Week 05 Lecture 21

Random Variable :- A random variable is a mapping from a random outcome ω to a space such as the space of integers.
The probability that S(random variable) takes the value s is then written as P[s].
The notation S(ω) refers to the value of S generated by the random event labeled by ω, and the sum is over all events for which S(ω) = s.

Some key statistics for discrete random variables include:

Quantity	Definition	Alias
mean	$\langle s\rangle = \sum_s P[s]s$	$\bar{s}$, $E[S]$
variance	$\mathrm{var}(S) = \langle s^2\rangle - \langle s\rangle^2 = \sum_s P[s]s^2 - \langle s\rangle^2$	σ_s^2, $V[S]$
covariance	$\langle s_1 s_2\rangle - \langle s_1\rangle\langle s_2\rangle = \sum_{s_1 s_2} P[s_1, s_2]s_1 s_2 - \langle s_1\rangle\langle s_2\rangle$	$\mathrm{cov}(S_1, S_2)$

- A discrete random variable is a real-valued function of the outcome of the experiment that can take a finite or countably infinite number of values.
- • A discrete random variable has an associated probability mass function (PMF), which gives the probability of each numerical value that the random variable can take.
- • A function of a discrete random variable defines another discrete random variable, whose PMF can be obtained from the PMF of the original random variable.
-

– A random variable is discrete if its domain consists of a finite (or countably infinite) set of values. A random variable is continuous if its domain is uncountably infinite.

Realisation of a random variable
:- The value of the random variable (that is, the function) *X* at a point $w \in \Omega$,

$$x = X(w)$$

is called a realization of *X*

Probability density function and probability Mass function:-

The probability mass function (PMF) of a discrete random variable X is the function f(·) that associates a probability with each x ∈ S. In other words, the PMF of X is the function that returns P(X = x) for each x in the domain of X. Any PMF must define a valid probability distribution, with the properties:

- f(x) = P(X = x) ≥ 0 for any x ∈ S
- $\Sigma_{x \varepsilon S} f(x)$=1

The probability density function (PDF) of a continuous random variable X is the function f(·) that associates a probability with each range of realizations of X. The area under the PDF between a and b returns P(a < X < b) for any a, b ∈ S satisfying a < b. Any PDF must define a valid probability distribution, with the properties:

- $f(x) \geq 0$ for any $x \in S$
- $\int_a^b f(x)dx = P(a < X < b) \geq 0$ for any $a, b \in S$ satisfying $a < b$
- $\int_{x \in S} f(x)dx = 1$

Crosscorrelation:- Dayan and Abbot, sections 2.1-2.2., Dayan and Abbot, section 1.2-1.3.

The cross-correlation between two different signals or functions or waveforms is defined as the measure of similarity or coherence between one signal and the time-delayed version of another signal. The cross-correlation between two different signals indicates the degree of relatedness between one signal and the time-delayed version of another signal.

Autocorrelation :-

The autocorrelation function is defined as the measure of similarity or coherence between a signal and its time delayed version. Therefore, the autocorrelation is the correlation of a signal with itself.

References:-

Dayan, Peter, and L. F. Abbott.*Theoretical Neuroscience: Computational and Mathematical Modeling of Neural Systems*. Cambridge, MA: MIT Press, 2001. ISBN: 9780262041997.

https://dlsun.github.io/probability

Week 05 Lecture 22

PSTH stands for "Peri-Stimulus Time Histogram." It's a method used in neuroscience to represent the timing of neural spikes (action potentials) relative to an external event or stimulus. The PSTH is useful for understanding how a neuron or a group of neurons respond to specific stimuli or events.

Stimulus Presentation: A stimulus (e.g., a light flash, sound, or touch) is presented to an animal or human subject multiple times, and the neural responses are recorded each time.

Spike Timestamps: For each presentation of the stimulus, the times at which the neuron fires (i.e., when action potentials or spikes occur) are recorded. These are often referred to as "spike timestamps."

