-2}}$$

Ans: option <u>A</u>

characteristic Equation

$$1 - K(z^{-1} + z^{-2}) = 0$$

$$z^2 - Kz - K = 0$$

$$z_{1,2} = \frac{K \pm \sqrt{K^2+4K}}{2}$$

if $\underline{\underline{K = 1}}$

$$z_{1,2} = \frac{1 \pm \sqrt{5}}{2} \times$$

$\underline{\underline{K = 2}}$

$$z_{1,2} = \frac{2 \pm \sqrt{12}}{2} \times$$

one pole is outside the unit circle.

Instead of finding magnitude substitute ,

03/09/2011
2 - 5.30 pm

observe page 100 relation

$$\boxed{z = e^{sT}}$$

$T \rightarrow$ sampling time

$$x(t) = e^{st}$$

$$\downarrow t = nT$$

$$x(nT) = e^{snT}$$

$$x[n] = e^{sT}$$

pg 101
eg: 7.4.1

$$\rightarrow H(s) = \frac{1}{s+a}$$

$$\downarrow \text{ILT}$$

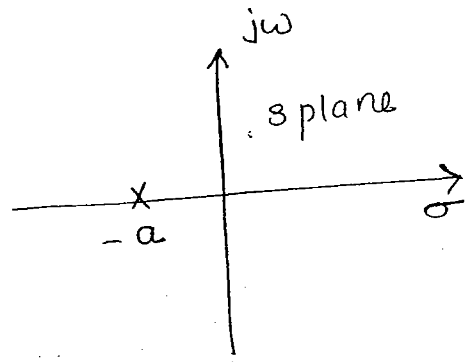
$$h(t) = e^{-at} u(t)$$

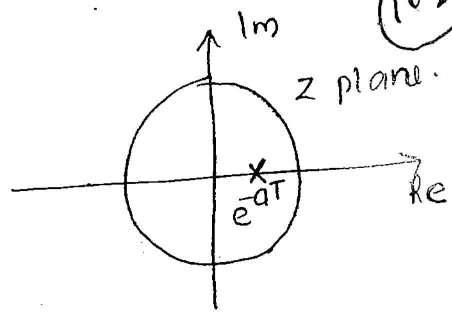
$$\downarrow t = nT$$

$$h(nT) = e^{-anT} u(nT)$$

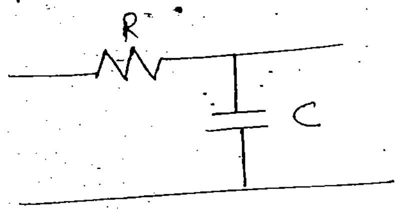
$$= (e^{-aT})^n u(nT)$$

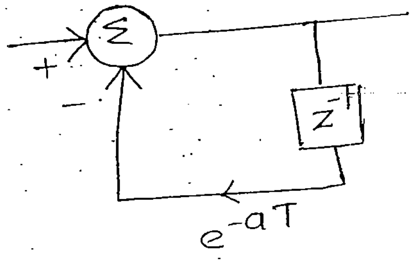
$$\Rightarrow H(z) = \frac{1}{1 - e^{-aT} z^{-1}}$$





As pole move towards $-\infty$ in s plane the pole approaches 0 in z plane. s/m becomes more stable.





$$\frac{1}{1+sRC} = \frac{1}{RC(s+1/RC)}$$

Design engg. concentrates on analog s/m.

pg 100
eg: 7.4.1

$f_s = 2\text{ Hz} \Longrightarrow T = 0.5\text{ sec}$

Poles at $s = -2 \pm j\pi \Longrightarrow z = e^{(-2\pm j\pi)(0.5)}$

$= 0 \Longrightarrow z = e^{(0)(0.5)} = 1$

Zeros at $s = \pm j\pi/2$

$= -2$

Realisation of Digital systems

The way we are implementing a TF/ Diff eqn.

