Network Science

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The bridges of Königsberg:

Can I have a path that crosses each bridge exactly once? The answer is no.

Computational Social Science:

Interdisciplinary field that uses computational methods to study social systems.

- How to connect nodes - Nodes have attributes - Quantity attached to an edge is a weight

If chatgpt can creates synthetic data, how to measure how good is.

A network is a graph. Network science is data driven Networks are collections of vertices joined by edges. Vertex are nodes. Edges are links. They can be referred as ties, bonds, connections, interactions.

A graph

$$G = (V, E)$$

where

$$V$$

is the set of vertices and

$$E$$

is the set of edges. Each edge

$$e \in E$$

is a tuple

$$(u, v)$$

where

$$u, v \in V$$

.

$$u$$

and

$$v$$

are the endpoints of the edge and are said to be adjacent.

The max number of edges in a graph is

$$|E| = \binom{|V|}{2}$$

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Lecture 2

How do we represent a graph?

- Visual - Adjacency matrix - Edge List (This is most common in projects)

Types of networks:

- Undirected, simple - Directed: also called - Network with self-edges - Multiedges - Weighted graphs - Hypergraphs - Trees - Bipartite networks - Planar networks

There are two types of transformation from Undirected to Directed:

- Co-citation networks - Bibliographic coupling networks

Degree of a node: number of edges incident to it.

Regular graph: all nodes have the same degree.

Vertex degree for directed graph

Forbidden Triad

——- Lecture 5

What do we know so far?

- A netfork is defined by a collection of nodes and edges - There are different types of networks - There are different network models - Erdős–Rényi Random graph - Barabási-Albert model - Watts-Strogatz model - We have some network metrics for the nodes - Clustering Coefficient - Node Degree - We have some network metrics for the network - Average Path Length - Average Shortest Path Length - Avg Clustering Coefficient - Density - Singleton - Degree Distribution - Diameter: Largest shortest path length - How do we compare nodes? - Clustering Coefficient - Node Degree - Node Betweenness Centrality

Betweenness Centrality: Extent that a node lies on the shortest path between other nodes.

$$X_i = \sum_{s \neq i \neq t} \frac{\sigma_{st}(i)}{\sigma_{st}}$$

Degree Centrality: The importance of a node is a function of the importance of its neighbors.

Kotz Centrality:

$$X_i = \sum_{j \in N(i)} X_j$$

Page Route

Closeness Centrality:

$$X_i = \frac{1}{\sum_{j \in N(i)} d_{ij}}$$

Similarity - Structural equivalence: nodes sharing same neighbords - Regultar equivalence: nodes sharing neighbors that are similar

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Link Prediction Approach: Look at the mechanism that connect the network - Preferential Attachment: calculating probability of connection based on degree - Common Neighbors: Multiple nodes in common between two nodes so it makes sense to create an edge between them. - Adamic-Adar - Resource Allocation - Hierarchical Random Graph: Connection nodes from different branches is less likely than nodes from the same branch (community) - Association Rules (GERM) - Many more approaches - Graph Embeddings - Based on Katz Centrality

Link Prediction in Multi Layer Networks Prediction of a link in which layer? - Simple Network - Social Balance Theory: The enemy of my enemy is my friend - Social Status Theory:

Metrics: - Frustration

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Communities Despite most of networks are sparse we can find communities

There are no ground truth communities

Modularity: A partition better thatn random chance

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Epidemic Spreading

SIS Model

SIR Model

Rumor Spreading

——- Paper 3

Methodologies Datasets Discussing Points

- Local comminities using Model

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- Information difusion - Threshold Model (Peer Pressure) - Independent Cascade Model (One-to-One influence)

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- Graph Mining Motifs: Subgraphs that are overrepresented in a network Isomorphism: Graphs that are structurally the same

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- Graphlets: Subgraphs that are overrepresented in a network - How to represent nodes, edges as networks - Node Embedding - Properties of Good Embeddings - Spectral Embeddings - Node Clustering Pooling - Random Walk Embeddings - node2Vec - Harp Embeddings - Node Roles