

1)

$$a) \quad i = 1, 2, \dots, m-1 \quad j = 1, 2, \dots, n-1$$

$$A[i, j] + A[i+1, j+1] \leq A[i, j+1] + A[i+1, j]$$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $k \quad 1 \quad 1 \quad k$

Let; $j+1 = 1$ and $i+1 = k$

for $i \Rightarrow 1, 2, \dots, n$ and $j \geq i$

for $k \Rightarrow 1, 2, \dots, m$ and $k \geq i$

So; $1 \leq j \leq i \leq n$, $1 \leq i \leq k \leq m$

AND $A[i, j] + A[k, i] \leq A[i, i] + A[k, j]$ is a special array.

b) pseudocode

for $i \leftarrow 0$ to length of array - 1 :

for $j \leftarrow 0$ to len of array[i]-1 :

if condition is not special array:
convert to special array.

→ Two for loops are searching the array and convert to special array. In example, I give an array which can be converted to special array with one move.

Part C)

I write a function that calculates the leftmost minimum in the given row. This function is looking to the left and right part of the row and then returns the minimum element.

I give the first row to this function and then call the remaining rows recursively (Decrease & Conquer). For this, I use a helper function.

Part D)

For findMin function: $T(n) = T(n/2) + 1 \rightarrow$ From Master The. $\rightarrow T(n) = O(\lg n)$

For helper Function : $T(n) = T(n-1) + \lg n \rightarrow \lg n$ is from findMin function.

$$T(n-1) = T(n-2) + \lg(n-1) + \lg(n) \rightarrow T(n) = T(n-k) + \lg(n * (n-1) * \dots * (n-k))$$

$$\text{Let, } k = n-1 \text{ and } T(1) = 1 \quad T(n) = T(1) + \lg(n!)$$

Using Stirling's approximation, $O(\lg n!) = O(n \lg n)$

2) the middle points of two arrays are m_1, m_2
According to $m_1 + m_2$, I call the functions recursively.

In every iteration, I call the function in length $/2$.

$$T(n) = T(n/2) + 1 \rightarrow \text{From M.T}$$

(0 ... mid or
mid ... last etc.)

$$\begin{matrix} a=1 \\ b=2 \\ d=0 \end{matrix} \Rightarrow \underline{O(\lg n)}$$

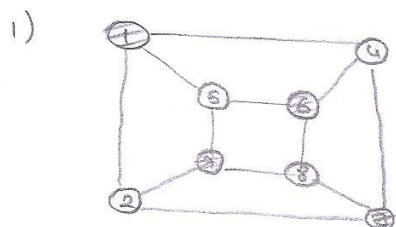
3) The list is divided in two parts (left and right).

$$T(n) = T(n/2) + 1 \rightarrow \text{From M.T} \Rightarrow O(\lg n)$$

Also, There is combine operation. For n element, this operation takes $O(n)$ time.

So, the complexity is $O(n \lg n)$.

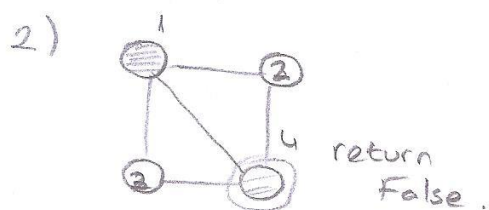
4) Ex:



$$V = [1, 2, 3, 4, 5, 6, 7, 8]$$

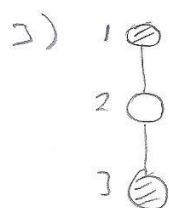
$$E = [[2, 4, 5] \dots [4, 5, 8] \dots]$$

I give the input like this.
Return True.



$$V = [1, 2, 3, 4]$$

$$E = [[2, 3, 4], [1, 4], [1, 4], [1, 2, 3]]$$

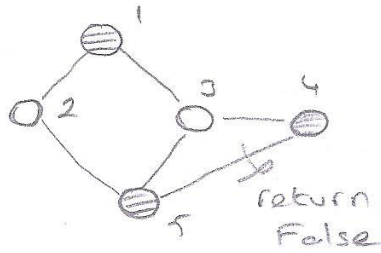


$$V = [1, 2, 3]$$

$$E = [[2], [1, 3], [2]]$$

Return True

Ex: 4)



$$V = [1, 2, 3, 4, 5]$$

$$E = [[2, 3], \dots, [2, 3, 4]]$$

→ For this part, I create the color array with length for vertex array. In initial, I give (-1) for "not visited."

I started from first index, I make it zero (black) and also its neighbour are (white).

And then, I call the function recursively without first index. This is the decrease & conquer part.

In next iterations, if there is similar colors in order (I mean, vertex is 1 and its neighbour is also one.)
zero.

return false.

At the end, if the list is empty, return true.

Running Time $\Rightarrow T(n) = T(n-1) + 1$

Worst Case: $T(n-1) = T(n-2) + 1 + 1$

$$T(n) = T(n-k) + k \quad \begin{matrix} \nearrow k = n-1 \\ T(1) = 1 \end{matrix}$$

$$T(n) = T(1) + n - 1 \Rightarrow T(n) \in O(n)$$

PART 5) For this part, I give the arrays. For (-) place, I give the -1 in examples. I trim this -1. and use helper function.

I return the maximum profit and their index.

For this, I divide the array from middle point, and search the maximum in right and left part.

$$T(n) = T(n/2) + 1 \rightarrow \text{From m.t} \Rightarrow O(\lg n)$$

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