2 forms

✓ IIR filter

$h[n]: n \to -\infty$ to $+\infty$

(better mag respons)

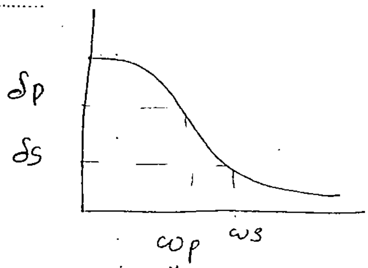
$$y(n) = \sum_{k=0}^{m} b_k x[n-k] - \sum_{k=1}^{n} a_k y(n-k]$$

FIR filter

$h[n]: \longrightarrow 0$ to $(m-1)$

(linear phase)

$$y(n) = \sum_{k=0}^{m} b_k x[n-k]$$



⇓

order

⇓

coefficients

⇓

stable.

DSP Processors

(MAC Inst)

Reason

Convolution } Multiplication
Correlation } & Addition.

DSP ⟹ Simultanous + & ×

Company

Analog Device – 24 DSP Process

Texas Instruments

TMS320C6713

⇓

1000 Instrn.

IIR Realisation

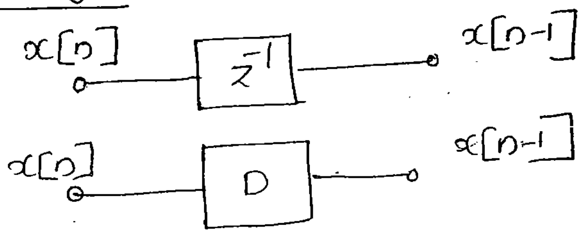
(i) Direct form I

(ii) II

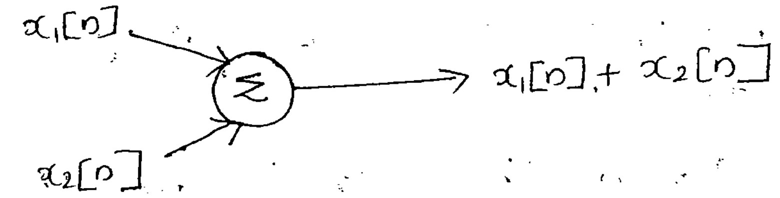
(iii) Cascaded form

(iv) Parallel form.

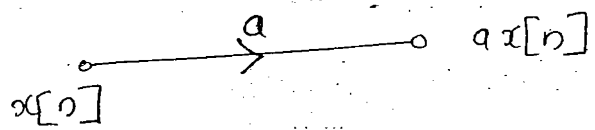
① Delay



(2) Adder



(3) Multiplier



Choice :

1. Memory requ.
2. Computation complexity
3. Finite word length effect

The way we are implementing the difference equation/ transfer function is realization. There are two forms of realization.

1) IIR filter which is used for better magnitude approximation

$$y[n] = \sum_{k=0}^{m} b_k\, x[n-k] - \sum_{k=1}^{n} a_k\, y[n-k]$$

2) FIR filter :- which is used for linear phase response.

$$y[n] = \sum_{k=0}^{m} b_k\, x[n-k]$$

- Generally all FIR filters are stable because h[n] is defined for finite number of samples.
- For the given specifications (pass and stop band gain, PB, SB frequencies)

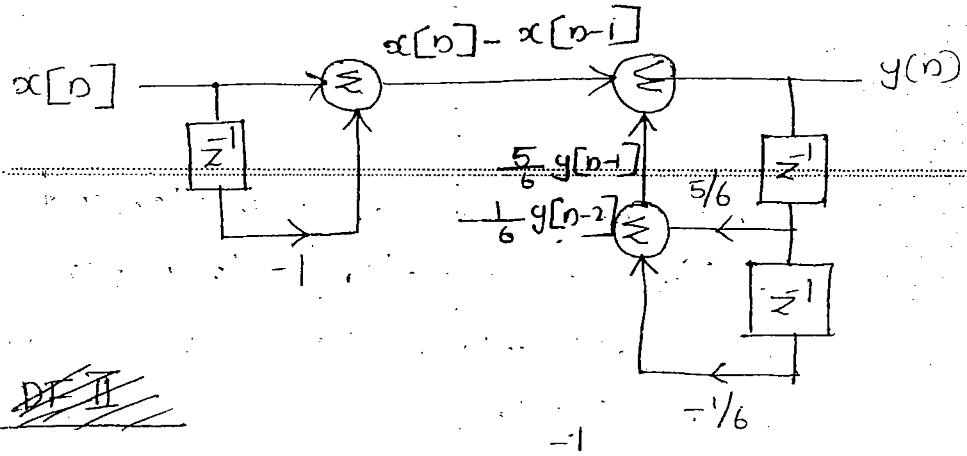
For the given specifications IIR requires less no of delays compared to FIR

