Propositional Logic – Part 4 – Valid Arguments and Inference Rules

Slide 1

In part 4 on propositional logic, we will discuss how to determine whether an argument is a valid one and then discuss a variety of important inference rules, each of which is a valid argument.

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We begin by explaining what is meant by an argument.

We define an argument as a series of premises or suppositions, followed by a single conclusion. All of the premises and the conclusion are propositions.

Considering an example will help clarify this definition. What this argument states is that given that the proposition p or q or r and the proposition not q are both true, we can conclude or argue that the proposition p or r is true.

The two propositions above the line, the quantity p or q, or r is one premise and not q is the other. The conclusion appears below the line, which in this case is p or r. The triangular three dot symbol is read as *therefore*.

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Next we discuss the first of two techniques for determining whether an argument is valid. We use the argument from the previous slide.

This technique requires that we identify what are called the critical rows of the truth table. They are the rows in which all the premises are true. There are three such rows in the above table, which are highlighted in blue. To be a valid argument, the conclusion must be true in each of these rows. Examining those rows reveals that requirement has been met, so we can conclude that this argument is a valid one.

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We now consider one first important inference rule.

Modus ponens, which is Latin for method of affirming, is undoubtedly the most commonly used inference rule. It states that given that p implies q is true and p is true, we can conclude q.

The above truth table confirms its validity using the technique that we just discussed. There is only one critical row in that truth table and in that row the conclusion is true as required.

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Next we examine another equivalent technique for determining whether an argument is valid.

In the above truth table we have written modus ponens as a compound proposition, which is done by inserting *and* between all the premises and an implication between the conjunction of all the premises and the conclusion.

Notice that the final column of this truth table is a tautology. Demonstrating that the compound proposition formed as described, is a tautology, is the other technique for validating an argument.

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The next inference rule we consider is modus tollens.

It is Latin for method of denying. It is commonly used but not nearly as often as modus ponens. It states that given that p implies q is true and not q is true, we can conclude that not p is true.

Applying the first technique, we notice that the above truth table has one critical row and in that row the conclusion is true demonstrating the validity of this argument.

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Let us illustrate the process of drawing inferences.

First we use modus ponens to illustrate this process.

Our first premise is the implication that if John studies hard, he will pass this course.

Our second premise is that John will study hard.

By modus ponens, we can conclude that John will pass this course.

Next we illustrate the same process with modus tollens.

Our first premise is the same as before.

But this time our second premise is that John did not pass the course.

Modus tollens allows us to conclude that John must have not studied hard.

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There are many other inference rules. We will consider a few more that are commonly used beginning with ones that involve *and* and *or*.

There are two rules referred to as disjunctive addition. Recall that the term *disjunctive* means that an *or* is involved. The two rules are similar. In the first case it states that if we know p is true, we also can conclude that p or q is true. In the second, if q is true then so is p or q.

The next two rules are referred to as conjunctive simplification. Recall the adjective *conjunctive* means that an *and* is involved. As before the two rules are very much alike. Both contain the premise that states that we are given that p and q is true. The first states that we can therefore conclude p is true and the second that q is true.

All four of these inference rules follow directly from the truth tables for *and* and *or*.

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Finally we consider two more inference rules that are used fairly often.

The first is called hypothetical syllogism. It states that if we know that p implies q and q implies r, we can conclude that p implies r. Algebraically, we could say that this rule confirms that implication is transitive.

The last rule is contradiction. We read that rule as follows: given that not p leads us to a contradiction, we can conclude that p must be true.

The contradiction inference rule is an important one because it is the rule that underlies a very important proof technique, which is proof by contradiction.