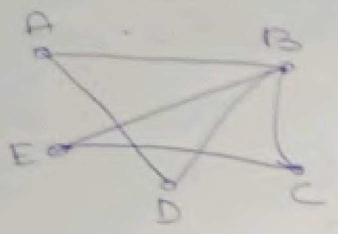
08/07/25
Monday

GRAPH THEORY

1) Represent the following scenario using a graph :
let A, B, C, D and E be people. A knows B and D, B knows C and D, E, C knows E.

=>



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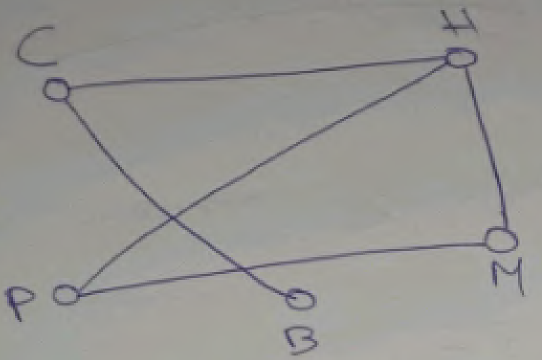
2) A country has a railway system between the following cities. chennai, banglore, hyderabad, mumbai and Pune. There are 3 trains per day from chennai to banglore, ~~To~~ 2 trains from Banglore to chennai. 1 train from chennai to hyderabad ~~and 2~~ 2 train from hyderabad to mumbai 1 train from mumbai to Pune and 2 trains from Pune to hyderabad.

i) model this network using a graph where an edge exists between cities that have atleast one train service. in either direction.

⇒



3 : C ⇒ B

2 : B ⇒ C

1 : C ⇒ H

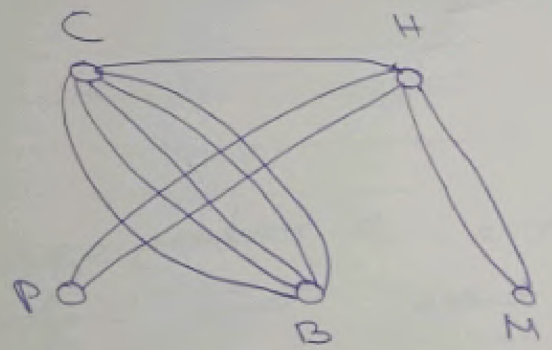
2 : H ⇒ M

1 : M ⇒ P

2 : P ⇒ H

ii) Model this network using a graph where each edge represents one train operating between cities (in either direction).

⇒



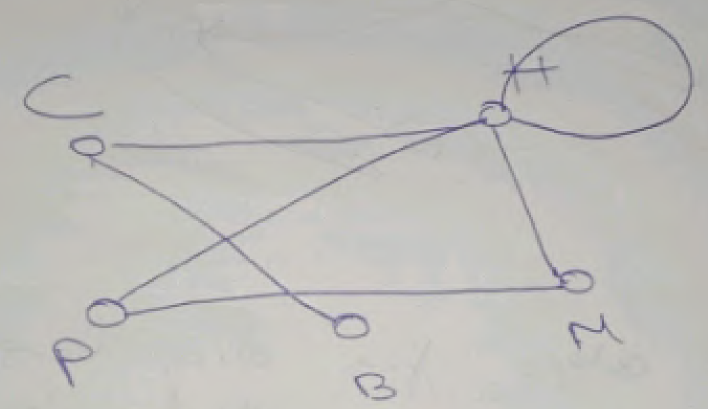
> ~~multiple~~ when there are more than one edge connecting two vertices the edges are called multiple edges / Parallel edges.

> graph that contain multiple edges are called <u>multi graphs</u>.

> graph without multiple edges or loops are called <u>simple graph</u>.

iii) model this network using a graph where an edge exists between cities that have atleast one train service in either direction plus a loop for a circular tourist train that starts and end at hyderabad.

⇒



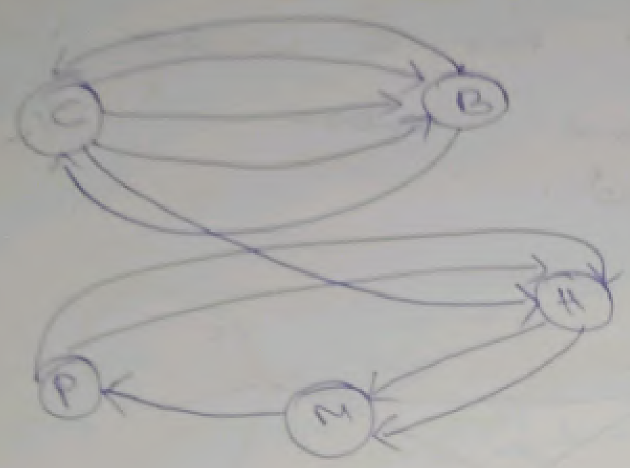
> The edges that start and ends at the same vertex is known as a loop.

> Any graph that contains loops are called Pseudo graph.

> Pseudo graph may or maynot contain multiple edges.

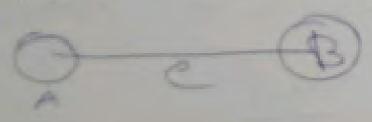
iv) model this network using a graph where edges indicate the direction of the train.

⇒



> If the edges has directions then the graphs are called directed graphs. or they are called undirected graphs.

Important TERMS



end vertices

→ edge e connects two vertices A and B then A and B are called end vertices of e. In such cases we also say e is incident on A and B. Since there is an edge connecting A and B, A and B will be called <u>adjacent</u> vertices.

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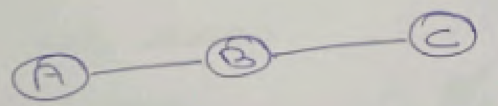
Informal Introduction to Graphs

A graph is a pictorial representation of relationships b/w objects. It helps us model connections, such as:

- Friendships in social networks
- routes between cities
- Flight connections
- network topologies in computers.

eg: consider 3 cities A, B and C. There are roads b/w A-B and B-C. we can represent this as



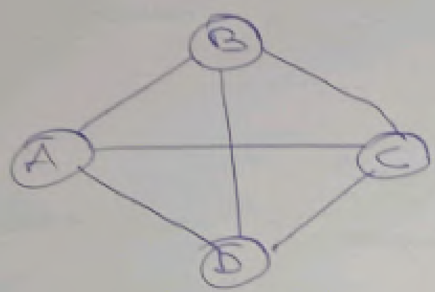
The Konigsberg Bridge Problem.

In the 18th century, the city of Konigsberg had seven bridges connecting different parts of land separated by a river. People wondered:

"Is it possible to take a walk that crosses each bridge exactly once and return to the starting point?".

Euler's insight: Represent the land areas as vertices and the bridges as edges, forming a graph. He showed that such a walk was not possible, laying the foundation of graph theory.

Graph Representation



Formal Definitions

Graph:

A graph G is an ordered Pair $G = (V, E)$ where V is a non-empty set of vertices and E is a set of edges (unordered or ordered Pairs from V).

Finite and Infinite Graph

A finite graph has a finite number of vertices and edges. An infinite graph has an infinite number of vertices and edges.

Simple graph

An undirected graph with no loops and no multiple edges b/w the same pair of vertices.

Multigraph

An undirected graph that may have multiple edges between the same pair of vertices but no loops.

Pseudograph

A graph that may contain both multiple edges and loops.

Undirected Graph

A graph where the edges are unordered pair of vertices (ie, edge $\{u,v\} = \{v,u\}$)

Directed graph

A graph where ~~the~~ edges are ordered pairs (arcs), ie (u,v) is directed from u to v.

Mixed graph

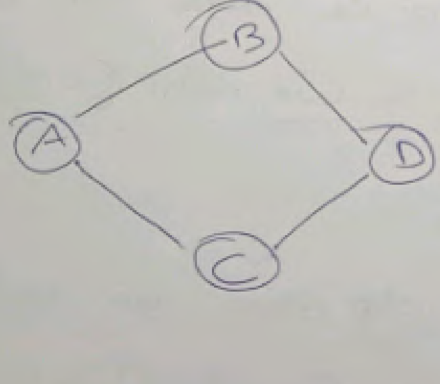
A graph that contains both ~~direction~~ directed and undirected edges.

example

1) Draw a graph that models the following situation: Let A, B, C, D be people. A is friends with B and C. B is also friends with D. C is friends with D. Draw an undirected graph representing this.

Vertices : A, B, C, D.

Edges : {A, B}, {A, C}, {B, D}, {C, D}



BASIC TERMS IN GRAPH THEORY

> Loop : A loop is an edge that connects a vertex to itself.

> Multiple edges : Multiple edges are two or more edges that connect the same pair of vertices.

> Endpoints of an edge : The endpoints of an edge are the two vertices that an edge connects.

> Adjacency : Two vertices u and v in an undirected graph G are said to be adjacent if there is an edge $e = \{u,v\}$ in G that connects u and v.

> Incidence : An edge e is said to be incident with a vertex v if v is one of the endpoints of e.

> Degree : The degree of a vertex v, denoted deg(v), is the number of edges incident with v in an undirected graph.

> Isolated vertex : A vertex with degree zero (not connected to any other vertex) is called an isolated vertex.

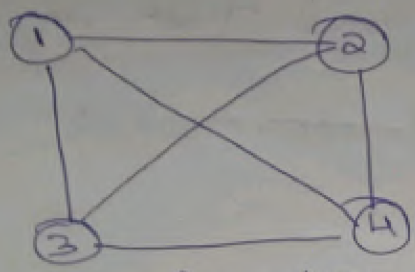
> Pendant vertex. A vertex with degree one is called a Pendant vertex / Leaf.

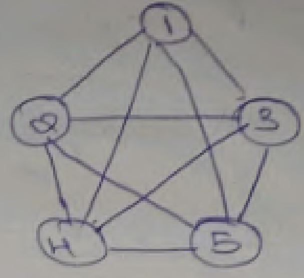
> Initial and Terminal vertex (Directed graph) :

In a directed graph, for an edge (u,v), u is the initial vertex and v is the terminal vertex

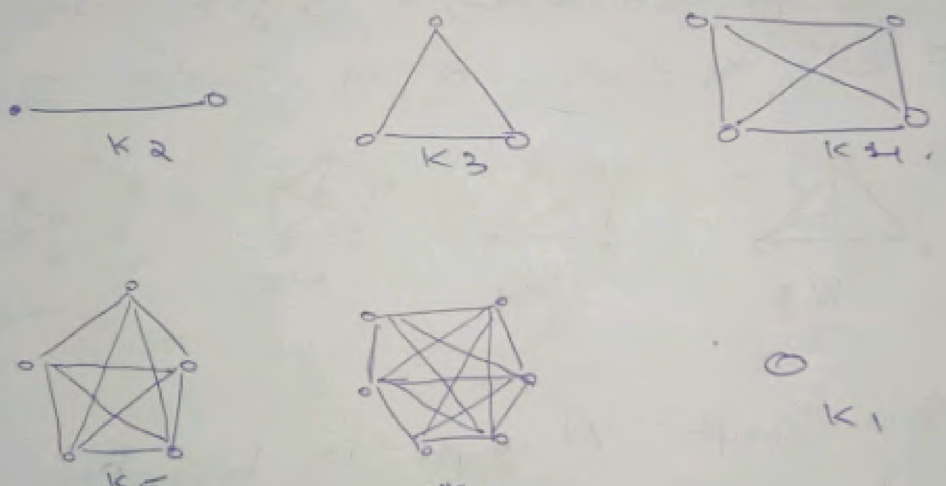
> In-degree and out-degree : In a directed graph, the in-degree of a vertex is the number of edges coming into the vertex, and the out-degree is the number of edges going out of the vertex.

> Regular Graph : A regular graph is a simple graph in which every vertex has the same degree. If every vertex has degree r, then the graph is called an r-regular graph.

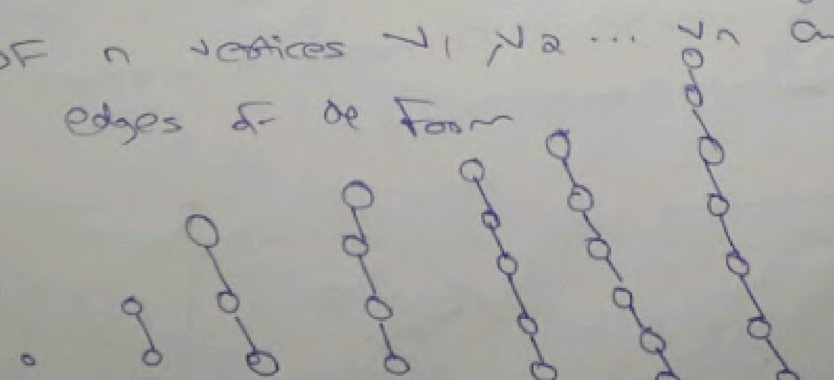




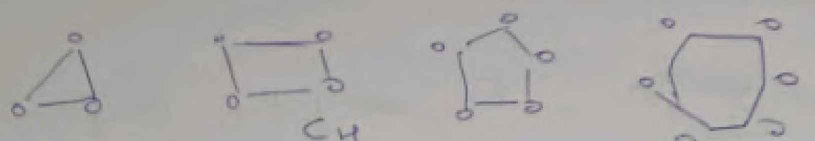
> Complete graph : A complete graph K_n, is a simple undirected graph in which every Pair of distinct vertices is connected by a unique edge.



> Path (P_n) A Path P_n is a simple graph consisting of n vertices $v_1, v_2 \dots v_n$ and $n-1$ edges of the Form

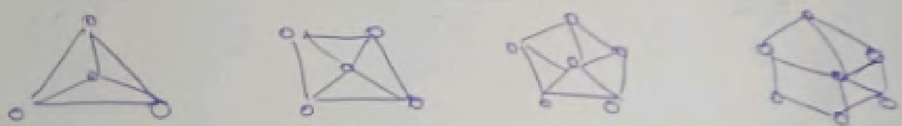


Cycle (C_n): A cycle is a closed path in which all edges and all intermediate vertices are distinct.



