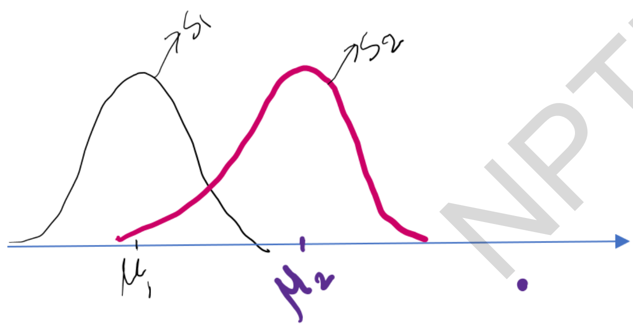
Week 07 lecture 01

Discrimination between stimuli based on response

- Correlational observations aim to uncover patterns of activity and potential relationships between neurons in response to various stimuli, tasks, or conditions.
However, it's important to note that correlation does not imply causation. Just because two neurons exhibit correlated activity does not necessarily mean that one neuron is causing the activity of the other.

Discrimination- Consider two stimuli S1 and S2, having responses (spike counts) following a Poisson. Now if the rates are high enough and in a fixed window size the response distribution is Gaussian.

Then,



Discrimination, $d^1 = \frac{\mu_1 - \mu_2}{\sigma}$ *(having same variance)*

If the variances are different, then

$$\frac{\sqrt{2}(\mu_1 - \mu_2)}{\sqrt{\sigma_1^2 - \sigma_2^2}}$$

Week 07 lecture 2

KL distance, also known as Kullback-Leibler divergence or relative entropy, is a mathematical measure used to quantify the difference between two probability distributions.

Given two probability distributions, P(x) and Q(x), defined over the same set of events or outcomes, the KL distance from P to Q is calculated as follows:

KL(P || Q) = ∑[P(x) * log(P(x) / Q(x))]

In this formula:

- P(x) and Q(x) are the probabilities of event x occurring according to distributions P and Q, respectively.
- The sum is taken over all possible events x for which both P(x) and Q(x) are non-zero.
- The logarithm is usually taken with base 2 or the natural logarithm (base e).

- KL distance for continuous gaussian distributions, with two different means, and equal std deviation .

$$P = \frac{1}{\sqrt{2\sigma^2}} \exp\left[\frac{(x-\mu_1)^2}{2\sigma^2}\right]$$

$$Q = \frac{1}{\sqrt{2\sigma^2}} \exp\left[\frac{(x-\mu_2)^2}{2\sigma^2}\right]$$

$$D_{kL}(p|_1 q) = \int_{-\infty}^{\infty} dx\, p(x) \log\left[\frac{\exp -\left(\frac{x-\mu_1}{2\sigma^2}\right)}{\exp -\left(\frac{x-\mu_2}{2\sigma_{\square}^2}\right)}\right]$$

$$= \int_{-\infty}^{\infty} \quad p(x)\, dx\, log\left[\frac{(x-\mu_2)/2\sigma^2}{(x-\mu_{1)/}2\sigma^2}\right]$$

$$\alpha\ (d^1)^2$$

- Consider two stimuli, S1 and S2, having different spike rates over time ,T .divided into M bins each of size Δ.
- Using the KL distance methods is it possible to discriminate between these two stimuli??

For each set of stimuli, depending on the size of the bins, Δ The number of spikes present varies. If $P_i[spike]$ be the probability of spiking on the $i^{th}\ bin$. M bins in corresponds to M random variables , each having a probability of spike

So, let

$$\overrightarrow{(p)} = [p_1, p_{2,} \ldots\ldots\ldots P_M]^T \text{ be the probabilities of } M \text{ bins for stimulus } S1$$

$$\overrightarrow{(q)} = [q_1, q_{2,} \ldots\ldots\ldots q_M]^T \text{ be the probabilities of } M \text{ bins for stimulus } S2$$

Assuming the spiking is independent in all bins, find the KL distance .

