MULTI-PERIOD PORTFOLIO OPTIMIZATION: MAXIMIZING SHARPE RATIO TO ALTERNATE RISK MEASURES

Yarden Fogel **Menachem Finkelstein**

INTRODUCTION

- We evaluated dynamic multi–period constrained optimization of investment portfolios with alternative risk measures compared to conventional mean–variance portfolio optimizations.

- We aimed to construct an optimal portfolio by considering different risk measures and definitions, exploring the boundaries of portfolio weights, various constraint functions, and comparing the performance of dynamically updated multi–period optimization (MPO) to that of single–period optimization (SPO)*. We also compared our results to those of uniform or equal–weighted portfolios.

- Our primary **objective function** was to achieve the highest possible risk–adjusted return, as measured by the Sharpe Ratio.

- Our **hypothesis** was that MPO portfolios would outperform SPO portfolios, and that optimized portfolios would outperform equal–weighted portfolios.

*Inspiration from Boyd, et al – https://stanford.edu/~boyd/papers/pdf/cvx_portfolio.pdf

METHODOLOGY

- **Data Collection:** We downloaded 13 years of daily stock price data from 2010-2022 on 29 securities using Yahoo! Finance API – 13 of the 29 were ETFs (Exchange-Traded Funds) covering broad asset categories like broad market, real estate, tech, commodities, bonds, emerging markets, etc, and the other 16 tickers were the largest market cap companies like Apple, Microsoft, etc. The tickers, names and market caps can be seen in the figure on the right.
- **Library Utilization:** We employed a portfolio optimization library, Riskfolio-Lib, to calculate the optimal portfolio weights based on historical returns for each year. We also ran optimizations with PyPortfolioOpt, CVXPortfolio, and others in our testing.
- **Multi-Period Optimization:** We adjusted the portfolio weights annually, taking into account updated market data and resulting risk measures, and all optimizations were based on trailing 5-year price data.
- **Testing and Evaluation:** We ran a litany of tests of analyses, but primarily focused on our Sharpe Ratio objective and compared the risk-adjusted returns of MPO, SPO and equal-weighted portfolios to to assess the benefits of our methodology.

Symbol	Company Name	Market Cap
GSG	iShares S&P GSCI Commodity-Indexed Trust	1.232B
EEM	iShares MSCI Emerging Markets ETF	29.908B
QQQ	Invesco QQQ Trust	143.941B
SPY	SPDR S&P 500 ETF Trust	400.621B
VNQ	Vanguard Real Estate Index Fund	30.077B
BND	Vanguard Total Bond Market Index Fund	-
IGOV	iShares International Treasury Bond ETF	-
GLD	SPDR Gold Shares	46.258B
XLF	Financial Select Sector SPDR Fund	29.048B
XLV	Health Care Select Sector SPDR Fund	26.146B
XRT	SPDR S&P Retail ETF	753.842M
FXI	iShares China Large-Cap ETF	3.552B
BTC-USD	Bitcoin USD	581.918B
AAPL	Apple Inc.	2.941T
TSLA	Tesla, Inc.	838.681B
MSFT	Microsoft Corporation	2.56T
GOOG	Alphabet Inc.	1.569T
AMZN	Amazon.com, Inc.	1.335T
META	Meta Platforms, Inc.	730.071B
V	Visa Inc.	478.34B
JPM	JPMorgan Chase & Co.	407.893B
XOM	Exxon Mobil Corporation	417.64B
WMT	Walmart Inc.	419.41B
JNJ	Johnson & Johnson	430.402B
UNH	UnitedHealth Group Incorporated	446.011B
BRK-B	Berkshire Hathaway Inc.	736.615B
BABA	Alibaba Group Holding Limited	228.951B
0700.HK	Tencent Holdings Limited	3.226T
MC.PA	LVMH Moët Hennessy - Louis Vuitton, Socié	417.665B

CONSTRAINTS / BOUNDARY CONDITIONS

- The constraints and boundary conditions we activated provided a more realistic active-management portfolio environment, along with added flexibility in constructing the portfolio and optimizing risk-adjusted returns over multiple periods.

- availability of constraint & objective functions differed between the two primary libraries we employed (Riskfolio-Lib & PyPortfolioOpt), and unfortunately were relatively limited with Riskfolio, which was our predominant optimization tool.

- In our optimizations with Riskfolio, we added the following constraints, enabling both shorting and leverage:
  - **Allowing shorts (negative weights):** profiting from falling prices in those securities.
  - **Exposure limits and leverage:** Upper bounds on the total sum of weights for each of longs and shorts of 200% and 100%, respectively, with a total portfolio budget constraint, where budget = upper limit on longs – upper limit on shorts. The upper bounds placed boundaries on the net exposure of the portfolio, along with the gross exposure or total leverage allowed.

SHARPE RATIO

- The Sharpe ratio is a widely-accepted, easily-calculated metric for evaluating the risk-adjusted return performance of an investment portfolio.
- It measured the excess return earned per unit of risk undertaken. “Excess return” is defined by the portfolio return in excess of the risk-free rate (typically US T-Bills or the like), while “units of risk” can be defined in many ways, with those varying definitions being one of the primary focal points of our project.
- Higher Sharpe Ratios indicate better risk-adjusted performance, which was the primary optimization objective function, i.e. “Max Sharpe”.