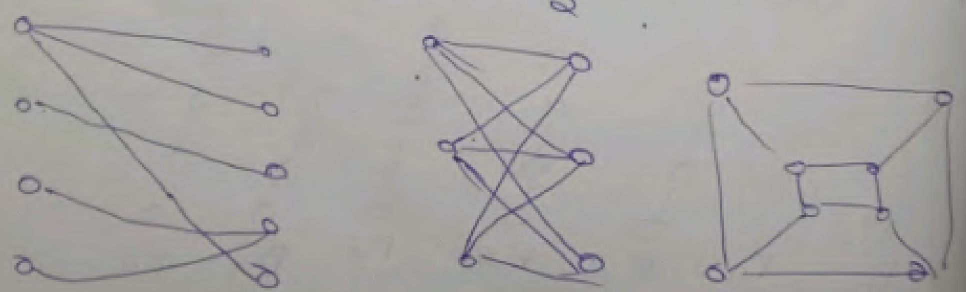
C_3 C_4 C_5 C_6

Wheel graph (W_n): A wheel graph W_n is formed by connecting a single vertex to all vertices of a cycle C_n.

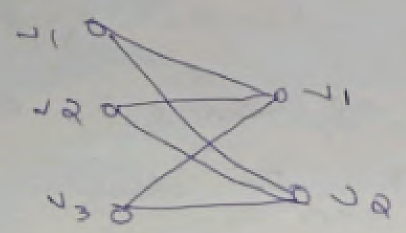


W_3 W_4 W_5 W_6

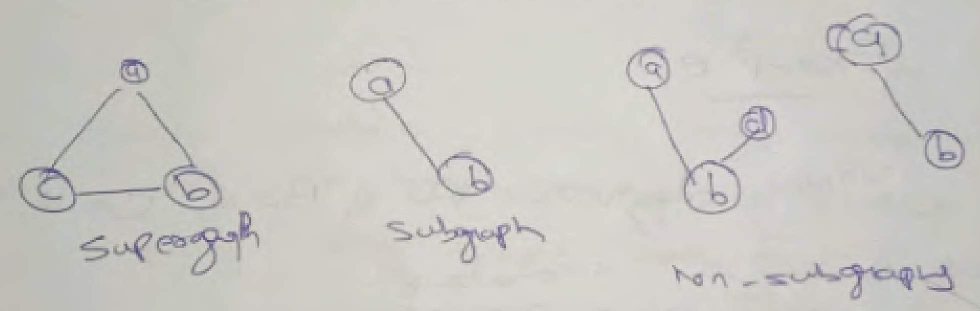
Bipartite Graph: A graph is bipartite if its vertex set can be divided into two disjoint sets V_1 and V_2 such that every edge connects a vertex in V_1 ~~to~~ to one in V_2.



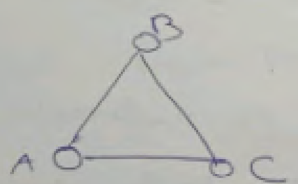
Complete Bipartite Graph : A complete bipartite graph $K_{m,n}$ is a bipartite graph where every vertex in V_1 is connected to every vertex in V_2.



Subgraph : A subgraph of a ~~~~ graph $G = (V, E)$ is a graph $H = (V', E')$ such that $V' \subseteq V$ and $E' \subseteq E$.



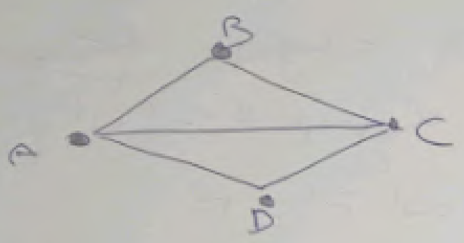
Is the following graph bipartite?



Answer : No it is not bipartite. A cycle of length 3 (odd length) cannot be bipartite - because we can't split the vertices into two distinct sets without connecting vertices within the same set.

HANDSHAKING LEMMA

Statement : In any undirected graph, the sum of the degrees of all vertices is equal to twice the number of edges.



Example :

Vertex degrees : $A = 2, B = 2, C = 2, D = 2$

Total $= 8$.

number of edges $= 4$.

Hence, $2|E| = \sum \deg(v) = 8$.

Proof : each edge in an undirected graph contributes exactly 2 to the total degree count (one for each endpoint), so the sum of degrees equals $2|E|$.

Theorem : Even Number of Odd Degree Vertices.

Statement : In any undirected graph the number of vertices with odd degree is even.

Proof : From the handshaking lemma, we know the total sum of all vertex degrees is even. Suppose there were an odd number of vertices with odd degrees, their sum would be odd, which contradicts the total being even. Hence, the number of odd-degree vertices must be even

1) Draw graph models, stating the type of graph used, to represent airline routes where every day there are <u>four</u> flights from <u>Boston to Newark</u>, <u>two</u> flights from <u>Newark to Boston</u>, <u>three</u> flights from <u>Newark to Miami</u>, <u>two</u> flights from <u>Miami to Newark</u>, <u>one</u> flight from <u>Newark to Detroit</u>, <u>two</u> flights from <u>Detroit to Newark</u>, <u>three</u> flights from <u>Newark to Washington</u>, <u>two</u> flights from <u>Washington to Newark</u> and <u>one</u> flight from <u>Washington to Miami</u>, with

a) an edge between vertices representing cities that have a flight between them (in either direction).

4 : B ⇒ N

2 : N ⇒ B

3 : N ⇒ M

2 : M ⇒ N

1 : N ⇒ D

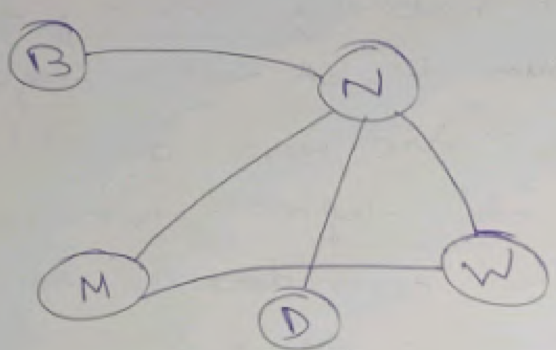
2 : D ⇒ N

3 : N ⇒ W

2 : W ⇒ N

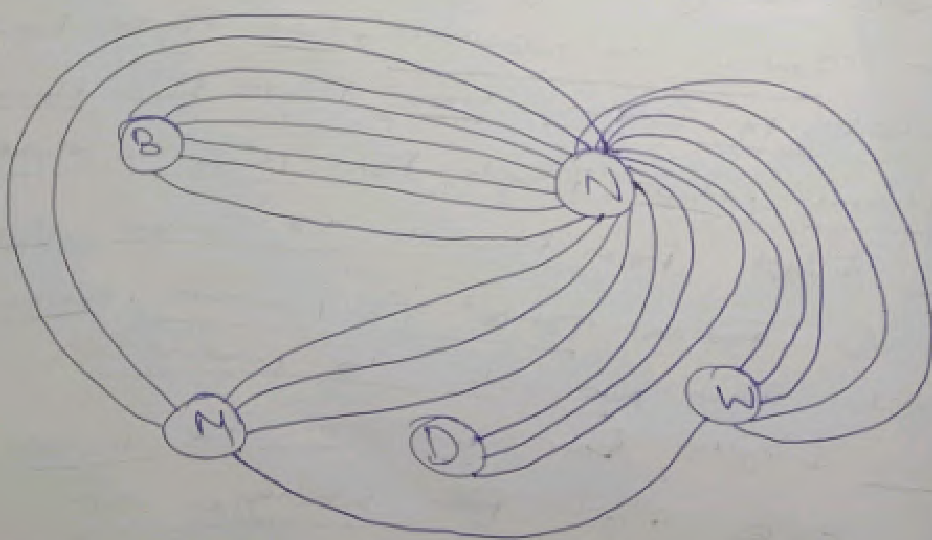
1 : W ⇒ M.

⇒



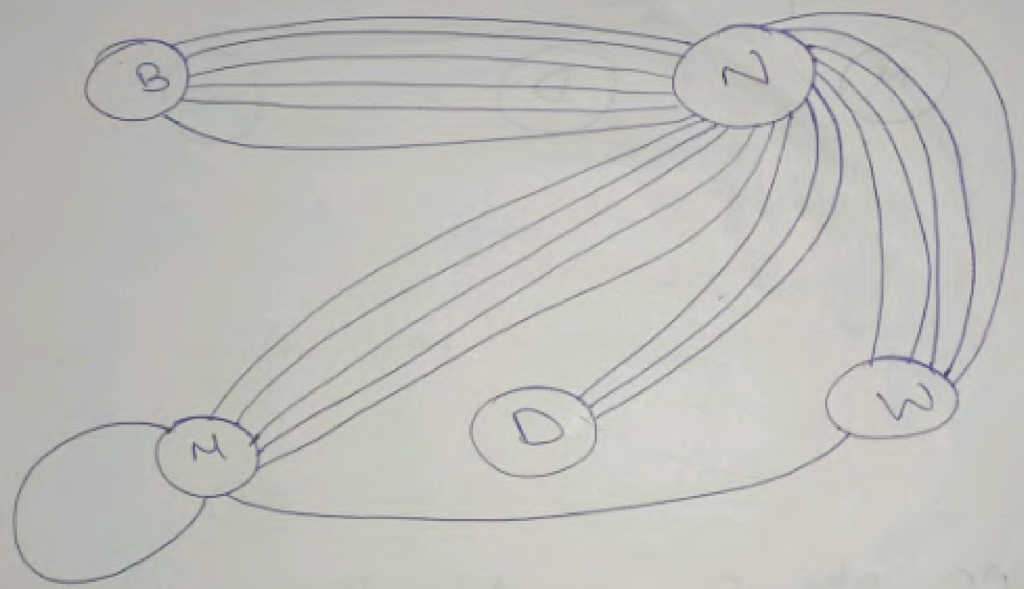
b) An edge between vertices representing cities for each flight that operates between them (in either direction).

⇒



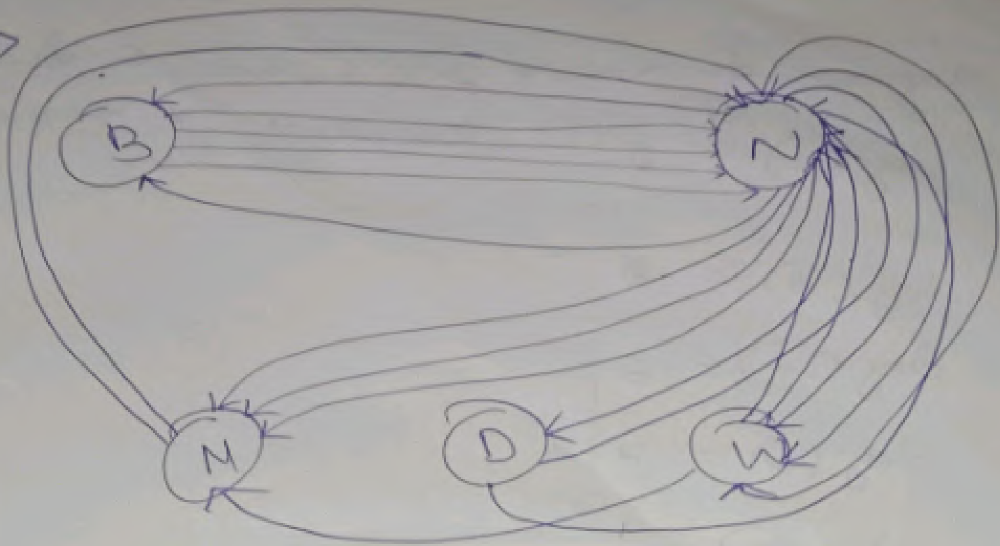
c) an edge between vertices representing cities for each flight that operates between them (in either direction), plus a loop for a special sightseeing trip that takes off and lands in Miami.

=>



d) An edge from a vertex representing a city where a flight starts to the vertex representing the city where it ends.

⇒



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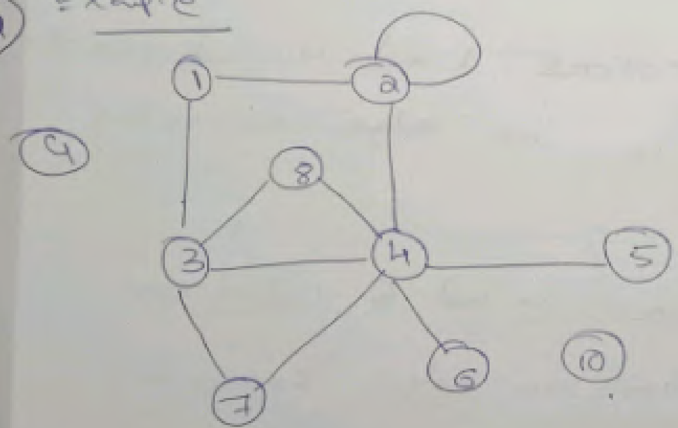
1) Draw a graph with isolated vertex ?

⇒

2) Example



> vertices 9 and 10 are isolated vertices.

> vertices 6 and 5 are Pendant vertices.

i) degree of vertex ⟹ The number of edges incident on a vertex.

deg (1) = 2

deg (2) = 2.

deg (4) = 6

deg (3) = 4

deg (5) = 1

deg (6) = 1

deg (7) = 2

deg (8) = 2

degree of vertex (2) is 4. A loop always adds 2 to the degree of the vertex.

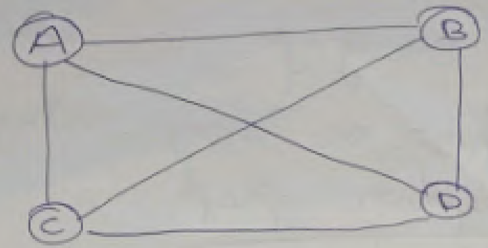
ii) Are the vertices 1 and 4 adjacent.

⟹ No, there is no edge connecting 1 and 4.

iii) Are the vertices 4 and 6 adjacent.

⟹ Yes, there is an edge connecting 4 and 6.

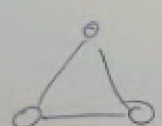
3) Can you draw a graph contain 4 vertices and any two pair have an edge connecting them?

=) 

complete graph (K_n) => A graph with n vertices, such that any pair of vertices has an edge connecting them is called a complete graph.

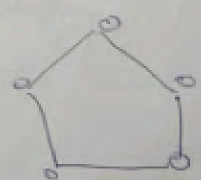
> K_1 is the smallest graph possible and it is a trivial graph.

> K_2 has two vertices and one vertices.

> 

K_3

K_4



K_5

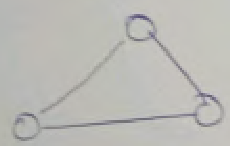
4) Can you draw a graph in which every vertex is got the Same degree ?

=) if we have a graph in which every vertex has a same degree is called regular graph.

