

**Autocallable Contingent Coupon Equity Linked
Securities
Linked to the VanEck Vectors[®] Junior Gold Miners
ETF
Due July 21, 2025
SEC Filing [Link to the Security](#)
Payments on the Securities Based on the Knock-in
Event**

-Valuation report by Yadvesh, Krish and Mayank

(I) Introduction

This report aims to value the Autocallable Contingent Coupon Equity Linked Securities Notes Linked to the VanEck Vectors[®] Junior Gold Miners ETF Due July 21, 2025, the payments of which are based on whether the Knock-in event happened or not. The note has discrete barriers as Contingent coupon payments on the [coupon dates](#) payable based on the index closing value of the underlying ETF on [observation dates](#). The Underwriter for this note is Citigroup Global Markets Inc. (“CGMI”). For detailed information about the note’s features and key dates, see the [Appendix](#).

Payoff at Maturity of the Note is:

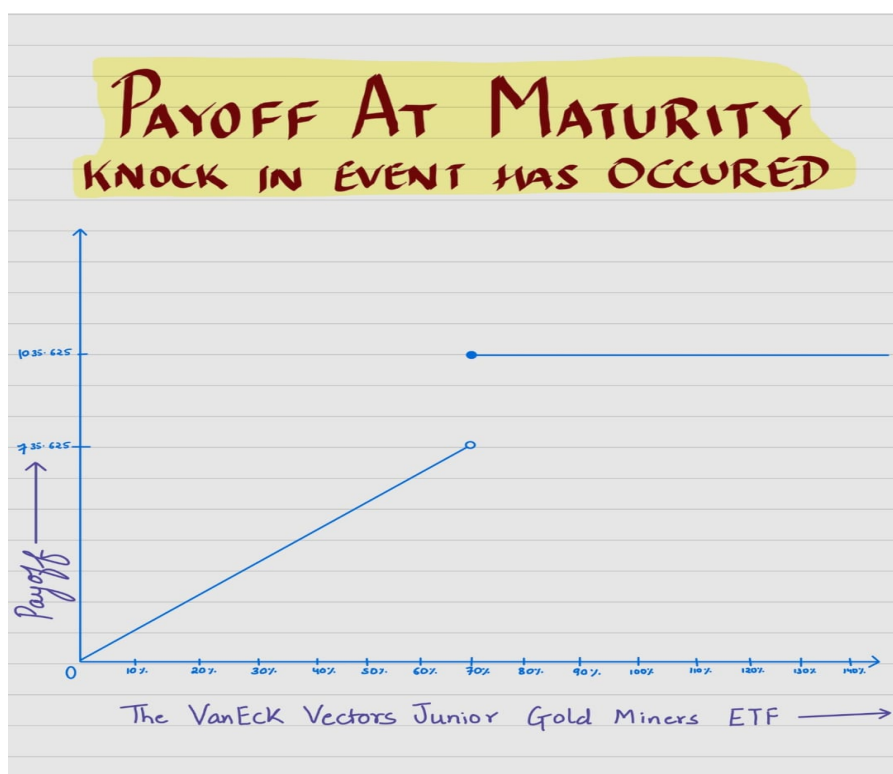


Figure 1: Payoff at Maturity if Knock-in Event has Occured

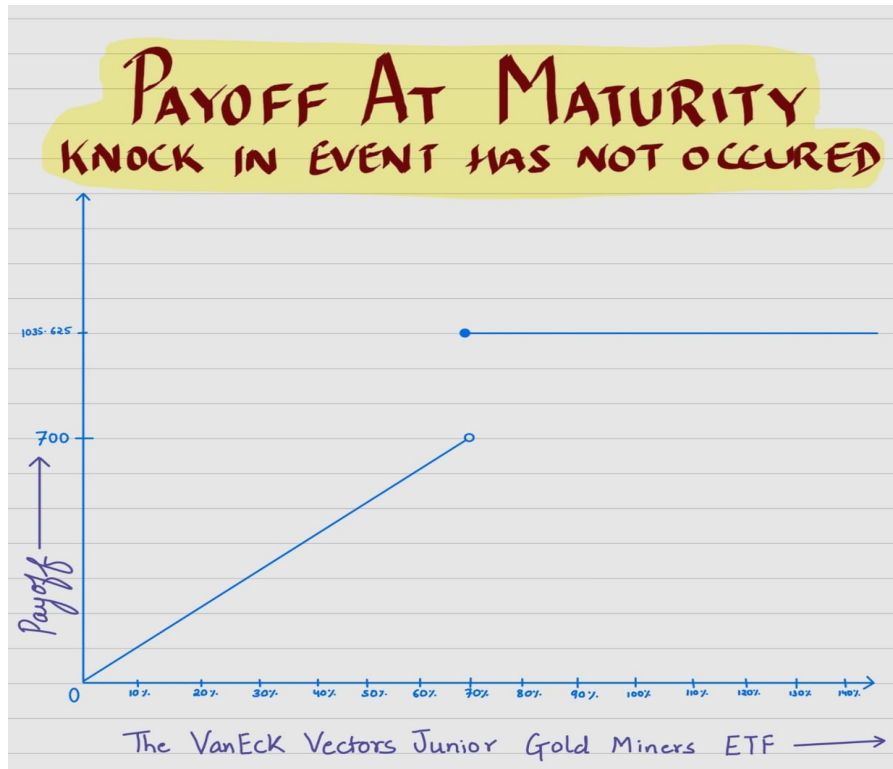


Figure 2: Payoff at Maturity if Knock-in Event has not Occured

Finite Differences Grid of the Note is:

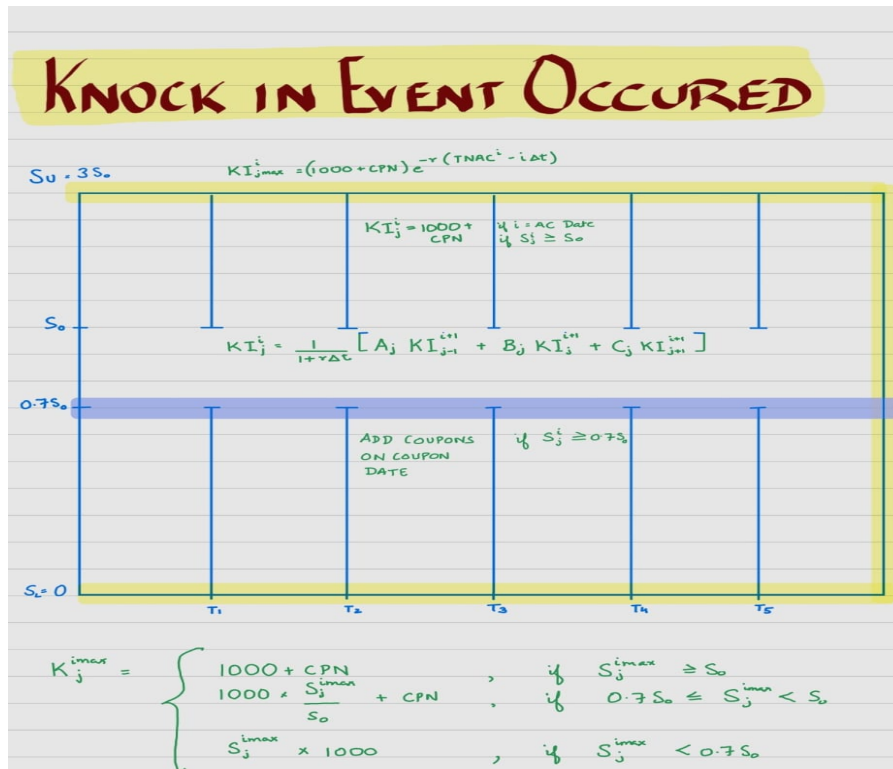


Figure 3: Finite Differences Grid if Knock-in Event has Occured

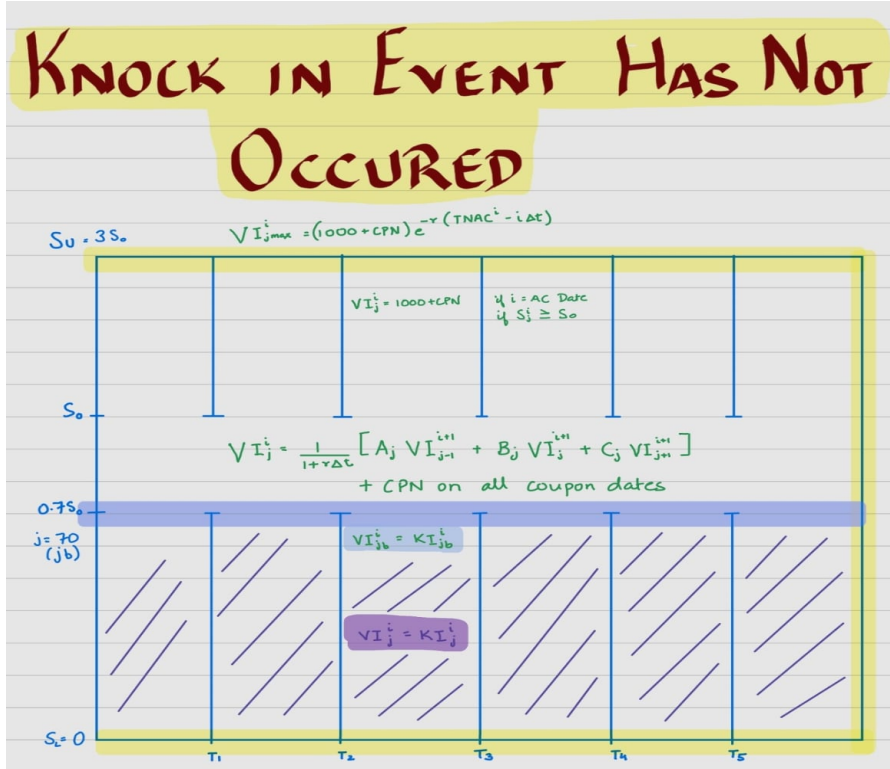


Figure 4: Finite Differences Grid if Knock-in Event has not Occured

The valuation method we chose to value this note was an **Explicit Finite Difference** method. The following steps were involved in this valuation:

1. **Calculation of Discounting (r_2) and Stochastic Process (r_1)** risk-free rates from the discount factors obtained from [OIS data](#).
2. **Building the finite difference scheme:**

- We use a finite difference grid with $j_{\max} = 300$ and i_{\max} determined from the stability condition with maximum implied volatility

$$\sigma_{\max} = 0.39547$$

(corresponding to 70% moneyness) so that the grid valuation remains stable across other volatility values:

$$i_{\max} = j_{\max}^2 \cdot \sigma_{\max}^2 \cdot T.$$

- The time step is $\Delta t = \frac{T}{i_{\max}}$ and the stock-price step is $\Delta S = \frac{S_U}{j_{\max}}$.
- **Terminal conditions at maturity** ($i = i_{\max}$) are defined in three regions:

$$\text{For } S \leq K : \quad V_{\text{terminal}} = 1000 \cdot \frac{S}{S_0} \cdot e^{-r_2(3/365)},$$

$$\text{For } K < S < S_0 : \quad V_{\text{terminal}} = \left(1000 \cdot \frac{S}{S_0} + \text{cpn} \right) \cdot e^{-r_2(3/365)},$$

$$\text{For } S \geq S_0 : \quad V_{\text{terminal}} = (1000 + \text{cpn}) \cdot e^{-r_2(3/365)}.$$

- **Finite difference coefficients** for the explicit scheme:

$$A_j = \left(\frac{1}{2} \sigma^2 j^2 + \frac{1}{2} (r_1 - D) j \right) \Delta t, \quad B_j = 1 - r_2 \Delta t - \sigma^2 j^2 \Delta t, \quad C_j = \left(\frac{1}{2} \sigma^2 j^2 - \frac{1}{2} (r_1 - D) j \right) \Delta t.$$

- **Backward induction for the knocked-in state:**

$$V_j^n = A_j V_{j+1}^{n+1} + B_j V_j^{n+1} + C_j V_{j-1}^{n+1}.$$

- **Backward induction for the not knocked-in state:**
 - Copy below-barrier values from the knocked-in state.
 - Recalculate above-barrier values using the explicit scheme.
- **Collect final result** at $t = 0$ for S_0 . Identify critical grid points:

$$j_{\text{crit}} = \frac{S_0}{\Delta S}, \quad j_{\text{critK}} = \frac{K}{\Delta S}.$$

- Repeat for different grid sizes to check convergence.

3. Performing sensitivity analyses by varying:

- Number of S -steps (mesh size), and
- Different implied volatilities at various moneyness levels

to find the volatility that best matches the note's **observed price** of \$977.75.

Note: We have used a continuous dividend yield in the *stochastic process* under the risk-neutral measure (GBM). A continuous dividend yield is included directly in the underlying asset's drift under the Black–Scholes framework.

Final Valuation: According to our methods, the estimated **fair value** of the note is \$964.714.

(II) Input Data Characteristics:

1. Risk-Free Rate (OIS):

- OIS rates are based on overnight lending rates which have very low credit risk and closely track central bank targets. They are typically very liquid, hence used as a proxy for risk-free rates.
- We linearly interpolate Discount Factors to find continuously compounded rates $r(0, T_1), r(0, T_2), r(0, T_3)$. The calculated values were:

$$r(0, T_1) = 5.4134\%, \quad r(0, T_2) = 4.19188\%, \quad r(0, T_3) = 4.18248\%.$$

- We also derive the forward rate $F(T_1, T_3)$ from T_1 to T_3 for discounting. And $r(0, T_2)$ is used in the stochastic process.
- Final values used:

$$r(0, T_2) = 4.191878\%, \quad F(T_1, T_3) = 4.175758\%.$$

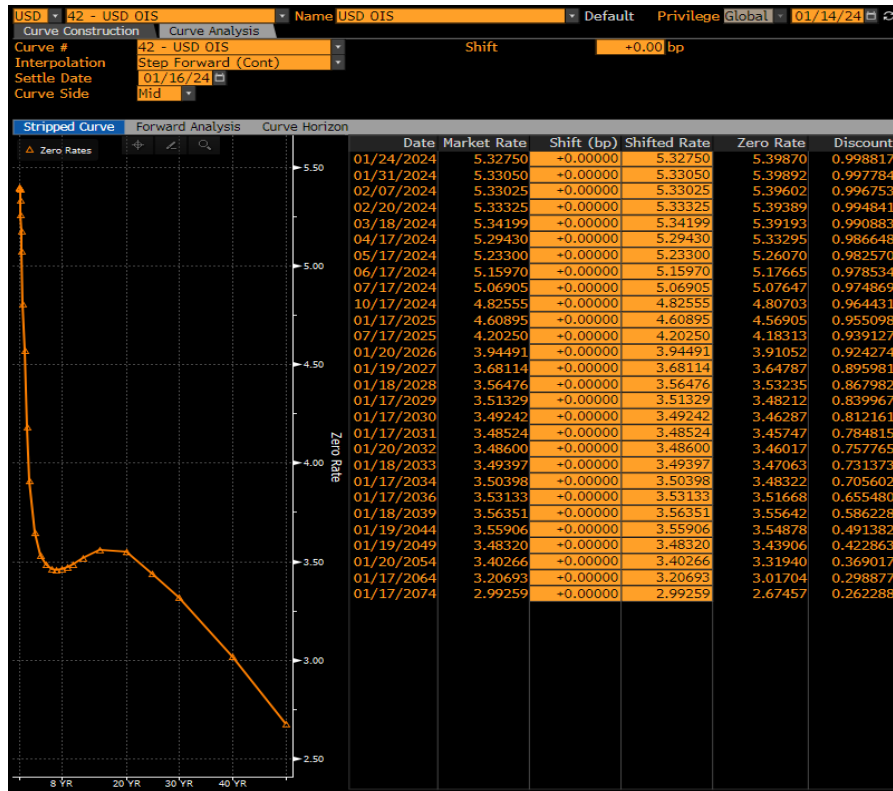


Figure 5: USD OIS at 01/16/2024

2. Implied Volatility Matrix:

- The Implied Volatility Matrix provides implied volatilities for different maturities and strikes (moneyness). For this note (maturity ≈ 1.5 years), we use the 18-month volatilities from 70% to 110% moneyness for the underlying GDXJ equity ETF.

Deal 124		Pricing		Scenario		Matrix		Volatility		Backtest																			
Underlying		GDGX US Equity		W/E 3R		Exercise		Direction		European		Trade		01/16/2024		to 16:30		Save Settings											
Und. Price		34.54 USD								Buy		Sell						Reset Settings											
Y-Axis X-Axis																													
Moneyness		Maturity																											
		70%		80%		90%		100%		110%		120%		130%		140%		150%		160%		170%		180%					
70.00 Volatility		42.564%		39.105%		38.734%		39.520%		39.516%		39.547%		39.522%		39.355%		39.147%		38.999%		38.879%		38.799%					
75.00 Volatility		40.096%		37.559%		37.475%		37.940%		38.005%		38.099%		38.164%		38.125%		37.891%		37.701%		37.591%		37.501%					
80.00 Volatility		37.889%		36.339%		36.479%		36.775%		36.822%		36.785%		36.772%		36.707%		36.679%		36.775%		36.775%		36.775%					
85.00 Volatility		36.192%		35.487%		35.747%		35.768%		35.819%		35.924%		36.136%		36.285%		36.258%		36.241%		36.241%		36.241%					
90.00 Volatility		34.991%		34.921%		35.308%		35.024%		35.081%		35.335%		35.578%		35.722%		35.827%		35.914%		35.914%		35.914%					
95.00 Volatility		34.568%		34.647%		34.882%		35.002%		35.164%		35.344%		35.519%		35.641%		35.740%		35.740%		35.740%		35.740%					
100.00 Volatility		34.358%		34.763%		34.961%		34.908%		35.049%		35.212%		35.321%		35.419%		35.545%		35.648%		35.648%		35.648%					
105.00 Volatility		34.776%		34.931%		35.337%		35.971%		35.838%		35.554%		35.328%		35.367%		35.509%		35.621%		35.621%		35.621%					
110.00 Volatility		35.275%		35.980%		35.225%		36.447%		36.323%		36.095%		35.762%		35.283%		35.410%		35.527%		35.527%		35.527%					

Figure 6: Implied Volatility Matrix for GDXJ at 01/16/2024

3. Continuous Dividend Yield:

- Including the dividend yield gives a more accurate representation of the underlying asset's price dynamics over time.
- For our project, we used a continuous dividend yield of $D = 1.807\%$ for GDXJ.