$$Sharpe = \frac{r_{portfolio} - r_{risk-free}}{risk_{portfolio}}$$

RISK MEASURES STUDIED AND OPTIMIZED

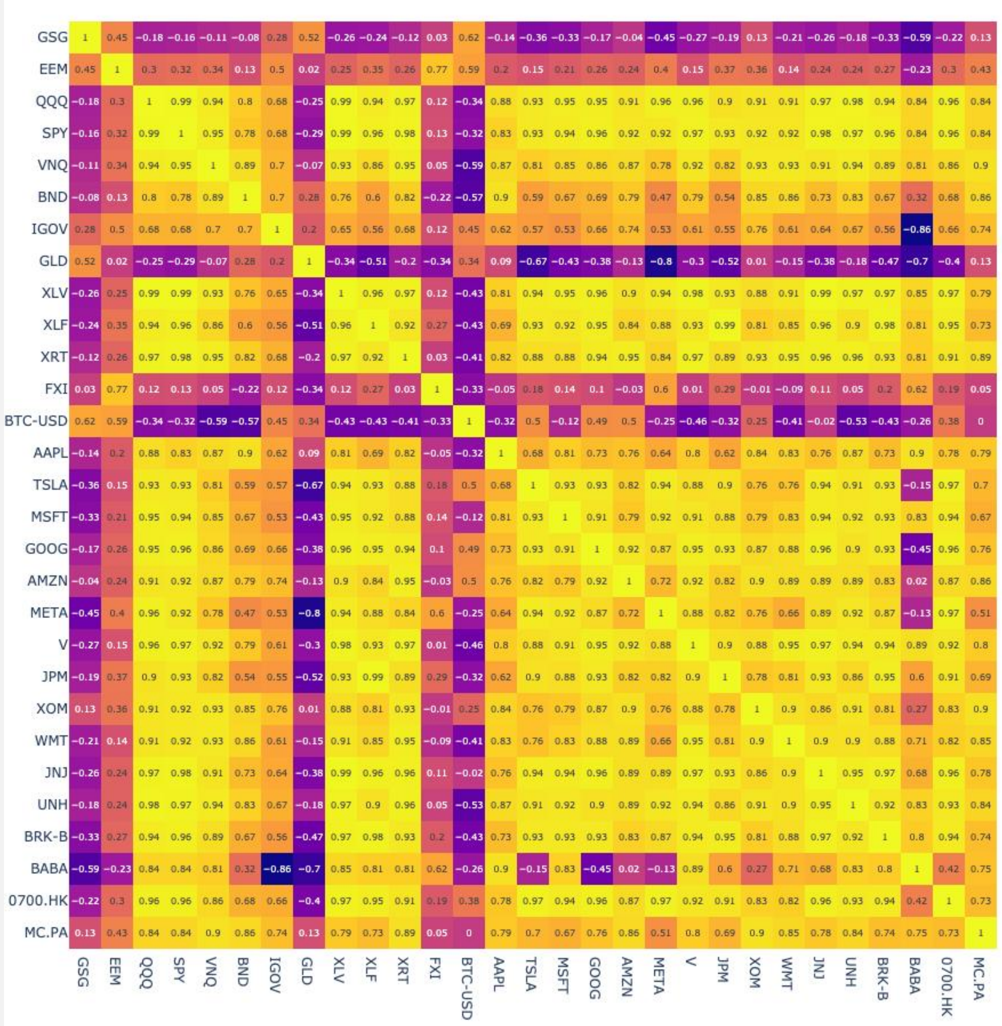
- **Dispersion Risk Measures:**
  - MV (Mean-Variance, standard deviation) : Quantifies the dispersion of returns around the mean and serves as a measure of total risk.
  - MAD (Mean Absolute Deviation): Calculates the average deviation of individual portfolio asset returns from the mean, providing a measure of dispersion and volatility.
- **Downside Risk Measures:**
  - FLPM (First Lower Partial Moment, Omega Ratio): Defined as the probability-weighted ratio of gains versus losses for some threshold return target.
  - CVaR (Conditional Value-at-Risk): Measures the expected loss when a specific threshold of the worst-case outcomes is exceeded.
- **Drawdown Risk Measures:**
  - UCI (Ulcer Index): Technical indicator that measures downside risk in terms of both the depth and duration of price declines. The index increases in value as the price moves farther away from a recent high and falls as the price rises to new highs.
  - CDaR (Conditional Drawdown at Risk): Measures the expected maximum loss a portfolio may experience during adverse market conditions at a specific confidence level, taking into account the severity and duration of potential drawdowns.
  - MDD (Maximum Drawdown, Calmar Ratio): Measures the largest peak-to-trough decline in portfolio value over a specific time period, indicating the maximum loss an investor could experience from a peak to subsequent low.