Bins: The time after each stimulus presentation is divided into small intervals or "bins." For example, if you divide the time into 10 ms bins, the first bin might be from 0-10 ms after the stimulus, the second bin from 10-20 ms, and so on.

Counting Spikes: For each bin, the number of spikes that occurred during that time interval is counted. This is done for each presentation of the stimulus.

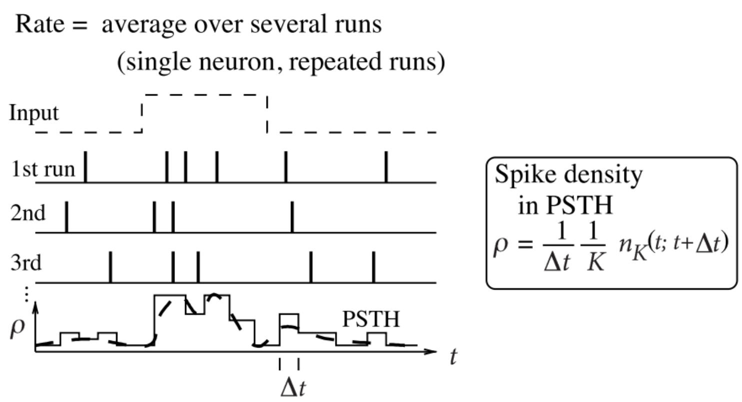
Averaging: The counts for each bin are then averaged across all presentations of the stimulus. This produces an average firing rate for the neuron in each time bin, relative to the stimulus presentation.

Visualization: The averaged data is plotted as a histogram, with time since stimulus onset on the x-axis and firing rate on the y-axis. Peaks in the PSTH indicate times when the neuron's firing rate is higher than its baseline rate, suggesting that the neuron is responding to the stimulus.

The PSTH provides a clear picture of how a neuron's firing rate changes in response to a specific event or stimulus. If a neuron consistently fires more frequently at a specific time after a stimulus, this will produce a peak in the PSTH, indicating that the neuron is likely involved in processing or representing information related to that stimulus.

The number of occurrences of spikes n_K(t;t + Δt) summed over all repetitions of the experiment divided by the number K of repetitions is a measure of the typical activity of the neuron between time t and t +Δt.

$$\rho(t) = \frac{1}{\Delta t} \frac{n_K(t; t + \Delta t)}{K}.$$



Ref:- ***** Shimazaki, H.; Shinomoto, S. (2007). "A method for selecting the bin size of a time histogram". *Neural Computation*.***
[Chapter 7 Variability of Spike Trains and Neural Codes | Neuronal Dynamics online book (epfl.ch)]

Spike train Statistics - the homogeneous poisson process

- stochastic relationship between a stimulus and response would require us to know the probabilities corresponding to every sequence of spikes that can be evoked by the stimulus