In Particular if a graph has vertices with degree k. Then the graph is called k-regular graph

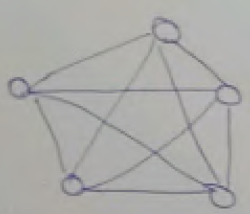
eg: regular graph.

2 regular



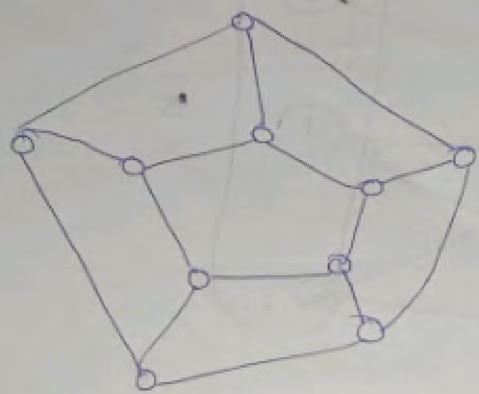
3 regular

4 -regular.



~~graph contain~~

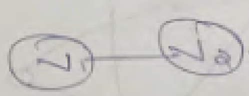
Example of 3 regular graph which are not complete.

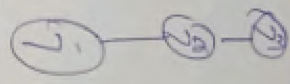


Classes of Graphs

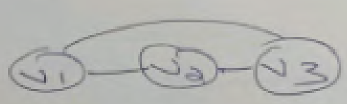
① Paths (Pn)

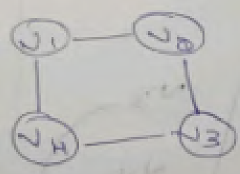
P1 

P2 

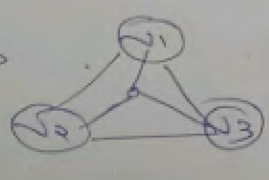
P3 

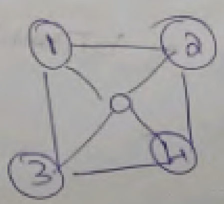
② cycles (cn)

c3 

c4 

③ wheels (wn)

w3 

w4 

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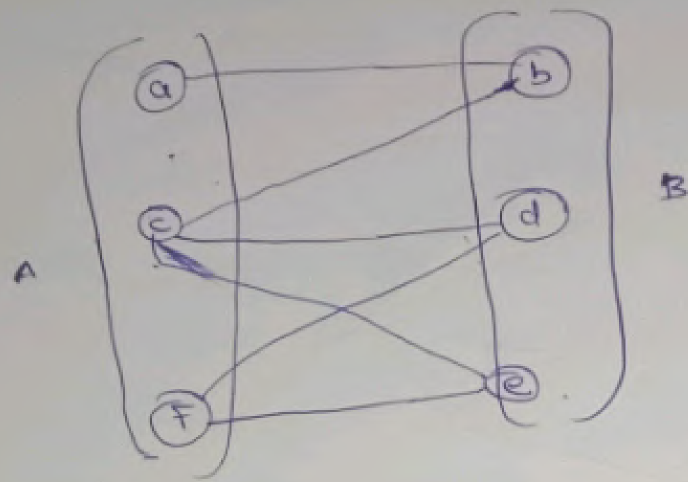
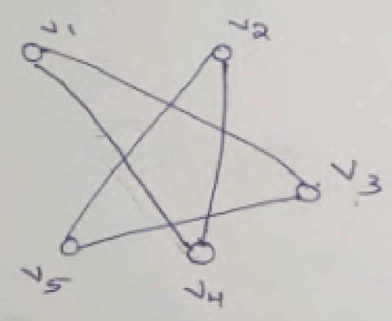
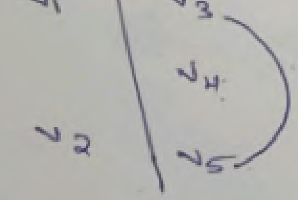
Bipartite Graph



1) check if the following graphs are bipartite

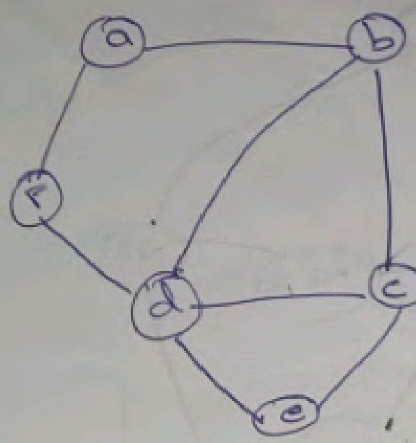


3)

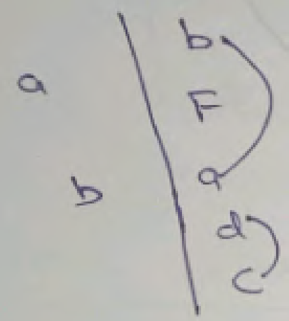


This is not a bipartite graph.

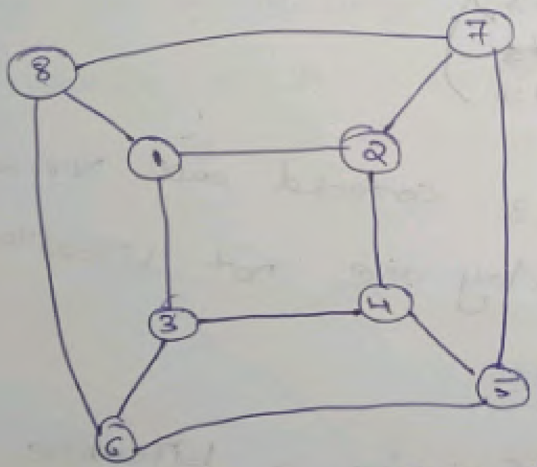
2)

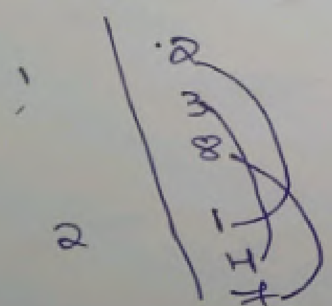


Let A and b be the partitions ~~set~~ of the vertex set if vertex c belong to ~~A~~ A then clearly b, d and e should belong to B. Since there are edges connecting b and a, d and c this is not bipartite graph.



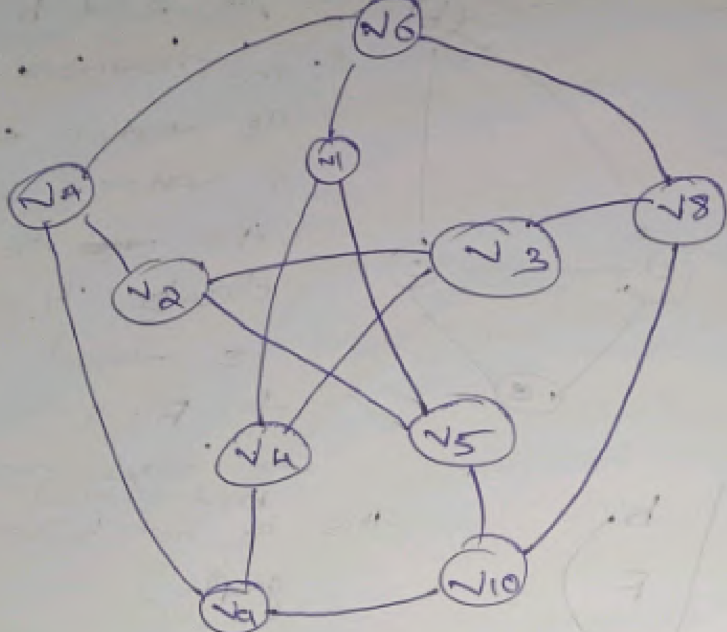
3)





Since there is an edge connecting 2 and 1, 3 and 4 and 8 and 7 ∴ it is not a bipartite graph.

4)

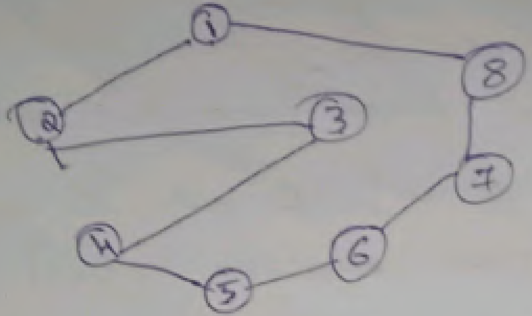


=>

v1	v5.
	v4
	v6
v2	v3
	~~v3~~
	v7

since v4 and v3 corrected and v6 ad v7 are corrected ∴ they are not bipartite graph.

5) Check IF C_8 is a bipartite graph,



1	8
	2
	1
2.	3

there is a correction between 2 and 1 so it is not bipartite graph.

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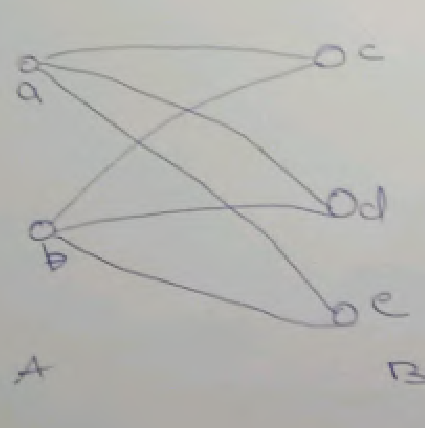
Complete Bipartite

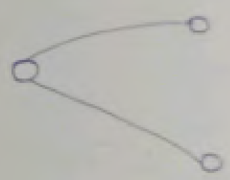
A complete Bipartite graph with bipartition A and B is a Bipartite graph in which every vertex of A is adjacent to every vertex of B and vice versa.



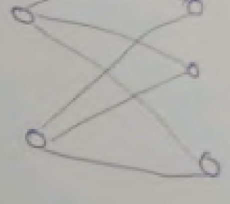
Note

If ~~G~~ G is a complete bipartite graph with bipartitions A and B such that A has m vertices and B has n vertices then the graph can be represented as $K_{m,n}$.

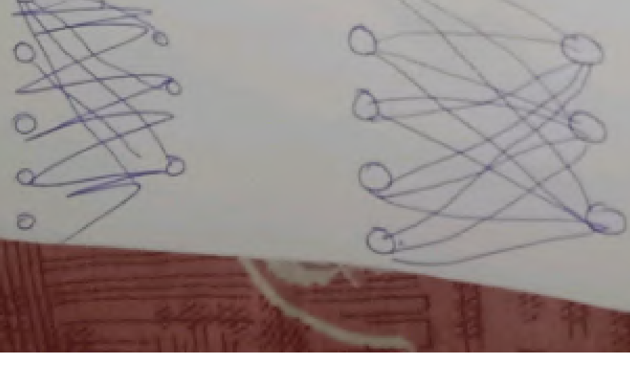
1) $K_{1,2}$



2) $K_{2,3}$



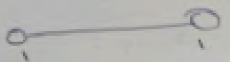
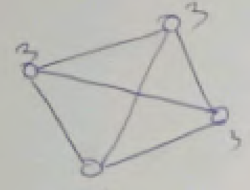
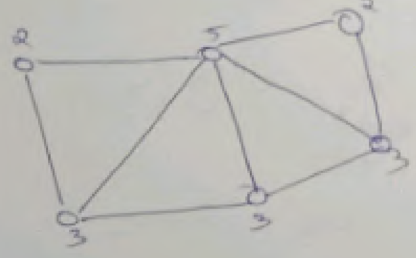
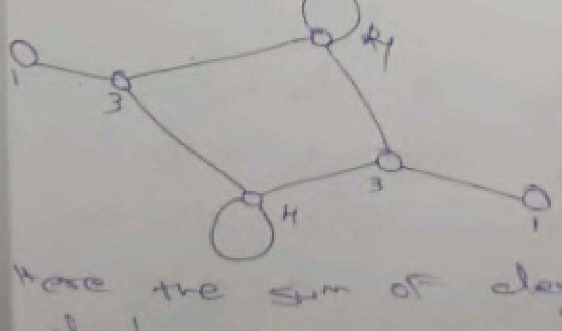
3) $K_{3,5}$



Hand staking lemma

for any undirecred graph sum of degrees of all the vertices is twice the number of edges of the graph

verify handshaking lemma in the following graph

	no of edges (m)	degree of term (n)
	1	2
	6	12
	9	18
	8	16

this is regular graph

Here the sum of degree of vertex is double the no of edges of graph

$2 \times$ the no of edges of graph $=$ sum of degree of vertices.

Theorem

In any undirected graph the number of odd vertices will always be even.

Proof

from handshaking lemma we know $\cdot\ 2 \times M = \sum\limits_{\text{vertices}} \deg(v)$

$$2m = \sum \deg(v)$$

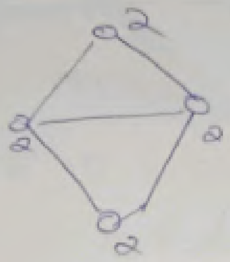
$$2m = \sum_{\text{even vertex}} \deg(v) + \sum_{\text{odd vertex}} \deg(v)$$

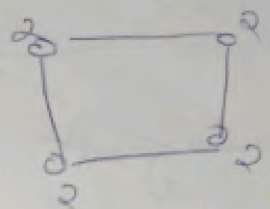
rearranging the terms we have.

$$2m - \sum_{\text{even vertex}} (v) = \sum_{\text{odd vertex}} \deg(v) .$$

Since every term on the ~~top~~ LHS is a multiple of two the RHS should also be the multiple of two. Since every term on the ~~RHS~~

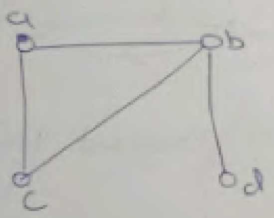
is an odd number, For RHS to be even we require even number of terms onde RHS. This implies there will be even number of odd vertices.





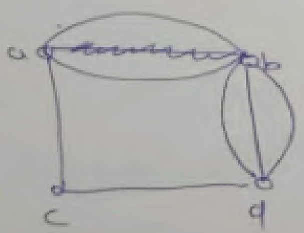
Determine the type of graph.

1)



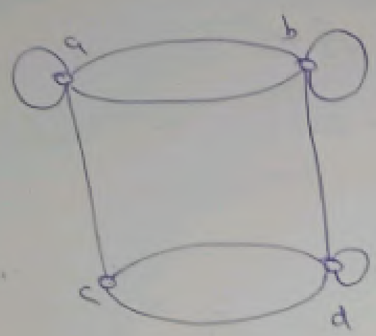
undirected simple graph.