*Q. Draw the direct form 1 & 2 realisation for the difference eqns

$$y[n] - \frac{5}{6} y[n-1] + \frac{1}{6} y[n-2] = x[n] - x[n-1]$$

DFI:

$$y[n] = x[n] - x[n-1] + \frac{5}{6} y[n-1] - \frac{1}{6} y[n-2]$$



~~DF II~~

$$\underbrace{H(z)}_{TF} = \frac{Y(z)}{X(z)} = \frac{1 - z^{-1}}{1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}}$$

Nr - input
Dr - output
Nr - same sign
Dr - opposite sign

Disadvantage

- Require more memory
- Execution time is more.

IIR
xared

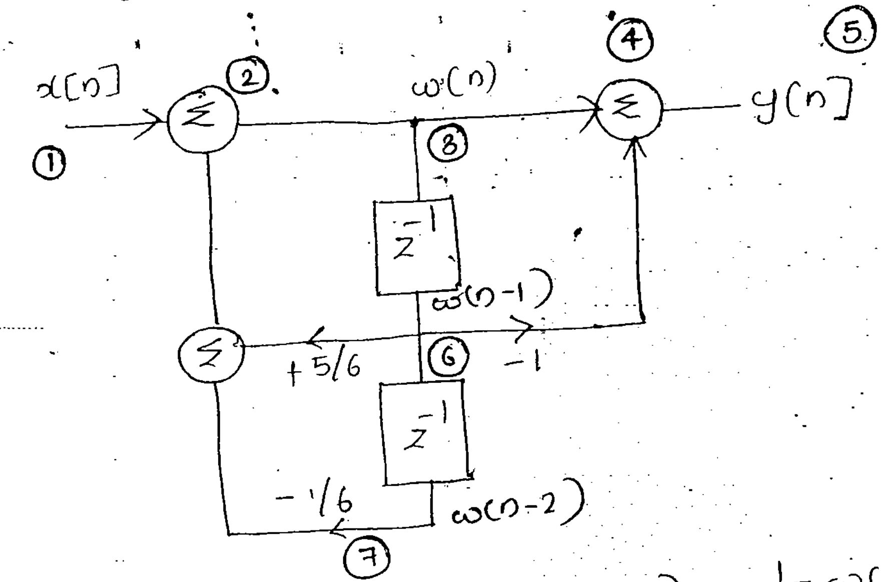
eqns
x[n-1]

- y(n)

DF II (Canonical form)

Minimum memory.

order = min no of delay required.



$$\omega(n) = x(n) + \frac{5}{6}\,\omega(n-1) - \frac{1}{6}\,\omega(n-2)$$

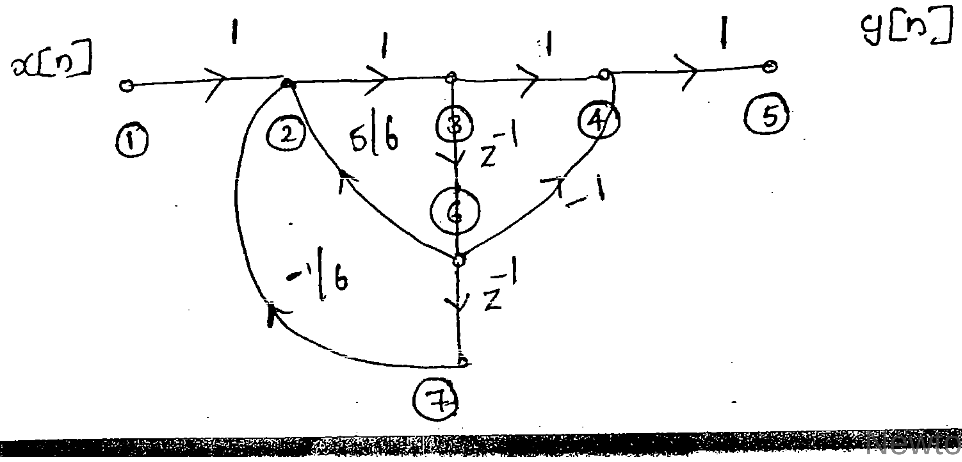
$$\frac{W(z)}{X(z)} = \frac{1}{1 - \frac{5}{6} z^{-1} + \frac{1}{6} z^{-2}}$$