RISKFOLIO OPTIMIZATION: UNDER THE HOOD

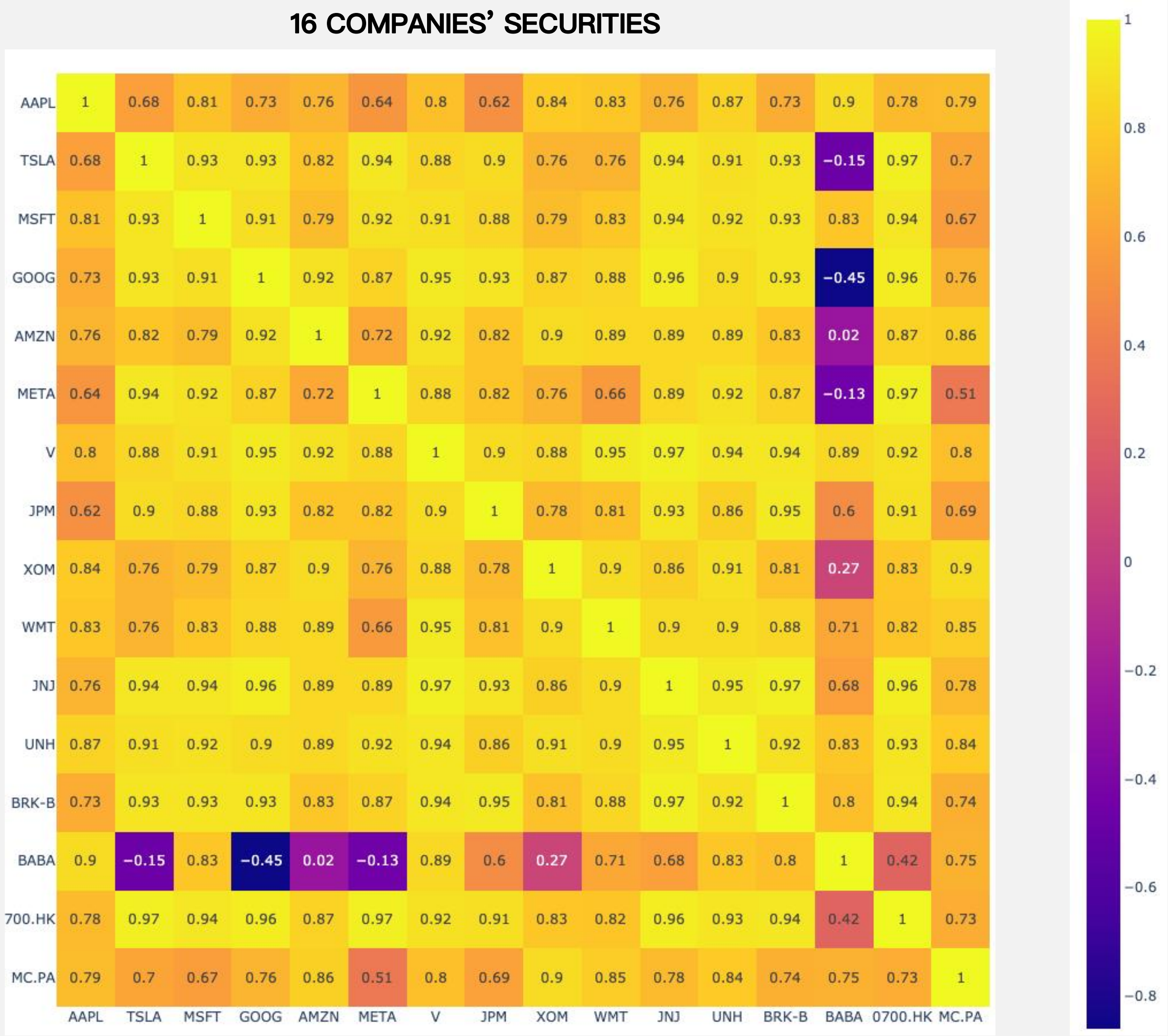
- Riskfolio-Lib utilizes the Hierarchical Risk Parity (HRP) algorithm for optimizing portfolios. HRP is a clustering-based approach that constructs a hierarchical structure of assets based on their pairwise correlations.
- By considering risk contributions and correlations, the algorithm allocates weights to clusters and individual assets, aiming to achieve risk parity. This ensures that each asset or cluster contributes an equal amount of risk to the portfolio, promoting diversification and optimizing risk-return characteristics.
- Riskfolio-Lib integrates HRP along with other optimization methods, providing users with a comprehensive toolkit for portfolio optimization and risk analysis.

