# Week 10 notes

## Lecture 46

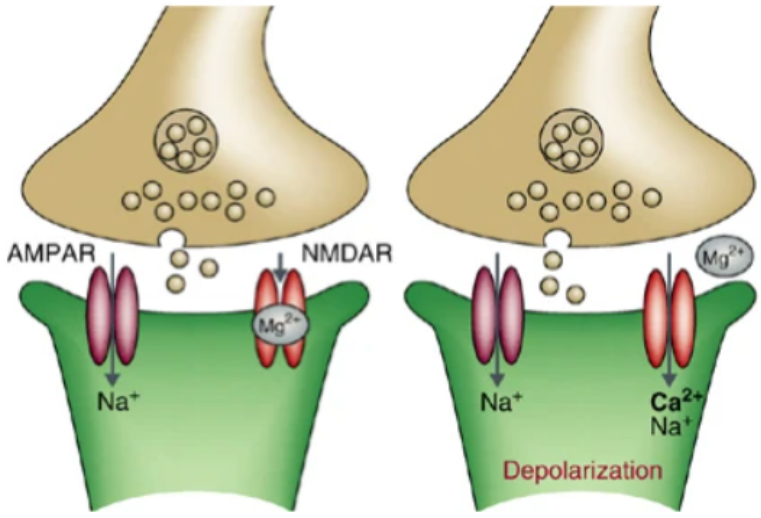
- Long term plasticity
- Protein synthesis in LTP and LTD
- Long-term potentiation (LTP) is an enhancement of synaptic strength that can be produced by pairing of presynaptic activity with postsynaptic depolarization.(https://pubmed.ncbi.nlm.nih.gov/7760933/)

Ref:-
https://www.nature.com/articles/1301559
At the post synaptic side , LTP

- causes upregulation of neurotransmitter receptors
- Increases the proteins involved in transport
- Increases the synaptic scaffolding protein PSD-95 (postsynaptic density 95)
- Increases the spine size , causing increase in teh cytoskeletal protein production



LTP mediated by NMDA




Model of synaptic transmission at excitatory synapses. During basal synaptic transmission (left panel), synaptically released glutamate binds both the NMDA and AMPARs. $Na^+$ flows through the AMPAR channel but not through the NMDAR channel because of the $Mg^{2+}$ block of this channel.

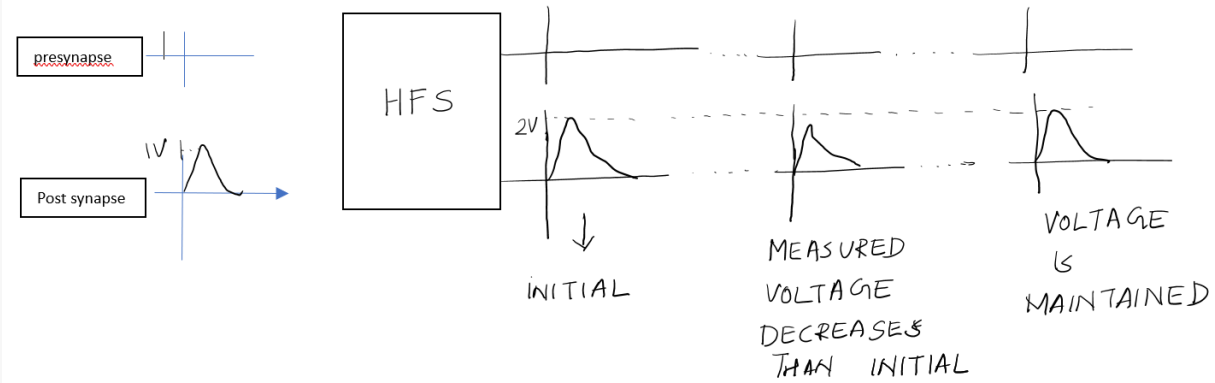
Depolarization of the postsynaptic cell (right) relieves the $Mg^{2+}$ block of the NMDAR channel and allows both $Na^+$ and $Ca^{2+}$ to flow into the dendritic spine. The resultant increase in $Ca^{2+}$ in the dendritic spine is necessary for triggering the subsequent events that drive synaptic plasticity.