For ref :- mrw.dvi (yale.edu) page no : 25-27

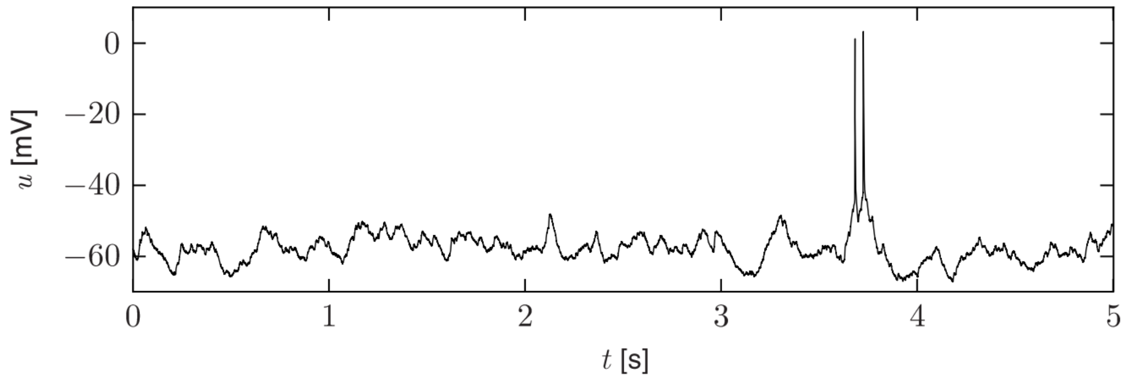
Week 05 Lecture 23

Sensory systems into "parameterizable" and "non-parameterizable" categories:
Gustatory Stimulus: Taste, like smell, is a bit more complex to parameterize.
Visual Stimulus: Several parameters can be used to describe visual stimuli, such as:wavelength, color, spatial frequency , orientation etc
Olfactory Stimulus: The perception of smell is complex, and the exact parameters that differentiate one smell from another are not as straightforward as with auditory stimuli.
Auditory Stimulus: This is perhaps one of the easiest stimuli to parameterize using frequency, pitch,ampliotute , phase, duration etc.

Receptive fields refer to the specific region of the sensory space (e.g., visual, auditory, or somatosensory space) in which a stimulus will modify the firing of a particular neuron.

Spontaneous spiking in neurons refers to the neuronal activity or action potentials that occur in the absence of external stimuli or input. In other words, even without any specific input or trigger, some neurons exhibit a baseline level of activity, firing action potentials at a certain rate.

During spontaneous activity, the voltage trajectory fluctuates considerably and intervals between one spike and the next exhibit a large degree of variability.



Eg of spontaneous activity :- Data courtesy of Sylvain Crochet and Carl Petersen (Crochet et al., 2011).

The rate of spontaneous spiking can vary between neurons, even among those in the same region or of the same type. Some neurons might have a high baseline firing rate, while others may rarely spike without external input..

Variability in spikes:
Can refer [7.1 Spike train variability | Neuronal Dynamics online book (epfl.ch)]

- A stimulus is played for N repetitions.let the baseline / spontaneous responses be denoted by b1,b2…bN
- The responses to stimulus be r1,r2…..rN
- The t-test is a statistical test used to determine if there's a significant difference between the means of two groups.
- When assessing the effect of a stimulus on neuronal responses, whether the mean firing rate (or some other measure of response) during stimulus presentation differs significantly from the baseline firing rate.
- Using a software package or statistical tool, input the two sets of data (baseline and stimulus-response) to perform a t-test. The software will output a t-statistic and a p-value.
- The p-value tells you the probability of observing the given difference (or a more extreme difference) between the two samples if there was no actual difference between the populations they represent (i.e., under the null hypothesis).

Week 05 Lecture 24

An LTI system refers to a "**Linear Time-Invarian**t" system.
Refer
https://eee.guc.edu.eg/Courses/Communications/COMM401%20Signal%20&%20System%20Theory/Alan%20V.%20Oppenheim,%20Alan%20S.%20Willsky,%20with%20S.%20Hamid-Signals%20and%20Systems-Prentice%20Hall%20%281996%29.pdf chapter two for detailed understanding

It's a fundamental concept in systems theory, signal processing, and control theory. The name "LTI" actually describes the two primary properties of the system: linearity and time-invariance.