$$y[n] = \omega[n] - \omega[n-1]$$

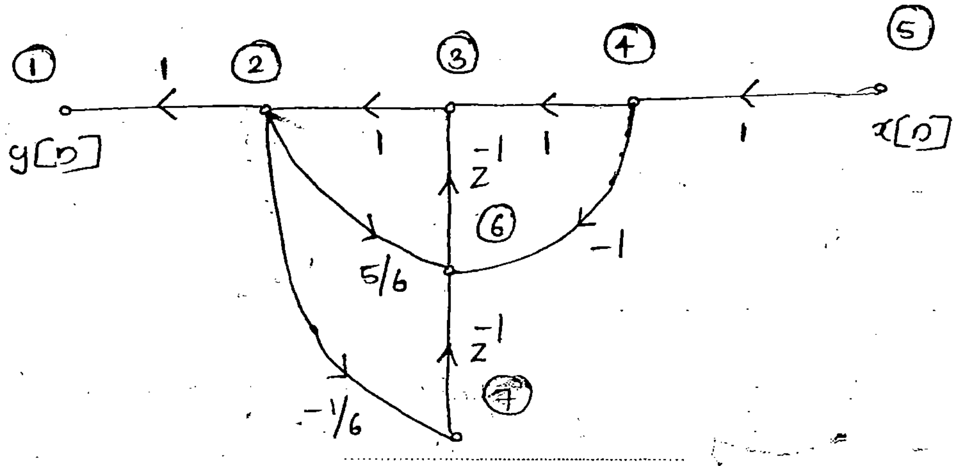
$$\frac{Y(z)}{W(z)} = 1 - z^{-1}$$

$$H(z) = \frac{Y(z)}{W(z)} \times \frac{W(z)}{X(z)}$$

Signal flow graph.



Transposed signal flow graph 7.5.2



Disa

Flow graph reversal theorem.

— If we interchange the direction of branch arrows, i/p & o/p the s/m fn remains unchanged.

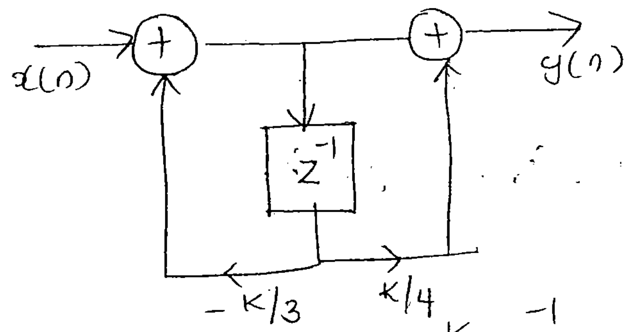
Pg 106

7.5.1 (B) 6 & 3

D II order of TF

7.5.3 (a)

