CS301 Homework 0

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1 Stable Marriage Problem

i. Define SMP as a computational problem: What is the input? What is the output?

Input: Two equal sized list of elements $< a_1, a_2.., a_n >$, $< b_1, b_2.., b_n >$ given an order of preferences for each element.

Output: Pair matching (Bijection) of each element in one list to an unique element in the other list $< (a_i, b_i), (a_k, b_k).., (a_m, b_m) >$ such that no pair is *Unstable**.

**Unstable: A matching (a_i, b_i) is Unstable if both:*

1. a_i prefers a different match b_k over b_i
2. b_k also prefers a_i to its existing match

hold true.

ii. Give an example for SMP

Two distinct set of men and women $< a_1, a_2.., a_n >$, $< b_1, b_2.., b_n >$ who want to get married.

Each candidate writes down a preference list, ranking each of the opposite gender candidate from most preferred to the least.

a_1	a_2	a_3	$..a_n$
b_i	b_l	b_t	b_n
b_j	b_k	b_h	b_i
b_k	b_i	b_i	b_l
..	..	..	..
b_l	b_j	b_l	b_k

Table 1: Preference List of Bachelors

b_1	a_2	a_3	$..b_n$
a_i	a_l	a_t	a_n
a_j	a_k	a_h	a_i
a_k	a_i	a_i	a_l
..	..	..	..
a_l	a_j	a_l	a_k

Table 2: Preference List of Bachelorettes

2 Gale - Shapley Algorithm

i. Present the Gale-Shapley algorithm with a pseudocode.

Algorithm 1: Gale - Shapley Algorithm

input : Two distinct sets of Males (M) and Women (W), with each having a preference list of the opposite gender candidates

output: Pairs of (m, w) where m $m \in$M and w$\in$W and there exists no two pairs of (m_i, w_i) and (m_j, w_j) such that w_j prefers m_i to her current match and w_i prefers m_j to her current match simultaneously.

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1  Initialize m ∈ M and w ∈ W to free ;
2  while ∃ free man m who has a woman w to propose to do
3  |  w := first woman on m's list to whom m has not yet proposed;
4  |  if ∃ some pair (m', w) then
5  |  |  if w prefers m to m' then
6  |  |  |  m' becomes free;
7  |  |  |  (m, w) become engaged;
8  |  |  end
9  |  else
10 |  |  (m, w) become engaged;
11 |  end
12 end
```

ii. Analyze the asymptotic time complexity of this algorithm. [Hint: Use the big-oh notation.]

Assume that the number of women and men is the same. $|M| = |W| = N$ and each $m \in M$ and $w \in W$ are already initialized to free.

In the algorithm, each man m iterates through his preference list of woman and proposes to her, where list size is N. (lines 2-12) In each iteration, woman w also iterates through her preference list, of size N, to see if the new proposal is preferred more (line 4). If so, she leaves her previous pair and engages to the current proposer. (lines 4-11)

Since outer While loop is $O(N)$ and inner operation takes $O(N)$ as explained above, this algorithm has $O(N^2)$ time complexity in *Worst Case Scenario.*