European Vanilla	
Parameters	Leg 1
Underlying	GDXJ US Equity
Und. Price	USD 44.55
Trade	01/16/2024 16:30
Settle	01/16/2024
Style	Vanilla European
Call/Put	Call
Direction / Position	Buy 1.00
Strike	% Money ATM 44.55
Expiry	03/18/2025 16:30
Time to Expiry	427 00:00
Model	BS - continuous
Vol	BVOL 36.881%
More Market Data	
Forward	Carry 35.7762
USRate	Semi 4.557%
Dvd Yld+Mgmt fee	1.807%
Discounted Div Flow	0.92
Borrow Cost	0.000%
Greeks	
Advanced Greeks	
Results	
Price (Total USD)	2.73
Price (Share)	2.7343
Price (%)	7.8888
Margin	Total 0.00

Figure 7: Dividend Yield for GDXJ at 01/16/2024

(III) Methodology

We value the note using an **Explicit Finite Difference** (EFD) method, which handles early-exercise and barrier conditions well. Below are the steps:

Assumptions

- **Risk-Neutral Valuation:** We assume a risk-neutral framework with drift $r - D$.
- **No Arbitrage:** The market is arbitrage-free, and the asset follows geometric Brownian motion.
- **Constant Parameters:** r_1, r_2, σ, D remain constant over the note's life.
- **Numerical Stability:** Step sizes satisfy the explicit scheme stability constraints.

1. Grid Setup and Initialization

- **Grid Construction:**

$$\Delta t = \frac{T}{i_{\max}}, \quad \Delta S = \frac{S_U}{j_{\max}}.$$

- **Critical Dates:**

$$\text{Observation dates } t_{\text{call}} = \frac{91, 182, 274, 366, 456, 547}{365},$$

$$\text{Payment dates } t_{\text{calls}} = \frac{94, 185, 279, 371, 461, 552}{365}.$$

- **Arrays:**

$V[\text{imax} + 1, \text{jmax} + 1]$ (knocked-in),

$VN[\text{imax} + 1, \text{jmax} + 1]$ (not knocked-in).

2. Finite Difference Coefficients

$$A_j = (0.5 \sigma^2 j^2 + 0.5 (r_1 - D) j) \Delta t, \quad B_j = 1 - r_2 \Delta t - \sigma^2 j^2 \Delta t, \quad C_j = (0.5 \sigma^2 j^2 - 0.5 (r_1 - D) j) \Delta t.$$

3. Terminal Conditions at Maturity ($i = i_{\max}$)

We define 3 regions:

- **Below Barrier** ($j \Delta S \leq K$):

$$V_{i,j} = 1000 \cdot \frac{j \Delta S}{S_0} \cdot \exp(-r_2 \cdot 3/365).$$

- **Between Barrier and Initial** ($K < j \Delta S < S_0$):

$$V_{i,j} = \left(1000 \cdot \frac{j \Delta S}{S_0} + \text{cpn} \right) \cdot \exp(-r_2 \cdot 3/365).$$

- **Above Initial** ($j \Delta S \geq S_0$):

$$V_{i,j} = (1000 + \text{cpn}) \cdot \exp(-r_2 \cdot 3/365).$$

4. Backward Induction Process

(a) Knocked-In State

- **From Maturity to First Call Date:**

– Lower boundary: $V[i, 0] = 0$.

– Interior points:

$$V_j^n = A_j V_{j+1}^{n+1} + B_j V_j^{n+1} + C_j V_{j-1}^{n+1}.$$

– Call date adjustments:

If $S > S_0 \implies$ full redemption + coupon.

If $K < S \leq S_0 \implies$ add contingent coupon.

- **Before First Call Date:**

– Same scheme.

– Additional coupon payments when $S > K$.

– Upper boundary includes sum of future contingent coupons.

(b) Not Knocked-In State

- **Below Barrier Region:**

$$VN[i, j] = V[i, j], \quad \text{for } j \leq j_n = \lfloor K/\Delta S \rfloor.$$

- **Above Barrier Region:**

$$VN[i_{\max}, j] = (1000 + \text{cpn}) e^{-r_2 (3/365)}.$$

Same finite difference scheme, with separate handling of call features and coupons.

5. Grid Convergence Analysis

- Vary grid sizes: $j_{\max} \in [j_{\min}, j_{\max}]$ with step j_{step} .
- Calculate critical points:

$$j_{\text{crit}} = \lfloor S_0 / \Delta S \rfloor, \quad j_{\text{critK}} = \lfloor K / \Delta S \rfloor + 1.$$

$$\lambda = \frac{(j_{\text{critK}} \cdot \Delta S) - K}{\Delta S}.$$

6. Final Value Calculation

The note value is

$$\text{Note Value} = VN[0, j_{\text{crit}}].$$

Output includes the number of steps, the note value, and λ adjustment.

(VI) Sensitivity Analysis and Possible Errors

Sensitivity Analysis

- **Grid Size Sensitivity with Varying Lambda (j_{\max}):** We analyze how the error profile and lambda values change with different grid sizes, ranging from $j_{\max} = 30$ to $j_{\max} = 300$ with a step size of 12. This helps understand the convergence properties of our finite difference scheme and its impact on pricing accuracy compared to the observed market price of \$977.75.