7.5.5



$$H(z) = \frac{1 - \frac{K}{4} z^{-1}}{1 + \frac{K}{3} z^{-1}}$$

for Causal & stable

$$\left| \frac{-K}{3} \right| < 1$$

$$\Rightarrow |K| < 3$$

7.5.2

$$y[n] = x[n-1]$$

$$H(z) = z^{-1}$$

$$H_1(z)\,H_2(z) = z^{-1}$$

$$H_2(z) = \frac{z^{-1}}{H_1(z)}$$

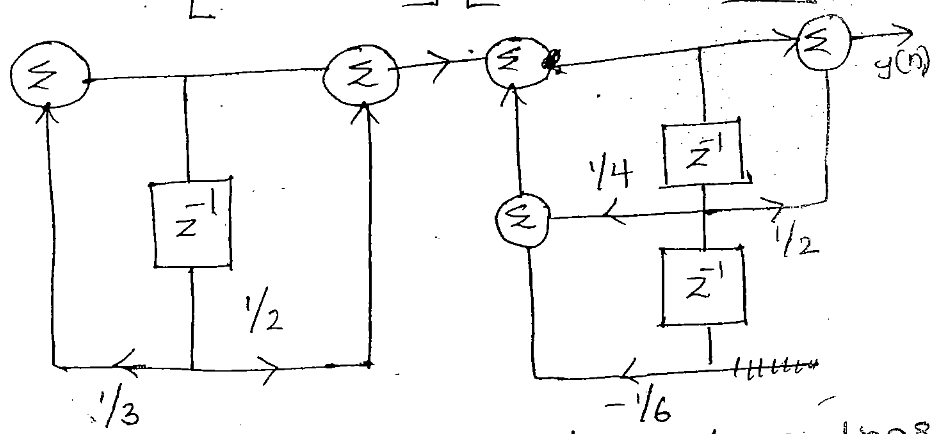
Disadv coeff. accuracy not maintained.

Cascaded form.

$$H(z) = H_1(z)\,H_2(z)\,H_3(z) \ldots$$

eg: $$H(z) = \frac{\left(1+\frac{1}{2}z^{-1}\right)^2}{\left(1-\frac{1}{3}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}+\frac{z^{-2}}{6}\right)}$$

$$= \left[\frac{1+\frac{1}{2}z^{-1}}{1-\frac{1}{3}z^{-1}}\right]\left[\frac{1+\frac{1}{2}z^{-1}}{1-\frac{1}{4}z^{-1}+\frac{z^{-2}}{6}}\right]$$



* nth order cascaded II^{nd} order sections

pairing

To implement an nth order cascaded by taking min. II^{nd} order section by taking of pairings possible sections, the no of ~~$\left(\frac{n}{2}!\right)^2$~~ are $\left(\frac{n}{2}!\right)^2$

Parallel form

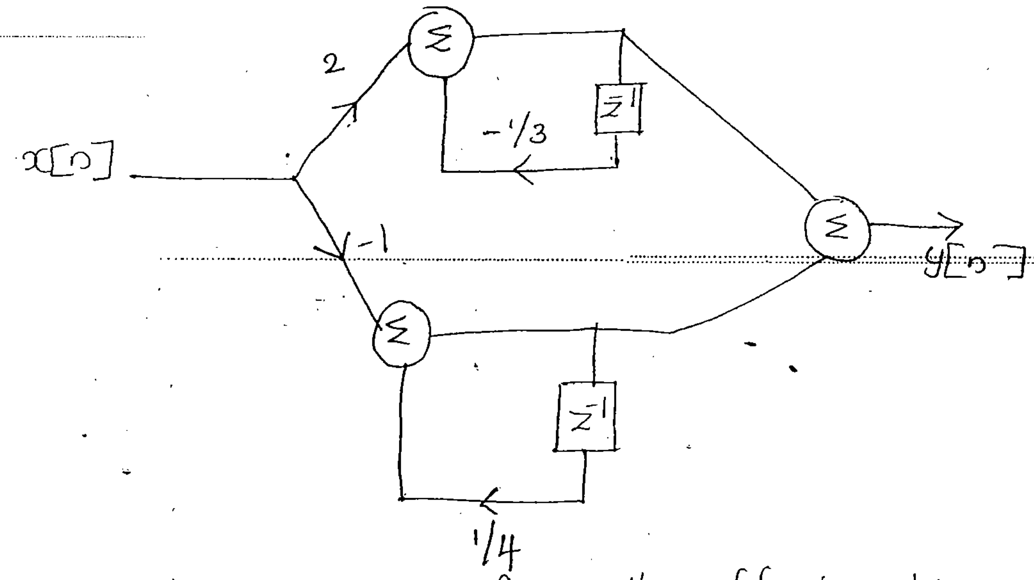
Pf expansion.