CORRELATION BETWEEN SECURITIES

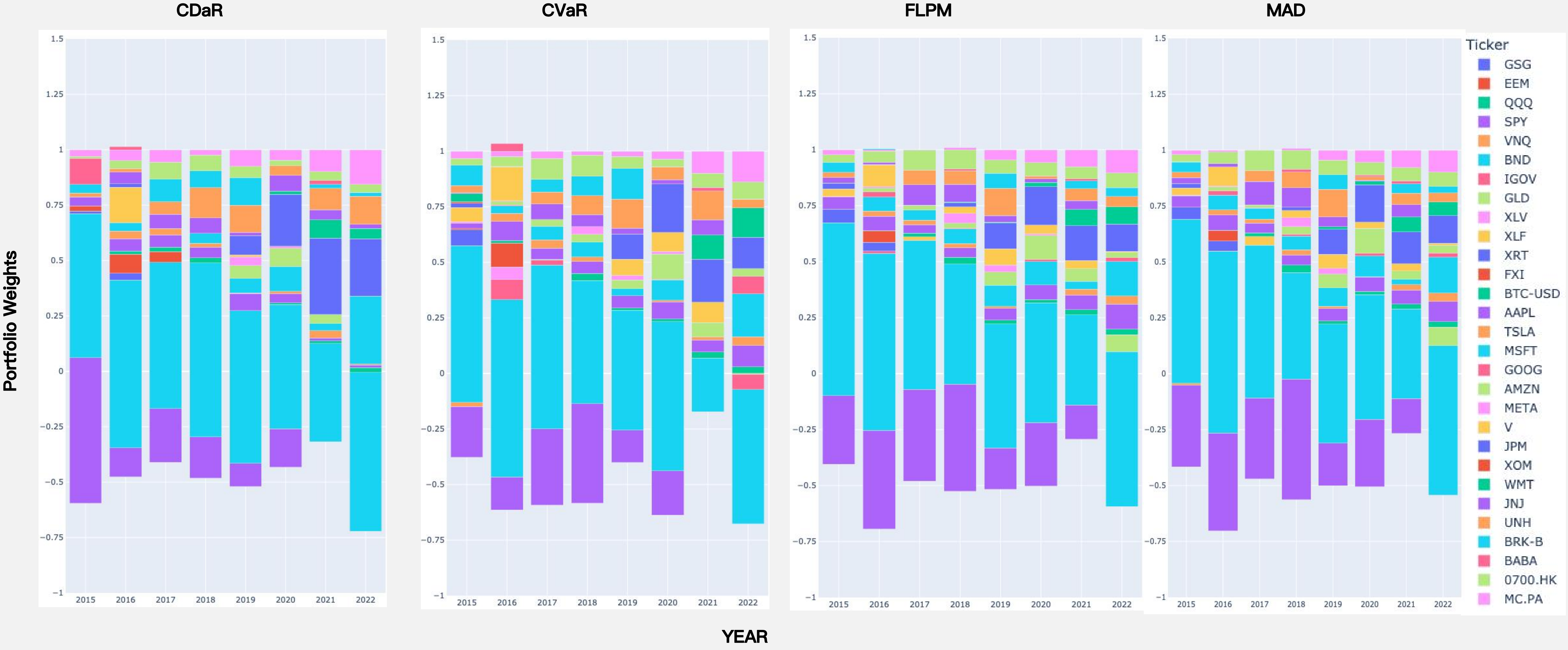
ALL 29 SECURITIES



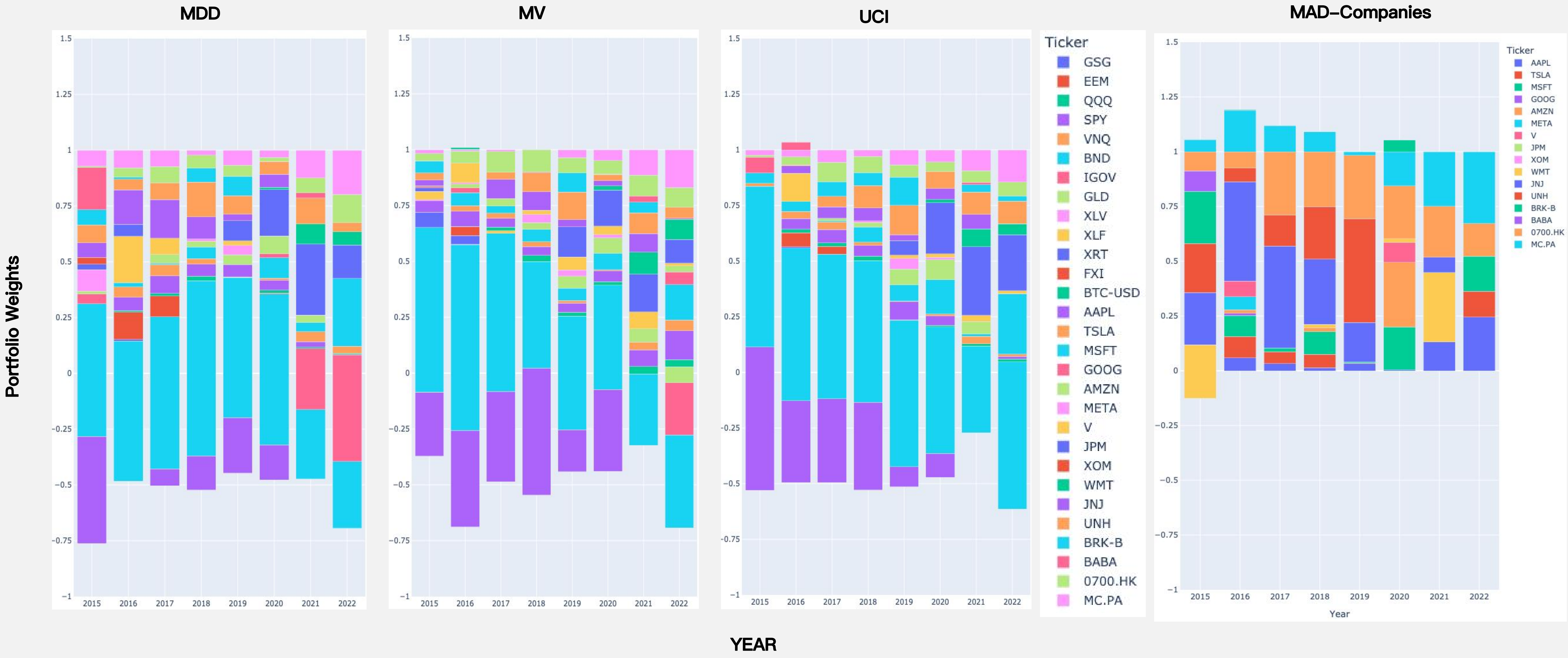
16 COMPANIES' SECURITIES



PORTFOLIO WEIGHTS DISTRIBUTION BY YEAR



PORTFOLIO WEIGHTS DISTRIBUTION BY YEAR



PORTFOLIO WEIGHTS BY RISK MEASURE: 2016 VS. 2020, ALL TICKERS VS. COMPANIES

2016 – ALL

	CVaR	MV	MDD	MAD	FLPM	UCI	CDaR
GSG	-9.43%	-10.40%	-9.34%	-9.37%	-8.45%	-6.97%	-6.39%
EEM	-34.31%	-24.77%	-21.84%	-25.72%	-25.26%	-18.26%	-17.95%
QQQ	-17.63%	-33.71%	-8.89%	-35.23%	-35.65%	-24.24%	-23.19%
SPY	16.01%	46.01%	0.00%	45.90%	45.41%	42.33%	20.57%
VNQ	-1.24%	-2.78%	-8.22%	-2.04%	-1.36%	-5.58%	-7.59%
BND	79.80%	83.17%	80.95%	81.18%	78.96%	87.67%	90.00%
IGOV	12.37%	1.71%	9.93%	-0.05%	4.26%	2.30%	3.87%
GLD	-3.41%	-0.57%	-6.08%	0.58%	-0.79%	-4.42%	-5.82%
XLV	23.41%	14.51%	5.26%	15.06%	14.16%	4.93%	7.68%
XLF	-17.12%	-15.50%	-27.25%	-15.48%	-16.53%	-21.90%	-20.03%
XRT	-0.49%	3.87%	0.69%	4.59%	3.90%	0.56%	3.12%
FXI	10.70%	4.07%	12.14%	4.65%	5.12%	6.36%	8.52%
BTC-USD	1.15%	-0.03%	0.81%	0.08%	0.10%	1.55%	1.48%
AAPL	8.61%	6.93%	5.89%	6.91%	6.44%	4.83%	5.58%
TSLA	3.65%	2.44%	4.76%	2.19%	2.33%	3.13%	3.50%
MSFT	3.32%	5.85%	5.67%	6.56%	6.20%	5.94%	6.73%
GOOG	0.39%	2.12%	-1.61%	2.01%	2.39%	-1.46%	-2.00%
AMZN	1.58%	1.76%	-0.65%	1.64%	1.67%	1.16%	0.27%
META	0.34%	0.55%	-1.60%	0.46%	0.58%	-1.15%	-1.25%
V	15.24%	9.97%	20.82%	9.22%	9.79%	13.06%	15.94%
JPM	5.46%	5.38%	6.67%	5.53%	6.18%	5.14%	6.22%
XOM	4.93%	0.38%	1.30%	0.45%	-0.19%	0.34%	1.27%
WMT	-4.01%	-6.26%	-2.53%	-6.68%	-5.85%	-5.88%	-5.86%
JNJ	-1.97%	1.99%	15.27%	2.95%	2.52%	3.41%	5.21%
UNH	-0.08%	3.30%	4.93%	4.00%	4.47%	3.14%	3.27%