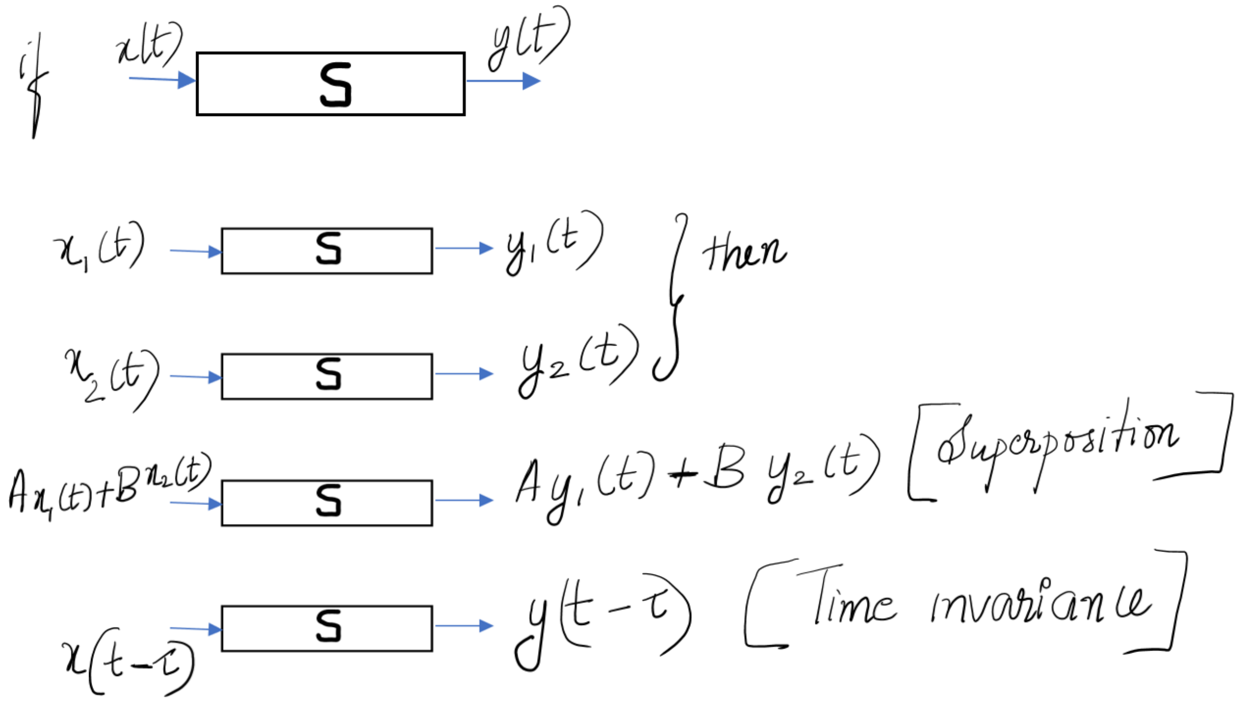
Linear:

This means that the system obeys the principles of superposition and homogeneity.
Superposition: If you input two different signals into the system and get two outputs, then inputting a weighted sum of those signals will produce an output that's the same weighted sum of the two outputs.
Homogeneity: If you scale the input signal, the output will scale by the same factor.
Time-Invariant:

This means that the system's behavior and characteristics don't change over time. If you input a signal into the system now and get a certain output, you would get the same output if you input the same signal at any other time.
Key characteristics of LTI systems:

Impulse Response:

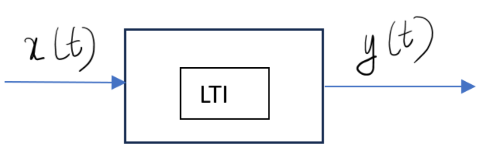
The impulse response of an LTI system, typically denoted as
h(t), is the output of the system when an impulse (a signal that's zero everywhere except at t=0) is the input. The impulse response is a fundamental characteristic because, due to the linearity property, the output for any input can be found by convolving the input with the system's impulse response.
– if it satisfies both the linearity and time-invariance properties.



⋮

Consider an LTI system with input x(t) and output y(t)

Details about $\delta(t)$ $:- \textit{unit impulse function}$
https://eee.guc.edu.eg/Courses/Communications/COMM401%20Signal%20&%20System%20Theory/Alan%20V.%20Oppenheim,%20Alan%20S.%20Willsky,%20with%20S.%20Hamid-Signals%20and%20Systems-Prentice%20Hall%20%281996%29.pdf
Page no 34, 35, 36