Ref:- https://www.nature.com/articles/1301559

Inducing plasticity with high frequency stimulation /Tectanic bursts

- Measurements are made from teh pre and post synaptic neurons with high frequency bursts.(100hz for sec)

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- Similarly for low frequency stimulations , supplied for minutes , causes long term depression
- An uncorrelated spiking activity between the pre and the post synaptic neurons will also cause long term depression
- Hebbian synapse:-
- Synapses that modify their strength are known as hebbian synapses. When synapses have correlated activity increases synaptic strength and decorrelated ones decreases the synaptic strength
- Neurons that fire together wire together

  Not all synapses follows these principles. They are known as Anti hebbian/ NOn hebbian

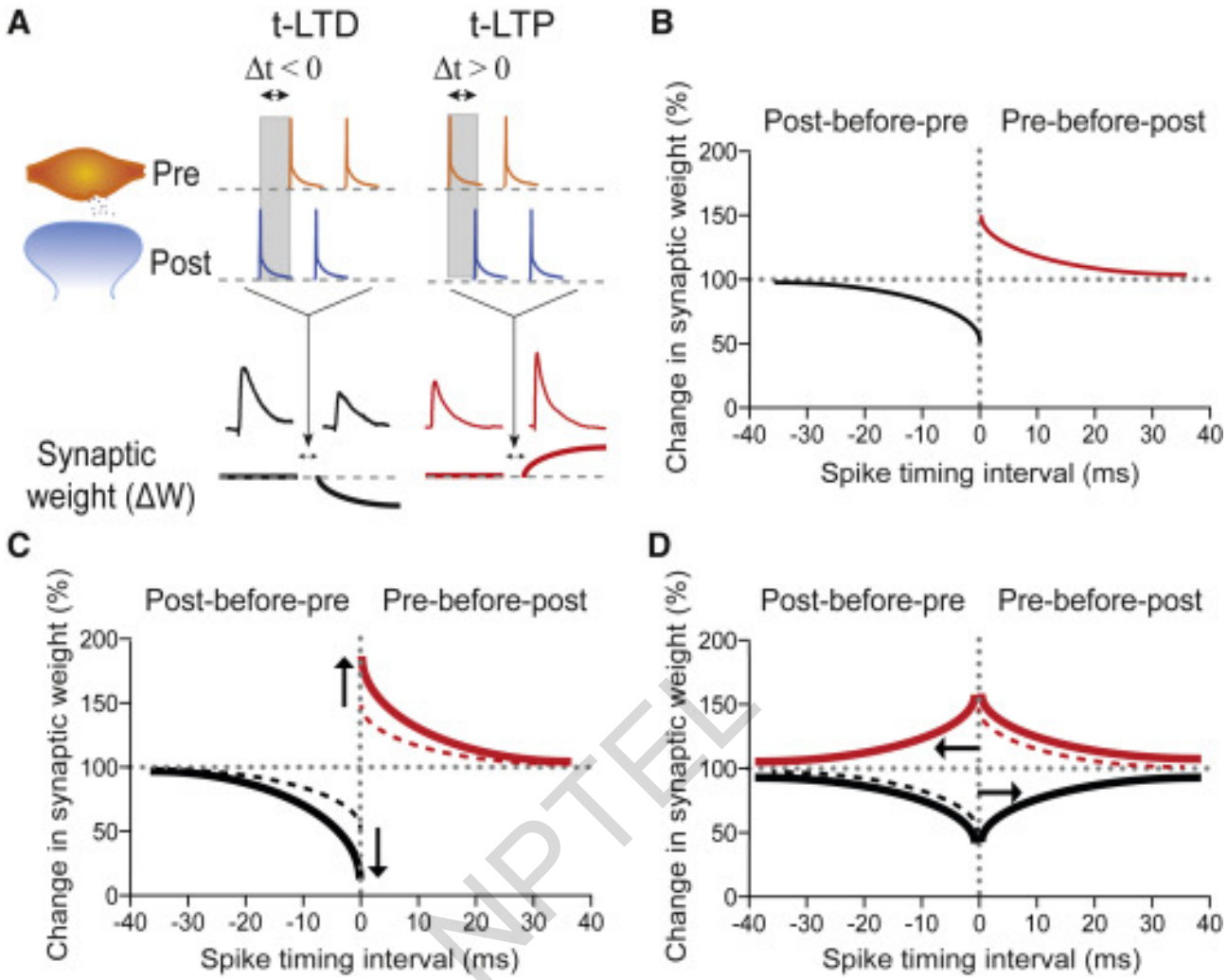
  -If pre synaptic spiking causes the post synaptic spike , it leads to long term potentiation

  -If post synaptic spikingoccurs prior to pre synaptic spike , it leads to long term depression.