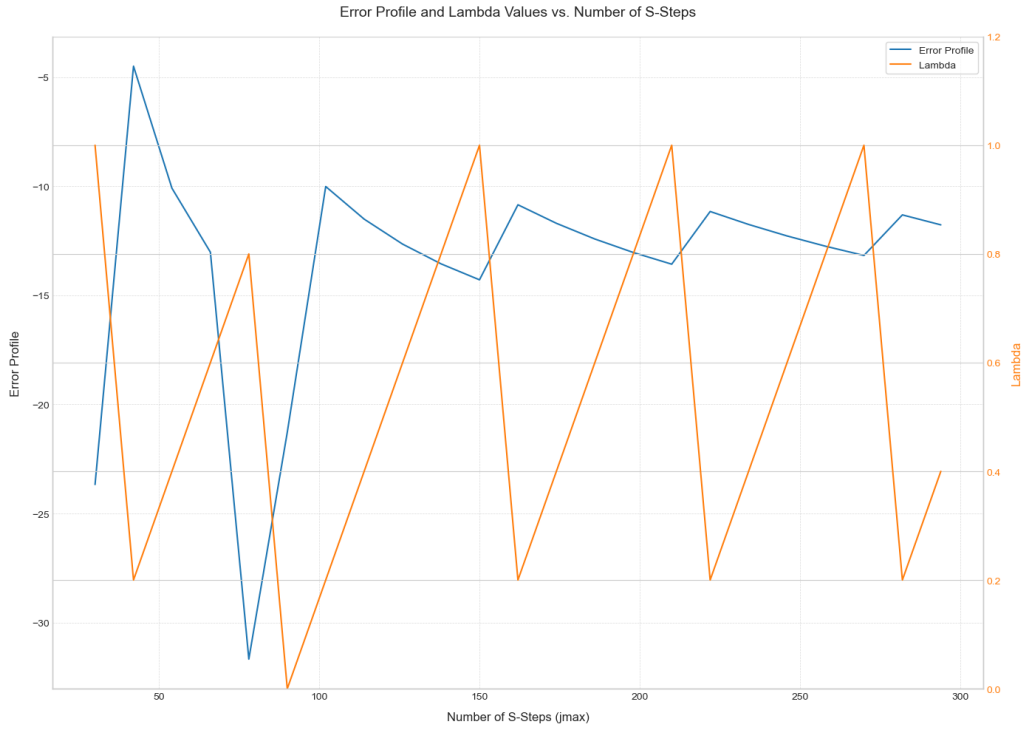


Figure 8: Error Profile and Lambda Values vs. Number of S-Steps (Varying λ)

The oscillating behavior in lambda values suggests grid sensitivity at certain critical points, potentially affecting the accuracy of our barrier option calculations.

- **Grid Size Sensitivity with Fixed Lambda:** To isolate the effect of grid size from lambda variations, we conduct a sensitivity analysis with fixed lambda values. Here, we vary j_{\max} from 10 to 300 with a step size of 30, maintaining:

$$i_{\max} = \text{ceil} \left(\frac{j_{\max}^2 \cdot \sigma_{\max}^2 \cdot T}{\text{total days}} \right) \cdot \text{total days}$$

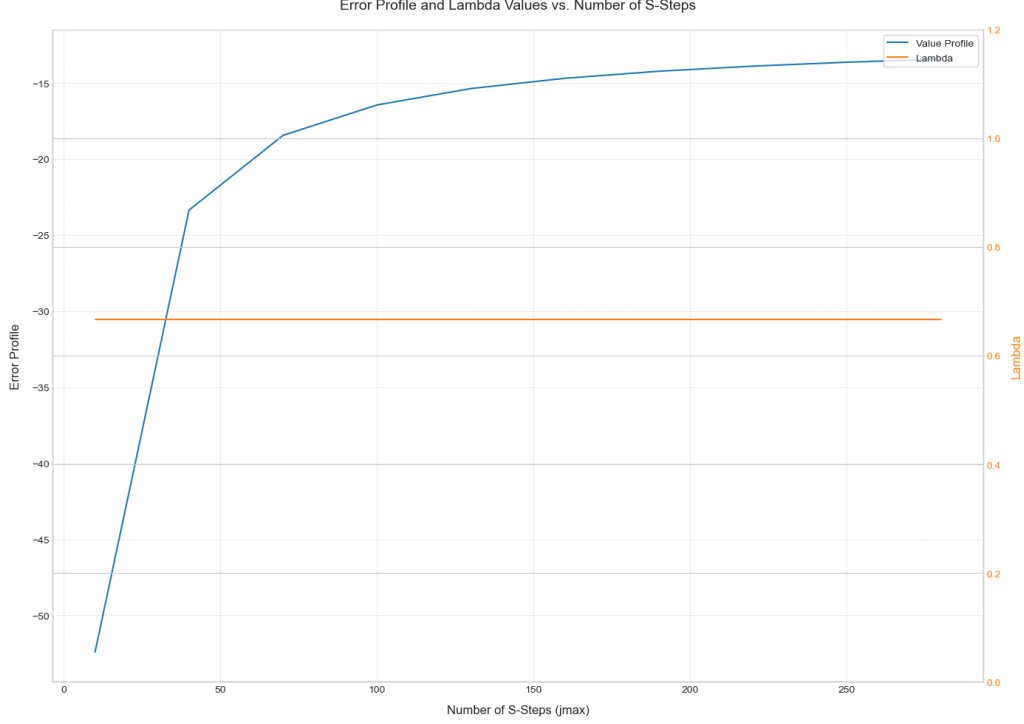


Figure 9: Error Profile and Lambda Values vs. Number of S-Steps (Fixed λ)