$H(z) = H_1(z) + H_2(z) + \cdots$

eg: $H(z) = \frac{2}{1+\frac{1}{3}z^{-1}} - \frac{1}{1-\frac{1}{4}z^{-1}}$

$H_1(z) + H_2(z)$

$\frac{Y_1(z)}{X(z)} + \frac{Y_2(z)}{X(z)}$.



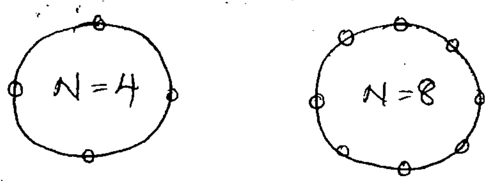
Change in coeff. will affect only the local segment.

used in Impulse - invariant form.

⇓

not using due to Aliasing.

parallel form is rarely used



freq. spacing $= \frac{2\pi}{N} = \frac{f_s}{N}$

2^m point

5pt $\Longrightarrow$ 8 pt

10 pt $\Longrightarrow$ 16 pt zero padding

$$X(K) = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}kn} \; ; \; K = 0 \text{ to } (N-1)$$

Phase factor $W_N = e^{-j\frac{2\pi}{N}}$

Properties

Periodicity : $W_N^{K+N} = W_N^K$

$$\left(e^{-j\frac{2\pi}{N}}\right)^{K+N} = e^{-j\frac{2\pi}{N}K} \cdot e^{-j2\pi}$$
$$= W_N^K$$

symmetry :

$\Longrightarrow W_N^{K+\frac{N}{2}} = -W_N^K$

$\Longrightarrow W_N^2 = W_{N/2}$

$$W_N^2 = \left(e^{-j2\pi/N}\right)^2 = e^{-j\frac{4\pi}{N}}$$
$$= e^{-j\frac{2\pi}{N/2}} = W_{N/2}$$

$$X(K) = \sum_{n=0}^{N-1} x[n]\, W_N^{Kn} \; ; \; K : 0 \text{ to } (N-1)$$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ \vdots \\ X(N-1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N^1 & W_N^2 & \cdots & W_N^{(N-1)} \\ 1 & W_N^2 & W_N^4 & \cdots & W_N^{2(N-1)} \\ \vdots & & & & \\ 1 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)^2} \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix}$$

Phase factor matrix

$$[X] = [W][x]$$

0 to (N-1)

$N = 2$

$$W_2 = \begin{bmatrix} 1 & 1 \\ 1 & W_2^1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$W_2^1 = e^{-j2\pi/2} = -1$$

$N = 4$

$$W_4 = e^{-j\frac{2\pi}{4}} = -j$$

$$W = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^4 & W_4^6 \\ 1 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix}$$

N/2

(N-1)

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^0 & W_4^2 \\ 1 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

*8.1.1

Ans $= \dfrac{10 \times 10^3}{1024}$

*8.1.2

$x[n] = \{0, 1, 2, 3\}$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \underbrace{\begin{bmatrix} \quad \end{bmatrix}}_{W} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

8.1.3

→ Area under the spectrum = Signal at origin

→ N^2 → Multiplications

→ $N(N-1)$ → Complex addition.

→ obeys conjugate symmetry with periodicity

periodic conjugate ppty

$$\boxed{X[K] = X^*[N-K]}$$

Relation b/w DFT & DFS

DFS:

$$x[n] = \sum_{k=0}^{N-1} C_k e^{jk\omega_0 n}$$

$$\omega_0 = \frac{2\pi}{N}$$

$$C_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$

C_k → DFS coefficient $\quad C_k = \frac{X(k)}{N}$

$$C_k = C_{k+N}$$

8.1.3

$$x(n) \xleftrightarrow{Npt} X(K)$$

$$x(n) \longrightarrow -x[N-1-n] \qquad X(0) = 0$$

Let $N = 4$

$$x[n] = -x[3-n]$$

$$x[0] = -x[3]$$

$$x[1] = -x[2]$$

$$X(0) = \sum_{n=0}^{N-1} x[n]$$

$$= \sum_{n=0}^{3} x[n]$$

$$= x(0) + x(1) + x(2) + x(3)$$

$$= \underline{\underline{0}}.$$

Area under odd s/l is zero.
(shows periodic odd s/mtry)