BRK-B	6.09%	-0.14%	12.75%	-0.48%	-1.31%	7.42%	8.24%
BABA	-10.31%	-5.84%	-11.98%	-4.96%	-4.61%	-10.14%	-9.91%
0700.HK	4.49%	5.39%	4.19%	5.22%	4.94%	3.61%	3.64%
MC.PA	2.45%	0.58%	7.98%	0.80%	0.59%	3.14%	4.91%

2020 – ALL

CVaR	MV	MDD	MAD	FLPM	UCI	CDaR
-4.57%	-1.39%	-1.22%	-1.82%	-2.52%	-0.54%	-1.37%
-6.09%	-8.97%	-9.29%	-9.32%	-8.20%	-9.85%	-7.29%
-53.09%	-33.55%	-37.29%	-39.42%	-39.55%	-36.77%	-34.64%
23.86%	41.20%	23.14%	34.23%	31.30%	14.85%	21.96%
-3.94%	-4.67%	-7.40%	-4.15%	-2.85%	-4.17%	-4.56%
87.82%	80.69%	97.04%	82.92%	81.20%	86.94%	90.39%
-5.71%	-3.06%	-13.09%	-3.56%	-4.70%	-8.34%	-8.58%
8.21%	5.82%	10.59%	6.48%	7.30%	10.35%	9.47%
-1.62%	-8.41%	-8.45%	-6.11%	-7.25%	-7.35%	-11.38%
-15.32%	-19.08%	-18.54%	-15.79%	-14.34%	-22.64%	-20.39%
-0.38%	-3.70%	0.00%	-3.63%	-3.40%	0.00%	-1.44%
-5.84%	-5.53%	0.45%	-4.36%	-5.34%	-2.12%	-1.95%
1.20%	1.60%	1.36%	1.42%	1.43%	0.77%	0.71%
7.71%	4.99%	4.53%	6.34%	6.51%	4.29%	4.22%
0.63%	0.35%	0.77%	0.19%	-0.01%	0.83%	0.91%
9.18%	7.24%	9.27%	9.25%	10.48%	16.91%	12.80%
0.26%	0.00%	1.66%	1.15%	0.84%	-1.48%	-1.60%
11.46%	6.99%	8.50%	11.29%	10.79%	8.97%	8.40%
1.08%	1.44%	-0.46%	0.00%	0.75%	1.00%	0.77%
8.64%	3.77%	-0.00%	2.71%	3.88%	1.62%	-0.00%
24.54%	23.92%	23.85%	24.75%	24.57%	27.34%	27.39%
-2.82%	-7.79%	-3.02%	-7.99%	-7.36%	-4.30%	-3.98%
0.00%	1.98%	0.96%	1.58%	1.81%	1.47%	1.49%
1.96%	2.28%	5.76%	0.39%	1.60%	4.90%	7.09%
6.45%	6.59%	7.04%	5.53%	5.67%	8.24%	7.14%
-0.61%	-3.85%	-1.24%	-3.85%	-4.48%	1.94%	-0.22%
0.02%	0.18%	-0.00%	0.72%	0.23%	-2.43%	-2.60%
3.42%	6.16%	1.77%	5.61%	5.97%	4.18%	2.58%
3.56%	4.80%	3.31%	5.44%	5.68%	5.42%	4.68%