The smoother convergence profile demonstrates more stable numerical behavior when lambda is held constant.

- **Implied Volatility Sensitivity:** We examine how different implied volatilities affect the note value across various moneyness levels using a fixed grid size of $j_{\max} = 300$. The complete volatility surface used in our analysis is:

Moneyiness	Implied Volatility
70%	39.547%
75%	38.099%
80%	36.785%
85%	35.924%
90%	35.335%
95%	35.164%
100%	35.212%
105%	35.554%
110%	36.095%

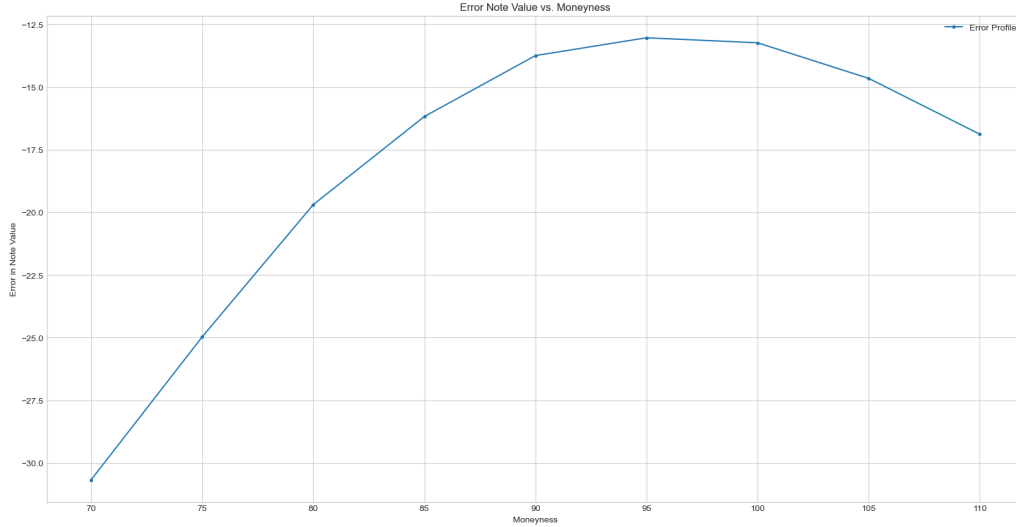


Figure 10: Error in Note Value vs. Moneyness

Key Findings

1. Grid Size Effects:

- Varying lambda analysis shows significant oscillations for $j_{\max} < 200$
- Error profile stabilizes beyond $j_{\max} = 200$
- Lambda fluctuations decrease with larger grid sizes

2. Fixed Lambda Analysis:

- Shows more consistent convergence behavior
- Error profile demonstrates clear asymptotic trend towards market price
- Computational efficiency improves with fixed lambda approach

3. Volatility Surface Impact:

- Highest sensitivity in 80-90% moneyness range
- Minimum error occurs near 95-100% moneyness
- Error increases for deep in-the-money (more than 105%) options
- Volatility smile effect visible in U-shaped volatility pattern

VII) Conclusion

The Explicit Finite Difference (EFD) method used in our analysis can exhibit instability under certain parameter configurations. This is particularly relevant for complex instruments like our note, which features discrete coupon barriers, knock-in events, and discontinuous payoffs at maturity. We discuss below the key considerations and measures taken to ensure accuracy in our valuation model.

Stability Analysis

In our EFD implementation, the coefficients $A[j]$, $B[j]$, and $C[j]$ must satisfy probability constraints $0 \leq A[j], B[j], C[j] \leq 1$. While $A[j]$ and $C[j]$ naturally tend to remain positive due to their formulation, $B[j] = 1 - r\Delta t - \sigma^2 j^2 \Delta t$ requires careful consideration. This leads to the stability condition:

$$i_{\max} > (r + \sigma^2 j^2)T$$

Based on this, we determined our grid parameters:

- $j_{\max} = 300$ for spatial discretization
- $i_{\max} = j_{\max}^2 \cdot \sigma_{\max}^2 \cdot T$ for temporal discretization
- Maximum volatility $\sigma_{\max} = 0.39547$ (at 70% moneyness)

This configuration ensures stability while maintaining computational efficiency.