# Lecture 47

Spike timing dependant plasticity (STDP)

- Spike-timing-dependent plasticity (STDP) refers to a form of associative synaptic plasticity in which the temporal order of the presynaptic and postsynaptic action potentials determines the direction of plasticity, that is, whether synaptic depression or potentiation is induced.
- In the most common form of STDP, long-term potentiation is induced if the presynaptic spike precedes the postsynaptic spike (pre→post), whereas long-term depression is induced if the postsynaptic spike precedes the presynaptic spike (post→pre). In addition to the order of the pre- and postsynaptic spike, STDP is sensitive to the interspike interval, the time elapsed between the two spikes.




A) After a stable baseline period, STDP is typically induced by repeated pairings of single presynaptic and postsynaptic spikes

(B) The classic Hebbian STDP window: induction protocols with positive (pre-before-post) spike-timing intervals induce synaptic potentiation; protocols with negative (post-before-pre) spike-timing intervals induce synaptic depression.

(C and D) The relative spike timing is not the sole determinant governing timing-dependent plasticity. Instead, STDP is malleable. Both the magnitude (C) and the temporal requirements for STDP (D) can be modulated.

Reference:- https://www.sciencedirect.com/science/article/pii/S0896627319304945

# Lecture 48

- Recall leaky integrate and fire model

- Let u and v be dimensionless measures of the corresponding neuronal firing rates or activities.
- $\tau_r$ is a time constant that controls the firing-rate response dynamics. Recall that $w_b$ is the synaptic weight that describes the strength of the synapse from presynaptic neuron b to the postsynaptic neuron, and w is the vector formed by all $N_u$ synaptic weights.

$$\tau_r \frac{dv}{dt} = -v + \mathbf{w} \cdot \mathbf{u} = -v + \sum_{b=1}^{N_u} w_b u_b \,,$$

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- The firing rate model describing the dynamics.
- The processes of synaptic plasticity are typically much slower than the dynamics characterized by equation above.
- If the stimuli are presented slowly enough to allow the network to attain its steady-state activity during training, we can replace the dynamic equation by v = w · u



Hebbs learning rule

$$\tau_w \frac{d\mathbf{w}}{dt} = v\mathbf{u} \,,$$

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- The rule implies that simultaneous pre- and postsynaptic activity increases synaptic strength. If the activity variables represent firing rates, the right side of this equation can be interpreted as a measure of the probability that the pre- and postsynaptic neurons both fire spikes during a small time interval.
- $\tau_w$ is the time constant controlling the rate of weight change

Averaged Hebb rule

- Synaptic plasticity is modelled as  a slow process, which modifies weights over time , during which inputs takes different values.
- So averaging different input pattern and computing the weight after averaging gives ,
- $$\tau_w \frac{d\mathbf{w}}{dt} = \langle v\mathbf{u} \rangle .$$
- Replace $v = uw \; or \; v = u^T w$.
- Thus
- $$\tau_w \frac{d\mathbf{w}}{dt} = < u\, u^T w > = < u\, u^T > w = Q.w$$

Q is the input correlation matrix

SO correlation based rule is given by

$$\tau_w \frac{d\mathbf{w}}{dt} = \mathbf{Q} \cdot \mathbf{w} \quad \text{or} \quad \tau_w \frac{dw_b}{dt} = \sum_{b'=1}^{N_u} Q_{bb'} w_{b'} ,$$

where $\mathbf{Q}$ is the input correlation matrix given by

$$\mathbf{Q} = \langle \mathbf{uu} \rangle \quad \text{or} \quad Q_{bb'} = \langle u_b u_{b'} \rangle .$$



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Basic Hebb rule is unstable(proof:- page 286:- https://boulderschool.yale.edu/sites/default/files/files/DayanAbbott.pdf

Covariance rule

- the basic Hebb rule describes only LTP

$$\tau_w \frac{d\mathbf{w}}{dt} = v(\mathbf{u} - \boldsymbol{\theta}_u) .$$

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$\theta_u$ is a vector of thresholds that determines the levels of presynaptic presynaptic activities above which LTD switches to LTP.