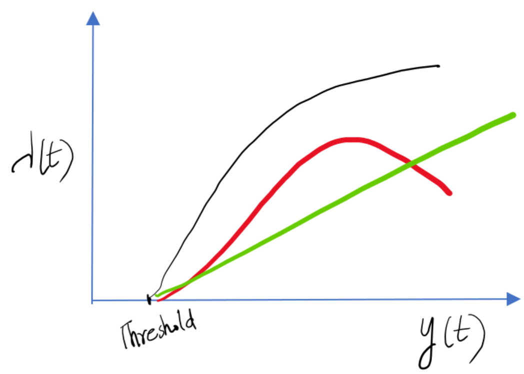
If $x(t) = \delta(t)$ and $y(t) = h(t)$

then $y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

$= x(t) * h(t)$

Let $\lambda(t)$ be the average of spike rates for a particular period or time window.
Probability of the spikes occurring in that window is proportional to the $\lambda(t)$.
$\lambda(t)$ is generated by the membrane potential of teh neurons, when it crosses a certain threshold. The membrane potential is expressed by y(t)

So y(t) -$\lambda(t)$ can have different possibilities like this



The system can then considered as a static nonlinearity with y(t) as input and (t) as output
Now taking x(t)n as input passes through an LTI system of impulse response producing y(t) , which akin to the membrane potential of the neuron.
So finally ,

$Y(t+\tau) = \int_{-\infty}^{\infty} X(t+\tau-u)\, h(u)\, du$

Then , $R_{YX}(\tau) = E[Y(t+\tau)X(t)]$

$= E[\int_{-\infty}^{\infty} X(t+\tau-u)\, h(u)\, du\, X(t)]$

$= \int_{-\infty}^{\infty} h(u)\, du\, E[X(t+\tau-u)X(t)]$

Where $E[X(t+\tau-u)X(t)]$ is teh autocorrelation function of X

Which leads to

$= \int_{-\infty}^{\infty} h(u)\, du\, R_{XX}(\tau)$,

Which implies (by convolution)

$$R_{YX}(\tau) = h(\tau) * R_{XX}(\tau) \text{———————————1}$$

Let x(t) be a Gaussian white noise

Then autocorrelation of gaussian white noise

$$E[X(t+\tau)X(t)] = 0 \; for \; all \;\; \tau not \; equal \; to \; 0$$

When τ=0 ;

$R_{XX}(\tau) = \delta(\tau).\sigma^2$ (here we also assume teh mean is zero)

As per eqn 1,

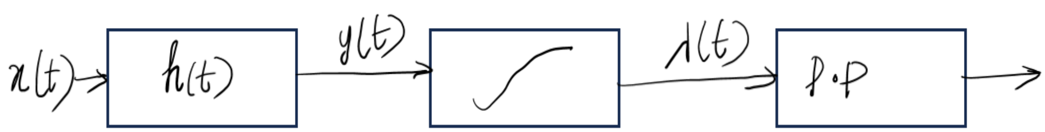
Then , $R_{YX}(\tau)$**=** $h(\tau)$ ***** $\delta(\tau).\sigma^2$

= $\int_{-\infty}^{\infty} \quad h(t)\,dt\,\delta(\tau - t).\sigma^2$

= $\sigma^2.h(\tau)$

$R_{YX}(\tau) \propto h(\tau)$ **[the cross correlation function is proportional to the impulse response]**

Consider the system,



- Output of a point process being teh spike trains (P(t)), $\Delta(bin\ width)$, $M\ bins$.
- The total time duration is given by T.
- The $\Delta(bin\ width)$ is so small that there can be max of only one spike per Δ.
- The bins are 0/1.
- Then the propabbility of $q_i, P(q_i = 1) = \lambda(\Delta_i).\Delta$
- Then the cross correlation of pulse train to teh input X,

  $$R_{PX} \simeq R_{qX}$$

  Since the pulse trains are not going to be for infinite duration , the cross correlation function can be an estimate ,

  Let the number of spikes be N_T

$$\hat{}R_{PX} = \frac{1}{N}\int_0^T X(t - \overline{\zeta})\sum_{i=1}^{N_T} \delta(t - t_i)dt$$

$= \frac{1}{N}\sum_{i=1}^{N_T} \int_0^T X(t - \overline{\zeta})\delta(t - t_i)dt$

$= \frac{1}{N}\sum_{i=1}^{N_T} X(t_i - \overline{\zeta})$