(ii) $x[n] = x[N-1-n]$ $\qquad \frac{N}{2}n.$

$$X\left[\frac{N}{2}\right] = \sum_{n=0}^{N-1} x[n]\, W_N^{\frac{N}{2}n}$$

$$W_N^{\frac{N}{2}n} = \left(e^{-j\frac{2\pi}{N}}\right)^{\frac{N}{2}n} = \underline{\underline{(-1)^n}}$$

n

with

8.2.1

108

$$= \sum_{n=0}^{N-1=3} x[n](-1)^n$$

$$= x[0] - x[1] + x[2] - x[3]$$

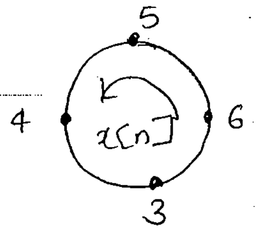
$$= 0$$

Even s/l $\longrightarrow$ Middle sample is equal to zero.

8.1.4

$x[n] = \{6, 5, 4, 3\}$

$x((n-2))_4 = ?$



8.2.2

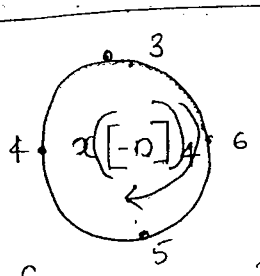
In $x((n-n_0))_N$ if n_0 is +ve, move last n_0 samples to the begining. if n_0 is -ve move 1st n_0 samples to the end.

$x[n+1]_4 = \{5, 4, 3, 6\}$.

$n_0 = -1$

Circular Reversal

(c)



$= \{6, 3, 4, 5\}$.

]

0.

move

ving

ples

1 (a) $X(0) = \int x(n) = \underline{-3}$

(b) $\sum_{K=0}^{5} X(K) = N\, x[0]$

$= 6 \times 1 = \underline{6}$

(c) $X(3) = X\left(\frac{N}{2}\right) = \sum_{n=0}^{5} x[n](-1)^n$

(d) $N\, \Sigma\, x[n] = \sum_{K=0}^{5} |x(K)|^2$

$= \sum_{K=0}^{5} (-1)^K x(K).$

(e) $N\, x\left[\frac{N}{2}\right] = 6\, x(3)$

$= 6(-4) = \underline{-24}$

8.2.2 $x_1[n] = \{0, a, b, c, d, e, 0, 0\}.$

$x_2[n] = \{d, e, 0, 0, 0, a, b, c\}.$

$x_2[n] = x_1((n-4))_8$

$= x_1\left[n - \frac{N}{2}\right]_N$

$\downarrow$ DFT

$X_2[K] = (-1)^K X_1(K)$

8.2.3 $x[n] = \{4, 3, 2, 1, 0, 0\}$

$N = 6.$

$Y(K) = W_6^{4K} X(K)$

$= W_N^{Kn_0} X(K)$

$\downarrow$ IDFT

$y[n] = x((n-4))_6$

$= \{2, 1, 0, 0, 4, 3\}.$

NPTEL
S. C. Dutta Roy

8.2.4

(109)