Non-linearity Error Management

Several features of our note contribute to non-linearity errors:

1. **Knock-in Feature:** The continuous knock-in barrier at 70% of initial value introduces non-linearity at the barrier level. We mitigate this by ensuring the barrier level aligns with grid points through appropriate choice of ΔS .
2. **Contingent Coupon Payments:** The discrete nature of coupon payments creates discontinuities at observation dates. Our grid construction with:

$$\lambda = \frac{(j_{\text{critK}} \cdot \Delta S) - K}{\Delta S}$$

helps minimize these errors.

3. **Maturity Payoff:** The discontinuous payoff structure at maturity (different regimes based on knock-in status and final level) is managed through careful grid alignment at critical levels (S_0 and K).

Convergence Analysis

Our sensitivity analyses demonstrate convergence properties:

- **Grid Size Convergence:** The error profile stabilizes beyond $j_{\max} = 200$, with diminishing improvements thereafter.
- **Lambda Consistency:** Fixed lambda analysis shows smooth convergence behavior, suggesting proper handling of barrier and payoff discontinuities.
- **Volatility Surface:** Error minimization near market moneyness (95-100%) validates our model's accuracy in the relevant pricing region.

Our model achieves a fair value estimate of \$964.714 compared to the observed market price of \$977.75, representing a pricing error of approximately 1.33%. This accuracy level, combined with our comprehensive stability and convergence analysis, suggests that our implementation effectively captures the note's complex features while maintaining computational efficiency.

Appendix

1) Below are the Key Dates of the Note:

Key Dates	
Pricing Date (T_0)	January 16, 2024
Original Issue Date (T_1)	January 19, 2024
Valuation Dates	April 16, 2024, July 16, 2024, October 16, 2024, January 16, 2025, April 16, 2025, and July 16, 2025
Final Valuation Date (T_2)	July 16, 2025
Maturity Date (T_3)	July 21, 2025

2) Below are the Note Offerings:

Note Offering				
Underlying Asset	Bloomberg Ticker	Initial Level	Coupon Barrier	Knock-in value
VanEck Vectors® Junior Gold Miners ETF	GDXJ	34.66	24.262 (70.00% of initial)	24.262 (70.00% of initial)
Contingent Coupon Rate: 14.25% per annum (equivalent to 3.5625% on Coupon Payment Dates)				
Underlying Return: $\frac{\text{Final underlying value} - \text{Initial underlying value}}{\text{Initial underlying value}}$				
CUSIP/ISIN: 17291TYM4 / US17291TYM43				

3) Below are the Observation Dates, Autocall Dates and Coupon Payment Dates:

Observation Dates, Autocall Dates and Coupon Payment Dates		
Observation Dates	Autocall Dates	Coupon Payment Dates
April 16, 2024	April 16, 2024	April 19, 2024
July 16, 2024	July 16, 2024	July 19, 2024
October 16, 2024	October 16, 2024	October 21, 2024
January 16, 2025	January 16, 2025	January 21, 2025
April 16, 2025	April 16, 2025	April 21, 2025
July 16, 2025 (final valuation date)	-	July 21, 2025 (maturity date)

Key Features of the Note:

1. Contingent Coupon Features:

- Rate: 14.25% per annum (3.5625% per quarter)
- Payable if: Closing value of underlying \geq coupon barrier value (70% of initial)
- Payment Dates: Third business day after each valuation date
- Initial Underlying Value: \$34.66

- Coupon Barrier Value: \$24.262 (70.00% of initial)

2. Autocall Feature:

- Trigger: If closing value \geq initial value (\$34.66) on any potential autocall date
- Monitored on quarterly valuation dates

3. Maturity Payoff Structure:

- **Scenario 1:** If final value \geq initial value
 - Return of principal
- **Scenario 2:** If final value less than initial value and no knock-in event
 - Return of principal
- **Scenario 3:** If final value less than initial value and knock-in event occurred
 - Principal reduced by underlying return
 - Underlying Return = $\frac{\text{Final Value} - \text{Initial Value}}{\text{Initial Value}}$

4. Knock-in Event Specifications:

- Knock-in Value: \$24.262 (70.00% of initial value)
- Observed continuously throughout the note's life
- Trigger: Occurs if underlying closes below knock-in value on any trading day

5. Market Information:

- CUSIP/ISIN: 17291TYM4 / US17291TYM43
- Listing: Not listed on any securities exchange
- Underwriter: Citigroup Global Markets Inc. (CGMI)
- Issue Price: \$1,000.00
- Estimated Value: \$971.20 per security
- Underwriting Fee: \$22.25
- Proceeds to Issuer: \$977.75

Note:

- *All payments are subject to the credit risk of the issuer and guarantor*
- *No participation in any appreciation of the underlying*
- *No dividend payments from the underlying are passed through*
- *Limited or no liquidity due to no exchange listing*
- *Early redemption limits potential returns in favorable market conditions*