P and q can be considered as a joint distribution with responses from first to last bin.
Then ,

$$= \sum_{1}^{2^M} \vec{p} \log \frac{\vec{p}}{\vec{q}}$$

$$= \sum_{i=1}^{2^M} \Pi p_i \log \frac{\Pi p_i}{\Pi q_i}$$

$$= \sum_{i=1}^{M} p_i \log \frac{p_i}{q_i}$$

$$= \sum D_{kL}\left(p_{i_{\|}} q_i\right)$$

Consider , teh bin size is very small , so that the number of spikes sin one bin will not exceed one .Also , probability of a spike occurring in i_{th} bin is a conditional probability depending on the spikes of the previous bins.
Then KL distance has to be calculated based on the conditional probability. The value in ith bin will depend on all the values prior say upto H bins prior.

- Consider , another case where $\Delta\ has\ 0,1,2,3, \dots k$ (larger bins)
- MΔ=T
- Then S1, $i_{th}\ bin\ the$ possibilities are from $p_i[0], p_i[1], \dots\dots p_i[k]$
- Similarly for S2 , $i_{th}\ bin\ the$ possibilities are from $q_i[0], q_i[1], \dots\dots q_i[k]$
- - bins are independent, for the ith bin the KL distance will be

$$D_{KL}(i) = \sum_{r=0}^{K} \quad p_i(r)\ log \frac{p_i(r)}{q_i(r)}$$

- The KL distances are calculated as the sum of each bin
- = $\sum_{i=1}^{M} \quad D_{KL}(i)$

Week 07 lecture 3

- Consider N stimuli, each having M repetitions(1,2,...M iterations)
- The dependency between the stimuli and the responses.
- Spike distribution matrix (D)
- Let $i = 1,2,3\ldots\ldots N$ and $j = 1,2,3\ldots\ldots M$
- Then
  r_{ij} *is the response or a function of response , ith stimulus ,* j_{th} *iteration*
- Compute the spike train distances of one stimulus , with its own iterations.
-
- $r_{11} = Mean[distance[r_{11}\ r_{1j^1}]$ *, where* j^1 varies from 1…M
- This is denoted by L_{11}(for stimulus 1, the distance of first spike train with the rest of its own iterations
- So,
  $L_{11}, L_{12\ldots} L_{1N}$ *be the distances of stimulus* 1 *with other set of stimulus*
- These distances are providing the estimate of teh stimulus
- If how many times the spike trains are associated to the stimulus 1,2,...N are plotted , with all the stimuli has equal probability of occurrence
-

$\hat{S}$ S	1	2	3	N

1	P(S = 1, $\hat{S}$ = 1)	P(S = 1, $\hat{S}$ = 2)	P(S = 1, $\hat{S}$ = 3)	P(S = 1, $\hat{S}$ = N)
2		P(S = 2, $\hat{S}$ = 2)		
3			P(S = 3, $\hat{S}$ = 3)	
N				P(S = N, $\hat{S}$ = N)

(Joint Distribution of S and $\hat{S}$)

- In every spike train , the correct stimulus is identified and the distance to other spike trains is always larger.
- Based on the responses the stimulus is correctly assigned, then all the values will be along the diagonal of this matrix, with 1/N elements along the diagonal
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-
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S \ $\hat{S}$	1	2	3	N
1	$1/N$	0	0	0
2	0	$1/N$	0	0
3	0	0	$1/N$	0
N	0	0	0	$1/N$

- Then $I(S,\widehat{S}) = \sum \quad \sum \quad P(S,\hat{S})\, log\, \frac{P(S,\hat{S})}{P(S)\, P(\hat{S})}$

  = $log\ N$ (maximum MI)

  - $\hat{S} = f(R)$(data processing inequality)
  - $S = f(R)\ \hat{S}$
  - $I(S,R) \geq I(S.\widehat{S})$

