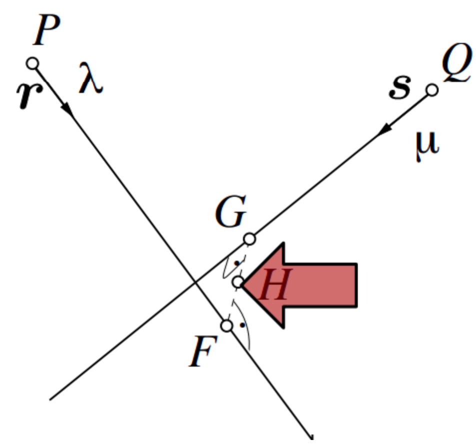
Bipasha Parui (BT19ECE019)
Yagnesh Devada (BT19ECE122)

Triangulation



Let P and Q be the points present in Image-1 and Image-2 respectively.

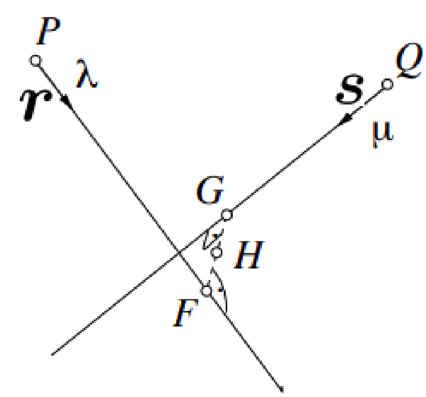
Ray r is formed by joining the point P in the image plane of Image-1 and its focal point, given the focal length of the camera.
Equation of Line: $f = p + \lambda r$

Similarly, the ray s is formed by joining the point Q in the image plane of Image-2 and its focal point, given the focal length of the camera.
Equation of Line: $g = q + \mu s$

Ideally, the corresponding 3D point is found if $f = g$.
But due to errors and quantization of pixels, we have to find as the rays r and s may not intersect at a point.

$$H = (G + F)/2$$

Where FG is the shortest length between the two lines which makes FG orthogonal to both the lines.



This leads to the following constraint:
$(f - g) \cdot r = 0$ and $(f - g) \cdot s = 0$

which leads to
$(q + \mu s - p - \lambda r) \cdot s = 0$ and

$(q + \mu s - p - \lambda r) \cdot r = 0$

Here we have 2 equations in μ and λ Thus we can solve the equations to find their values.

Now simply find the values of F and G would lead to H.

Performing triangulation for every corresponding feature point would their corresponding 3D coordinate.

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Calibration of camera using Zhang's Method

The projection matrix to transform a 3D point to 2D plane consists of intrinsic and extrinsic parameters forming the relation given below.

In the case of a checkerboard, it is known that all the grid corners are coplanar for each individual frame's reference coordinate system. And their location i.e X and Y coordinates known.

$$z \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{00} & r_{01} & r_{02} & t_0 \\ r_{10} & r_{11} & r_{11} & t_1 \\ r_{20} & r_{21} & r_{22} & t_2 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Using this point it can be assumed that the z coordinate of all corners in

a frame is zero Z=0.

This renders the 3rd row of the pose matrix to be of no significance and leads to the transformed equation:

$$z \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{00} & r_{01} & t_0 \\ r_{10} & r_{11} & t_1 \\ r_{20} & r_{21} & t_2 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Here the matrices together form the H matrix

H = K [r1 r2 T] while the rest of the parameters i.e x, y, X, Y are known.

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Hence making use of a minimum of 1 point (X, Y) from each image frame of the checkerboard, observing the same point, we can find the H matrix using 3 image frames.

Hence we find the homography matrix H using 3 points(from 3 frames) such that for each point we have (i=1,2,3) and solve the following

a^T_{xi} h = 0 ; a^T_{yi} h = 0 where h = vec(H^T)

a= other parameters consisting from x, y, X, Y

As H = k [r1 r2 t] we exploit the conditions that

a. K is invertible
b. $r1^T r2 = 0$
c. ||r1|| = ||r2|| = 1

To solve for K using linear algebra and further using single value decomposition.

To be noted:

Here K =

$$\begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

Where f_x, f_y are focal lengths i.e the distance between the pinhole and the image plane and c_x , c_y are the camera centre.