2)



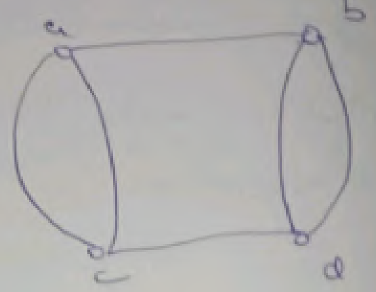
multigraph

3)



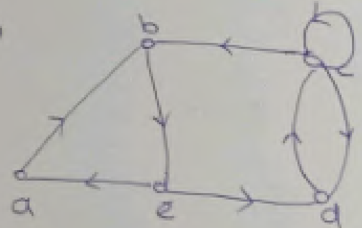
Pseudograph.

4)



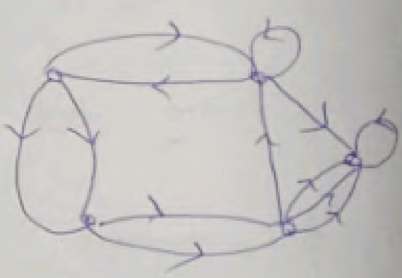
Multigraph

5)



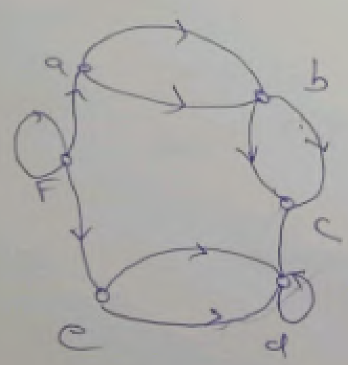
Pseudograph (directed)

6)



directed Pseudograph

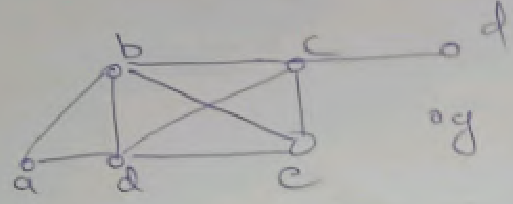
7)



Directed Pseudograph

2) What are the degrees and neighbourhood of the vertices in the following graph.

i)



A: dg (a) = 2 : b, F

deg (b) = 4 : a, F, e, ~~g~~ c

dg (c) = 5 : b, F, e, d

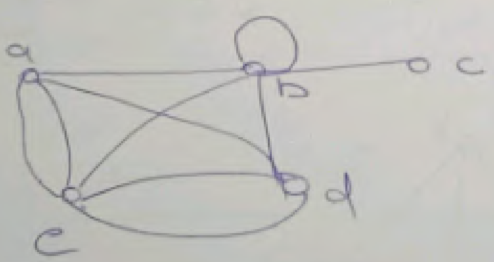
dg (d) = 1 : c

deg (e) = 3 : c, b, F

deg (F) = 4 : a, b, c, e

deg (g) = 0 .

ii)



A: deg (a) = 3 : b, e, d

deg (b) = 5 : c, d, e, a

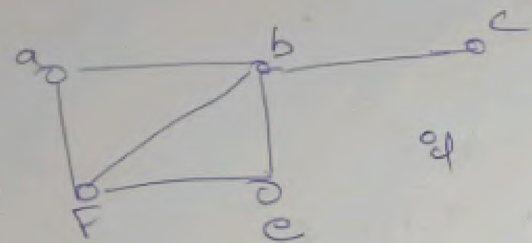
deg (c) = 1 : b .

deg (d) = 3 : b, e, a

deg (e) = 4 : b, d, a

3) In the following graphs, find the no of vertices, edges and degrees of each vertex. identify all isolated and Pendant vertex.

i)

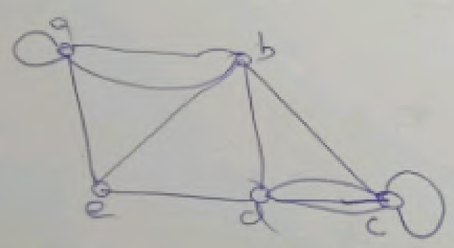


A : vertices : 6
edges : 6

Pendant vertice : c
Isolated vertice : d

deg(a) : 2
deg(b) = 4
deg(c) = 1
deg(d) = 0.
deg(e) = 2
deg(F) = 3

ii)



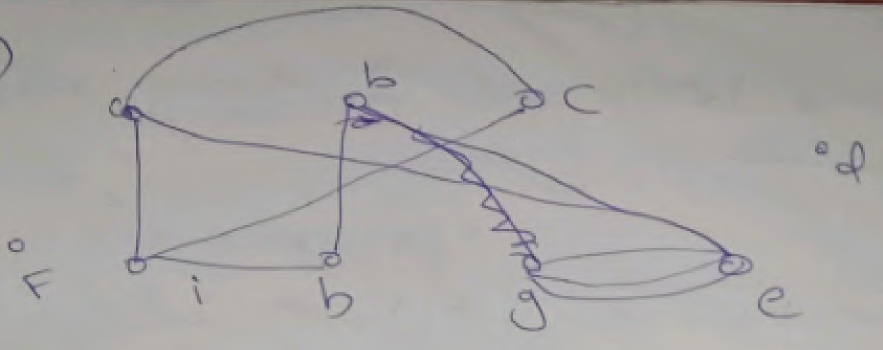
vertices = 5
edges = 11

deg(e) = 6

deg(a) = 5

deg(b) = 6
deg(c) = 6
deg(d) = 5

iii)



A: vertices : 9

edges = 12

isolated vertice = F, d

deg (a) = 3
deg (b) = 2
deg (c) = 4
deg (d) = 0
deg (e) = 6
deg (F) = 0
deg (h) = 2
deg (i) = 3
deg (g) = 4.

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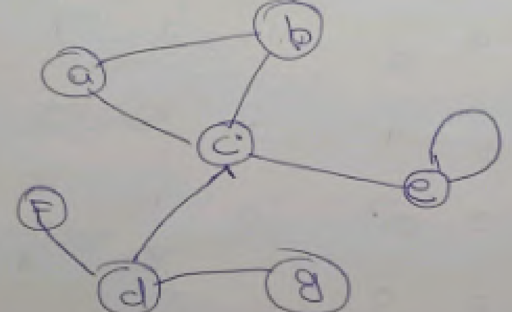
Representation of A graph

Adja.



① adjacency list

Vertices	
a	b, c
b	a, c
c	a, b, e, d
d	F, c, g
e	c, e
F	d
g	d.

not consider multiple edges

② adjacency matrix

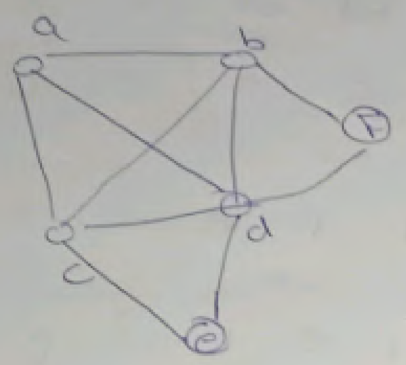
$$\begin{array}{c|ccccccc} & a & b & c & d & e & F & g \\ \hline a & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ b & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ c & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ d & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ e & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ F & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ g & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{array}$$

③ incidence matrix

$$\begin{array}{c|ccccccc} & ab & ac & bc & ce & cd & dF & dg \\ \hline a & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ b & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ c & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ d & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ e & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ F & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ g & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}$$

No loops considered

2) Draw adjacency list, adjacency matrix, and incidence matrix.



adjacency list

Vertices	
a	b, c, d
b	a, d, F, c
c	a, b, d, e
d	a, b, c, F, e
e	c, d
F	b, d.

adjacency matrix.

$$
\begin{array}{c|cccccc}
 & a & b & c & d & e & F \\
\hline
a & 0 & 1 & 1 & 1 & 0 & 0 \\
b & 1 & 0 & 1 & 1 & 0 & 1 \\
c & 1 & 1 & 0 & 1 & 1 & 0 \\
d & 1 & 1 & 1 & 0 & 1 & 1 \\
e & 0 & 0 & 1 & 1 & 0 & 0 \\
F & 0 & 1 & 0 & 1 & 0 & 0
\end{array}
$$

~~incidence matrix~~

~~ab ac ad bd bf bc~~.

incidence matrix

$$
\begin{array}{c|c}
 & \begin{matrix} Fb & ab & ac & ad & bc & bd & cd & de & dF & ec \end{matrix} \\
\begin{matrix} a \\ b \\ c \\ d \\ e \\ F \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}
\end{array}
$$

3) Adjascency matrix

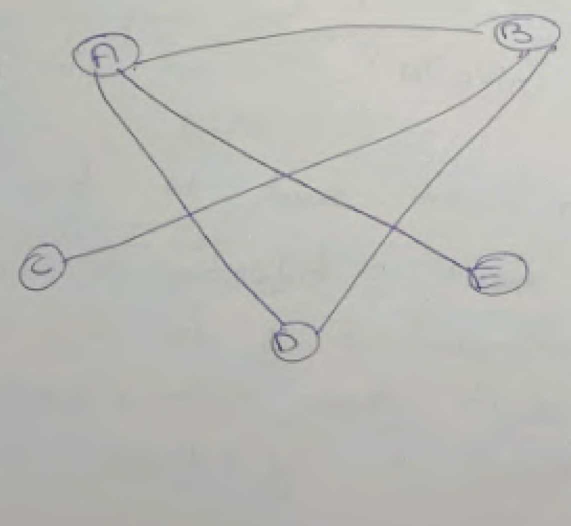
$$
\begin{array}{c|c}
 & \begin{matrix} a & b & c & d \end{matrix} \\
\begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}
\end{array}
$$

⇒

incidence matrix

$$\begin{array}{c|ccccc} & AD & AB & BC & AE & BD \\ \hline A & 1 & 1 & 0 & 1 & 0 \\ B & 0 & 1 & 1 & 0 & 1 \\ C & 0 & 0 & 1 & 0 & 0 \\ D & 1 & 0 & 0 & 0 & 1 \\ E & 0 & 0 & 0 & 1 & 0 \end{array}$$

⇒



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Draw the graph.

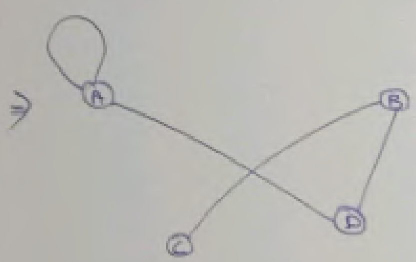
1) Adjascency matrix.

$$
\begin{array}{c|cccc}
 & A & B & C & D \\
\hline
A & 1 & 0 & 0 & 1 \\
B & 0 & 0 & 1 & 1 \\
C & 0 & 1 & 0 & 0 \\
D & 1 & 1 & 0 & 0
\end{array}
$$

2) Incidence matrix

$$
\begin{array}{c|cccc}
 & AB & BC & CD & AD \\
\hline
A & 1 & 0 & 0 & 1 \\
B & 1 & 1 & 0 & 0 \\
C & 0 & 1 & 1 & 0 \\
D & 0 & 0 & 1 & 1
\end{array}
$$

$\Rightarrow$



$\Rightarrow$

ISOMORPHISM GRAPH

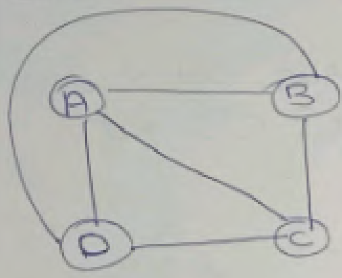
Two graphs g and h are said to be isomorphic if there exist a bijective function F from the vertex set of g to the vertex set of h such that whenever U and V are adjascent in g then F(U) and F(V) are adjascent in h.

and vice versa.

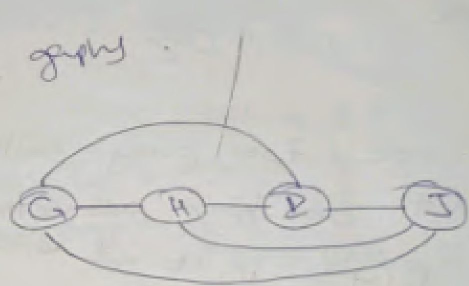
NOTE: If G and H are isomorphic graph then they must have the same number of vertices, the same number of edges and their degree sequence (list of degrees of each vertex) must be identical.

example 1

Consider the following graphs.



Graph I



Graph II.

Since the number of vertices and edges of G and H are the same and degree sequence of both the graphs is $(3,3,3,3)$ there is a possibility that graph are isomorphic.
let the bijective function from $V(G)$ to $V(H)$ be as follows

$F(A) = G$

$F(B) = H$

$F(C) = I$

$F(D) = J$

UV	F(U) F(V)	Present in H
AB	GH	✓
BC	HZ	✓
BD	HJ	✓
CD	ZJ	✓
AD	GJ	
AC	GZ	✓

Since for every adjacent vertices U and V is G, we have adjacent vertices F(U) and F(V) in H and vice-versa.

G and H are isomorphic.

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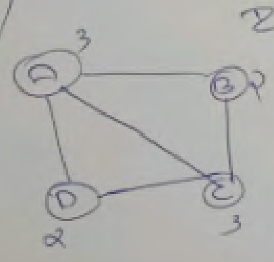
Check if the graph are isomorphic

1)

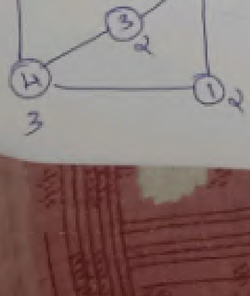
2



no of vertices = 4

no of edges = 5

degree sequence (2, 2, 3, 3)



F

A → 2 ✓ H

B → 3 1

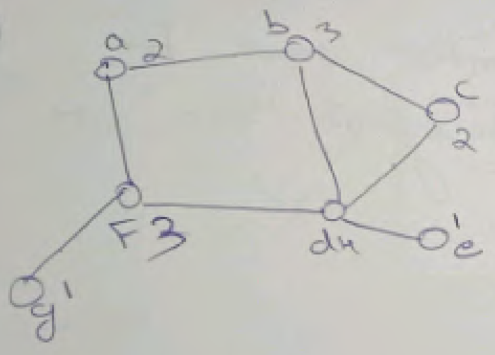
C → 4

D → 1

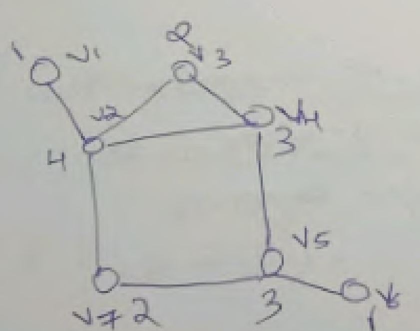
UV	F(u) · F(v)	Present in H ?
AB	23	✓
BC	34	✓
CD	41	✓
DA	21	✓
AC	24	✓

∴ F is the bijective function that satisfies the condition described in the definition of isomorphic graph. Hence the graphs are isomorphic.