Week 07 lecture 4

- Recall the neuron model (HH model) and leaky integrate and fire model
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- The time constant (for the decay of EPSC), varies with neurons

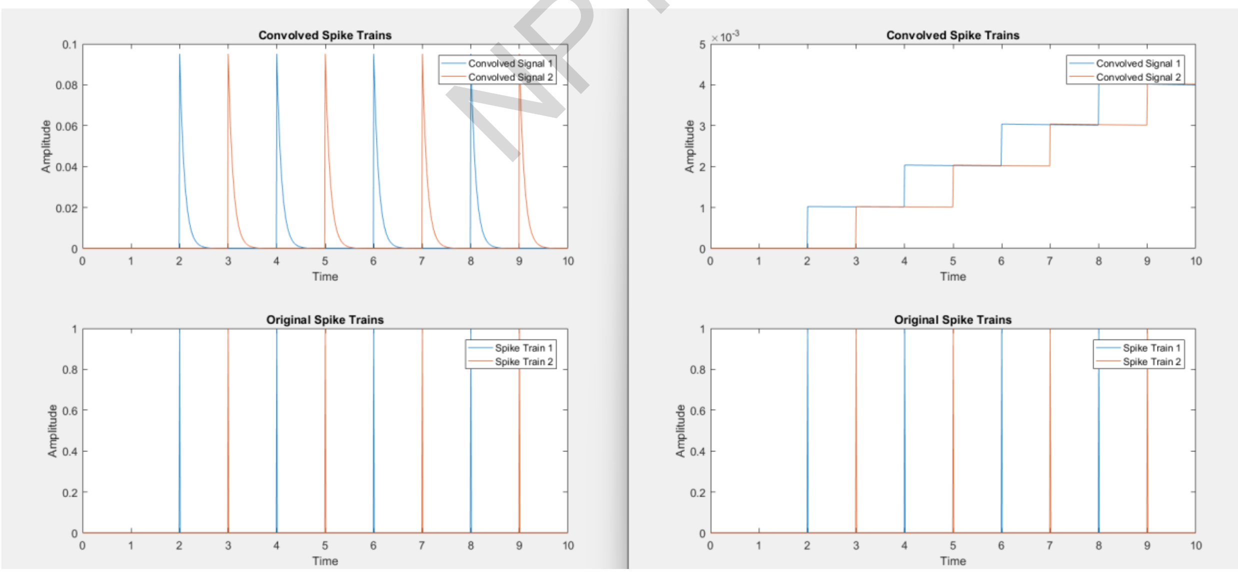
Consider
case 1:- S1 and S2 being two different stimuli having just one spike each , with spike timing difference and tau being extremely small.then the post synaptic neuron will be able to differentiate the stimulus.
Case 2:- - S1 and S2 being two different stimuli having just one spike each , with spike timing difference and tau being very large, then the postsynaptic current will not be differentiable

Discrimination measure based on tau considering the post synaptic potential
Ref: - A novel spike distance , Neural Comput., 13 (2001), p. 751

