Assignment 5

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Problem 1:

To convert the original optimization problem into a decision version, we can introduce new positive integer B denoting a lower bound for profit gained by completing a subset of projects by spending total effort of at most c. Thus, the decision version of the original problem has the following definition, input, and output:

Decision Version of the Original Problem: Is there a subset T' of T with $k \leq |T|$ distinct elements such that $\sum_{1 \leq i \leq k}^{n}(e_i) \leq c$ where e_i denotes the effort spent on project $t_i \in T'$ and $\sum_{1 \leq i \leq k}^{n}(p_i) > B$ where p_i denotes the profit gained from project $t_i \in T$?

Input: Given a set of projects T where each project $t \in T$ is associated with a positive integer e_t denoting the amount of effort required to complete the project t, and a positive integer p_t denoting the profit gained by completing the project t. Given a positive integer c denoting the maximum amount of effort that can be spend of these projects, and a positive integer B denoting a lower bound for profit gained by completing a subset of projects by spending total effort of at most c.

Output: Either *'Yes'* or *'No'* since this is a decision problem.

Problem 2:

To prove that the decision version of the problems belongs to NP, we can show either of the following:

1. Showing that the problem is solvable by a non-deterministic Turing Machine in Polynomial Time.
2. Showing that the problem is verifiable by a deterministic Turing Machine in Polynomial Time.

In our case, we will use the latter.

- Assume that the subset T' of set T with $k \leq |T|$ distinct projects is a candidate, also called a witness, for this problem.
- To verify this candidate is a solution for the problem, do the following:
  - Calculate total profit $p_{sum} = \sum_{i=1}^{k}(p_i)$, where p_i is a profit gained from $t_i \in T$.
  - Calculate total effort $e_{sum} = \sum_{i=1}^{k}(e_i)$, where e_i is a profit gained from $t_i \in T$.
  - Check that total profit $p_{sum} > Q$, and total effort $e_{sum} < c$.
- Make sure that finding a witness is polynomial time: this is correct since selecting a subset of projects from a set of projects T is polynomial time in the size of input $|T|$.
- Make sure that given a witness, checking the correctness (evaluating to true) is polynomial time: this is correct as well since for calculating the sum of profits p_{sum} and sum of efforts e_{sum} requires traversing the projects only once, and checking $p_{sum} > B$ and $e_{sum} < c$ are one-time operations, thus checking the correctness is polynomial time in the size of input $|T|$.

As a result, we can confirm that indeed the decision version of the original problem is in NP.