2)



no of vertices = ~~8~~ 7
no of edges = ~~9~~ 8
degree sequence = (2, 3, 2, 1, 4, 3, 1)



	F
a	v3, v7
b	
c	
d	→ v2
e	→ 1
F	
g	→

correction ✓
so no adjacency

Since vertex $\cancel{d}$ V and $\cancel{\;}$ V_2 are the only vertices of degree 4 in G and h respectively $F(\cancel{x})$ should be V_2. Now since a is the vertex of degree 2 it can be mapped to either V_3 or V_7 but note that since a and b are not adjacent $F(d) = V_2$ and $F(a)$ should not be adjacent. But both V_3 and V_7 are adjacent to V_2. Hence a bijective function satisfying condition described in the definition of isomorphism cannot be constructed. Hence the graph are not isomorphic.

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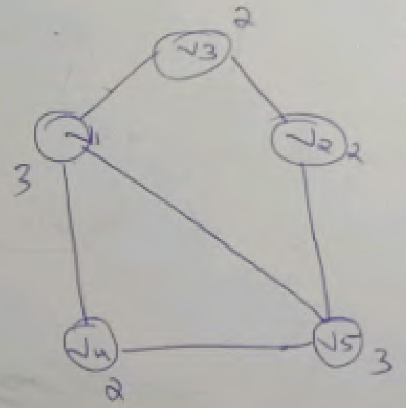
1)

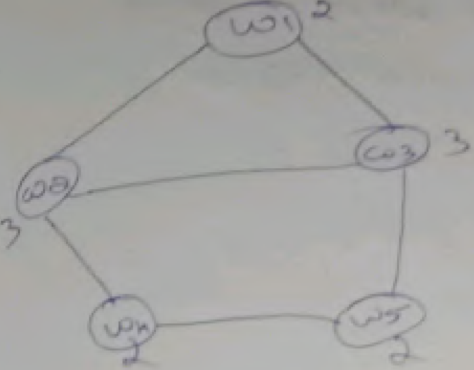


no of vertices = 5

no of edges = 6

degree sequence

$= (2, 2, 2, 3, 3)$

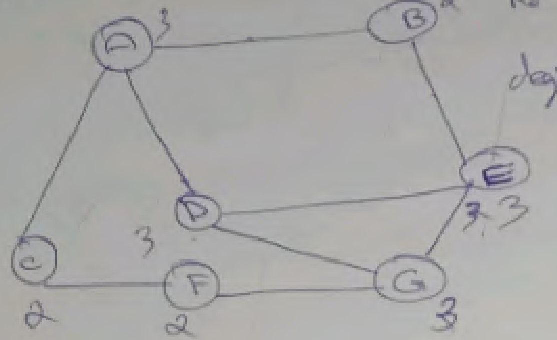


v_1	w_3
v_5	w_2
v_2	w_1 w_4 w_5
v_3	w_1 w_5
v_4	w_1

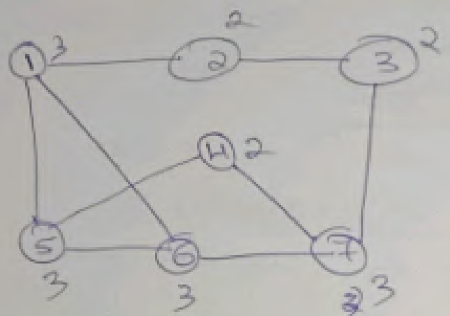
uv	F(u)F(v)	
$v_1 v_3$	$w_3 w_5$	✓
$v_3 v_2$	$w_5 w_4$	✓
$v_2 v_5$	$w_4 w_2$	✓
$v_5 v_4$	$w_2 w_1$	✓
$v_4 v_1$	$w_1 w_3$	✓
$v_1 v_5$	$w_3 w_2$	✓

It satisfies the bijective function therefore the graphs are bijective function.

2) .

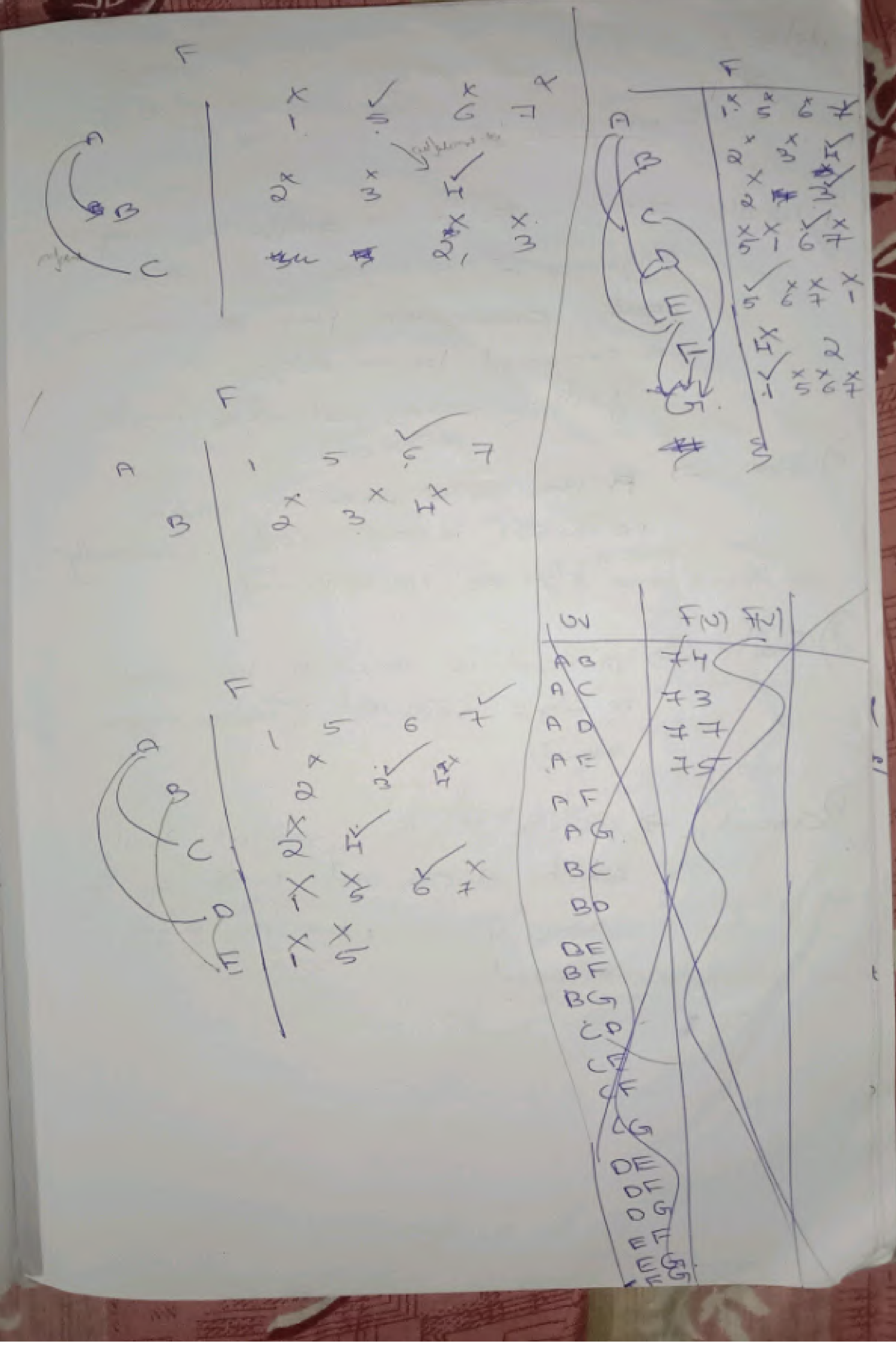


no of edges ⇒ ~~8~~ 9
no of vertices ⇒ 7
degree sequence ⇒
(3, 3, 3, 3, 2, 2, 2)



1. left side is adjacent to n, right is adjacent
(4 not adjacent to 1)

		F			
(deg 3)	A	1 ✓	5	6	7
(deg 2)	B	2 ✓	3 ×	4 ×	
(deg 2)	C	3 ×	4 ×		
	D				
	E				
	F				
	G				


UV
F(U) F(V)

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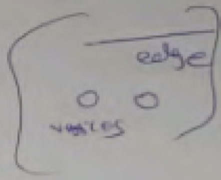


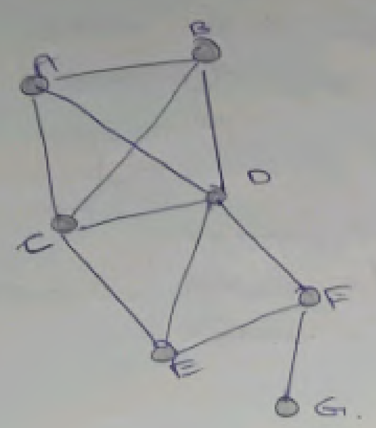
* CONNECTIVITY

1) Walk ⇒ A walk in a graph is any sequence of vertices such that each consecutive pair of vertices is connected by an edge in the graph. (vertices and edges maybe repeated).

2) Path ⇒ A Path is a walk in which no vertex is repeated. (naturally edges are not repeated).

3) Trail → A trail is a walk in which no edges repeated (but vertices maybe repeated).

4) Circuit → A circuit is a closed trail which starts and ends at the same vertex, but no edges repeated.



ABDFECADFG → walk

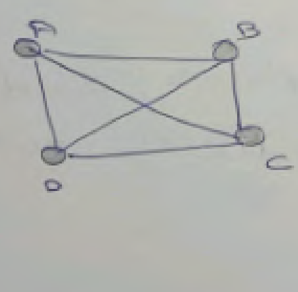
ABDEFG → Path

ABDACDF → trail.

ABDCA → circuit.

5) Hamiltonian Path → A Path that visits each vertex exactly once is called a hamiltonian Path.
(the edges can never be repeated). every vertex should be included.

6) Hamiltonian Circuit Eg. → A hamiltonian Path that is closed. It starts and ends at the same vertex, visiting every vertex exactly once in between.

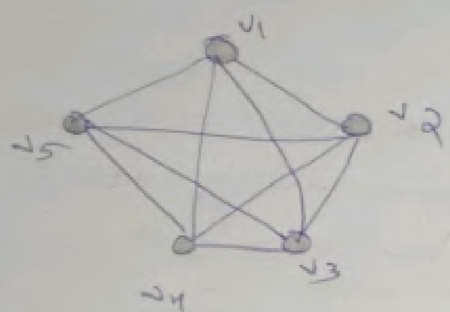


ABCD → Hamiltonian Path

ABCDA → Hamiltonian Circuit

7) Euler trail ⇒ A trail that uses every edge exactly once.

8) Euler circuit ⇒ An eulerian trail that is closed - starts and ends at the same vertex.



euler circuit ⇒ ~~v1 v2 v3 v3 v4 v5~~ $v_2 v_4 v_1$

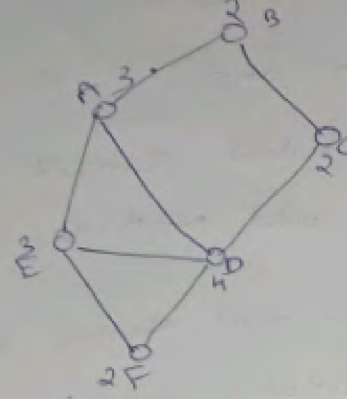
$v_1 v_2 v_3 v_4 v_5 v_2 v_4 v_1 v_3 v_5 v_1$

*

9) Hamiltonian graph ⇒ A graph is said to be hamiltonian if it contains a hamiltonian circuit.

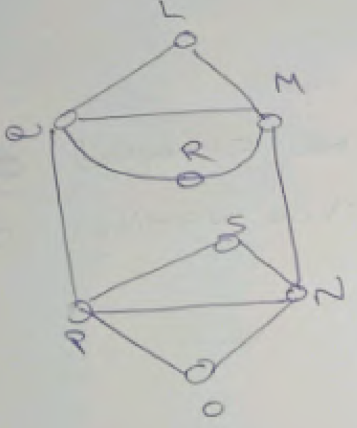
10) ~~E~~ euler graph ⇒ A graph is said to be euler graph if it contains an eulers circuit

14/2/25

1) 

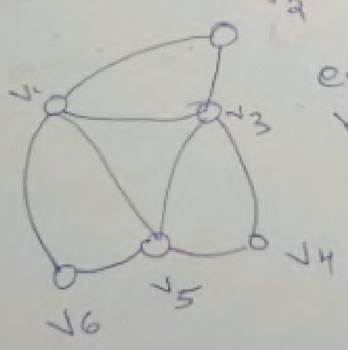
It is not an Euler graph. Cause it has two odd edges.

E.

2) 

LMNOPSNPQRMQL.

~~[illegible]~~

3) 

eulers circuit is possible because of all even edges.

theorems :

All vertices of the graph are .
IF the graph contains an Euler Circuit .
A graph has exactly two odd vertices .

A graph has exactly two odd vertices IF and only IF the graph contains an . euler trail .

→ Connected graph .

A graph is said to be connected if there connecting any two vertices of the graph .

Weighted graph

A graph is said to be weighted graph IF the . and .a edges are assigned costs Kindly .

DIJKSTRAS

step1 : construct a table with columns corresponding to every vertex of the graph .

Step 2 : In the first row assign the value zero to the source vertex (origin) and infinity to all other vertices.

Step 3 : To fill the next row look at all the vertices adjacent to the current vertex. update the columns corresponding to its neighbours with the dual edge weight from the source. If smaller than the existing value.

Step 4 : From the newest row chose the vertex with the smallest value that has not been permanently marked.

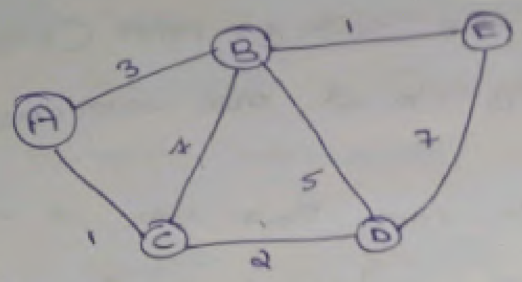
Step 5 : construct this next row by assigning all neighbours of S1. For each neighbour update its column, with the smallest of following two values.