Let

$$\tau = 0,1,2,3.\ldots.\infty$$

Consider two cases of tau(small and large) , for S1 and S2 ,

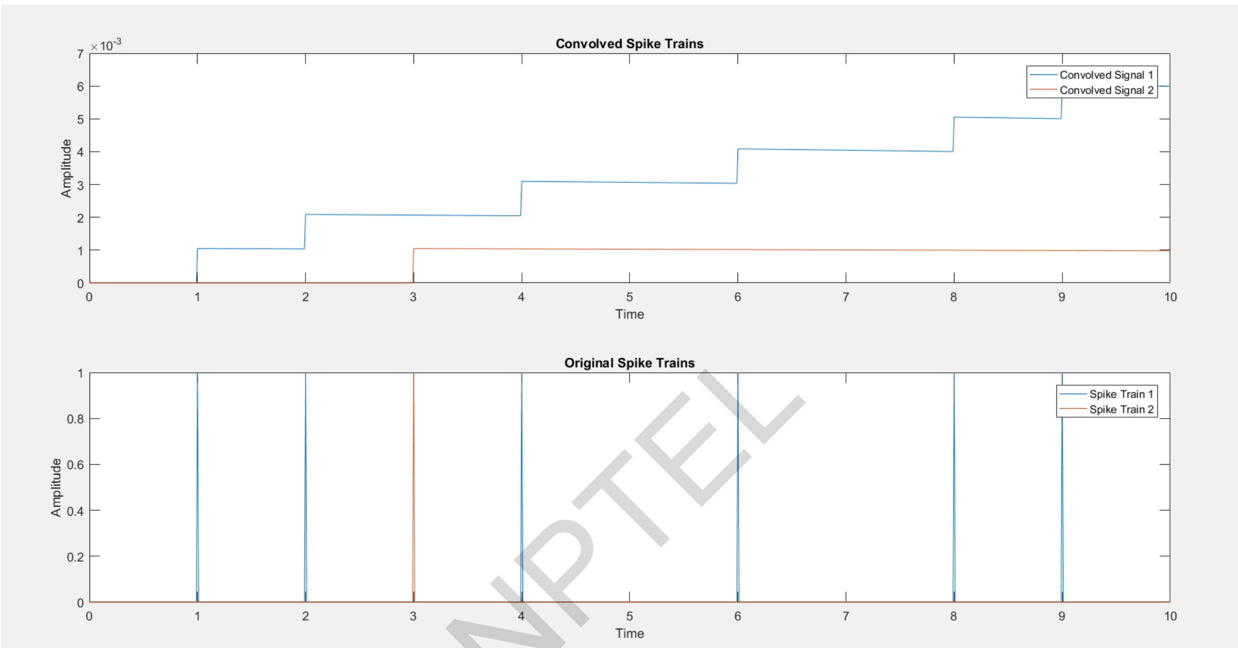


In the left figure, you can see the two “EPSCs” one marked in red and other in blue. In both the spike trains, number of spikes is same. There is only a difference in spike timing for each spike in 2 spike trains.

So, when you convolve with a exponential of smaller time constant like 0.1, you get convolved signals as shown in top left. And van rossum distance between them is around 0,05.

But when you convolve both the spike trains with a larger time constant like 100, you get convolved signals as shown in top right. And the van rossum distance between them falls down to 0.002.

Essentially, because there is a difference between 2 spike trains only in timing, it is possible to capture the difference only with smaller tau. But when you use a larger tau, the difference in their timings can't be captured. Hence you observe a smaller van rossum distance.



But if you see another case, where there is a huge difference in number of spikes(rate). One of them has only one spike and other has 6 spikes(biologically not possible due to refractory period, just given for sake of example), you can see that how a large tau like 100 can show two distinct convolved signals, which was not visible in previous case even with large tau because the number of spikes was same in previous figure.