2016 – COMPANIES

	CVaR	MV	MDD	MAD	FLPM	UCI	CDaR
AAPL	0.84%	6.82%	13.52%	5.87%	4.46%	18.53%	23.30%
TSLA	12.53%	10.38%	6.90%	9.68%	9.72%	9.54%	8.59%
MSFT	4.67%	11.89%	-0.00%	9.67%	10.72%	21.21%	24.66%
GOOG	-0.00%	2.48%	-4.57%	1.01%	1.79%	2.84%	0.00%
AMZN	3.83%	1.43%	-0.00%	1.67%	0.40%	-4.76%	-5.46%
META	8.91%	6.52%	-0.00%	5.82%	6.79%	-2.77%	1.77%
V	61.81%	51.70%	65.40%	61.83%	61.97%	73.47%	65.94%
JPM	-20.76%	-16.50%	-48.01%	-18.68%	-18.61%	-28.56%	-32.09%
XOM	-47.93%	-39.49%	-36.60%	-33.98%	-37.56%	-24.42%	-29.26%
WMT	-9.13%	-1.82%	-0.00%	-1.97%	-2.65%	-3.17%	3.09%
JNJ	49.22%	47.52%	65.04%	45.33%	46.40%	35.50%	41.23%
UNH	35.36%	34.85%	24.56%	32.91%	32.77%	22.90%	19.40%
BRK-B	-0.00%	-6.33%	17.60%	-7.11%	-4.79%	9.42%	7.63%
BABA	-12.84%	-17.83%	-0.00%	-19.48%	-18.47%	-27.20%	-21.17%
0700.HK	22.83%	26.42%	6.97%	26.20%	24.97%	6.59%	4.39%
MC.PA	-9.35%	-18.02%	-10.81%	-18.78%	-17.93%	-9.12%	-12.02%