The sum of the smallest value from the previous row and the edge weight connecting S1 to that neighbour.

The existing ~~entry~~ entry in that column.

Repeat steps 4 and 5 until shortest distance from the destination vertex is determined or until all vertices are permanently marked.

18/8/05 1) Find the shortest Path from A to E.

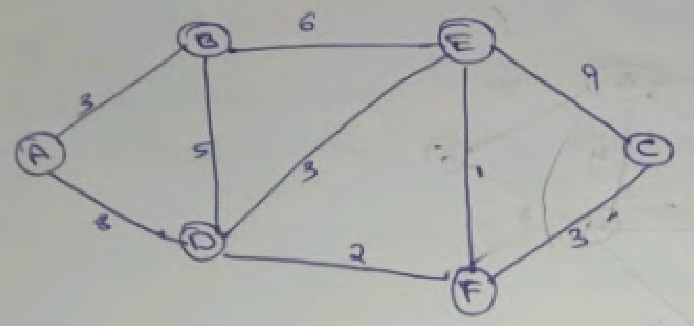


$\Rightarrow$

	A	B	C	D	E
A (0)	0	∞	∞	∞	∞
	0	3	1	∞	∞
C (1)	0 (0, (1+1)) min	3 (3, 8) min	1	3 (∞, 2+1) min	∞
B (3)	0 (0, 0) min	3	1 (1, (4+3)) min	3 (3, (5+3))	4 (∞, 4) min
D (3)	0	3 (3, 5+3)	1 (1, (2+3))	3	4 (4, (7+3))
E (4)	0	3 (3, (1+1))	1	3 (3, (7+4))	4,

Shortest distance is 4//.

2) Find the shortest distance from A to all other vertices of the graph.



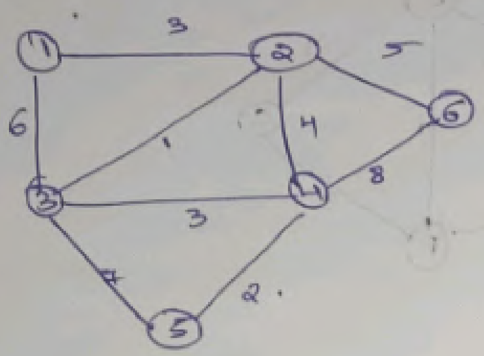
⇒

	A	B	C	D	E	F
A	0	∞	∞	∞	∞	∞
A(0)	0	3	∞	8	∞	∞
B(3)	0 (0,(3+3))	3	∞	8 (8,(5+3))	9 (∞,(6+3))	∞
D(8)	0	3	∞	8	9 (9,(8+3)) min	10 (∞,(8+2)) min
E(9)	0	3	18 (∞,18)	8 (8,(9+3))	9	10
F(10)	0	3	13 (18,13)	8 (8,12)	9 (9,11)	10
C(13)	0	3	13	8	9 (9,22)	10 (10,16)

Shortest distances.

∴ Shortest distance from A to B = 3, A to C = 13, A to D = 8, A to E = 9 and A to F = 10, A to A = 0.

3) Find the shortest distance from vertex 1 to all other vertices of the graph. Also mention the path to be followed to reach there.



⇒

	1	2	3	4	5	6
1	0	∞	∞	∞	∞	∞
1(0)	0	3	6	∞	∞	∞
2(3)	0	3	4 (6,(3+1))	~~8~~ 7 (∞,(4+3))	∞	8 (∞,8)
3(4)	0	3 (3,5)	4	7 (8,7)	11	8
4(7)	0	3 (3,11)	4	7	9 (11,9)	8 (8,15)
6(8)	0	3	4	7	9	8
5(9)	0	3	4 (4,11)	7 (7,11)	9	8

the shortest distance from vertex 1 to 2 = 3
1 to 3 = 4, 1 to 4 = 7, 1 to 5 = 9, 1 to 6 = 8

Path

1 (3)
1 (3)
1 (3)
1 (3)
1 (3)

19/8/25

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STEPS

Step1

Step 2

Step 3

Step4

Step5

Path

1 $\xrightarrow{(3)}$ 2

1 $\xrightarrow{(3)}$ 2 $\xrightarrow{(1)}$ 3

1 $\xrightarrow{(3)}$ 2 $\xrightarrow{(4)}$ 4

1 $\xrightarrow{(3)}$ 2 $\xrightarrow{(4)}$ 4 $\xrightarrow{(2)}$ 5

1 $\xrightarrow{(3)}$ 2 $\xrightarrow{(5)}$ 6

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Travelling salesman Problem (TSP)

The TSP is the Problem of ~~Creation~~ Finding a hamiltonian cycle / circuit of minimal total cost in a weighted graph.

STEPS

Step1 : choose the starting vertex as the one with maximum degree. mark it as visited.

Step 2 : From the current vertex look at all the edges leading to vertices that are not yet visited.

Step 3 : select the edge with minimal weight among them.

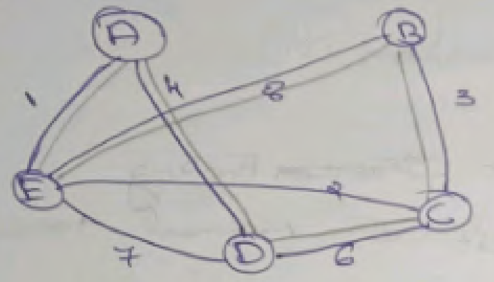
Step4 : move to that vertex and mark it as visited.

Step 5 : repeat the steps 2 to 4 untill all vertices have been visited exactly once.

Step 6 : Return to the starting vertex by choosing the edge from the last visited vertex ~~back to the start~~

Step 7 : sum up all the choses edge weights to find the total travel coust.

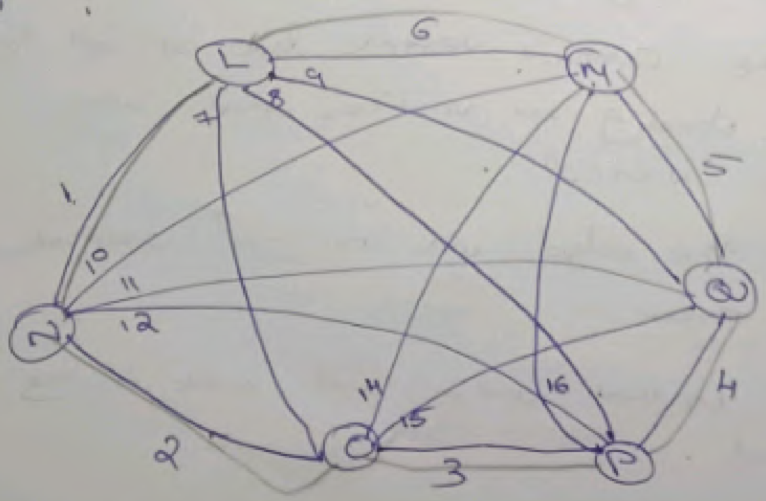
=> 1)



$E \xrightarrow{1} A \xrightarrow{4} D \xrightarrow{6} C \xrightarrow{3} B \xrightarrow{8} E$

$1+4+6+3+8 = 22$ //.

2)

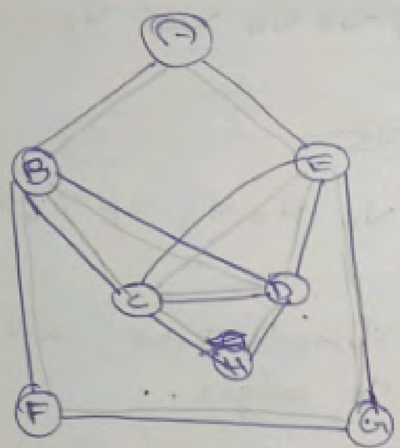


$L \xrightarrow{1} N \xrightarrow{2} O \xrightarrow{3} P \xrightarrow{4} Q \xrightarrow{5} M \xrightarrow{6} L$

$1+2+3+4+5+6 = 21$ //.

Chinese Postman Problem (CPP)

> solve CPP for the following graph.



ABFGEA

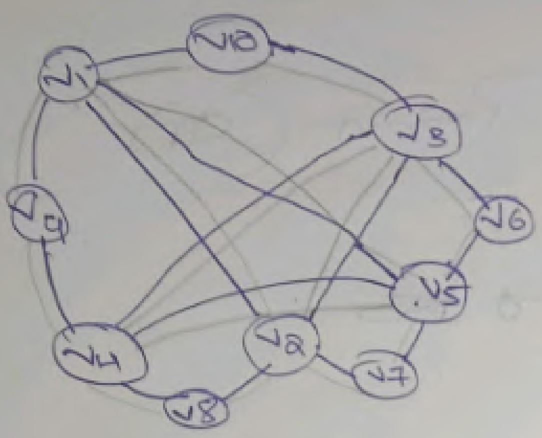
ECHDE

ABFGECHDEA

CDBC

ABFGECDBCHDEA

2)



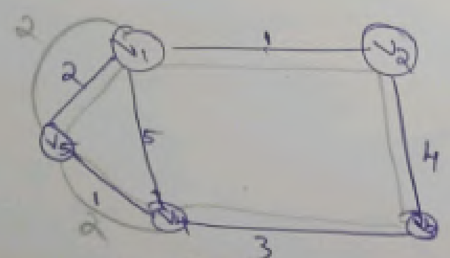
V1 V10 V3 V6 V5 V7 V2 V8 V4 V9 V1

~~V1V10~~ V5 V4 V3 V2 V1 V5

V1 V10 V3 V6 V5 V4 V3 V2 V1 V5 V7 V2 V8 V4 V9 V1

NOTE: length of the above shortest paths will be sum of weights of all the edges..

3)



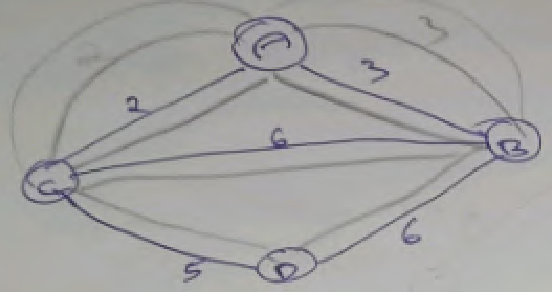
V1 V5 V4 Short → 3

~~V1 V2 V3 V4 V1 V5 V4 V5~~

V1 V2 V3 V4 V5

V1 V2 V3 V4 V1 V5 V4 V5

3)



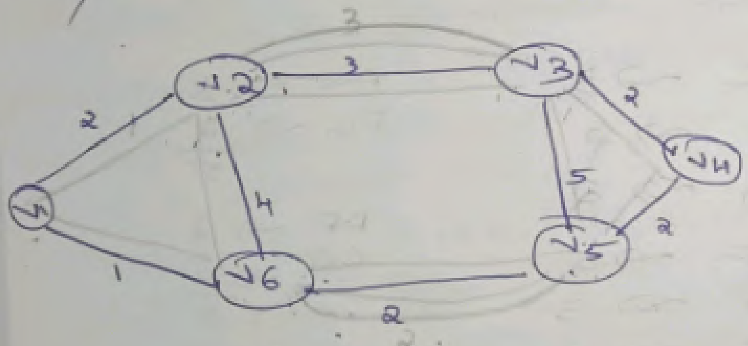
A B D C A

~~A B B D C A~~

A B C A B D C A

26/8/25

1) solve the CPP for the following



odd vertex 2 {v2 v3 v5 v6}

v2 v5 v3 v6 ✗
5 + 6 = 11

v2 v6 v3 v5 = 7 ✗
3 + 4

v5 v6 → 2 ✓
v2 v3 → 3
5

v2 v3 → v2 v3 → 3

v2 v5 → v2 v1 v6 v5 → 5

v2 v6 → v2 v1 v6 → 3

v3 v5 → v3 v4 v5 → 4

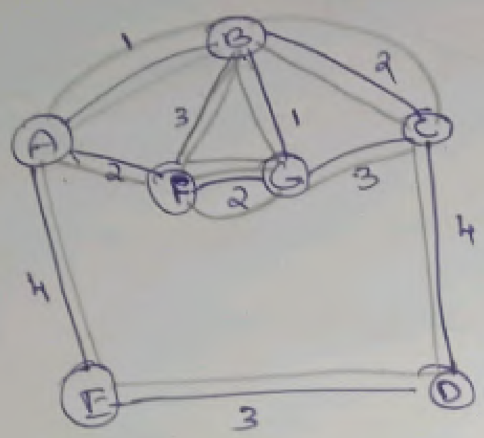
v3 v6 → v3 v4 v5 v6 → 6

v5 v6 → v5 v6 → 2.

v1 v2 v3 v4 v5 v6 v1

v1 v2 v3 v5 v6 v2 v3 v4 v5 v6 v1 = 26

2)



I {A, C, F, G}.

AC → ABC → 3
AF → AF → 2
AG → ABG → 2
CF → CGF → 5
CG → CG → 3
FG → FG → 2

AC → 3
FG → 2
5

AF → 2
CG → 3
5

AG → 2
CF → 5
7

Shortest Path =

A B C D E

ABCGFABCDE

ABCGFABFGBCDE = 30

Module 2

Counting Techniques

Addition Rule

If an event can happen in 'm' ways and another different event can happen in 'n' ways, then the total number of ways to choose one of the events is m+n ways (the events must be disjoined). eg. ~~[illegible]~~

A student wants to choose a subject where he has three different choices in mathematics and 4 different choices in science. Hence the student has a total 3+4 = 7 choices. Since choosing math OR science are seperate choices.

Multiplication Rule

If a tasks can be done in 'm' ways and for each of these another tasks can be done in 'n' different ways, then both the tasks can be done together in ~~[illegible]~~ $m \times n$ ways

example: ~~[illegible]~~ if a meal consist of two choices of starters and three choices of main dishes there are $2 \times 3 = 6$ meal choices (since a choice has to be made from starters and main dishes).

2/9/25

Permutation.

A Permutation is an arrangement of objects in order, if order matters, we use Permutation.

The no of ways to arrange 'r' objects from n distinct objects is given by the formula.

$$\boxed{{}^nP_r = P(n,r) = \frac{n!}{(n-r)!}}$$

where $n! = 1\times2\times3\times \ldots\ldots \times n$.

eg: no of ways in which the letters of word BEST can be used to form words with 3 letters is given by $4P_3 = \frac{4!}{(4-3)!}$

$$= \frac{4!}{1!} = \frac{1\times2\times3\times4}{1} = \underline{\underline{24}}$$

BES BET EST SBE SET TSB
BSE BTE ETS SEB STE TBS
BST EBS EBT STB TBE TES
BTS ESB ETB SBT TEB TSE.