$$x_1(n) = \{1, 2, 1, 1, 2, 1, 1, 2\}$$
$$x_2(n) = \{0, 1, 3, 2, 0, 0, 0, 0\}$$

1 2 1 1 2 1 1 2
0 1 3 2 0 0 0 0

0 0 0 0 0 0 0 0
1 2 1 1 2 1 1 2
3 6 3 3 6 3 3 6
2 4 2 2 4 2 2 4

lin conv: 0 1 5 9 8 7 9 8, 7 8 4

Wrap around: 7 8 4

Circular conv. 7 9 9 9 8 7 9 8

$X_3(2) = 9$

normal

Decimat by

Decima by

Linear Convolution using DFT

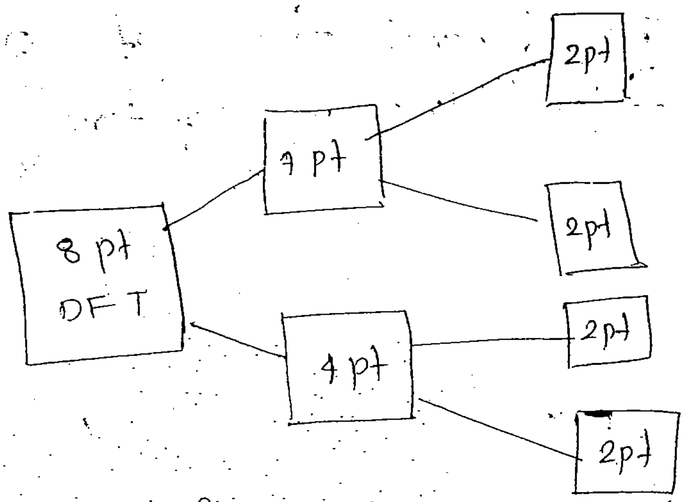
Block dia

F.F.T

Algorithm

Inventors: Cooley - Tukey (brothers)

	Npt DFT	FFT
×	N^2	$\frac{N}{2}\log_2\frac{N}{2}$
+	$N(N-1)$	$N\log_2 N$



Divide & Conqour approach – Even & odd samples.

$N = 8pt$

normal seq	0	1	2	3	4	5	6	7
Decimate by 2	0	2	4	6	1	3	5	7
Decimate by 2	0	4	2	6	1	5	3	7

$\underbrace{\qquad\qquad\qquad}_{\text{Bit reversal.}}$

	normal	BR	
0	0 0 0	0 0 0	0
1	0 0 1	1 0 0	~~1~~ 4
2	0 1 0	0 1 0	2
3	0 1 1	1 1 0	3

* percentage saving.

* 256 pt $\Longrightarrow$ 8 bits.

normal $\Longrightarrow$ 6

$\Downarrow$

B.R ?

* Comparison ?

The General forms of Digital filter transfer function.

$y[n] = x[n] + x[n-1] \Rightarrow LPF$

$y[n] = x[n] - x[n-1] \Rightarrow HPF$

FIR	IIR
$H_{LP}(z) = 1 + z^{-1}$	$H_{LP}(z) = \left(\frac{1-\alpha}{2}\right)\frac{1+z^{-1}}{1-\alpha z^{-1}}$
Replace 'z' by '-z'	$\lvert\alpha\rvert < 1$
$H_{HP}(z) = 1 - z^{-1}$	$H_{HP}(z) = \left(\quad \right)\frac{1-z^{-1}}{1+\alpha z^{-1}}$
$H_{BP}(z) = 1 - z^{-2}$	
$H_{BS}(z) = 1 + z^{-2}$	$H_{BP}(z) = \frac{1-z^{-2}}{1-\alpha(1+\beta)z^{-1}+\alpha z^{-2}}$
All poles are located at origin.	$H_{BS}(z) = \frac{1-\cancel{\alpha z^{-1}}+z^{-2}}{1-\alpha(1+\beta)z^{-1}+\alpha z^{-2}}$

Notc freq. $\omega = \pi/2$

$$H_{APF} = \frac{z^{-N} Q_N(z)}{Q_N(z^{-1})}$$

$z = e^{j\omega}$

$$\left.\begin{array}{l} \omega = 0 \\ \omega = \pi/2 \\ \omega = \pi \end{array}\right\} \text{To decide the nature of filter.}$$

ital

$\frac{1+z^{-1}}{1-\alpha z^{-1}}$

$|\alpha| < 1$

$\frac{1-z^{-1}}{1+\alpha z^{-1}}$

z^{-2}

$) z^{-1} + \alpha z^{-2}$

$+ z^{-2}$

$) z^{-1} + \alpha z^{-2}$

Notch freq.
$\omega = \pi/2$

Synopsis

	CTFT	LT	DTFT	ZT	DFT
Def					
inv					
pptries					

CTFT	LT
	$u(t)$ $-u(t)$
DTFT	$a^n u(n)$ $a^n \{-u(-n-1)\}$