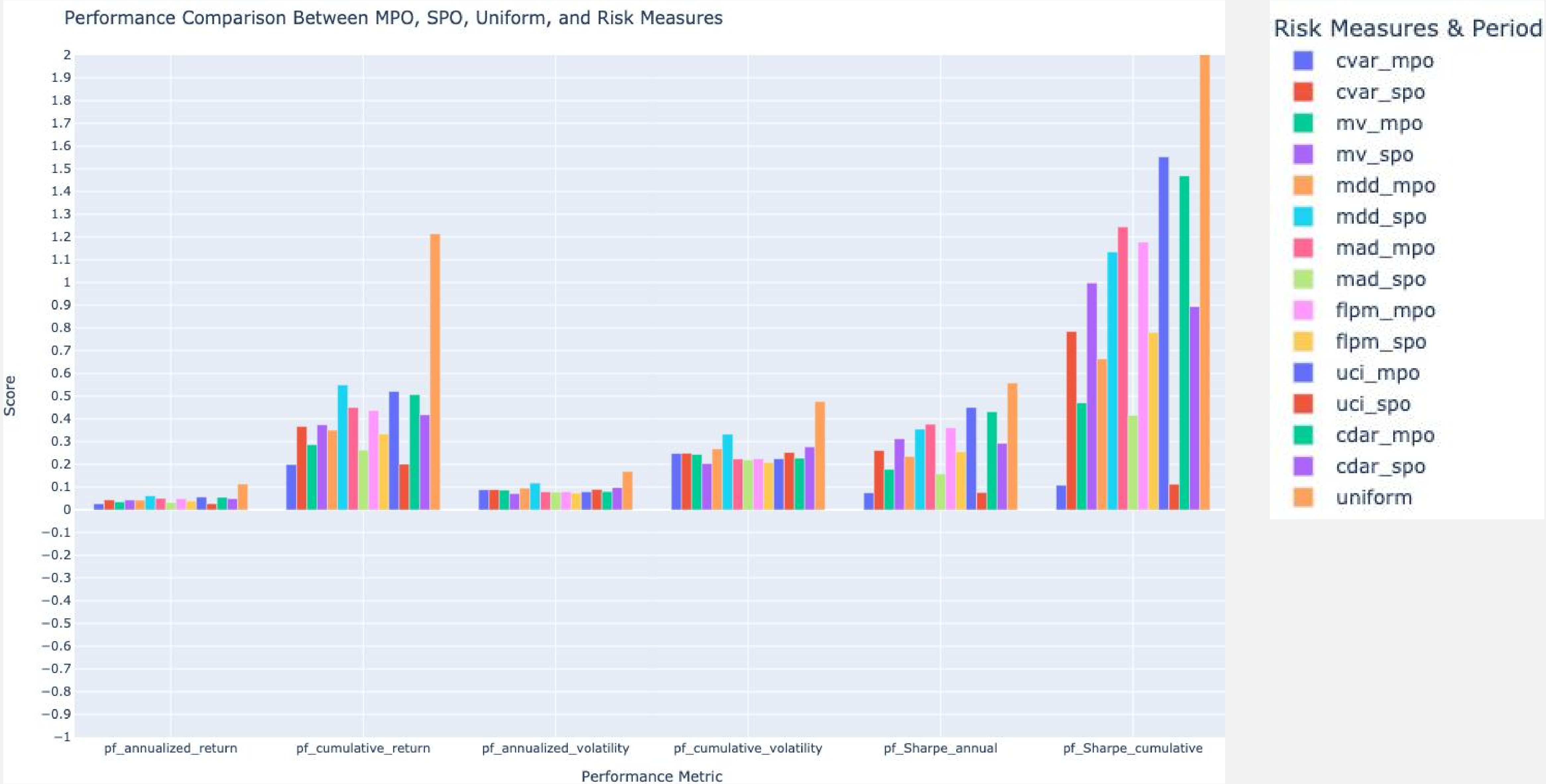
2020 – COMPANIES

CVaR	MV	MDD	MAD	FLPM	UCI	CDaR
1.80%	3.82%	6.48%	3.80%	2.44%	1.31%	0.00%
-8.25%	-3.43%	-13.03%	-3.48%	-3.15%	-6.48%	-12.57%
17.12%	18.06%	31.63%	26.70%	25.78%	52.65%	42.76%
-2.32%	-6.85%	0.00%	-7.05%	-5.10%	-8.17%	1.86%
41.13%	30.35%	-0.00%	35.78%	37.71%	13.18%	4.96%
-10.98%	-0.29%	-0.00%	-6.33%	-7.11%	-6.70%	-0.15%
0.00%	14.42%	33.89%	11.32%	11.35%	34.26%	33.85%
18.11%	32.61%	-0.00%	35.85%	36.17%	10.17%	7.79%
-40.67%	-44.06%	-56.74%	-38.13%	-35.82%	-46.73%	-54.53%
2.60%	9.02%	9.04%	8.11%	8.46%	8.87%	15.41%
14.90%	13.27%	43.70%	10.89%	7.30%	9.21%	25.61%
48.52%	30.28%	35.07%	27.79%	27.48%	37.64%	37.24%
-15.51%	-28.35%	-0.00%	-30.23%	-32.13%	-5.90%	-1.65%
-22.28%	-17.01%	-30.22%	-14.78%	-16.69%	-26.03%	-31.10%
27.38%	26.56%	33.38%	24.14%	26.06%	21.81%	23.27%
28.45%	21.60%	6.83%	15.61%	17.25%	10.89%	7.25%

PERFORMANCE COMPARISON TABLE

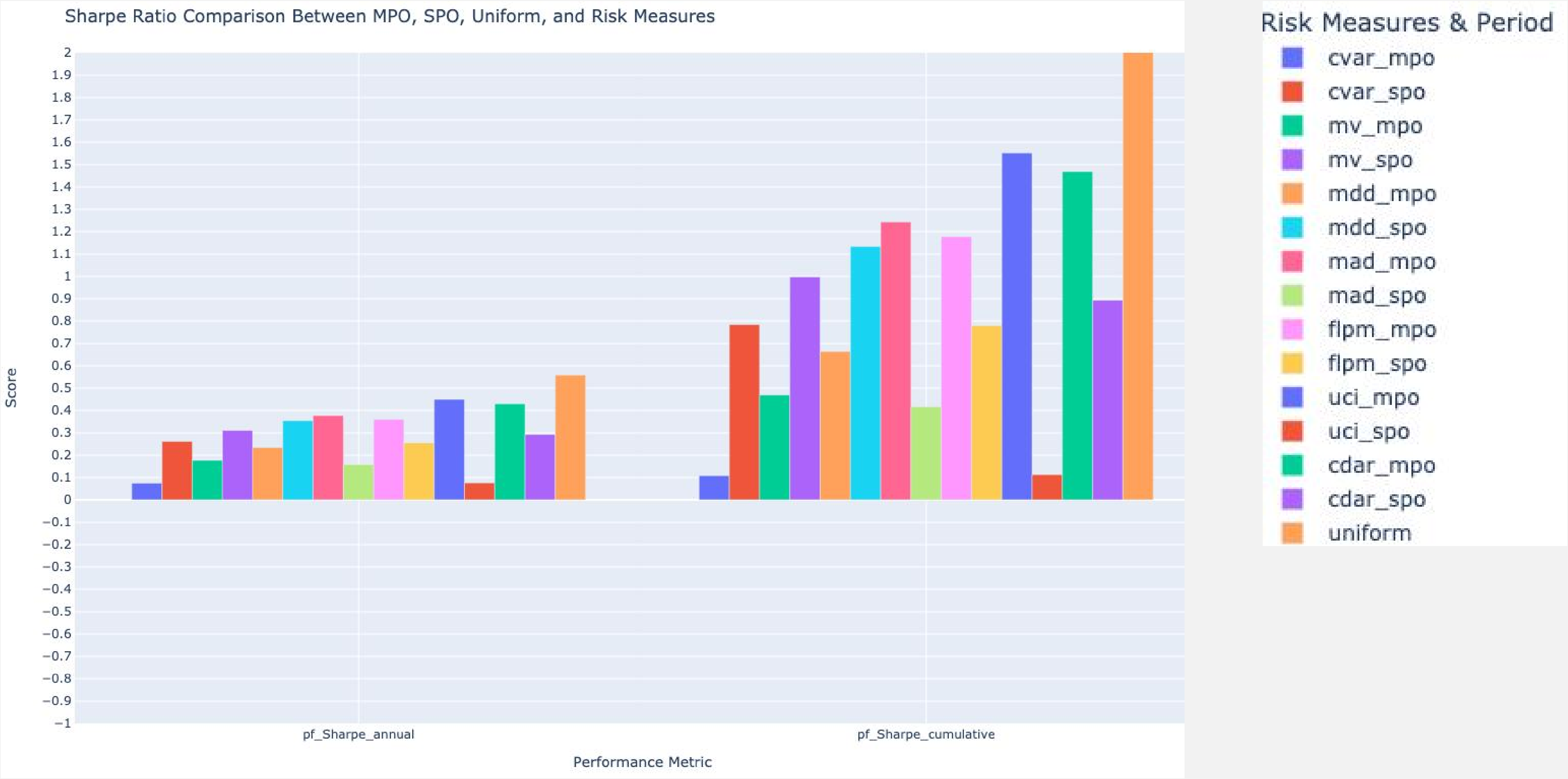
Portfolio	pf_annualized_return	pf_cumulative_return	pf_annualized_volatility	pf_cumulative_volatility	pf_Sharpe_annual	pf_Sharpe_cumulative
cvar_mpo	0.027	0.198	0.087	0.247	0.075	0.108
cvar_spo	0.043	0.366	0.088	0.248	0.261	0.784
mv_mpo	0.035	0.286	0.086	0.243	0.178	0.470
mv_spo	0.042	0.374	0.072	0.203	0.312	0.998
mdd_mpo	0.042	0.350	0.095	0.268	0.234	0.663
mdd_spo	0.062	0.549	0.118	0.333	0.354	1.134
mad_mpo	0.050	0.450	0.079	0.224	0.377	1.244
mad_spo	0.032	0.263	0.077	0.219	0.158	0.416
flpm_mpo	0.049	0.436	0.079	0.225	0.360	1.177
flpm_spo	0.039	0.333	0.073	0.208	0.255	0.779
uci_mpo	0.056	0.520	0.079	0.225	0.450	1.552
uci_spo	0.027	0.200	0.089	0.252	0.076	0.113
cdar_mpo	0.055	0.505	0.080	0.227	0.431	1.468
cdar_spo	0.049	0.418	0.098	0.276	0.292	0.894
uniform	0.114	1.214	0.168	0.476	0.558	2.192