COMBINATION

A combination is selection of objects without caring its order of selection. If order does not matter we use combination.

The number of ways to choose r objects From 'n' distinct objects is given by.

$$\boxed{{}^nC_r = \binom{n}{r} = \frac{n!}{(n-r)!\, r!}}$$

eg:

Two students From a class of 5 students can be chosen in

$$5C_2 = \frac{5!}{(5-2)!\, 2!}$$

$$= \frac{5!}{3!\, 2!} = \frac{1\times2\times3\times4\times5}{(1\times2\times3)(1\times2)}$$

$$= \frac{20}{2} = 10$$

ways

i) If A, B, C, D and E are the 5 students of the class. two students could be chosen in one of the following ways.

AB BC CD DE
AC BD CE
AD BE
AE

can choose to read.

1) A student can either a maths books (with four choices) or an english books (with 5 choices). How many different choices does the student have.

⇒ using addition theorem no of choices $= 4+5 = 9$ //.

2) In How many ways can 5 students sit in a row of 3 chairs.

$$\Rightarrow {}^5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{1\times2\times3\times4\times5}{1\times2} = 60 //$$

3) How many different 4 digit numbers can be formed using the digits 1, 2, 3, 4, 5 ; if repetition of digits is not allowed

$$\Rightarrow {}^5P_4 = \frac{5!}{(5-4)!} = \frac{1\times2\times3\times4\times5}{1} = \underline{\underline{120}}$$

4) How many different arrangements of letters of the word MATHS are possible ?

$$\Rightarrow {}^5P_5 = \frac{5!}{(5-5)!} = \frac{120}{0!} = \frac{120}{1} =$$

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5) From a group of 10 players, How many ways can a team of 4 Players be selected ?

6) A school ID Number consists of two letters followed by three digits. How many different ID numbers are possible. ? (where character repitition is not allowed).

7) A commitee of two Boys and three Girls. is to be Formed From 5 Boys and 6 girls. How many such commitees can be formed ?

8) In How many ways can the First 3 Prizes be awarded to 10 competetors ?

9) A Box contains 10 red, 8 Blue, and 6 green Balls. we have to select 2 Balls, How many ways can you select these, if the colour doesnt matter. also In How many ways can you do this selection if both the balls are of the same colour.

⇒ 5) $n = 10,\ r = 4.$

$$^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$$

$$^{10}C_{4} = \frac{10!}{(10-4)!\ 4!} = \frac{10}{6!\,4!} = \frac{10\times9\times\cancel{8\times7\times6\times5\times4\times3\times2\times1}\,7\times8\times}{1\times2\times3\times4} = \frac{5040}{24} = 210$$

6).

alphabet numbers

$^{26}P_2$ $^{10}P_3$

$= {}^{26}P_2 \times {}^{10}P_3$.

$= \frac{26!}{(26-2)!} \times \frac{10!}{(10-3)!} = \frac{26!}{24!} \times \frac{10! \; 8\times9\times10}{7!} \; \frac{720}{}$

$= \cancel{}$
650×720

$= \underline{\underline{468000}}$

7) 5 boys and 6 girls, 2 boys & 3 girls.

$^5C_2 \times {}^6C_3$.

$= \frac{5!}{3! \times 2!} \times \frac{6! \; 1\times2\times3\times4\times5\times6}{3! \times 3!}$

$= \cancel{}$ $\cancel{20 \times 120}$ 10×20

$= \cancel{}$ $= 200$//.

variation of question number 7. no
generate the school 20 number & characters
have to be chosen ... word characters
can be chosen in 26 different ways

when ...

8) $n = 10, r = 3$

$$^{10}P_3 = \frac{10!}{(10-3)!} = \frac{\cancel{10!}^{8\times9\times10}}{\cancel{7!}} = 720$$

9)

... balls whose color not significant

$^{24}C_2$ ways.

$R_1 R \dots \quad B_1, B_2, B_3 \dots G_1 G_2 \dots G_n$

2 red	2 blue	2 green
$^{10}C_2$	$^{8}C_2$	$^{6}C_2$.

$^{10}C_2 + {}^{8}C_2 + {}^{6}C_2 = 45 + 28 + 15 = 88$.

11/9/25

1) What is the minimum number of students that should be Present in the class. to guarantee that atleast 2 students recieve the same score ~~on~~ in the Final exam, If the exam is ~~scored~~ graded on a scale from 0 to 100?.

2) How many People should be Present in a class such that there must be atleast 2 People with the same birthday?

3) How many words must be written such that there must be atleast 2 words that begin with the same letter?

⇒ 1) 101 ⟶ 102 students

2) 366 ⟶ 367 People

3) 26 words but needed 27 words:

26 ⟶ 27 words

Pigeon hole Principal

(K+1) ~~[illegible]~~

If case of Positive integer and one or more objects are Placed into K boxes then there is atleast one box containing two or more of the objects. → There Should be atleast 2 rings into box

4) How many strings of length are can be Formed From the english alphabets.

⟹

5) A group of 30 People have been trained as astronauts to go on the first mission to Mars. How may ways are there to select a Crew of 6 People to go ~~[illegible]~~ on this mission

⟹ $n=30$, $r=6$. $30C6$∥

6) In how many ways can you Select 5 Cards from a standard deck of 52 Cards.

⟹ $52C5$∥

7) Let $A = \{A, B, C, D\}$ How many Subsets of order two are there in the set A.

⟹ $4C_2 = 6$∥

8) Suppose the Salesman has to visit 8 different Cities such that she must begin her trip in the specified city

$\Rightarrow {}^{7}P_{7}$ (orders no, ways)

Recurrence relation

1) What is recurrence relation?

A recurrence relation is an equation that defines each term of a seq as a function of its previous term. It expresses the nth term of a sequence in terms of earlier terms.

eg: $a(n) = a(n-1) + 2$ means any term is obtained by adding 2 to the previous term.

2) General form of a Recurrence Relation?

General form: $a(n) = F(a(n-1), a(n-2), \ldots a(n-k))$

where $a(n)$ = nth term, F = some function, K = order of the recurrence

3) Solution of a Recurrence Relation

A solution is a formula that gives the value $a(n)$ directly without recursion.

eg. $a(n) = a(n-1) + 2$ with $a(0) = 1$ has

solution $a(n) = 2n+1$.

4) method 1 : Iteration (substitution) method

Steps

1) write the recurrence and initial condition.

eg: $a(n) = a(n-1) + 2$, $a(0) = 1$.

2. expand step by step:

$a(n) = a(n-1) + 2 = (a(n-2)+2) + 2 =$

$a(n-2) + 4 \ldots$

after k steps : $a(n) = a(n-k) + 2k$.

3) stop when you reach the initial condition

$a(0) = 1$. thus $a(n) = 1 + 2n$.

. example (Iteration method)

Reccurrence : $a(n) = a(n-1) + 5$, $a(0) = 4$

Soln : $a(n) = 4 + 5n$.

Recurrence : $a(n) = 2a(n-1)$, $a(0) = 1$

Solution : $a(n) = 2^n$.

5) Method 2 : characteristic eqn method.

used for linear recurrence relations with constant co-efficients.

general form : $a(n) = c_1 \cdot a(n-1) + c_2 \cdot a(n-2) \ldots c_k \, a(n-k)$

steps

1. write recurrence in standard form.

2. form the characteristic polynomial by assuming $a(n) = r^n$.

3. ~~Factorize~~ solve for r (roots of the polynomial).

4. write general solution depending on type of roots.

5. use initial conditions to solve for constants.

Case 1 : Distinct Roots.

If roots are distinct $r_1, r_2, \ldots r_k$ then

$a(n) = A_1 \cdot r_1^n + A_2 \cdot r_2^n + \cdots + A_k \cdot r_k^n$

eg: $a(n) = 3a(n-1) + 4a(n-2)$, $a(0) = 1$, $a(1) = 7$,

characteristic eqn $r^2 - 3r - 4 = 0 \Rightarrow$ roots $4, -1$.

general soln $a(n) = A \cdot 4^n + B(-1)^n$.

using condn $\Rightarrow A = 8/5$, $B = 3/5$.

Final solution : $a(n) = (8/5) 4^n - (3/5)(-1)^n$.

Case 2 : Repeated Roots

If a root r has

10/9/25

1) Find the first 5 terms of the sequence defined by each of the following recurrence relations and initial conditions ?

⇒ i). $a_n = 6a_{n-1}$, where $a_0 = 2$.

$a_1 = 6 \times a_0 = 6 \times 2 = 12$ //

$a_2 = 6 \times a_1 = 6 \times 12 = 72$ //

$a_3 = 6 \times a_2 = 6 \times 72 = 432$ //

$a_4 = 6 \times a_3 = 6 \times 432 = 2592$

$a_5 = 6 \times a_4 = 6 \times 2592 = 15552$

ii) $a_n = a_{n-1}^2$; $a_1 = 2$.

$a_1 = 2$

$a_2 = a_1^2 = 4$ //

$a_3 = a_2^2 = 16$ //

$a_4 = a_3^2 = 256$ //

$a_5 = a_4^2 = 256^2$
$= 65536$

iii) $a_n = a_{n-1} + 3a_{n-2}$.
where $a_0 = 1, a_1 = 2$

$a_2 = a_1 + 3a_0$
$= 2 + 3$
$= 5$ //

$a_3 = a_2 + 3a_1$
$= 5 + 3\times 2$
$= 5+6 = 11$

$a_4 = a_3 + 3a_2$
$= 11 + 3\times 5$
$= 11 + 15$
$= \underline{\underline{26}}$

$a_5 = a_4 + 3a_3$
$= 26 + 3\times 11$
$= 26 + 33$
$= \underline{\underline{59}}$

ii) $a_n = n\,a_{n-1} + n^2 a_{n-2}$
$a_0 = 1 \quad a_1 = 1$.

$a_2 = 2\times a_1 + 2^2 \times a_0$
$= 2\times 1 + 4\times 1$
$= 6$

$a_3 = 3\times a_2 + 3^2 \times a_1$
$= 3\times 6 + 9\times 1$
$= 18 + 9$
$= 27$

$a_4 = 4\times a_3 + 4^2\times a_2 = 4\times 27 + 16\times 6 = 204$
$a_5 = 5\times a_4 + 5^2\times a_3 = 5\times 204 + 25\times 27 =$
$1020 + 675 = 1695$

v) $a_n = a_{n-1} + a_{n-3}$

$a_0 = 1, \quad a_1 = 2, \quad a_2 = 0.$

$a_3 = a_2 + a_0$

$= 0 + 1 = 1$ //

$a_4 = a_3 + a_1$

$= 1 + 2 = \cancel{2}3$ //

$a_5 = a_4 + a_2$

$= 2 + \cancel{2} = \cancel{4}.3$

vi) let $a_n = 2^n + 5 \times 3^n$.

For $n = 0, 1, 2, 3 \ldots\ldots$

a) Find a_0, a_1, a_2, a_3 and a_4.

b) show that $a_2 = 5 \times a_1 - 6 \times a_0$,

$a_3 = 5a_2 - 6a_1$ and $a_4 = 5a_3 - 6a_2$.

=> a) $a_0 = 2^0 + 5 \times 3^0$

$= 1 + 5$

$= 6$ //

$a_1 = 2^1 + 5 \times 3^1$

$= 2 + 15$

$= \cancel{16}\ 17$ //

$a_2 = 2^2 + 5 \times 3^2$

$= 4 + 5 \times 9 = 49$

$=$

$a_3 = 2^3 + 5 \times 3^3$

$= 8 + 135$

$= \underline{\underline{143}}$

$a_4 = 2^4 + 5 \times 3^4$

$= 16 + 5 \times 81$

$= 405 + 16$

$= \underline{\underline{421}}$

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b) $a_2 = 5a_1 - 6a_0$

LHS $a_2 = 49$ —— (1)

RHS

$5a_1 + 6a_0$

$\left(\begin{array}{l} a_1 = 17 \\ a_0 = 6 \\ a_2 = 49 \end{array} \right)$

$= 5(17) - 6(6)$

$= 85 - 36 = 49$ //.

$a_3 = 5a_2 - 6a_1$

LHS —— (1)

$a_3 = 143$ //

RHS

$5a_2 - 6a_1 = (5 \times 49) - (6 \times 17)$ —— (2)

$= 245 - 102 = \underline{\underline{143}}$

$a_4 = 5a_3 - 6a_2$

LHS

$a_4 = 421$

RHS

$5 \times 143 - 6 \times 49$

$= 715 - 294$

$= 421$ //.

iii) $a_n = 5a_{n-1} - 6a_{n-2}$.

LHS

$a_n = 2^n + 5.3^n$ ——— (1)

RHS

$5a_{n-1} - 6a_{n-2}$

$$\boxed{\begin{array}{l} a^{m+n} = a^m . a^n \\ a^{-m} = \dfrac{1}{m} \end{array}}$$

$= 5(2^{n-1} + 5.3^{n-1}) - 6(2^{n-2} + 5.3^{n-2})$

$= (5 \times 2^{n-1}) + (25 \times 3^{n-1}) - (6 \times 2^{n-2}) - (30 \times 3^{n-2})$

$= 5 \times 2^n \times 2^{-1}) + (25 \times 3^n \times 3^{-1}) - (6 \times 2^n . 2^{-2}) - (30 \times 3^n \times 3^{-2})$

$$\left(5\times 2^n\times 2^{-1}\right)+\left(25\times 3^n\times 3^{-1}\right)-\left(6\times 2^n\times 2^{-2}\right)-\left(30\times 3^n\times 3^{-2}\right)$$

$$=\left(5\times 2^n\times \frac{1}{2}\right)+\left(25\times 3^n\times\frac{1}{3}\right)\left(-6\times 2^n\times\frac{1}{4}\right)\left(30\times 3^n\times\frac{1}{9}\right)$$

$$=\left(5\times 2^n\times 1/2\right)-\left(6\times 2^n\times 1/4\right)+\left(25\times 3^n\times\frac{1}{3}\right)-\left(30\times 3^n\times\frac{1}{9}\right)$$

$$=2^n\left[\frac{5}{2}-\frac{6}{4}\right]+3^n\left[\frac{25}{3}-\frac{\cancel{30}\,10}{\cancel{9}\,3}\right]$$

$$=2^n[1]+3^n[5]$$

$$=2^n+3^n\cdot 5 \text{ ——— } (2)$$

(1) = (2) ∴ LHS = RHS Hence Proved.

iv) Check if the sequence $a_n = 3n$ for every non-negative integer is a solution of the recurrence relation $a_n = 2a_{n-1} - a_{n-2}$ for $n = 2, 3, 4$.

