Assignment 5

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Problem 1

This version of this problem is not a decision version because the output of this problem is not 'yes' or 'no'. The current inputs of this problem are **a set** T where each project $t \in T$, **a positive integer** e_t denotes the amount of effort needed to complete the project t, **a positive integer** p_t denotes the profit from completing the project t, **a positive integer** c denotes the maximum amount of effort that can be spent.
The current output of this problem is an integer that denotes the largest amount of profit by completing a subset of projects by at most spending an effort equal to c.
In order to change this problem into a decision problem we need to introduce another **positive integer Q** denotes a lower bound for the profit gained by completing projects by spending at most effort c. The inputs of our problem are the ones listed above. So, our problem converts to this

> Is there a subset T' of T that has $k \leq |T|$ distinct elements such that $\sum_{1 \leq i \leq k} e_i \leq c$ where e_i denotes the effort spent on project $t_i \in T'$ and $\sum_{1 \leq i \leq k} p_i > Q$ where p_i denotes the profit gained from project $t_i \in T'$?

And the output of this problem is either 'yes' or 'no' as a decision problem's output should be.

Problem 2

To prove that this problem is in NP, we need to show either problem is verifiable by an deterministic Turing machine in polynomial time or it should be solvable by non-deterministic Turing machine in polynomial time. In this case, to prove the NP membership of this problem, showing problem can be verifiable by deterministic Turing machine will be used.

- Assume that subset T' of set T which includes $k \leq |T|$ distinct projects is a candidate solution (or a witness) for this problem.
- To verify this witness is a solution for the problem, we need to
  - Find total effort p_{sum} needed to finish all projects such that $t \in T'$
  - Find total profit e_{sum} gained from all projects such that $t \in T'$
  - Check that total profit $p_{sum} > Q$ and total effort $e_{sum} \leq c$.

  Finding total effort e_{sum} and total profit p_{sum} given witness can be done in polynomial time since only traversing the candidate set once is needed. Also, checking total profit $p_{sum} > Q$ and total effort $e_{sum} \leq c$ can be done in polynomial time since they are single if checks.

As a result, this decision problem is a member of NP class.