ALL PERFORMANCE COMPARISONS BY RISK MEASURES

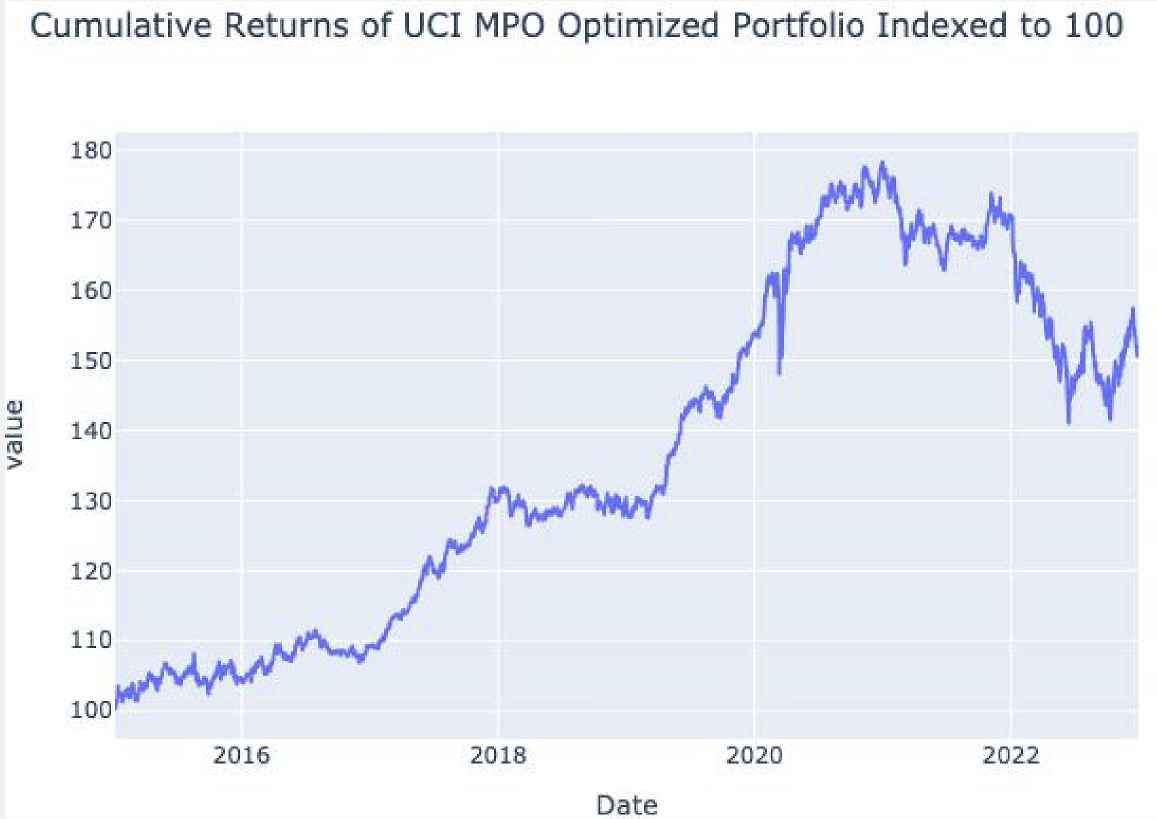


SHARPE RATIO COMPARISONS: MPO VS. SPO

SHARPE RATIO COMPARISONS BY RISK MEASURES



RETURNS OF TOP-PERFORMING MPO PORTFOLIOS



CONCLUSIONS

- Perhaps the clearest and most significant result, unfortunately for our purposes, was that single-period, equal-weighted portfolio significantly outperformed all other portfolio and optimization constructs, which was effectively the exact opposite of what we predicted!
- However, comparing the MPO vs. SPO portfolios, the multi-period optimizations performed significantly better on a handful of risk-measure optimizations, including the last 4 risk measures, MAD, FLPM, UCI, and CDaR, with UCI MPO and SPO Sharpe Ratios of 1.55 and 0.11, respectively.
- One additional point to consider is that of the securities we studied, including broad-based ETF's, with some particularly "low-beta", low-risk ETFs like the bond-focused BND and IGOV. Disproportionately high weights to those, along with other broad-market ETFs like the SPY, may have dramatically skewed the performance of the optimized portfolios. We tested a "companies-only" optimization run, and the results were indeed closer together, though much further work remains to fully understand these dynamics.

GITHUB REPO – CODE, DECK, AND CHARTS

- For your viewing pleasure, our code, slides, & many charts available at our GitHub repo:

https://github.com/yafo1948/Numerical-Optimization_Reichman_Spring2023/tree/main/3327_Final_Project_YF.MF_Jun2023

THANKS FOR TUNING IN!