$\Rightarrow$ LHS : $a_n = 3n$ ——— (1)

RHS

$$2a_{n-1} - a_{n-2}.$$

$a_n = 3n$

$\therefore a_{n-1} = 3(n-1)$

$a_{n-2} = 3(n-2)$

$$2\left(3(n-1)\right) - 3(n-2)$$

$= ~~10~~$

$6n - 6 - 3n + 6$

$= 3n$ —— (2)

hence $\{a_n = 3n\}$ is a solution of the recurrence relation.

v) $\{a_n\} = \{2^n\}$ —— (1)

$a_n = 2a_{n-1} - a_{n-2}$ for $n = 2, 3, 4$

$a_n = 2[2^n - 1] - [2^n - 2]$

$= [4^n - 2] - [2^n - 2]$

$= 4^n - 2^n = 2^n$ —— (2)

$a_n = 2^n$
$a_{n-1} = 2^{n-1}$
$a_{n-2} = 2^{n-2}$

LHS = RHS

Hence $\{a_n\} = \{2^n\}$ is a solution of the recurrence relation.

$> 2(2^{n-1}) - 2^{n-2}$

$= 2(2^n \times 2^{-1}) - 2^n \times 2^{-2}.$

$2 \times 2^n \times \frac{1}{2} - 2^n \times \frac{1}{4}$

$2^n \left[1 - \frac{1}{4}\right] = \frac{3}{4} \cdot 2^n$

LHS $\neq$ RHS.

Hence it is not a recursive soln.

19/9/25

1) check if the sequence $\{a_n\}$ is a solution of the recurrence relation ~~$a_n = -3a_{n-1}$~~

$a_n = -3a_{n-1} + 4a_{n-2}$.

if a) $a_n = 0$

b) $a_n = 1$

c) $a_n = (-4)^n$

d) $2(-4)^n + 3$.

a)

LHS $a_n = 0$

RHS $-3a_{n-1} + 4a_{n-2}$

$= -3(0) + 4(0) = 0$ //.

b) LHS $a_n = 1$

RHS

$-3a_{n-1} + 4a_{n-2}$

$= -3(1) + 4(1) = -3+4 = 1$ //.

c) LHS $a_n = (-4)^n$. ——— ①

RHS

$-3a_{n-1} + 4a_{n-2}$.

$-3\left((-4)^{n-1}\right) + 4\left((-4)^{n-2}\right)$

$= -3\left[(-4)^n(-4)^{-1}\right] + 4\left[(-4)^n(-4)^{2}\right]$

$= -3\left[(-4)^n \frac{1}{(-4)}\right] + 4\left[(-4)^n\left(\frac{1}{-4}\right)^2\right]$

$= \frac{-3}{-4}(-4)^n + \frac{4}{16}(-4)^n$.

$= (-4)^n\left[\frac{-3}{-4} + \frac{1}{4}\right] = 1$

$= (-4)^n$ // ——— ②

① = ② LHS = RHS ∴ the given solution is a recurrence relation.

d) $2(-4)^n + 3$.

Solutions to recurrence relation

· Characteristic equation

Linear Homogeneous Recurrence Relation with constant co-efficients.

The general form of a LHRR is given by

$a_n = c_1 \cdot a_{n-1} + c_2 \cdot a_{n-2} + \dots + c_k \cdot a_{n-k}$.

where $c_1, c_2, c_3 \dots c_k$ are constants.

The recurrence relation in the definition is the relation

i) linear because the right hand side is a sum of previous terms of the sequence (or a multiple of it.

ii) Homogeneous because no terms occur that are not multiples of the previous terms of the sequence.

iii) the co-efficient of the terms of the sequence are all constants rather than functions that depend on n.

iv) In a recurrence relation if a_n is expressed in terms of the previous k terms of the seq then degree of the recurrence relation is said to be k.

2) Determine which of the following are linear homogeneous recurrence relation with constant co-efficients. Also find the degree of those ~~...~~ that are.

i) $a_n = 3a_{n-1} + 4a_{n-2} + 5a_{n-3}$ $(k=3)$

ii) $a_n = 2n \cdot a_{n-1} + a_{n-2}$

iii) $a_n = a_{n-1} + a_{n-4}$

iv) $a_n = a_{n-1} + 2$

v) $a_n = a_{n-2}^2$

vi) $a_n = a_{n-1} + a_{n-2}$

vii) $a_n = a_{n-1} + n$.

viii) $a_n = 3a_{n-2}$

ix) $a_n = a^2_{n-1}$

x) $a_n = 3$

xi) $a_n = a_{n-1} + \cancel{5}\, 2a_{n-3}$

xii) $a_n = \frac{a_{n-1}}{n}$

xiii) $a_n = a_{n-1} + a_{n-2} + n + 3$

xiv) $a_n = 4a_{n-2} + 5a_{n-4} + 9a_{n-7}$.

(If linear
RHS dependent
= 1)

solve

1) $a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 2$, $a_1 = 7$.

$a_n = r^n$

$r^n = r^{n-1} + 2r^{n-2}$

$1 = 1/r + 2/r^2$

$r^2 = r + 2$

$\overset{a}{r^2} - \overset{b}{r} - \overset{c}{2} = 0$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1+8}}{2}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4 \times -2}}{2 \times 1}$$

$$= \frac{-1 \pm \sqrt{1+8}}{2}$$

$= \frac{1 \pm 3}{2} = 2, -1$ //

$r_1 = 2, \; r_2 = -1$

$r_1, r_2, r_3 \ldots\ldots$

$a_n = c_1(r_1)^n + c_2(r_2)^n + c_3(r_3)^n \ldots$

$a_n = c_1(2)^n + c_2(-1)^n$ — (1)

$a_0 = c_1 \cdot 1 + c_2 \cdot 1$

$\boxed{2 = c_1 + c_2}$ — (A)

$a_1 = c_1(2) + c_2(-1)$

$\boxed{7 = 2c_1 - c_2}$ — (B)

$2 = c_1 + c_2$

$7 = 2c_1 - c_2$

(A)+(B) $9 = 3c_1$

$c_1 = 3$ //

(A) $= 2 = 3 + c_2$

$c_2 = -1$ //

$a_n = 3(2^n) - 1(-1)^n$

2) $a_n = a_{n-1} + 6 \times a_{n-2}$. com $a_0 = 3$, $a_1 = 6$.

$a_n = r^n$.

$r^n = r^{n-1} + 6 \times r^{n-2}$.

$\frac{r^n}{r^n} = \frac{r^{n-1}}{r^n} + \frac{6 r^{n-2}}{r^n}$

$1 = r^{-1} + 6r^{-2}$

$1 = 1/r + 6 \cdot 1/r^2$.

$r^2 = r + 6$

$r^2 - r - 6 = 0$

$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times -6}}{2 \times 1}$

$= \frac{-1 \pm \sqrt{1 + 24}}{2}$

$= \frac{1 \pm 5}{2} = 3, -2$.

$a_n = C_1 (3)^n + C_2 (-2)^n$ —— ①

$a_0 = 3$
$a_1 = 6$

$n = 0 - 3 = C_1 + C_2$ —— ⓐ

$n = 1 = 6 = 3C_1 - 2C_2$ —— ⓑ

Solving ① and ②

$a \times 2 = 6 = 2c_1 + 2c_2$

$b \quad = \dfrac{6 = 3c_1 - 2c_2}{12 = 5c_1 \quad 0}$

$c_1 = 12/5$ //

$3 = 12/5 + c_2$

$c_2 = 3 - 12/5$

$= 3/5$

Substitute in ①

$a_n = \frac{12}{5}(3)^n + 3/5\,(-2)^n$

solve the recurrence relation

$a_n = a_{n-1} - 1/4\, a_{n-2}$

where $a_0 = 1,\ a_1 = 2$.

$\Rightarrow$ i) $r^n = r^{n-1} - 1/4\, r^{n-2}$

$1 = 1/r - 1/4 \cdot 1/r^2$

$r^2 = r - 1/4$

$r^2 - r + 1/4 = 0$

$4r^2 - 4r + 1 = 0$

soly, $r = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$$= \frac{(-4) \pm \sqrt{4^2 - 4 \times 4 \times 1}}{2 \times 4}$$

$$= \frac{4 \pm \sqrt{16-16}}{8} = \frac{4}{8} = 1/2$$

$r = 1/2, 1/2$

$a_n = (c_1 + n c_2)(1/2)^n$.

$n = 0 \Rightarrow a_0 = c_1 (1/2)^0 = c_1 \Rightarrow 1 = c_1$

$n = 1 \Rightarrow a_1 = (c_1 + c_2)(1/2) \Rightarrow 2 = (1 + c_2) 1/2$

$4 = 1 + c_2 \Rightarrow c_2 = 3$

$\Rightarrow$ solve $a_n = -6a_{n-1} - 9a_{n-2}$.

IF $a_0 = 2$ $a_1 = 3$.

As : $a_n = r^n$

$r^n - 6r^{n-1} - 9r^{n-2}$

$$1 = \frac{6r^{n-1}}{r^n} - \frac{9r^{n-2}}{r^n}$$

$$1 = \frac{-6}{r} - \frac{9}{r^2}$$

$r^2 = -6r - 9$

$r^2 + 6r + 9 = 0$

solving $r = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-6 \pm \sqrt{6^2-4\times1\times9}}{2\times1}$

$= \frac{-6 \pm \sqrt{36-36}}{2}$

$= \frac{-6}{2}, -3, -3$ //

$a_n = (c_1 + n c_2)(-3)^n$

$n=0 \quad a_0 = (c_1)(-3)^0 = c_1$

$a_0 = 2 \quad c_1 = 2.$

$n=1$

$a_1 = (c_1+c_2)(-3) = (2+c_2)(-3)$

$= 3 - 6 - 3c_2$

$9 = -3c_2$

$c_2 = -3$ //

$\therefore a_n = (2-3n)(-3)^n$

Unit 3

$A = \{a, b, c\}$

$P(A) = \{\{a, b, c\}, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$

Powerset of A.

$|A| = 3$.

$|P(A)| = 2^3 = 8$ //.

2) list the members of these set.

⇒ i) $\{x : x \text{ is a real number} : x^2 = 1\}$

ii) $\{x : x \text{ is a positive integer less than } 12\}$

iii) $\{x : x \text{ is the square of an integer and } x < 100\}$

iv) $\{x : x \text{ is an integer such that } x^2 = 2\}$

v) $\{x : x \text{ is a non-negative integer less than } 5\}$

3) write the set builder form for the following sets.

i) $\{0, 3, 6, 9, 12\}$.

ii) $\{0, -3, -2, -1, 1, 2, 3\}$.

iii) { m, n, o, p }

II Ques

i) ⇒ { 1, -1 }.

ii) ⇒ { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 }

iii) ⇒ { 1, 4, 9, 16, 25, 36, 49, 64, 81 }

iv) { }.

v) { 0, 1, 2, 3, 4 }.

III i) { x / 0 ≤ x ≤ 12, x is a whole number and a multiple of 3 }.

ii) { x : x is an integer : -3 ≤ x ≤ 3 }

iii) { x : x is an alphabet. blw l and q. }.

10) Determine whether each of these pairs of sets are equal.

i) $\{1, 3, 3, 3, 5, 5, 5, 5, 5\}$.

$\{5, 3, 1\}$

ii) $\{1, \{1\}\}$, $\{\{1\}\}$

=> i) members or elements are same. ∴ sets are same.

ii) they are not same cause one is an element 1 and a set of element 1.

11) Suppose $A = \{2, 4, 6\}$, $B = \{2, 6\}$
$C = \{4, 6\}$ and $D = \{4, 6, 8\}$.

determine which of these sets are subsets of which other of these sets.

=> A is not subset of B cause 4 not present.
B is a subset of A.
C is a subset of D.

7.10.25

Relations

A relation is the subset of the cartisian product.

Representing Relations

Representing using matrices

A relation b/w finite sets.
→ a set contains finite number of elements

can be represented using a matrix. suppose R is a relation from $A = \{a_1, a_2 \ldots a_m\}$. To $B = \{b_1, b_2 \ldots b_n\}$.
A has got m number of elements and B has got n number of elements. The Relation R can be represented by the matrix. $M_R = \{m_{ij}\}$

where $m_{ij} \begin{cases} 1, & (i,j) \in R \\ 0, & (i,j) \in R \end{cases}$

1) let $A = \{1, 2, 3\}$
$B = \{1, 2\}$

let R be the relation from A to B

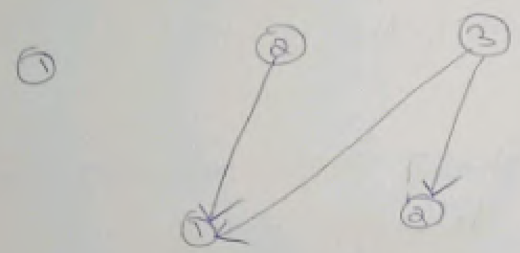
such that $R = \{(2,1), (3,1), (3,2)\}$

What is the Matrix representation of R.

$$\Rightarrow \begin{array}{cc} & B \\ A & \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{array} \end{array}$$

(2,1), (3,1) (3,2) is related

graphic representation.



2) let R be a relation from set A to B. which is represented by the following matrix. identify the ordered pairs that are presented R.

$$M_R = \begin{array}{cc} & B \\ A & \begin{array}{c|ccccc} & b_1 & b_2 & b_3 & b_4 & b_5 \\ \hline a_1 & 0 & 1 & 0 & 0 & 0 \\ a_2 & 1 & 0 & 1 & 1 & 0 \\ a_3 & 1 & 0 & 1 & 0 & 1 \end{array} \end{array}$$

let $A = \{a_1, a_2, a_3\}$

$B = \{b_1, b_2, b_3, b_4, b_5\}$

$R = \{(a_1, b_2)\ (a_3, b_3)\ (a_2, b_1)\ (a_3, b_1)\ (a_2, b_3)\ (a_3, b_5)\ (a_2, b_4)\ (a_3, b_1)\}$

3) Find the matrix representing the following relation defined on set $\{1, 2, 3, 4\}$

$R = \{(1,1), (1,3), (2,1)\ (2,3), (2,4)\ (3,1)\ (3,2)\ (4,1)\}$

4) Find the ordered pairs in the relation R represented by the following graphs.