Scientific Computing in Finance

This course covers scientific computing methods and techniques, and their applications in Finance. The course is designed to have a strong focus in quantitative Finance practices, topics are often presented with real world examples and common industry practices.

Text book:
1. D. Bindel and J. Goodman: Principles of Scientific Computing, 2009.
2. Netlib repository: www.netlib.org

References:
1. P. Glasserman: Monte Carlo method in Financial Engineering
2. L. Anderson and V. Piterbarg: Interest Rate Modeling, Volume I and II 2010.

Course Syllabus:
1. Introduction: (1 week: Yadong)
   a. overview of the class
   b. introduction to computer architecture, memory hierarchy and performance
   c. a quick overview of good software practices
   d. source of errors, IEEE standard of floating point numbers
   e. error propagation and condition number

2. Linear Algebra (2 weeks, Yadong)
   a. review of linear algebra
   b. solving linear system: Gaussian elimination, LU factorization
   c. correlation and covariance matrix, Cholesky decomposition
   d. mean/variance portfolio optimization
   e. QR decomposition and Householder transformation
   f. eigen values and eigen vectors
   g. least square
   h. PCA/SVD analysis and its applications in Finance
   i. Conditioning, condition number for linear systems

3. Interpolation, curve building and root search (1 week, Yadong)
   a. Overview of 1-D root search methods: Bisection, Secant, and Brent
   b. Types of curves and curve building in Finance
   c. Piecewise linear/constant interpolation
   d. Cubic spline, Tension spline
   e. Bootstrapping and iterative algorithm
   f. Perturbation and locality

4. derivatives and hedging (1 week, Yadong)
   a. Order of accuracy in finite difference
   b. Derivative and deltas: Jacobian matrix for delta transformation
   c. Hedging with bespoke instruments

   d. Hedge optimization
   e. Numerical integration and its accuracy

5. Optimization (2 weeks, Hongwei)
   a. Introduction---A brief survey of optimization problems
      I. Unconstrained, Constrained
      II. Linear programming, Quadratic programming
      III. Integer programming, Mixed Integer programming
      IV. Nonlinear programming, Convex programming
      V. Dynamic programming, Stochastic Programming
   b. Simplex Method --- Asset/Liability Cash Flow Matching
   c. Nonlinear programming
      I. Optimality conditions, Lagrange multipliers, KKT.
      II. Volatility Estimation with GARCH
      III. Dynamic programming --- Option Pricing, Structured ABS Securities
   d. Optional: Stochastic programming

6. Monte Carlo simulation (2 weeks, Yadong)
   a. Pseudo random number generators
   b. MC error, sample statistics, curse of dimensionality
   c. Discretization of SDEs
   d. Least square Monte Carlo and American options
   e. Variance reduction techniques
   f. Quasi random sequence

7. Maximum entropy and allocation (1 week, Yadong)
   a. Information entropy
   b. Maximum entropy and the dual problem
   c. Risk capital allocation methods: Shapley, Aumann-Shapley, Euler and C-Shapley

8. ODE/PDE (2 weeks, Hongwei)
   a. ODEs, Runge Kutta method (e.g. the ODE from affine term structure models)
   b. Parabolic PDE in finance from valuation problems
   c. Tree method, Finite difference method
   d. Stability, Convergence
   e. Explicit/Implicit Method, Crank Nicolson
   f. Optional: multi-factor PDE, operator split and ADI method

9. Project presentation (1 week, everyone)