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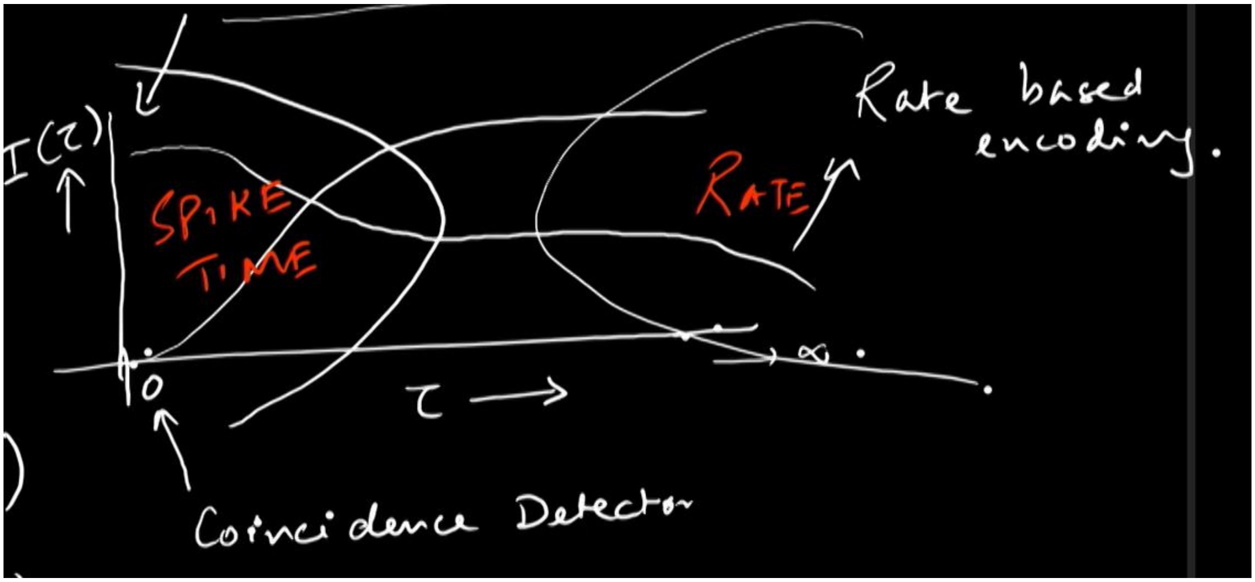

Hence, as discussed in lecture, with a smaller tau, when you are able to differentiate between spike trains better(mathematically equivalent to high mutual information of confusion matrix), it is spike time based encoding. Neuron is encoding the difference between 2 stimuli by timing of spikes. But when you are able to differentiate between 2 spike trains better with large tau, then it means neurons are encoding the difference between 2 stimuli by number of spikes(rate).

Week 07 Lecture 5

Signal Correlation and Noise Correlation:

Signal Correlation: This is a measure of how similarly two signals vary in time or space. High signal correlation between two neurons, for instance, would indicate that they often fire together in response to a stimulus. Signal correlation reflects underlying shared features in the stimulus or shared processing mechanisms between the neurons or signal pathways.

Noise Correlation: This refers to the correlation in the variability of two signals, above and beyond any simultaneous changes due to a shared stimulus. For example, if two neurons mostly fire independently but occasionally show simultaneous spikes that are not explained by the stimulus, they would be considered noise-correlated. This type of correlation is often thought to arise from shared or connected internal network dynamics rather than external factors.

Possible physiological reasons for High Signal Correlation and High Noise Correlations:

High Signal Correlation

1. Similar Inputs: When two neurons receive highly similar inputs, their outputs (i.e., spike trains) are likely to be correlated.
2. Functional Connectivity: Neurons that are part of the same functional circuit or are tuned to similar features of a stimulus can show high signal correlation.

3. Common Driving Force: If an external stimulus or internal state variable strongly drives multiple neurons, they may show high signal correlation.

High Noise Correlation

1. Network Connectivity: Neurons that are directly connected or part of a local network often display correlated noise.
2. Common Modulatory Inputs: If different neurons receive shared, non-specific background inputs, they can exhibit noise correlation.
3. Synchrony in Network Dynamics: Intrinsic network properties like rhythmic activity can cause high noise correlations.

Effects of high correlations between 2 neuron responses in Encoding and Decoding:

High Signal Correlation

1. Redundancy: High signal correlation could mean that multiple neurons are conveying similar information, making the system less efficient for information coding.
2. Ambiguity in Source: It can be challenging to identify which part of the circuit or which specific neuron is responsible for a particular response.

High Noise Correlation

1. Reduced Independent Information: High noise correlation implies that the spikes from different neurons are not entirely independent, reducing the richness of the coded information.
2. Decoding Complexity: Correlated noise can make decoding algorithms more complex and computationally intensive, as it's harder to separate the 'true' signal from noise.

You can also refer to this review article(relevant till Fig 2)
https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3586814/