

## HW 2: Parallel Merge Sort Using Threads

You are provided with a program `sort_list.c` that sorts a list of integers in ascending order using an iterative variant of the merge sort algorithm. A recursive approach to merge sort consists of splitting the list into two sublists, sorting the two sublists recursively, and then merging them. The merged list is constructed one element at a time by comparing pairs of elements – one from each sublist – and placing the smaller of the two in the merged list. The pointer that keeps track of the sublist elements to be compared advances to the next when its element is placed in the merged list.

The merging process is inherently serial. To develop a parallel merge sort, it is worthwhile to consider a bottom-up approach which results in an iterative algorithm. Starting from an unsorted list, pairs of adjacent elements are “merged” into sorted sublists of size 2. This is followed by merging adjacent lists into a larger list that is twice the size. The algorithm iterates through  $k = \log_2(n)$  steps, or levels, where each level consists of merging adjacent sublists into a list twice the size. The last level merges two halves of the original list into a fully sorted list, and is equivalent to the first step in the recursive version described earlier.

To develop a parallel merge sort with  $p = 2^q$  threads, we begin at level  $(k - q)$  of the iterative algorithm described in the preceding paragraph. At this level, we have  $p$  sorted sublists, each consisting of  $n/p$  elements (for simplicity assume both  $n$  and  $p$  are powers of 2). Let us denote the sublist for thread  $i$  by  $L_i$ . The remaining  $q$  steps of the algorithm result in merging pairs of sublists for the last  $q$  levels as described in the iterative algorithm.

In order to employ all the threads in the merger process, thread  $i$  is given the responsibility of determining the location of every element of its own sublist  $L_i$  in the merged list, and placing it directly in that location. For an element  $v$ , it is sufficient to figure out how many elements in the paired list  $L_{i-1}$  (or  $L_{i+1}$ ) are smaller than  $v$ , which can be determined by either a linear search or binary search for  $v$  in of  $L_{i-1}$ . Note that at higher levels where the sublists are larger than  $L_i$ , thread  $i$  remains responsible for  $n/p$  elements originally assigned to it; however, now it has to consider larger sublists when computing the location of an element  $v$ . This is a bit involved but should be evident when you examine the code provided to you. See figure at the end for additional clarification.

While the code is serial, it is written in a way that identifies the work to be done by each thread, which should allow straightforward parallelization using threads. The only difference from the iterative algorithm is that sublists  $L_i$  at level  $(k - q)$  are sorted by quick-sort instead of merge-sort.

To compile and execute the code, use the commands:

```
module load intel
icx -o sort_list.exe sort_list.c
./sort_list.exe <k> <q>
```

where  $<k>$  and  $<q>$  are integer arguments that specify the number of elements in the list  $n = 2^k$ , and the number of threads  $p = 2^q$ . The output of a sample run is shown below.

```
./sort_list.exe 20 4
```

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```
List Size = 1048576, Threads = 16, error = 0, time (sec) = 0.1414,  
qsort_time = 0.1260
```

Note that in its present form, the code is not multi-threaded. Thus `Threads` is merely reporting the value 2<sup>4</sup> for the input `q`.

1. (70 points) Revise the code to implement a thread-based parallel merge sort. The code should compile successfully and should report `error=0` for the following instances:

```
./sort_list.exe 4 1  
./sort_list.exe 4 2  
./sort_list.exe 4 3  
./sort_list.exe 20 4  
./sort_list.exe 24 8
```

2. (20 points) Plot speedup and efficiency for all combinations of `k` and `q` chosen from the following sets:  $k = 12, 20, 28$ ;  $q = 0, 1, 2, 4, 6, 8, 10$ . Comment on how the results of your experiments align with or diverge from your understanding of the expected behavior of the parallelized code.
3. (10 points) Your code should demonstrate speedup when sorting lists of appropriate sizes. Determine two values of `k` for which your code shows speedup as `q` is varied. Present the timing results for your code along with speedup and efficiency obtained to convince the reader that you have a well-designed parallel merge sort. You may use results from experiments in previous problems or identify new values `k` and `q` to illustrate how well your code has been parallelized.

### Submission

Upload two files to Canvas:

1. Submit the file `sort_list.c` that implements your thread-based parallel merge sort for Problem 1.
2. Submit a single PDF document with your responses for Problems 2 and 3.

### Helpful Information

1. You may use Grace for this assignment.
2. Load the compiler module prior to compiling your program. Use:  
`module load intel`
3. Compile C programs using `icx`. Link the `pthreads` library when using `pthreads`. For example, to compile `code.c` to create the executable `code.exe`, use  
`icx -o code.exe code.c -lpthread`
4. The run time of a code should be measured when it is executed in dedicated mode. Create a batch file and submit your code for execution via the batch system on the system.