- Alternatively, the below rule can also be used

$$\tau_w \frac{d\mathbf{w}}{dt} = v(\mathbf{u} - \boldsymbol{\theta}_u) .$$

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- A convinient threshold for the above equations is $\theta_u = < u >$ or $\theta_v = < v >$.
- On putting $v = \vec{w}.\vec{u}$, the rule can be simplified as

$$\tau_w \frac{d\mathbf{w}}{dt} = \mathbf{C} \cdot \mathbf{w} ,$$

where $\mathbf{C}$ is the input covariance matrix,

$$\mathbf{C} = \langle (\mathbf{u} - \langle \mathbf{u} \rangle)(\mathbf{u} - \langle \mathbf{u} \rangle) \rangle = \langle \mathbf{uu} \rangle - \langle \mathbf{u} \rangle^2 = \langle (\mathbf{u} - \langle \mathbf{u} \rangle)\mathbf{u} \rangle .$$

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- Note that this rule is also unstable as $< d|w|^2/dt >$ is proportional to variance of $v$, which is always positive indicating weights keep on increasing.
- This rule is also not competitive like Basic Hebb rule.



# Lecture 49

BCM Rule:

The BCM rule is given by

$$\tau_w \frac{d\mathbf{w}}{dt} = v\mathbf{u}\,(v - \theta_v)\ .$$

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The critical condition for stability is that $\theta_v$ must grow more rapidly than $v$ as the output activity grows large.

But if the threshold is fixed, the rule is still unstable. To fix that, the threshold is kept sliding according to

$$\tau_\theta \frac{d\theta_v}{dt} = v^2 - \theta_v\ .$$

$\tau_\theta$ sets the time scale for modification of the threshold.

- $\tau_{\theta} < \tau_w$

BCM rule implements competition between synapses because strengthening some synapses increases the postsynaptic firing rate, which raises the threshold and makes it more difficult for other synapses to be strengthened or even to remain at their current strengths.



# Lecture 50

## Synaptic Normalization

- Subtractive rule
- Multiplicative rule /Ojas rule

Subtractive rule

- Sum of weight vectors is considered to be a constant
- where n is an $N_u$-dimensional vector with all its components equal to

- $\sum w_u = n.w$

$$\tau_w \frac{d\mathbf{w}}{dt} = v\mathbf{u} - \frac{v(\mathbf{n}\cdot\mathbf{u})\mathbf{n}}{N_u}.$$

If sum of weights is a constant then $d(n.w)/dt = 0$

$$\tau_w \frac{d\mathbf{n}\cdot\mathbf{w}}{dt} = v\mathbf{n}\cdot\mathbf{u}\left(1 - \frac{\mathbf{n}\cdot\mathbf{n}}{N_u}\right) = 0.$$

$$\mathbf{n}\cdot\mathbf{n} = N_u.$$

Multiplicative rule /Ojas rule

$$\tau_w \frac{d\mathbf{w}}{dt} = v\mathbf{u} - \alpha v^2\mathbf{w}$$

Where $\alpha > 0$

- The stability of the Oja rule can be established by taking the dot product of teh above equation with the weight vector w to

$$\frac{d\|\mathbf{w}\|^2}{dt} = 2\mathbf{w}^T\frac{d\mathbf{w}}{dt} = \frac{2}{\tau_w}\mathbf{w}^T(\mathbf{u}v - \alpha v^2\mathbf{w}) = \frac{2}{\tau_w}(v^2 - \alpha v^2\mathbf{w}^T\mathbf{w})$$

$$\text{i.e., } \tau_w\frac{d\|\mathbf{w}\|^2}{dt} = 2v^2(1-\alpha\|\mathbf{w}\|^2)$$

$$\text{At steady state : } \|\mathbf{w}\|^2 = \frac{1}{\alpha}. \quad (\|\mathbf{w}\| = \frac{1}{\sqrt{\alpha}})$$

- prevents the weights from growing without bound- hence stable
- It also induces competition between the different weights, because when one weight increases, the maintenance of a constant length for the weight vector forces other weights to decrease

– STDP based learning rule

- Simulating the spike-timing dependence of synaptic plasticity requires a spiking model. However, an approximate model can be constructed on the basis of firing rates.
- A function $H(\tau)$ determines the rate of synaptic modification that occurs due to postsynaptic activity separated in time from presynaptic activity by an interval $\tau$
- total rate of synaptic modification is determined by integrating over all time differences $\tau$
- the rate of synaptic modification is proportional to the product of the pre- and postsynaptic rates, as it is for a Hebbian rule, the rate of change timing-based rule of the synaptic weights at time t is given by

$$\tau_w \frac{d\mathbf{w}}{dt} = \int_0^\infty d\tau \ (H(\tau)v(t)\mathbf{u}(t-\tau) + H(-\tau)v(t-\tau)\mathbf{u}(t)) \ .$$

If H(τ) is positive for positive τ and negative for negative τ, the first term on the right side of this equation represents LTP, and the second, LTD

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