Propositional Logic – Part 2 – Logical Operators

Slide 1

In part 2 on propositional logic, we will provide the truth tables for each of the logical operators and describe two techniques for constructing truth tables for compound propositions.

Slide 2

We begin with the three elementary operators *and*, *or* and *not*.

Because both *and* and *or* are binary operators, two elementary propositions p and q are required. That means we need four rows in our truth table. In general if there are n elementary propositions, the truth table will required 2^n rows.

We begin with the column of the table for *and*. This table should be easy to remember because the result is only true when both are true.

Next we consider the column for *or*. Recall that in part 1 we mentioned that in propositional logic *or* is assumed to be the inclusive or. So again this one is also easy to remember because the result is only false when both are false. By contrast an exclusive or is true when either is true but not both.

The final column is *not*. Because not is a unary operator we have only put entries in two or the four rows. not reverses logical values turning true into false and false into true.

Slide 3

There are two commonly used approaches for constructing truth tables for compound propositions, so we now consider the first of these two techniques.

On the last slide we described the exclusive or in English, now we see how it can be written as a compound proposition. The quantity p or q and not the quantity p and q.

With this first technique we begin by placing values under each of the elementary propositions. Recall that with two elementary proposition the truth table requires four rows.

The remaining columns of the table are constructed by filling in the column under each operator. The process is done from inside out and from left to right.

We start with the left-most quantity p or q

We construct the column for p or q under the symbol for *or*.

We proceed to the next inner-most quantity p and q.

We fill in the column for p and q under the symbol for *and*.

We apply the not to the column under the *and* and fill in the column under the *not*.

Finally we apply the *and* operation to the column under the *or* and the one under the *not* and fill in the column under the left-most *and*. Because this operator is the last one evaluated, this column represents the final result for the whole compound proposition.

Slide 4

Next we illustrate an alternate technique for constructing truth tables to evaluate compound propositions.

Notice first that this propositions has three elementary propositions, p, q and r, which means the truth table must have 2^3 or 8 rows. With this technique we list the values of the elementary propositions only once in the left-most columns. Then as before we evaluate the proposition from inside out and from left to right. This time, however, we list the result of the subpropositions in separate columns in the order that they are evaluated.

We begin with the left-most inner proposition, which is p and r. We place the result in the next column.

Then we evaluate q and r and fill the column under that subproposition with its logical values.

Finally we evaluate the whole compound proposition by oring together the previous two columns. With this technique the final result is always in the last column. Either approach is fine and will produce the same final result.

Slide 5

Now we examine the truth tables for the remaining two logical connectives, the conditional and biconditional.

First we consider the conditional. Notice that the result is only false when the supposition p is true and the conclusion q is false. Of all the truth tables, this one may seem the least intuitive. To understand why the result is true in the last two rows, one must realize that when one begins with a false supposition, anything can be concluded. Whether we accept that as reasonable or not, realize that this table represents a definition, and as such, we have the freedom to define the meaning of this symbol.

The other connective is the biconditional. From the truth table we see the result is true whenever the both p and q are the same and false when they are different. That suggests that this connective is defining logical equivalence.

Slide 6

Finally we consider a compound proposition that defines the biconditional.

Notice that this compound proposition is the conjunction, the result of forming an *and* of conditionals in both directions.

The final result that appears under the *and* is the same as the definition of the biconditional. Recognizing that the biconditional is true when the conditionals are true in both directions offers some reassurance that the truth table for the conditional was a sensible one.